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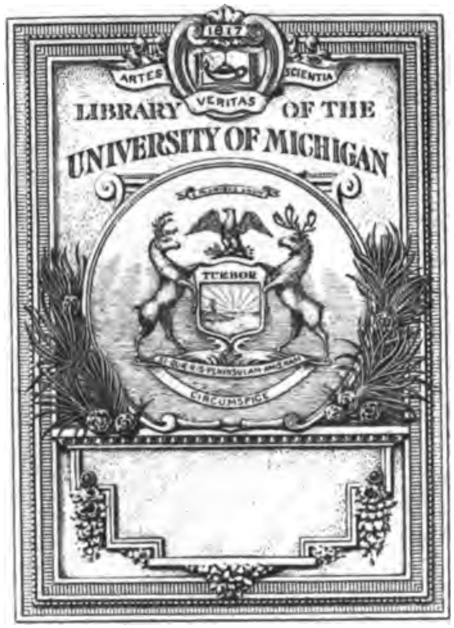

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PHILOSOPHICAL AND MATHEMATICAL

DICTIONARY:

CONTAINING

AN EXPLANATION OF THE TERMS, AND AN ACCOUNT OF THE SEVERAL SUBJECTS,

COMPRISED UNDER THE HEADS

MATHEMATICS, ASTRONOMY, AND PHILOSOPHY

BOTH NATURAL AND EXPERIMENTAL;

WITH AN

HISTORICAL ACCOUNT OF THE RISE, PROGRESS, AND PRESENT STATE OF THESE SCIENCES;

ALSO

MEMOIRS OF THE LIVES AND WRITINGS OF THE MOST EMINENT AUTHORS,

BOTH ANCIENT AND MODERN,

WHO BY THEIR DISCOVERIES OR IMPROVEMENTS HAVE CONTRIBUTED TO THE ADVANCEMENT OF THEM.

BY CHARLES HUTTON, LL.D.

FELLOW OF THE ROYAL SOCIETIES OF LONDON AND EDINBURGH, AND OF THE PHILOSOPHICAL SOCIETIES OF HAARLEM
AND AMERICA; AND EMERITUS PROFESSOR OF MATHEMATICS IN THE ROYAL
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M.

M, In Astronomical Tables, &c, is used for meridional or southern; and sometimes for meridian, or mid-day.—In the Roman numeration, it denotes a thousand.

MACHIN (JOHN), a very distinguished mathematician, was some time professor of astronomy at Gresham-college, (to which he was elected May 16, 1713, on the resignation of Dr. Torriano,) and secretary of the Royal Society, died June 9, 1751. His papers in the Philos. Trans. were, 1. To find the curve which a descending body describes in the shortest time, vol. 30; 2. On a Distempered Skin, vol. 37; 3. A Solution of Kepler's Problem. Besides these, by an approximating series of Dr. Halley's, Mr. Machin computed the circumference of the circle to 100 places of figures. And another ingenious approximating series of his own is given in Mr. Jones's Synopsis Palmariorum Matheseos, 1706, the investigation of which was first given in Dr. Hutton's Mensuration, 1772. Mr. Machin's Laws of the Moon's Motion were printed in Motte's translation of Newton's Principia.

MACHINE, denotes any thing that serves to augment, or to regulate moving powers: or it is any body destined to produce motion, so as to save either time or force. The word, in Greek, signifies an invention, or art: and hence, in strictness, a machine is something that consists more in art and invention, than in the strength and solidity of the materials; for which reason it is that the inventors of machines are called Ingenieurs, or Engineers.

Machines are either simple or compound. The simple machines are the 6 mechanical powers, viz, the lever, pulley, wheel-and-axle, inclined plane, wedge, and screw; which are otherwise called the simple mechanic powers. The balance also is a lever.

These simple machines serve for different purposes, according to their different structures; and it is the business of the skilful mechanist to choose and combine them, in the manner that may be best adapted to produce the desired effect. The lever is a very useful machine for many purposes, its power being readily varied as the occasion may require; when weights are to be raised only a little way, such as stones out of quarries, &c. On the other hand, the wheel-and-axle serves to raise weights from the greatest depth, or to the greatest height. Pulleys, being easily carried, are therefore much employed in ships. The balance is useful for ascertaining an equality of weight. The wedge is very useful for separating the parts of bodies; and being impelled by the force of percussion, it is incomparably greater than any of the other powers. The screw is useful for compressing or squeezing bodies together, and also for raising very heavy weights to a small height; its great friction is even of considerable use, to preserve the effect already produced by the machine.

Compound MACHINE, is formed from these simple machines, combined together for different purposes. The number of compound machines is almost infinite; and yet it would seem that the ancients went far beyond the moderns in the powers and effects of them; especially their machines of war and architecture.

Accurate descriptions and drawings of machines would be a very curious and useful work. But to make a collection of this kind as beneficial as possible, it should contain also an analysis of them; pointing out their advantages and disadvantages, with the reasons of the constructions; also the general problems implied in these constructions, with their solutions &c, should be noticed. Though a complete work of this kind be still wanting, yet many curious and useful particulars may be gathered from Strada, Besson, Beroaldus, Augustinus de Ramellis, Bockler, Leupold, Beyer, Limpergh, Van Zyl, Perault, and others; a short account of whose works may be found in Wolfii Commentatio de Præcipuis Scriptis Mathematicis; Elem. Mathes. Univ. tom. 5, pa. 84. To these may be added, Belidor's Architecture Hydraulique, Desaguliers's Course of Experimental Philosophy, Emerson's Mechanics, and Dr. Gregory's Mechanics, which contains a description of a great number of the most useful and modern machines. The Royal Academy of Sciences at Paris have also given a collection of machines and inventions approved of by them. This work, published by M. Gallon, consists of 6 vols. in 4to, containing engraved draughts of the machines, with their descriptions annexed.

MACHINE, *Architectonical*, is an assemblage of pieces of wood so disposed as that, by means of ropes and pulleys, a small number of men may raise great loads, and lay them in their places; such as cranes, &c.—It is hard to conceive what sort of machines the ancients must have used to raise those immense stones found in some of the antique buildings; as some of those still found in the walls of Balbeck in Turkey, the ancient Heliopolis, which are 63 feet long, 12 feet broad, and 12 feet thick, and which must weigh 6 or 7 hundred tons a-piece.

Blowing MACHINE. See BELLOWS.

Boyleian MACHINE. Mr. Boyle's *Air-Pump.*

Electrical MACHINE. See ELECTRICAL *Machine.*

Wind MACHINE. See ANEMOMETER, and WIND *Machine.*

Hydraulic or *Water* MACHINE, is used either to denote a simple machine, serving [illegible] conduct or raise water, as a sluice, pump, and the like, or several of these acting together, to produce some extraordinary effect; as the

MACHINE *of Marli.* See FIRE-*engine*, STEAM-*engine*, and WATER-*works.*

Military MACHINES, among the ancients, were of three kinds: the first serving to throw arrows, as the scorpion; or javelins, as the catapult; or stones, as the balista; or fiery darts, as the pyrabolus: the 2d kind serving to beat down walls, as the battering-ram and terebra: and the 3d sort to shelter those who approach the enemy's wall, as the tortoise or testudo, the vinea, and the towers of wood. See the respective articles.

The machines of war now in use, consist in artillery, including cannon, mortars, petards, &c.

MACLAURIN (COLIN), a very eminent mathematician and philosopher, was the son of a clergyman, and born at Kilmoddan in Scotland, in the year 1698. He was sent to the university of Glasgow in 1709; where he continued 5 years, applying to his studies in a very intense manner, and particularly to the mathematics. His great genius for mathematical learning discovered itself so early as at 12 years of age; when, having accidentally met with a copy of Euclid's Elements in a friend's chamber, he became in a few days master of the first 6 books without any assistance: and it is certain, that in his 16th year he had invented many of the propositions which were afterwards published as part of his work, entitled Geometria Organica. In his 15th year he took the degree of master of arts; on which occasion he composed and publicly defended a thesis on the power of gravity, with great applause. After this he quitted the university, and retired to a country-seat of his uncle, who had the care of his education, his parents having died while he was very young. Here he spent two or three years in pursuing his favourite studies; but in 1717, at 19 years of age only, he offered himself a candidate for the professorship of mathematics in the Marischal College of Aberdeen, and obtained it after a ten days trial, against a very able competitor.

In 1719, Mr. Maclaurin visited London, where he left his Geometria Organica to print, and where he became acquainted with Dr. Hoadley then bishop of Bangor, Dr. Clarke, Sir Isaac Newton, and other eminent men; at which time also he was admitted a member of the Royal Society: and in another journey, in 1721, he contracted an intimacy with Martin Folkes, esq. the president of it, which continued during his life.

In 1722, Lord Polwarth, plenipotentiary of the king of Great Britain at the congress of Cambray, engaged Maclaurin to go as a tutor and companion to his eldest son, who was then to set out on his travels. After a short stay at Paris, and visiting other towns in France, they fixed in Lorrain; where he wrote his piece, On the Percussion of Bodies, which gained him the prize of the Royal Academy of Sciences for the year 1724. But his pupil dying soon after at Montpelier, he returned immediately to his profession at Aberdeen. He was hardly settled here, when he received an invitation to Edinburgh; the curators of that university being desirous that he should supply the place of Mr. James Gregory, whose great age and infirmities had rendered him incapable of teaching. He had here some difficulties to encounter, arising from competitors, who had good interest with the patrons of the university, and also from the want of an additional fund for the new professor; which however at length were all surmounted, principally through the means of Sir Isaac Newton. Accordingly, in Nov. 1725, he was introduced into the university; after which the mathematical classes soon became very numerous, there being generally upwards of 100 students attending his lectures every year; who being of different standings and proficiency, he was obliged to divide them into four or five classes, in each of which he employed a full hour every day from the first of November to the first of June. In the first class he taught the first 6 books of Euclid's Elements, Plane Trigonometry, Practical Geometry, the Elements of Fortification, and an Introduction to Algebra. The second class studied Algebra, with the 11th and 12th books of Euclid, Spherical Trigonometry, Conic Sections, and the general Principles of Astronomy. The third studied Astronomy and Perspective, and read a part of Newton's Principia, he having performed a course of experiments for illustrating them: he afterwards read and demonstrated the Elements of Fluxions. Those in the fourth class read a System of Fluxions, the Doctrine of Chances, and the remainder of Newton's Principia.

In 1734, Dr. Berkeley, bishop of Cloyne, published a piece called the Analyst; in which he took occasion, from some disputes that had arisen concerning the grounds of the fluxionary method, to explode the method itself; and also to charge mathematicians in general with infidelity in religion. Maclaurin thought himself included in this charge, and began an answer to Berkeley's book: but other answers coming out, and as he proceeded, so many discoveries, so many new theories and problems occurred to him, that instead of a vindicatory pamphlet, he produced a Complete System of Fluxions, with their application to the most considerable problems in Geometry and Natural Philosophy. This work was published at Edinburgh in 1742, 2 vols. 4to; and as it cost him infinite pains, so it is the most considerable of all his works, and will do him immortal honour, being indeed the most complete treatise on that science that has yet appeared.

In the mean time, he was continually presenting the public with some observation or performance of his own, several of which were published in the 5th and 6th volumes of the Medical Essays at Edinburgh. Many of them were likewise published in the Philosophical Transactions; as the following: 1. On the Construction and Measure of Curves, vol. 30.—2. A New Method of describing all kinds of Curves, vol. 30.—3. On Equations with Impossible Roots, vol. 34.—4. On the Roots of Equations, &c, vol. 34.—5. On the Description of Curve Lines, vol. 39.—6. Continuation of the same, vol. 39.—7. Observations on a Solar Eclipse, vol. 40.—8. A Rule for finding the Meridional Parts of a Spheroid with the same Exactness as in a Sphere, vol. 41.—9. An Account of the Treatise of Fluxions, vol. 42.—10. On the Bases of the Cells where the Bees deposit their Honey, vol. 42.

In the midst of these studies, he was always ready to lend his assistance in contriving and promoting any scheme which might contribute to the public service. When the earl of Morton went, in 1739, to visit his estates in Orkney and Shetland, he requested Mr. Maclaurin to assist him in settling the geography of those countries, which was very erroneous in all the maps; to examine their natural history, to survey the coasts, and to take the measure of a degree of the meridian. Maclaurin's family affairs would not permit him to comply with this request: he however drew up a memorial of what he thought necessary to be observed, and furnished proper instruments for the work, recommending Mr. Short, the celebrated optician, as a proper operator for the management of them.

Mr. Maclaurin had still another scheme for the improvement of geography and navigation, of a more extensive nature; which was the opening a passage from Greenland to the South Sea by the north pole. That such a passage might be found, he was so fully persuaded, that he used to say, if his situation could admit of such adventures, he would undertake the voyage, even at his own charge. But when schemes for finding it were laid before the parliament in 1741, and he was consulted by several persons of high rank concerning them, and before he could finish the memorials he proposed to send, the premium was limited to the discovery of a north-west passage: he regretted much that the word west was inserted, because he thought that passage, if at all to be found, must lie not far from the pole.

In 1745, having been very active in fortifying the city of Edinburgh against the rebel army, he was obliged to fly from thence into England, where he was invited by Dr. Herring, archbishop of York, to reside with him during his stay in this country. In this expedition however, being exposed to cold and hardships, and naturally of a weak and tender constitution, which had been much more enfeebled by close application to study, he laid the foundation of an illness which put an end to his life, in June 1746, at 48 years of age, leaving his widow with two sons and three daughters.

Mr. Maclaurin was a very good, as well as a very great man, and worthy of love as well as admiration. His peculiar merit as a philosopher was, that all his studies were accommodated to general utility; and we find, in many places of his works, an application even of the most abstruse theories, to the perfecting of mechanical arts. For the same purpose, he had resolved to compose a course of Practical Mathematics, and to rescue several useful branches of the science from the ill treatment they often met with in less skilful hands. These intentions however were prevented by his death; unless we may reckon, as a part of his intended work, the translation of Dr. David Gregory's Practical Geometry, which he revised, and published with additions, in 1745.

In his lifetime, however, he had frequent opportunities of serving his friends and his country by his great talents. Whatever difficulty occurred concerning the constructing or perfecting of machines, the working of mines, the improving of manufactures, the conveying of water, or the execution of any public work, he was always ready to resolve it. He was employed to terminate some disputes of consequence that had arisen at Glasgow, concerning the gauging of vessels; and for that purpose presented to the commissioners of the excise two elaborate memorials, with their demonstrations, containing rules by which the officers now act. He made also calculations relating to the provision, now established by law, for the children and widows of the Scotch clergy, and of the professors in the universities, entitling them to certain annuities and sums, on the voluntary annual payment of a certain sum by the incumbent. In contriving and adjusting this wise and useful scheme, he bestowed a great deal of labour, and contributed not a little towards bringing it to perfection.

Of his works, we have mentioned his Geometria Organica, in which he treats of the description of curve lines by continued motion; as also of his piece which gained the prize of the Royal Academy of Sciences in 1724. In 1740, he likewise shared the prize of the same academy, with the celebrated D. Bernoulli and Euler, for resolving the problem relating to the motion of the tides from the theory of gravity: a question which had been given out the former year, without receiving any solution. He had only 10 days to draw this paper up, and could not find leisure to transcribe a fair copy; so that the Paris edition of it is incorrect. He afterwards revised the whole, and inserted it in his Treatise of Fluxions; as he did also the substance of the former piece. These, with the Treatise of Fluxions, and the pieces printed in the Medical Essays and the Philosophical Transactions, a list of which is given above, are all the writings which our author lived to publish. Since his death, however, two more volumes have appeared; his Algebra, and his Account of Sir Isaac Newton's Philosophical Discoveries. The Algebra, though not finished by himself, is yet allowed to be excellent in its kind; containing, in a moderate volume, a complete elementary treatise on that science, as far as it had then been carried; besides some neat analytical papers on curve lines. His Account of Newton's Philosophy was occasioned in the following manner:—Sir Isaac dying in the beginning of 1728, his nephew, Mr. Conduitt, proposed to publish an account of his life, and desired Mr. Maclaurin's assistance. The latter, out of gratitude to his great benefactor, cheerfully undertook, and soon finished the History of the Progress which Philosophy had made before Newton's time; and this was the first draught of the work in hand; which not going forward, on account of Mr. Conduitt's death, was returned to Mr. Maclaurin. To this he afterwards made great additions, and left it in the state in which it now appears. His main design seems to have been, to explain only those parts of Newton's Philosophy, which have been controverted; and this is supposed to be the reason why his grand discoveries concerning light and colours are but transiently and generally touched upon; for it is known, that whenever the experiments, on which his doctrine of light and colours is founded, had been repeated with due care, this doctrine had not been contested; while his accounting for the celestial motions, and the other great appearances of nature, from gravity, had been misunderstood, and even attempted to be ridiculed.

MACULÆ, in Astronomy, are dark spots appearing on the luminous surfaces of the sun and moon, and even some of the planets. The solar maculæ are dark spots of an irregular and changeable figure, observed in the face of the sun. These are said to have been first observed in November and December of the year 1610, by Galileo in Italy, and Harriot in England, unknown to, and independent of each other, soon after the invention of telescopes. But Montucla, in his History of the Mathematics, says that the honour of this discovery is due to J. Fabricius, as appears from his work published at Wittenberg, in June 1611, entitled, De Maculis in Sole visis, et earum cum sole revolutione narratio. They were afterwards also observed by Scheiner, Hevelius, Flamsteed, Cassini, Kirch, and others. See **Maculæ**, **Nebulous**, **Spots**, &c.

MADRIER, in Artillery, is a thick plank, armed with plates of iron, and having a cavity sufficient to receive the mouth of a petard, with which it is applied against a gate, or any thing else intended to be broken down.—This term is also applied to certain flat beams, fixed to the bottom of a moat, to support a wall.—There are also madriers lined with tin, and covered with earth; serving as defences against artificial fires, in lodgments, &c, where there is need of being covered overhead.

MÆSTLIN (**Michael**), in Latin Mæstlinus, a noted astronomer of Germany, was born in the duchy of Wittemberg; but spent his youth in Italy, where he made a speech in favour of Copernicus's system, which brought Galileo over from Aristotle and Ptolemy, to whom he was before wholly devoted. He afterwards returned to Germany, and became professor of mathematics at Tubingen; where, among his other scholars, he taught the celebrated Kepler, who has commended several of his ingenious inventions, in his Astronomia Optica. Mæstlin published many mathematical and astronomical works, and died in 1590.—Though Tycho Brahé did not assent to Mæstlin's opinion, yet he allowed him to be an extraordinary person, and deeply skilled in the science of astronomy.

MAGAZINE, a place in which stores are kept, of arms, ammunition, provisions, &c.

Artillery **Magazine**, or the magazine to a field battery, is made about 25 or 30 yards behind the battery, towards the parallels, and at least 3 feet under ground, to receive the powder, loaded shells, port-fires, &c.—Its roof and sides should be well secured with boards, to prevent the earth from falling in: it has a door, and a double trench or passage sunk from the magazine to the battery, the one to enter, and the other to go out at, to prevent confusion. Sometimes traverses are made in the passages, to prevent ricochet shot from entering the magazine.

Powder-**Magazine**, is the place where powder is kept in large quantities. Authors differ very much with regard to the situation and construction of these magazines; but all agree, that they ought to be arched and bomb-proof. In fortifications, they were formerly placed in the rampart; but of late they have been built in different parts of the town. The first powder-magazines were made with Gothic arches: but M. Vauban thinking these too weak, constructed them of a semicircular form, the dimensions being 60 feet long within, and 25 feet broad; the foundations are 8 or 9 feet thick, and 8 feet high from the foundation to the spring of the arch; also the floor 2 feet from the ground, to preserve it from the damp.

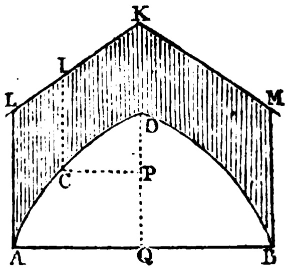
It is a constant observation, that after the centering of semicircular arches is struck, they settle at the crown, and rise up at the haunces, even with a straight horizontal extrados; and still more so in powder-magazines, where the outside at top is formed, like the roof of a house, by inclined planes joining in an angle over the top of the arch, to give a proper descent to the rain; which effects are exactly what might be expected from the true theory of arches. Now, this shrinking of the arches, as it must be attended with very bad consequences, by breaking the texture of the cement, after it has in some degree been dried, and also by opening the joints of the vousoirs at one end, I have provided a remedy for this inconvenience, with regard to bridges, by the arch of equilibration, in my Tracts, vol. 1: but as the ill consequences of it are much greater in powder-magazines, in question 96 of my Mathematical Miscellany, I proposed to find an arch of equilibration for them also; which question was there resolved by Mr. Wildbore and myself, both upon general principles, and which I illustrated by an application to a particular case, there constructed, and accompanied with a table of numbers for that purpose. From that solution it appears that the general value of the ordinate PC or y, is $y =$

$$b \times \frac{\log. \frac{w + \sqrt{(w^2 - a^2)}}{a}}{\log. \frac{c + \sqrt{(c^2 - a^2)}}{a}};$$ where (in the following figure)

$y =$ PC, $a =$ KD, $b =$ AQ $= \frac{1}{2}$ AB, $c =$ AL, and $w =$ IC; from which equation PC may be found, when IC is given. But if, on the other hand, PC be given, to find IC, which will be the more convenient way, then from the former equation will be found,

$w = \frac{a^2 + n^2}{2n} =$ CI; where $n = 2{\cdot}718281828$ the number whose hyp. log. is $cy +$ A, and A $=$ log. of a, also C $= \frac{1}{b} \times$ log. of $\frac{c + \sqrt{(c^2 - a^2)}}{a}$. Thus for example, in the following figure, representing a transverse vertical section of the arch, if the span AB $= 20$, the pitch or height DQ $= 10$, the thickness at top DK $= 7$, and the angle at top LKM $=$ 112° 37′; then for every different value of PC, the last equation will give the following correspondent values of CI. That is, if ALKMB represent a vertical transverse section of the arch, the roof forming an angle LKM of 112° 37′, also PC an ordinate parallel to the horizon taken in any part, and IC perpendicular to the same, and the other dimensions as above; then for properly constructing the curve so as to be the strongest, or an arch of equilibration in all its parts, the corresponding values of PC and CI will be as in the following table, where those numbers may denote any lengths whatever, either inches, or feet, or half-yards.

Value of CP.	Value of CI
1	7·0310
2	7·1243
3	7·2806
4	7·5015
5	7·7838
6	8·1452
7	8·5737
8	9·0781
9	9·6628
10	10·3333



See my Tracts, vol. 1, pa. 57 &c.

Magazine, or *Powder-Room*, on ship-board, is a close room or store-house, built in the fore or after part of the hold, in order to preserve the gunpowder for the use of the ship. This apartment is strongly secured against fire, and no person is allowed to enter it with a lamp or candle; it is therefore lighted, as occasion requires, by means of the candles or lamps in the light-room contiguous to it.

MAGELLANIC Clouds, whitish appearances like clouds, seen in the heavens towards the south pole, and having the same apparent motion as the stars. They are three in number, two of them near each other.—The largest lies far from the south pole; but the other two are not many degrees more remote from it than the nearest conspicuous star, that is, about 11 degrees. Mr. Boyle conjectures that if these clouds were seen through a good telescope, they would appear to be multitudes of small stars, like the milky way. Dr. Herschel thinks rather that nebulæ are often owing to a self-luminous fluid. See Philos. Trans. an. 1791, pa. 71, and an. 1811, pa. 269.

MAGIC LANTERN, an optical machine, by means of which small painted images are represented on the wall of a dark room, magnified to any size at pleasure. This machine was contrived by Kircher (see his Ars Magna Lucis et Umbræ, pa. 768); and it was so called, because the images were made to represent strange phantasms, and terrible apparitions, which have been taken for the effect of magic, by such as were ignorant of the secret.

This machine is composed of a concave speculum, from 4 to 12 inches diameter, reflecting the light of a candle through the small hole of a tube, at the end of which is fixed a double convex lens of about 3 inches focus. Between the two are successively placed, many small plain glasses, painted with various figures, usually such as are the most formidable and terrifying to the spectators, when represented at large on the opposite wall.

Thus (pl. 17, fig. 14) ABCD is a common tin lantern, to which is added a tube FG to draw out. In H is fixed the metallic concave speculum, from 4 to 12 inches diameter; or else, instead of it, near the extremity of the tube, there must be placed a convex lens, consisting of a segment of a small sphere, of but a few inches in diameter. The use of this lens is to throw a strong light upon the image; and sometimes a concave speculum is used with the lens, to render the image still more vivid. In the focus of the concave speculum or lens, is placed the lamp L; and within the tube, where it is soldered to the side of the lantern, is placed a small lens, convex on both sides, being a portion of a small sphere, having its focus about the distance of 3 inches. The extreme part of the tube FM is square, and has an aperture quite through, so as to receive an oblong frame NO passing into it; in which frame there are round holes, of an inch or two in diameter. Answering to the magnitude of these holes there are circles drawn on a plain thin glass; and in these circles are painted any figures, or images, at pleasure, with transparent water-colours. These images fitted into the frame, in an inverted position, at a small distance from the focus of the lens I, will be projected on an opposite white wall of a dark room, in all their colours, greatly magnified, and in an erect position. By having the instrument so contrived, that the lens I may move on a slide, the focus may be made, and consequently the image appear distinct, at almost any distance.

Or thus: Every thing being managed as in the former case, into the sliding tube FG, insert another convex lens K, the segment of a sphere rather larger than I. Now, if the picture be brought nearer to I than the distance of the focus, diverging rays will be propagated as if they proceeded from the object; therefore, if the lens K be so placed, as that the object be very near its focus, the image will be exhibited on the wall, greatly magnified.

MAGIC *Square*, is a square figure, formed of a series of numbers in arithmetical progression, so disposed in parallel and equal ranks, as that the sums of each row, taken either perpendicularly, horizontally, or diagonally, are equal to one another. As the following square, formed of these nine numbers, 1, 2, 3, 4, 5, 6, 7, 8, 9, where the sum of the three figures in every row, in all the directions, is always the same number, viz 15. But if the same numbers be placed in their natural order, in form of a square, the first being 1, and the last of them a square number, they will form what is called a natural square, whose two diagonals, as also its middle column, and middle horizontal line, will have the same sum as all the rows of the magic square, viz. 15.

Natural Square.

1	2	3
4	5	6
7	8	9

Magic Square.

4	9	2
3	5	7
8	1	6

Or in the two following squares of the first 25 numbers,

Natural Square.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

Magic Square.

16	14	8	2	25
3	22	20	11	9
15	6	4	23	17
24	18	12	10	1
7	5	21	19	13

where every row and diagonal in the magic square makes just the sum 65, being the same as the two diagonals of the natural square, as well as of the middle row and middle column.

It is probable that these magic squares were so called both because of this property in them, viz, that the ranks in every direction making the same sum, appeared extremely surprising, especially in the more ignorant ages, when mathematics passed for magic; and because also of the superstitious operations they were employed in, as the construction of talismans, &c; for, according to the childish philosophy of those days, which ascribed virtues to numbers, what might not be expected from numbers so seemingly wonderful!

The magic square was held in great veneration among the Egyptians, and the Pythagoreans their disciples, who, to add more efficacy and virtue to this square, dedicated it to the then known 7 planets divers ways, and engraved it on a plate of the metal that was esteemed in sympathy with the planet. The square thus dedicated, was inclosed by a regular polygon, inscribed in a circle, which was divided into as many equal parts as there were units in the side of the square; with the names of the angles of the planet, and the signs of the zodiac written upon the void spaces between the polygon and the circumference of the circumscribed circle. Such a talisman or metal they vainly imagined would, upon occasion, befriend the person who carried it about him.

To Saturn they attributed the square of 9 places or cells, the side being 3, and the sum of the numbers in every row 15: to Jupiter the square of 16 places, the side being 4, and the amount of each row 34: to Mars the square of 25 places, the side being 5, and the amount of each row 65: to the sun the square with 36 places, the side being 6, and the sum of each row 111: to Venus the square of 49 places, the side being 7, and the amount of each row 175: to Mercury the square with 64 places, the side being 8, and the sum of each row 260: and to the moon the square of 81 places, the side being 9, and the amount of each row 369. Finally, they attributed to imperfect matter, the square with 4 divisions, having 2 for its side; and to God the square of only one cell, the side of which is also an unit, which multiplied by itself, undergoes no change.

However, what was at first the vain practice of conjurers and makers of talismans, has since become the subject of a serious research among mathematicians. Not that they imagine it will lead them to any thing of solid use or advantage; but rather as it is a kind of play, in which the difficulty makes the merit, and it may chance to produce some new views or properties of numbers, which mathematicians might probably turn to some account.

It would seem that Eman. Moschopulus, a Greek author of no high antiquity, was the first now known of, who has spoken of magic squares: he has left some rules for their construction; though, by the age in which he lived, there is reason to imagine he did not look upon them merely as a mathematician.

In the treatise of Cornelius Agrippa, so much accused of magic, are found the squares of 7 numbers, viz, from 3 to 9 inclusive, disposed magically; and it is not to be supposed that those 7 numbers were preferred to all others without some good reason: indeed it is because their squares, according to the system of Agrippa and his followers, are planetary. The square of 3, for instance, belongs to Saturn; that of 4 to Jupiter; that of 5 to Mars; that of 6 to the sun; that of 7 to Venus; that of 8 to Mercury; and that of 9 to the moon.

M. Bachet applied himself to the study of magic squares, on the hint he had taken from the planetary squares of Agrippa, as being unacquainted with Moschopulus's work, which is only in manuscript in the French king's library; and, without the assistance of any author, he found out a new method for the squares of uneven numbers; for instance, 25, or 49, &c; but he could not succeed with those that have even roots.

M. Frenicle next engaged in this subject. It was the opinion of some, that though the first 16 numbers might be disposed 20922789888000 different ways in a natural square, yet they could not be disposed more than 16 ways in a magic square; but M. Frenicle showed, that they might be thus disposed in 878 different ways. To this business he thought fit to add a difficulty that had not yet been considered; which was, to take away the marginal numbers quite around, or any other circumference at pleasure, or even several of such circumferences, and yet that the remainder should still be magical. Again he inverted that condition, and required that any circumference taken at pleasure, or even several circumferences, should be inseparable from the square; that is, that it should cease to be magical when they were removed, and yet continue magical after the removal of any of the rest. M. Frenicle however gives no general demonstration of his methods, and it often seems that he has no other guide but chance. It is true, his book was not published by himself, nor did it appear till after his death, viz, in 1693.

In 1703, M. Poignard, canon of Brussels, published a treatise on sublime magic squares. Before his time there had been no magic squares made, but for series of natural numbers that formed a square; but M. Poignard made two very considerable improvements. 1st, Instead of taking all the numbers that fill a square, for instance the 36 successive numbers, which would fill all the cells of a natural square whose side is 6, he only takes as many successive numbers as there are units in the side of the square, which in this case are 6; and these six numbers alone he disposes in such manner, in the 36 cells, that none of them occur twice in the same rank, whether it be horizontal, vertical, or diagonal; whence it follows, that all the ranks, taken all the ways possible, must always make the same sum; and this method M. Poignard calls repeated progressions. 2d, Instead of being confined to take these numbers according to the series and succession of the natural numbers, that is in arithmetical progression, he takes them likewise in a geometrical progression; and even in an harmonical one, the numbers of all the ranks always following the same kind of progression: he makes squares of each of these three progressions repeated.

M. Poignard's book gave occasion to M. Lahire to turn his thoughts to the same subject, which he did with such success, that he greatly extended the theory of magic squares, as well for even numbers as those that are uneven; as may be seen at large in the Memoirs of the Royal Academy of Sciences, for the years 1705, and in 1710 by M. Sauveur. See also Saunderson's Algebra, vol. 1, pa. 354, &c; as also my Mathematical Recreations, translated from Ozanam and Montucla, giving the following easy method of filling up a magic square.

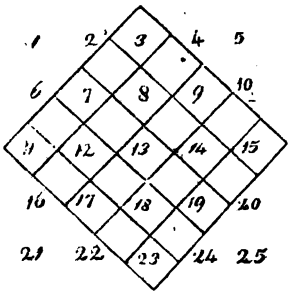
To form a Magic Square of an Odd Number of Terms in the Arithmetic Progression 1, 2, 3, 4, &c. Place the least term 1 in the cell immediately under the middle, or central one, and the rest of the terms, in their natural order, in a descending diagonal direction, till they run off either at the bottom, or on the side: when the number runs off at the bottom, carry it to the uppermost cell that is not occupied, of the same column that it would have fallen in below, and then proceed descending diagonalwise again as far as you can, or till the numbers either run off at bottom or side, or are interrupted by coming at a cell already filled: now when any number runs off at the right-hand side, then bring it to the farthest cell on the left hand of the same row or line it would have fallen in towards the right hand: and when the progress diagonalwise is interrupted by meeting with a cell already occupied by some other number, then descend diagonally to the left from this cell till an empty one is met with, where enter it; and thence proceed as before.

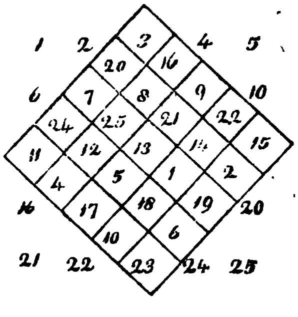
Thus, to make a magic square of the 49 numbers 1, 2, 3, 4, &c. First place the 1 next below the centre cell, and thence descend to the right till the 4 runs off at the bottom, which therefore carry to the top corner on the same column as it would have fallen in; but as it runs off at the side, bring it to the beginning of the second line, and thence descend to the right till you arrive at the cell occupied by 1; carry the 8 therefore to the next diagonal cell to the left, and so proceed till 10 run off at the bottom, which carry there-

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

fore to the top of its column, and so proceed till 13 runs off at the side, which therefore bring to the beginning of the same line, and thence proceed till 15 arrives at the cell occupied by 8; from this therefore descend diagonally to the left; but as 16 runs off at the bottom, carry it to the top of its proper column, and thence descend till 21 run off at the side, which is therefore brought to the beginning of its proper line; but as 22 arrives at the cell occupied by 15, descend diagonally to the left, which brings it into the 1st column, but off at the bottom, and therefore it is carried to the top of that column; thence descending till 29 runs off both at the bottom and side, which therefore carry to the highest unoccupied cell in the last column; and here, as 30 runs off at the side, bring it to the beginning of its proper column, and thence descend till 35 runs off at the bottom, which therefore carry to the beginning or top of its own column; and here, as 36 meets with the cell occupied by 29, it is brought from thence diagonally to the left; thence descending, 38 runs off at the side, and therefore it is brought to the beginning of its proper line; thence descending, 41 runs off at the bottom, which therefore is carried to the beginning or top of its column; whence descending, 43 arrives at the cell occupied by 36, and therefore it is brought down from thence to the left; thence descending, 46 runs off at the side, which therefore is brought to the beginning of its line; but here, as 47 runs off at the bottom, it is carried to the beginning or top of its column, whence descending with 48 and 49, the square is completed, the sum of every row and column and diagonal making just 175.—There are many other ways of filling up such squares, but none that are easier than that above described; unless perhaps the following mechanical way, communicated by an ingenious friend, Mr. J. B. Wise, of Boyn Hill, near Maidenhead, Berks, which is as follows for an odd number, suppose 5, in the progression 1, 2, 3, 4, 5, &c.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25





First set down the 25 numbers in the form of a square, as in the first diagram here above. Next draw lines cutting off the three figures at each corner, viz, the 1, 2, 6, at the upper left-hand corner; the 4, 5, 10, at the upper right-hand corner; and in like manner 16, 21, 22, and 20, 24, 25 at the bottom corners; which four lines will form a square; then draw inner lines parallel to these, and they will divide the figure into 25 cells, as in the 2d diagram above, in which 13 of the cells will be occupied by as many of the 25 numbers, the other 12 cells being empty, and the filled and empty cells mutually alternating with each other, in every one of the 10 bands.—Lastly, the 12 corner numbers at first cut off, are to be carried into the aforesaid 12 empty cells, in this manner: viz, each of these 12 external numbers is to be carried to the farthest distant empty cell in the band opposite to which it stands; thus, the 2 is carried along its opposite band to the empty cell below the No. 14; in like manner, the 1 is carried to the cell next below 13, the 6 to the cell next below 18, the 4 along its band to the cell next below 12, the 5 to the cell below 13, the 10 to the cell below 18, the 16 to the cell above 8, the 21 to the cell above 13, the 22 to the cell above 14, the 20 to the cell above 8, the 24 to the cell above 12, and the 25 to the cell above 13; thus completing the magic square, as in the 3d diagram, which is a perfect square, the sum of the numbers in every band, and in both the diagonals, making up the same quantity, 65.

For the purpose of perspicuity, in the above process, three diagrams have been employed, in order to exhibit distinctly the several stages of the process; though in fact one diagram only is quite sufficient in practice. And the method is the same for squares of any other odd number of cells.

The same learned friend communicated a great many more of very ingenious constructions of squares, that are much larger and more curious than any that have yet been published, but are too extensive for this place; but it is to be wished that he will himself give them entire to the public in a connected state.

It was observed before, that the sum of the numbers in the rows, columns and diagonals, was 15 in the square of 9 numbers, 34 in a square of 16, 65 in a square of 25, &c; hence then is derived a method of finding the sums of the numbers in any other square, viz, by taking the successive differences till they become equal, and then adding them successively to produce or find out the amount of the following sums. Thus, having ranged the sides and cells in two columns, and a few of the first sums in a third column, take the first differences of these, which will be 1, 4, 10, 19, &c, as in the 4th column; and of these take the differences 0, 3, 6, 9, 12, &c, as in the 5th column; and again of these, the differences 3, 3, 3 &c, as in the 6th or last column. Then, returning back again, add always 3, the constant last or 3d difference, to the last found of the 2d differences, which will complete the remainder of the column of these, viz, 15, 18, 21, 24, &c: then add these 2d differences to the last found of the 1st differences, which will complete the column of these, viz, giving 31, 46, 64, &c: lastly, add always these corresponding 1st differences to the last found number or amount of the sums, and the column of sums will thus be completed.

Side.	Cells.	Sums.	Diffs.		
0	0	0	1	0	3
1	1	1	4	3	3
2	4	5	10	6	3
3	9	15	19	9	3
4	16	34	31	12	3
5	25	65	46	15	3
6	36	111	64	18	3
7	49	175	85	21	3
8	64	260	109	24	3
9	81	369	136	27	3
10	100	505		30	

Again, like as the terms of an arithmetical progression arranged magically, give the same sum in every row &c, so the terms of a geometrical series arranged magically give the same product in every row &c, by multiplying the numbers continually together; so this progression 1, 2, 4, 8, 16, &c, arranged as in the margin, gives, for each continual product, 4096 in every row &c, which is just the cube of the middle term, 16.

8	256	2
4	16	64
128	1	32

Also, the terms of an harmonical progression being ranged magically, as in the margin, have the terms in each row &c in harmonical progression.

1260	840	630
504	420	360
315	280	252

The ingenious Dr. Franklin, it seems, carried this curious speculation further than any of his predecessors in the same way. He constructed both a large Magic Square of Squares, and a Magic Circle of Circles, the description of which may be seen in the collection of his works, with many curious properties; but the square is omitted here, as an imperfection has been detected in the diagonals.

The Magic Circle of Circles, fig. 2, pl. 19, by this author, is composed of a series of numbers, from 12 to 75 inclusive, divided into 8 concentric circular spaces, and ranged in 8 radii of numbers, with the number 12 in the centre; which number, like the centre, is common to all these circular spaces, and to all the radii.

The numbers are so placed, that 1st, the sum of all those in either of the concentric circular spaces above mentioned, together with the central number 12, amount to 360, the same as the number of degrees in a circle.

2. The numbers in each radius also, together with the central number 12, make just 360.

3. The numbers in half of any of the above circular spaces, taken either above or below the double horizontal line, with half the central number 12, make just 180, or half the degrees in a circle.

4. If any four adjoining numbers be taken, as if in a square, in the radial divisions of these circular spaces; the sum of these, with half the central number, make also the same 180.

5. There are also included four sets of other circular spaces, bounded by circles that are excentric with regard to the common centre; each of which sets contain five spaces; and their centres being at A, B, C, D. For distinction, these circles are drawn with different marks, some dotted, others by short unconnected lines, &c; or still better with inks of divers colours, as blue, red, green, yellow.

These sets of excentric circular spaces intersect those of the concentric, and each other; and yet, the numbers contained in each of the excentric spaces, taken all around through any of the 20, which are excentric, make the same sum as those in the concentric, namely 360, when the central number 12 is added. Their halves also, taken above or below the double horizontal line, with half the central number, make up 180. It is observable, that there is not one of the numbers but what belongs at least to two of the circular spaces; some to three, some to four, some to five: and yet they are all so placed, as never to break the required number 360, in any of the 28 circular spaces within the primitive circle. They have also other properties. See Franklin's Exp. and Obs. pa. 350, edit. 4to, 1769; or Ferguson's Tables and Tracts, 1771, pa 318; or my Recreations, vol. 1, pa. 183.

In Dr. Franklin's magic square, above-mentioned, three accidentally erroneous numbers have been detected by Isaac Dalby, Esq. first-professor at the Royal Military College, which he has communicated, as well as the discovery of a radical imperfection in the square, owing to an inequality in the two diagonals: for though the half diagonals have the proper sums, yet the whole diagonals have not. Mr. Dalby has therefore constructed another perfect magic square of magic squares, engraven on pl. 19, fig. 1. the properties of which are as below.

This magic square, made with the series 1, 2, 3, 4, &c, to 256, is composed of 16 magic squares of 16 cells each; the 4 numbers in each column, or diagonal of each of these squares, make 514, and consequently each column or diagonal of the great square is 514 × 4 or 2056.

The principal properties are

1. The sum of the 4 numbers in any 4 contiguous cells forming a square (the square of 2) is 514; thus, 226 + 32 + 255 + 1 = 514; and 1 + 242 + 240 + 31 = 514, &c. Consequently

2. The sum of the 16 numbers in any 16 cells making a square (the square of 4) is 514 × 4 = 2056:— in any square of 36 cells (the square of 6) the sum is 514 × 9: —and 514 × 16 in a square of 64 cells:—514 × 25 in that of 100 cells:—514 × 36 in a square of 144 cells:— and 514 × 49 in any square having 14 for its side. Thus if a square hole just the size of 16 cells be cut in a paper, and the paper laid any where upon the great square, the sum of the 16 numbers appearing through the hole will always be 514 × 4. If the hole takes in 36 cells, the sum is 514 × 9: &c.

3. The sum of the 4 corner numbers of either of these squares will always be 514.

4. The sum of the 16 numbers in any bent row whose halves are parallel to the diagonals is 2056: thus, from 26 to 36, and from 173 to 151, is a bent row; also, from 74 to 22, and from 227 to 191 another bent row, &c.

5. If the square be divided horizontally, or vertically, through the middle, the halves may change places, and the properties of the square will remain as before.

6. If the square be cut into the 16 squares, it is manifest that any four of them will make a magic square of 64 cells; and any nine another magic square of 144 cells; consequently as many different magic squares of 64 cells, and also of 144 cells, may be made with the 16 squares, as there are combinations of 4 in 16, and of 9 in 16.

The construction of the great square evidently depends upon that of a magic square of 16 cells having the sum of the 4 numbers in any square composed of 4 cells always the same. To construct such a square with the series 1, 2, 3, 4, &c. to 16. *First*, $\frac{(16+1)\times 8}{4} = 34$ the sum in each column or diagonal, or in the 4 cells. Now arrange the 16 numbers as in fig. 1., then by the nature of the progression, each diagonal will contain 34: and the excess above 34 in the 4th perpendicular column on the right is equal to the defect in the 1st column on the left; and the excess and defect in the lower and upper horizontal columns are also

Fig. 1.

1	5	9	13
2	6	10	14
3	7	11	15
4	8	12	16

the same; hence (the corner numbers remaining) if 2 and 15, 3 and 14, 5 and 12, 9 and 8 respectively change places, we have the magic square fig. 2. In this square, however, only the middle square of 4 cells, and the 4 corner ones contain 34 each: but in magic squares we readily obtain varieties by shifting the columns: thus in fig. 2, the 2d vertical column from the left may be made the last on the right, or the 3d from the left made the 1st, and the two upper, or the two lower horizontal columns take place of one another; either of these arrangements will answer: thus, let the 3d vertical column be brought on the left, and change the two upper horizontal columns one for the other, and we get the magic square fig. 3, having 34 in any square of 4 cells.

Fig. 2.

1	12	8	13
15	6	10	3
14	7	11	2
4	9	5	16

Fig. 3.

10	15	6	3
8	1	12	13
11	14	7	2
5	4	9	16

It will now be perceived that the numbers in fig. 3, consist of pairs situated alternately, the sums being 18 and 16 respectively; thus 10 + 8, 15 + 1, 6 + 12, &c, are pairs. Hence, to make the great square with the series 1, 2, 3, 4, &c. to 256, let the numbers be arranged thus

256 255 254 253 252 251 &c.
2 1 4 3 6 5

the pairs making 258, 256, 258, &c, and call the upper or greater numbers the complements of the lower or lesser numbers: then arrange the first 32 numbers of the lower series (consisting of the numbers from 1 to 128) as in the margin: next, place the 8 upper numbers with their complements in a square of 16 cells in the same order, from the least to the greatest, as they stand in fig. 3, and we shall have the first or corner square of the great square on the left at top.

1	16	17	32
2	15	18	31
3	14	19	30
4	13	20	29
5	12	21	28
6	11	22	27
7	10	23	26
8	9	24	25

In fig. 3.	1	2	3	4	5	6	7	8	9	10	11	12	13
Correspond. numb.	1	2	15	16	17	18	31	32	225	226	239	240	241
In fig. 3.	14	15	16										
Correspond. numb.	242	255	256										

The next 8 numbers, or 3, 4, 13, 14, 19, 20, 29, 30, with their complements, make the 2d square downwards, and so on for the four first squares on the left. And for the next 4 squares, the numbers from 32 to 64 are arranged as above: and two more arrangements, viz, from 64 to 96, and from 96 to 128, in the same manner, will complete the 16 squares.

It is already remarked that the pairs fall alternately in fig. 3; and the same order necessarily results in the great square; the pairs making 258 and 256 throughout: and 258 + 256 = 514; therefore when two numbers with their complements occupy a square of 4 cells, it is obvious their sum is 514; but the sum will also be equal to 258 + 256, when the 4 numbers belong to 4 pair, the excess on one side being equal to the defect on the other; thus, taking the numbers in the 2d and 3d horizontal columns by fours, 32 + 239 = 271, 1 + 242 = 243, now 271 − 256 = 15, and 258 − 243 = 15, therefore 271 + 243 = 258 + 256; and the like of any other 4 adjacent numbers in the two ranks.

But Dr. Franklin, instead of taking 256 and 258 alternately, begins with two pair of 258 in succession: thus, 200 + 58 = 258, the first pair at top; then 198 + 60 = 258, the next under; and these are followed by four pair of 256 each; thus, 201 + 55 = 256, the first pair; then 203 + 53 = 256 the next under, &c; and the two pair at the bottom of the column are 258 each, or 196 + 62, and 194 + 64. His 2d vertical column begins at top with two pair of 256 each, and these are followed by four pair of 258. In this manner he has a regular alternation of 2 pair and 4 pair, throughout his square; but this construction will not answer diagonally, and therefore his square is radically wrong, and cannot be mended but by a new one.

The doctor's smaller square too, of 64 cells (in plate 5 in his Works), is constructed in a manner similar to his large one, namely, by pairs of numbers, whose sums are 66 and 64; but, instead of these sums or pairs being alternate, we find 66 first, then two of 64 each; next 66 (taking the first vertical column on the left, and proceeding downwards): this order runs through the whole square, and in consequence it fails diagonally, one diagonal being 292, the other 228.

Last year Mr. Youle, schoolmaster at Sheffield, published an Arithmetic, to which is added a tract on magic squares: in this we are informed, that the Rev. Mr. Watson of Whitby, in a small treatise lately published, has noticed the deficiencies in Doctor Franklin's square, and has given a magic square of 256 cells, with the following additional property, viz—"the 4 corner numbers of every interior square whose root is an even number, whether concentric or not to the great square, always make 514."

Watson has also analysed the doctor's magic circle of circles, which is made with the numbers of a magic square of 64 cells (the numbers from 12 to 75).

The square in Youle's book is constructed with pairs of 256 and 258, as in Dr. F.'s and mine: but Youle's theory and mine are totally different.

MAGICAL *Picture,* in Electricity, was first contrived by Mr. Kinnersley, and is thus constructed: Having a large mezzotinto with a frame and glass, as of the king for instance, take out the print, and cut a pannel out of it, near two inches distant from the frame all around; then with thin paste or gum-water, fix the border that is cut off on the inside of the glass, pressing it smooth and close; then fill up the vacancy by gilding the glass well with leaf-gold or brass. Gild likewise the inner edge of the back of the frame all around, except the top part, and form a communication between that gilding and the gilding behind the glass; then put in the board, and that side is finished. Next turn up the glass, and gild the foreside exactly over the back gilding, and when it is dry, cover it by pasting on the pannel of the picture that has been cut out, observing to bring the corresponding parts of the border and picture together, by which means the picture will appear entire, as at first, only part behind the glass, and part before.

Hold the picture horizontally by the top, and place a small moveable gilt crown on the king's head. If now the picture be moderately electrified, and another person take hold of the frame with one hand, so that his fingers touch its inside gilding, and with the other hand endeavour to take off the crown, he will receive a violent blow, and fail in the attempt. If the picture were highly charged, the consequence might be as fatal as that of high treason. The operator, who holds the picture by the upper end, where the inside of the frame is not gilt, to prevent its falling, feels nothing of the shock, and may touch the face of the picture without danger. And if a ring of persons take the shock among them, the experiment is called the conspirators. See Franklin's Exper. and Observ. pa. 30.

MAGINI (JOHN-ANTHONY), or MAGINUS, professor of mathematics in the university of Bologna, was born at Padua in the year 1536. Magini was remarkable for his great assiduity in acquiring and improving the knowledge of the mathematical sciences, with several new inventions for these purposes, and for the extraordinary favour he obtained from most princes of his time. This doubtless arose partly from the celebrity he had in matters of astrology, to which he was greatly addicted, making horoscopes, and foretelling events, both relating to persons and things. He was invited by the Emperor Rodolphus to come to Vienna, where he promised him a professor's chair, about the year 1597; but not being able to prevail on him to settle there, he nevertheless gave him a handsome pension.

It is said, he was so much addicted to astrological predictions, that he not only foretold many good and evil events relative to others with success; but even foretold his own death, which came to pass the same year: all which he represented as under the influence of the stars. Tomasini says, that Magini, being advanced to his 61st year, was struck with an apoplexy, which ended his days; and that a long while before, he had told him and others, that he was afraid of that year. And Roffeni, his pupil, says, that Magini died under an aspect of the planets, which, according to his own prediction, would prove fatal to him; and he mentions Riccioli as affirming that he said, the figure of his nativity, and his climacteric year, doomed him to die about that time; which happened in 1618, in the 62d year of his age.

His writings however do honour to his memory, as they were very considerable, and on learned subjects. The principal were the following: 1. His Ephemeris, in 3 volumes, from the year 1580 to 1630.—2. Tables of Secondary Motions.—3. Astronomical, Gnomonical, and Geographical Problems.—4. Theory of the Planets, according to Copernicus.—5. A Confutation of Scaliger's Dissertation concerning the Precession of the Equinox. —6. A Primum Mobile, in 12 books.—7. A Treatise of Plane and Spherical Trigonometry.—8. A Commentary on Ptolemy's Geography.—9. A Chorographical Description of the Regions and Cities of Italy, illustrated with 60 maps; with some papers on astrological subjects.

MAGNET, MAGNES, the *Loadstone;* a kind of ferruginous stone, resembling iron ore in weight and colour, though rather harder and heavier; and is endued with divers extraordinary properties, attractive, directive, inclinatory, &c. See MAGNETISM.

The magnet is also called lapis Heraclæus, from Heraclea, a city of Magnesia, a port of the ancient Lydia, where it was said to have been first found, and from which it is usually supposed that it took its name. Though some derive the word from a shepherd named Magnes, who first discovered it on mount Ida with the iron of his crook. It is also called lapis nauticus, from its use in navigation: also siderites, from its virtue in attracting iron, which the Greeks call σιδηϱος.

The magnet is usually found in iron-mines, and sometimes in very large pieces, half magnet, half iron. Its colour is different, as found in different countries. Norman observes, that the best are those brought from China and Bengal, which are of a rusty or sanguine colour; those of Arabia are reddish; those of Macedonia, blackish; and those of Hungary, Germany, England, &c, the colour of unwrought iron. Neither its figure nor bulk are constant or determined; being found of all shapes and sizes.

The ancients reckoned five kinds of magnets, different in colour and virtue: the Ethiopic, Magnesian, Bœotic, Alexandrian, and Natolian. They also fancied it to be male and female: but the chief use they made of it was in medicine; especially for the cure of burns and defluxions of the eyes.—The moderns, more happy, take it to conduct them in their voyages.

The most distinguishing properties of the magnet are, That it attracts iron, and that it points towards the poles of the world; and in other circumstances also dips or inclines to a point beneath the horizon, directly under the pole; it also communicates these properties, by contact, to iron. By means of which, are obtained the mariner's needles, both horizontal and inclinatory or dipping needles.

The Attractive Power of the MAGNET, was known to the ancients, and is mentioned even by Plato and Euripides, who call it the Herculean stone, because it commands iron, which subdues every thing else: but the knowledge of its directive power, by which it disposes its poles along the meridian of every place, or nearly so, and causes needles, pieces of iron, &c, touched with it, to point nearly north and south also, is of a much later date; though the discoverer himself, and the exact time of the discovery, be not now known. The first mention of it is about 1260, when it has been said that Marco Polo, a Venetian, introduced the mariner's compass; though not as an invention of his own, but as derived from the Chinese, who it seems had the use of it long before; though some imagine that the Chinese rather borrowed it from the Europeans.

But Flavio de Gira, a Neapolitan, who lived in the 13th century, is the person usually supposed to have the best title to the discovery; and yet Sir G. Wheeler mentions, that he had seen a book of astronomy much older, which supposed the use of the needle; though not as applied to the purposes of navigation, but of astronomy. And in Guiot de Provins, an old French poet, who wrote about the year 1180, there is an express mention made of the loadstone and the compass; and their use in navigation obliquely hinted at.

The Variation of the MAGNET, or needle, or its deviation from the pole, was first discovered by Sebastian Cabot, a Venetian, in 1500; and the variation of that variation, or change in its direction, by Mr. Henry Gellibrand, professor of astronomy in Gresham-college, about the year 1625. Lastly, the dip or inclination of the needle, when at liberty to play vertically, to a point beneath the horizon, was first discovered by another of our countrymen, Mr. Robert Norman, about the year 1576.

The Phenomena of the MAGNET, are as follow: 1, In every magnet there are two poles, of which the one points northwards, the other southwards, when it is freely suspended; and if the magnet be divided into ever so many pieces, the two poles will be found in each piece. The poles of a magnet may be found by holding a very fine short needle over it; for where the poles are, the needle will stand upright, but no where else.—2, These poles, in different parts of the globe, are differently inclined towards a point under the horizon.—3, And, though contrary to each other, do help mutually towards the magnet's attraction and suspension of iron.—4, If two magnets be spherical, one will turn or conform itself to the other, so as either of them would do to the earth; and after they have so conformed or turned themselves, they endeavour to approach or join each other; but if placed in a contrary position, they avoid each other.—5, If a magnet be cut through the axis, the segments or parts of the stone, which before were joined, will now avoid and fly each other.—6, If the magnet be cut perpendicular to its axis, the two points, which before were conjoined, will become contrary poles; one in each segment.—7, Iron receives virtue from the magnet by application to it, or barely from an approach near it, though it do not touch it; and the iron receives this virtue variously, according to the parts of the stone it is made to touch, or even approach to.—8, If an oblong piece of iron be in any manner applied to the stone, it receives virtue from it only lengthways.—9, The magnet loses none of its own virtue by communicating any to the iron; and this virtue it can communicate to the iron very speedily: though the longer the iron joins or touches the stone, the longer will it maintain its communicated virtue; and a better magnet will communicate more of it, and sooner, than one not so good.—10, Steel receives virtue from the magnet better than iron.—11, A needle touched by a magnet will turn its ends the same way towards the poles of the world, as the magnet itself does.—12, Neither loadstone nor needles touched by it conform their poles exactly to those of the world, but have usually some variation from them: and this variation is different in different places, and at divers times in the same places.—13, A loadstone will take up much more iron when armed, or capped, than it can alone. (A loadstone is said to be armed, when its poles are surrounded with plates of steel: and to determine the quantity of steel to be applied, try the magnet with several steel bars; and the greatest weight it takes up, with a bar on, is to be the weight of its armour.) And though an iron ring or key be suspended by the loadstone, yet this does not hinder the ring or key from turning round any way, either to the right or left.—14, The force of a loadstone may be variously increased or lessened, by variously applying to it, either iron, or another loadstone.—15, A strong magnet at the least distance from a smaller or weaker one, cannot draw to it a piece of iron adhering actually to such smaller or weaker stone; but if it touch it, it can draw it from the other: but a weaker magnet, or even a small piece of iron, can draw away or separate a piece of iron contiguous to a larger or stronger magnet.—16, In these northern parts of the world, the south pole of a magnet will raise up more iron than its north pole.—17, A plate of iron only, but no other body interposed, can impede the operation of the loadstone, either as to its attractive or directive quality.—18, The power or virtue of a loadstone may be impaired by lying long in a wrong position, as also by rust, wet, &c; and may be quite destroyed by fire, lightning, &c.—19, A piece of iron wire well touched, on being bent round in a ring, or coiled round on a stick, &c, will always have its directive virtue diminished, and often quite destroyed. And yet if the whole length of the wire were not entirely bent, so that the ends of it, though but for the length of one-tenth of an inch, were left straight, the virtue will not be destroyed in those parts; though it will in all the rest.—20, The sphere of activity of magnets is greater and less at different times. Also, the variation of the needle from the meridian, is various at different times of the day.—21, By twisting a piece of wire touched with a magnet, its virtue is greatly diminished; and sometimes so disordered and confused, that in some parts it will attract, and in others repel; and even, in some places, one side of the wire seems to be attracted, and the other side repelled, by one and the same pole of the stone.—22, A piece of wire that has been touched, on being split, or cleft in two, the poles are sometimes changed, as in a cleft magnet; the north pole becoming the south, and the south the north: and yet sometimes one half of the wire will retain its former poles, and the other half will have them changed.—23, A wire being touched from end to end with one pole of a magnet, the end at which you begin will always turn contrary to the pole that touched it: and if it be again touched the same way with the other pole of the magnet, it will then be turned the contrary way.—24, If a piece of wire be touched in the middle with only one pole of the magnet, without moving it backwards or forwards; in that place will be the pole of the wire, and the two ends will be the other pole.—25, If a magnet be heated red-hot, and again cooled either with its south pole towards the north in a horizontal position, or with its south pole downwards in a perpendicular position, its poles will be changed.—26, Mr. Boyle (to whom we are indebted for the following magnetical phenomena) found he could presently change the poles of a small fragment of a loadstone, by applying them to the opposite vigorous poles of a large one.—27, Hard iron tools well tempered, when heated by a brisk attrition, as filing, turning, &c, will attract thin filings or chips of iron, steel, &c; and hence we observe that files, punches, augres, &c, have a small degree of magnetic virtue.—28, The iron bars of windows, &c, which have stood a long time in an erect position, grow permanently magnetical; the lower ends of such bars being the north pole, and the upper end the south pole.—29, A bar of iron that has not stood long in an erect posture, if it be only held perpendicularly, will become magnetical, and its lower end the north pole, as appears from its attracting the south pole of a needle: but then this virtue is transient, and by inverting the bar, the poles change their places. In order therefore to render the quality permanent in an iron bar, it must continue a long time in a proper position. But fire will produce the effect in a short time: for as it will immediately deprive a loadstone of its attractive virtue; so it soon gives a verticity to a bar of iron, if, being heated red hot, it be cooled in an erect posture, or directly north and south. Even tongs and fire-forks, by being often heated, and set to cool again in a posture nearly erect, have gained this magnetic property. Sometimes iron bars, by long standing in a perpendicular position, have acquired the magnetic virtue in a surprising degree. A bar about 10 feet long, and three inches thick, supporting the summer beam of a room, was able to turn the needle at 8 or 10 feet distance, and exceeded a loadstone of 3½ pounds weight: from the middle point upwards it was a north pole, and downwards a south pole. And Mr. Martin mentions a bar, which had been the beam of a large steelyard that had several poles in it.—30, Mr. Boyle found, that by heating a piece of English oker red-hot, and placing it to cool in a proper posture, it manifestly acquired a magnetic virtue. And an excellent magnet, belonging to the same ingenious gentleman, having lain near a year in an inconvenient posture, had its virtue greatly impaired, as if it had been by fire.—31, A needle well touched, it is known, will point north and south: if it have one contrary touch of the same stone, it will be deprived of its faculty; and by another such touch, it will have its poles interchanged.—32, If an iron bar have gained a verticity by being heated red-hot and cooled again, north and south, and then hammered at the two ends; its virtue will be destroyed by two or three smart blows on the middle.—33, By drawing the back of a knife, or a long piece of steel-wire, &c, leisurely over the pole of a loadstone, carrying the motion from the middle of the stone to the pole; the knife or wire will attract one end of a needle; but if the knife or wire be passed from the said pole to the middle of the stone, it will repel the same end of the needle.—34, Either a magnet or a piece of iron being laid on a piece of cork, so as to float freely on water; it will be found, that, whichever of the two is held in the hand, the other will be drawn to it: so that iron attracts the magnet as much as it is attracted by it; action and re-action being always equal. In this experiment, if the magnet be set afloat, it will direct its two poles to the poles of the world nearly.—35, A knife &c touched with a magnet, acquires a greater or less degree of virtue, according to the part it is touched on. It receives the strongest virtue, when it is drawn leisurely from the handle towards the point over one of the poles. And if the same knife thus touched, and thus possessed of a strong attractive power, be retouched in a contrary direction, viz, by drawing it from the point towards the handle over the same pole, it immediately loses all its virtue.—36, A magnet acts with equal force in vacuo as in the open air.—37, The smallest magnets have usually the greatest power in proportion to their bulk. A large magnet will seldom take up above 3 or 4 times its own weight, while a small one will often take up more than ten times its weight. A magnet worn by Sir Isaac Newton in a ring, and which weighed only 3 grains, would take up 746 grains, or almost 250 times its own weight. A magnetic bar made by Mr. Canton, weighing 10 oz. 12 dwts, took up more than 79 ounces; and a flat semicircular steel magnet, weighing 1 oz. 13 dwts, took up an iron wedge of 90 ounces.

Armed MAGNET, denotes one that is capped, cased, or set in iron or steel, to make it take up a greater weight, and also more readily to distinguish its poles. For the methods of doing this, see Mr. Michell's book on this subject.

Artificial MAGNET, is a bar of iron or steel, impregnated with the magnetic virtue, so as to possess all the properties of the natural loadstone, and be used instead of it. How to make magnets of this kind, by means of a natural magnet, and even without the assistance of any magnet, was suggested many years since by Mr. Savary, and particularly described in the Philos. Trans. No. 414. See also my Abridgment, vol. 7, pa. 400. But as his method was tedious and operose, though capable of communicating a very considerable virtue, it was little practised. Dr. Gowin Knight first brought this kind of magnets to their present state of perfection, so as to be even of much greater efficacy than the natural ones. But as he foolishly refused to discover his methods on any terms whatever, these curious and valuable secrets in a great measure died with him. The result of his method however was first published in the Philos. Trans. for 1744, art. 8, and for 1745, art. 3. See also the vol. for 1747, art. 2. And in the 69th vol. Mr. Benjamin Wilson has given a process, which at least discovers one of the leading principles of Dr. Knight's art. The method, according to Mr. Wilson, was as follows. Having provided a great quantity of clean iron filings, he put them into a large tub that was more than one-third filled with clean water; he then, with great labour, shook the tub to and fro for many hours together, that the friction between the grains of iron, by this treatment, might break or rub off such small parts as would remain suspended in the water for some time. The water being thus rendered very muddy, he poured it into a clean iron vessel, leaving the filings behind; and when the water had stood long enough to become clear, he poured it out carefully, without disturbing such of the sediment as still remained, which now appeared reduced almost to impalpable powder. This powder was afterwards removed into another vessel, to dry it: and having, by several repetitions of the process, procured a sufficient quantity of this very fine powder; the next thing was to make a paste of it, and that with some vehicle containing a good quantity of the phlogistic principle; for this purpose, he had recourse to linseed oil, in preference to all other fluids: and with these two ingredients only, he made a stiff paste, and took great care to knead it well before he moulded it into convenient shapes. Sometimes, while the paste continued in its soft state, he would put the impression of a seal; one of which is in the British Museum. This paste so moulded was then set upon wood, or a tile, to dry or bake it before a moderate fire, being placed at about one foot distance. He found that a moderate fire was most proper, because a greater degree of heat would make the composition crack in many places. The time requisite for the baking or drying of this paste, was usually about 5 or 6 hours, before it attained a sufficient degree of hardness. When that was done, and the several baked pieces were become cold, he gave them their magnetic virtue in any direction he pleased, by placing them between the extreme ends of his large magazine of artificial magnets, for a few seconds. The virtue they acquired by this method was such, that, when any of those pieces were held between two of his best ten-guinea bars, with its poles purposely inverted, it immediately of itself turned about to recover its natural direction, which the force of those very powerful bars was not sufficient to counteract. Philos. Trans. vol. 65, for 1779.

Methods for artificial magnets were also discovered and published by the Rev. Mr. John Michell, in a Treatise on Artificial Magnets, printed in 1750, and by Mr. John Canton, in the Philos. Trans. for 1751. The process for the same purpose was also found out by other persons, particularly by Du Hamel, Hist. Acad. Roy. 1745 and 1750, and by Marul Uitgeleeze Natuurkund. Verhand. tom. 2, p. 261.

Mr. Canton's method is as follows: Procure a dozen of bars; 6 of soft steel, and 6 of hard; the former to be each 3 inches long, a quarter of an inch broad, and 1-20th of an inch thick; with two pieces of iron, each half the length of one of the bars, but of the same breadth and thickness; and the 6 hard bars to be each 5½ inches long, half an inch broad, and 3-20ths of an inch thick, with two pieces of iron of half the length, but the whole breadth and thickness of one of the hard bars; and let all the bars be marked with a line quite around them at one end. Then take an iron poker and tongs (fig. 1, plate 20), or two bars of iron, the larger they are, and the longer they have been used, the better; and fixing the poker upright between the knees, hold to it, near the top, one of the soft bars, having its marked end downwards by a piece of sewing-silk, which must be pulled tight by the left hand, that the bar may not slide: then grasping the tongs with the right hand, a little below the middle, and holding them nearly in a vertical position, let the bar be stroked by the lower end, from the bottom to the top, about ten times on each side, which will give it a magnetic power sufficient to lift a small key at the marked end: which end, if the bar were suspended on a point, would turn towards the north, and is therefore called the north pole; and the unmarked end is, for the same reason, called the south pole. Four of the soft bars being impregnated in this manner, lay the two (fig. 2) parallel to each other, at a quarter of an inch distance, between the two pieces of iron belonging to them, a north and a south pole against each piece of iron: then take two of the four bars already made magnetical, and place them together, so as to make a double bar in thickness, the north pole of one even with the south pole of the other; and the remaining two being put to these, one on each side, so as to have two north and two south poles together, separate the north from the south poles at one end by a large pin, and place them perpendicularly with that end downward on the middle of one of the parallel bars, the two north poles towards its south end, and the two south poles towards its north end: slide them three or four times backward and forward the whole length of the bar; then removing them from the middle of this bar, place them on the middle of the other bar as before directed, and go over that in the same manner; then turn both the bars the other side upwards, and repeat the former operation: which being done, take the two from between the pieces of iron; and, placing the two outermost of the touching bars in their stead, let the other two be the outermost of the four to touch these with; and this process being repeated till each pair of bars have been touched three or four times over, which will give them a considerable magnetic power. Put the half-dozen together after the manner of the four (fig. 3), and touch them with two pair of the hard bars placed between their irons, at the distance of about half an inch from each other; then lay the soft bars aside, and with the four hard ones let the other two be impregnated (fig. 4), holding the touching bars apart at the lower end near ⅕th of an inch; to which distance let them be separated after they are set on the parallel bar, and brought together again before they are taken off: this being observed, proceed according to the method described above, till each pair have been touched two or three times over. But as this vertical way of touching a bar, will not give it quite so much of the magnetic virtue as it will receive, let each pair be now touched once or twice over in their parallel position between the irons (fig. 5), with two of the bars held horizontally, or nearly so, by drawing at the same time the north end of one from the middle over the south end, and the south of the other from the middle over the north end of a parallel bar; then bringing them to the middle again, without touching the parallel bar, give three or four of these horizontal strokes to each side. The horizontal touch, after the vertical, will make the bars as strong as they possibly can be made, as appears by their not receiving any additional strength, when the vertical touch is given by a great number of bars, and the horizontal by those of a superior magnetic power.

This whole process may be gone through in about half an hour; and each of the large bars, if well hardened, may be made to lift 28 troy ounces, and sometimes more. And when these bars are thus impregnated, they will give to a hard bar of the same size its full virtue in less than two minutes; and therefore will answer all the purposes of magnetism in navigation and experimental philosophy, much better than the loadstone, which has not a power sufficient to impregnate hard bars. The half dozen being put into a case (fig. 6), in such a manner as that no two poles of the same name may be together, and their irons with them as one bar, they will retain the virtues they have received; but if their power should, by making experiments, be ever so far impaired, it may be restored without any foreign assistance in a few minutes. And if perchance a much larger set of bars should be required, these will communicate to them a sufficient power to proceed with; and they may, in a short time, by the same method, be brought to their full strength.

MAGNETISM, the quality or constitution of a body, by which it is rendered magnetical, or a magnet, sensibly attracting iron, and giving it a meridional direction.—This is a transient power, capable of being produced, destroyed, or restored.

The Laws of MAGNETISM.—These laws are laid down by Mr. Whiston in the following propositions.—1. The loadstone has both an attractive and a directive power united together, while iron touched by it has only the former; i. e. the magnet not only attracts needles, or steel filings, but also directs them to certain different angles, with respect to its own surface and axis; whereas iron, touched with it, does little or nothing more than attract them; still suffering them to lie along or stand perpendicular to its surface and edges in all places, without any such special direction.

2. Neither the strongest nor the largest magnets give a better directive touch to needles, than those of a less size or virtue: to which may be added, that whereas there are two qualities in all magnets, an attractive and a directive one, neither of them depend on, or are any proof of, the strength of the other.

3. The attractive power of magnets, and of iron, will greatly increase or diminish the weight of needles on the balance; nay, it will overcome that weight, and even sustain some other additional also: while the directive power has a much smaller effect. Gassendus indeed, as well as Mersennus and Gilbert, assert that it has none at all: but this is a mistake; for Whiston found, from repeated trials on large needles, that after the touch they weighed less than before. One of 4584⅛ grains, lost 2⅝ grains by the touch; and another of 65726 grains weight, no less than 14 grains.

4. It is probable that iron consists almost wholly of the attractive particles; and the magnet, of the attractive and directive together; mixed, probably, with other heterogeneous matter; as having never been purged by the fire, which iron has; and hence may arise the reason why iron, after it has been touched, will lift up a much greater weight than the loadstone that touched it.

5. The quantity and direction of magnetic powers,

communicated to needles, are not properly, after such communication, owing to the magnet which gave the touch; but to the goodness of the steel that receives it, and to the strength and position of the terrestrial loadstone, whose influence alone those needles are afterwards subject to, and directed by: so that all such needles, if good, move with the same strength, and point to the same angle, whatever loadstone they may have been excited by, provided it be but a good one Nor does it seem that the touch does much more in magnetical cases, than attrition does in electrical ones; i. e. serving to rub off some obstructing particles, that adhere to the surface of the steel, and opening the pores of the body touched, thus making way for the entrance and exit of such effluvia as occasion or assist the powers we are speaking of. Hence Mr. Whiston takes occasion to observe, that the directive power of the loadstone seems to be mechanical, and to be derived from magnetic effluvia, circulating continually about it.

6. The absolute attractive power of different armed loadstones, is, cæteris paribus, not according to either the diameters or solidities of the loadstones, but according to the quantity of their surfaces, or in the duplicate proportion of their diameters.

7. The power of good unarmed magnets, sensibly equal in strength, similar in figure and position, but unequal in magnitude, is sometimes a little greater, sometimes a little less, than in the proportion of their similar diameters.

8. The loadstone attracts needles that have been touched, and others that have not been touched, with equal force at unequal distances, viz. when the distance of the former is to the distance of the latter, as 5 to 2.

9. Both poles of a magnet equally attract needles, till they are touched; then it is, and then only, that one pole begins to attract one end, and repel the other: though the repelling pole will still attract upon contact, and even at very small distances.

10. The attractive power of loadstones, in their similar position to, but different distances from, magnetic needles, is in the sesquiduplicate proportion of the distances of their surfaces from their needles reciprocally; or as the mean proportionals between the squares and the cubes of those distances reciprocally; or inversely as the square roots of the 5th powers of those distances. Thus, the magnetic force of attraction, at twice the distance from the surface of the loadstone, is between a 5th and 6th part of the force at the first distance; at thrice the distance, the force is between the 15th and 16th part; at four times the distance, the power is the 32d part of the first; and at six times the distance, it is the 88th part. Where it is to be noted, that the distances are not counted from the centre, as in the laws of gravity, but from the surface: as experience teaches us, that the magnetic power resides chiefly, if not wholly, in the surfaces of the loadstone and iron; without any particular relation to any centre whatever. The proportion here laid down was determined by Mr. Whiston from a great number of experiments by Mr. Hawksbee, Dr. Brook Taylor, and by Mr. Whiston himself; measuring the force by the chords of those arcs by which the magnet at several distances draws the needle out of its natural direction; to which chords, as he demonstrates, it is always proportional. The numbers in some of their most accurate trials, he gives in the following table, setting down the half chords, or the sines of half those arcs of declination, as the true measures of the force of magnetic attraction.

Distances in inches.	Degrees of inclination.	Sines of $\frac{1}{2}$ arcs.	Sesquiduplicate ratio.
20	2	175	466
$14\frac{8}{9}$	4	349	216
$13\frac{5}{8}$	6	523	170
$12\frac{3}{8}$	8	697	138
$11\frac{1}{8}$	10	871	105
$10\frac{1}{4}$	12	1045	87
$9\frac{1}{4}$	14	1219	70

Other persons however have found some variations in the proportions of the magnetic force with respect to distance: Thus, Newton supposes it to decrease nearly in the triplicate ratio of the distance: Mr. Martin observes, that the power of his loadstone decreases in the sesquiduplicate ratio of the distances inversely: but Dr. Helsham and Mr. Michell found it to be as the square of the distance inversely: while others, as Dr. Brook Taylor and M. Muschenbroek, are of opinion, that this power follows no certain ratio at all, and that the variation is different in different stones.

11. An inclinatory, or dipping-needle of 6 inches radius, and of a prismatic or cylindric figure, when it oscillates along the magnetic meridian, performs there every mean vibration in about $6''$ or $360'''$, and every small oscillation in about $5''\frac{1}{2}$, or $330'''$: and the same kind of needle, 4 feet long, makes every mean oscillation in about $24''$, and every small one in about $22''$.

12. The whole power of magnetism in this country, as it affects needles a foot long, is to that of gravity, nearly as 1 to 300; and as it affects needles 4 feet long, as 1 to 600.

13. The quantity of magnetic power accelerating the same dipping-needle, as it oscillates in different vertical planes, is always as the cosines of the angles made by those planes with the magnetic meridian, taken on the horizon.

Thus, in estimating the quantity of force in the horizontal and in the vertical situations of needles at London, it is found that the latter, in needles of a foot long, is to the whole force along the magnetic meridian, as 96 to 100; and in needles 4 feet long, as 9667 to 10000: whereas, in the former, the whole force in needles of a foot long, is as 28 to 100; and in those of 4 feet long, as 256 to 1000. Whence it follows, that the power by which horizontal needles are governed in these parts of the world, is but the quarter of the power by which the dipping-needle is moved.

Hence also, as the horizontal needle is moved only by a part of the power that moves the dipping-needle; and as it only points to a certain place in the horizon, because that place is the nearest to its original tendency of any that its situation will allow it to tend to; whenever the dipping-needle stands exactly perpendicular to the horizon, the horizontal needle will not respect one point of the compass more than another, but will wheel about any way uncertainly.

14. The time of oscillation and vibration, both in dipping and horizontal needles, that are equally good, is as their length directly; and the actual velocities of their points along their arcs are always unequal. And hence, magnetical needles are, cæteris paribus, still better, the longer they are; and that in the proportion of their lengths.

Of the Causes of MAGNETISM. Though many authors have proposed hypotheses concerning the cause of magnetism, as Plutarch, Descartes, Boyle, Newton, Gilbert, Hartsoeker, Halley, Whiston, Knight, Beccaria, &c; nothing however has yet appeared that can be called a satisfactory solution of its phenomena. It is certain indeed, that both natural and artificial electricity will give polarity to needles, and even reverse their poles; but though from this it may appear probable that the electric fluid is also the cause of magnetism, yet in what manner the fluid acts while producing the magnetical phenomena, seems to be quite unknown.

Dr. Knight, indeed, from several experiments deduces the following propositions, which he offers, not so much to explain the nature of the cause of magnetism, as the manner in which it acts: the magnetic matter of a loadstone, he says, moves in a stream from one pole to the other internally, and is then carried back in a curve line externally, till it arrive again at the pole where it first entered, to be again admitted: the immediate cause why two or more magnetical bodies attract each other, is the flux of one and the same stream of magnetical matter through them; and the immediate cause of magnetic repulsion, is the conflux and accumulation of the magnetic matter. Philos. Trans. vol. 44, pa. 665. Mr. Michell rejects the motion of a subtile fluid; but though he proposed to publish a theory of magnetism established by experiments, no such theory ever appeared.

Signor Beccaria, from observing that a sudden stroke of lightning gives polarity to magnets, conjectures, that a regular and constant circulation of the whole mass of the electric fluid, from north to south, may be the original cause of magnetism in general. But this current he does not suppose to arise from one source, but from several, in the northern hemisphere of the earth: the aberration of the common centre of all the currents from the north point, may be the cause of the variation of the needle; the period of this declination of the centre of the currents, may be the period of the variation; and the obliquity with which the currents strike into the earth, may be the cause of the dipping of the needle, and also why bars of iron more easily receive the magnetic virtue in one particular direction. Lettre dell' Elettricismo, pa. 269; or Priestley's Hist. Elec. vol. 1, pa. 409. See also Cavallo's Treatise on Magnetism, and the article VARIATION in this volume.

MAGNIFYING, is the making of objects appear larger than they usually and naturally appear to the eye; whence convex lenses, which have the power of doing this, are called Magnifying Glasses.

The magnifying power of dense mediums of certain figures, was known to the ancients; though they were far from understanding the cause of this effect. Seneca says, that small and obscure letters appear larger and brighter through a glass globe filled with water; and he absurdly accounts for it by saying, that the eye slides in the water, and cannot lay hold of its object. And Alexander Aphrodisensis, about two centuries after Seneca, says, that the reason why apples appear large when immersed in water, is, that the water which is contiguous to any body is affected with the same quality and colour; so that the eye is deceived in imagining the body itself larger. But the first distinct account we have of the magnifying power of glasses, is in the 12th century, in the writings of Roger Bacon, and Alhazen; and it is not improbable that from their observations the construction of spectacles was derived. In the Opus Majus of Bacon, it is demonstrated, that if a transparent body, interspersed between the eye and an object, be convex towards the eye, the object will appear magnified.

MAGNIFYING *Glass*, in Optics, is a small spherical convex lens; which, in transmitting the rays of light, inflects them more towards the axis, and so exhibits objects viewed through them larger than when viewed by the naked eye. See MICROSCOPE.

MAGNITUDE, any thing made up of parts locally extended, or continued; or that has several dimensions; as a line, surface, solid, &c. Quantity is often used as synonymous with magnitude. See QUANTITY.

Geometrical MAGNITUDES, are usually, and most properly, considered as generated or produced by motion; as lines by the motion of points, surfaces by the motion of lines, and solids by the motion of surfaces.

Apparent MAGNITUDE, is that which is measured by the optic or visual angle, intercepted between rays drawn from its extremes to the centre of the pupil of the eye. It is a fundamental maxim in optics, that whatever things are seen under the same or equal angles, appear equal; and vice versa.—The apparent magnitudes of an object, at different distances, are in a ratio less than that of their distances reciprocally.

The apparent magnitudes of the two great luminaries, the sun and moon, at rising and setting, are a phenomenon that has greatly embarrassed the modern philosophers. According to the ordinary laws of vision, they should appear the least when nearest the horizon, being then farthest from the eye; and yet it is found that the contrary is true in fact. Thus, it is well known that the mean apparent diameter of the moon, at her greatest height in the meridian, is nearly 31′ in round numbers, subtending then an angle of that quantity as measured by any instrument. But, being viewed when she rises or sets, she seems to the eye as two or three times as large as before; and yet when measured by the instrument, her diameter is not found increased at all, but diminished.

Ptolemy, in his Almagest, lib. 1, cap. 3, taking for granted, that the angle subtended by the moon was really increased, ascribed the increase to a refraction of the rays by vapours, which actually enlarge the angle under which the moon appears, just as the angle is enlarged by which an object is seen from under water: and his commentator Theon explains distinctly how the dilatation of the angle in the object immersed in water is caused. But it being afterwards discovered, that there is no alteration in the angle, another solution was started by the Arab Alhazen, which was followed and improved by Bacon, Vitello, Kepler, Peckham, and others. According to Alhazen, by sight we apprehend the surface of the heavens to be flat, and judge of the stars as of ordinary visible objects extended on a wide plain; the eye sees them under equal angles indeed, but withal perceives a difference in their distances, and (on account of the semidiameter of the earth, which is interposed in one case, and not in the other) it is hence induced to judge those that appear more remote to be greater. Further improvement was made in this explanation by Mr. Hobbes, though he fell into some mistakes in his application of geometry to this subject: for he observes, that this deception operates gradually from the zenith to the horizon; and that if the apparent arch of the sky be divided into any number of equal parts, those parts, in descending towards the horizon, will subtend an angle that is gradually less and less. And he was the first who expressly considered the vaulted appearance of the sky as a real portion of a circle.

Descartes, and from him Dr. Wallis, and most other authors, account for the appearance of a different distance under the same angle, from the long series of objects interposed between the eye and the extremity of the sensible horizon; which makes us imagine it more remote than when in the meridian, where the eye sees nothing in the way between the object and itself. This idea of a great distance makes us imagine the luminary the larger; for an object being seen under any certain angle, and believed at the same time very remote, we naturally judge it must be very large, to appear under such an angle at such a distance. And thus a pure judgment of the mind makes us see the sun, or the moon, larger in the horizon than in the meridian; notwithstanding their diameters measured by any instrument are really less in the former situation than the latter.

James Gregory, in his Geom. Pars Universalis, pa. 141, subscribes to this opinion: Father Mallebranche also, in the first book of his Recherche de la Verité, has explained this phenomenon almost in the expression of Descartes: and Huygens, in his Treatise on the Parhelia, translated by Dr. Smith, Optics, art. 536, has approved, and very clearly illustrated, the received opinion. The cause of this fallacy, says he, in short, is this; that we think the sun, or any thing else in the heavens, farther from us when it is near the horizon, than when it approaches towards the vertex, because we imagine every thing in the air that appears near the vertex to be farther from us than the clouds that fly over our heads; whereas, on the other hand, we are used to observe a large extent of land lying between us and the objects near the horizon, at the farther end of which the convexity of the sky begins to appear; which therefore, with the objects that appear in it, are usually imagined to be much farther from us. Now when two objects of equal magnitude appear under the same angle, we always judge that object to be larger which we think is remoter. And this, according to them, is the true cause of the deception in question.

Gassendus was of opinion, that this effect arises from hence; that the pupil of the eye, being always more open as the place is more dark, as in the morning and evening, when the light is less, and besides the earth being then covered with gross vapours, through a longer column of which the rays must pass to reach the horizon; the image of the luminary enters the eye at a greater angle, and is really painted there larger than when the luminary is higher. See APPARENT *Diameter* and *Magnitude*.

F. Gouge advances another hypothesis, which is, that when the luminaries are in the horizon, the proximity of the earth, and the gross vapours with which they then appear enveloped, have the same effect with regard to us, as a wall, or other dense body, placed behind a column; which in that case appears larger than when insulated, and encompassed on all sides with an illuminated air.

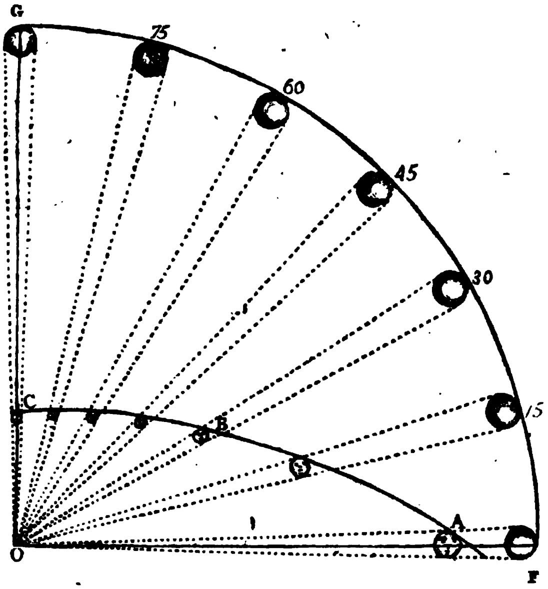
The commonly received opinion has been disputed, not only by F. Gouge, who observes, Acad. Sci. 1700, pa. 11, that the horizontal moon appears equally large across the sea, where there are no objects to produce the effect ascribed to them; but also by Mr. Molyneux, who says, Philos. Trans. Abr. vol. 3, pa. 365, that if this hypothesis be true, we may at any time increase the apparent magnitude of the moon, even in the meridian; for, in order to divide the space between it and the eye, we need only to look at it behind a cluster of chimneys, the ridge of a hill, or the top of a house, &c. He makes also the same observation with F. Gouge, above mentioned, and further observes, that when the height of all the intermediate objects is cut off; by looking through a tube, the imagination is not helped, and yet the moon seems still as large as before.

M. Biot, however, in his treatise of Physical Astronomy, seems to be of a contrary opinion, for he says, that as soon as the moon is viewed through a tube, or even through a small hole pierced in a card, so as to take off the view of intervening objects, the deception ceases; and the diameter appears no larger than when it is observed in the zenith.

Bishop Berkeley supposed, that the moon appears larger near the horizon, because she then appears fainter, and her beams affect the eye less. And Mr. Robins has pretty fully recited some other opinions on this subject, Math. Tracts, vol. 2, pa. 242, &c.

Dr. Desaguliers has illustrated the doctrine of the horizontal moon, Philos. Trans. Abr. vol 8, pa. 105, on the supposition of our imagining the visible heavens to be only a small portion of a spherical surface, and consequently supposing the moon to be farther from us in the horizon than near the zenith; and by several ingenious contrivances he demonstrated how liable we are to such deceptions. The same idea is pursued still further by Dr. Smith, in his Optics, where he determines, that the centre of the apparent spherical segment of the sky lying much below the eye, or the horizon, the apparent distance of its parts near the horizon was about 3 or 4 times greater than the apparent distance of its parts over head; from which reason it is, he infers, that the moon always appears the larger as she is lower, and also that we always think the height of a celestial object to be more than it really is. Thus, he determined, by measuring the actual height of some of the heavenly bodies, when to his eye they seemed to be half way between the horizon and the zenith; that their real altitude was then only 23°: when the sun was about 30° high, the upper portion always appeared less than the under; and he thought that it was constantly greater when the sun was 18° or 20° high. Mr. Robins, in his Tracts, vol. 2, pa. 245, shows how to determine the apparent concavity of the sky in a more accurate and geometrical manner; by which it appears, that if the altitude of any of the heavenly bodies be 20°, at the time when it seems to be half way between the horizon and the zenith, the horizontal distance will be hardly less than 4 times the perpendicular distance; but if that altitude be 28°, it will be little more than 2 and a half.

Dr. Smith, having determined the apparent figure of the sky, thus applies it to explain the phenomenon of the horizontal moon, and other similar appearances in the heavens. Suppose the arc ABC to represent the apparent concavity of the heavens: then the diameter of the sun and moon would seem to be greater in the horizon than at any altitude, measured by the angle AOB, in the ratio of its apparent distances, AO, BO. The numbers that express these proportions he reduced into the annexed table, answering to the corresponding altitudes of the sun or moon, which are also exactly represented to the eye in the figure, in which the moon, placed in the quadrantal arc FG described about the centre O, are all equal to each other, and represent the body of the moon in the heights there noted, and the unequal moons in the concavity ABC are terminated by the visual rays coming from the circumference of the real moon, at those heights to the eye, at o.



Dr. Smith also observes, that the apparent concave of the sky, being less than a hemisphere, is the cause that the breadths of the colours in the inward and outward rainbows, and the interval between the bows, appear least at the top, and greater at the bottom. This theory of the horizontal moon is also confirmed by the appearances of the tails of comets, which, whatever be their real figure, magnitude, and situation, in absolute space, do always appear to be an arc of the concave of the heavens. Dr. Smith however justly acknowledges that, at different times, the moon appears of very different magnitudes, even in the same horizon, and occasionally of an extraordinary large size; which he is not able to give a satisfactory explanation of. Smith's Optics, vol. 1, pa. 63, &c, Remarks, pa. 53.

The alt. of the sun or moon in degrees.	Apparent diameters or distances.
00	100
15	68
30	50
45	40
60	34
75	31
90	30

MAIGNAN (Emanuel), a religious minim, and one of the greatest philosophers of his age, was born at Thoulouse in 1601. Like the famous Pascal, he became a complete mathematician without the assistance of a teacher; and filled the professor's chair at Rome in 1636, where, at the expense of Cardinal Spada, he published his book De Perspectiva Horaria, in 1648. Maignan returned to Thoulouse in 1650, and was created Provincial. His knowledge in mathematics, and physical experiments, was very early known; especially from a dispute which arose between him and father Kircher, about a catoptrical invention.

The king, who in 1660 amused himself with the machines and curiosities in the father's cell, made him offers by Cardinal Mazarin, to draw him to Paris; but he humbly desired to spend the remainder of his days in a cloister.—He published a Course of Philosophy, in 4 volumes 8vo, at Thoulouse, in 1652; to the second edition of which, in folio, 1673, he added two treatises; the one against the vortices of Descartes, the other on the speaking-trumpet invented by Sir Samuel Morland.—He also formed a machine, which showed, by its movements, that Descartes's supposition concerning the manner in which the universe was formed, or might have been formed, and concerning the centrifugal force, was entirely without foundation.

Thus this great philosopher and divine passed a life of tranquillity, in writing books, making experiments, and reading lectures. He was frequently consulted by the most eminent philosophers on different subjects, which he answered either by writing or otherwise; and no person was certainly more industrious: it is said that he even studied in his sleep; for his very dreams employed him in problems, which he pursued sometimes till he came to a solution or demonstration; and he has frequently been awaked out of his sleep of a sudden, by the exquisite pleasure which he felt on discovery of it. The excellence of his manners, and his unspotted virtues, rendered him no less worthy of esteem, than his genius and learning.—It is said that he composed with great ease, and without making any alterations. He died at Thoulouse in 1676, at 75 years of age.

MALLEABLE, the property of a solid ductile body, from which it may be beaten, forged, and extended under the hammer, without breaking, which is a property of all metals.

MANFREDI (Eustachio), a celebrated atsronomer and mathematician, born at Bologna in 1674. His genius was always above his age: for he was a tolerable poet and wrote ingenious verses while he was but a child: and while very young he formed in his father's house an academy of youth of his own age, who became there the Academy of Sciences, or the Institute. He was professor of mathematics at Bologna in 1698, and superintendant of the waters there in 1704. The same year he was placed at the head of the College of Montalte, founded at Bologna for young men intended for the church: and in 1711 he obtained the office of astronomer to the Institute of Bologna. He became member of the Academy of Sciences of Paris in 1726, and of the Royal Society of London in 1729; and died the 15th of February 1739, at 65 years of age.—His works are:

1. Ephemerides Motuum Cœlestium ab anno 1715 ad annum 1750; 4 volumes in 4to.—The first volume is an excellent introduction to astronomy; and the other three contain numerous calculations. His two sisters greatly assisted him in composing this work.

2. De Transitu Mercurii per Solem, anno 1723. Bologna 1724, in 4to.

3. De Annuis Inerrantium Stellarum Aberrationibus, Bologna 1729, in 4to.—Besides a number of papers in the Memoirs of the Academy of Sciences, and in other places.

MANILIUS (Marcus), a Latin astronomical poet, who lived in the reign of Augustus Cæsar. He wrote an ingenious poem relating to the stars and the sphere, called Astronomicon; which, not being mentioned by any of the ancient poets, was unknown, till about two centuries since, when it was found buried in some German library, and published by Poggius. There is no account to be found of this author, but what can be drawn from his poem; which contains a system of the ancient astronomy and astrology, together with the philosophy of the Stoics. It consists of five books; though there was a sixth, which has not been recovered. In this work, Manilius hints at some opinions, which later ages have been ready to glory in as their own discoveries. Thus he defends the fluidity of the heavens, against the hypothesis of Aristotle; and asserts that the fixed stars are not all in the same concave superficies of the heavens, and equally distant from the centre of the world: he maintains that they are all of the same nature and substance with the sun, and that each of them has a particular vortex of its own: and lastly, he says that the milky way is only the united lustre of a great many small imperceptible stars; which indeed the moderns now see to be such through their telescopes. The best editions of Manilius are that of Joseph Scaliger, in 4to, 1600; that of Bentley, in 4to, 1738, and that of Edmund Burton, esq. in 8vo, 1783.

MANOMETER, or MANOSCOPE, an instrument to show or measure the alterations in the rarity or density of the air.—The manometer differs from the barometer in this, that the latter only serves to measure the Weight of the atmosphere, or of the column of air over it; but the former, the Density of the air in which it is found; which density depends not only on the weight of the atmosphere, but also on the action of heat and cold, &c. Authors however often confound the two together; and Mr. Boyle himself has given a very good manometer of his contrivance, under the name of a Statical Barometer, consisting of a bubble of thin glass, about the size of an orange, which being counterpoised when the air was in a mean state of density, by means of a nice pair of scales, sunk when the atmosphere became lighter, and rose as it grew heavier.

The manometer used by Captain Phipps, in his voyage towards the north pole, consisted of a tube of a small bore, with a ball at the end. The barometer being at 29·7, a small quantity of quicksilver was put into the tube, to take off the communication between the external air, and that confined in the ball and the part of the tube below this quicksilver. A scale is placed on the side of the tube, which marks the degrees of dilatation arising from the increase of heat in this state of the weight of the air, and has the same graduation as that of Fahrenheit's thermometer, the point of freezing being marked 32. In this state therefore it will show the degrees of heat in the same manner as a thermometer. But when the air becomes lighter, the bubble inclosed in the ball, being less compressed, dilates itself, and occupies a space as much larger as the compressing force is less; therefore the changes arising from the increase of heat, are proportionably larger; and the instrument shows the differences in the density of the air, arising from the changes in its weight and heat. Mr. Ramsden found, that a heat equal to that of boiling water, increased the magnitude of the air, from what it was at the freezing point, by $\frac{414}{1000}$ of the whole. Hence it follows, that the ball and the part of the tube below the beginning of the scale, is of a magnitude equal to almost 414 degrees of the scale. If the height of both the manometer and thermometer be given, the height of the barometer may be thence deduced, by this rule:

as the height of the manometer increased by 414,
to the height of the thermometer increased by 414,
so is 29·7, to the height of the barometer;

or if m denote the height of the manometer, and t the height of the thermometer; then

$m + 414 : t + 414 :: 29{\cdot}7 : \frac{t + 414}{m + 414} \times 29{\cdot}7$, which is the height of the barometer.

Another kind of manometer was made use of by Colonel Roy, in his attempts to correct the errors of the barometer; which is described in the Philos. Trans. vol. 67, pa. 689.

MANTELET, a kind of moveable parapet, or screen, of about 6 feet high, set upon trucks or little wheels, and guided by a long pole; so that in a siege it may be driven before the pioneers, and serve as blinds, or screens, to shelter them from the enemy's small shot. Mantelets are made of different materials, so as to render them musket proof; some consisting of strong boards nailed together, and covered with tin; or of thick leather, or of layers of rope, &c, firmly bound together.

There are also other kinds of mantelets, covered on the top, used by the miners in approaching the walls or works of an enemy. The double mantelets form an angle, and stand square, making two fronts. It appears from Vegetius, that mantelets were in use among the ancients, under the name of Vineæ.

MAP, a plane figure representing the surface of the earth, or some part of it on a plane; being a projection of the globular surface of the earth, exhibiting countries, seas, rivers, mountains, cities, &c, in their due positions, or nearly so.

Maps are either universal or partial.

Universal MAPS are such as exhibit the whole surface of the earth, or the two hemispheres.

Particular, or *Partial* MAPS, are those that exhibit some particular region, or part of the earth.

Both kinds are usually called geographical, or land-maps, as distinguished from hydrographical, or sea-maps, which represent only the seas and sea coasts, and are properly called Charts.

Anaximander, the scholar of Thales, it is said, about 400 years before Christ, first invented geographical tables, or maps The Pentingerian Tables, published by Cornelius Pentinger of Augsburg, contain an itinerary of the whole Roman empire; all places, except seas, woods, and deserts, being laid down according to their measured distances, but without any mention of latitude, longitude, or bearing.

The maps published by Ptolemy of Alexandria, about the 144th year of Christ, have meridians and parallels, the better to define and determine the situation of places, and are great improvements on the construction of the more ancient maps. Though Ptolemy himself owns that his maps were copied from some that were made by Marinus, Tirus, &c, with the addition of some improvements of his own. But from his time till about the 14th century, during which geography and most sciences were neglected, no new maps were published. Mercator was the first of any note among the moderns, and next to him Ortelius, who undertook to make a new set of maps, with the modern divisions of countries and names of places; for want of which, those of Ptolemy were become almost useless. After Mercator, many others published maps, but for the most part they were mere copies of his. Towards the middle of the 17th century, Bleau in Holland, and Sanson in France, published new sets of maps, with many improvements from the travellers of those times, which were afterwards copied, with little variation, by the English, French, and Dutch: but the best of these were those of Vischer and Dewitt. And later observations have furnished us with still more accurate and copious sets of maps, by Delisle, Robert, Wells, &c, &c. Concerning maps, see Varenius's Geog, lib. 3, cap. 3, prop. 4; Fournier's Hydrog. lib. 4, cap. 24; Wolfius's Elem. Hydrog. cap. 9; John Newton's Idea of Navigation; Mead's Construction of Globes and Maps; Wright's Construction of Maps, &c, &c.

Construction of MAPS.—Maps are constructed by making a projection of the globe, either on the plane of some particular circle, or by the eye placed in some particular point, according to the rules of perspective, &c; of which there are several methods.

First, to construct a Map of the World, or a general Map.

1st *Method.*—A map of the world must represent two hemispheres; and they must both be drawn upon the plane of that circle which divides the two hemispheres. The first way is to project each hemisphere upon the plane of some particular circle, by the rules of orthographic projection, forming two hemispheres on one common base or circle. When the plane of projection is that of a meridian, the maps will be the east and west hemispheres, the other meridians will be ellipses, and the parallel circles will be right lines. On the plane of the equinoctial, the meridians will be right lines crossing in the centre, which will represent the pole, and the parallels of latitude will be circles having that common centre, and the maps will be the northern and southern hemispheres. The fault of this way of drawing maps, is, that near the outside the circles are too near one another; and therefore equal spaces on the earth are represented by very unequal spaces on the map.

2d *Method.*—Another way is to project the same hemispheres by the rules of stereographic projection; in which way, all the parallels are represented by circles, and the meridians by circles or right lines. And here the contrary fault happens, viz, the circles towards the outsides are too far asunder, and about the middle they are too near together.

3d *Method.*—To remedy the faults of the two former methods, proceed as follows. First, for the east and west hemispheres, describe the circle PENQ for the meridian (pl. 21, fig. 1) or plane of projection; through the centre of which draw the equinoctial EQ, and axis PN perpendicular to it, making P and N the north and south pole. Divide the quadrants PE, EN, NQ, and QP into 9 equal parts, each representing 10 degrees, beginning at the equinoctial EQ: divide also CP and CN into 9 equal parts; beginning at EQ; and through the corresponding points draw the parallels of latitude. Again, divide CE and CQ into 9 equal parts; and through the points of division, and the two poles P and N, draw circles, or rather ellipses, for the meridians. So shall the map be prepared to receive the several places and countries of the earth.

Secondly, For the north or south hemisphere, draw AQBE, for the equinoctial (fig. 2), dividing it into the four quadrants EA, AQ, QB, and BE; and each quadrant into 9 equal parts, representing each 10 degrees of longitude; and then, from the points of division, draw lines to the centre C, for the circles of longitude. Divide any circle of longitude, as the first meridian EC, into 9 equal parts, and through these points describe circles from the centre C, for the parallels of latitude, numbering them as in the figure.

In this 3d method, equal spaces on the earth are represented by equal spaces on the map, as near as any projection will bear; for a spherical surface can no way be represented exactly on a plane. Then the several countries of the world, seas, islands, sea-coasts, towns, &c, are to be entered in the map, according to their latitudes and longitudes.

In filling up the map, all places representing land are filled with such things as the countries contain; but the seas are left blank; the shores adjoining to the sea being shaded. Rivers are marked by strong lines, or by double lines, drawn winding in form of the rivers they represent; and small rivers are expressed by small lines. Different countries are best distinguished by different colours, or at least the borders of them. Forests are represented by trees; and mountains shaded to make them appear as such. Sands are denoted by small points or specks; and rocks under water by a small cross. In any void space, draw the mariner's compass, with the 32 points or winds.

II. *To draw a Map of any particular Country.*

1st *Method.*—For this purpose its extent must be known, as to latitude and longitude; as suppose Spain, lying between the north latitudes 36 and 44, and extending from 10 to 23 degrees of longitude; so that its extent from north to south is 8 degrees, and from east to west 13 degrees.

Draw the line AB for a meridian passing through the middle of the country (fig. 3), on which set off 8 degrees from B to A, taken from any convenient scale; A being the north, and B the south point. Through A and B draw the perpendiculars CD, EF, for the extreme parallels of latitude. Divide AB into 8 parts, or degrees, through which draw the other parallels of latitude, parallel to the former.

For the meridians; divide any degree in AB into 60 equal parts, or geographical miles. Then, because the length of a degree in each parallel decreases towards the pole, from the table showing this decrease, under the article DEGREE, take the number of miles answering to the latitude of B, which is $48\frac{1}{2}$ nearly, and set it from B, 7 times to E, and 6 times to F; so is EF divided into degrees. Again, from the same table take the number of miles of a degree in the latitude A, viz $43\frac{1}{6}$ nearly; which set off, from A, 7 times to C, and 6 times to D. Then from the points of division in the line CD, to the corresponding points in the line EF, draw so many right lines, for the meridians. Number the degrees of latitude up both sides of the map, and the degrees of longitude on the top and bottom. Also, in some vacant place make a scale of miles; or of degrees, if the map represent a large part of the earth; to serve for finding the distances of places from each other.

Then make the proper divisions and subdivisions of the country: and knowing the latitudes and longitudes of the principal places, it will be easy to set them down in the map; for, any town, &c, must be placed where the circles of its latitude and longitude intersect. For instance, Gibraltar, whose latitude is 36° 11', and longitude 12° 27', will be at G: and Madrid, whose lat. is 40° 10', and long. 14° 44', will be at M. In like manner the mouth of a river must be set down; but to describe the whole river, the latitude and longitude of every turning must be marked down, and the towns and bridges by which it passes. And so for woods, forests, mountains, lakes, castles, &c. The boundaries will be described, by setting down the remarkable places on the sea-coast, and drawing a continued line through them all: which method is very proper for small countries.

2d *Method.*—Maps of particular places are but portions of the globe, and therefore may be drawn after the same manner as the whole is drawn: that is, such a map may be drawn either by the orthographic or stereographic projection of the sphere, as in the last prob. But in partial maps, an easier way is as follows. Having drawn the meridian AB (fig. 3), and divided it into equal parts as in the last method, through all the points of division draw lines perpendicular to AB, for the parallels of latitude; CD, EF being the extreme parallels. Then to divide these, set off the degrees in each parallel, diminished after the manner directed for the two extreme parallels CD, EF, in the last method: and through all the corresponding points draw the meridians, which will be curve lines; which were right lines in the last method; because only the extreme parallels were divided by the table. This method is proper for a large tract, as Europe, &c: in which case the parallels and meridians need only be drawn to every 5 or 10 degrees: and it is also much used in drawing maps; as all the parts are nearly of their due magnitude, but a little distorted towards the outside, from the oblique intersections of the meridians and parallels.

3d *Method.*—Draw PB of a convenient length, for a meridian; divide it into 9 equal parts, and through the points of division, describe as many circles for the parallels of latitude, from the centre P, which represents the pole. Suppose AB (fig. 4) the height of the map; then CD will be the parallel passing through the greatest latitude, and EF will represent the equator. Divide the equator EF into equal parts, of the same dimension as those in AB, both ways, beginning at B. Divide also all the parallels into the same number of equal parts, but lesser, in proportion to the numbers for the several latitudes, as directed in the last method for the rectilineal parallels. Then through all the corresponding divisions, draw curve lines, to represent the meridians, the extreme ones being EC and FD. Lastly, number the degrees of latitude and longitude, and place a scale of equal parts, either of miles or degrees, for measuring distances.—This is a very good way of drawing large maps, and is called the globular projection; all the parts of the earth being represented nearly of their due magnitude, excepting that they are a little distorted towards the outsides.

When the place which the map is to represent, is but small, as if a county was to be exhibited; the meridians, as to sense, will be parallel to one another, and the whole will differ very little from a plane. Such a map will be made more easily than by the preceding rules. It will here be sufficient to measure the distances of places in miles, and so lay them down in a plane rectangular map. But this belongs more properly to surveying.

The Use of MAPS is obvious from their construction. The degrees of the meridians and parallels show the latitudes and longitudes of places, and the scale of miles annexed, their distances; the situation of places, with regard to each other, as well as to the cardinal points, appears by inspection; the top of the map being always the north, the bottom the south, the right hand the east, and the left hand the west; unless the compass, usually annexed, show the contrary.

MARALDI (JAMES PHILIP), a learned astronomer and mathematician, was born in 1665, at Perinaldo in the county of Nice, a place already honoured by the birth of his maternal uncle the celebrated Cassini. Having made a considerable progress in mathematics, at the age of 22, his uncle who had been a long time settled in France, invited him there, that he might himself cultivate the promising genius of his nephew. Maraldi no sooner applied himself to the contemplation of the heavens, than he conceived the design of forming a catalogue of the fixed stars, the foundation of all the astronomical edifice. In consequence of this design, he applied himself to observe them with the most constant attention; and he became by this means so intimate with them, that on being shown any one of them, however small, he could immediately tell what constellation it belonged to, and its place in that constellation. He has been known to discover those small comets, which astronomers often take for the stars of the constellation in which they are seen, for want of knowing precisely what stars the constellation consists of, when others, on the spot, and with eyes directed equally to the same part of the heavens, could not for a long time see any thing of them.

In 1700 he was employed under Cassini in prolonging the French meridian to the northern extremity of France, and had no small share in completing it. He then set out for Italy, where Clement the 11th invited him to assist at the assemblies of the Congregation then sitting in Rome to reform the calendar. Bianchini also availed himself of his assistance to construct the great meridian of the Carthusian church in that city. And in 1718 Maraldi, with three other academicians, prolonged the French meridian to the southern extremity of that country. He was admitted a member of the Academy of Sciences of Paris in 1699, in the department of astronomy, and communicated to it a great multitude of papers, which are printed in their Memoirs, in almost every year from 1699 to 1729, and usually several papers in each of the years; for he was indefatigable in his observations of every thing that was curious and useful in the motions and phenomena of the heavenly bodies. As to the catalogue of the fixed stars, it was not quite completed by him: for just as he had placed a mural quadrant on the terrace of the observatory, to observe some stars towards the north and the zenith, he fell sick, and died the 1st of December 1729.

MARCH, *Martius*, the 3d month of the year, according to the common way of computing, and consists of 31 days. The sun enters the sign Aries about the 20th or 21st day of this month.—Among the Romans, March was the first month; and in some ecclesiastical computations, that order is still preserved. In England, before the alteration of the stile, March was the 1st month in order, the year always commencing with the 25th day of the month. It has been said that it was Romulus who first divided the year into months; to the first of which he gave the name of his supposed father Mars. It is observed by Ovid, however, that the people of Italy had the month of March before the time of Romulus; but that they placed it differently; some making it the third, some the 4th, some the 5th, and others the 10th month of the year.

MARINE BAROMETER, See BAROMETER.

MARINERS-COMPASS. See COMPASS.

MARINUS. See PROCLUS.

MARIOTTE (EDME), an eminent French philosopher and mathematician, was born at Dijon, and admitted a member of the Academy of Sciences of Paris in 1666. His works however are better known than his life. He was a good mathematician, and the first French philosopher who applied much to experimental physics. The law of the shock or collision of bodies, the theory of the pressure and motion of fluids, the nature of vision, and of the air, particularly engaged his attention. He carried into his philosophical researches, that spirit of scrutiny and investigation so necessary to those who would make any considerable progress in improvement. He died in 1684.—He communicated a number of curious and valuable papers to the Academy of Sciences, which were printed in the collection of their Memoirs dated 1666, viz, from volume 1 to volume 10. And all his works were collected into 2 volumes in 4to, and printed at Leyden in 1717.

MARS, one of the ancient seven primary planets, and the first of the superior ones, being placed immediately next above the earth. It is usually denoted by this character ♂, being a mark rudely formed from a man holding a spear protruded, representing the god of war of the same name.

The mean distance of Mars from the sun, is 1524 of those parts, of which the distance of the earth from the sun is 1000; his excentricity 142; and his real distance 145 millions of miles. The inclination of his orbit to the plane of the ecliptic, is 1° 52′; the length of his year, or the period of one revolution about the sun, is $686\frac{23}{24}$ of our days, or $667\frac{1}{4}$ of his own days, which are 40 minutes longer than ours, the revolution on his axis being performed in 24 hours 40 minutes. His mean diameter is 4444 miles; and the same seen from the sun is 11″: the inclination of the axis to his orbit 0° 0′; place of the aphelion ♍ 2° 24′; place of his ascending node ♉ 17° 3′; and his parallax, according to Dr. Hooke and Mr. Flamsteed, is scarce 30 seconds.

Dr. Hooke, in 1665, observed several spots in Mars; which having a motion, he concluded the planet turned round its centre. In 1666, M. Cassini observed several spots in the two faces or hemispheres of Mars, which he found made one revolution in 24 hours 40 minutes. These observations were repeated in 1670, and confirmed by Maraldi in 1704 and 1719: whence both the motion and period, or natural day, of that planet, were determined.

In the Philos. Trans. for 1781, Dr. Herschel gave a series of observations on the rotation of this planet about its axis, from which he concluded that one mean sidereal rotation was between 24 h. 39 m. 5 sec. and 24 h. 39 m. 22 sec.; and in the Philos. Trans. for 1784, is given a paper by the same gentleman, on the remarkable appearances at the polar regions of the planet Mars, the inclination of its axis, the position of its poles, and its spheroidical figure; with a few hints relating to its real diameter and atmosphere, deduced from his observations taken from the year 1777 to 1783 inclusively. He also observed several remarkable bright spots near both poles, which had a small motion; and the results of his observations are as follow; viz,

"Inclination of axis to the ecliptic, 59° 22′.

The node of the axis is in ♓ 17° 47′.

Obliquity of the planet's ecliptic 28° 42′.

The point Aries on Mars's ecliptic answers to our ♐ 19° 28′.—The figure of Mars is that of an oblate spheroid, whose equatorial diameter is to the polar one, as 1355 to 1272, or as 16 to 15 nearly.—The equatorial diameter of Mars, reduced to the mean distance of the earth from the sun, is 9″ 8‴—And the planet has a considerable, but moderate atmosphere, so that its inhabitants probably enjoy a situation in many respects similar to ours."—Mars always appears with a ruddy troubled light; owing, it is supposed, to the nature of his atmosphere, through which the light passes.—In the acronical rising of this planet, or when in opposition to the sun, it is 5 times nearer to us than when in conjunction with him; and therefore appears much larger and brighter than at other times.—Mars, having his light from the sun, and revolving round it, has an increase and decrease like the moon: it may also be observed almost bisected, when in the quadratures, or in perigæon; but is never seen cornicular, as the inferior planets.

MARTIN (Benjamin), was born in 1704, and became one of the most celebrated mathematicians and opticians of his time. He first taught a school in the country; but afterwards came up to London, where he read lectures on experimental philosophy for many years, and carried on a very extensive trade as an optician and globemaker in Fleet-street, till the growing infirmities of old age compelled him to withdraw from the active part of business. Trusting too fatally to what he thought the integrity of others, he unfortunately, though with a capital more than sufficient to pay all his debts, became a bankrupt. The unhappy old man, in a moment of desperation from this unexpected stroke, attempted to destroy himself; and the wound, though not immediately mortal, hastened his death, which happened the 9th of February 1782, at 78 years of age.

Mr. M. had a valuable collection of fossils and curiosities of almost every species; which after his death were almost given away by public auction. He was indefatigable as an artist, and as a writer he had a very happy method of explaining his subject, and wrote with clearness, and even considerable elegance. He was chiefly eminent in the science of optics; but he was well skilled in the whole circle of the mathematical and philosophical sciences, and wrote useful books on every one of them; though he was not distinguished by any remarkable inventions or discoveries of his own. His publications were very numerous, and generally useful: some of the principal of them were as follow:

The Philosophical Grammar; being a View of the present State of Experimental Physiology, or Natural Philosophy, 1735, 8vo.—A new, complete, and universal System or Body of Decimal Arithmetic, 1735, 8vo.—The Young Student's Memorial Book, or Pocket Library, 1735, 8vo.—Description and Use of both the Globes, the Armillary Sphere and Orrery, Trigonometry, 1736, 2 vols. 8vo.—System of the Newtonian Philosophy, 1759, 3 vols.—New Elements of Optics, 1759—Mathematical Institutions, 1764, 2 vols.—Philologic and Philosophical Geography, 1759.—Lives of Philosophers, their Inventions, &c. 1764.—Young Gentleman and Lady's Philosophy, 1764, 3 vols.—Miscellaneous Correspondence, 1764, 4 vols.—Institutions of Astronomical Calculations, 3 parts, 1765.—Introduction to the Newtonian Philosophy, 1765.—Treatise of Logarithms.—Treatise on Navigation.—Description and Use of the Air-pump.—Description of the Torricellian Barometer.—Appendix to the Use of the Globes.—Philosophia Britannica, 3 vols.—Principles of Pump-work.—Theory of the Hydrometer.—Description and Use of a Case of Mathematical Instruments.—Ditto of a Universal Sliding Rule.—Micrographia, on the Microscope.—Principles of Perspective.—Course of Lectures.—Optical Essays.—Essay on Electricity.—Essay on Visual Glasses or Spectacles.—Horologia Nova, or New Art of Dialling.—Theory of Comets.—Nature and Construction of Solar Eclipses.—Venus in the Sun.—The Mariner's Mirror.—Thermometrum Magnum—Survey of the Solar System.—Essay on Island Chrystal.—Logarithmologia Nova, &c. &c.

MASKELYNE (NEVIL), D. D. F. R. S. Astronomer Royal, &c, was born in London, on the 6th of October 1732, of an ancient family, which had been long established in the west of England. At 9 years of age he was placed at Westminster school, where his diligence speedily distinguished him. He acquired an early taste for astronomy and optics; but it was the solar eclipse of 1748 which decided his vocation. Perceiving how necessary the mathematics were, in the course he proposed to take, he determined on the study of them, and acquired in a few months the elements of geometry and algebra. This first success was the earnest of what he could not avoid obtaining, by reading the chief works on astronomy and the higher analysis, which he habitually studied. About this time he went to Cambridge, and entered in Catharine-hall, but afterwards in Trinity-college, where he received, with applause, the degree of bachelor of arts.

In 1755, he accepted of a curacy in the vicinity of London, where he resided some years, employing his leisure time in his favourite study. This situation also facilitated his acquaintance with the then astronomer-royal Bradley, for whom it appears that he made some calculations of importance. In 1758, he became fellow of Trinity-college, Cambridge, and the next year a fellow of the Royal Society.

But it was in the year 1761 that his real astronomical career began, when he was chosen to go to the island of St. Helena, to observe the transit of Venus over the sun's disk, and the parallax of the star Sirius, which had often been observed by Lacaille at the Cape of Good Hope. From calculating these observations, Dr. M. thought he saw proofs for the existence of a parallax of 4″½.

Clouds prevented the observation of the transit of Venus, the first object of the voyage. But being furnished with an excellent pendulum clock of Shelton's, which had been regulated at Greenwich by Dr. Bradley, he determined the number of oscillations which it made less at St. Helena than at London, in order thence to deduce the diminution of gravity.

The secondary object of the voyage, the parallax of Sirius also failed, through the fault of the suspension of the plumb-line, by a loop from the neck of a central pin; which had likewise been the fault of Lacaille's instrument. This disappointment gave occasion to an improvement in the construction of these astronomical instruments. Several other observations however in part indemnified Dr. M. for those disappointments; such as the observation of the tides at St. Helena, the variation of the compass, and the moon's horary parallaxes, &c. Also, in going out and returning home, he practised the method of finding the longitude by the lunar distances taken with a Hadley's quadrant, making out rules for the use of the seamen, and taught the method to the officers on board the ship. The same he afterwards explained in a letter to Dr. Birch, the secretary to the Royal Society, which was inserted in the Philos. Trans. vol. 52, for the year 1762; and still more fully in the British Mariner's Guide, which he published soon after his return from St. Helena, and which contained, among various new and practical articles in nautical astronomy, rules and examples for working the lunar observations.

In 1763, Dr. Maskelyne went to the island of Barbadoes, to settle the longitude of the place, and compare Mr. Harrison's watch with the time there, when he should arrive at the island with it. In this voyage also, Dr. M. tried observations on board of ship with Irwin's marine chair, which was found not to answer the purpose. Dr. M. made also several other astronomical observations, and among the rest, many relating to the moon's horary parallaxes. See Astronomical Observations at St. Helena and Barbadoes, in the Philos. Trans. vol. 54. Dr. M. returned from Barbadoes in the autumn of 1764, and made the report on Mr. H's watch, which, though favourable in general to the celebrated artist, was far from satisfying Mr. H. who attacked him in a pamphlet, to which Dr. M. wrote a reply.

In 1765, Dr. M. succeeded Mr. Bliss, as astronomer-royal at Greenwich Observatory, where for 47 years he diligently watched the heavens, and rendered innumerable benefits to the nation, as well as to individuals, in all the arts and sciences connected with astronomy and navigation. Immediately on his appointment to that office, he recommended the lunar method of finding the longitude to the Board of Longitude, and proposed to them to cause a nautical almanac to be calculated, and published, to facilitate the method; which they agreed to; the first of which was published for 1767, and which was continued under his direction, with the greatest credit, through 48 successive years. He also published a useful collection of tables requisite to be used with the nautical almanacs; as well as edited or encouraged the publication of other works, of like accessory usefulness; as, Taylor's Logarithms, the improved lunar tables of Mayer, and Mason, &c, &c. He procured also, at the expense of the Royal Society, the regular publication of all his own astronomical observations, made at the Observatory, forming a vast body of valuable matter, in 4 large folio volumes.

Dr. M. thus continued indefatigable in making observations for 47 years, hardly ever quitting the Observatory, except once a-week, in attending the meetings of the Royal Society one part of the year. In 1769, he remained in it to observe the transit of Venus, and he drew up instructions for the astronomers sent out by Great Britain to different countries. He collected their observations, and deduced from them the sun's parallax, and his distance from the earth. At his observatory, he made many of the most interesting and most difficult observations himself, as those of the moon; but necessarily confided to his assistant, those which were less essential and more easy. He followed closely the methods established by his celebrated predecessor Bradley, whom he even surpassed in the exactness of his daily observations. He brought to perfection Flamsteed's method of determining, at once, the right ascensions of stars and of the sun. He gave a catalogue of stars, not numerous indeed, but determined with particular care, which has served almost solely, during these thirty years, for the foundation of all astronomical researches.

Dr. M. did not publish much himself, being otherwise more usefully employed on his observations: but he was the cause and promoter of many publications by others. He corresponded with almost all the astronomers and philosophers in the world; and he was the medium of many of their communications to the Royal Society. The writings he produced, are remarkable for just ideas and an enlightened criticism. Such is a Dissertation on the

Equation of Time, where he has delicately noticed a mistake of Lacaille, and another less important mistake of Lalande. Some doubts having been raised, respecting the difference in latitude and longitude between the observatories of Greenwich and Paris, Dr. M., to whom the observations were sent, showed, with his usual moderation, that the doubts were improper; but he did not oppose the methods proposed to obviate them.

It was owing to the exertions of Dr. M., that a very satisfactory experiment was made to ascertain the general attraction of matter, and the medium density of all the matter in the earth. By a memoir presented to the Royal Society, he recommended it to that body, to try the experiment on the attraction of some hill in the British dominions. A convenient one having been found, viz, the Mountain Schihallien in Scotland, at the request of the Society, Dr. M. himself repaired to the place, and superintended the necessary measurements and observations, with his usual attention and correctness. His survey furnished the just plan and numerous sections of the hill, and his zenith sector showed $5''\cdot8$ for the mean deviation of the plumb-line by the attraction. From these materials it was, that the laborious calculations of Dr. Hutton showed, for the first time, that the mean density of the whole earth was about 5 times the density of water, a determination most likely very near the truth.

Dr. M. was particularly attentive to the care of his instruments, and made many improvements in them, and the modes of employing them. He greatly improved the suspension of the plumb-line of the zenith sectors. He contrived a micrometer composed of a prism, which moves according to the axis of the telescope. He made the eye-piece moveable, in order to avoid all parallax in bringing the eye opposite to each of the five wires, which the star passes in succession. He discovered also the inconvenience of narrow openings, used in all observatories: he enlarged the size of those at Greenwich, after having shown the necessity of placing the telescopes as much as possible in the open air: besides many more optical and mechanical improvements.

Dr. M. had good church preferment from his college; and his paternal estates, of which he was the last male heir, were also considerable. Having experienced a gradual decline of his health for some months, he at length expired on the 9th of Februay 1811, in the 79th year of his age; leaving a widow and daughter, as also his sister, the relict of Robert, late Lord Clive.

The principal works which Dr. M. left, besides his 4 vols. in folio of observations, the memoirs before noticed, and the first 48 volumes of the Nautical Almanac, calculated under his direction, and revised by him, are, his British Mariner's Guide; the Tables requisite for the Usage of the Nautical Almanac; Dissertations on Nautical Astronomy and the Use of the Octant; with at least 30 learned memoirs presented to the Royal Society, and printed in the Philos. Transactions, between the years 1762 and 1794; and finally, his posthumous works, of the contents of which we are as yet ignorant, but which astronomers will be very anxious to receive from the hands of Professor Vince, to whose care it seems they have been left. Indeed it would be well becoming the respect of his relict, to cause a collected and uniform edition of all his works to be made, for the honour of his memory and the greater convenience of the scientific public.

Thus we have described the philosopher; but the man, the father, the friend, was not less valuable. Every astronomer, every philosopher, found in him a brother. Of a character friendly and amiable, he gained the affections of all those who had the good fortune to know him, and his death was honoured with their regret.

MASS.—The quantity of matter in any body. This is rightly estimated by its weight; whatever be its figure, or whether its bulk or magnitude be large or small.

MATERIAL, relating to Matter.

MATHEMATICAL, relating to Mathematics.

MATHEMATICAL *Sect*, is one of the two leading philosophical sects, which arose about the beginning of the 17th century; the other being the metaphysical sect. The former directed its researches by the principles of Gassendi, and sought after truth by observation and experience. The disciples of this sect denied the possibility of erecting on the basis of metaphysical and abstract truths, a regular and solid system of philosophy, without the aid of assiduous observation and repeated experiments, which are the most natural and effectual means of philosophical progress and improvement. The advancement and reputation of this sect, and of natural knowledge in general, were much owing to the plan of philosophizing proposed by Lord Bacon, to the establishment of the Royal Society in London, to the genius and industry of Mr. Boyle, and to the unparalleled researches and discoveries of Sir Isaac Newton. Barrow, Wallis, Locke, and many other great luminaries in learning also adorned this sect.

MATHEMATICS, the science of quantity; or a science that considers magnitudes either as computable or measurable. The word in its original, *μαθησίς*, *mathesis*, signifies Discipline or Science in general; and, it seems, has been applied to the doctrine of quantity, either by way of eminence, or because, this being the first of all other sciences, the rest took their common name from it. As to the origin of the mathematics, Josephus dates it before the flood, and makes the sons of Seth observers of the course and order of the heavenly bodies: he adds, that to perpetuate their discoveries, and secure them from the injuries either of a deluge or a conflagration, they had them engraven on two pillars, the one of stone, the other of brick; the former of which, he says, was yet standing in Syria in his time.

Indeed it is pretty generally agreed that the first cultivators of mathematics, after the flood, were the Assyrians and Chaldeans; from whom, Josephus adds, the science was carried by Abraham to the Egyptians; who became so celebrated for their knowledge, that Aristotle even fixes the first rise of mathematics among them. From Egypt, 584 years before Christ, mathematics passed into Greece, being carried thither by Thales; who having learned geometry of the Egyptian priests, taught it in his own country. After Thales, came Pythagoras; who, among other mathematical arts, paid a particular regard to arithmetic; drawing the greatest part of his philosophy from numbers. He was the first, according to Laertius, who abstracted geometry from matter; and to him we owe the doctrine of incommensurable magnitude, and the five regular bodies, besides the first principles of music and astronomy. To Pythagoras succeeded Anaxagoras, Oenopides, Briso, Antipho, and Hippocrates of Scio; all of whom particularly applied themselves to the quadrature of the circle, the duplication of the cube, &c; but the efforts of the latter were the most successful: he is also mentioned by Proclus, as the first who compiled elements of mathematics.

Democritus excelled in mathematics as well as physics; though none of his works in either kind are extant; the destruction of which is by some authors ascribed to Aristotle. The next in order is Plato, who not only improved geometry, but introduced it into physics, and so laid the foundation of a solid philosophy. From his school arose a number of mathematicians. Proclus mentions 13 of note; among whom was Leodamus, who improved the analysis first invented by Plato; Theætetus, who wrote Elements; and Archytas, who has the credit of being the first that applied mathematics to the useful purposes of life. These were succeeded by Neocles and Theon, the last of whom contributed to the elements. Eudoxus excelled in arithmetic and geometry, and was the first founder of a system of astronomy. Menechmus invented the conic sections, and Theudius and Hermotimus improved the elements.

For Aristotle, his works are so stored with mathematics, that Blancanus compiled out of them an entire book on mathematics. Eudemus and Theophrastus were of this school; the first of whom wrote upon numbers, geometry, and invisible lines; and the latter composed a mathematical history. To Aristeus, Isidorus, and Hypsicles, we owe the books of Solids; which, with the other books of Elements, were improved, collected, and methodised by Euclid, who died 284 years before the birth of Christ.

A hundred years after Euclid, Eratosthenes and Archimedes became celebrated for their extensive knowledge, particularly the latter, who was contemporary with Conon, a geometrician and astronomer. Soon after which flourished Apollonius Pergæus, whose excellent treatise on conics is still extant. To him are also ascribed the 14th and 15th books of Euclid, and which, it is said, were contracted by Hypsicles. Hipparchus and Menelaus wrote on the subtenses of the arcs in a circle; and the latter also on spherical triangles. Theodosius's 3 books of Spherics are still extant. All these authors, Menelaus excepted, lived before Christ.

Ptolemy of Alexandria, a celebrated geometrician, and the prince of astronomers, was born about 70 years after Christ. To him succeeded the philosopher Plutarch, some of whose mathematical problems are still extant. After him, in the order of time, was Eutocius, who commented on Archimedes, and occasionally mentions the inventions of Philo, Diocles, Nicomedes, Sporus, and Heron, on the duplication of the cube. To Ctesebes of Alexandria we are indebted for pumps; and Geminus, who lived soon after, is preferred by Proclus to Euclid himself.

Diophantus of Alexandria was a great master of numbers, and the first Greek writer on algebra that we know of. Among others of the ancients, Nicomachus is celebrated for his arithmetical, geometrical, and musical works: Serenus, for his books on the section of the cylinder: Proclus, for his commentaries on Euclid; and Theon, who has been said to be the author of the books of Elements ascribed to Euclid. The last to be named among the ancients, is Pappus of Alexandria, who flourished about the year of Christ 400, and is justly celebrated for his books of Mathematical Collections, still extant.

Mathematics are commonly distinguished into speculative and practical, pure and mixed.

Speculative MATHEMATICS, is that which barely contemplates the properties of things: and

Practical MATHEMATICS, that which applies the knowledge of those properties to some useful purposes.

Pure MATHEMATICS is that branch which considers quantity abstractedly, and without any relation to matter or bodies.

Mixed MATHEMATICS considers quantity as subsisting in material beings; for instance, length in a pole, depth in a river, height in a tower, &c.

Pure Mathematics, again, either considers quantity as discrete, and so computable, as arithmetic; or as concrete, and so measurable, as geometry.

Mixed Mathematics are very extensive, and are distinguished by various names, according to the different subjects it considers, and the different views in which it is taken; such as astronomy, geography, optics, hydrostatics, navigation, &c, &c.

Pure mathematics has one peculiar advantage, that it occasions no contests among wrangling disputants, as is the case in other branches of knowledge: and the reason is, because the definitions of the terms are premised, and every person that reads a proposition has the same idea of every part of it. Hence it is easy to put an end to all mathematical controversies, by showing, either that our adversary is not constant with his definitions, or has not established the true premises, or that he has drawn false conclusions from true principles; and in case we are not able to do either of these, we must acknowledge the truth of what he has proved. It is true, that in mixed mathematics, where we reason mathematically upon physical subjects, such just definitions cannot be given as in geometry: we must therefore be content with descriptions; which will be of the same use as definitions, provided we be consistent with ourselves, and always mean the same thing by those terms we have once explained.

Dr. Barrow gives a very elegant description of the excellence and usefulness of mathematical knowledge, in his inaugural oration, on being appointed professor of mathematics at Cambridge. The mathematics, he observes, effectually exercise, not vainly delude, nor vexatiously torment studious minds with obscure subtleties, but plainly demonstrate every thing within their reach, draw certain conclusions, instruct by profitable rules, and unfold pleasant questions. These disciplines likewise enure and corroborate the mind to a constant diligence in study; they wholly deliver us from a credulous simplicity, most strongly fortify us against the vanity of scepticism, effectually restrain us from a rash presumption, most easily incline us to a due assent, and perfectly subject us to the government of right reason. While the mind is abstracted and elevated from sensible matter, distinctly views pure forms, conceives the beauty of ideas, and investigates the harmony of proportions; the manners themselves are sensibly corrected and improved, the affections composed and rectified, the fancy calmed and settled, and the understanding raised and excited to more divine contemplations.

For the history of mathematics, consult Wallis, Montucla, Kæstner, Bossut, Bailey, &c, and the names of its several branches in this Dictionary.

MATTER, an extended substance. Other properties of matter are, that it resists, is solid, divisible, moveable, passive, &c: and it forms the principles of which all bodies are composed. Matter and form, the two simple and original principles of all things, according to the ancients, composing some simple natures, which they called Elements; from the various combinations of which all natural things were afterwards composed.

Dr. Woodward was of opinion, that matter is originally and really various, being at its first creation divided into several ranks, sets, or kinds of corpuscles, differing in substance, gravity, hardness, flexibility, figure, size, &c; and from the various compositions and combinations of which, he thinks, arise all the varieties in bodies as to colour, hardness, gravity, tastes, &c. But it is Sir Isaac Newton's opinion, that all those differences result from the various arrangements of the same matter; which he accounts homogeneous and uniform in all bodies.

The quantity of matter in any body, is its measure arising from the joint consideration of the magnitude and density of the body: as, if one body be twice as dense as another, and also occupy twice the space, then will it contain 4 times the matter of the other. This quantity of matter is best discovered by the weight or gravity of the body, to which it is always proportional.

Newton observes, that "it seems probable, God, in the beginning, formed matter in solid, massy, hard, impenetrable, moveable particles, of such sizes, figures, and with such other properties, and in such proportion to space, as most conduced to the end for which he formed them; and that these primitive particles, being solid, are incomparably harder than any porous bodies compounded of them; even so very hard, as never to wear, and break in pieces: no ordinary power being able to divide what God himself made one in the first creation. While the particles continue entire, they may compose bodies of one and the same nature and texture in all ages; but should they wear away, or break in pieces, the nature of things depending on them would be changed. Water and earth, composed of old worn particles, would not be of the same nature and texture now with water and earth composed of entire particles in the beginning. And therefore, that nature may be lasting, the changes of corporeal things are to be placed only in the various separations and new associations and motions of these permanent particles; compound bodies being apt to break, not in the midst of solid particles, but where those particles are laid together, and touch in a few points. It seems farther," he continues, "that these particles have not only a vis inertiæ, accompanied with such passive laws of motion as naturally result from that force, but also that they are moved by certain active principles, such as is that of gravity, and that which causeth fermentation, and the cohesion of bodies. These principles are to be considered not as occult qualities, supposed to result from the specific forms of things, but as general laws of nature, by which the things themselves are formed; their truth appearing to us by phenomena, though their causes are not yet discovered."

Hobbes, Spinoza, &c, maintain that all the beings in the universe are material, and that their differences arise from their different modifications, motions, &c. Thus they conceive that matter extremely subtile, and in a brisk motion, may think, &c. Dr. Berkeley, on the contrary, argues against the existence of matter itself; and endeavours to prove that it is a mere ens rationis, and has no existence out of the mind.

Some late philosophers have advanced a new hypothesis concerning the nature and essential properties of matter. The first of these who suggested, or at least published an account of this hypothesis, was M. Boscovich, in his Theoria Philosophiæ Naturalis. He supposes that matter is not impenetrable, but that it consists of physical points only, endued with powers of attraction and repulsion, taking place at different distances, that is, surrounded with various spheres of attraction and repulsion; in the same manner as solid matter is generally supposed to be. Provided therefore that any body move with a sufficient degree of velocity, or have sufficient momentum to overcome any power of repulsion that it may meet with, it will find no difficulty in making its way through any body whatever. If the velocity of such a body in motion be sufficiently great, Boscovich contends, that the particles of any body through which it passes, will not even be moved out of their place by it. With a degree of velocity something less than this, they will be considerably agitated, and ignition might perhaps be the consequence, though the progress of the body in motion would not be sensibly interrupted; and with a still less momentum it might not pass at all.

Mr. Michell, Dr. Priestley, and some others of our own country, are of the same opinion. See Priestley's History of Discoveries relating to Light, pa. 390.—In conformity to the above hypothesis, our author maintains, that matter is not that inert substance that it has been supposed to be; that powers of attraction or repulsion are necessary to its very being, and that no part of it appears to be impenetrable to other parts. Accordingly, he defines matter to be a substance, possessed of the property of extension, and of powers of attraction or repulsion, which are not distinct from matter, and foreign to it, as it has been generally imagined, but absolutely essential to its very nature and being; so that when bodies are divested of these powers, they become nothing at all. In another place, Dr. Priestley has given a somewhat different account of matter; according to which it is only a number of centres of attraction and repulsion; or more properly of centres, not divisible, to which divine agency is directed; and as sensation and thought are not incompatible with these powers, solidity, or impenetrability, and consequently a vis inertiæ only having been thought repugnant to them, he maintains, that we have no reason to suppose that there are in man two substances absolutely distinct from each other. See Disquisitions on Matter and Spirit.

But Dr. Price, in a correspondence with Dr. Priestley, published under the title of A Free Discussion of the Doctrines of Materialism and Philosophical Necessity, 1778, has suggested a variety of strong objections against this hypothesis of the penetrability of matter, and against the conclusions that are drawn from it. The vis inertia of matter, he says, is the foundation of all that is demonstrated by natural philosophers concerning the laws of the collision of bodies. This, in particular, is the foundation of Newton's philosophy, and especially of his three laws of motion. Solid matter has the power of acting on other matter by impulse; but unsolid matter cannot act at all by impulse; and this is the only way in which it is capable of acting, by any action that is properly its own. If it be said, that one particle of matter can act upon another without contact and impulse, or that matter can, by its own proper agency, attract or repel other matter which is at a distance from it, then a maxim hitherto universally received must be false, that "nothing can act where it is not." Newton, in his letters to Bentley, calls the notion, that matter possesses an innate power of attraction, or that it can act upon matter at a distance, and attract and repel by its own agency, an absurdity into which he thought no one could possibly fall. And in another place he expressly disclaims the notion of innate gravity, and has taken pains to show that he did not take it to be an essential property of bodies: and by pursuing the same kind of reasoning, it must appear, that matter has not the power of attracting and repelling; that this power is the power of some foreign cause, acting upon matter according to stated laws; and consequently that attraction and repulsion, not being actions, much less inherent qualities of matter, as such, it ought not to be defined by them. And if matter has no other property, as Dr. Priestley asserts, than the power of attracting and repelling, it must be a non-entity; because this is a property that cannot belong to it. Besides, all power is the power of something; and yet if matter is nothing but this power, it must be the power of nothing; and the very idea of it is a contradiction. If matter be not solid extension, what can it be more than mere extension?

Further, matter that is not solid, is the same with pore; and therefore it cannot possess what philosophers mean by the momentum or force of bodies, which is always in proportion to the quantity of matter in bodies, void of pore.

MAUNDY THURSDAY, is the Thursday in Passion week; which was called Maunday or Mandate Thursday, from the command which Christ gave his apostles to commemorate him in the Lord's Supper, which he instituted on this day; or from the new commandment which he gave them to love one another, after he had washed their feet as a token of his love to them.

MAUPERTUIS (PETER LOUIS MORCEAU DE), a celebrated French mathematician and philosopher, was born at St. Malo in 1698, and was there privately educated till he attained his 16th year, when he was placed under the celebrated professor of philosophy, M. le Blond, in the college of la Marche, at Paris; he had also M. Guisnée, of the Academy of Sciences, for his instructor in mathematics. For this science he soon discovered a strong inclination, and particularly for geometry. He also practised instrumental music in his early years with great success; but fixed on no profession till he was 20, when he entered into the army; in which he remained about 5 years, during which time he pursued his mathematical studies with great vigour; and it was soon remarked by M. Freret and other academicians, that nothing but mathematics could satisfy his active soul and unbounded thirst for knowledge.

In the year 1723, he was received into the Royal Academy of Sciences, and read his first performance, which was a memoir on the construction and form of musical instruments. During the first years of his admission, he did not wholly confine his attention to mathematics; he dipped into natural philosophy, and discovered great knowledge and dexterity in observations and experiments on animals.

If the custom of travelling into remote countries, like the sages of antiquity, in order to be initiated into the learned mysteries of those times, had still subsisted, no one would have conformed to it with more eagerness than Maupertuis. His first gratification of this passion was to visit the country which had given birth to Newton; and during his residence in London he became as zealous an admirer and follower of that philosopher as any one of his own countrymen. His next excursion was to Basil in Switzerland, where he formed a friendship with the celebrated John Bernoulli and his family, which continued till his death. At his return to Paris, he applied himself to his favourite studies with greater zeal than ever. And how well he fulfilled the duties of an academician, may be seen by running over the Memoirs of the academy from the year 1724 to 1744; where it appears that he was neither idle, nor occupied on objects of small importance. The most sublime questions in the mathematical sciences, received from his hand that elegance, clearness, and precision, so remarkable in all his writings.

In the year 1736, he was sent to the polar circle, to measure a degree of the meridian, in order to ascertain the figure of the earth; in which expedition he was accompanied by Mess. Clairaut, Camus, Monnier, Outhier, and Celsus, the celebrated professor of astronomy at Upsal. This business rendered him so eminent, that on his return he was admitted a member of almost every academy in Europe; though it has been since found that their deductions have been considerably erroneous.

In the year 1740, Maupertuis had an invitation from the king of Prussia to go to Berlin; which was too flattering to be refused. His rank among men of letters had not wholly effaced his love for his first profession, that of arms He followed the king to the field, but at the battle of Molwitz was deprived of the pleasure of being present, wher victory declared in favour of his royal patron, by a singulai kind of adventure. His horse, during the heat of the action, running away with him, he fell into the hands of th enemy; and was at first but roughly treated by the Austrian hussars, to whom he could not make himself know for want of language; but being carried prisoner to Vienna, he received such honours from the emperor as neve were effaced from his memory. Maupertuis lamented ver much the loss of a watch of Mr. Graham's, the celebrate English artist, which they had taken from him; but th emperor, who happened to have another by the same artis enriched with diamonds, presented it to him, saying, "th hussars meant only to jest with you, they have sent m your watch, and I return it to you."

He went soon after to Berlin; but as the reform of th academy which the king of Prussia then meditated was nc yet matured, he repaired to Paris, where his affairs calle him, and he was there chosen, in 1742, director of th Academy of Sciences. In 1743 he was received into th French Academy; which was the first instance of the sam person being a member of both the academies at Paris the same time. Maupertuis again assumed the soldier the siege of Fribourg, and was pitched upon by Marsh Coigny and the count d'Argenson to carry the news to tl French king of the surrender of that citadel.

Maupertuis returned to Berlin in the year 1744, whe a marriage was negotiated and brought about, by the goc offices of the queen-mother, between our author ar Mademoiselle de Borck, a lady of great beauty and meri and nearly related to M. de Borck, at that time minist of state. This determined him to settle at Berlin, as l was extremely attached to his new spouse, and regarde this alliance as the most fortunate circumstance of his lif

In the year 1746, Maupertuis was declared, by the ki of Prussia, president of the Royal Academy of Sciences Berlin, and soon after by the same prince was honour with the Order of Merit. However, all these accumulat honours and advantages, so far from lessening his ardo for the sciences, seemed to furnish new allurements to l bour and application. Not a day passed but he produc some new project or essay for the advancement of knowledge. Nor did he confine himself to mathematical studies only: metaphysics, chemistry, botany, polite literature, all shared his attention, and contributed to his fame. At the same time he had, it seems, a strange inquietude of spirit, and dark melancholy humour, which rendered him miserable amid honours and pleasures. Such a temperament did not promise a pacific life, and he was in fact engaged in several quarrels. One of these was with Koenig, the professor of philosophy at Franeker, and another more terrible with Voltaire. Maupertuis had inserted in the volume of Memoirs of the Academy of Berlin for 1746, a discourse on the laws of motion; which Koenig was not content with attacking, but attributed to Leibnitz. Maupertuis, stung with the imputation of plagiarism, engaged the academy of Berlin to call upon him for his proof, which Koenig failing to produce, his name was struck out of the academy, of which he was a member. Several pamphlets were the consequence of this measure; and Voltaire, for some reason or other, engaged in the quarrel against Maupertuis. We say, for some reason or other; because Maupertuis and Voltaire were apparently on the most amicable terms; and the latter respected the former as his master in the mathematics. Voltaire on this occasion, however, exerted all his wit and satire against him; and on the whole was so much transported beyond what was thought right, that he found it expedient in 1753 to quit the court of Prussia.

Our philosopher's constitution had long been considerably impaired by the great fatigues of various kinds in which his active mind had involved him; though to the amazing hardships he had undergone, in his northern expedition, most of his bodily sufferings may be traced. The intense sharpness of the air could only be supported by means of strong liquors, which helped but to lacerate his lungs, and bring on a spitting of blood, which began at least 12 years before his death. Yet still his mind seemed to enjoy the greatest vigour; for the best of his writings were produced, and most sublime ideas developed, during the time of his confinement by sickness, when he was unable to occupy his presidential chair at the academy. He took several journeys to St. Malo, during the last years of his life, for the recovery of his health: and though he always received benefit by breathing his native air, yet still, on his return to Berlin, his disorder likewise returned with greater violence. His last journey into France was undertaken in the year 1757; when he was obliged, soon after his arrival there, to quit his favourite retreat at St. Malo, on account of the danger and confusion which that town was thrown into by the arrival of the English in its neighbourhood. From thence he went to Bourdeaux, hoping there to meet with a neutral ship to carry him to Hamburgh, in his way back to Berlin; but being disappointed in that hope, he went to Toulouse, where he remained seven months. He had then thoughts of going to Italy, in hopes a milder climate would restore him to health; but finding himself grow worse, he rather inclined towards Germany, and went to Neufchatel, where for three months he enjoyed the conversation of lord Marischal, with whom he had formerly been much connected. At length he arrived at Basil, October 16, 1758, where he was received by his friend Bernoulli and his family with the utmost tenderness and affection. He at first found himself much better here than he had been at Neufchatel: but this amendment was of short duration; for as the winter approached, his disorder returned, accompanied by new and more alarming symptoms. He languished here many months, during which he was attended by M. de la Condamine; and died in 1759, at 61 years of age.

The works which he published were collected into 4 volumes 8vo, published at Lyons in 1756, where also a new and elegant edition was printed in 1768. These contain the following works:—1. Essay on Cosmology.—2. Discourse on the different Figures of the Stars.—3. Essay on Moral Philosophy.—4. Philosophical Reflections on the Origin of Languages, and the Signification of Words.—5. Animal Physics, concerning Generation &c.—6. System of Nature, or the Formation of Bodies.—7. Letters on various Subjects.—8. On the Progress of the Sciences.—9. Elements of Geography.—10. Account of the Expedition to the Polar Circle, for determining the Figure of the Earth; or the Measure of the Earth at the Polar Circle.—11. Account of a Journey into the Heart of Lapland, to search for an Ancient Monument.—12. On the Comet of 1742.—13. Various Academical Discourses pronounced in the French and Prusian Academies.—14. Dissertation on Languages.—15. Agreement of the Different Laws of Nature, which have hitherto appeared incompatible.—16. On the Laws of Motion.—17. On the Laws of Rest.—18. Nautical Astronomy.—19. On the Parallax of the Moon.—20. Operations for determining the Figure of the Earth, and the Variations of Gravity.—21. Measure of a Degree of the Meridian at the Polar Circle.

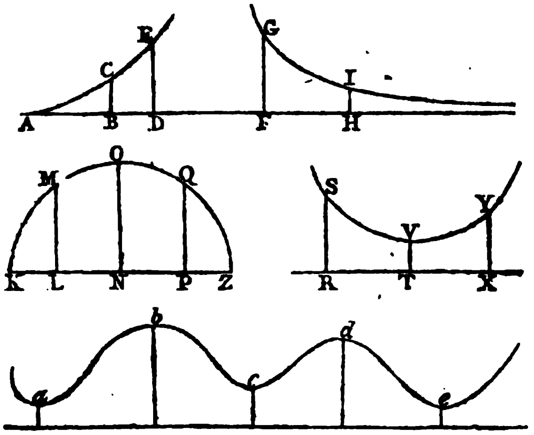
Besides these works, Maupertuis was author of a great number of interesting papers, particularly those printed in the Memoirs of the Paris and Berlin Academies, far too numerous here to mention; viz, in the Memoirs of the Academy at Paris, from the year 1724, to 1749; and in those of the Academy of Berlin, from the year 1746, to 1756.

MAUROLICO (Francis), was born at Messina in 1494, and became abbot of St. Maria del Porto in Sicily, and taught mathematics with reputation in his native country, having possessed the happy art of rendering the most abstract questions plain, by his clearness of expression; and he applied particularly to the summation of several series, such as those of the natural numbers, triangular numbers, &c. He died in 1575.—His works chiefly are, 1. An edition of the Spherics of Theodosius.—2. Emendatio et Restitutio Conicorum Apollonii Pergæi.—3. Archimedis Monumenta omnia.—4. Euclidis Phenomena, &c. And he introduced the use of the secants into trigonometry.

MAXIMUM, denotes the greatest state or quantity attainable in any given case, or the greatest value of a variable quantity. By which it stands opposed to Minimum, which is the least possible quantity in any case. As in the algebraical expression $a^2 - bx$, where a and b are constant or invariable quantities, and x a variable one. Now it is evident that the value of this remainder or difference, $a^2 - bx$, will increase as the term bx, or x, decreases; and therefore that remainder will be the greatest when x is the smallest; that is, $a^2 - bx$ is a maximum, when x is the least, or nothing at all. Again, the expression or difference $a^2 - \frac{b}{x}$, evidently increases as the fraction $\frac{b}{x}$ diminishes; and this diminishes as x increases; therefore the given expression will be the greatest, or a maximum, when x is the greatest, or infinite.

Also, if along the diameter KZ (*the 3d fig. below*) of a circle, a perpendicular ordinate LM be conceived to move, from K towards Z; it is evident that, from K it increases continually till it arrive at the centre, in the position NO, where it is at the greatest state; and from thence it continually decreases again, as it moves along from N to Z, and quite vanishes at the point Z. So that the maximum state of the ordinate is NO, equal to the radius of the circle.

Methodus de MAXIMIS *et* MINIMIS, a method of finding the greatest or least state or value of a variable quantity.



Some quantities continually increase, and therefore have no maximum but what is infinite; as the ordinates BC, DE of the parabola ACE: Some continually decrease, and have therefore their least or minimum state in nothing; as the ordinates FG, HI, to the asymptotes of the hyperbola. Others increase to a certain magnitude, which is their maximum, and then decrease again; as the ordinates LM &c of the circle. And others again decrease to a certain magnitude TV, which is their minimum, and then increase again; as the ordinates of the curve SVY. While others admit of several maxima and minima; as the ordinates of the curve *abcde*, where at *b* and *d* they are maxima, and at *a*, *c*, *e*, minima. And thus the maxima and minima of all other variable quantities may be conceived; expressing those quantities by the ordinates of some curves.

The first ideas of maxima and minima are found in the Elements of Euclid, or flow immediately from them: thus, it appears, by the 5th prop. of book 2, that the greatest rectangle that can be made of the two parts of a given line, any how divided, is when the line is divided equally in the middle; prop. 7, book 3, shows that the greatest line that can be drawn from a given point within a circle, to its circumference, is that which passes through the centre; and that the least line that can be so drawn, is the continuation of the same to the other side of the circle: prop. 8, ib. shows the same for lines drawn from a point without the circle: and thus many instances of a similar nature might be pointed out in the Elements.—Other writers on the maxima and minima, are, Apollonius, in the whole 5th book of his Conic Sections; and in the preface or dedication of that book, he says that others had then also treated the subject, though in a slighter manner.—Archimedes; as in prop. 9 of his treatise on the Sphere and Cylinder, where he demonstrates that, of all spherical segments under equal superficies, the hemisphere is the greatest.—Serenus, in his 2d book, or that on the Conic Sections.—Pappus, in many parts of his Mathematical Collections; as in lib. 3, prop. 28 &c, lib. 6, prop. 31 &c, where he treats of some curious cases of variable geometrical quantities, showing how some increase and decrease both ways to infinity; while others proceed only one way, by increase or decrease, to infinity, and the other way to a certain magnitude; and others again both ways to a certain magnitude, giving a maximum and minimum; also lib. 7, prop. 13, 14, 165, 166, &c. And all these are the geometrical maxima and minima of the ancients; to which may be added some others of the same kind, viz Viviani De Maximis et Minimis Geometrica Divinatio in quintum Conicorum Apollonii Pergæi, in fol. at Flor. 1659; also an ingenious little tract in Simpson's Geometry, on the maxima and minima of Geometrical Quantities. See also vol. 3 of my Course of Mathematics.

But the subject of maxima and minima is treated in a more general way by the modern analysis; the first among which perhaps may be placed that of Fermat. This, and other methods, are best referred to, and explained by the ordinates of curves: for when the ordinate of a curve increases to a certain magnitude, where it is greatest, and afterwards decreases again, it is evident that two ordinates on the contrary sides of the greatest ordinate may be equal to each other; and when the ordinates decrease to a certain point, where they are at the least, and afterwards increase again; there may also be two equal ordinates, one on each side of the least ordinate. Hence then an equal ordinate corresponds to two different abscisses, o for every value of an ordinate there are two values o abscisses. Now as the difference between the two abscisses is conceived to become less and less, it is evident that th two equal ordinates, corresponding to them, approach nearer and nearer together; and when the differences o the abscisses are infinitely small, or nothing, then the equa ordinates unite in one, which is either the maximum o minimum. The method hence derived then, is this: Fin two values of an ordinate, expressed in terms of the abscisses: put those two values equal to each other, cancel ing the parts that are common to both, and dividing al the remaining terms by the difference between the abscisses which will be a common factor in them: next, supposin the abscisses to become equal, that the equal ordinate may concur in the maximum or minimum, that differenc will vanish, as well as all the terms of the equation tha include it; and therefore, striking those terms out of th equation, the remaining terms will give the value of th absciss corresponding to the maximum or minimum.

For example, Suppose it were required to find th greatest ordinate in a circle KMQ. Put the diameter K $= a$, the absciss KL $= x$, the ordinate LM $= y$; hence th other part of the diameter is LZ $= a - x$, and consequentl by the nature of the circle KL × LZ being equal LM^2, $\times (a - x)$ or $ax - x^2 = y^2$. Again, put another absci KP $= x + d$, where d is the difference LP, the ordina PQ, being equal to LM or y; here then again KP × PZ PQ^2, or $(x + d) \times (a - x - d) = ax - x^2 - 2dx + ad$ $d^2 = y^2$: put now these two values of y^2 equal to ea other, so shall $ax - x^2 = ax - x^2 - 2dx + ad - d^2$; canc the common terms ax and x^2, then $0 = - 2dx + ad$ d^2, or $2dx + d^2 = ad$; divide all by d, so shall $2x + d =$ a general equation derived from the equality of the tv ordinates. Now, bringing the two equal ordinates t gether, or making the two abscisses equal, their diff ence d vanishes, and the last equation becomes barely $2x = a$, or $x = \frac{1}{2}a$, $=$ KN, the value of the absciss KN when the ordinate NO is a maximum, viz, the greatest ordinate bisects the diameter. And the operation and conclusion it is evident will be the same, to divide a given line into two parts, so that their rectangle shall be the greatest possible.

For a second example, let it be required to divide the given line AB into two such parts, that the one part drawn into the square of the other may be the greatest possible. Putting the given line AB $= a$, and one part AC $= x$; then the other part CB will be $a - x$, and therefore $x^2 \times (a - x) = ax^2 - x^3$ is the product of one part by the square of the other. Again, let one part be AD $= x + d$, then the other part is $a - x - d$, and $(x + a)^2 \times (a - x - d) = ax^2 - x^3 - 3dx^2 + (2ad - 3d^2) \,.\, x + ad^2 - d^3$. Then, putting these two products equal to each other, cancelling the common terms $ax^2 - x^3$, and dividing the remainder by d, there results $0 = -3x^2 + (2a - 3d) \,.\, x + ad - d^2$; hence, canceling all the terms that contain d, there remains $0 = -3x^2 + 2ax$, or $3x = 2a$, and $x = \frac{2}{3}a$; that is, the given line must be divided into two parts in the ratio of 3 to 2. See Fermat's Opera Varia, pa. 63, and his Letters to Mersenne.

The next method was that of John Hudde, given by Schooten among the additions to Descartes's Geometry, near the end of the 1st vol. of his edition. This method is also drawn from the property of an equation having two equal roots. He there demonstrates that, having ranged the terms of an equation, that has two roots equal, according to the order of the exponents of the unknown quantity, taking all the terms over to one side, and so making them equal to nothing on the other side; if then the terms in that order be multiplied by the terms of any arithmetical progression, the resulting equation will still have one of its roots equal to one of the two equal roots of the former equation. Now since, by what has been said of the foregoing method, when the ordinate of a curve, admitting of a maximum or minimum, is expressed in terms of the abscissa, that abscissa, or the value of x, will be two-fold, because there are two ordinates of the same value; that is, the equation has at least two unequal roots or values of x: but when the ordinate becomes a maximum or minimum, the two abscisses unite in one, and the two roots, or values of x, are equal; therefore, from the above-said property, the terms of this equation for the maximum or minimum being multiplied by the terms of any arithmetical progression, the root of the resulting equation will be one of the said equal roots, or the value of the absciss x when the ordinate is a maximum.

Though the terms of any arithmetic progression may be used for this purpose, some are more convenient than others; and Mr. Hudde directs to make use of that progression which is formed by the exponents of x, viz, to multiply each term by the exponent of its power, and putting all the resulting products equal to nothing; which, it is evident, is exactly the same process as taking the fluxions of all the terms, and putting them equal to nothing; being the common process now used for the same purpose.

Thus, in the former of the two foregoing examples, where $ax - x^2$, or y^2, is to be a maximum;
mult. by 1 2
gives $ax - 2x^2 = 0$; hence $2x = a$, and $x = \frac{1}{2}a$, as before.

And in the 2d example, where $ax^2 - x^3$, is to be a maximum; mult. by - - 2 3
gives - - - - - $2ax^2 - 3x^3 = 0$;
hence $2a - 3x = 0$, or $3x = 2a$, and $x = \frac{2}{3}a$, as before.

The next general method, and which is now usually practised, is that of Newton, or the method of Fluxions, which is founded on a principle different from that of the two former methods of Fermat and Hudde. These are derived from the idea of the two equal ordinates of a curve uniting into one, at the place of the maximum and minimum; but Newton's from the principle, that the fluxion or increment of an ordinate is nothing, at the point of the maximum or minimum; a circumstance which immediately follows from the nature of that doctrine: for, since a quantity ceases to increase at the maximum, and to decrease at the minimum, at those points it neither increases nor decreases; and since the fluxion of a quantity is proportional to its increase or decrease, therefore the fluxion is nothing at the maximum or minimum. Hence this rule: Take the fluxion of the algebraical expression denoting the maximum or minimum, and put it equal to nothing; and that equation will determine the value of the unknown letter or quantity in question.—So, in the first of the two foregoing examples, where it is required to determine x when $ax - x^2$ is a maximum: the fluxion of this is $a\dot{x} - 2x\dot{x} = 0$; which divided by $\dot{x}$, gives $a - 2x = 0$, or $a = 2x$, and $x = \frac{1}{2}a$. Also, in the 2d example, where $ax^2 - x^3$ is to be a maximum: the fluxion is $2ax\dot{x} - 3x^2\dot{x} = 0$; hence $2a - 3x = 0$, or $2a = 3x$, and $x = \frac{2}{3}a$.

When a quantity becomes a maximum or minimum, and is expressed by two or more affirmative and negative terms, in which only one variable letter is contained; it is evident that the fluxion of the affirmative terms will be equal to the fluxion of the negative ones; since their difference is equal to nothing.

And when, in the expression for the fluxion of a maximum or minimum, there are two or more fluxionary letters, each contained in both affirmative and negative terms; the sum of the terms containing the fluxion of each letter, will be equal to nothing: For, in order that any expression be a maximum or minimum, which contains two or more variable quantities, it must produce a maximum or minimum, if but one of those quantities be supposed variable. So if $ax - 2xy + by$ denote a minimum; its fluxion is $a\dot{x} - 2y\dot{x} - 2x\dot{y} + b\dot{y}$; hence $a\dot{x} - 2y\dot{x} = 0$, and $b\dot{y} - 2x\dot{y} = 0$; from the former of these $y = \frac{1}{2}a$, and from the latter $x = \frac{1}{2}b$. Or, in such a case, take the fluxion of the whole expression, supposing only one quantity variable; then take the fluxion again, supposing another quantity only variable: and so on, for all the several variable quantities; which will give the same number of equations for determining those quantities. So, in the above example, $ax - 2xy + by$, the fluxion is $a\dot{x} - 2y\dot{x} = 0$, supposing only x variable; which gives $y = \frac{1}{2}a$: and the fluxion is $-2x\dot{y} + b\dot{y} = 0$, when y only is variable; which gives $x = \frac{1}{2}b$; the same as before.

Farther, when any quantity is a maximum or minimum, all the powers or roots of it will be so too; as will also be the result, when it is increased or decreased, or multiplied, or divided by a given or constant quantity; and the logarithm of the same will be also a maximum or minimum.

To find whether a proposed algebraic quantity admits of a maximum or minimum.—Every algebraic expression does not admit of a maximum or minimum, properly so called; for it may either increase continually to infinity, or decrease continually to nothing; in both which cases there is neither a proper maximum nor minimum: for the true maximum is that value to which an expression increases, and after which it decreases again; and the minimum is that value to which the expression decreases, and after that it increases again. Therefore when the expression admits of a maximum, its fluxion is positive before that point, and negative after it; but when it admits of a minimum, its fluxion is negative before, and positive after it. Hence, take the fluxion of the expression immediately before the fluxion is equal to nothing, and a little after it; if the first fluxion be positive, and the last negative, the middle state is a maximum; but if the first fluxion be negative, and the last positive, the middle state is a minimum. Thus, for example, see Maclaurin's Fluxions, book 1, chap. 9, and book 2, chap. 5, art. 859.

We shall add here a few problems, as a farther illustration of this method.

Prob. 1. To divide a given number a into two parts, x, and y, so that $x^m y^n$ may be a maximum.

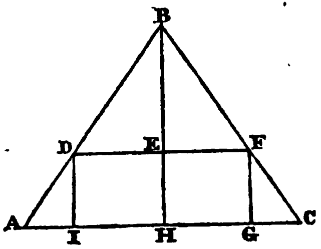
Since $x + y = a$, and $x^m y^n =$ max. the fluxion of each $= 0$, the former because it is constant, and the latter because it is a maximum; therefore $\dot{x} + \dot{y} = 0$ and $my^n x^{m-1}\dot{x} + nx^m y^{n-1}\dot{y} = 0$; hence $\dot{x} = -\dot{y}$, and $\dot{x} = -\frac{nx^m y^{n-1}\dot{y}}{my^n x^{m-1}} = -\frac{nx\dot{y}}{my}$; therefore $-\dot{y} = -\frac{nx\dot{y}}{my}$, or $my = nx$, that is $m : n :: x : y$. And since $y = \frac{nx}{m}$; therefore $x + \frac{nx}{m} = a$, and consequently $x = \frac{ma}{m+n}$, and $y = \frac{na}{m+n}$.

If $m = n$, the two parts are equal.

Prob. 2. To inscribe the greatest parallelogram in a given triangle.

Let ABC, Fig. 1, be the given triangle, and DFGI the required parallelogram; draw BH perpendicular to AC; and put AC $= a$, BH $= b$, BE $= x$; then EH $= b - x$; and by similar triangles $b : a :: x : \frac{ax}{b} =$ DF; hence the area DFGI $= \frac{ax}{b} \times (b - x) =$ max. or $bx - x^2 =$ max. Therefore, taking the fluxion, we have, $b\dot{x} - 2x\dot{x} = 0$, or $x = \frac{1}{2}b$; and hence EH $= \frac{1}{2}$BH.

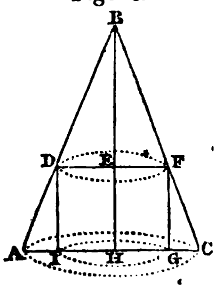
Fig. 1.



Prob. 3. To inscribe the greatest cylinder in a given cone.

Let ABC, Fig. 2, represent the given cone, and DFGI the cylinder required. Put $p = \cdot78539$ &c; then the same notation remaining, as in the foregoing problem, we have by similar triangles, $b : a :: x : \frac{ax}{b} =$ DF; therefore the area of the end of the cylinder $= \frac{pa^2x^2}{b^2}$; and hence, by the question, $\frac{pa^2x^2}{b^2} \times (b - x) =$ a max. or $x^2 \times (b - x) = bx^2 - x^3 =$ a max. And this being thrown into fluxions, we have $2bx\dot{x} - 3x^2\dot{x} = 0$, or $x = \frac{2}{3}b$; therefore EH $= \frac{1}{3}$BH, when the cylinder is the greatest possible.

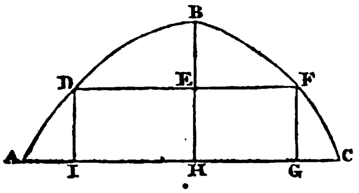
Fig. 2.



Prob. 4. To inscribe the greatest possible parallelogram in a given parabola.

Let ABC, Fig. 3, represent the given parabola, and DFGI the required parallelogram. Also put BH $= a$, parameter $= p$, and BE $= x$; then by the property of the parabola, DE$^2 = px$; therefore DE $= \sqrt{px}$, and DF $= 2\sqrt{px}$; hence the area of the parallelogram DFGI $= 2\sqrt{px} \times (a - x)$ a max. or $ax^{\frac{1}{2}} - x^{\frac{3}{2}} =$ a max.; and this in fluxions gives $\frac{1}{2}ax^{-\frac{1}{2}}\dot{x} - \frac{3}{2}x^{\frac{1}{2}}\dot{x} = 0$, or $\frac{a}{x^{\frac{1}{2}}} = 3x^{\frac{1}{2}}$ or $a = 3x$, consequently $x = \frac{1}{3}a$; that is EH $= \frac{2}{3}$ BH, when the inscribed parallelogram is a maximum.

Fig. 3.



Prob. 5. To determine the dimensions of a cylindric vessel open at top, that shall contain a given quantity o liquor, under the least possible superficies.

Let the altitude be represented by x, and the diamete by y, also put $\cdot78539$ &c $= p$, consequently $3\cdot1416 = 4p$ and the content $= a$; then $4pxy$ will be the cylindric surface, and py^2 the area of the bottom. Hence these tw equations $py^2x = a$, and $4pxy + py^2 =$ a minimum from the first $x = \frac{a}{py^2}$, which being substituted for x i the second, gives $\frac{4a}{y} + py^2 =$ a minimum; and this i fluxions, gives $\frac{-4a\dot{y}}{y^2} + 2py\dot{y} = 0$, or $2py = \frac{4a}{y^2}$; henc $y^3 = \frac{2a}{p}$, and $y = \sqrt[3]{\frac{2a}{p}}$ the diameter; also since $x = \frac{a}{py^2}$ or $x^3 = \frac{a^3}{p^3y^6} = \frac{a^3}{p^3} \times \frac{p^2}{4a^2} = \frac{a}{4p} = \frac{2a}{8p}$; therefore $x = \frac{1}{2}\sqrt[3]{\frac{2a}{p}}$: that is, the diameter is double the altitud when the surface is a minimum.

Prob. 6. Of all right-angled triangles having the san hypothenuse, to determine the dimensions of that who area is a maximum.

Let the hypothenuse be represented by a, and the le by x and y. Then we have these two equations,

$$x^2 + y^2 = a^2, \text{ and } \tfrac{1}{2}xy = \text{max.}$$

The fluxion of each of these is equal to 0, the first b cause it is constant, and the second because it is a ma mum; we have therefore

$2x\dot{x} + 2y\dot{y} = 0$, and $\frac{1}{2}x\dot{y} + \frac{1}{2}y\dot{x} = 0$; from the fi $\dot{x} = \frac{-y\dot{y}}{x}$ and from the second $\dot{x} = \frac{-x\dot{y}}{y}$, therefore $\frac{y\dot{y}}{x} = \frac{x\dot{y}}{y}$; hence $x = y$, that is, the area is a maximu when the legs are equal.

For various examples of this kind, see Simpson's, Ma laurin's, Emerson's, and Vince's Fluxions.

MAXIMUM *Effect of Machines.* See MECHANICS.

MAY, *Maius*, the fifth month in the year, reckoni from our first, or January; but the third, counting t year to begin with March, as the Romans did ancientl It was called Maius by Romulus, in respect to the ser tors and nobles of his city, who were named Majores; as the following month was called Junius, in honour of the youth of Rome, in honorem juniorum, who served him in the war. Though some say it has been thus called from Maia, the mother of Mercury, to whom they offered sacrifice on the first day of this month: and Papias derives the name from Madius, eo quod tunc terra madeat.—In this month the sun enters the sign Gemini, and the plants of our hemisphere begin mostly to flower.

MAYER (Tobias), one of the greatest astronomers and mechanists of the 18th century, was born at Maspach, in the duchy of Wirtemberg, 1723. He taught himself mathematics, and at 14 years of age designed machines and instruments with the greatest dexterity and exactness. These pursuits, however, did not hinder him from cultivating the belles lettres; for he acquired the Latin tongue, and wrote it with elegance. In 1750, the university of Gottingen chose him for their mathematical professor; and every year of his short life was thenceforward marked with some considerable discoveries in geometry and astronomy. He published several works on these subjects, which are all accounted excellent of their kind; and some papers are inserted in the second volume of the Memoirs of the University of Gottingen. He was very accurate and indefatigable in his astronomical observations; indeed his labours seem to have very early exhausted him; for he died worn out in 1762, at no more than 39 years of age.

His Table of Refractions, deduced from his own astronomical observations, accurately agrees with that of Doctor Bradley; and his Theory of the Moon, and Astronomical Tables and Precepts, were so well esteemed, that they were rewarded by the English Board of Longitude, with the premium of 3000 pounds, which sum was paid to his widow after his death. These tables and precepts were published by the Board of Longitude in 1770.

MEAN, a middle state between two extremes: as a mean motion, mean distance, arithmetical mean, geometrical mean, &c.

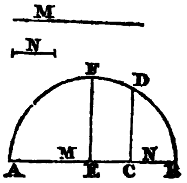
Arithmetical **Mean**, is half the sum of the extremes. So, 4 is an arithmetical mean between 2 and 6, or between 3 and 5, or between 1 and 7; also an arithmetical mean between a and b is $\frac{a+b}{2}$ or $\frac{1}{2}a + \frac{1}{2}b$.

Geometrical **Mean**, commonly called a mean proportional, is the square root of the product of the two extremes; so that, to find a mean proportional between two given extremes, multiply these together, and extract the square root of the product. Thus, a mean proportional between 1 and 9, is $\sqrt{(1 \times 9)} = \sqrt{9} = 3$; a mean between 2 and $4\frac{1}{2}$ is $\sqrt{(2 \times 4\frac{1}{2})} = \sqrt{9} = 3$; also the mean between 4 and 6 is $\sqrt{(4 \times 6)} = \sqrt{24}$; and the mean between a and b is $\sqrt{ab}$.

The geometrical mean is always less than the arithmetical mean, between the same two extremes. So the arithmetical mean between 2 and $4\frac{1}{2}$ is $3\frac{1}{4}$, but the geometrical mean is only 3. To prove this generally; let a and b be any two terms, a the greater, and b the less; then, universally, the arithmetical mean $\frac{1}{2}a + \frac{1}{2}b$ shall be greater than the geometrical mean $\sqrt{ab}$, or $a + b$ greater than $2\sqrt{ab}$. For, by

squaring both, they are $a^2 + 2ab + b^2 > 4ab$;
subtr. $4ab$ from each, then $a^2 - 2ab + b^2 > 0$,
that is - - - $(a - b)^2 > 0$.

To find a Mean Proportional Geometrically, between two given lines M and N. Join the two given lines together at C in one continued line AB; on the diameter AB describe a semicircle, and erect the perpendicular CD; which will be the mean proportional between AC and CB, or M and N. This, it is evident, is always less than the arithmetical mean, AE or EB or EF; except when the two lines are equal; for then the two means are equal also.



To find two Mean Proportionals between two given extremes. Multiply each extreme by the square of the other, viz, the greater extreme by the square of the less, and the less extreme by the square of the greater; then extract the cube root out of each product, and the two roots will be the two mean proportionals sought. That is, $\sqrt[3]{a^2b}$ and $\sqrt[3]{ab^2}$ are the two means between a and b. So, between 2 and 16, the two mean proportionals are 4 and 8; for $\sqrt[3]{(2^2 \times 16)} = \sqrt[3]{64} = 4$, and $\sqrt[3]{(2 \times 16^2)} = \sqrt[3]{512} = 8$.

In a similar manner we proceed for three means, or four means, or five means, &c. From all which it appears that the series of the several numbers of mean proportionals, between a and b, will be as follows: viz,

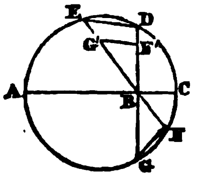
one mean, $\sqrt{ab}$;
two means, $\sqrt[3]{a^2b}$, $\sqrt[3]{ab^2}$;
three means, $\sqrt[4]{a^3b}$, $\sqrt[4]{a^2b^2}$, $\sqrt[4]{ab^3}$;
four means, $\sqrt[5]{a^4b}$, $\sqrt[5]{a^3b^2}$, $\sqrt[5]{a^2b^3}$, $\sqrt[5]{ab^4}$;
five means, $\sqrt[6]{a^5b}$, $\sqrt[6]{a^4b^2}$, $\sqrt[6]{a^3b^3}$, $\sqrt[6]{a^2b^4}$, $\sqrt[6]{ab^5}$;
&c, &c.

Harmonical **Mean**, is double a fourth proportional to the sum of the extremes, and the two extremes themselves a and b: thus, as $a + b : a :: 2b : \frac{2ab}{a+b} = m$ the harmonical mean between a and b. Or it is the reciprocal of the arithmetical mean between the reciprocals of the given extremes; that is, take the reciprocals of the extremes a and b, which will be $\frac{1}{a}$ and $\frac{1}{b}$; then take the arithmetical mean between these reciprocals, or half their sum, which will be $\frac{1}{2a} + \frac{1}{2b}$ or $\frac{a+b}{2ab}$; lastly, the reciprocal of this is $\frac{2ab}{a+b} = m$ the harmonical mean: for, arithmeticals and harmonicals are mutually reciprocals of each other;
so that if $a, m, b,$ &c be arithmeticals,
then shall $\frac{1}{a}, \frac{1}{m}, \frac{1}{b},$ &c be harmonicals;
or if the former be harmonicals, the latter will be arithmeticals.

For example, to find a harmonical mean between 2 and 6; here $a = 2$, and $b = 6$; therefore $\frac{2ab}{a+b} = \frac{2 \times 2 \times 6}{2+6} = \frac{24}{8} = 3 = m$ the harmonical mean sought between 2 and 6.

It is remarkable that the three means, viz, the arithmetical, the geometrical, and the harmonical, between any two quantities, a and b, are in continued geometric progression; for it is evident that $\frac{a+b}{2} : \sqrt{ab} :: \sqrt{ab} : \frac{2ab}{a+b}$.

To place the said three Means in a Circle.—On the sum (AC) of the two means (AB, BC), as a diameter, describe a circle; in which erect BD the given mean, and apply BE = $\frac{1}{2}$AC the arithm. mean, then produce EB to F, so shall BF be the harmonical mean. For, produce DB to



G, then BG = BD; also join DE and FG, making the triangles BDE, BFG similar: for the opposite angles at B are equal, also the angles E and G are equal, standing on the same arc DF; hence BE : BD : : BG = BD : BF, which is therefore the harmonical mean.

Otherwise; Having drawn BE and BD, the arithm. and geom. means; take BG′ = BD, and draw G′F′ parallel to ED; then is BF′ = BF the harm. mean also. For, by sim. triangles, BE : BD : : BG′ = BD : BF′.

In the 3d book of Pappus's Mathematical Collections, we have a very good tract on all the three kinds of mean proportionals, beginning at the 5th proposition. He observes, that the ancients could not resolve, in a geometrical way, the problem of finding two mean proportionals; and because it is not easy to describe the conic sections in plano, for that purpose, they contrived easy and convenient instruments, by which they obtained good mechanical constructions of that problem; as appears by their writings; as in the Mesolabe of Eratosthenes, of Philo, with the Mechanics and Catapultics of Hero. For these, rightly deeming the problem a solid one, effected the construction only by instruments, and Apollonius Pergæus by means of the conic sections; which others again performed by the loci solidi of Aristæus; also Nicomedes solved it by the conchoid, by means of which likewise he trisected an angle: and Pappus himself gave another solution of the same problem.

Pappus adds definitions of the three foregoing different kinds of means, with many problems and properties concerning them; and, among others, this curious similarity of them, viz, a, m, b, being three continued terms, either arithmeticals, geometricals, or harmonicals; then in the

Arithmeticals, $a : a :: a - m : m - b$;
Geometricals, $a : m :: a - m : m - b$;
Harmonicals, $a : b :: a - m : m - b$.

Mean-*and-Extreme Proportion*, or *Extreme-and-Mean Proportion*, is when a line, or any quantity, is so divided, that the less part is to the greater, as the greater is to the whole.—This is easily performed geometrically, as is done in Euclid.

But it cannot be done arithmetically in rational numbers: for if a denote a given number, to be divided in extreme-and-mean ratio; then the two parts are $\frac{\sqrt{5}-1}{2}a$ and $\frac{3-\sqrt{5}}{2}a$, which cannot be given in rational numbers, on account of the radical $\sqrt{5}$.

Mean *Anomaly*, of a planet, is an angle, which is always proportional to the time of the planet's motion from the aphelion or perihelion, or proportional to the area described by the radius vector; that is, as the whole periodic time in one revolution of the planet, is to the time past the aphelion or perihelion, so is 360° to the mean anomaly. See Anomaly.

Mean *Axis*, in Optics. See Axis.

Mean *Conjunction* or *Opposition*, is when the mean place of the sun is in conjunction, or opposition, with the mean place of the moon in the ecliptic.

Mean *Diameter*, in Gauging, is a mean between the diameters at the head and bung of a cask.

Mean *Distance*, of a planet from the sun, is an arithmetical mean between the planet's greatest and least distances; and this is equal to the semitransverse axis of the elliptic orbit in which it moves, or to the right line drawn from the sun or focus to the extremity of the conjugate axis of the same.

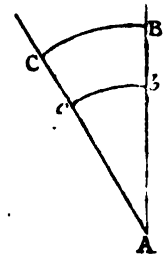
Mean *Motion*, is that by which a planet is supposed to move equably in its orbit; and it is always proportional to the time.

Mean *Time*, or Equal Time, is that which is measured by an equable motion, as a clock; as distinguished from apparent time, arising from the unequal motion of the earth or sun.

MEASURE, denotes any quantity, assumed as unity, or one, to which the ratio of other homogeneous or like quantities may be expressed.

Quantities are not always necessarily measured by quantities of the same kind. See vol. 3 of my Course of Mathematics, pa. 87, the note.

Measure *of an Angle*, is an arc of a circle described from the angular point as a centre, and intercepted between the legs or sides of the angle: and it is usual to estimate and express the measure of the angle by the number of degrees and parts contained in that arc, of which 360 make up the whole circumference. So, the measure of the angle BAC, is the arc BC to the radius AB, or the arc bc to the radius Ab.



Hence, a right angle is measured by a quadrant, or 90 degrees; and any angle, as BAC, is to a right angle, as the arc BC is to a quadrant, or as the degrees in BC are to 90 degrees.

Common **Measure**. See Common *Measure*.

Measure *of a Figure*, or Plane Surface, is a square inch, or square foot, or square yard, &c, that is, a square whose side is an inch, or a foot, or a yard, or some other determinate length; and this square is called the *measuring unit*.

Measure *of a Line*, is any right line taken at pleasure, and considered as unity; as an inch, or a foot, or a yard, &c.

Line of **Measures**. See Line *of Measures*.

Measure *of a Mass*, or *Quantity of Matter*, is its weight.

Measure *of a Number*, is any number that divides it, without leaving a remainder. So, 2 is a measure of 4, of 8, or of any even number; and 3 is a measure of 6, or of 9, or of 12, &c.

Measure *of a Ratio*, is its logarithm, in any system of logarithms; or it is the exponent of the power to which the ratio is equal, the exponent of some given ratio being assumed as unity. So, if the logarithm or measure of the ratio of 10 to 1, be assumed equal to 1; then the measure of the ratio of 100 to 1, will be 2, because 100 is $= 10^2$, or because 100 to 1 is in the duplicate ratio of 10 to 1; and the measure of the ratio of 1000 to 1, will be 3, because 1000 is $= 10^3$, or because 1000 to 1 is triplicate of the ratio of 10 to 1.

Measure *of a Solid*, is the number of cubic inches, or cubic feet, or cubic yards, &c, that are contained in it.

Measure *of a Superficies*, is the number of square inches, or square feet, or square yards, &c, contained in it.

Measure *of Velocity*, is the space uniformly passed over by a moving body in a given time.

Universal or *Perpetual* **Measure**, is a kind of measure unalterable by time or place, to which the measures of different ages and nations might be reduced, and by which they may be compared and estimated. Such a measure would be very useful, if it could be attained; since, being used at all times, and in all places, a greal deal of confusion and error would be avoided.

Measures of length appear to be the originals for all others, both for surfaces and solids or capacities, as well as for weights. The long measures of all nations seem, from their names, to have been originally taken from some part of the human body; as the foot, the hand, the cubit or elbow, the span, the fathom, &c. But as these measures must differ according to the different sizes of men, standards of some durable substance have been adopted in all civilised countries; which are found however to differ universally from each other, to the great inconvenience of all commerce. In order to remedy this inconvenience, different methods have been proposed for establishing a universal and perpetual standard, unalterable by time or place, to which the measures of all nations might be reduced, and by which they might be occasionally adjusted. But as all material substances are liable to decay and alteration, an invariable standard can be obtained only from some stable principle in nature, such as the action of gravitation, the motions of the heavenly bodies, or the magnitude of the earth, &c; and accordingly several of such methods have been proposed, of which the two following only have been attended with any degree of success; viz, 1. The length of a pendulum that vibrates seconds of mean time; 2. The length of a certain division or arc of the meridian.

The first of these methods is liable to this inconvenience, that the length of a seconds pendulum varies in different latitudes, increasing from the equator to the poles, owing to the spheroidical figure of the earth. The second method is liable to a similar inconvenience; as, from the same cause, the degrees of the meridian must also increase from the equator to the poles. Sir I. Newton calculated that the equatorial diameter of the earth is to the polar diameter, as 230 to 229; and therefore that on different parts of the earth's surface the weight of the same body is different, according as it is more or less distant from the centre of the earth; so that the length of a pendulum, vibrating any equal portions of time, must increase from the equator to the poles; and the degrees of the meridian must also increase on account of the curvature of the oblate spheroid.

Several measurements have been made in different latitudes, both of the lengths of the pendulum vibrating seconds, and of the degrees of the meridian; and they have been found nearly to agree with the above theory. Hence it appears, that a universal standard cannot be obtained from any of these methods, unless all nations were to agree that the trial or measurement should be made in some particular latitude;—an agreement that is never likely to take place. Such methods however may be usefully applied, to preserve the standards already established sufficiently correct for all practical purposes.

Huygens, in his Horol. Oscil. proposes, for this purpose, the length of a pendulum that should vibrate seconds, measured from the point of suspension to the point of oscillation: the 3d part of such a pendulum to be called horary foot, and to serve as a standard to which the measure of all other feet might be referred. But this measure, in order to its being universal, supposes that the action of gravity is the same on every part of the earth's surface, which is contrary to fact: for which reason it would really serve only for places under the same parallel of latitude: so that, if all latitudes were to have its foot equal to the 3d part of the pendulum vibrating seconds there, every different latitude would still have a different length of foot. And besides, the difficulty of measuring exactly the distance between the centres of motion and oscillation are such, that hardly any two measures would make it the same quantity.

Since that time, various other expedients have been proposed for establishing a universal measure. In 1779, a method was proposed to the Society of Arts, &c, by a Mr. Hatton, in consequence of a premium, which had been 4 years advertised by that institution, of a gold medal, or 100 guineas, 'for obtaining invariable standards for weights and measures, communicable at all times and to all nations.' Mr. Hatton's plan consisted in the application of a moveable point of suspension to one and the same pendulum, in order to produce the full and absolute effect of two pendulums, the difference of whose lengths was the intended measure. Mr. Whitehurst much improved on this idea, by very curious and accurate machinery, in his tract published 1787, entitled An Attempt towards obtaining invariable Measures of Length, Capacity, and Weight, from the Mensuration of Time, &c. Mr. Whitehurst's plan is, to obtain a measure of the greatest length that conveniency will permit, from two pendulums whose vibrations are in the ratio of 2 to 1, and whose lengths coincide with the English standard in whole numbers. The numbers he has chosen show great ingenuity. On a supposition that the length of a seconds pendulum, in the latitude of London, is 39·2 inches, the length of one vibrating 42 times in a minute, must be 80 inches; and of another vibrating 84 times in a minute, must be 20 inches; their difference, 60 inches or 5 feet, is his standard measure. By his experiments, however, the difference in the lengths of the two pendulums was found to be 59·892 inches, instead of 60, owing to the error in the assumed length of the seconds pendulum, 39·2 inches being greater than the truth. Mr. Whitehurst has however so far accomplished his design, as to show how an invariable standard may, at all times, be found for the same latitude. He has also ascertained a fact, as accurately as human powers seem capable of ascertaining it, of great consequence in natural philosophy: which is, that the difference between the lengths of the rods of two pendulums whose vibrations are known, is a datum from which may be derived the true length of pendulums, the spaces through which heavy bodies fall in a given time, with many other particulars relative to the doctrine of gravitation, the figure of the earth, &c, &c. The result deduced from this experiment is, that the length of a seconds pendulum, vibrating in a circular arc of 3° 20′, is 39·119 inches very nearly; but vibrating in the arc of a cycloid it would be 39·136 inches; and hence, heavy bodies will fall, in the first second of their descent, 16·094 feet, or 16 feet $1\frac{1}{8}$ inch, very nearly.

The other method, of deriving a standard from an arc of the meridian, has been lately executed in France; and it is said to possess the advantage over the pendulum method, of being on a larger scale; as any error in this operation must be dimininished by subdivision; whereas, an error in the small standard must be increased by multiplication. But this method is objected to, on account of the inequality in the earth's surface; for it has been found that the degrees of the meridian vary in different longitudes, even in the same latitude.

The mathematicians who adopted this plan, objected to the pendulum method, as depending on two different elements, viz, gravitation and time. But gravitation is uniform in the same latitude; and time is universally so, as depending on the regularity of the earth's diurnal rotation on its axis, which has never been found to vary, notwithstanding the inequality in its periodical revolution.—Thus it appears that superior accuracy cannot be ascribed to the meridian method; and as the chief use of an original standard from nature, is to restore lost measures; if two methods are equally correct, that which can be performed with the greater convenience, ought to be preferred; and in this view the pendulum must have a decided preference, as affording the readier means of recurring to the original.

Be that however as it may, the fundamental standard adopted in France, for the new system of weights and measures, is a quadrant of the meridian. This quadrant is divided into ten millions of equal parts; and one of these parts or divisions is called a Metre, which is adopted as the unit of length; and from which, by decimal multiplication and division, all other measures are derived. The length of the quadrant has been computed, by measuring an arc of the meridian, between the parallels of Dunkirk and Barcelona; and its length has been thus found equal to 5130740 French toises. This quantity, divided by ten millions, gives 443·296 lines = 36·9413 French inches = 39·3702 English inches, for the length of the metre.

In order to express certain decimal proportions, the following vocabulary has been adopted. The word Deca prefixed, means 10 times; Hecto, 100 times; Kilo, 1000 times; and Myria 10,000 times. On the other hand, the word Deci expresses the 10th part; Centi, the 100th part; and Milli, the 1000th part: thus a decametre, means 10 metres; a decimetre, the 10th part of a metre; a hectometre, 100 metres; a centimetre, the 100th part of a metre; and so of the rest.

The metre then being the element of long measures; the Are, which is a square decametre, is the element of superficial measures; the Stere, which is a cubic metre, is the element of solid measures; the Litre, which is a cubic decimetre, is the element of liquid measures, and of all other measures of capacity; and lastly, the Gramme, which is the weight of a cubic centimetre of distilled water, is the element for all weights.

A third standard has been proposed, viz, the space that a heavy body would freely fall through in a second of time, which in the latitude of London has been found to be nearly $16\frac{11}{120}$ feet. But this, like the above standards, must vary in different latitudes; and the operation is besides extremely difficult to be performed with accuracy.

The ancients mostly adjusted their standards by the dimensions of some durable buildings. In Egypt, the base of one of the pyramids was used; and it is stated by Paucton, that a degree of the meridian was also measured there at a very early period, by which the Greeks and Romans adjusted their standards.

MEASURE, in a legal, commercial, and popular sense, denotes a certain quantity or proportion of any thing, bought, sold, valued, or the like. The regulation of weights and measures ought to be universally the same throughout the nation, and indeed all nations; and they should therefore be reduced to some fixed rule or standard. Measures are various, according to the various kinds or dimensions of the things measured. Hence arise

Lineal or *Longitudinal* MEASURES, for lines or lengths:

Square MEASURES, for areas or superficies: and

Solid or *Cubic* MEASURES, for the solid contents and capacities of bodies.

The standards of English weights and measures, like those of all other countries, are uncertain in their origin. That of long measure is said to have been fixed in the year 1101, by Henry the 1st, who ordained that the ancient ulna or arm, which answers to the modern yard (the Saxon gyrd or girth), should be adjusted to the length of his arm. This standard is subdivided into feet, inches, and barley-corns; and multiplied into poles, furlongs, miles, &c. The standards of English weights appear to have been originally from grains of wheat; 32 of which were directed, by the Compositio Mensurarum, to make a pennyweight, and 20 pennyweights an ounce.

The standards, both of English weights and measures, are chiefly kept in the Exchequer at Westminster, from which copies are taken, and committed to the care of magistrates and other officers, in different parts of the kingdom, who are empowered to examine the weights and measures of their respective districts, and to condemn such as are found erroneous. From the Exchequer, standards are obtained for public offices, and also for individuals, with indentures or licences for sizing, adjusting, and vending weights and measures. The principal office of this kind is at Guildhall, London, where several ancient standards are kept, and occasionally compared with those of the Exchequer. Here the avoirdupois weights, which are cast by the Founders' Company, for the use of the city and for other purchasers, are sized and sealed; and measures of capacity are likewise adjusted. Standards are also kept at the Tower, particularly for troy weight. By these regulations a uniformity of weights and measures is established throughout the kingdom; but measures of capacity, particularly those for corn, vary considerably in different places.

In the year 1758, a committee of the House of Commons was appointed, to inquire into the standards of English weights and measures. It was composed chiefly of men of science; who were assisted in their researches by several eminent mathematicians and mechanists. The report of this committee, which is printed in the minutes of the house, contains the most full and authentic statement of the English weights and measures perhaps ever published; and as no alteration in them has since taken place, the substance of the report is here given, with some account of the proceedings of the committee.

From the report it appears, that the subdivisions of the original standards, at the Exchequer and at Guildhall, d not perfectly agree in their various combinations. Thei differences however are very small, and are of the less im portance, as the principal standards of long measures an of weights are sufficiently correct.

With respect to the measures of capacity, considerabl differences were found to exist in the subdivisions; as we as a great diversity in the corn bushel in different parts the kingdom, notwithstanding the numerous acts of pa liament which had been passed to enforce uniformity. all these acts, the Winchester bushel is stated to be t only legal one. This is the bushel now used at the port of London, at Mark-lane, and at Guildhall; and yet it does not exactly agree, either in shape or contents, with the standard bushel at the Exchequer.

As to the different kinds of weights, the committee recommended that the troy pound should be made the unit or standard, by which the avoirdupois and other weights should be regulated; because it is the weight best known to our laws, and that which has been longest in use; that by which our coins are weighed, and which is best known to the rest of the world; that to which our learned countrymen have referred, in comparing ancient and modern weights; and that which has been divided into the smallest proportions or parts.

Indeed this pound (called by the Romans the pondus or weight, and also the libra or balance,) is the most general standard or unit for weights, as the foot is for measures; and it is remarkable that both have been divided into the same number of equal parts, and that their divisions were anciently called by the same name, uncia, which signifies the 12th part of a whole. Hence, the ounce and inch have one common derivation, the former being called uncia libræ, and the latter uncia pedis.

The committee, having found some variations in the divisions and multiples of the standard troy pound at the Tower, caused it to be divided into halves, quarters, eighths, &c, down to the 1000th part of a grain. These divisions were made with so much accuracy, as to answer their due proportions in every possible combination; and for the purpose of ascertaining them with the greatest correctness, a very curious weighing apparatus was constructed by Mr. Bird, which is still carefully preserved in the Mint. It is adapted to five different beams, which ascertain the weights from 12 ounces down to 1 grain, and with so much exactness as to discern an error to the 2000th part of a grain. By this apparatus it has been found that the brass standard avoirdupois pound, kept in the Mint, weighed exactly 7000 grains; and it was further ascertained that this pound perfectly agreed with the bell standard pound (of 1588) at the Exchequer, and also with the bell standard pound at Guildhall.

The Royal Society of London have paid very laudable attention to the subject of weights and measures, at different periods, particularly in 1768, under the management of Dr. Maskelyne the astronomer-royal, and in 1798, under that of Sir George Shuckburgh; as may be seen in the Philos. Trans. of those years. And the same has been done by the Society of Arts in London, as before noticed. In 1802, M. Pictet, professor of philosophy at Geneva, made the following trials of the different English standards of length, by a scale constructed with great accuracy by Mr. Troughton, of London; and by means of a comparer made by the same ingenious artist, capable of showing differences to the 10,000th part of an inch. The following results were from trials made on several standard yards, in the temperature of 62° of Fahrenheit's thermometer.

	Inches.
Parliamentary standard of 1758, by Bird	36·00023
Royal Society's ditto, also made by Bird	35·99955
Ditto, by Mr. Graham - - - -	36·00130
Exchequer standard - - - -	35·99380
Tower standard - - - -	36·00400
General Roy's do. (for the trigon. survey)	36·00036

The above statement was presented by M. Pictet to the National Institute of France; when, by several trials with the same apparatus, the new French metre was found to be 39·371 English inches, which in 1800 had been found by the Royal Society of London to be 39·3702, from a comparison with two toises sent by M. Lalande to Dr. Maskelyne. See the article **Weights**, also Kelly's Universal Cambist.

In the spring of the year 1814, the English parliament again took into consideration the forming general standards of uniform measures and weights, which might be conveniently used in all the British dominions. For this purpose, a committee of their members was appointed, who, after consulting and examining some learned scientific men, delivered in their report, the results of which are the following:

1. The committee recommended that the brass standard yard kept in the Court of Exchequer should be adopted and considered as the original standard measure; as divided into 3 feet, and each foot into 12 inches, or the yard 36 inches.—2. The committee then assert that the simple pendulum vibrating seconds in the latitude of London, in the temperature of 56½ degrees of Fahrenheit's thermometer, has been found to measure 39·13 such inches.—3. That a cubical foot of pure or distilled water weighs just 1000 ounces avoirdupois; and that therefore the avoirdupois weight will be the most convenient to adopt for the general weight of the country, the pound being divided into 16 ounces, and the ounce into 16 drachms. That, as a cubical foot of water weighs 1000 ounces, therefore, by proportion, the pound or 16 ounces of water will contain 27·648 cubic inches; from which all other weights, above and below the pound, are to be estimated proportionally.—4. That, as the standard weights are thus derived from the lineal measures, so the measures of capacity are recommended to be derived from the standard weights, in this manner; viz, that the gallon measure shall contain 10 pounds weight of water; consequently that the table of measures of capacity will be as follows:

The gallon of 10 lb. = 276·48 cubic inches.
The bushel = 8 gallons = 80 lb. = 2211·84 cub. inch.
The quart = ¼ gallon = 40 oz. = 69·12 cub. inch.
The pint = ⅛ gallon = 20 oz. = 34·56 cub. inch.
The half pint = 10 oz. = 17·28 cub. in. or $\frac{1}{100}$ of a cub. ft.

For the committee's report at large, see the Philosophical Magazine, vol. 44, pa. 171.

The several measures used in England, are as in the following tables:

1. *English Long Measure.*

Barley Corns
3 = 1 Inch
36 = 12 = 1 Foot
108 = 36 = 3 = 1 Yard
594 = 198 = 16½ = 5½ = 1 Pole
23760 = 7920 = 660 = 220 = 40 = 1 Furlong
190082 = 63360 = 5280 = 1760 = 320 = 8 = 1 Mile

Also, 4 Inches = 1 Hand
6 Feet, or 2 yds = 1 Fathom
3 Miles = 1 League
60 Nautical or Geograph. Miles = 1 Degree
or $69\frac{1}{15}$ Statute Miles = 1 Degree nearly
360 Degrees, or 24,864 Miles nearly = the circumference of the meridian.

2. *Cloth Measure.*

Inches
2¼ = 1 Nail
9 = 4 = 1 Quarter
36 = 16 = 4 = 1 Yard
27 = 12 = 3 = 1 Ell Flemish
45 = 20 = 5 = 1 Ell English
54 = 24 = 6 = 1 Ell French.

3. *Square Measure.*

Inches
144 = 1 Foot
1296 = 9 = 1 Yard
39204 = 272¼ = 30¼ = 1 Pole
1568160 = 10890 = 1210 = 40 = 1 Rood
6272640 = 43560 = 4840 = 160 = 4 = 1 Acre.

4. *Solid*, or *Cubical Measure.*

Inches
1728 = 1 Foot
46656 = 27 = 1 Yard

5. *Wine Measure.*

Pints
2 = 1 Quart
8 = 4 = 1 Gallon = 231 Cubic Inches
336 = 168 = 42 = 1 Tierce
504 = 252 = 63 = 1½ = 1 Hogshead
672 = 336 = 84 = 2 = 1⅓ = 1 Puncheon
1008 = 504 = 126 = 3 = 2 = 1½ = 1 Pipe
2016 = 1008 = 252 = 6 = 4 = 3 = 2 = 1 Tun.

Also, 231 Cubic Inches = 1 Gallon
10 Gallons = 1 Anker
18 Gallons = 1 Runlet
31½ Gallons = 1 Barrel.

6. *Ale and Beer Measure.*

Pints
2 = 1 Quart
8 = 4 = 1 Gallon = 282 Cubic Inches
72 = 36 = 9 = 1 Firkin
144 = 72 = 18 = 2 = 1 Kilderkin
288 = 144 = 36 = 4 = 2 = 1 Barrel
432 = 216 = 54 = 6 = 3 = 1½ = 1 Hogshead
576 = 288 = 72 = 8 = 4 = 2 = 1⅓ = 1 Puncheon
864 = 432 = 108 = 12 = 6 = 3 = 2 = 1½ = 1 Butt

Note, The Ale gallon contains 282 inches.

7. *Dry Measure.*

Pints
8 = 1 Gallon = 268⅘ Cubic Inches
16 = 2 = 1 Peck
64 = 8 = 4 = 1 Bushel
256 = 32 = 16 = 4 = 1 Coom
512 = 64 = 32 = 8 = 2 = 1 Quarter
2560 = 320 = 160 = 40 = 10 = 5 = 1 Wey
5120 = 640 = 320 = 80 = 20 = 10 = 2 = 1 Last.

Also, 268⅘ Cubic Inches = 1 Gallon.
and 36 Bushels of Coals = 1 Chaldron.

8. *Proportion of the Long Measures of several Nations to the English Foot.*

		Thousandth Parts.	Inches.			Thousandth Parts.	Inches.
English	foot	1000	12·000	Amsterdam	ell	2269	27·228
Paris	foot	1065¾	12·792	Antwerp	ell	2273	27·276
Rynland, or Leyden	foot	1033	12·396	Rynland, or Leyden	ell	2260	27·120
Amsterdam	foot	942	11·304	Frankfort	ell	1826	21·912
Brill	foot	1103	13·236	Hamburg	ell	1905	22·860
Antwerp	foot	946	11·352	Leipsic	ell	2260	27·120
Dort	foot	1184	14·208	Lubeck	ell	1908	22·896
Lorrain	foot	958	11·496	Noremburg	ell	2227	26·724
Mechlin	foot	919	11·028	Bavaria	ell	954	11·448
Middleburg	foot	991	11·892	Vienna	ell	1053	12·636
Strasburg	foot	920	11·040	Bononia	ell	2147	25·764
Bremen	foot	964	11·568	Dantzic	ell	1903	22·836
Cologn	foot	954	11·448	Florence	Brace or ell	1913	22·956
Frankfort ad Mænum	foot	948	11·376	Spanish, or Castile	palm	751	9·012
Spanish	foot	1001	12·012	Spanish	vare	3004	36·040
Toledo	foot	899	10·788	Lisbon	vare	2750	33·000
Roman	foot	967	11·604	Gibraltar	vare	2760	33·120
On the monument of Cestius Statilius	foot	972	11·664	Toledo	vare	2685	32·220
				Naples	palm	861	10·332
Bononia	foot	1204	14·448		brace	2100	25·200
Mantua	foot	1569	18·838		canna	6880	82·560
Venice	foot	1162	13·944	Genoa	palm	830	9·960
Dantzic	foot	944	11·328	Milan	calamus	6544	78·528
Copenhagen	foot	965	11·580	Parma	cubit	1866	22·392
Prague	foot	1026	12·312	China	cubit	1016	12·192
Riga	foot	1831	21·972	Cairo	cubit	1824	21·888
Turin	foot	1062	12·744	Old Babylonian	cubit	1520	18·240
The Greek	foot	1007	12·084	Old Greek	cubit	1511	18·132
Old Roman	foot	970	11·640	Old Roman	cubit	1458	17·496
Lyons	ell	3967	47·604	Turkish	pike	2200	26·400
Bologna	ell	2076	24·912	Persian	arash	3197	38·364

MEASURING, the same as Mensuration.

MECHAIN (Pierre-François-André), a member of the National Institute, and of the Board of Longitude, F. R. S. Lond. &c, was born at Laon, in the department of Aisne, April 16, 1744, and died in the province of Valencia in Spain, of an epidemic disorder, as he was prosecuting the measurement of an arc of the meridian, Sept. 20, 1805.—Having shown at an early age a good taste for mathematics, Mechain was sent to Paris, where he was patronized by M. Lalande, was entered in the depôt of the marine, and afterwards made two voyages with M. Bretonniere, to survey the coasts of France from Neuport to St. Maloes. The first memoirs which made him known as an astronomer, were on the occultation of Aldebaran, which had been observed in 1744, on the great eclipse of the sun in 1778, and on the opposition of Jupiter in 1779. After this period, he rapidly advanced to celebrity, became in 1782 a member of the French Academy, and in 1785 was intrusted with the direction of the Connoissance des Temps, the volumes of which, from 1788 to 1794, were greatly enriched by his memoirs.—He was also united with MM. Cassini and Legendre in measuring a series of triangles, to correspond with those of the English mathematicians, for the purpose of rectifying the relative positions of the observatories of Greenwich and Paris; of which Mechain gave details in the Memoirs of the Academy. When the Academy also was consulted by the Constituent Assembly, on the choice of a new system of measures, and proposed for the base of this measure, a quarter of the earth's meridian, the length of which arc was to be ascertained with the greatest possible exactness, M. Mechain was one of those who were appointed to this important undertaking. The arc proposed to be measured extended from Dunkirk to Barcelona, which he continued to labour upon till 1798, when he returned to Paris. But, to complete that work, he wished to continue it as far as the Balearic islands, for which purpose he set out again in 1803. He had already, with vast difficulty, revisited all the stations, and completed the observations at three of them, when he was cut off by a fever which prevails every year on the coast of Valencia, by reason of the morasses produced from the overflowing of the rivers.

In his character, M. Mechain is represented as remarkably modest and silent, seldom speaking at the meetings of the society. Before his last expedition it seems he intrusted to M. Delambre all his registers and manuscripts, extracts from which, it is said; will contribute more to Mechain's praise than the most eloquent oration.

MECHANICS, a mixed mathematical science, that treats of forces, motion, and moving powers, with their effects in machines, &c. The science of mechanics is distinguished, by Sir Isaac Newton, into practical and rational; the former treats of the mechanical powers, and of their various combinations; the latter, or rational mechanics, comprehends the whole theory and doctrine of forces, with the motions and effects produced by them.

That part of mechanics which treats of the weight, gravity, and equilibrium of bodies and powers, is called statics; as distinguished from that part which considers the mechanical powers, and their application, which is properly called mechanics.

Some of the principles of statics were established by Archimedes, in his Treatise on the Centre of Gravity of Plane Figures: besides which, little more on mechanics is to be found in the writings of the ancients, except what is contained in the 8th book of Pappus's Mathematical Collections, concerning the five mechanical powers. Galileo laid the best foundation of mechanics, when he investigated the descent of heavy bodies; and since his time, by the assistance of the new methods of computation, a great progress has been made, especially by Newton, in his Principia, which is a general treatise on rational and physical mechanics, in its largest extent. Other writers on this science, or some branch of it, are, Guido Ubaldus, in his Liber Mechanicorum; Torricelli, Libri de Motu Gravium naturaliter Descendentium et Projectorum; Balianus, Tractatus de Motu naturali Gravium; Huygens, Horologium Oscillatorium, and Tractatus de Motu Corporum ex Percussione; Leibnitz, Resistentia Solidorum, in Acta Eruditor. an. 1684; Guldinus, De Centro Gravitatis; Wallis, Tractatus de Mechanica; Varignon, Projet d'une Nouvelle Mechanique, and his papers in the Memoir. Acad. an. 1702; Borelli, Tractatus De Vi Percussionis, De Motionibus Naturalibus a Gravitate pendentibus, and De Motu Animalium; De Chales, Treatise on Motion; Pardies, Discourse of Local Motion; Parent, Elements of Mechanics and Physics; Casatus, Mechanica; Oughtred, Mechanical Institutions; Rohault, Tractatus de Mechanica; Lamy, Mechanique; Keill, Introduction to true Philosophy; Labire, Mechanique; Mariotte, Traité du Choc des Corps; Ditton, Laws of Motion; Herman, Phoronomia; Gravesande, Physics; Euler, Tractatus de Motu; Musschenbroek, Physics; Bossut, Mechanique; Desaguliers, Mechanics; Rowning, Natural Philosophy; Emerson, Mechanics; Parkinson, Mechanics; Lagrange, Mechanique Analytique; Nicholson, Introduction to Natural Philosophy; Enfield, Institutes of Natural Philosophy, &c, &c. As to the Description of Machines, see Strada, Zeisingius, Besson, Augustine de Ramellis, Boetler, Leopold, Sturmy, Perrault, Limberg, Emerson, Royal Academy of Sciences, Gregory's Mechanics, &c.

In treating of machines, we should consider the weight that is to be raised, the power by which it is to be raised, and the instrument or engine by which this effect is to be produced. And, in treating of these, there are two principal problems that present themselves: the first is, to determine the proportion which the power and weight ought to have to each other, that they may just be in equilibrio; the second is, to determine what ought to be the proportion between the power and weight, that a machine may produce the greatest effect in a given time. All writers on mechanics treat on the first of these problems; but few have considered the second, though not less useful than the other.

As to the first problem, this general rule holds in all powers, namely, that when the power and weight are reciprocally proportional to the distances of the directions in which they act, from the centre of motion; or when the product of the power by the distance of its direction, is equal to the product of the weight by the distance of its direction; this is the case in which the power and weight sustain each other, and are in equilibrio; so that the one would not prevail over the other, if the engine were at rest; and if it were in motion, it would continue to proceed uniformly, if it were not for the friction of its parts, and other resistances. And, in general, the effect of any power, or force, is as the product of that force multiplied by the distance of its direction from the centre of motion, or the product of the power and its velocity when in motion, since this velocity is proportional to the distance from that centre.

Maximum effects of Machines.—This second general problem in mechanics, is, to determine the proportion between the power and weight, so that when the power prevails, and the machine is in motion, the greatest effect possible may be produced by it in a given time. It is manifest, that this is an inquiry of the greatest importance, though few have treated of it. When the power is only a little greater than what is sufficient to sustain the weight, the motion usually is too slow; and though a greater weight be raised in this case, it is not sufficient to compensate for the loss of time. On the other hand, when the power is much greater than what is sufficient to sustain the weight, this is raised in less time; but it may happen that this is not sufficient to compensate for the loss arising from the smallness of the load. It ought therefore to be determined when the product of the weight multiplied by its velocity, is the greatest possible; for this product measures the effect of the engine in a given time, which is always the greater in proportion both as the weight is greater, and as its velocity is greater. For some calculations on this problem, see Maclaurin's Account of Newton's Discoveries, pa. 171, &c; also his Fluxions, art. 908 &c; Gregory's Mechanics; also vol. 3 of my Course of Mathematics, chap. xi. And, for the various properties in mechanics, see the several terms MOTION, FORCE, MECHANICAL POWERS, LEVER, &c.

MECHANIC, or MECHANICAL, something relating to mechanics, or regulated by the nature and laws of motion.

MECHANICAL is also used in mathematics, to signify a construction or proof of some problem, not done in an accurate and geometrical manner, but coarsely and unartfully, or by the assistance of instruments; as are most problems relating to the duplicature of the cube, and the quadrature of the circle.

MECHANICAL *Affections*, such properties in matter, as result from their figure, bulk, and motion.

MECHANICAL *Causes*, are such as are founded on Mechanical Affections.

MECHANICAL *Curve*, called also *Transcendental*, is one whose nature cannot be expressed by a finite algebraical equation.

MECHANICAL *Philosophy*, also called the *Corpuscular Philosophy*, is that which explains the phenomena of nature, and the operations of corporeal things, on the principles of mechanics; viz, the motion, gravity, figure, arrangement, disposition, greatness, or smallness of the parts which compose natural bodies.

MECHANICAL *Solution*, of a problem, is either when the thing is done by repeated trials, or when the lines used in the solution are not truly geometrical, or by organical construction.

MECHANICAL *Powers*, are certain simple machines which are used for raising greater weights, or overcoming greater resistances than could be effected by the natural strength without them.

These simple machines are usually accounted six in number, viz, the lever, the wheel-and-axle, or axis in peritrochio, the pulley, the inclined plane, the wedge, and the screw. Of the various combinations of these simple powers do all engines, or compound machines consist: and in treating of them, so as to settle their theory and properties, they are considered as mathematically exact, or void of weight and thickness, and moving without friction. See the properties and demonstrations of each of these under the several words LEVER, &c. To which may be added the following general observations on them all, in a connective way.

1. A *Lever*, the most simple of all the mechanic powers, is an engine chiefly used to raise large weights to small heights; such as a handspike, when of wood; and a crow, when of iron. In theory, a lever is considered as an inflexible line, like the beam of a balance, and subject to the same proportions; only that the power applied to it, is commonly an animal power; and from the different ways of using it, or applying it, it is called a lever of the first, second, or third kind: viz, of the 1st kind, when the weight is on one side of the prop, and the power on the other; of the 2d kind, when the weight is between the prop and the power; and of the 3d kind, when the power is between the prop and the weight.

Many of the instruments in common use, are levers of one of the three kinds; thus, pincers, sheers, forceps, snuffers, and such like, are compounded of two levers of the first kind; for the joint about which they move, is the fulcrum, or centre of motion; the power is applied to the handles, to press them together; and the weight is the body which they pinch or cut. The cutting-knives used by druggists, patten-makers, block-makers, and some other trades, are levers of the 2d kind: for the knife is fixed by a ring at one end, which makes the fulcrum, or fixed point; while the other end is moved by the hand, or power; and the body to be cut, or the resistance to be overcome, is the weight. Doors are levers of the 2d kind; the hinges being the centre of motion; the hand applied to the lock is the power; while the door or weight lies between them. A pair of bellows consists of two levers of the 2d kind; the centre of motion is where the ends of the boards are fixed near the pipe; the power is applied at the handles; and the air pressed out from between the boards, by its resistance, acts against the middle of the boards like a weight. The oars of a boat are levers of the 2d kind: the fixed point is the blade of the oar in the water; the power is the hand acting at the other end; and the weight to be moved is the boat. And the same of the rudder of a vessel. Spring sheers and tongs are levers of the 3d kind; where the centre of motion is at the bow-spring at one end; the weight or resistance is acted on by the other end; and the hand or power is applied between the ends. A ladder raised by a man against a wall, is a lever of the 3d kind: and so are also almost all the bones and muscles of animals.

In all levers, the effect of any power or weight, is both proportional to that power or weight, and also to its distance from the centre of motion. And hence it is that, in raising great weights by a lever, we choose the longest levers; and also rest it upon a point as far from the hand or power, and as near to the weight, as possible. Hence also there will be an equilibrium between the power and weight, when those two products are equal, viz, the power multiplied by its distance, equal to the weight multiplied by its distance; that is, when the weight and power are to each other reciprocally as their distances from the fulcrum or fixed parts.

2. *The Axis in Peritrochio*, or *Wheel and Axle*, is a simple engine consisting of a wheel fixed upon the end of an axle, so that they both turn round together in the same time. This engine may be referred to the lever: for the centre of the axis, or wheel, is the fixed point; the radius of the wheel is the distance of the power, acting at the circumference of the wheel from that point; and the

radius of the axle is the distance of the weight from the same point. Hence the effect of the power, independent of its own natural intensity, is as the radius of the wheel; and the effect of the weight is as the radius of the axle: so that the two will be in equilibrio, when the two products are equal, which are made by multiplying each of these, the weight and power, by the radius, or distance at which it acts; and then also, the weight and power are reciprocally proportional to those radii.

In practice, the thickness of the rope, that winds upon the axle, and to which the weight is fastened, is to be considered: which is done, by adding half its thickness to the radius of the axis, for its distance from the fixed point, when there is only one fold of rope upon the axle; or as many times the thickness as there are folds, wanting only one half when there are several folds of the rope, one over another: which is the reason that more power must be applied when the axis is thus thickened; as often happens in drawing water from a deep and narrow well, over which a long axle cannot be placed.

If the rope to which the power is affixed, be successively applied to different wheels, whose diameters are larger and larger; the axis will be turned with still more and more ease, unless the intensity of the power be diminished in the same proportion; and if so, the axis will always be drawn with the same strength by a power continually diminishing: as is the case in spring clocks and watches; where the spiral spring, which is strongest in its action when first wound up, draws the fuzee, or continued axis in peritrochio, first by the smaller wheels, and as it unbends and becomes weak, acts upon the larger wheels, in such a manner that the machinery is always carried round with the same force.

As a small axis would be too weak for very great weights, and a large wheel would be not only expensive, but also inconvenient in its application, requiring more room than perhaps could be spared fot it; therefore, in order that the action of the power may be increased, without incurring either of those inconveniences, a compound axis in peritrochio is used, which is effected by combining wheels and axles by means of pinions, or small wheels, upon the axles, the teeth of which take hold of teeth made in the large wheels; as is seen in clocks, jacks, and other compound machines. And in such a combination of wheels and axles, the effect of the power is increased in the ratio of the continual product of all the axles, or small wheels, to that of all the large ones. Thus, if there be two small wheels and an axle, turning three large wheels; the axle being 2 inches diameter, and each of the small wheels 4 inches, while the large ones are 2 feet or 24 inches diameter; then $2 \times 4 \times 4 = 32$ is the continual product of the small diameters, and $24 \times 24 \times 24 = 13824$ is that of the large ones; therefore 13824 to 32, or 432 to 1, is the ratio in which the power is increased: and if the power be a man, whose natural strength is equal, suppose, to 150 pounds weight, then $432 \times 150 = 64800$lb, or 28ton 18cwt 64lb, is the weight he would be able to balance, suspended about the axle.

3. *A Single Pulley*, is a small wheel, moveable round an axis, called its centre pin; which of itself is not properly one of the mechanical powers, because it produces no mechanical advantage, except convenience; for as the weight hangs by one end of the cord that passes over the pulley, and the power acts at the other end of the same, these act at equal distances from the centre or axis of motion, and consequently the power is equal to the weight when in equilibrio. So that the chief use of the single pulley is to change the direction of the power from upwards to downwards, &c, and to convey bodies to a great height or distance, without a person moving from his place.—But by combining several single pulleys together a considerable gain of power is made, and that in proportion to the additional number of ropes made to pass over them; while it possesses at the same time the properties of a single pulley, by changing the direction of the action in any manner.

4. *The Inclined Plane*, is made by planks, bars, or beams, laid aslope; by which large and heavy bodies may be more easily raised or lowered, by sliding them up or down the plane; and the gain in power is in proportion as the length of the plane to its height, or as radius to the sine of the angle of inclination of the plane with the horizon.—In drawing a weight up an inclined plane, the power acts to the greatest advantage, when its direction is parallel to the plane.

5. *The Wedge*, which resembles a double inclined plane, is very useful to drive in below very heavy weights, to raise them but a small height, also in cleaving and splitting blocks of wood, and stone, &c; and the power gained, is in proportion of the slant side to half the thickness of the back. So that, if the back of a wedge be 2 inches thick, and the side 20 inches long, any weight pressing on the back will balance 20 times as much acting on the side. But the great advantage of a wedge lies in its being urged, not by pressure, but usually by percussion, as the blow of a hammer or mallet; by which means a wedge may be driven in below, and so be made to lift, almost any the greatest weight, as the largest ship, by a man striking the back of a wedge with a mallet.—To the wedge may be referred the axe or hatchet, the teeth of saws, the chisel, the auger, the spade and shovel, knives and swords of all kinds, as also the bodkin and needle, and in short all sorts of instruments which, beginning from edges or points, become gradually thicker as they lengthen; the manner in which the power is applied to such instruments, being different according to their different shapes, and the various uses for which they have been contrived.

6. *The Screw*, is a kind of perpetual or endless inclined plane; the power of which is still farther assisted by the addition of a handle or lever, where the power acts; so that the gain in power, is in the proportion of the circumference described or passed through by the power, to the distance between thread and thread in the screw.—The uses to which the screw is applied, are various; as, the pressing of bodies close together; such as the press for napkins, for bookbinders, for packers, hotpressers, &c.—In the screw, and the wedge, the power has to overcome both the weight, and also a very great friction in those machines; such indeed as amounts sometimes to as much as the weight to be raised, or more. But then this friction is of use in retaining the weight and machine in its place, even after the power is taken off.

If machines or engines could be made without friction, the least degree of power added to that which balances the weight, would be sufficient to raise it. In the lever, the friction is little or nothing; in the wheel-and-axle, it is but small; in pulleys, it is very considerable; and in the inclined plane, wedge, and screw, it is very great.

It is a general property in all the mechanical powers, that when the weight and power are regulated so as to balance each other, and if they be then put in motion, the power and weight will be to each other reciprocally as the velocities of their motion, or the power is to the weight as the velocity of the weight is to the velocity of the power; so that their two momenta are equal, viz, the product of the power multiplied by its velocity, equal to the product of the weight multiplied by its velocity. And hence too, universally, what is gained in power, is lost in time; for the weight moves as much slower as the power is larger.

Hence also it is plain, that the force of the power is not at all increased by engines; only the velocity of the weight, either in lifting or drawing, is so diminished by the application of the instrument, as that the momentum of the weight is not greater than the force of the power. Thus, for instance, if a force can raise a pound weight with a given velocity, it is impossible by the application of that force to any engine to raise 2 pounds weight with the same velocity: but it may be made to raise 2 pounds weight with half the velocity, or even 1000 times the weight with the 1000th part of the velocity.

See Maclaurin's Account of Newton's Philos. Discov. book 2, chap. 3; Hamilton's Philos. Ess. 1; Philos. Trans. 53, pa. 116; or Landen's Memoirs, vol. 1, pa. 1; or Gregory's Mechanics, vol. 1.

MECHANISM, either the construction or the machinery employed in any thing; as the mechanism of the barometer, of the microscope, &c.

MEDIETATES, a term greatly used by Pappus, and some other authors, for sets of proportionals, both arithmetical, geometrical, and harmonical. See Pappus, lib. 3, prop. 1 to prop. 27; also Viviani de Solidis Locis, lib. 3, pa. 81 to 100; and Blondel Resolution des 4 princip. Problemes d'Architecture, pa. 37.

MEDIUM, the same as Mean, either arithmetical, geometrical, or harmonical.

MEDIUM denotes also that space, or region, or fluid, &c, through which a body passes in its motion towards any point. Thus, the air, or atmosphere, is the medium in which birds and beasts live and move, and in which a projectile moves; water is the medium in which fishes move; and ether is a supposed subtile medium in which the planets move. Glass is also called a medium, being that through which the rays of light move and pass. Mediums resist the motion of bodies moving through them, in proportion to their density or specific gravity.

Subtile or *Æthereal* MEDIUM, is an universal one, whose existence is by Newton rendered probable. He makes it universal; and vastly more rare, subtile, elastic, and active than air; and by that means freely permeating the pores and interstices of all other mediums, and diffusing itself through the whole creation. By the intervention of this subtile medium he thinks it is that most of the great phenomena of nature are effected. See ÆTHER.

This medium it would seem he has recourse to, as the first and most remote physical spring, and the ultimate of all natural causes: by the vibrations of this medium, he supposes that heat is propagated from lucid bodies; as also the intenseness of heat increased and preserved in hot bodies, and from them communicated to cold ones. By means of this medium, he supposes that light is reflected, inflected, refracted, and put alternately into fits of easy reflection and transmission; which effects he also elsewhere ascribes to the power of attraction; so that it would seem, the ethereal medium is the source and cause even of attraction itself.

Again, this medium being much rarer within the heavenly bodies, than in the heavenly spaces, and growing denser as it recedes farther from them, he supposes this is the cause of the gravitation of these bodies towards each other, and of the parts towards the bodies.

Again, from the vibrations of the same medium, excited in the bottom of the eye by the rays of light, and thence propagated through the capillaments of the optic nerves into the sensorium, he supposes that vision is performed: and so likewise hearing, from the vibrations of this or some other medium, excited in the auditory nerves by the tremors of the air, and propagated through the capillaments of those nerves into the sensorium: and so of the other senses.

And again, he conceives that muscular motion is performed by the vibrations of the same medium, excited in the brain at the command of the will, and thence propagated through the capillaments of the nerves into the muscles; and thus contracting and dilating them.

The elastic force of this medium, he shows, must be prodigiously great. Light moves at the rate of considerably more than 10 millions of miles in a minute; yet the vibrations and pulsations of this medium, to cause the fits of easy reflection and transmission, must be swifter than light, which is yet 7 hundred thousand times swifter than sound. Its elastic force therefore, in proportion to its density, must be above 490,000 million of times greater than the elastic force of the air, in proportion to its density; the velocities and pulses of the elastic mediums being in a subduplicate ratio of the elasticities, and the rarities of the mediums, taken together. And thus it may be conceived that the vibration of this medium is the cause also of the elasticity of bodies.

Farther, the particles of which it is composed being supposed indefinitely small, even smaller than those of light; if they be likewise supposed, like our air, endued with a repelling power, by which they recede from each other, the smallness of the particles may exceedingly contribute to the increase of the repelling power, and consequently to that of the elasticity and rarity of the medium; by that means fitting it for the free transmission of light, and the free motions of the heavenly bodies, in which the planets and comets may revolve without any considerable resistance. If it be 700,000 times more elastic, and as many times rarer, than air, its resistance will be above 600 million times less than that of water; a resistance that would cause no sensible alteration in the motion of the planets in ten thousand years.

MEIBOMIUS (MARCUS), a very learned person, of a family in Germany which had long been famous for learned men, was born at Helmstadt in 1590. He devoted himself to literature and criticism, but particularly to the learning of the ancients; such as their music, the structure of their galleys, &c. In 1652 he published a collection of seven Greek authors, who had written upon ancient music, to which he added a Latin version by himself. This work he dedicated to Queen Christina of Sweden; in consequence of which he received an invitation to that princess's court, like several other learned men, which he accepted. The queen engaged him one day to sing an air of ancient music, while a person danced the Greek dances to the sound of his voice; but the immoderate mirth which this occasioned in the spectators, so covered him with ridicule, and disgusted him so vehemently, that he abruptly left the court of Sweden immediately, after heartily battering with his fists the face of Bourdelot, the favourite physician and buffoon to the queen, who had persuaded her to exhibit that spectacle.

Meibomius pretended that the Hebrew copy of the Bible was full of errors, and undertook to correct them by means of a metre, which he fancied he had discovered in those ancient writings; but this it seems drew upon him no small raillery from the learned. Nevertheless, besides the work above mentioned, he produced several others, which showed him to be a good scholar; witness his Notes upon Diogenes Laertius in Menage's edition; his Liber de Fabrica Triremium, 1671, in which he thinks he discovered the method in which the ancients disposed their banks of oars; his edition of the Ancient Greek Mythologists; and his Dialogues on Proportions, a curious work, in which the interlocutors, or persons represented as speaking, are Euclid, Archimedes, Apollonius, Pappus, Eutocius, Theo, and Hermotimus. This last work was opposed by Langius, and by Dr. Wallis, in a considerable tract, printed in the first volume of his works. Meibomius died in 1668.

MELODY, is the agreeable effect of different musical sounds, ranged or disposed in a proper succession, being the effect only of one single part, voice, or instrument; by which it is distinguished from harmony, which properly results from the union of two or more musical sounds heard together.

MENISCUS, a lens or glass, convex on one side, and concave on the other. Sometimes also called a lune or lunula. See its figure under the article Lens.

To find the Focus of a Meniscus, the rule is, as the difference between the diameters of the convexity and concavity, is to either of them, so is the other diameter, to the focal length, or distance of the focus from the meniscus. So that, having given the diameter of the convexity, it is easy to find that of the concavity, so as to remove the focus to any proposed distance from the meniscus. For, if D and d be the diameters of the two sides, and f the focal distance; then since,

by the rule $D - d : D :: d : f$,
therefore $d : D :: f - d : f$,
or $f - d : f :: d : D$.

Hence, if D the diameter of the concavity be double to d that of the convexity, f will be equal to D, or the focal distance equal to the diameter; and therefore the meniscus will be equivalent to a plano-convex lens.—Again, if $D = 3d$, or the diameter of the concavity triple to that of the convexity, then will $f = \frac{1}{2}D$, or the focal distance equal to the radius of concavity; and therefore the meniscus will be equivalent to a lens equally convex on either side.—But if $D = 5d$, then will $f = \frac{1}{4}D$; and therefore the meniscus will be equivalent to a sphere.—Lastly, if $D = d$, then will f be infinite; and therefore a ray falling parallel to the axis, will still continue parallel to it after refraction.

MENSTRUUM, Solvent, or Dissolvent, any fluid that will dissolve hard bodies, or separate their parts. Sir Isaac Newton accounts for the action of menstruums from the acids with which they are impregnated; the particles of acids being endued with a strong attractive force, in which their activity consists, and by virtue of which they dissolve bodies. By this attraction they gather together about the particles of bodies, whether metallic, stony, or the like, and adhere very closely to them, so as scarce to be separated from them by distillation, or sublimation. Thus strongly attracting, and gathering together on all sides, they raise, disjoin, and shake asunder the particles of bodies, i. e. they dissolve them; and by the attractive power with which they rush against the particles of the bodies, they move the fluid, and so excite heat, shaking some of the particles to that degree, as to convert them into air, and so generating bubbles.

Dr. Keill has given the theory or foundation of the action of menstruums, in several propositions. See Attraction. From those propositions are perceived the reasons of the different effects of different menstruums; why some bodies, as metals, dissolve in a saline menstruum; others again, as resins, in a sulphureous one; &c: particularly why silver dissolves in aqua fortis, and gold only in aqua regis; all the varieties of which are accounted for, from the different degrees of cohesion, or attraction in the parts of the body to be dissolved, the different diameters and figures of its pores, the different degrees of attraction in the menstruum, and the different diameters and figures of its parts.

MENSURABILITY, the fitness of a body for being applied, or conformable to a certain measure.

MENSURATION, the act, or art, of measuring figured extension and bodies; or of finding the dimensions and contents of bodies, both superficial and solid.

Every different species of mensuration is estimated and measured by others of the same kind: so, the solid contents of bodies are measured by cubes, as cubic inches, or cubic feet, &c; surfaces by squares, as square inches, feet, &c; and lengths or distances by other lines, as inches, feet, &c.

The contents of rectilinear figures, whether plane or solid, can be accurately determined, or expressed; but of many curved ones, this is not possible. So the quadrature of the circle, and cubature of the sphere, are problems that have never yet been accurately solved. See the various kinds of mensuration, as well as that of the different figures, under their respective terms.

The first writers on geometry were chiefly writers on mensuration; as Euclid, Archimedes, &c. See Quadrature; also the Preface to my Mensuration, for more ample information on this subject.

MERCATOR (Gerard), an eminent geographer and mathematician, was born in 1512, at Ruremonde in the Low-Countries. He applied himself with such industry to the sciences of geography and mathematics, that it has been said he often forgot to eat and sleep. The emperor Charles the 5th encouraged him much in his labours; and the duke of Juliers made him his cosmographer. He composed and published a Chronology; a large and small Atlas; and some Geographical Tables; besides some books in philosophy and divinity. He was also so curious, as well as ingenious, that he engraved and coloured his maps himself. He made various maps, globes, and other mathematical instruments for the use of the emperor; and gave the most ample proofs of his uncommon skill in what he professed. His method of laying down charts is still used, which bear the name of Mercator's Charts; also a part of navigation is from him called Mercator's Sailing.—He died at Duisbourg in 1594, at 82 years of age.—See Mercator's *Chart*, below.

Mercator (*Nicholas*), an eminent mathematician and astronomer, whose name in High-Dutch was Hauffman, was born, about the year 1640, at Holstein in Denmark. From his works we learn, that he had an early and liberal education, suitable to his distinguished genius, by which he was enabled to extend his researches into the mathematical sciences, and to make very considerable improvements: for it appears from his writings, as well as from the character given of him by other mathematicians, that his talent rather lay in improving, and adapting any discoveries and improvements to use, than invention. However, his genius for the mathematical sciences was very conspicuous, and introduced him to public regard and esteem in his own country, and facilitated a correspondence with such as were eminent in those sciences, in Denmark, Italy, and England. Some of his correspondents gave him an invitation to this country, which he accepted, and he afterwards continued in England till his death. He had not been long here before he was admitted F. R. S. and gave frequent proofs of his close application to study, as well as of his eminent abilities in improving some branch or other of the sciences. But he is charged sometimes with borrowing the inventions of others, and adopting them as his own. And it appeared on some occasions, that he was not of an over liberal mind in scientific communications. Thus, it had some time before him been observed, that there was an analogy between a scale of logarithmic tangents and Wright's protraction of the nautical meridian line, which consisted of the sums of the secants; though it does not appear by whom this analogy was first discovered. It appears however to have been first published, and introduced into the practice of navigation, by Henry Bond, who mentions this property in an edition of Norwood's Epitome of Navigation, printed about 1645; and he again treats of it more fully in an edition of Gunter's Works, printed in 1653, where he teaches, from this property, to resolve all the cases of Mercator's Sailing by the logarithmic tangents, independent of the table of meridional parts. This analogy had only been found to be nearly true by trials, but not demonstrated to be a mathematical property. Such demonstration seems to have been first discovered by Mercator, who, desirous of making the most advantage of this and another concealed invention of his in navigation, by a paper in the Philosophical Transactions for June 4, 1666, invites the public to enter into a wager with him, on his ability to prove the truth or falsehood of the supposed analogy. This mercenary proposal it seems was not taken up by any one, and Mercator reserved his demonstration. Our author however distinguished himself by many valuable pieces on philosophical and mathematical subjects. His first attempt was, to reduce astrology to rational principles, which proved a vain attempt. But his writings of more particular note, are as follow:

1. Cosmographia, sive Descriptio Cœli et Terræ in Circulos, qua fundamentum sternitur sequentibus ordine Trigonometriæ Sphericorum Logarithmicæ, &c, a Nicolao Hauffman Holsato; printed at Dantzic, 1651, 12mo.

2. Rationes Mathematicæ subductæ anno 1653; Copenhagen, in 4to.

3. De Emendatione annua Diatribæ duæ, quibus exponuntur et demonstrantur Cycli Solis et Lunæ, &c; in 4to.

4. Hypothesis Astronomica nova, et Consensus ejus cum Observationibus; Lond. 1664, in folio.

5. Logarithmotechnia, sive Methodus Construendi Logarithmos nova, accuratá, et facilis; scripto antehac communicata anno sc. 1667 nonis Augusti; cui nunc accedit, Vera Quadratura Hyperbolæ, et Inventio summæ Logarithmorum. Auctore Nicolao Mercatore Holsato è Societate Regia. Huic etiam jungitur Michaelis Angeli Riccii Exercitatio Geometrica de Maximis et Minimis, hic ob argumenti præstantiam et exemplarium raritatem recusa; Lond. 1668, in 4to.

6. Institutionum Astronomicarum libri duo, de Motu Astrorum communi et proprio, secundum hypotheses veterum et recentiorum præcipuas; deque Hypotheseon ex observatis constructione, cum tabulis Tychonianis, Solaribus, Lunaribus, Lunæ-solaribus, et Rudolphinis Solis, Fixarum et quinque Errantium, earumque usu præceptis et exemplis commonstrato. Quibus accedit Appendix de iis, quæ novissimis temporibus cœlitus innotuerunt; Lond. 1676, 8vo.

7. Euclidis Elementa Geometrica, novo ordine ac methodo fere, demonstrata. Una cum Nic. Mercatoris in Geometriam Introductione brevi, qua Magnitudinum Ortus ex genuinis Principiis, et Ortarum Affectiones ex ipsa Genesi derivantur. Lond. 1678, 12mo.

His papers in the Philosophical Transactions, are,

1. A Problem on some Points in Navigation: vol. 1.
2. Illustrations of the Logarithmo-technia: vol. 3.
3. Considerations concerning his Geometrical and Direct Method for finding the Apogees, Excentricities, and Anomalies of the Planets: vol. 5, pa. 1168.

Mercator died in 1694, about 54 years of age.

MERCATOR's *Churt*, or *Projection*, is a projection of the surface of the earth in plano, so called from Gerrard Mercator, a Flemish geographer, who first published maps of this sort in the year 1556; though it was Edward Wright who first gave the true principles of such charts, with their application to navigation, in 1699.

In this chart or projection, the meridians, parallels, and rhumbs, are all straight lines, the degrees of longitude being every-where increased so as to be equal to one another, and having the degrees of latitude also increased in the same proportion; namely, at every latitude or point on the globe, the degrees of latitude, and of longitude, or the parallels, are increased in the proportion of radius to the sine of the polar distance, or cosine of the latitude; or, which is the same thing, in the proportion of the secant of latitude to radius; a proportion which has the effect of representing the parallel circles by parallel and equal right lines, and all the meridians by parallel lines also, but increasing infinitely towards the poles.

From this proportion of the increase of the degrees of the meridian, viz, that they increase as the secant of the latitude, it is very evident that the length of an arch of the meridian, beginning at the equator, is proportional to the sum of all the secants of the latitude, i. e. that the increased meridian, is to the true arch of it, as the sum of all those secants, to as many times the radius. But it is not so evident that the same increased meridian is also analogous to a scale of the logarithmic tangents, which however it is. "It does not appear by whom, nor by what accident, the analogy was discovered between a scale of logarithmic tangents and Wright's protraction of the nautical meridian line, which consisted of the sums of the secants. It appears however to have been first published, and introduced into the practice of navigation, by Mr. Henry Bond, who mentions this property in an edition of Norwood's Epitome of Navigation, printed about 1645; and he again treats of it more fully in an edition of Gunter's Works, printed in 1653, where he teaches, from this property, to resolve all the cases of Mercator's Sailing by the logarithmic tangents, independent of the table of meridional parts. This analogy had only been found however to be nearly true by trials, but not demonstrated to be a mathematical property. Such demonstration, it seems, was first discovered by Mr. Nicholas Mercator, which he offered a wager to disclose; but this not being accepted, Mercator reserved his demonstration; as mentioned in the account of his life in the foregoing page. The proposal however excited the attention of mathematicians to the subject, and demonstrations were not long wanting. The first was published about 2 years after, by James Gregory, in his Exercitationes Geometricæ; from hence, and other similar properties there demonstrated, he shows how the tables of logarithmic tangents and secants may easily be computed from the natural tangents and secants.

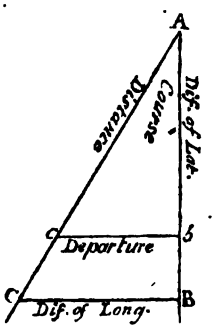
"The same analogy between the logarithmic tangents and the meridian line, as also other similar properties, were afterwards more elegantly demonstrated by Dr. Halley, in the Philos. Trans. for Feb. 1696, and various methods given for computing the same, by examining the nature of the spirals into which the rhumbs are transformed in the stereographic projection of the sphere on the plane of the equator: the doctrine of which was rendered still more easy and elegant by the ingenious Mr. Cotes, in his Logometria, first printed in the Philos. Trans. for 1714, and afterwards in the collection of his works published 1732, by his cousin Dr. Robert Smith, who succeeded him as Plumian professor of philosophy in the university of Cambridge."

The learned Dr. Isaac Barrow also, in his Lectiones Geometricæ, Lect. xi, Append. first published in 1672, delivers a similar property, namely, "that the sum of all the secants of any arc, is analogous to the logarithm of the ratio of $r + s$ to $r - s$, viz, radius plus sine to radius minus sine; or, which is the same thing, that the meridional parts answering to any degree of latitude, are as the logarithms of the ratios of the versed sines of the distances from the two poles." Preface to my Logarithms, pa. 100.

The meridian line in Mercator's Chart, is a scale of logarithmic tangents of the half colatitudes. The differences of longitude on any rhumb, are the logarithms of the same tangents, but of a different species; those species being to each other, as the tangents of the angles made with the meridian. Hence any scale of logarithmic tangents is a table of the differences of longitude, to several latitudes, on some one determinate rhumb; and therefore, as the tangent of the angle of such a rhumb, is to the tangent of any other rhumb, so is the difference of the logarithms of any two tangents, to the difference of longitude on the proposed rhumb, intercepted between the two latitudes, of whose half complements the logarithmic tangents were taken.

It was the great study of our predecessors to contrive such a chart in plano, with straight lines, on which all, or any parts of the world, might be truly set down, according to their longitudes and latitudes, bearings and distances. A method for this purpose was hinted by Ptolemy, near 2000 years since; and a general map, on such an idea, was made by Mercator; but the principles were not demonstrated, and a ready way shown of describing the chart, till Wright explained how to enlarge the meridian line by the continual addition of secants; so that all degrees of longitude might be proportional to those of latitude, as on the globe: which renders this chart, in several respects, far more convenient for the navigator's use, than the globe itself; and which will truly show the course and distance from place to place, in all cases of sailing.

Mercator's *Sailing*, or more properly *Wright's* Sailing, is the method of computing the cases of sailing on the principles of Mercator's chart, which principles were laid down by Edward Wright in the beginning of the 17th century; or the art of finding on a plane the motion of a ship upon any assigned course, that shall be true as well in longitude and latitude, as distance; the meridians being all parallel, and the parallels of latitude straight lines.



In the right-angled triangle A*bc*, let A*b* be the true difference of latitude between two places, the angle *b*AC the angle of the course sailed, and AC the true distance sailed; then will *bc* be what is called the departure, as in plane sailing: produce A*b* till AB be equal to the meridional difference of latitude, and draw BC parallel to *bc*; so shall BC be the difference of longitude.

Now from the similarity of the two triangles A*bc*, ABC, when three of the parts are given, the rest may be found; as in the following analogies: As
Radius : sin. course : : distance : departure;
Radius : cos. course : : distance : dif. latitude;
Radius : tan. course : : merid. dif. lat : dif. longitude.

And by means of these analogies, all the cases of Mercator's Sailing may be resolved.

MERCURY, in Astronomy, the smallest of the inferior planets, and the nearest to the sun, about which it is carried with a very rapid motion. Hence it was, that the Greeks called this planet after the name of the nimble messenger of the gods, and represented it by the figure of a youth with wings at his head and feet; whence is derived ☿, the character in present use for denoting this planet.—The mean distance of Mercury from the sun, is to that of the earth from the sun, as 387 to 1000; and therefore his distance is about 37 millions of miles, or little more than one-third of the earth's distance from the sun. Hence the sun's diameter will appear at Mercury, near 3 times as large as at the earth; and hence also the sun's light and heat received there are about 7 times those at the earth; a degree of heat more than sufficient to make water boil. Such a degree of heat therefore must render Mercury not habitable to creatures of our constitution: and if bodies on its surface be not inflamed, and set on fire, it must be because their degree of density is proportionably greater than that of bodies on our globe.

The diameter of Mercury is also more than ⅓ of the diameter of the earth, or about 3222 miles. Hence the surface of Mercury is nearly 1-9th, and his magnitude or bulk 1-27th of that of the earth.

The inclination of his orbit to the plane of the ecliptic, is 7° 0′; his period of revolution round the sun, 87 days 23 hours; his greatest elongation from the sun 28° 20′; the excentricity of his orbit ⅓ of his mean distance, which is far greater than that of any of the other planets; and he moves in his orbit about the sun at the amazing rate of 95,000 miles an hour.

The place of his aphelion is ♐ 14° 32′; place of ascending node ♉ 14° 44′, and consequently that of the descending node ♏ 14° 44′. His length of day, or rotation on his axis, and inclination of axis to his orbit, are unknown.

Mercury changes his phases, like the moon, according to his various positions with regard to the earth and sun; except only, that he never appears quite full, because his enlightened side is never turned directly towards us, unless when he is so near the sun as to be lost to our sight in his beams. And as his enlightened side is always towards the sun, it is plain that he shines not by any light of his own; for if he did, he would constantly appear round.

The best observations of this planet are those made when it is seen on the sun's disc, called its transit; for in its lower conjunction, it sometimes passes before the sun like a little spot, eclipsing a small part of the sun's body, only observable with a telescope. That node from which Mercury ascends northward above the ecliptic, is in the 15th degree of Taurus, and the opposite in the 15th degree of Scorpio. The earth is in those parts on the 6th of November, and 4th of May, new style; and when Mercury comes to either of his nodes at his inferior conjunction about these times, he will appear in this manner to pass over the sun's disc. But in all other parts of his orbit, his conjunctions are invisible, because he goes either above or below the sun. The first observation of this kind was made by Gassendi, in November 1631. Several following observations of the like transits are collected in Du Hamel's Hist. of the Royal Acad. of Sciences, pa. 470, ed. 2. And Mr. Whiston has given a list of several periods at which Mercury may be seen on the sun's disc, viz, in 1782, Nov. 12, at $3^h\ 44^m$ afternoon; in 1786, May 4th, at $6^h\ 57^m$ in the forenoon; in 1789, Dec. 6th, at $3^h\ 55^m$ afternoon; and in 1799, May 7th, at $2^\circ\ 34^m$ afternoon. There are also several intermediate transits, but none of them visible at London. See Dr. Halley's account of the Transits of Mercury and Venus, in the Philos. Trans. No. 193.

Mercury, a metal of a silvery white colour, and is otherwise called quicksilver. This metal is always fluid at the usual temperature of the atmosphere, but freezes and becomes fixed at the temperature of −39° of Fahrenheit's thermometer, that is 39° below 0, or 71° below the freezing point of water; and it contracts about $\frac{1}{20}$ of its bulk in the moment of freezing. Its boiling point is 660°; it may therefore be totally evaporated, or distilled from one vessel into another, by which means it is purified from various other metallic matters. The vapour of mercury is invisible, and elastic, like common air; like air too, its elasticity is indefinitely increased by heat, so that it breaks the strongest vessel, with an explosion as loud as a cannon.

MERIDIAN, in Astronomy, is a great circle of the celestial sphere, passing through the poles of the world, and both the zenith and nadir, crossing the equinoctial at right angles, and dividing the sphere into two equal parts, or hemispheres, the one eastern, and the other western. Or, the meridian is a vertical circle passing through the poles of the world.—It is called meridian, from the Latin Meridies, mid-day or noon, because when the sun comes to the south part of this circle, it is noon to all those places situated under it.

Meridian, in Geography, is a great circle passing through the north and south poles, and any given place: thus, the meridian of London, is that circle which passes over London and through the poles of the earth; and it lies exactly under, or in the plane of, the celestial meridian.—These meridians are various, and change according to the longitude of places; so that their number may be said to be infinite, since all places from east to west have their several meridians. Farther, as the meridian invests the whole earth, there are many places situated under the same meridian. Also, as it is noon whenever the centre of the sun is in the celestial meridian; and as the meridian of the earth is in the plane of the former; it follows, that it is noon at the same time, in all places situated under the same meridian.

First **Meridian**, is that from which the rest are counted, reckoning both east and west, and is the beginning of longitude. The fixing of the first meridian, is a matter merely arbitrary; and hence different persons, nations, and ages, have fixed it differently: from which circumstance some confusion has arisen in geography. The rule among the ancients was, to make it pass through the place farthest to the west that was known. But the moderns knowing that there is no such place on the earth as can be esteemed the most westerly, the way of computing the longitudes of places from one fixed point is much laid aside.

Ptolemy assumed the meridian that passes through the farthest of the Canary Islands, as his first meridian; that being the most western place of the world then known. After him, as more countries were discovered in that quarter, the first meridian was removed farther off. The Arabian geographers chose to place the first meridian on the utmost shore of the western ocean. Some fixed it to the island of St. Nicholas near the Cape Verd; Hondius to the isle of St. James; others to the island of Del Corvo, one of the Azores; because on that island the magnetic needle at that time pointed directly north, without any variation; and it was not then known that the variation of the needle is itself subject to variation. The latest geographers, particularly the Dutch, have pitched on the Peak of Teneriffe; others on the Isle of Palm, another of the Canaries; and lastly, the French, by order of the king, on the island of Fero, another of the Canaries.

But, without much regard to any of these rules, geographers and constructors of maps often assume the meridian of the place where they live, or the capital of their country, or its chief observatory, for a first meridian; and from it reckon the longitudes of places, east and west.

Astronomers, in their calculations, usually choose the meridian of the place where their observations are made, for their first meridian: as Ptolemy at Alexandria; Tycho Brahé at Uranibourg; Riccioli at Bologna; Flamsteed at the Royal Observatory at Greenwich; and the French at the observatory at Paris.

There is a suggestion in the Philos. Trans. that the meridians vary in time. And it has been said that this is rendered probable, from the old meridian line in the church of St. Petronio at Bologna, which is said to vary no less than 8 degrees from the true meridian of the place at this time: and from the meridian of Tycho at Uranibourg, which M. Picart observes, varies 18 minutes from the modern meridian. If there be any thing of truth in this hint, Dr. Wallis says, the alteration must arise from a change of the terrestrial poles (here on earth, of the earth's diurnal motion), not of their pointing to this or that of the fixed stars: for if the poles of the diurnal motion remain fixed to the same place on the earth, the meridians, which pass through these poles, must remain the same.

But the notion of the changes of the meridian seems to be much weakened by an observation of M. Chazelles, of the French Academy of Sciences, who, when in Egypt, found that the four sides of a pyramid, built 3000 years ago, still pointed very exactly to the four cardinal points: a position which cannot be considered as merely fortuitous. But here again it may be asked, If the meridians vary, may it not be by an oscillatory motion, similar to that of the variation of the magnetic needle, so that at a distance of 3000 years, the observations made in any particular place may agree, though during that period a constant and successive variation may have taken place, vibrating as it were between certain limits, as is now generally known to be the case in other planetary variations, such as the acceleration of the moon, the variation in obliquity of the ecliptic to the equator, &c. And under this point of view the observation of Chazelles would not affect the truth of the other assertions. For measuring an arc of the meridian, see the article Degree.

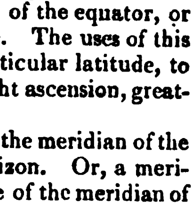
Meridian *of a Globe*, or *Sphere*, is the brazen circle, in which the globe hangs and turns. It is divided into four 90's, or 360 degrees, beginning at the equinoctial: on it, each way, from the equinoctial, on the celestial globes, is counted the north and south declination of the sun, moon, or stars; and on the terrestrial globe, the latitude of places, north and south. There are two points on this circle called the poles; and a diameter, continued from thence through the centre of either globe, is called the axis of the earth, or heavens, on which it is supposed they revolve.

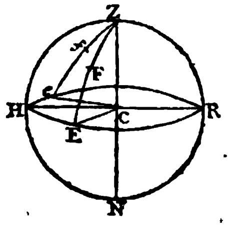
On the terrestrial globes there are usually drawn 36 meridians, one through every tenth degree of the equator, or through every 10th degree of longitude. The uses of this circle are, to set the globes, in any particular latitude, to show the sun's or a star's declination, right ascension, greatest altitude, &c.

Meridian *Line*, an arch, or part, of the meridian of the place, terminated each way by the horizon. Or, a meridian line is the intersection of the plane of the meridian of the place with the plane of the horizon, often called a north-and-south line, because its direction is from north to south.

The meridian line is of most essential use in astronomy, geography, dialling, &c; and the greatest pains are taken by astronomers to fix it at their observatories to the utmost precision. M. Cassini has distinguished himself by a meridian line drawn on the pavement of the church of St. Petronio, at Bologna; being extended to 120 feet in length. In the roof of this church, 1000 inches above the pavement, is a small hole, through which the sun's image, when in the meridian, falling upon the line, marks his progress all the year. When finished, M. Cassini, by a public writing, quaintly informed the mathematicians of Europe, of a new oracle of Apollo, or the sun, established in a temple, which might be consulted, with entire confidence, as to all difficulties in astronomy. See Gnomon.

To draw a Meridian Line.—There are many ways of doing this; but some of the easiest and simplest are as follow: 1. On an horizontal plane describe several concentric circles AB, *ab*, &c; and on the common centre C erect a stile, or gnomon, perpendicular to the horizontal plane of about a foot in length. About the 21st of June, between the hours of 9 and 11 in the morning, and between 1 and 3 in the afternoon, observe the points A, *a*, B, *b*, &c, in the circles, where the shadow of the stile terminates. Bisect the arches AB, *ab*, &c, in D, *d*, &c. If then the same right line DE bisect all these arches, it will be the meridian line sought.—As it is not easy to determine precisely the extremity of the shadow, it will be best to make the stile flat at top, and to drill a small hole through it, noting the lucid point projected by it on the arches AB and *ab*, instead of marking the extremity of the shadow itself.





2. Another method is thus: Knowing the south quarter pretty nearly, observe the altitude FE of some star on the east side of it, and not far from the meridian HZRN: then keeping the quadrant firm on its axis, so as the plummet may still cut the same degree, direct it to the western side of the meridian, and wait till you find the star has the same altitude as before, as *fe*. Lastly, bisect the angle EC*e*, formed by the intersection of the two planes in which the quadrant has been placed at the time of the two observations, by the right line HR, which will be the meridian sought.

Many other methods are given by authors, of describing a meridian line; as by the pole star, or by equal altitudes of the sun, &c; by Schooten in his Exercitationes Geometricæ; Grey, Derham, &c, in the Philos. Trans. and by Ferguson in his Lectures on Select Subjects.

From what has been said it is evident that whenever the shadow of the stile covers the meridian line, the centre of the sun is in the meridian, and therefore it is then noon. And hence the use of a meridian line in adjusting the motion of clocks to the sun. If another stile be erected perpendicularly on any other horizontal plane, and a signal be given when the shadow of the former stile covers the meridian line drawn on another plane, noting the apex or extremity of the shadow projected by the second stile, a line drawn through that point and the foot of the stile will be a meridian line at the 2d place. Or, instead of the 2d stile, a plumb-line may be hung up, and its shadow noted on a plane, upon a signal given that the shadow of another plummet, or of a stile, falls exactly in another meridian line, at a little distance; which shadow will give the other meridian line parallel to the former.

Meridian *Line*, on a dial, is a right line arising from the intersection of the meridian of the place with the plane of the dial. This is the line of noon, or 12 o'clock, and from hence the division of the hour-line begins.

Meridian *Line*, on Gunter's Scale, is divided unequally towards 87 degrees, the same as the meridian in Mercator's chart is divided and numbered. This line is very useful in navigation. For, 1st, It serves to graduate a sea-chart according to the true projection. 2d, Being joined with a line of chords, it serves for the protraction and resolution of such rectilineal triangles as are concerned in latitude, longitude, course, and distance, in the practice of sailing; as also in pricking the chart truly at sea.

Magnetical Meridian, is a great circle passing through or by the magnetical poles; to which meridians the magnetical needle conforms itself.

Meridian *Altitude*, of the sun or stars, is their altitude when in the meridian of the place where they are observed.

MERIDIONAL *Distance*, in Navigation, is the same with the departure, or easting and westing, or distance between two meridians.

MERIDIONAL *Parts*, *Miles*, or *Minutes*, in Navigation, are the parts of the increased or enlarged meridian, in the Mercator's chart. Tables of these parts are found in most books of navigation; and they serve both for constructing that sort of charts, and for working that kind of navigation.

Under the article MERCATOR'S *Chart*, it is shown that the parts of the enlarged meridian increase in proportion as the cosine of the latitude to radius, or, which is the same thing, as radius to the secant of the latitude; and therefore it follows, that the whole length of the enlarged nautical meridian, from the equator to any point, or latitude, will be proportional to the sum of all the secants of the several latitudes up to that point of the meridian. And on this principle was the first table of meridional parts constructed, by the inventor of it, Mr. Edward Wright, and published in 1599; viz, he took the meridional parts

of $1' =$ the sec. of $1'$;
of $2' =$ sec. of $1' +$ sec. of $2'$;
of $3' =$ secants of 1, 2, and 3 min.
of $4' =$ secants of 1, 2, 3, and 4 min.

and so on by a constant addition of the secants.

The tables of meridional parts, so constructed, are perhaps exact enough for ordinary practice in navigation; but they would be more accurate if the meridian were divided into more or smaller parts than single minutes; and the smaller the parts, so much greater the accuracy. But, as a continual subdivision would greatly augment the labour of calculation, other ways of computing such a table have been devised, and treated of, by Bond, Gregory, Oughtred, Sir Jonas Moor, Dr. Wallis, Dr. Halley, and others. See MERCATOR'S *Chart*, and Robertson's Navigation, vol. 2, book 8. The best of these methods was derived from this property, viz. that the meridian line, in a Mercator's chart, is analogous to a scale of logarithmic tangents of half the complements of the latitudes; from which property also a method of computing the cases of Mercator's Sailing has been deduced, by Dr. Halley. Vide ut supra, also the Philos. Trans. vol. 46, pa. 559.

To find the MERIDIONAL PARTS *to any Spheroid, with the same exactness as in a Sphere.*

Let the semidiameter of the equator be to the distance of the centre from the focus of the generating ellipse, as m to 1. Let A represent the latitude for which the meridional parts are required, and s the sine of the latitude, to the radius 1: Find the arc B, whose sine is $\frac{s}{m}$; take the logarithmic tangent of half the complement of B, from the common tables; subtract the log. tangent from 10·0000000, or the log. tangent of 45°; multiply the remainder by the number 7915·7044679, and divide the product by m; then the quotient subtracted from the meridional parts in the sphere, computed in the usual manner for the latitude A, will give the meridional parts, expressed in minutes, for the same latitude in the spheroid, when it is the oblate one.

Example. If $mm : 1 :: 1000 : 22$, then the greatest difference of the meridional parts in the sphere and spheroid is 76·0929 minutes. In other cases it is found by multiplying the remainder above mentioned by the numb 1174·078.

When the spheroid is oblong, the difference in the r ridional parts between the sphere and spheroid, for t same latitude, is then determined by a circular arc. S Philos. Trans. No. 461, sect. 14. Also Maclaurin's Fl ions, art 895, 899. And Murdoch's Mercator's Sailing &

MERLON, in Fortification, that part of the para which lies between two embrasures.

MERSENNE (MARTIN), a learned French auth was born at Bourg of Oyse, in the province of Mai 1588. He studied at La Fleche at the same time wi Descartes; with whom he contracted a strict friendshi which continued till death. He afterwards went to Pa and studied at the Sorbonne; and in 1611 entered hims among the Minims. He became well skilled in Hebre philosophy, and mathematics. From 1615 to 1619, taught philosophy and theology in the convent of Neve and became the superior of that convent. But being d sirous of applying himself more freely and closely to stud he resigned all the posts he enjoyed in his order, and tired to Paris, where he spent the remainder of his lif excepting some short excursions which he occasional made into Italy, Germany, and the Netherlands.

Study and literary conversation were afterwards h whole employment. He held a correspondence with mo of the learned men of his time; being as it were the ve centre of communication between literary men of countries, by the mutual correspondence which he m naged between them; being in France what Mr. Colli was in England. He omitted no opportunity to enga them to publish their works; and the world is obliged him for several excellent discoveries, which would prob bly have been lost, but for his encouragement; and c all accounts he had the reputation of being one of th best men, as well as philosophers, of his time. No pe son was more curious in penetrating into the secrets nature, nor more anxious to bring all the arts and scienc to perfection. He was the chief friend and literary age of Descartes at Paris; giving him advice and assistance o all occasions, and informing him of all that passed a Paris and elsewhere. For, being a person of univers learning, but particularly excelling in physical and ma thematical knowledge, Descartes scarcely ever did an thing, or at least was not perfectly satisfied with an thing he had done, without first knowing what Mersenn thought of it. It is even said, that when Mersenne gav out in Paris, that Descartes was erecting a new system o physics on the foundation of a vacuum, and found th public very indifferent to it on that very account, he immediately sent notice to Descartes, that a vacuum wa not then the fashion at Paris; upon which, that philosopher changed his system, and adopted the old doctrine o a plenum.

Mersenne was a man of good invention also himself and he had a peculiar talent in forming curious questions though he did not always succeed in resolving them; however, he at least gave occasion to others to do it. It is said he invented the cycloid, otherwise called the roulette. Presently the chief geometricians of the age engaged in the contemplation of this new curve, among whom Mersenne himself held a distinguished rank. After a very studious and useful life, he died at Paris in 1648, at 60 years of age.

Mersenne was author of many useful works, particularly the following:

1. Questiones celeberrimæ in Genesim.
2. Harmonicorum Libri.
3. De Sonorum Natura, Causis, et Effectibus.
4. Cogitata Physico-Mathematica; 2 vols. 4to.
5. La Verité des Sciences.
6. Les Questions inouies.

Besides many letters in the works of Descartes and other authors.

MESOLABE, or **Mesolabium**, a mathematical instrument invented by the ancients, for finding two mean proportionals mechanically, which they could not perform geometrically. It consists of three parallelograms, moving in a groove to certain intersections. Its figure is described by Eutocius, in his Commentary on Archimedes. See also Pappus, lib. 3.

METEORIC *Stones*, or Aerolites, certain semi-metallic masses which sometimes fall from the atmosphere. See Aerolite.

METEOROLOGY, is that part of physics which treats of the state of the weather, and the various phenomena of the atmosphere; such as hail, rain, snow, thunder, lightning, &c. Which see under the respective articles.

METIUS (Adrian), a native of Alcmaer, was professor of mathematics and medicine at Franeker, where he died in 1636. He was author of several mathematical works: as, 1. Doctrina Sphæricæ; 2. Astronomiæ universæ Institutio, 8vo; 3. Arithmeticæ et Geometricæ practica, 4to; 4. De Gemino usu utriusque Globi, 4to; 5. Geometrices per usum Circini nova praxis, 8vo.—The ratio 113 to 355, for the diameter to the circumference of a circle, is ascribed to this author.

METIUS (James), brother to Adrian above-mentioned, who invented prospective glasses, or telescopes, one of which he presented to the States-General, in 1609. Tubes, with several pipes had long been used for directing the sight to distant objects, and rendering it more distinct; but those tubes were without glasses, and it seems it was James Metius who first added them. The story goes, that he discovered this method by chance, from seeing some school-boys playing on the ice, who used the covers of their copy-books for tubes, and having in sport placed bits of glass at the ends of these tubes, were much surprised to find, that distant objects, by that means, appeared nearer.

METO, or **Meton**, the son of Pausanias, a famous mathematician of Athens, who flourished 432 years before Christ. In the first year of the 87th Olympiad, he observed the solstice at Athens: and published his Anneadecatoride, that is, his cycle of 19 years; by which he endeavoured to adjust the course of the sun to that of the moon, and to make the solar and lunar years begin at the same point of time. See Cycle.

Metonic Cycle, called also the Golden Number, and Lunar Cycle, or Cycle of the Moon, that which was invented by Meton the Athenian; being a period of 19 years. See Cycle.

METOPE, or **Metopa**, in Architecture, the square space between the triglyphs of the Doric frize; which among the ancients used to be adorned with the heads of beasts, basons, vases, and other instruments used in sacrificing.

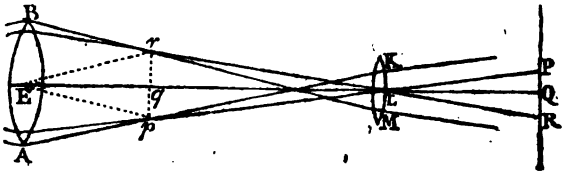
A *Demi-Metope* is a space somewhat less than half a metope, at the corner of the Doric frize.

MICHAELMAS, the feast of St. Michael the archangel; held on the 29th of September.

MICROCOUSTICS, the same with Microphones.

MICROMETER, is an instrument usually fitted to a telescope, in the focus of the object-glass, for measuring small angles or distances; as the apparent diameters of the planets, &c. There are several kinds of these instruments, upon different principles; the origin of which has been disputed. The general principle is, that the instrument moves a fine wire parallel to itself, in the plane of the picture of an object, formed in the focus of a telescope, and so with great exactness to measure its perpendicular distance from a fixed wire in the same plane: and thus are measured small angles, subtended by remote objects at the naked eye.

For example. Let a planet be viewed through the telescope; and when the parallel wires are opened to such a distance as to appear exactly to touch two opposite points in the circumference of the planet, it is evident that the perpendicular distance between the wires is then equal to the diameter of the picture of the planet, formed in the focus of the object-glass. Let this distance, whose measure is given by the mechanism of the micrometer, be represented by the line pq; then, since the measure of the focal distance $q\text{L}$ may also be known, the ratio of $q\text{L}$ to qp, that is, of radius to the tangent of the angle $q\text{L}p$, will give the angle itself, by a table of tangents; and this angle is equal to the opposite angle PQL, which the real diameter of the planet subtends at L, or at the naked eye.



With respect to the invention of the micrometer; Mess. Azout and Picard have the credit of it in common fame, as being the first who published it, in the year 1666; but Mr. Townley, in the Philos. Trans. reclaims it for one of our own countrymen, Mr. Gascoigne. He relates that, from some scattered papers and letters of this gentleman, he had learnt that before our civil wars he had invented a micrometer, of as much effect as that since made by M. Azout, and had made use of it for some years, not only in taking the diameters of the planets, and distances on land, but in determining other matters of nice importance in the heavens; as the moon's distance, &c. Mr. Gascoigne's instrument also fell into the hands of Mr. Townley, who says further, that by the help of it he could make above 40,000 divisions in a foot. This instrument being shown to Dr. Hooke, he gave a drawing and description of it, and proposed several improvements in it; which may be seen in the Philos. Trans. vol. 1, pa. 63. Mr. Gascoigne divided the image of an object, in the focus of the object-glass, by the approach of two pieces of metal, ground to a very fine edge; instead of which, Dr. Hooke would substitute two fine hairs, stretched parallel to each other: and two other methods of Dr. Hooke, different from this, are described in his posthumous works, pa. 497, &c. An account of several curious observations which Mr. Gascoigne made by the help of his micrometer, particularly in measuring the diameter of the moon and other planets, may be seen in the Philos. Trans. vol. 48, pa. 190; where Dr. Bevis refers to an original letter of Mr. Gascoigne, to Mr. Oughtred, written in 1641, for an account given by the author of his own invention, &c.

Mons. Lahire, in a discourse on the æra of the inventions of the micrometer, pendulum clock, and telescope, read before the Royal Academy of Sciences in 1717, makes M. Huygens the inventor of the micrometer. That author, he observes, in his Observations on Saturn's Ring, &c, published in 1659, gives a method of finding the diameters of the planets by means of a telescope, viz, by putting an object, which he calls a virgula, of a size proper to take in the distance to be measured, in the focus of the convex object-glass: in this case, says he, the smallest object will be seen very distinctly in that place of the glass. By such means, he adds, he measured the diameter of the planets, as he there delivers them. See Huygens's System of Saturn.

This micrometer, M. Lahire observes, is so very little different from that published by the Marquis De Malvasia, in his Ephemerides, 3 years after, that they ought to be esteemed the same: and the micrometer of the marquis differed yet less from that published 4 years after his, by Azout and Picard. Hence, Lahire concludes, that it is to Huygens the world is indebted for the invention of the micrometer; without taking any notice of the claim of our countryman Gascoigne, which however is many years prior to any of them.

Lahire says, that there is no method more simple or commodious for observing the digits of an eclipse, than a net in the focus of the telescope. These, he says, were usually made of silken threads; and for this particular purpose 6 concentric circles had also been used, drawn upon oiled paper: but he advises to draw the circles on very thin pieces of glass, with the point of a diamond. He also gives some particular directions to assist persons in using them. In another memoir, he shows a method of making use of the same net for all eclipses, by using a telescope with two object-glasses, and placing them at different distances from each other. Mem. 1701 and 1717.

M. Cassini invented a very ingenious method of ascertaining the right ascensions and declinations of stars, by fixing 4 cross hairs in the focus of the telescope, and turning it about its axis, so as to make them move in a line parallel to one of them. But the later improved micrometers will answer this purpose with greater exactness. Dr. Maskelyne has published directions for the use of it, extracted from Dr. Bradley's papers, in the Philos. Trans. vol. 62. See also Smith's Optics, vol 2, pa. 343.

Wolfius describes a micrometer of a very easy and simple structure, first contrived by Kirchius.

Dr. Derham tells us, that his micrometer is not put into a tube, as is usual, but is contrived to measure the spectres of the sun on paper, of any radius, or to measure any part of them. By this means he can easily, and very exactly, with the help of a fine thread, take the declination of a solar spot at any time of the day; and, by his half-seconds watch, measure the distance of the spot from either limb of the sun.

J. And. Segner proposed to enlarge the field of view in these micrometers, by making them of a considerable extent, and having a moveable eye-glass, or several eye-glasses, placed opposite to different parts of it. He thought however, that two would be quite sufficient, and he gives particular directions how to make use of such micrometers in astronomical observations. See Comm. Gotting. vol. 1, pa. 27.

A considerable improvement in the micrometer was communicated to the Royal Society, in 1743, by Mr. S. Savary; an account of which, extracted from the minutes by Mr. Short, was published in the Philos. Trans. for 1753. The first hint of such a micrometer was suggested by M. Roemer, in 1675: and M. Bouguer proposed a construction similar to that of M. Savary, in 1748; for which see HELIOMETER. The late Mr. Dollond made a further improvement in this kind of micrometer, an account of which was given to the Royal Society by Mr. Short, and published in the Philos. Trans. vol. 48. Instead of two object-glasses, he used only one, which he neatly cut into two semicircles, and fitted each semicircle in a metal frame, so that their diameters sliding in one another, by means of a screw, may have their centres brought together in such a manner as to appear like one glass, and so form one image; or by their centres receding, may form two images of the same object: it being a property of such glasses, for any segment to exhibit a perfect image of an object, though not so bright as the whole glass would give it. If proper scales are fitted to this instrument, showing how far the centres recede, relative to the focal length of the glass, they will also show how far the two parts of the same object are asunder, relative to its distance from the object-glass; and consequently give the angle under which the distance of the parts of that object are seen. This divided object-glass micrometer, which was applied by the late Mr. Dollond to the object end of a reflecting telescope, and has been with equal advantage adapted by his son to the end of an achromatic telescope, is of so easy use, and affords so large a scale, that it is generally considered by astronomers as the most convenient and exact instrument for measuring small angles in the heavens. However, the common micrometer is peculiarly adapted for measuring differences of right ascension, and declination, of celestial objects, but less convenient and exact for measuring their absolute distances; whereas the object-glass micrometer is peculiarly fitted for measuring distances, though generally supposed improper for the former purpose. But Dr. Maskelyne has found that this may be applied with very little trouble to that purpose also; and he has furnished the directions necessary to be followed, when it is used in this manner. The addition requisite for this purpose, is a cell, containing two wires, intersecting each other at right angles, placed in the focus of the eye-glass of the telescope, and moveable about, by the turning of a button. For the description of this apparatus, with the method of applying and using it, see Dr. Maskelyne's paper on the subject, in the Philos. Trans. vol. 61, pa. 536, &c.

After all, the use of the object-glass micrometer is attended with many difficulties, arising from the alterations in the focus of the eye, which are apt to cause it to give different measures of the same angle at different times. To obviate these difficulties, Dr. Maskelyne, in 1776, contrived a prismatic micrometer, consisting of two achromatic prisms, or wedges, applied between the object-glass and eye-glass of an achromatic telescope, by moving of which wedges nearer to or farther from the object-glass, the two images of an object produced by them appeared to approach to, or recede from, each other, so that the focal length of the object-glass becomes a scale for measuring the angular distance of the two images. The rationale and use of this micrometer are explained in the Philos. Trans. vol. 67, pa. 799, &c. And a similar invention by the Abbé Rochon, which was afterwards improved by the Abbé Boscovich, was also communicated to the Royal Society, and published in the same volume of the Transactions, pa. 789, &c.

Mr. Ramsden invented two other micrometers, which he has contrived for remedying the defects of the object-glass micrometer. One of these is a catoptric micrometer, which, besides the advantage it derives from the principle of reflection, of not being disturbed by the heterogeneity of light, avoids every defect of other instruments of this kind, and can have no aberration, nor any defect arising from the imperfection of materials, or of execution; as the great simplicity of its construction requires no additional mirrors or glasses, to those necessary for the telescope; and the separation of the image being effected by the inclination of the two specula, and not depending on the focus of a lens or mirror, any alteration in the eye of an observer cannot affect the angle measured. It has peculiar to itself the advantages of an adjustment, to make the images coincide in a direction perpendicular to that of their motion; and also of measuring the diameter of a planet on both sides of the zero; which will appear no inconsiderable advantage to observers who know how much easier it is to ascertain the contact of the external edges of two images, than their perfect coincidence.

The other micrometer invented and described by Mr. Ramsden, is adapted to the principle of refraction. It is applied to the erect eye-tube of a refracting telescope, and is placed in the conjugate focus of the first eye-glass, as the image is considerably magnified before it comes to the micrometer, any imperfection in its glass will be magnified only by the remaining eye-glasses, which in any telescope seldom exceeds 5 or 6 times; and besides, the size of the micrometer glass will not be the 100th part of the area which would be necessary, if it were placed at the object-glass; and yet the same extent of scale is preserved, and the images are uniformly bright in every part of the field of the telescope. See the description and construction of these two micrometers in the Philos. Trans. vol. 69, part 2, art. 27.

In vol. 72 of the Philos. Trans. for the year 1782, Dr. Herschel, after explaining the defects and imperfections of the parallel-wire micrometer, especially for measuring the apparent diameter of stars, and the distances between double and multiple stars, describes one, for these purposes, which he calls a lamp micrometer; one that is free from such defects, and has the advantage of a very enlarged scale. In speaking of the application of this instrument, he says, "It is well known to opticians and others, who have been in the habit of using optical instruments, that we can with one eye look into a microscope or telescope, and see an object much magnified, while the naked eye may see a scale upon which the magnified picture is thrown. In this manner I have generally determined the power of my telescopes; and any one who has acquired a facility of taking such observations, will very seldom mistake so much as one in 50 in determining the power of an instrument, and that degree of exactness is fully sufficient for the purpose.

"The Newtonian form is admirably adapted to the use of this micrometer; for the observer stands always erect, and looks in a horizontal direction, notwithstanding the telescope should be elevated to the zenith.—The scale of the micrometer at the convenient distance of 10 feet from the eye, with the power of 460, is above a quarter of an inch to a second; and by putting on my power of 932, I obtain a scale of more than half an inch to a second, without increasing the distance of the micrometer; whereas the most perfect of my former micrometers, with the same instrument, had a scale of less than the 2000th part of an inch to a second.

"The measures of this micrometer are not confined to double stars only, but may be applied to any other objects that require the utmost accuracy, such as the diameters of the planets or their satellites, the mountains of the moon, the diameters of the fixed stars, &c."

The micrometer has not only been applied to telescopes, and employed for astronomical purposes; but there have been various contrivances for adapting it to microscopical observations. Mr. Leeuwenhoek's method of estimating the size of small objects, was by comparing them with grains of sand, of which 100 in a line took up an inch. These grains he laid upon the same plate with his objects, and viewed them at the same time. Dr. Jurin's method was similar to this; for he found the diameter of a piece of fine silver wire, by wrapping it very close upon a pin, and observing how many rings made an inch: and he used this wire in the same manner as Leeuwenhoek used his sand. Dr. Hooke used to look upon the magnified object with one eye, while at the same time he viewed other objects, placed at the same distance, with the other eye. In this manner he was able, by the help of a ruler, divided into inches and small parts, and laid on the pedestal of the microscope, as it were to cast the magnified appearance of the object upon the ruler, and thus exactly to measure the diameter which it appeared to have through the glass; and this being compared with the diameter as it appeared to the naked eye, easily determined the degree in which it was magnified. A little practice, says Mr. Baker, will render this method exceedingly easy and pleasant.

Mr. Martin, in his Optics, recommends such a micrometer for a microscope as had been applied to telescopes; for he advises to draw a number of parallel lines on a piece of glass, with the fine point of a diamond, at the distance of one 40th of an inch from one another, and to place it in the focus of the eye-glass. By this method, Dr. Smith contrived to take the exact draught of objects viewed by a double microscope; for this purpose he advises the observer to get a lattice, made with small silver wires or squares, drawn upon a plain glass by the strokes of a diamond, and to put it into the place of the image formed by the object-glass. Then, by transferring the parts of the object, seen in the squares of the glass or lattice, upon similar corresponding squares drawn on paper, the picture may be exactly taken. Mr. Martin also introduced into compound microscopes another micrometer, consisting of a screw. See both these methods described in his Optics, pa. 277.

A very accurate division of a scale is performed by Mr. Coventry, of Southwark. The micrometers of his construction are parallel lines drawn on glass, ivory, or metal, from the 10th to the 10,000th part of an inch. These may be applied to microscopes, for measuring the size of minute objects, and the magnifying power of the glasses; and to telescopes, for measuring the size and distance of objects, and the magnifying power of the instrument. To measure the size of an object in a single microscope; lay it on a micrometer, whose lines are seen magnified in the same proportion with it, and they will give at one view the real size of the object. For measuring the magnifying power of the compound microscope, the best and readiest method is the following: On the stage in the focus of the object-glass, lay a micrometer, consisting of an inch divided into 100 equal parts; count how many divisions of the micrometer are taken into the field of view; then lay a two-foot rule parallel to the micrometer: fix one eye on the edge of the field of light, and the other eye on the end of the rule, which move, till the edge of the field of light and the end of the rule correspond; then the distance from the end of the rule to the middle of the stage, will be half the diameter of the field: ex. gr. If the distance be 10 inches, the whole diameter will be 20, and the number of the divisions of the micrometer contained in the diameter of the field, is the magnifying power of the microscope. For measuring the height and distance of objects by a micrometer in the telescope, see **Telescope**.

Mr. Adams has applied a micrometer, that shows immediately the magnifying power of any telescope.

In the Philos. Trans. for 1791, a very simple scale micrometer for measuring small angles with the telescope is described by Mr. Cavallo. This micrometer consists of a thin and narrow slip of mother-of-pearl finely divided, and placed in the focus of the eye-glass of a telescope, just where the image of the object is formed; whether the telescope is a reflector or a refractor, provided the eye-glass be a convex lens. This substance Mr. Cavallo, after many trials, found much more convenient than either glass, ivory, horn, or wood, as it is a very steady substance, the divisions very easy marked upon it, and when made as thin as common writing-paper it has a very useful degree of transparency.

On this subject, see M. Azout's Tract, contained in Divers Ouvrages de Mathematique et de Phisique; par Messieurs de l'Academie Royal des Sciences; M. de la Hire's Astronomicæ Tabulæ; Mr. Townley, in the Philos. Trans. No. 21; Wolfius, in his Elem. Astron. § 508; Dr. Hooke, and many others, in the Philos. Trans. No. 29, &c; Hevelius, in the Acta Eruditorum, ann. 1708; Mr. Balshaser, in his Micrometria; also several volumes of the Paris Memoirs, &c.

MICROPHONES, instruments contrived to magnify small sounds, as microscopes do small objects.

MICROSCOPE, an optical instrument, composed of lenses or mirrors, by means of which small objects are made to appear larger than they do to the naked eye.

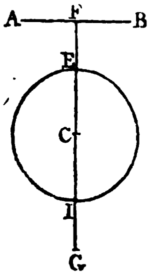
Microscopes are distinguished into simple and compound, or single and double.

Simple, or *Single* Microscopes, are such as consist of a single lens, or a single spherule. And a

Compound Microscope consists of several lenses properly combined.—As optics have been improved, other varieties have been contrived in this instrument: Hence reflecting microscopes, water microscopes, &c. It is not certainly known when, or by whom, microscopes were first invented; though it is probable they would soon follow on the use of telescopes, since a microscope is like a telescope inverted. We are informed by Huygens, that one Drebell, a Dutchman, had the first microscope, in the year 1621, and that he was reputed the inventor of it: though F. Fontana, a Neapolitan, in 1646, claims the invention to himself, and dates it from the year 1618. Be this as it may, it seems they were first used in Germany about 1621; and according to Peter Borelli, they were invented by Zacharias Jansen and his son, who presented the first microscopes they had constructed to prince Maurice, and to Albert arch-duke of Austria. William Borelli, who gives this account in a letter to his brother Peter, says, that when he was ambassador in England, in 1619, Cornelius Drebell showed him a microscope, which he said was the same that the archduke had given him, and had been made by Jansen himself. Borelli De vero Telescopii inventore, pa. 35, also on the Microscope. Se Lens.

Theory and Foundation of Microscopes.

If an object be placed in the focus of the convex len of a single microscope, and the eye be very near on th other side, the object will appear distinct in an erec situation, and be magnified in the ratio of the focal dis tance of the lens, to the ordinary distance of distinct vi sion, viz, about 8 inches. So, if the object AB be placed in the focus F, of a small glass sphere, and the eye behind it, as in the focus G, the object will appear distinct, and in an erect posture, its diameter being increased in the ratio of $\frac{3}{4}$ of the diameter EI to 8 inches. If, ex. gr. the diameter EI of the small sphere be $\frac{1}{10}$ of an inch; then $CE = \frac{1}{20}$, and $FE = \frac{1}{2}CE = \frac{1}{40}$, so that $CF = \frac{3}{40}$; then as $\frac{3}{40} : 8$, or as $3 : 320$, or as $1 : 106\frac{2}{3}$ :: the natural size to the magnified appea ance; in which case, the object is magnified about 1(times.



Hence the smaller the spherule or the lens is, so mu the more is the object magnified. But then, so much t less part is comprehended at one view, and so much t less distinct is the appearance of the object.—Equal a pearances of the same object, formed by different com nations, become obscure in proportion as the number rays constituting each pencil decreases, that is, in prop tion to the smallness of the object-glass. Therefore, if t diameter of the object-glass exceeds the diameter of t pupil, as many times as the diameter of the appeara exceeds the diameter of the object; the appearance w be as clear and distinct as the object itself.

But the diameter of the object-glass cannot be so mu increased, without increasing at the same time the fo distances of all the glasses, and consequently the length the instrument: otherwise the rays would fall too' liquely on the eye-glass, and the appearance become c fused and irregular.

There are several kinds of single microscopes; of wh the following is the most simple. AB (plate 22, fig. is a small tube, to one end of which, BC, it fitted a pl glass; and to this any object, as a gnat, the wing of an sect, or the like, is applied; to the other end AD, proper distance from the object, is applied a lens, con on both sides, of about an inch in diameter: the pl glass is turned to the sun, or the light of a candle, and object is seen magnified. And if the tube be made draw out, lenses or segments of different spheres may used.

Again, a lens, convex on both sides, is inclosed in a AC (fig. 2), and held there by the screw H. Through stem or pedestal CD passes a long screw EF, carryi stile or needle EG. In E is a small tube; on which, on the point G, the various objects are to be dispo Thus, lenses of various spheres may be applied.

A good simple instrument of this kind is Mr. Wilson's pocket microscope, which has 9 different magnifying glasses, 8 of which may be used with two different instruments, for the better applying them to various objects. One of these instruments is represented at AABB (fig. 3), which is made either of brass or ivory. There are three thin brass plates at E, and a spiral spring H of steel wire within it: to one of the thin plates of brass is fixed a piece of leather F, with a small furrow G, both in the leather, and brass to which it is fixed: in one end of this instrument there is a long screw D, with a convex glass C, placed at the end it; at the other end of the instrument there is a hollow screw *oo*, in which any of the magnifying glasses, M, are screwed, when they are to be made use of. The 9 different magnifying glasses are all set in ivory, 8 of which are set in the manner expressed at M. The greatest magnifier is marked upon the ivory, in which it is set; number 1, the next number 2, and so on to number 8; the 9th glass is not marked, but is set in the manner of a little barrel box of ivory, as at *b*. At *ee* is a flat piece of ivory, of which there are 8 belonging to this sort of microscopes (though any one who has a mind to keep a register of objects may have as many of them as he pleases); in each of them there are 3 holes *fff*, in which 3 or more objects are placed between two thin glasses, or talcs, when they are to be used with the greater magnifiers.

The use of this instrument AABB is as follows: A handle W, from fig. 4, being screwed upon the button S, take one of the flat pieces of ivory or sliders *ee*, and slide it between the two thin plates of brass at E, through the body of the microscope, so that the object to be viewed be just in the middle; observing to put that side of the plate *ee*, where the brass rings are, farthest from the end AA: then screw into the hollow screw, *oo*, the 3d, 4th, 5th, 6th, or 7th magnifying glass M; which being done, put the end AA close to your eye, and while looking at the object through the magnifying glass, screw in or out the long screw D, and this moving round upon the leather F, held tight to it by the spiral wire H, will bring the object to the true distance; which may be known by seeing it clearly and distinctly.

Thus may be viewed all transparent objects, dusts, liquids, crystals of salts, small insects, such as fleas, mites, &c. If they be insects that will creep away, or such objects as are to be kept, they may be placed between the two register glasses *ff*. For, by taking out the ring that keeps in the glasses *ff*, where the object lies, they will fall out of themselves; so the object may be laid between the two hollow sides of them, and the ring put in again as before; but if the objects be dusts or liquids, a small drop of the liquid, or a little of the dust laid on the outside of the glass *ff*, and applied as before, will be seen very easily.

As to the 1st, 2d, and 3d magnifying glasses, being marked with a + upon the ivory in which they are set, they are only to be used with those plates or sliders that are also marked with a +, in which the objects are placed between two thin talcs; because the thickness of the glasses in the other plates or sliders, hinders the object from approaching to the true distance from these greater magnifiers. But the manner of using them is the same with the former.

For viewing the circulation of the blood at the extremities of the arteries and veins, in the transparent parts of fishes' tails, &c, there are two glass tubes, a larger and a smaller, as expressed at *gg*, into which the animal is put. When these tubes are to be used, turn the end screw D in the body of the microscrope, until the tube *gg* can be easily received into that little cavity G of the brass plate fastened to the leather F under the other two thin plates of brass at E. When the tail of the fish lies flat on the glass tube, set it opposite to the magnifying glass, and bringing it to the proper distance by screwing in or out the end screw D, and you will then clearly perceive the circulation of the blood.

To view the blood circulating in the foot of a frog; choose such a frog as will just go into the tube; then with a little stick expand its hinder foot, which apply close to the side of the tube, observing that no part of the frog hinders the light from coming on its foot; and when it is brought to the proper distance, by means of the screw D, the rapid motion of the blood will be seen in its vessels, which are very numerous, in the transparent thin membrane or web between the toes. For this object, the 4th and 5th magnifiers will do very well; but the circulation may be seen in the tails of water-newts in the 6th and 7th glasses, because the globules of the blood of those newts are as large again as the globules of the blood of frogs or small fish, as has been remarked in No. 280 of the Philos. Trans. pa. 1184.

The circulation cannot so well be seen by the 1st, 2d, and 3d magnifiers, because the thickness of the glass tube, containing the fish, hinders the approach of the object to the focus of the magnifying glass. Fig. 4 is another instrument for this purpose.

In viewing objects, one ought to be careful not to hinder the light from falling upon them by the hat, hair, or any other thing, especially in looking at opaque objects; for nothing can be seen with the best of glasses, unless the object be at a due distance, with a sufficient light. The best lights for the plates or sliders, when the object lies between the two glasses, is a clear sky-light, or where the sun shines on something white, or the reflection of the light from a looking-glass. The light of a candle is also very proper for viewing small objects, though it be a little uneasy to those who are not practised in the use of microscopes.

To cast small Glass Spherules for MICROSCOPES.—There are several methods for this purpose. Hartsoeker first improved single microscopes by using small globules of glass, melted in the flame of a candle; by which he discovered the animalculæ in semine masculino, and thereby laid the foundation of a new system of generation. Wolfius describes the following method of making such globules: A small piece of very fine glass, sticking to the wet point of a steel needle, is to be applied to the extreme blueish part of the flame of a lamp, or rather of spirits of wine, which will not black it; being there melted, and run into a small round drop, it is to be removed from the flame, on which it instantly ceases to be fluid. Then folding a thin plate of brass, and making very small smooth perforations, so as not to leave any roughness on the surfaces, and also smoothing them over to prevent any glaring, fit the spherule between the plates against the apertures, and put the whole in a frame, with objects convenient for observation.

Mr. Adams gives another method, thus: Take a piece of fine window-glass, and rase it, with a diamond, into as many lengths as you think needful, not more than 1-8th of an inch in breadth; then holding one of those lengths between the fore-finger and thumb of each hand, over a very fine flame, till the glass begins to soften, draw it out till it be as fine as a hair, and break; then applying each of the ends into the purest part of the flame, you presently have two spheres, which may be made greater or less at pleasure: if they remain long in the flame, they will have spots; so that they must be drawn out immediately after they are turned round. Break the stem off as near the globule as possible; and, lodging the remainder of the stem between the plates, by drilling the hole exactly round, all the protuberances are buried between the plates; and the microscope performs to admiration.

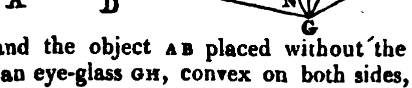
Mr. Butterfield gave another manner of making these globules, in No. 141 of Philos. Trans.

In any of these ways may the spherules be made much smaller than any lens; so that the best single microscopes, or such as magnify the most, are made of them. Leeuwenhoeck and Musschenbroek have succeeded very well in spherical microscopes, and their greatest magnifiers enlarged the diameter of an object about 160 times; Philos. Trans. vol. 7, pa. 129, and vol. 8, pa. 121. But the smallest globules, and consequently the highest magnifiers for microscopes, were made by F. de Torre of Naples, who, in 1765, sent four of them to the Royal Society. The largest of them was only two Paris points in diameter, and magnified a line 640 times; the second was the size of one Paris point, and magnified 1280 times; and the 3d no more than half a Paris point, or the 144th part of an inch in diameter, and magnified 2560 times. But since the focus of a glass globule is at the distance of one-4th of its diameter, and therefore that of the 3d globule of de Torre, above mentioned, only the 576th part of an inch distant from the object, it must be with the utmost difficulty that globules so minute as those can be employed to any purpose; and Mr. Baker, to whose examination they were referred, considers them as matters of curiosity rather than of real use. Philos. Trans. vol. 55, pa. 246, vol. 56, pa. 67.

Water MICROSCOPE. Mr. S. Gray, and, after him, Wolfius and others, have contrived water microscopes, consisting of spherules or lenses of water, instead of glass. But since the distance of the focus of a lens or sphere of water is greater than that in one of glass, the spheres of which they are segments being the same, consequently water microscopes magnify less than those of glass, and therefore are less esteemed. Mr. Gray first observed, that a small drop or spherule of water, held to the eye by candle-light or moon-light, without any other apparatus, magnified the animalcules contained in it, vastly more than any other microscope. The reason is, that the rays coming from the interior surface of the first hemisphere, are reflected so as to fall under the same angle on the surface of the hinder hemisphere, to which the eye is applied, as if they came from the focus of the spherule; whence they are propagated to the eye in the same manner as if the objects were placed without the spherule in its focus.

Hollow glass spheres of about half an inch diameter, filled with spirit of wine, are often used for microscopes; but they do not magnify near so much.

Theory of Compound or Double MICROSCOPES.—Suppose an object-glass ED, the segment of a very small sphere, and the object AB placed without the focus F. Suppose an eye-glass GH, convex on both sides, and the segment of a sphere greater than that of DE, though not too great; and, the focus being at K, let it be so disposed behind the object, that CF : CL :: CL : CK. Lastly suppose LK : LM :: LM : LI. If then O be the place where an object is seen distinct with the naked eye; the eye in this case, being placed in I, will see the object AB distinctly, in an inverted position, and magnified in the compound ratio of MK × LC to LK × CO; as is proved by the laws of dioptrics; that is, the image is larger than the object, and we are able to view it distinctly at a less distance. For example—If the image be 20 times larger than the object, and by the help of the eye-glass we are able to view it 5 times nearer than we could have done with the naked eye, it will, on both these accounts, be magnified 5 times 20, or 100 times.



Laws of Double MICROSCOPES.

1. The more an object is magnified by the microscope, the less is its field, i. e. the less of it is taken in at one view.

2. To the same eye-glass may be successively applied object-glasses of various spheres, so as that both the entire objects, but less magnified, and their several parts, much more magnified, may be viewed through the same microscope. In which case, on account of the different distance of the image, the tube in which the lenses are fitted should be made to draw out.

3. Since it is proved, that the distance of the image LK, from the object-glass DE, will be greater, if another lens, concave on both sides, be placed before its focus; it follows, that the object will be magnified the more, if such a lens be here placed between the object-glass DE, and the eye-glass GH. Such a microscope is much commended by Conradi, who used an object-lens, convex on both sides, whose radius was 2 digits, its aperture equal to a mustard-seed; a lens, concave on both sides, from 12 to 16 digits; and an eye-glass, convex on both sides, of 6 digits.

4. Since the image is projected to the greater distance, the nearer another lens, of a segment of a larger sphere, is brought to the object-glass; a microscope may be composed of three lenses, which will magnify to a prodigious extent.

5. From these considerations it follows, that the object will be magnified the more, as the eye-glass is the segment of a smaller sphere; but the field of vision will be the greater, as the same is a segment of a larger sphere. Therefore if two eye-glasses, the one a segment of a larger sphere, the other of a smaller one, be so combined, as that the object appearing very near through them, i.e. not farther distant than the focus of the first, be yet distinct; the object, at the same time, will be vastly magnified, and the field of vision much greater than if only one lens was used; and the object will be still more magnified, and the field enlarged, if both the object-glass and eye glass be double. But because an object appears dim when viewed through so many glasses, part of the rays being reflected in passing through each, it is not adviseable greatly to multiply glasses; so that, among compound microscopes, the best are those which consist of one object-glass and two eye-glasses.

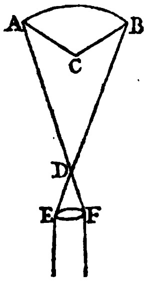
Dr. Hooke, in the preface to his Micrography, says, that in most of his observations he used a microscope of this kind, with a middle eye-glass of a considerable diameter, when he wanted to see much of the object at one view, and took it out when he would examine the small parts of an object more accurately; for the fewer refractions there are, the more light and clear the object appears.

For a miscroscope of three lenses De Chales recommends an object-glass of $\frac{1}{3}$ or $\frac{1}{4}$ of a digit; and the first eye-glass he makes 2 or $2\frac{1}{2}$ digits; and the distance between the object-glass and eye-glass about 20 lines. Conradi had an excellent miscroscope, whose object-glass was half a digit, and the two eye-glasses (which were placed very near) 4 digits; but it answered best when, instead of the object-glass, he used two glasses, convex on both sides, their sphere about a digit and a half, and at most 2, and their convexities touching each other within the space of half a line. Eustachio Divini, instead of an object-glass convex on both sides, used two plano-convex lenses, whose convexities touched. Grindeli did the same; only that the convexities did not quite touch. Zahnius made a binocular microscope, with which both eyes were used. But the most commodious double microscope, it is said, is that of our countryman Mr. Marshal; though some improvement was made in it by Mr. Culpepper and Mr. Scarlet. These are exhibited in figures 5 and 6.

It is observed, that compound microscopes sometimes exhibit a fallacious appearance, by representing convex objects concave, and vice versa. Philos. Trans. No. 476, pa. 387.

To fit microscopes, as well as telescopes, to short-sighted eyes, the object-glass and the eye-glass must be placed a little nearer together, so that the rays of each pencil may not emerge parallel, but may fall diverging upon the eye.

Reflecting Microscope, is that which magnifies by reflection, as the foregoing ones do by refraction. The inventor of this microscope was Sir Isaac Newton. The structure of such a microscope may be conceived thus: near the focus of a concave speculum AB, place a minute object C, that its image may be formed larger than itself in D; to the speculum join a lens, convex on both sides, EF, so as the image D may be in its focus. The eye will here see the image inverted, but distinct, and enlarged; consequently the object will be larger than if viewed through the lens alone.



Any telescope is changed into a microscope, by removing the object-glass to a greater distance from the eye-glass. And since the distance of the image is various, according to the distance of the object from the focus; and it is magnified the more, as its distance from the object-glass is greater; the same telescope may be successively changed into microscopes which magnify the object in different degrees. See some instruments of this sort described in Smith's Optics, Remarks, pa. 94.

Solar Microscope, called also the Camera Obscura Microscope, was invented by Mr. Lieberkuhn, in 1738 or 1739, and consists of a tube, a looking-glass, a convex lens, and a Wilson's microscope. The tube (fig. 7) is brass, near 2 inches in diameter, fixed in a circular collar of mahogany, with a groove on the outside of its periphery, denoted by 2, 3, and connected by a cat-gut to the pulley 4 on the upper part; which turning round at pleasure, by the pin 5 within, in a square frame, may be easily adjusted to a hole in the shutter of a window, by the screws I, I, so closely, that no light can enter the room but through the tube of the instrument. The mirror G is fastened to the frame by hinges, on the side that goes without the window: this glass, by means of a jointed brass wire, 6, 7, and the screw H 8, coming through the frame, may be moved either vertically or horizontally, to throw the sun's rays through the brass tube into the darkened room. The end of the brass tube without the shutter has a convex lens, 5, to collect the rays thrown on it by the glass G, and bring them to a focus in the other part, where D is a tube sliding in and out, to adjust the object to a due distance from the focus. And to the end G of another tube F, is screwed one of Wilson's simple pocket microscopes, containing the object to be magnified in a slider; and by tube F, sliding on the small end E, of the other tube D, it is brought to a true focal distance.

The solar microscope has been introduced into the small and portable camera obscura, as well as the large one: and if the image be received on a piece of half-ground glass, shaded from the light of the sun, it will be sufficiently visible. Mr. Lieberkuhn made considerable improvements in his solar microscope, particularly in adapting it to the viewing of opaque objects; and M Aepinus, Nov. Com. Petrop. vol. 9, pa. 326, has contrived, by throwing the light upon the foreside of any object, before it is transmitted through the object-lens, to represent all kinds of objects by it with equal advantage. In this improvement, the body of the common solar microscope is retained, and only an addition made of two brass plates, AB, AC (fig. 8), joined by a hinge, and held at a proper distance by a screw. A section of these plates, and of all the necessary parts of the instrument, may be seen in fig. 9, where *a c* represent rays of the sun converging from the illuminating lens, and falling upon the mirror *bd*, which is fixed to the nearer of the two brass plates. From this they are thrown upon the object at *cf*, and are thence transmitted through the object-lens at K, and a perforation in the farther plate, upon a screen, as usual. The use of the screen *n* is to vary the distance of the two plates, and thereby to adjust the mirror to the object with the greatest exactness. M. Euler also contrived a method of introducing vision by reflected light into this microscope.

The Microscope *for Opaque Objects* was also invented by M. Lieberkuhn, about the same time with the former, and it remedies the inconvenience of having the dark side of an object next the eye; for by means of a concave speculum of silver, highly polished, having a magnifying lens placed in its centre, the object is so strongly illuminated, that it may be examined with ease. A convenient apparatus of this kind, with 4 different speculums and magnifiers of different powers, was brought to perfection by Mr. Cuff. Philos. Trans. No. 458, § 9.

Microscopic *Objects.* All things too minute to be viewed distinctly by the naked eye, are proper objects for the microscope. Dr. Hooke has distinguished them into these three general kinds; viz, exceeding small bodies, exceeding small pores, or exceeding small motions. The small bodies may be seeds, insects, animalcules, sands, salts, &c.: the pores may be the interstices between the solid parts of bodies, as in stones, minerals, shells, &c. or the mouths of minute vessels in vegetables, or the pores of the skin, bones, and other parts of animals; the small motions, may be the movements of the several parts or members of minute animals, or the motion of the fluids, contained either in animal or vegetable bodies. Under one or other of these three general heads, almost every thing about us affords matter of observation, and may conduce both to our amusement and instruction.

Great caution is to be used in forming a judgment on what is seen by the microscope, if the objects are extended or contracted by force or dryness. Nothing can be determined about them, without making the proper allowances; and different lights and positions will often show the same object as very different from itself. There is no advantage in any greater magnifier than such as is capable of showing the object in view distinctly; and the less the glass magnifies, the more pleasantly the object is always seen.—The colours of objects are very little to be depended on, as seen by the microscope; for their several component particles, being thus removed to great distances from one another, may give reflections very different from what they would, if seen by the naked eye.—The motions of living creatures too, or of the fluids contained in their bodies, are by no means to be hastily judged of from what we see by the microscope, without due consideration; for as the moving body, and the space in which it moves, are magnified, the motion must also be magnified; and therefore that rapidity with which the blood seems to pass through the vessels of small animals, must be judged of accordingly. Baker on the Microscope, pa. 52, 62, &c. See also an elegant work on this subject, published by that ingenious optician, the late Mr. George Adams.

The following directions are given for using the New Universal Pocket Microscope, made and sold by W. & S. Jones, opticians, Holborn, London. See fig. 4, pl. 33.

"This microscope is adapted to the viewing of all sorts of objects, whether transparent, or opake; and for insects, flowers, animalcules, and the infinite variety of the *minutiæ* of nature and art, will be found the most complete and portable, for the price, of any hitherto contrived.

"Place the square pillar of the microscope in the square socket at the foot D, and fasten it by the pin, as shown in the figure. Place also in the foot, the reflecting mirror C. There are three lenses at the top shown at A, which serve to magnify the objects. By using these lenses separately or combined, you make seven different powers. When transparent objects, such as are in the ivory sliders No. 4, are to be viewed, you place the sliders over the spring, at the underside of the stage B; then looking through the lens or magnifier, at A, at the same time reflect up the light, by moving the mirror C below, and move gently, upwards or downwards as may be necessary, the stage B on its square pillar, till you see the object illuminated and distinctly magnified; and in this manner for the other objects.

"For animalcules, you unscrew the brass box that is fitted at the stage B, containing two glasses, and leave the undermost glass upon the stage, to receive the fluids. If you wish to view thereon any moving insect, &c, it may be confined by screwing on the cover: of the two glasses, the concave is best for fluids. Should the objects be opake, such as seeds, &c; they are to be placed upon the black and white ivory round piece, No. 3, which is fitted also to the stage B. If the objects are of a dark colour, you place them contrastedly on the white side of the ivory. If they are of a white, or a light colour, upon the blackened side. Some objects will be more conveniently viewed, by sticking them on the point of No. 2; or between the nippers at the other end, which open by pressing the two little brass pins. This apparatus is also fitted to a small hole in the stage, made to receive the support of the wire.

"The brass forceps, No. 1, serve to take up any small object by, in order to place them on the stage for view. The instrument may be readily converted into a hand microscope, to view objects against the common light; and which, for some transparent ones, is better so. It is done by only taking out the pillar from its foot in D, turning it half round, and fixing it in again; the foot then becomes a useful handle, and the reflector C is laid aside. The whole apparatus packs into a fish-skin case, 4¼ inches long, 2¼ inches broad, and 1½ inches deep.

"For persons more curious and nice in these instruments, there is contrived a useful adjusting screw to the stage, represented at *e*. It is first moved up and down like the other, to the focus nearly, and made fast by the small screw. The utmost distinctness of the object is then obtained, by gently turning the long fine-threaded screw, at the same time you are looking through the magnifiers A. In this case, there may be also added an extraordinary deep magnifier, and a concave silver speculum, with a magnifier to screw on at A, which will serve for viewing the very small and opake objects in the completest manner, and render the instrument as comprehensive in its uses and powers, as those formerly sold under the name of Wilson's Microscope."

MIDDLE *Latitude*, is half the sum of two given latitudes; or the arithmetical mean, or the middle between two parallels of latitude. Therefore, if the latitudes be of the same name, either both north or both south, add the one number to the other, and divide the sum by 2; the quotient is the middle latitude, which is of the same name with the two given latitudes. But if the latitudes be of different names, the one north and the other south; subtract the less from the greater, and divide the remainder by 2, so shall the quotient be the middle latitude, of the same name with the greater of the two.

	Ex. 1.			*Ex.* 2.	
One lat.	35°	27′ N.		35°	27′ S.
the other	21	13 N.		21	13 N.
2)	56	40	2)	14	14
Mid. lat.	28	20 N.	Mid. lat.	7	7 S.

MIDDLE *Latitude Sailing*, is a method of resolving the cases of globular sailing, by means of the middle latitude, on the principles of plain and parallel sailing jointly. This method is not quite accurate, yet often agrees pretty nearly with Mercator's sailing, and is founded on the following principle, viz, that the departure is accounted a meridional distance in the middle latitude between the latitude sailed from and the latitude arrived at.—This artifice seems to have been invented, on account of the easy manner in which the several cases may be resolved by the traverse table, and to serve where a table of meridional parts are wanting. It is sufficiently near the truth either when the two parallels are near the equator, or not far distant from each other, in any latitude. It is performed by these two rules:

1. As the cosine of the middle latitude :
   Is to radius : :
   So is the departure :
   To the difference of longitude .
2. As the cosine of the middle latitude :
   Is to the tangent of the course : :
   So is the difference of latitude :
   To the difference of longitude .

Ex. A ship sails from latitude 37° north, steering constantly N. 33° 19′ east, for 8 days, when she was found in latitude 51° 18′ north; required her difference of longitude.

	51°	18′		51°	18′
	37	00		37	00
2)	88	18	Diff. lat.	14	18 = 858 m.
As cos. mid. l.	44	09	-	0·14417	
To tang. cour.	33	19	-	9·81776	
So diff. lat.	858		-	2·93349	
To diff. long.	786		-	2·89542	

or 13° 6′ diff. of long. sought.

Middle *Region*. See Region.

MID Heaven, *Medium Cœli*, is that point of the ecliptic which culminates, or is highest, or is in the meridian at any time.

MIDSUMMER-*Day*, is held on the 24th of June, the same day as the nativity of St. John the Baptist.

MILE, a long measure, by which the English, Italians, and some other nations, use to express the distance between places: the same as the French use the word league. The mile is of different lengths in different countries. The geographical, or Italian mile, contains 1000 geometrical paces, mille passus, whence the term mile is derived. The English mile consists of 8 furlongs, each furlong of 40 poles, and each pole of 16½ feet; so that the mile is = 8 furlongs = 320 poles = 1760 yards = 5280 feet.

The following table shows the length of the mile, or league, in the principal nations of Europe, expressed in geometrical paces, the pace being accounted equal to $4\frac{4}{10}$ feet.

	Geomet. Paces.	Yards.
Mile of Russia	750	1100
of Italy	1000	1467
of England	1200	1760
of Scotland and Ireland	1500	2200
Old league of France	1500	2200
Small league, ibid.	2000	2933
Mean league of France	2500	3667
Great league, ibid.	3000	4400
Mile of Poland	3000	4400
of Spain	3428	5028
of Germany	4000	5867
of Sweden	5000	7333
of Denmark	5000	7333
of Hungary	6000	8800

MILITARY *Architecture*. The same with Fortification.

MILKY Way, *Via Lactea*, or *Galaxy*, a broad track or path, encompassing the whole heavens, distinguishable by its white appearance, whence it obtains the name. It extends itself in some parts by a double path, but for the most part it is single. Its course lies through the constellations Cassiopeia, Cygnus, Aquila, Perseus, Andromeda, part of Ophiucus and Gemini, in the northern hemisphere; and in the southern, it takes in part of Scorpio, Sagittarius, Centaurus, the Argonavis, and the Ara. There are some traces of the same kind of light about the south pole, but they are small in comparison with this: these are called by some, luminous spaces and Magellanic clouds; but they seem to be of the same kind with the milky way.

The milky way has been ascribed to various causes. The ancients fabled, that it proceeded from a stream of milk, spilt from the breast of Juno, when she pushed away the infant Hercules, whom Jupiter laid to her breast to render him immortal. Some again, as Aristotle, &c, imagined that this path consisted only of a certain exhalation hanging in the air; while Metrodorus, and some Pythagoreans, thought the sun had once gone in this track, instead of the ecliptic; and consequently that its whiteness proceeds from the remains of his light. But it is now found, by the help of telescopes, that this track in the heavens consists of an immense multitude of stars, seemingly very close together, whose mingled light gives this appearance of whiteness; by Milton beautifully described as a path "powdered with stars." Dr. Herschel accounts it a stratum of nebulous matter.

MILL properly denotes a machine for grinding corn, &c; but in a more general signification, is applied to all machines whose action depends on a circular motion. Of these there are several kinds, according to the various methods of applying the moving power; as water-mills, wind-mills, horse-mills, hand-mills, &c, and even steam-mills, or such as are worked by the force of steam, as that noble structure that was erected near Blackfriars Bridge, called the Albion Mills, which was unfortunately destroyed by fire.

The water acts both by its impulse and weight in an overshot water-mill, but only by its impulse in an undershot one; but here the velocity is greater, because the water is suffered to descend to a greater depth before it strikes the wheel. Mr. Ferguson observes, that where there is but a small quantity of water, and a fall great enough for the wheel to lie under it, the bucket or overshot wheel is always used: but where there is a large body of water, with a little fall, the breast or float-board wheel must take place: and where there is a large supply of water, as a river, or large stream or brook, with very little fall, then the undershot wheel is the easiest, cheapest, and most simple structure.

Dr. Desaguliers, having had occasion to examine many undershot and overshot mills, generally found that a well made overshot mill ground as much corn, in the same time, as an undershot mill does with ten times as much water; supposing the fall of water at the overshot to be 20 feet, and at the undershot about 6 or 7 feet: and he generally observed that the wheel of the overshot mill was of 15 or 16 feet diameter, with a head of water of 4 or 5 feet, to drive the water into the buckets with some momentum.

In water-mills, some persons have given the preference to the undershot wheel, but most writers prefer the overshot one. M. Belidor greatly preferred the undershot to any other construction. He had even concluded, that water applied in this way would do more than 6 times the work of an overshot wheel; while Dr. Desaguliers, in overthrowing Belidor's position, determined that an overshot wheel would do 10 times the work of an undershot wheel, with an equal quantity of water. So that between these two celebrated authors, there is a difference of no less than 60 to 1. In consequence of such striking disagreement, Mr. Smeaton began the course of experiments mentioned below.

In the Philos. Trans. vol. 51, for the year 1759, we have a large paper with experiments on mills turned both by water and wind, by that ingenious and experienced engineer Mr. Smeaton. From those experiments it appears, pa. 129, that the effects obtained by the overshot wheel are generally 4 or 5 times as great as those with the undershot wheel, in the same time, with the same expense of water, descending from the same height above the bottom of the wheels; or that the former performs the same effect as the latter, in the same time, with an expense of only one-4th or one-5th of the water, from the same head or height. And this advantage seems to arise from the water lodging in the buckets, and so carrying the wheel about by their weight. But, in pa. 140, Mr. Smeaton reckons the effect of overshot only double to that of the undershot wheel. And hence he infers, in general, "that the higher the wheel is in proportion to the whole descent, the greater will be the effect; because it depends less upon the impulse of the head, and more upon the gravity of the water in the buckets. However, as every thing has its limits, so has this; for thus much is desirable, that the water should have somewhat greater velocity, than the circumference of the wheel, in coming upon it; otherwise the wheel will not only be retarded, by the buckets striking the water, but dashing a part of it over, so much of the power is lost." He is further of opinion, that the best velocity for an overshot wheel is when its circumference moves at the rate of about 3 feet in a second of time. See WIND-MILL.

Considerable differences have also arisen as to the mathematical theory of the force of water striking the floats of a wheel in motion. M. Parent, Maclaurin, Desaguliers, &c, have determined, by calculation, that a wheel works to the greatest effect, when its velocity is equal to one-third of the velocity of the water which strikes it; or that the greatest velocity that the wheel acquires, is one-third of that of the water. And this determination, which has been followed by all mathematicians, till very lately, necessarily results from a position which they assume, viz, that the force of the water against the wheel, is proportional to the square of its relative velocity, or of the difference between the absolute velocity of the water and that of the wheel. And this position is itself an inference which they make from the force of water striking a body at rest, being as the square of the velocity, because the force of each particle is as the velocity it strikes with, and the number of particles or the whole quantity that strikes is also as the same velocity. But when the water strikes a body in motion, the quantity of it that strikes is still as the absolute velocity of the water, though the force of each particle be only as the relative velocity, or that with which it strikes. Hence it follows, that the whole force or effect is in the compound ratio of the absolute and relative velocities of the water; and therefore is greater than the above-mentioned effect or force, in the ratio of the absolute to the relative velocity. The effect of this correction is, that the maximum velocity of the wheel becomes one half the velocity of the water, instead of one-third of it only: a determination which nearly agrees with the best experiments, as those of Mr. Smeaton.

This correction has been lately made by Mr. W. Waring, in the 3d volume of the Transactions of the American Philosophical Society, pa. 144. This ingenious writer says, 'Being lately requested to make some calculations relative to mills, particularly Dr. Barker's construction as improved by James Rumsey, I found more difficulty in the attempt than I at first expected. It appeared necessary to investigate new theorems for the purpose, as there are circumstances peculiar to this construction, which are not noticed, I believe, by any author; and the theory of mills, as hitherto published, is very imperfect, which I conceive to be the reason it has been of so little use to practical mechanics.

'The first step, then, toward calculating the power of any water-mill (or wind-mill), or proportioning their parts and velocities to the greatest advantage, seems to be,

'*The Correction of an Essential Mistake adopted by Writers on the Theory of Mills.*

'This is attempted with all the deference due to eminent authors, whose ingenious labours have justly raised their reputation and advanced the sciences; but when any wrong principles are successively published by a series of such pens, they are the more implicitly received, and more particularly claim a public rectification; which must be pleasing, even to these candid writers themselves.'

A very ingenious writer in England, 'in his masterly treatise on the rectilinear motion and rotation of bodies, published so lately as 1784, continues this oversight, with its pernicious consequences, through his propositions and corollaries (pa. 275 to 284), although he knew the theory was suspected: for he observes (pa. 382) "Mr. Smeaton in his paper on mechanic power (published in the Philosophical Transactions for the year 1776) allows, that the theory usually given will not correspond with matter of fact, when compared with the motion of machines; and seems to attribute this disagreement, rather to deficiency in the theory, than to the obstacles which have prevented the application of it to the complicated motion of engines, &c. In order to satisfy himself concerning the reason of this disagreement, he constructed a set of experiments, which, from the known abilities and ingenuity of the author, certainly deserve great consideration and attention from every one who is interested in these inquiries." And notwithstanding the same learned author says, "The evidence upon which the theory rests is scarcely less than mathematical;" I am sorry to find, in the present state of the sciences, one of his abilities concluding (pa. 380), "It is not probable that the theory of motion, however incontestable its principles may be, can afford much assistance to the practical mechanic," although indeed his theory, compared with the above-cited experiments, might suggest such an inference. But to come to the point, I would just premise these

'*Definitions.*

'If a stream of water impinge against a wheel in motion, there are three different velocities to be considered, appertaining thereto, viz,

'First, the absolute velocity of the water;

'Second, the absolute velocity of the wheel;

'Third, the relative velocity of the water to that of the wheel, i. e. the difference of the absolute velocities, or the velocity with which the water overtakes or strikes the wheel.

'Now the mistake consists in supposing the momentum or force of the water against the wheel, to be in the duplicate ratio of the relative velocity: Whereas,

'PROP. I.

'The force of an Invariable Stream, impingeing against a Mill-wheel in Motion, is in the Simple Direct Proportion of the Relative Velocity.

'For, if the relative velocity of a fluid against a single plane be varied, either by the motion of the plane, or of the fluid from a given aperture, or both, then the number of particles acting on the plane in a given time, and likewise the momentum of each particle, being respectively as the relative velocity, the force on both these accounts, must be in the duplicate ratio of the relative velocity, agreeably to the common theory, with respect to this single plane: but, the number of these planes, or parts of the wheel acted on in a given time, will be as the velocity of the wheel, or inversely as the relative velocity; therefore, the moving force of the wheel must be in the simple direct ratio of the relative velocity. Q. E. D.

'Or the proposition is manifest from this consideration; that, while the stream is invariable, whatever be the velocity of the wheel, the same number of particles or quantity of the fluid, must strike it somewhere or other in a given time; consequently the variation of force is *only* on account of the varied impingent velocity of the same body, occasioned by a change of motion in the wheel; that is, the momentum is as the relative velocity. Now, this true principle substituted for the erroneous one in use, will bring the theory to agree remarkably with the notable experiments of the ingenious Smeaton, before-mentioned, published in the Philosophical Transactions of the Royal Society of London for the year 1760, vol. 51, for which the honorary annual medal was adjudged by the society, and presented to the author by their president. An instance or two of the importance of this correction may be adduced as below.'

PROP. II.

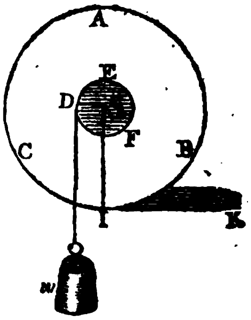
'The velocity of a wheel, moved by the impact of a stream, must be half the velocity of the fluid, to produce the greatest possible effect.—For let

V = the velocity, m = the momentum of the fluid;
v = the velocity, p = the power of the wheel.

Then $V - v$ = the relative velocity, by def. 3d; and as $V : V - v :: m : \frac{m}{V} \times (V - v) = p$ (prop. 1); this multiplied by v, gives $pv = \frac{m}{V} \times (Vv - v^2)$ = a maximum; hence $Vv - v^2$ = a maximum, and its fluxion (v being the variable quantity) is $V\dot{v} - 2v\dot{v} = 0$; therefore $v = \frac{1}{2}V$, that is, the velocity of the wheel = half that of the fluid, at the place of impact, when the effect is a maximum. Q. E. D.—The usual theory gives $v = \frac{1}{3}V$; where the error is not less than one third of the true velocity of the wheel.

'This proposition is applicable to undershot-wheels, and corresponds with the accurate experiments before cited, as appears from the author's conclusion (Philos. Trans. for 1776, pa. 457), viz, "The velocity of the wheel, which according to M. Parent's determination, adopted by Desaguliers and Maclaurin, ought to be no more than one third of that of the water, varies at the maximum in the experiments of table 1, between one third and one half; but in all the cases there related, in which the most work is performed in proportion to the water expended, and which approach the nearest to the circumstances of great works when properly executed, the maximum lies much nearer one half than one third, one half seeming to be the true maximum, if nothing were lost by the resistance of the air, the scattering of the water carried up by the wheel, &c." Thus he fully shows the common theory to have been very defective; but, I believe, none have since pointed out wherein the deficiency lay, nor how to correct it; and now we see the agreement of the true theory with the result of his experiments.' For another problem.

PROP. III.



'Given, the momentum (m) and velocity (V) of the fluid at I, the place of impact; the radius (R = IS) of the wheel ABC; the radius (r = DS) of the small wheel DEF on the same axle or shaft; the weight (w) or resistance to be overcome at D; and the friction (f) or force necessary to move the wheel without the weight; required the velocity (v) of the wheel, &c.'

'Here we have $V : V - v :: m : m \times \frac{V - v}{V}$ = the acting force at I in the direction IK, as before (prop. 2). Now $R : r :: w : \frac{rw}{R}$ = the power at I necessary to counterpoise the weight w; hence $\frac{rw}{R} + f$ = the whole resistance opposed to the action of the fluid at I; which deducted from the moving force, leaves $m \times \frac{V - v}{V} - \frac{rw}{R} - f$ = the accelerating force of the machine; which, when the motion becomes uniform, will be evanescent or = 0; therefore $m \times \frac{V - v}{V} = \frac{rw}{R} + f$, which gives

$v = V \times (1 - \frac{rw}{mR} - \frac{f}{m})$ = the true velocity required; or, if we reject the friction, then $v = V \times (1 - \frac{rw}{mR})$ is the theorem for the velocity of the wheel. This, by the common theory, would be $v = V \times (1 - \sqrt{\frac{rw}{mR}})$, which is too little by $V\sqrt{\frac{rw}{mR}} - V\frac{rw}{mR}$. No wonder why we have hitherto derived so little advantage from the theory.'

'COROL. 1.—If the weight (w) or resistance be required, such as just to admit of that velocity which would produce the greatest effect; then, by substituting $\frac{1}{2}V$ for its equivalent v (by prop. 2), we have $\frac{1}{2}V = V \times (1 - \frac{rw}{mR} - \frac{f}{m})$; hence $w = \frac{\frac{1}{2}m - f}{r} \times R$; or, if $f = 0$, $w = \frac{mR}{2r}$; but theorists make this $\frac{4mR}{9r}$, where the error is $\frac{mR}{18r}$.'

'COROL. 2. We have also $r = \frac{\frac{1}{2}m - f}{w} \times R$; or, rejecting friction, $r = \frac{mR}{2w}$, when the greatest effect is produced, instead of $r = \frac{4mR}{9w}$, as has been supposed: this is an important theorem in the construction of mills.'

In the same volume of the American Transactions, pa. 185, is another ingenious paper, by the same author, on the power and machinery of Dr. Barker's mill, as improved by Mr. James Rumsey, with a description of it. This is a mill turned by the resisting force of a stream of water that issues from an orifice, the rotatory part, in which that orifice is, being impelled the contrary way by its reaction against the stream that issues from it.

Mr. Ferguson has given the following directions for constructing water-mills in the best manner; with a table of the several corresponding dimensions proper to a great variety of perpendicular falls of the water. When the float-boards of the water-wheel move with a 3d part (it should be $\frac{1}{2}$) of the velocity of the water that acts upon them, the water has the greatest power to turn the mill: and when the millstone makes about 60 turns in a minute, it is found to perform its work the best: for, when it makes but about 40 or 50, it grinds too slowly; and when it makes more than 70, it heats the meal too much, and cuts the bran so small that a great part of it mixes with the meal, and cannot be separated from it by sifting or boulting. Consequently the utmost perfection of mill-work lies in making the train so as that the millstone shall make about 60 turns in a minute when the water-wheel moves with a 3d part of the velocity of the water. To have it so, observe the following rules:

1. Measure the perpendicular height of the fall of water, in feet, above the middle of the aperture, where it is let out to act by impulse against the float-boards on the lowest side of the undershot wheel.

2. Multiply that height of the fall in feet by the constant number $64\frac{1}{3}$, and extract the square root of the product, which will be the velocity of the water at the bottom of the fall, or the number of feet the water moves per second.

3. Divide the velocity of the water by 3 (or 2); and the quotient will be the velocity of the floats of the wheel in feet per second.

4. Divide the circumference of the wheel in feet, by the velocity of its floats; and the quotient will be the number of seconds in one turn or revolution of the great water-wheel, on the axis of which is fixed the cog-wheel that turns the trundle.

5. Divide 60 by the number of seconds in one turn of the water-wheel or cog-wheel; and the quotient will be the number of turns of either of these wheels in a minute.

6. Divide 60 (the number of turns the millstone ought to have in a minute) by the abovesaid number of turns; and the quotient will be the number of turns the millstone ought to have for one turn of the water or cog-wheel. Then,

7. As the required number of turns of the millstone in a minute, is to the number of turns of the cog-wheel in a minute, so must the number of cogs in the wheel, be t the number of staves or rounds in the trundle on th axis of the millstone, in the nearest whole number tha can be found.

By these rules the following table is calculated; i which, the diameter of the water-wheel is supposed 18 feet and consequently its circumference $56\frac{4}{7}$ feet, and the dia meter of the millstone is 5 feet.

The Mill-Wright's Table.

Perpendicular height of the fall of water.	Velocity of the water in feet per second.	Velocity of the wheel in feet per second.	Number of turns of the wheel in a minute.	Required number of turns of the millstone for each turn of the wheel.	Nearest number of cogs and staves for that purpose.		Number of turns of the millstone for one turn of the wheel by these cogs and staves.	Number of turns of the millstone in a minute by these cogs and staves.
					Cogs.	Staves.		
1	8·02	2·67	2·83	21·20	127	6	21·17	59·91
2	11·40	3·78	4·00	15·00	105	7	15·00	60·00
3	13·89	4·63	4·91	12·22	98	8	12·25	60·14
4	16·04	5·35	5·67	10·58	95	9	10·56	59·87
5	17·93	5·98	6·34	9·46	85	9	9·44	59·84
6	19·64	6·55	6·94	8·64	78	9	8·66	60·10
7	21·21	7·07	7·50	8·00	72	9	8·00	60·00
8	22·68	7·56	8·02	7·48	67	9	7·44	59·67
9	24·05	8·02	8·51	7·05	70	10	7·00	59·57
10	25·35	8·45	8·97	6·69	67	10	6·70	60·09
11	26·59	8·86	9·40	6·38	64	10	6·40	60·16
12	27·77	9·26	9·82	6·11	61	10	6·10	59·90
13	28·91	9·64	10·22	5·87	59	10	5·80	60·18
14	30·00	10·00	10·60	5·66	56	10	5·60	59·36
15	31·05	10·35	10·99	5·46	55	10	5·40	60·48
16	32·07	10·69	11·34	5·29	53	10	5·30	60·10
17	33·06	11·02	11·70	5·13	51	10	5·10	59·67
18	34·02	11·34	12·02	4·99	50	10	5·00	60·10
19	34·95	11·65	12·37	4·85	49	10	4·80	60·61
20	35·86	11·92	12·68	4·73	47	10	4·70	59·59

For the theory, &c, of wind-mills, see WIND-*Mill*.

MILLION, the number of ten hundred thousand, or a thousand times a thousand.

MINE, in Fortification, &c, is a subterraneous canal or passage, dug under any place or work intended to be blown up by gunpowder. The passage of a mine leading to the powder is called the Gallery; and the extremity, or place where the powder is placed, is called the Chamber. The line drawn from the centre of the chamber perpendicular to the nearest surface, is called the Line of lea Resistance; and the pit or hole, made by the mine wh sprung, or blown up, is called the Excavation. T mines made by the besiegers in the attack of a place, a called simply Mines; and those made by the besiege Counter-mines.

The fire is conveyed to the mine by a pipe or ho made of coarse cloth, of about an inch and half in di meter, called Saucisson, extending from the powder in t chamber to the beginning or entrance of the gallery, to the end of which is fixed a match, that the miner who sets fire to it may have time to retire before it reaches the chamber.

It is found by experiments, that the figure of the excavation made by the explosion of the powder, is nearly a paraboloid, having its focus in the centre of the powder, and its axis the line of least resistance; its diameter being more or less according to the quantity of the powder, to the same axis, or line of least resistance. Thus, M. Belidor lodged 7 different quantities of powder in as many different mines, of the same depth, or line of least resistance, 10 feet; the charges and greatest diameters of the excavation, measured after the explosion, were as follow:

	Powder.	Diam.
1st	120lb	$22\frac{2}{3}$ feet
2d	160	26
3d	200	29
4th	240	$31\frac{1}{4}$
5th	280	$33\frac{1}{2}$
6th	320	36
7th	360	38

From which experiments it appears, that the excavation, or quantity of earth blown up, is in the same proportion with the quantity of powder; whence the charge of powder necessary to produce any other proposed effect, will be had by the rule of proportion.

Mine-*Dial*, is a box and needle, with a brass ring divided into 360 degrees, with several dials graduated upon it, commonly made for the use of miners.

MINERALOGY, is that branch of philosophy which treats of the physical and chemical properties of unorganized bodies; commonly called crude matter, or minerals; by which we are enabled to determine their distinctive characters, and their particular rank in the general system: and is thus distinguished from geology, which treats, more particularly, of the reciprocal position of the different species of minerals, and of the masses composed of two or more of these species.

MINUTE, is the 60th part of a degree, or of an hour. The minutes of a degree are marked with the acute accent, thus $'$; the seconds by two, $''$; the thirds by three, $'''$. The minutes, seconds, thirds, &c, in time, are sometimes marked the same way; but, to avoid confusion, the better way is, by the initials of the words; as minutes^{m}, seconds^{s}, thirds^{t}, &c.

Minute, in Architecture, usually denotes the 60th part of a module, but sometimes only the 30th part.

MIRROR, a speculum, looking-glass, or any other polished body, the use of which is to form the images of distinct objects by reflexion of the rays of light. Mirrors are either plane, convex, or concave. The first sort reflects the rays of light in a direction exactly similar to that in which they fall upon it, and therefore represents bodies in their natural magnitude. But the convex ones make the rays diverge much more than before reflexion, and therefore greatly diminish the images of those objects which they exhibit: while the concave ones, by collecting the rays into a focus, not only magnify the objects they show, but will also burn very fiercely when exposed to the rays of the sun; and hence they are commonly known by the name of Burning Mirrors.

In ancient times, the mirrors were made of some kind of metal; and from a passage in the Mosaic writings we learn, that the mirrors used by the Jewish women, were made of brass; a practice doubtless learned from the Egyptians. Any kind of metal, when well polished, will reflect very powerfully; but of all others, silver reflects the most, but it is too expensive a material for common use. Gold is also very powerful; and all metals, or even wood, gilt and polished, will act with considerable effect as burning mirrors. Even polished ivory, or straw nicely plaited together, will form mirrors capable of burning, if on a large scale.

Since the invention of glass, and the application of quicksilver to it, have become generally known, it has been universally employed for those plane mirrors used as ornaments to houses; but in making reflecting telescopes they have been found much inferior to metallic ones. It does not appear however that the same superiority belongs to the metallic burning mirrors, considered merely as burning speculums; since the mirror with which Mr. Macquer melted platina, though only 22 inches diameter, and made of quicksilvered glass, produced much greater effects than M. Villette's metal speculum, which was of a much larger size. It is very probable, however, that M. Villette's mirror was not so well polished as it ought to have been; as the art of preparing the metal for taking the finest polish, has but lately been discovered, and published in the Philos. Transactions, by Dr. Mudge of Plymouth, and, after him, by Mr. Edwards, Dr. Herschel, &c.

Some of the more remarkable laws and phenomena of plane mirrors, are as follow:—1. A spectator will see his image of the same size, and erect, but reversed as to right and left, and as far beyond the speculum as he is before it. As he moves to or from the speculum, his image will, at the same time, move towards or from the speculum also on the other side. In like manner if, while the spectator is at rest, an object be in motion, its image behind the speculum will be seen to move at the same rate. Also when the spectator moves, the images of objects that are at rest will appear to approach or recede from him, after the same manner as when he moves towards real objects.

2. If several mirrors, or several fragments or pieces of mirrors, be all disposed in the same plane, they will only exhibit an object once.

3. If two plane mirrors, or speculums, meet in any angle, the eye, placed within that angle, will see the image of an object placed within the same, as often repeated as there may be perpendiculars drawn determining the places of the images, and terminated without the angle. Hence, as the more perpendiculars, terminated without the angle, may be drawn as the angle is more acute; the acuter the angle, the more numerous the images. Thus, Z. Traber found, at an angle of $\frac{1}{3}$d of a circle, the image was represented twice, at $\frac{1}{4}$th thrice, at $\frac{1}{6}$th five times, and at $\frac{1}{12}$th eleven times.

Further, if the mirrors be placed upright, and so contracted; or if you retire from them, or approach to them, till the images reflected by them coalesce, or run into one, they will appear monstrously distorted. Thus, if they be at an angle somewhat greater than a right one, the image of one's face will appear with only one eye; if the angle be less than a right one, you will see 3 eyes, 2 noses, 2 mouths, &c. At an angle still less, the body will have two heads. At an angle somewhat greater than a right one, at the distance of 4 feet, the body will be headless, &c. Again, if the mirrors be placed, the one parallel to the horizon, the other inclined to it, or declined from it, it is easy to perceive that the images will be still more romantic. Thus, one being declined from the horizon to an angle of 144 degrees, and the other inclined to it, a man sees himself standing with his head to another's feet.

Hence it appears how mirrors may be managed in gardens, &c, so as to convert the images of those near them into monsters of various kinds: and since glass mirrors will reflect the image of a lucid object twice or thrice, if a candle, &c, be placed in the angle between two mirrors, it will be multiplied a great number of times.

Laws of Convex Mirrors.

1. In a spherical convex mirror, the image is less than the object. And hence the use of such mirrors in the art of painting, where objects are to be represented less than the life.

2. In a convex mirror, the more remote the object, the less its image; also the smaller the mirror, the less the image.

3. In a convex mirror, the right hand is turned to the left, and the left to the right; and bodies perpendicular to the mirror appear inverted.

4. The image of a right line, perpendicular to the mirror, is a right line; but that of a right line oblique or parallel to the mirror, is convex.

5. Rays reflected from a convex mirror, diverge more than if reflected from a plane mirror; and the smaller the sphere, the more the rays diverge.

Laws of Concave Mirrors.

The effects of concave mirrors are, in general, the reverse of those of convex ones; rays being made to converge more, or diverge less than in plane mirrors; the image is magnified, and the more so as the sphere is smaller; &c, &c.

MITRE, in Architecture, is the workmen's term for an angle that is just 45 degrees, or half a right angle. And if the angle be the half of this, or a quarter of a right angle, they call it a half-mitre.

Mixt *Angle*, or *Figure*, is one contained by both right and curved lines.

Mixt *Number*, is one that is partly an integer, and partly a fraction; as 3¼.

Mixt *Ratio*, or *Proportion*, is when the sum of the antecedent and consequent is compared with the difference of the antecedent and consequent;

$$\text{As if}\left\{\begin{array}{l} 4 : 3 :: 12 : 9 \\ a : b :: c : d \end{array}\right.$$

$$\text{then}\left\{\begin{array}{l} 7 : 1 :: 21 : 3 \\ a + b : a - b :: c + d : c - d. \end{array}\right.$$

MOAT, in Fortification, a deep trench dug round a town or fortress, to be defended, on the outside of the wall, or rampart. The breadth and depth of a moat often depend on the nature of the soil; according as it is marshy, rocky, or the like. The brink of the moat next the rampart, is called the scarp; and the opposite side, the counterscarp.

Dry Moat, is one that is without water; on which account it ought to be deeper than one that has water, called a wet moat. A dry moat, or one that has a little water, has often a small ditch running all along the middle of its bottom, called a cuvette.

Flat-bottomed Moat, is that which has no sloping, its corners being somewhat rounded.

Lined Moat, is that whose scarp and counterscarp are cased with a wall of mason's work lying aslope.

MOBILE, *Primum*, in the Ancient Astronomy, was a 9th heaven, or sphere, conceived above those of the planets and fixed stars. It was supposed that this was the first mover, and carried all the lower spheres about with it; by its rapidity communicating to them a motion carrying them round in 24 hours. But the diurnal apparent revolution of the heavens is now better accounted for, by the rotation of the earth on its axis, without the assistance of any such primum mobile.

MOBILITY, an aptitude or facility to be moved.

The mobility of mercury is owing to the smallness and sphericity of its particles; and these also render its fixation so difficult. The hypothesis of the mobility of the earth is the most plausible, and is universally admitted by modern astronomers.—Pope Paul V. appointed commissioners to examine the opinion of Copernicus with regard to the mobility of the earth. The result of their inquiry was, a prohibition to assert, not that the mobility was possible, but that it was really true: that is, they allowed the mobility of the earth to be held as an hypothesis, which gives an easy and sensible solution of the phenomena of the heavenly motions; but forbade this doctrine to be maintained as a thesis, or real effective thing; because they conceived it contrary to Scripture.

MODILLIONS, small inverted consoles under the soffit or bottom of the drip, or of the corniche, seeming to support the projecture of the larmier, in the Ionic, Composite, and Corinthian orders.

MODULAR *Ratio*, a term invented by Mr. Cotes, to denote the ratio or number whose logarithm is what he calls the modulus. This ratio is the ratio

$$\text{of } 1 + \frac{1}{1} + \frac{1}{2} + \frac{1}{2.3} + \frac{1}{2.3.4} + \frac{1}{2.3.4.5} \text{ \&c to } 1, \text{ or}$$

$$\text{of } 1 \text{ to } 1 - \frac{1}{1} + \frac{1}{2} - \frac{1}{2.3} + \frac{1}{2.3.4} - \frac{1}{2.3.4.5} \text{ \&c};$$

that is, the ratio of 2·718281828459 &c to 1,
or the ratio of 1 to 0·367879441171 &c.

See Modulus, and Cotes's Logometria.

MODULE, or Little Measure, in Architecture, a certain measure, taken at pleasure, for regulating the proportions of columns, and the symmetry or distribution of the whole building. Architects usually choose the diameter, or the semidiameter, of the bottom of the column, for their module; which they subdivide into minutes; for estimaitng all the other parts of the building by.

MODULUS, of a system of logarithms, a term used by Mr. Cotes, to denote the log. of the modular ratio. All the logs. in any system, are proportional to this modulus, which in the hyperbolic or Napier's logs. is 1, and in the common or Briggs's logs. is 0·4342944819 &c. See Modular *Ratio*, and Cotes's Logometria.

MOINEAU, a flat bastion raised before a curtin when it is too long, and the bastions of the angles too remote to be able to defend each other. Sometimes the moineau is joined to the curtin, and sometimes it is divided from it by a moat. Here musquetry are placed to fire each way.

MOIVRE, De. See Demoivre.

MOLYNEUX (William), an excellent mathematician and astronomer, was born at Dublin in 1656. After the usual grammar education, which he had at home, he was entered of the college of that city. Here he distinguished himself by the probity of his manners, as well as by the strength of his genius; and having made a remarkable progress in academical learning, and particularly in the new philosophy, as it was then called, after four years spent in this university, he was sent over to London, where he was admitted into the Middle Temple in 1675. Here he spent three years, in the study of the laws of his coun try. But the bent of his genius lay strongly toward mathematical and philosophical studies; and even at the university he conceived a dislike to scholastic learning, and fell into the methods of lord Bacon.

Returning to Ireland in 1678, he shortly after married Lucy the daughter of sir William Domville, the king's attorney-general. Being master of an easy fortune, he continued to indulge himself in prosecuting such branches of natural and experimental philosophy as were most agreeable to his fancy; in which astronomy having the greatest share, he began, about 1681, a literary correspondence with Mr. Flamsteed, the astronomer royal, which was continued for several years. In 1683 he formed a design of erecting a Philosophical Society at Dublin, in imitation of the Royal Society at London; and, by the countenance and encouragement of Sir William Petty, who accepted the office of president, began a weekly meeting that year, when our author was appointed their first secretary.

Mr. Molyneux's reputation for learning recommended him, in 1684, to the notice and favour of the first great duke of Ormond, then lord-lieutenant of Ireland; by whose influence chiefly he was appointed that year, jointly with sir William Robinson, surveyor-general of the king's buildings and works, and chief engineer.

In 1685, he was chosen fellow of the Royal Society at London; and the same year he was sent by the government to view the most considerable fortresses in Flanders. Accordingly he travelled through that country and Holland, as also of Germany and France; and carrying with him letters of recommendation from Flamsteed to Cassini, he was introduced to him and others, the most eminent astronomers in the several places through which he passed. Soon after his return from abroad, he published at Dublin, in 1686, his Sciothericum Telescopium, containing a Description of the Structure and Use of a Telescopic Dial, invented by him: another edition of which was published at London in 1700.

In 1688 the Philosophical Society of Dublin was broken up and dispersed by the confusion of the times. Mr. Molyneux had distinguished himself as a member of it from the beginning, and presented several discourses upon curious subjects, some of which were transmitted to the Royal Society at London, and afterwards printed in the Philosophical Transactions. In 1689, among great numbers of other Protestants, he withdrew from the disturbances in Ireland, occasioned by the severities of Tyrconnel's government; and after a short stay at London, he fixed himself with his family at Chester. In this retirement, he employed himself in putting together the materials he had some time before prepared for his Dioptrics, in which he was much assisted by Mr. Flamsteed; and in August 1690, he went to London to put it to the press, where the sheets were revised by Dr. Halley, who, at our author's request, gave leave for printing, in the appendix, his celebrated Theorem for finding the Foci of Optic Glasses. Accordingly the book came out, 1692, in 4to, under the title of "Dioptrica Nova: a Treatise of Dioptrics, in two parts; wherein the various effects and appearances of spherical glasses, both convex and concave, single and combined, in telescopes and microscopes, together with their usefulness in many concerns of human life, are explained." He gave it the title of Dioptrica Nóva, both because it was almost wholly new, very little being borrowed from other writers, and because it was the first book that appeared in English upon the subject. The work contains several of the most generally useful propositions for practice, demonstrated in a clear and easy manner, for which reason it was for many years used by the artificers: and the second part is very entertaining, especially in the history which he gives of the several optical instruments, and of the discoveries made by them.

As soon as the public tranquillity was settled in his native country, he returned home; and, on the convening of a new parliament in 1692, was chosen one of the representatives for the city of Dublin. In the next parliament, in 1695, he was chosen to represent the university there, and continued to do so to the end of his life; that learned body having lately conferred on him the degree of doctor of laws. He was also nominated by the lord-lieutenant one of the commissioners for the forfeited estates, to which employment was annexed a salary of 500*l*. a year; but considering it as an invidious office, he declined it.

In 1698, he published "The Case of Ireland stated, in regard to its being bound by Acts of Parliament made in England;" in which it is supposed he has delivered all, or most, that can be said upon this subject, with great clearness and strength of reasoning.

Among many learned persons with whom he maintained correspondence and friendship, Mr. Locke was in a particular manner dear to him, as appears from their letters. In the above-mentioned year, which was the last of our author's life, he made a journey to England, on purpose to pay a visit to that great man; and not long after his return to Ireland, he was seized with a fit of the stone, which terminated his existence.

Besides the three works already mentioned, viz, the Sciothericum Telescopium, the Dioptrica Nova, and the Case of Ireland stated; he published a great number of pieces in the Philosophical Transactions, which are contained in the volumes 14, 15, 16, 18, 19, 20, 21, 22, 23, 26, 29, several papers commonly in each volume.

MOLYNEUX (*Samuel*), son of the former, was born at Chester in July 1689; and educated with great care by his father, according to the plan laid down by Locke on that subject. When his father died, he was left to the management of his uncle, Dr. Thomas Molyneux, an excellent scholar and physician at Dublin, and also an intimate friend of Mr. Locke, who executed his trust so well, that Mr. Molyneux became afterwards a most polite and accomplished gentleman, and was made secretary to George the 3d when prince of Wales. Astronomy and optics being his favourite studies, as they had been his father's, he projected many schemes for the advancement of them, and was particularly employed in the years 1723, 1724, and 1725, in perfecting the method of making telescopes; one of which instruments, of his own making, he had presented to John the 5th, king of Portugal.

Being soon after appointed a commissioner of the admiralty, he became so engaged in public affairs, that he had not leisure to pursue those inquiries any further, as he intended. He therefore gave his papers to Dr. Robert Smith, professor of astronomy at Cambridge, whom he invited to make use of his house and apparatus of instruments, in order to finish what he had left imperfect. But Mr. Molyneux dying soon after, Dr. Smith lost the opportunity; he however supplied what was wanting from M. Huygens and others, and published the whole in his "Complete Treatise of Optics."

MOMENT, in Time, is sometimes taken for an extremely small part of duration; but, more properly, it is only an instant or termination or limit in time, like a point in geometry. Maclaurin's Fluxions, vol. 1, pa. 245.

Moments, in the new Doctrine of Infinites, denote the indefinitely small parts of quantity; or they are the same with what are otherwise called infinitesimals, and differences, or increments and decrements; being the momentary increments or decrements of quantity considered as in a continual flux. Moments are the generative principles of magnitude: they have no determined magnitude of their own; but are only inceptive of magnitude. Hence, as it is the same thing, if, instead of these moments, the velocities of their increases and decreases be made use of, or the finite quantities that are proportional to such velocities; the method of proceeding which considers the motions, changes, or fluxions of quantities, is denominated, by Sir Isaac Newton, the method of fluxions.

Leibnitz, and most foreigners, considering these infinitely small parts, or infinitesimals, as the differences of two quantities; and thence endeavouring to find the differences of quantities, i. e. some moments, or quantities indefinitely small, which taken an infinite number of times shall equal given quantities; call these moments, differences; and the method of procedure, the differential calculus.

Moment, or *Momentum*, in Mechanics, is the same thing with Impetus, or the quantity of motion in a moving body. In comparing the motions of bodies, the ratio of their momenta is always compounded of the quantity of matter and the celerity of the moving body: so that the momentum of any such body, may be considered as the rectangle or product of the quantity of matter and the velocity of the motion. As, if b denote any body, or the quantity or mass of matter, and v the velocity of its motion; then bv will express, or be proportional to, its momentum m. Also if B be another body, and V its velocity; then its momentum M, is as BV. So that, in general, $M : m :: BV : bv$, i. e. the momenta are as the products of the mass and velocity. Hence, if the momenta M and m be equal, then shall the two products BV and bv be equal also; and consequently $B : b :: v : V$, or the bodies will be to each other in the inverse or reciprocal ratio of their velocities; that is, either body is so much the greater as its velocity is less. And this force of momentum is of a different kind from, and incomparably greater than, any mere dead weight, or pressure whatever.

The momentum also of any moving body, may be considered as the aggregate or sum of all the momenta of the parts of that body; and therefore when the magnitudes and number of particles are the same, and also moved with the same celerity, then will the momenta of the wholes be the same also.

MONDAY, the second day in the week.

MONADES. Digits, indivisible things.

MONNIER, (Peter Charles le) the son of Peter le Monnier, professor of philosophy at Paris, was born at Paris, November 20, 1715, and died at Lizieux in Normandy, April 2, 1799, in the 84th year of his age, and then the oldest astronomer in Europe. His observations and memoirs, to a vast number, are chiefly contained in the memoirs of the Royal Academy of Sciences; besides which, he published the Histoire Celeste, 1741, in 4to. In this work is twice found, but only as a fixed star, Dr. Herschel's new planet. From his earliest years he devoted himself to astronomy; when a youth of 16 he made his first observations, viz. of the opposition of Saturn. At 20, he was nominated a member of the Royal Academy of Sciences. In 1735, he accompanied Maupertuis in the expedition to Lapland, to measure a degree of the meridian: and he was the first astronomer who had the satisfaction of measuring the diameter of the moon on the sun's disk. In 1750, he drew a meridian at the Royal Château at Bellevue, where the king often made observations. Le Monnier was naturally of a very irritable temper; as ardently as he loved his friends, as easily could he be offended; and his hatred was then implacable. Lalande, who had been his pupil, had the misfortune to incur his displeasure; and he never after could regain his favour. At the time of Le Monnier's death, he had amassed a vast quantity of observations, which he could never be prevailed on to publish, but concealed them in a place, which it was feared he had forgotten; so that it has been supposed they are lost to the world, unless the place should happen to be known to the celebrated mathematician Lagrange, who married one of his daughters in 1792.

MONOCEROS, the *Unicorn*, one of the new constellations of the northern hemisphere, or one of those which Hevelius has added to the 48 old asterisms, and formed out of the stellæ informes, or those which were not comprised within the outlines of any of the others. In Hevelius's catalogue, the Unicorn contains 19 stars, but in the Britannic catalogue 31.

MONOCHORD, a musical instrument with only one string, used by the ancients to try the variety and proportion of sounds. It was formed of a rule, divided and subdivided into several parts, on which there is a moveable string stretched over two bridges at the extremes of it. In the interval between these is a sliding or moveable bridge, by means of which, in applying it to the different divisions of the line, the sounds are found to bear the same proportion to each other, as the division of the line cut by the bridge. This instrument is also called the Harmonical Canon, or the Canonical Rule, because it serves to measure the degrees of gravity or acuteness. Ptolemy examines his harmonical intervals by the monochord. When the chord was divided into two equal parts, so that the parts were as 1 to 1, they called them Unisons; but if they were as 2 to 1, they called them Octaves or Diapasons; when they were as 3 to 2, they called them Diapentes, or Fifths; if they were as 4 to 3, they called them Diatessarons, or Fourths; if the parts were as 5 to 4, they called them Diton, or Major-third; but if they were as 6 to 5, they were called a Demi-diton, or Minor-third; and lastly, if the parts were as 24 to 25, a Demitone, or Dieze.

The monochord, being thus divided, was properly what they called a system, of which there were many kinds, according to the different divisions of the monochord.

Monochord is also used for any musical instrument consisting of only one chord or string. Such is the trump-marine.

MONOTRIGLYPH, a term in Architecture, denoting the space of one triglyph between two pilasters, or two columns.

MONSOON, a regular or periodical wind, that blows one way for 6 months together, and the contrary way the other 6 months of the year. These prevail in several parts of the eastern and southern oceans.

MONTH, the 12th part of the year, and is so called from the moon, by whose motions it was formerly regulated; being properly the time in which the moon runs through the zodiac. The lunar month is either illuminative, periodical, or synodical.

Illuminative MONTH, is the interval between the first appearance of one new moon and that of the next following. As the moon appears sometimes sooner after one change than after another, the quantity of the illuminative month is not always the same. The Turks and Arabs reckon by this month.

Lunar Periodical MONTH, is the time in which the moon runs through the zodiac, or returns to the same point again; the quantity of which is 27 days 7hrs 43m. 8 sec.

Lunar Synodical MONTH, called also a Lunation, is the time between two conjunctions of the moon with the sun, or between two new moons; the quantity of which is 29 days 12 hours 44 m. 2 sec. 48 thirds. The ancient Romans used lunar months, and made them alternately of 29 and 30 days: they marked the days of each month by three terms, viz, Calends, Nones, and Ides.

Solar MONTH, is the time in which the sun runs through one entire sign of the ecliptic, the mean quantity of which is 30 days 10 hours 29 min. 5 sec. being the 12th part of 365 ds. 5 hrs. 49 min. the mean solar year.

Astronomical or *Natural* MONTH, is that measured by some exact interval corresponding to the motion of the sun or moon. Such are the lunar and solar months abovementioned.

Civil or *Common* MONTH, is an interval of a certain number of whole days, approaching nearly to the quantity of some astronomical month. These may be either lunar or solar. The

Civil Lunar MONTH, consists alternately of 29 and 30 days. Thus will two civil months be equal to two astronomical ones, abating for the odd minutes; and so the new moon will be kept to the first day of such civil months for a long time together. This was the month in civil or common use among the Jews, Greeks, and Romans, till the time of Julius Cæsar. The

Civil Solar MONTH, consisted alternately of 30 and 31 days, excepting one month of the twelve, which consisted only of 29 days, but every 4th year of 30 days. And this form of civil months was introduced by Julius Cæsar. Under Augustus, the 6th month, till then from its place called sextilis, received the name Augustus, now August, in honour of that prince; and, to make the compliment still the greater, a day was added to it; which made it consist of 31 days, though till then it had only contained 30 days; to compensate for which, a day was taken from February, making it consist of 28 days, and 29 every 4th year. And such are the civil or calendar months now used through Europe.

MONTUCLA (JOHN STEPHEN), member of the National Institute, and of the Academy of Berlin, censor royal of mathematical books, was born at Lyons, the 5th of September 1725. His father was a banker, by whom he was intended for the same profession; but the science of calculations, to which he was early introduced, soon produced a discovery of the natural bent of his mind. In the Jesuits-college at Lyons he laid a good foundation in the ancient languages, as well as in the mathematical sciences, which enabled him afterwards easily to acquire a competent acquaintance with the Italian, the German, the Dutch, and the English, which he not only read, but also spoke very well.

At 16 years of age Montucla lost his father; and his grandmother, who had been left guardian of his education, died 4 years after. Having finished his studies at Lyons, he went to Toulouse to study the law, a branch of study deemed necessary in the liberal education of every person not destined for the profession of arms.

From hence he repaired to Paris, to enjoy in that capital all the benefits it afforded to the studious, in the lessons of the best masters, in the rich collections of the productions of nature and art, in the best libraries of books, and in the united societies of the literati, among whom he found friends for the rest of his life, and which fixed and determined his choice and pursuit of the mathematical and philosophical sciences, in which he afterwards distinguished himself in so eminent a degree. It was only in relaxing and unbending his mind, from such severe exercises, that he could sometimes occupy himself privately on subjects of less magnitude; such as when he in a manner made an entire new book of Ozanam's Mathematical Recreations, by the multitude of articles added, abridged, or substituted: on which occasion he had so closely concealed from every person the secret of his concern in that neat and improved edition, that the work was actually sent to him to examine and authorize in his capacity of public censor for mathematical books, an honorary office to which he had some time before been appointed. To the last edition of those Recreations however, he set the initials of his name,

Many other pieces were in the like anonymous manner composed by Montucla; among which may be here noticed an ingenious and learned History of Researches relating to the quadrature of the circle, published in 1754; a work very interesting, on account of the number of speculators who have gone astray after that seducing phantom, and of the curious properties which the researches have given rise to.

On occasion of introducing into France, in 1756, the practice of inoculation, which had been introduced into England in 1721, by lady Montague, on her return from Constantinople, Montucla made a translation from the English of the principal writings on that subject, which he added to the Memoire of la Condamine.

In the year 1758, came out Montucla's grand work, the History of Mathematics, in 2 large volumes in 4to: a work of profound reading and learning, and upon which, young as he was, he had spent a great many years of his life. This performance, of immense labour and erudition, published at 33 years of his age, justly procured to the author a most distinguished place in the learned world. This history, so truly admirable, whether we consider the extreme clearness and precision with which the subjects are treated, or the profound learning it exhibits, having been long out of print, the author's employment under the government, as first commissary of the king's buildings, for many years prevented him from fully yielding to the solicitations of his learned friends, to continue the work through the 18th century, in a new and enlarged edition. But the unfortunate loss of his fortune and employment, by the late revolution in France, left him but too much leisure for that purpose. The consequence, happy in this instance for the sciences, has been a new edition in 4 large volumes; in which the history is continued down to the end of the 18th century, and the former parts also very much enlarged and corrected.

In 1755, Montucla was elected an associated member of the Academy at Berlin. And in 1761 he was placed at Grenoble as secretary to the office of intendance, where he united in a happy marriage with Maria Françoise Romand.

The duke de Choiseul having ordered, in 1764, a colony to be formed at Cayenne, Montucla went out there as first secretary to the commission, to which appointment was joined also that of astronomer royal. The affairs of the colony not proving successful, after 15 months Montucla returned again to Grenoble, bringing with him many useful observations and specimens in botany and natural history, which proved beneficial both to the sciences and to the public at large. This voyage also furnished him with those curious observations on the shining of the sea in many places, and of various luminous insects, which are inserted near the end of the 4th volume of his Recreations.

Soon after his return, Montucla was appointed at Versailles to the honourable and profitable office of first commissioner of the royal and public buildings; an employment which he executed with great ability and usefulness during more than 25 years, till the overthrow of the monarchy put an end at once to this office, and the little fortune his regularity and œconomy had enabled him to save, throwing him again on the world, in his old age, naked and stript of every thing except his integrity, and the love and respect of his friends.

The modesty and integrity of Montucla were not less remarkable than his erudition. He was offered a place in the Academy of Sciences of Paris; which through delicacy he refused, as he felt he should not have leisure sufficient properly to attend to the duties of it. The portions of time which others would give to their pleasures, or amusements in their families, he always devoted to the details of the duties of his office, or to his studies. The translation from the English, of Carver's Travels in North America, was the sole monument of his pen, during that long interval. And even this was produced properly in the faithful discharge of the public duties with which he was charged. Being particularly intrusted by the government with the correspondence relating to the voyages which it ordered, he made it his duty and care to collect all the accounts he could find relating to such enterprises by other countries. With this view, at first only amusing his family with the reading of Carver's Travels, finding it entertaining and instructive, he completed and published the whole translation.

Montucla was named a member of the National Institute from the time of its commencement. And the government of 1795 employed him in examining and analysing the treatises deposited in the national archives. He was named professor of mathematics of the central school at Paris; but the bad state of his health would not permit him to accept it; and the department honoured him with a place in the jury of central instruction. But a place in the office for the national lottery was the only resource for his family during two years; a pension of 2400 francs (100*l.*) given him by the minister Neufchateau on the death of Saussure, and which he enjoyed only four months before his decease, which happened the 18th of December 1799. It was chiefly occasioned, as it often happens to literary and sedentary men, by a retention of urine: leaving a widow, as also a daughter, married in 1783, and a son employed in the office of the minister for the interior.

Montucla was one of, the many considerable mathematicians of the 18th century; being well acquainted with all the branches and improvements in those abstruse sciences. His taste however, always chaste and clear, led him to prefer the pure and luminous methods of the ancient mathematicians, and to blame, in the French and the Germans, the great neglect of the same principles, which they showed on all occasions by their preference ot the more modern analysis.

In the qualities of his heart too Montucla was truly estimable: remarkably modest in his manner and deportment; benevolent far beyond the means of his small fortune: of a very respectable personal appearance; he spok with ease and precision, but unassuming and with simplicity; related anecdotes and stories in a pleasant and playful manner; and breathing, in all his conduct and deport ment the sweetness of virtue, and the delicacy of a fin taste.

MOON, *Luna*, ☾, one of the heavenly bodies, bein a satellite, or secondary planet to the earth; considere as a primary planet, about which she revolves in an ellipti orbit, or rather the earth and moon revolve about a com mon centre of gravity, which is as much nearer to th earth's centre than to the moon's, as the mass of the fo mer exceeds that of the latter.

The mean time of a revolution of the moon about th earth, from one new moon to another, when she overtake the sun again, is 29d. 12h. 44m. 2s. 48th.; but she mov once round her own orbit in 27d. 7h. 43m. 8s. movin about 2290 miles every hour; and turns once round he axis exactly in the time that she goes round the eartl which is the reason that she shows always the same sid towards us; and that her day and night taken togeth are just as long as our lunar month.

The mean distance of the moon from the earth is 6 radii, or 30 diameters, of the earth; which is abo 237,500 miles. The mean excentricity of her orbit $\frac{55}{1000}$ or $\frac{1}{18}$th nearly of her mean distance, amounting about 13,000 miles. Her diameter is to that of the eart as 20 to 73, or nearly as 3 to 11, or 1 to $3\frac{2}{3}$; and ther fore it is equal to 2180 miles: her mean apparent di meter is 31′ 16″½, that of the sun being 32′ 12″. Tl surface of the moon is to the surface of the earth, as to $13\frac{1}{3}$, or as 3 to 40; so that the earth reflects 13 tim as much light upon the moon, as she does upon the eartl and her solid content to that of the earth, as 3 to 146, as 1 to $48\frac{2}{3}$. The density of the moon's body is to th of the earth, as 5 to 4; and therefore her quantity matter to that of the earth, as 1 to 39 very nearly; t force of gravity on her surface, is to that on the eart as 100 to 293. The moon has little or no difference seasons; because her axis is almost perpendicular to t ecliptic.

Phenomena and Phases of the MOON. The moon bei a dark, opaque, spherical body, only shining with t light she receives from the sun, hence only that si turned towards him, at any instant, can be illuminat the opposite side remaining in its native darkness: and, the face of the moon visible on our earth, is that part her body turned towards us; so, according to the vari positions of the moon, with respect to the earth and su we perceive different degrees of illumination; sometin a large and sometimes a less portion of the enlighten surface being visible: And hence the moon appears son times increasing, then waning; sometimes horned, th half-round; sometimes gibbous, then full and rou This may be easily illustrated by means of an ivory b which being before a candle in various positions, will present a greater or less portion of its illuminated hemisphere to the view of the observer, according to its situation in moving it round the candle.

The same phases may be otherwise exhibited thus: Let S represent the sun, T the earth, and ABCD &c the moon's orbit. (Plate 19, fig. 3.) Now, when the moon is at A, in conjunction with the sun S, her dark side being entirely turned towards the earth, she will be invisible, as at *a*, and is then called the new moon. When she comes to her first octant at B, or has run through the 8th part of her orbit, a quarter of her enlightened hemisphere will be turned towards the earth, and she will then appear horned, as at *b*. When she has run through the quarter of her orbit, and arrived at C, she shows us the half of her enlightened hemisphere, as at *c*, and she is then said to be at the half. At D she is in her 2d octant, and by showing us more of her enlightened hemisphere than at C, she appears gibbous, as at *d*. At her opposition at E her whole enlightened side is turned towards the earth, when she appears round, as at *e*, and she is said to be full; having increased all the way round from A to E. On the other side she decreases again all the way from E to A: thus, in her 3d octant at F, part of her dark side being turned towards the earth, she again appears gibbous, as at *f*. At G she appears still farther decreased, showing again just one half of her illuminated side, as at *g*. But when she comes to her fourth octant, at H, she presents only a quarter of her enlightened hemisphere, and again appears horned, as at *h*. And at A, having now completed her course, she again disappears, or becomes a new moon again, as at first. The earth also presents exactly the same phases to a spectator in the moon, as she does to us, but only in a contrary order, the one being full when the other changes, &c.

The Motions of the MOON are most of them very irregular. The only equable motion she has, is her revolution on her own axis, in the space of a month, or time in which she moves round the earth; which is the reason that she always turns the same face towards us. This exposure of the same face is not however so uniform, but that she turns sometimes a little more of the one side, and sometimes of the other, called the moon's libration; and also shows sometimes a little more towards one pole and sometimes towards the other, by a motion like a kind of wavering, or vacillation. The former of these motions happens from this circumstance: the moon's rotation on her axis is equable or uniform; while her motion in her orbit is unequal, being quickest when the moon is in her perigee, and slowest when in the apogee, like all other planetary motions; whence it happens that sometimes more of one side is turned to the earth, and sometimes of the other. And the other irregularity arises from this: that the axis of the moon is not perpendicular, but a little inclined to the plane of her orbit: and as this axis maintains its parallelism, in the moon's motion round the earth; it must necessarily change its situation, in respect to an observer on the earth; whence it happens that sometimes the one, and sometimes the other pole of the moon, becomes visible.

The very orbit of the moon is changeable, and does not always preserve the same figure: for though her orbit be elliptical, or nearly so, having the earth in one focus, the excentricity of the ellipse is varied, being sometimes increased, and sometimes diminished; viz, being greatest when the line of the apses coincides with that of the syzygies, and least when these lines are at right angles to each other. Nor is the apogee of the moon without an irregularity; being found to move forward, when it coincides with the line of the syzygies; and backward, when it cuts that line at right angles. Neither is this progress or regress uniform; for in the conjunction or opposition, it goes briskly forward; and in the quadratures, it either moves slowly forward, stands still, or goes backward.— The motion of the nodes is also variable; being quicker and slower in different positions.

The Physical Cause of the MOON's *Motion*, about the earth, is the same as that of all the primary planets about the sun, and of the satellites about their primaries, viz, the mutual attraction between the earth and moon. As for the particular irregularities in the moon's motion, to which the earth and other planets are not subject, they arise from the sun, which acts on, and disturbs her in her ordinary course through her orbit; and are all mechanically deducible from the same great law by which her general motion is directed, viz, the law of gravitation and attraction. The other secondary planets which attend on Jupiter, Saturn, &c, are also subject to the like irregularities with the moon; as they are exposed to the same perturbating or disturbing force of the sun; but their distance secures them from being so greatly affected as the moon is, and also from being so well observed by us.

For a familar idea of this matter, it must first be considered, that if the sun acted equally on the earth and moon, and always in parallel lines, this action would serve only to restrain them in their annual motions round the sun, and no way affect their actions on each other, or their motions about their common centre of gravity. But because the moon is nearer the sun, in one half of her orbit, than the earth is, and farther off in the other half of her orbit; and because the power of gravity is always less at a greater distance: it follows, that in one half of her orbit the moon is more attracted than the earth towards the sun, and less attracted than the earth in the other half: and hence irregularities necessarily arise in the motions of the moon; the excess of attraction in the first case, and the defect in the second, becoming a force that disturbs her motion: and besides, the action of the sun, on the earth and moon, is not directed in parallel lines, but in lines that meet in the centre of the sun; which makes the effect of the disturbing force still the more complex and embarrassing. And hence, as well as from the various situations of the moon, arise the numerous irregularities in her motions, and the equations, or corrections, employed in calculating her places, &c.

Newton, as well as others, has computed the quantities of these irregularities, from their causes. He finds that the force added to the gravity of the moon in her quadratures, is to the gravity with which she would revolve in a circle about the earth, at her present mean distance, if the sun had no effect on her, as 1 to $178\frac{29}{40}$: he finds that the force subducted from her gravity in the conjunctions and oppositions, is double of this quantity; and that the area described in a given time in the quarters, is to the area described in the same time in the conjunctions and oppositions, as 10973 to 11073: and he finds that, in such an orbit, her distance from the earth in her quarters, would be to her distance in the conjunctions and oppositions, as 70 to 69. On these irregularities, see Maclaurin's Account of Newton's Discoveries, book 4, chap. 4; as also most books of astronomy. Other particulars relating to the moon's motions, &c, have been stated as follow: The power of the moon's influence, as to the tides, is to that of the sun, as 4·4815 to 1, according to Sir I. Newton; but different according to others.

As to the figure of the moon, supposing her at first to have been a fluid, like the sea, Newton calculates, that the earth's attraction would raise the water there near 90 feet high, as the attraction of the moon raises our sea 12 feet: whence the figure of the moon must be a spheroid, whose greatest diameter extended, will pass through the centre of the earth; and will be longer than the other diameter, perpendicular to it, by 180 feet; and hence it comes to pass, that we always see the same face of the moon; for she cannot rest in any other position, but always endeavours to conform herself to this situation: Princip. lib. 3, prop. 38.

Newton estimates the mean apparent diameter of the sun at 32′ 12″; as the moon is 31′ 17″. The density of the moon he concludes is to that of the earth, as 9 to 5 nearly; and that the mass, or quantity of matter, in the moon, is to that of the earth, as 1 to 26 nearly. The plane of the moon's orbit is inclined to that of the ecliptic, and makes with it an angle of about 5 degrees: but this inclination varies, being greatest when she is in the quarters, and least when in her syzygies.

As to the inequality of the moon's motion, she moves swifter, and, by the radius drawn from her to the earth, describes a greater area in proportion to the time, also has an orbit less curved, and by that means comes nearer to the earth, in her syzygies or conjunctions, than in the quadratures, unless the motion of her excentricity prevents it: which excentricity is the greatest when the moon's apogee falls in the conjunction, but least when this falls in the quadratures: her motion is also swifter in the earth's aphelion, than in its perihelion. The apogee also goes forward swifter in the conjunction, and slower at the quadratures: but her nodes are at rest in the conjunctions, and recede swiftest of all in the quadratures. The moon also perpetually changes the figure of her orbit, or the species of the ellipse she moves in.

There are also some other inequalities in the motion of this planet, which it is very difficult to reduce to any certain rule; as the velocities or horary motions of the apogee and nodes, and their equations, with the difference between the greatest eccentricity in the conjunctions, and the least in the quadratures; and that inequality which is called the variation of the moon. All these increase and decrease annually, in a triplicate ratio of the apparent diameter of the sun: and this variation is increased and diminished in a duplicate ratio of the time between the quadratures; as is proved by Newton in many parts of his Principia. He also found that the apogees in the moon's syzygies, go forward in respect of the fixed stars, at the rate of 23′ each day; and backwards in the quadratures 16′$\frac{1}{4}$ per day: and therefore the mean annual motions he estimates at 40 degrees.

The gravity of the moon towards the earth, is increased by the action of the sun, when the moon is in the quadratures, and diminished in the syzygies: and, from the syzygies to the quadrature, the gravity of the moon towards the earth is continually increased, and she is continually retarded in her motion: but from the quadrature to the syzygy, the moon's motion is perpetually diminished, and the motion in her orbit is accelerated.

The moon is less distant from the earth at the syzygi and more at the quadratures. As radius is to $\frac{3}{2}$ of the si of double the moon's distance from the syzygy, so is t addition of gravity in the quadratures, to the force whi accelerates or retards the moon in her orbit. And radius is to the sum or difference of $\frac{1}{2}$ the radius and $\frac{3}{2}$ t cosine of double the distance of the moon from t syzygy, so is the addition of gravity in the quadratures, the decrease or increase of the gravity of the moon at th distance.

The apses of the moon go forward when she is in t syzygies, and backward in the quadratures. But, in a who revolution of the moon, the progress exceeds the regre In a whole revolution, the apses go forward the fastest wh the line of the apses is in the nodes; and in the same ca they go back the slowest of all in the same revolutic When the line of the apses is in the quadratures, the ap are carried in consequentia, the least of all in the syzygi but they return the swiftest in the quadratures; and in t case the regress exceeds the progress, in one entire revol tion of the moon.

The excentricity of the orbit undergoes various chang every revolution. It is the greatest of all when the line the apses is in the syzygies, and the least when that line in the quadratures.—Considering one entire revolution the moon, cæteris paribus, the nodes move in anteceden swiftest of all when she is in the syzygies; then slow and slower, till they are at rest, when she is in the quad tures.—The line of nodes acquires successively all possi situations in respect of the sun; and every year it go twice through the syzygies, and twice through the quad tures.—In one whole revolution of the moon, the nodes back very fast when they are in the quadratures; th slower till they come to rest, when the line of nodes is the syzygies.

The inclination of the plane of the orbit is changed the same force with which the nodes are moved; bei increased as the moon recedes from the node, and minished as she approaches it. The inclination of t orbit is the least of all when the nodes are come to t syzygies. For in the motion of the nodes from the syz gies to the quadratures, and in one entire revolution of t moon, the force which increases the inclination, excee that which diminishes it; therefore the inclination is i creased; and it is the greatest of all when the nodes are the quadratures.

The moon's motion being considered in general: gravity towards the earth is diminished on her coming n the sun, and the periodical time is the greatest; as a the distance of the moon, cæteris paribus, the great when the earth is in the perihelion. All the errors the moon's motion are something greater in the co junction than in the opposition. All the disturbi forces are inversely as the cube of the distance of the s from the earth; which when it remains the same, they as the distance of the moon from the earth. Consideri all the disturbing forces together, the diminution of gravi prevails.

The figure of the MOON'S *path*, about the earth, is, as h been said, nearly an ellipse; but her path, in moving, gether with the earth about the sun, is made up of a ser or repetition of epicycloids, and is in every point conca towards the earth. See Maclaurin's Account of Newto Discov. pa. 336, 4to. Ferguson's Astron. pa. 129, & and Rowe's Flux. pa. 225, edit. 2.

Astronomy of the Moon.

To determine the Periodical and Synodical Months; or the period of the moon's revolution about the earth, and the period between one opposition or conjunction and another. In the middle of a lunar eclipse, the moon is in opposition to the sun: compute therefore the time between two such eclipses, at some considerable distance of time from each other; and divide this by the number of lunations that have passed in the mean time; so shall the quotient be the quantity of the synodical month. Compute also the sun's mean motion during the time of this synodical month, which add to 360°. Then, as the sum is to 360°, so is the synodical to the periodical month.

For example, Copernicus observed two eclipses of the moon, the one at Rome on November 6, 1500, at 12 at night, and the other at Cracow on August 1, 1523, at 4 h. 25 min. the difference of meridians being 0 h. 29 min.: hence the quantity of the synodical month is thus determined:

2d Observ.	1523^y	237^d	4^h	25^m
1st Observ.	1500	310	0	29
Difference	22	292	3	56
Add intercalary days		5		
Exact interval	22	297	3	56

which divided by 282, the number of lunations in that time, gives the synodical month $29^d\ 12^h\ 41^m$.

From two other observations of eclipses, the one at Cracow, the other at Babylon, the same author determines more accurately the quantity of the synodical month to be $29^d\ 12^h\ 43^m$ &c; and from other observations, probably more accurate still, the same is fixed at $29^d\ 12^h\ 44^m$.

The sun's mean motion in that time $29°\ 6'\ 24''\ 18'''$, added to 360°, gives the moon's motion 389 6 24 18; Therefore the periodical month is $27^d\ 7^h\ 43^m\ 5^s$.

According to the observations of Kepler,

the mean synodical month is	29^d	12^h	44^m	3^s	2^{th},
and the mean periodical month	27	7	43	8	

Hence, 1st, the quantity of the periodical month being given, the moon's diurnal or horary motion, &c, may be found: and thus may tables of the mean motion of the moon be constructed.

2. If the mean diurnal motion of the sun be subtracted from that of the moon, the remainder will give the moon's diurnal motion from the sun: and thus may a table of this motion be constructed.

3. Since the moon is in the node at the time of a total eclipse, if the sun's place be found for that time, and 6 signs be added to the same, the sum will give the place of that node.

4. By comparing the ancient observations with the modern, it appears, that the nodes have a motion, and that they proceed in antecedentia, or backwards, from Taurus to Aries, from Aries to Pisces, &c. Therefore, if the diurnal motion of the nodes be added to the moon's diurnal motion, the sum will be the motion of the moon from the node; and thence by simple proportion may be found in what time the moon goes 360° from the dragon's head, or ascending node, or in what time she goes from, and returns to it; that is, the quantity of the dracontic month.

5. If the motion of the apogee be subtracted from the mean motion of the moon, the remainder will be the moon's mean motion from the apogee; and hence, by the rule of three, the quantity of the anomalistic month is determined.

Thus, according to Kepler's observations,

The mean synodical month is -	29^d	12^h	44^m	3^s	2^{th}
The periodical month - -	27	7	43	8	
The place of the apogee for the year 1700 Jan. 1, old style, was	11^s	$8°$	$57'$	$1''$	
The place of the ascending node -	4	27	39	17	
Mean diurnal motion of the moon -		13	10	35	
Diurnal motion of the apogee -			6	41	
Diurnal motion of the nodes -			3	11	
Theref. diurnal mot. from the latter -		13	13	46	
And the diurnal motion from the apogee - - - -		13	3	54	

Lastly, the excentricity is 4362, of such parts as the semidiameter of the excentric is 100,000.

To find nearly the Moon's *Age or Change.*

To the epact add the number and day of the month; their sum, abating 30 if it be above that number, is the moon's age; and her age taken from 30, shows the day of the change.—The numbers of the months, or monthly epacts, are the moon's age at the beginning of each month, when the solar and lunar years begin together; and are thus:

0	2	1	2	3	4	5	6	8	8	10	10
Jan.	Feb.	Mar.	Ap.	Ma.	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.

Ex. To find the moon's age, Oct. 14, 1813.

Here, the epact is	9
Number of the month	8
Day of the month	14
The sum is	31
Subtract or abate	30
Leaves moon's age	1
Taken from	30
Days till the change	29
Answering to Nov. 12	

To find nearly the Moon's *Southing*, or coming to the meridian. Take $\frac{4}{5}$ or $\frac{8}{10}$ of her age, for her southing nearly; after noon, if it be less than 12 hours; but if greater, the excess is the time after the foregoing midnight.

Ex. Oct. 23, 1814.

The moon's age is 10 days

$\frac{8}{10}$ of which is 8^h the sou. afternoon.

Mr. Ferguson, in his Select Exercises, pa. 135, &c, has given very easy tables and rules for finding the new and full moons near enough the truth for any common almanac. But the Nautical Almanac, which is now always published for several years before-hand, in a great measure supersedes the necessity of these and other such contrivances.

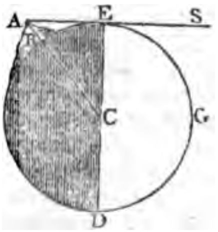
Of the Spots and Mountains, &c, in the Moon.

The face of the moon is greatly diversified with inequalities, and parts of different colours, some brighter and some darker than the other parts of her disc. When viewed through a telescope, her face is evidently diversified with hills and valleys: and the same is also shown by the edge or border of the moon appearing jagged, when so viewed, especially about the confines of the illuminated part when the moon is either horned or gibbous.

The astronomers, Florenti, Langreni, Hevelius, Grimaldi, Riccioli, Cassini, and Delahire, &c, have drawn the face of the moon as viewed through telescopes; noting all the more shining parts, and, for the better distinction, marking them with some proper name; some of these authors calling them after the names of philosophers, astronomers, and other eminent men; while others denominate them from the known names of the different countries, islands, and seas on the earth. The names adopted by Riccioli however are mostly followed, as the names of Hipparchus, Tycho, Copernicus, &c. Fig. 4, plate 19, is a rather exact representation of the full moon in her mean libration, with the numbers to the principal spots according to Riccioli, Cassini, Mayer, &c, which denote the names as in the following list of them: also the asterisk refers to one of the volcanoes observed by Herschel.

*	Herschel's Volcano
1	Grimaldi
2	Galileo
3	Aristarchus
4	Kepler
5	Gassendi
6	Schikard
7	Harpalus
8	Heraclides
9	Lansberg
10	Reinhold
11	Copernicus
12	Helicon
13	Capuanus
14	Bulliald
15	Eratosthenes
16	Timocharis
17	Plato
18	Archimedes
19	Insula Sinus Medii
20	Pitatus
21	Tycho
22	Eudoxus
23	Aristotle
24	Manilius
25	Menelaus
26	Hermes
27	Possidonius
28	Dionysius
29	Pliny
30	Catharina Cyrillus, Theophilus
31	Fracastor
32	Promontorium auctum, Censorinus
33	Messala
34	Promontorium Somnii
35	Proclus
36	Cleomedes
37	Snell and Furner
38	Petavius
39	Langrenus
40	Taruntius
A	Mare Humorum
B	Mare Nubium
C	Mare Imbrium
D	Mare Nectaris
E	Mare Tranquillitatis
F	Mare Serenitatis
G	Mare Fœcunditatis
H	Mare Crisium

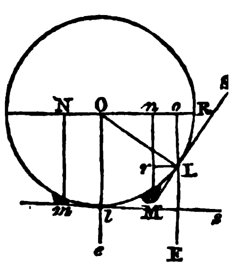
That the spots in the moon, which are taken for mountains and valleys, are really such, is evident from their shadows. For in all situations of the moon, the elevated parts are constantly found to cast a triangular shadow in a direction from the sun; and, on the contrary, the cavities are always dark on the side next the sun, and illuminated on the opposite one; which is exactly conformable to what we observe of hills and valleys on the earth. And as the tops of these mountains are considerably elevated above the other parts of the surface, they are often illuminated when they are at a considerable distance from the confines of the enlightened hemisphere, and by this means afford us a method of determining their heights.



Thus, let ED be the moon's diameter, ECD the boundary of light and darkness; and A the top of a hill in the dark part beginning to be illuminated; with a telescope take the proportion of AE to the diameter ED: then there are given the two sides AE, EC of a right-angled triangle ACE, the squares of which being added together give the square of the third side AC, and the root extracted is that side itself; from which subtracting the radius BC, leaves AB the height of the mountain. In this way, Ri cioli observed the top of the hill called St. Catharine, o the 4th day after the new moon, to be illuminated when was distant from the confines of the enlightened hem sphere about one 16th part of the moon's diameter; an thence found its height must be near 9 miles.

It is probable however that this determination is t much. Indeed, Galileo makes AE to be only one 20th ED, and Hevelius makes it only one 26th of ED; the form of these would give 5½ miles, and the latter only 3¾ mile for AB, the height of the mountain: and probably it shou be still less than either of these.

Accordingly, they are greatly reduced by the observ tions of Herschel, whose method of measuring them w given in the Philos. Trans. an. 1780, pa. 507, or n Abridg. v. 14, pa. 717; and which is as follows. Th method is for any time whatever of the moon's age; where the method used by Hevelius, as above explained, will ser for the time of the quadrature only; in all other positio the projection of the hills must appear much shorter tha it really is. Let SLM, or *slm*, be a line drawn from th sun to the mountain, touching the moon at L or *l*, and the mountain at M or *m*. Then, to an observer at E or *e*, the lines LM, *lm*, will not appear of the same length, though the mountains should be of an equal height; for LM will be projected into *on*, and *lm* into ON. But these are the quantities that are taken by the micrometer, when we observe a mountain to project from the line of illumination From the observed quantity *on*, when the moon is not i her quadrature, to find LM, we have the following analogy: the triangles LO*o*, LM*r*, are similar; therefor L*o* : LO : : L*r* : LM; but LO is the radius of the moon, an L*r*, or *on*, is the observed distance of the mountain's projection; and L*o* is the sine of the angle ROL = *o*LS, whic we may take to be the distance of the sun from the moon without any material error, and which therefore we ma find at any given time from an ephemeris.



In this manner Dr. H. measured the height of many o the lunar prominences, and draws at last the followin conclusions:—"From these observations I believe it i evident, that the height of the lunar mountains in genera is greatly over-rated; and that, when we have excepted few, the generality do not exceed half a mile in their perpendicular elevation." And this is confirmed by th measurement of several mountains, as may be seen in th place above quoted.

As the moon has on her surface mountains and valley in common with the earth, some modern astronomers hav discovered a still greater similarity, viz, that some of thes are really volcanoes, emitting fire, as those on the eart do. An appearance of this kind was discovered some few years ago by Don Ulloa in an eclipse of the sun. It was a small bright spot like a star near the margin of the moon, and which he at that time supposed to be a hole or valley with the sun's light shining through it. Succeeding observations, however, have induced astronomers to attribute appearances of this kind to the eruption of volcanic fire; and Dr. Herschel has particularly observed several eruptions of the lunar volcanoes, the last of which he gives an account of in the Philos. Trans. for 1787, April 19, 10h. 6m. sidereal time, "I perceived," says he, "three volcanoes in different places of the dark part of the new moon. Two of them are either already nearly extinct, or otherwise in a state of going to break out; which perhaps may be decided next lunation. The third shows an actual eruption of fire or luminous matter: its light is much brighter than the nucleus of the comet which M. Mechain discovered at Paris the 10th of this month." The following night he found it burnt with greater violence; and by measurement he found that the shining or burning matter must be more than 3 miles in diameter; being of an irregular round figure, and very sharply defined on the edges. The other two volcanoes resembled large faint nebulæ, that are gradually much brighter in the middle; but no well-defined luminous spot was discovered in them. He adds, "the appearance of what I have called the actual fire, or eruption of a volcano, exactly resembled a small piece of burning charcoal when it is covered by a very thin coat of white ashes, which frequently adhere to it when it has been some time ignited; and it had a degree of brightness about as strong as that with which a coal would be seen to glow in faint day-light.

In a letter by M. Lalande, it is said that, the 13th inst. from 7 to 9 in the evening, Dom. Nouet, one of the astronomers of the Royal Observatory, perceived, in the unenlightened part of the moon, what Dr. Herschel has called a volcano, like a star of the sixth magnitude, or one of the cloudy ones, the brightness of which increased from time to time, as if by flashes. Other astronomers have perceived it, and M. de Villeneuve had seen it before, on the 22d of May, 1787. We cannot therefore doubt of the existence of this volcano in the moon. Dr. Herschel saw it the 4th of May, 1783, and particularly the 19th of April, 1787. In the eclipse of the 24th of June, 1778, M. d'Ulloa, a well-known Spanish astronomer, had seen on the dark disc of the moon, a bright point; and in the total eclipse of 1715, certain curious observers had perceived some flashes of light. There is no sensible atmosphere in the moon, it is true, and chemists may dispute about the name of volcanoes being given to such apparent eruption; but the name after all is of no consequence, and we must certainly subscribe to Dr. Herschel's opinion. This volcano is situated in the north-east part of the moon, about three minutes from the moon's border, towards the spot called Helicon, marked No. 12 in the figure of the moon in Lalande's astronomy. On the next day, March the 14th, Jupiter had been eclipsed by the moon. This rare and curious phenomena has been observed by all astronomers.

It has been disputed whether the moon has any atmosphere. The following arguments have been urged by those who deny it. 1. The moon, say they, constantly appears with the same brightness when our atmosphere is clear; which could not be the case if she were surrounded with an atmosphere like ours, so variable in its density, and so often obscured by clouds and vapours. 2. In an appulse of the moon to a star, when she comes so near it that a part of her atmosphere comes between our eye and the star, refraction would cause the latter to seem to change its place, so that the moon would appear to touch it later than by her own motion she would do. 3. Some philosophers are of opinion, that because there are no seas or lakes in the moon, there is therefore no atmosphere, as there is no water to be raised up in vapours.

But all these arguments have been answered by other astronomers in the following manner. It is denied that the moon appears always with the same brightness, even when our atmosphere appears equally clear. Hevelius relates, that he has several times found in skies perfectly clear, when even stars of the 6th and 7th magnitude were visible, that at the same altitude of the moon with the same elongation from the sun, and with the same telescope, the moon and her maculæ do not appear equally lucid, clear, and conspicuous at all times; but are much brighter and more distinct at some times than at others. And hence it is inferred that the cause of this phenomenon is neither in our air, in the tube, in the moon, nor in the spectator's eye; but must be looked for in something existing about the moon. An additional argument is drawn from the different appearances of the moon in total eclipses, which it is supposed are owing to the different constitutions of the lunar atmosphere.

To the 2d argument Dr. Long replies, that Newton has shown (Princip. prop. 37, cor. 5), that the weight of any body upon the moon is but a third part of what the weight of the same would be upon the earth: now the expansion of the air is reciprocally as the weight that compresses it; therefore the air surrounding the moon, being pressed together by a weight of one-third, or being attracted towards the centre of the moon by a force equal only to one-third of that which attracts our air towards the centre of the earth, it thence follows, that the lunar atmosphere is only one-third as dense as that of the earth, which is too little to produce any sensible refraction of the star's light. Other astronomers have contended, that such refraction was sometimes very apparent. M. Cassini says, he often observed that Saturn, Jupiter, and the fixed stars, had their circular figures changed into an elliptical one, when they approached either to the moon's dark or illuminated limb, though they own that, in other occultations, no such change could be observed. And, with regard to the fixed stars, it has been urged that, granting the moon to have an atmosphere of the same nature and quantity as ours, no such effect as a gradual diminution of light ought to take place; at least none that we could be capable of perceiving. At the height of 44 miles, our atmosphere is so rare as to be incapable of refracting the rays of light: this height is the 180th part of the earth's diameter; but since clouds are never observed higher than 4 miles, it appears that the vapourous or obscure part is only the 1980th part. The mean apparent diameter of the moon is 31′ 29″, or 1889″: therefore the obscure parts of her atmosphere, when viewed from the earth, must subtend an angle of less than one second; which space is passed over by the moon in less than two seconds of time. It can therefore hardly be expected that observation should generally determine whether the supposed obscuration takes place or not.

As to the 3d argument, it concludes nothing, because it is not known that there is no water in the moon; nor, though this could be proved, would it follow that the lunar atmosphere answers no other purpose than the raising of water into vapour. There is however a strong argument in favour of the existence of a lunar atmosphere, taken from the appearance of a luminous circle round the moon in the time of total solar eclipses; a circumstance that has been observed by many astronomers; especially in the total eclipse of the sun which happened May 1, 1706.

These are the arguments that have been advanced for and against the hypothesis of the existence of a lunar atmosphere; but this question seems to be at last settled, by the accurate and long continued observations of S. Piazzi, a celebrated astronomer, who has proved, as satisfactorily as the nature of the subject seems to allow, that the moon has really an atmosphere, though much less dense than ours, and the height of it scarcely exceeding some of the highest of the lunar mountains.

Of the Harvest Moon. It is remarkable that the moon, during the week in which she is at the full about the time of harvest, rises sooner after sun-setting, than she does in any other full-moon week in the year. By this means she affords an immediate supply of light after sun-set, which is very beneficial for the harvest and gathering in the fruits of the earth: and hence this full moon is distinguished from all the others in the year, by calling it the harvest-moon.

To conceive the reason of this phenomena, it may first be considered, that the moon is always opposite to the sun when she is full; that she is at the full in the signs Pisces and Aries in our harvest months, those being the signs opposite to Virgo and Libra, the signs occupied by the sun about the same season; and because those parts of the ecliptic rise in a shorter space of time than others, as may easily be shown and illustrated by the celestial globe: consequently, when the moon is about her full in harvest, she rises with less difference of time, or more immediately after sun-set, than when she is full at other seasons of the year.

In our winter, the moon is in Pisces and Aries about the time of her first quarter, when she rises about noon; but her rising is not then noticed, because the sun is above the horizon.—In spring, the moon is in Pisces and Aries about the time of her change; at which time, as she gives no light, and rises with the sun, her rising cannot be perceived.—In summer, the moon is in Pisces and Aries about the time of her last quarter; and then, as she is on the decrease, and rises not till midnight, her rising usually passes unobserved.—But in autumn, the moon is in Pisces and Aries at the time of her full, and rises soon after sunset for several evenings successively; which makes her regular rising very conspicuous at that time of the year.

And this would always be the case, if the moon's orbit lay in the plane of the ecliptic. But as her orbit makes an angle of 5° 18′ with the ecliptic, and crosses it only in the two opposite points called the nodes, her rising when in Pisces and Aries will sometimes not differ above 1 h. and 40 min. through the whole of 7 days; and at other times, in the same two signs she will differ 3 hours and a half in the time of her rising in a week, according to the different positions of the nodes with respect to these signs; which positions are constantly changing, because the nodes go backward through the whole ecliptic in 18 years and 225 days.

This revolution of the nodes will cause the harvest moons to go through a whole course of the most and least beneficial states, with respect to the harvest, every 19 years. The following table shows in what years the harvest moons are least beneficial as to the times of their rising, and in what years they are most beneficial, from the year 1790 to 1861; the column of years under the letter L, are those in which the harvest-moons are least beneficial, because they fall about the descending node; and those under the letter M are the most beneficial, because they fall about the ascending node.

Harvest Moons.

L	M	L	M	L	M	L	M
1790	1798	1807	1816	1826	1835	1844	1853
1791	1799	1808	1817	1827	1836	1845	1854
1792	1800	1809	1818	1828	1837	1846	1855
1793	1801	1810	1819	1829	1838	1847	1856
1794	1802	1811	1820	1830	1839	1848	1857
1795	1803	1812	1821	1831	1840	1849	1858
1796	1804	1813	1822	1832	1841	1850	1859
1797	1805	1814	1723	1833	1842	1851	1860
	1806	1815	1824	1834	1843	1852	1861
			1825				

As to the Influence of the Moon, on the changes of the weather, and the constitution of the human body, it may be observed, that the vulgar doctrine concerning it is very ancient, and has also gained much credit among the learned, though perhaps without sufficient examination. The common opinion is, that the lunar influence is chiefly exerted about the time of the full and change, but more especially the latter; and it would seem that long experience has in some degree established the fact: hence, persons observed at those times to be a little deranged in their intellects, are called lunatics; and hence many persons anxiously look for the new moon to bring a change in the weather. The moon's influence on the sea, in producing tides, being agreed upon on all hands, it is argued that she must also produce similar changes in the atmosphere, but in a much higher degree; which changes and commotions there, must, it is inferred, have a considerable influence on the weather, and on the human body.

Besides the observations of the ancients, which tend to establish this doctrine, several among the modern philosophers have defended the same opinion, and that upon the strength of experience and observation; while others as strenuously deny the fact. The celebrated Dr. Mead was a believer in the influence of the sun and moon on the human body, and published a book on this subject, intitled, De Imperio Solis ac Lunæ in Corpore Humano. The existence of such influence was however opposed by bishop Horsley, in a learned paper on this subject, in the Philos. Trans. for the year 1775; where he gives a specimen of arranging tables of meteorological observations, so as to deduce from them facts, that may either confirm or refute this popular opinion; recommending it to the learned, to collect a large series of such observations, as no conclusions can be drawn from one or two only. On the other hand professor Toaldo, and some French philosophers, take the opposite side of the question; and, from the authority of a long series of observations, pronounce decidedly in favour of the Lunar Influence.

Acceleration of the Moon. See Acceleration.

Moon-*Dial*. See Dial.

Horizontal Moon. See *Apparent* Magnitude.

MOORE (Sir Jonas), a very respectable mathematician, and surveyor-general of the ordnance, was born at Whitbee in Lancashire, about the year 1620. After enjoying the advantages of good school education, he bent his studies principally to the mathematics, to which he had always a strong inclination. In the expeditions of King Charles the 1st into the northern parts of England, our author was introduced to him, as a person studious and learned in those sciences; when the king expressed much approbation of him, and promised him encouragement; which indeed laid the foundation of his fortune. He was afterwards appointed mathematical master to the king's second son James, to instruct him in arithmetic, geography, the use of the globes, &c. During Cromwell's government it seems he followed the profession of a public teacher of mathematics; for I find him styled, in the title-page of some of his publications, "professor of the mathematics." After the return of Charles the 2d, he found great favour and promotion, becoming at length surveyor-general of the king's ordnance. He was also a great favourite both with the king and the duke of York, who often consulted him, and were advised by him on many occasions. And it must be owned that he often employed his interest with the court to the advancement of learning and the encouragement of merit. Thus, it was through his interest that Flamsteed-house was built in 1675, as a public observatory, recommending Mr. Flamsteed to be the king's astronomer, to make the observations there: and being surveyor-general of the ordnance himself, he made the salary of the astronomer-royal payable out of the office of ordnance, as it still continues. Being also a governor of Christ's-hospital, he prevailed on the king to found the mathematical school there, allowing a handsome salary for a master to instruct a certain number of the boys in mathematics and navigation, to qualify them for the sea-service. Here he soon found an opportunity of exerting his abilities in a manner somewhat answerable to his wishes, namely, that of serving the rising generation. And considering with himself the benefit the nation might receive from a mathematical school, if rightly conducted, he made it his utmost care to promote the improvement of it. But though the school was established, there still wanted a methodical institution from which the youths might receive such necessary helps as their studies required: a laborious work, from which his other great and assiduous employments might very well have exempted him, had not a predominant regard to a more general usefulness engaged him to devote all the leisure hours of his declining years to the improvement of so useful and important a seminary of learning.

Having thus engaged himself in the prosecution of this general design, he next sketched out the plan of a course or system of mathematics for the use of the school, and then drew up and published several parts of it himself, when death put an end to his labours, before the work was completed, about the middle of 1681, the year in which the work was published by his sons-in-law, Mr. Hanway and Mr. Potinger. Of this work, the Arithmetic, Practical Geometry, Trigonometry, and Cosmography, were written by Sir Jonas himself, and printed before his death. The Algebra, Navigation, and the books of Euclid were supplied by Mr. Perkins, at that time master of the mathematical school. And the Astronomy, or Doctrine of the Sphere, was written by Mr. Flamsteed, the astronomer-royal.

Further, as he was the king's constant counsellor in all matters of science, it was doubtless by his advice that the Royal Society also was founded in the year 1662.

The list of Sir Jonas's works, as far as I have seen them, is as follows:

1. The New System of Mathematics; above mentioned, in 2 vols 4to, 1681.

2. Arithmetic in two books, viz, Vulgar Arithmetic and Algebra. To which are added two Treatises, the one, A new Contemplation Geometrical, upon the Oval Figure called the Ellipsis; the other, The two first books of Mydorgius, his Conical Sections analized &c. 8vo, 1660.

3. A Mathematical Compendium; or Useful Practices in Arithmetic, Geometry, and Astronomy, Geography and Navigation, &c, &c. 12mo, 4th edition in 1705.

4. Modern Fortification, &c, 1673, in 8vo.

5. A General Treatise of Artillery: or, Great Ordnance. Written in Italian by Tomaso Moretii of Brescia. Translated into English, with notes thereupon, and some additions out of French for Sea-Gunners. By Sir Jonas Moore, Kt. 8vo, 1683.

MORELAND or **MORLAND** (Sir Samuel), an ingenious mechanist and philosopher. He was master of mechanics to king Charles the 2d, and he invented several useful machines; as, the speaking-trumpet, a fire-engine, and a capstan for heaving up anchors, &c. He published also a respectable book on Arithmetic, in 1674. Three papers of his are inserted in the Philos. Trans.; one on the speaking-trumpet above-mentioned; another on a scheme for raising water; and a third on a successful operation for the hydrops pectoris.

This author was the son of another Sir Samuel Morland, a great statesman, and under-secretary to the minister Thurlow. He was employed by Cromwell in several embassies, and had received the title of baronet for services rendered to King Charles the 1st.

In 1675, Sir S. got a patent for a certain powerful engine to raise water, which project was, in the preceding year, announced in the Philos. Trans. of the Royal Society. This machine, by the strength of 8 men, would force water, in a continual stream, from the river Thames, to the top of Windsor Castle, and 60 feet higher, at the rate of 60 barrels an hour; which experiment was repeated several times, in the year 1681, before the king, queen, and court; when his majesty presented to Sir S. a medal, with his effigy set round with diamonds, and constituted him his master of mechanics, &c. So that it seems it has not always been the practice to present to this office, without some view to public utility.—To Sir S. also it appears, is due the first account of the steam-engine; on which subject, he wrote a book, in which he not only showed the practicability of the plan, but went so far as to calculate the power of different cylinders. This book is now extant in manuscript, in the Harleian collection of MSS. in the British Museum, described in the improved Harleian catalogue, vol. iii, No. 5771, and it is also pointed out in the preface to that volume, sect. 32. The author dates his invention in 1682; consequently 17 years prior to Savery's patent. It was presented to the French king in 1683, at which time experiments were actually shown at St. Germain's. As Mr. S. held places under Charles the 2d, we must naturally conclude that he would not have gone over to France to offer his invention to Louis the 14th, had he not found it slighted at home. The project seems to have remained obscure in both countries till 1699, when Savery, who probably knew more of Morland's invention than he owned, obtained a patent; and in the very same year, M. Amontons proposed something similar to the French Academy, seemingly as his own.

MORTALITY. *Bills of Mortality*, are accounts or registers specifying the numbers born and buried, and sometimes married, in any town, parish, or district. These are of great use, not only in the doctrine of life annuities, but in showing the degrees of healthiness and prolificness, with the progress of population in the places where they are kept. It is therefore much to be wished that such accounts had always been correctly kept in every kingdom, and regularly published at the end of every year. We should then have had under inspection the comparative strength of every kingdom, as far as it depends on the number of inhabitants, and its increase or decrease at different periods.

Such accounts are rendered still more useful, when they include the ages of the dead, and the distempers of which they have died. In this case they convey some of the most important instructions, by furnishing the means of ascertaining the law which governs the waste of human life, the values of annuities dependent on the continuance of any lives, or any survivorships between them, and the favourableness or unfavourableness of different situations to the duration of life.

There are however but few registers of this kind; nor has this subject, though so interesting to mankind, ever engaged much attention till lately. Indeed, bills of mortality for the several parishes of the city of London have been kept from the year 1592, with little interruption; and a very ample account of them has been published down to the year 1759, by Dr. Birch, in a large 4to vol. which is perhaps the most complete work of the kind extant; containing besides the bills of mortality, with the diseases and casualties, several other valuable tracts on the subject of them, and on political arithmetic, by several other authors, as Capt. John Graunt, F. R. S.; Sir William Petty, F. R. S.; Corbyn Morris, Esq. F. R. S.; and J. P. Esq. F. R. S.; the whole forming a valuable repository of materials; and it would be well if a continuation were published, down to the present date, and so continued from time to time.

Bills, containing the ages of the dead, were long since published for the town of Breslaw in Silesia. It is well known what use has been made of these by Dr. Halley, and after him by Mr. Demoivre. A table of the probabilities of the duration of human life at every age, deduced from them by Dr. Halley, was published in the Philos. Trans. vol. 17, and has been inserted in this work under the article LIFE-*Annuities*; which is the first table of the kind that has been published. Since the publication of this table, similar bills have been established in many other places, in England, Germany, Switzerland, France, Holland, &c, but more particularly in Sweden; the results of some of which may be seen in the large comparative table of the duration of life, under our article LIFE-*Annuities*, as well as in the writings of Dr. Price, baron Maseres, Mr. Baily, &c.

MORTAR, or **MORTAR-PIECE**, a short piece of ordnance, thick and wide, proper for throwing bomb-shells, carcases, stones, grape-shot, &c. It is thought that the use of mortars is prior to that of cannon: for they were employed in the wars of Italy, to throw balls of red-hot iron, and stones, long before the invention of shells: and it is generally believed that the Germans were the first inventors. The practice of throwing red-hot balls out of mortars, was first practised at the siege of Stralsund in 1675, by the elector of Brandenburg; though some say, in 1653, at the siege of Bremen.

Mortars are made either of brass or iron, and it is usual to distinguish them by the diameter of the bore; as the 13 inch, the 10 inch, or the 8 inch mortar: there are some of a smaller sort, as Coehorns of 4·6 inches, and Royals of 5·8 inches in diameter. As to the larger sizes, as 18 inches, &c, they are now disused by the English, as well as most other European nations. For the circumstances relating to mortars, see Muller's Artillery.

Coehorn-**MORTAR**, a small kind of one, invented by the celebrated engineer baron Coehorn, to throw small shells or grenades. These mortars have been sometimes fixed, to the number of a dozen, on a block of oak, at the elevation of 45°.

MOTION, or *Local* **MOTION**, is a continued and successive change of place. Borelli defines it, the successive passage of a body from one place to another, in a determinate time, by becoming successively contiguous to al the parts of the intermediate space. Or motion is tha affection of matter by which it is transferred from on point of space to another.

In order that the doctrine of mechanics may be brough within the boundaries of mathematical investigation, it i necessary, not only that the quantities it proposes fo discussion should be measurable, either in themselves o in their effects, but also that some general principle should be established, the truth of which should be incontrovertible, and to which the student may at all time appeal in the course of his researches. Such gener principles were first distinctly proposed by Sir I. Newto in his Principia, and they have since his time been receive as mechanical axioms, or, as they are commonly calle Laws of Motion, which are as follows:

1. Every body continues in its state of rest or unifor motion in a right line, until a change is effected in it, b the agency of some external force.

2. Any change affected in the quiescence, or motion a body, is in the direction of the force impressed, and proportional to the quantity of it.

3. Action and reaction are equal and contrary; the mutual actions of two bodies on each other, are a ways equal, and directed to contrary parts.

Continuation of **MOTION**, or the cause why a body, on in motion, continues to persevere in it, is a subject, th has been much controverted by many celebrated philos phers; we must, however, be content with knowing that is one of the fundamental laws of nature, which is beyo the comprehension of the human mind; and by whic motion once begun, would be continued in infinitum, we it to meet with no interruption from external caus such as the power of gravity, the resistance of mediun &c, &c.

Communication of **MOTION**, or how a body in moti communicates the same to a body at rest, by coming contact with it, is also a subject which has been as mu controverted by philosophers as the former, and after a is as little understood as the continuation of motion, t cause of gravity, and other speculative inquiries of a sin lar nature.

Motion, as we before observed, is the proper subject mechanics, and these are the basis of all natural philo phy; and hence the denomination, Mechanical, or Ex rimental Philosophy.

In effect, all the phenomena of nature, all the chan that happen in the system of bodies, are owing to moti and are directed according to the laws of it. Hence modern philosophers have applied themselves with pecu ardour to consider the doctrine of motion; to investig the properties and laws of it; by observation and exp ment, aided by the use of geometry. And to this is ow the great advantage of the modern philosophy over tha the ancients; who generally founded their systems philosophy on some absurd hypothesis of their own invention; whereas the moderns, by deducing theirs from experiments, carefully and frequently repeated, are enabled to proceed from effects to their causes in a much more rational manner.

Motion is considered as of various kinds, viz, Absolute, Relative, Equable, Accelerated, Retarded, &c.

Absolute Motion, is an absolute change of place, in any moving body, considered independently of any other motion, the celerity of which will therefore be measured by the quantity of absolute space which the moveable body has passed over. And

Relative Motion, is the change of the relative place of a moving body, considered with respect to some other body also in motion, and the celerity of it is estimated by the quantity of relative space run through. This may be illustrated by considering two vessels, sailing either in the same, or in contrary directions, but with different velocities in the former case; both of which are in absolute motion with regard to the port whence they sailed, or any other fixed point, but in relative motion with respect to each other.

Among the ancients, there is nothing extant on motion, excepting some things in Archimedes's books De Æquiponderantibus, and in Aristotle's. We are indebted to Galileo for a great part of the doctrine of motion: he first discovered the general laws of it, and particularly of the descent of heavy bodies, both perpendicularly and on inclined planes; the laws of the motion of projectiles; the vibration of pendulums, and of stretched cords, with the theory of resistances, &c: things which the ancients had little notion of.

Torricelli considerably improved on the discoveries of his master, Galileo; and added many experiments concerning the force of percussion, and the equilibrium of fluids. Huygens extended the doctrine of pendulums; and both he and Borelli the effects of percussion. Lastly, Newton, Leibnitz, Varignon, Mariotte, &c, have brought the doctrine of motion still nearer to perfection.

The general laws of motion were first brought into a system, and analytically demonstrated together, by Dr. Wallis, Sir Christopher Wren, and M. Huygens, all much about the same time; the first in bodies not elastic, and the two latter in elastic bodies. Lastly, the whole doctrine of motion, including all the discoveries both of the ancients and moderns on that head, was given by Dr. Wallis in his Mechanica, sive De Motu, published in 1670.

Quantity of Motion, is the same as Momentum, which see. It is a principle maintained by the Cartesians, and some others, that the Creator at the beginning impressed a certain quantity of motion on bodies; and that under such laws, as that no part of it should be lost, but the same portion of motion should be constantly preserved in matter: and hence they conclude, that if any moving body strike another body, the former loses no more of its motion than it communicates to the latter. This position however has been opposed by other philosophers, and perhaps justly, unless the preservation of motion be understood only of the quantity of it as estimated always in the same direction; in which case the principle will hold good. However, the reasoning ought to have proceeded in the contrary order; by first observing from experiment, or otherwise, that when two bodies act upon each other, the one gains exactly the motion which is lost by the other, in the same direction; and hence have drawn the inference, that there is therefore the same quantity of motion preserved in the universe, as was created by God in the beginning; since no body can act upon another, without being itself equally acted on in the opposite or contrary direction.

Equable Motion, is that by which the moving body proceeds with exactly the same velocity or celerity; passing always over equal spaces in equal times.

The Laws of Equable Motion, are these: 1. The spaces described, or passed over, are in the compound ratio of the velocities, and the times of describing those spaces. So that, if V and v be any two uniform velocities, S and s the spaces described or passed over by them, in the respective times T and t:

then is $S : s :: TV : tv$,
or $20 : 12 :: 4 \times 5 : 3 \times 4$;
taking $T = 4$, $t = 3$, $V = 5$, and $v = 4$.

2. In uniform motions, the time is as the space directly, and as the velocity reciprocally; or as the space divided by the velocity. So that

$$T : t :: \frac{S}{V} : \frac{s}{v} \text{ or } :: Sv : sV.$$

3. The velocity is as the space directly, and the time reciprocally; or as the space divided by the time. That is, $V : v :: \frac{S}{T} : \frac{s}{t}$ or $:: St : sT$.

Accelerated Motion, is that which continually receives fresh accessions of velocity. And it is said to be uniformly accelerated, when its accessions of velocity are equal in equal times; such as that which is produced by the continual action of one and the same force, like the force of gravity, &c.

Retarded Motion, is that whose velocity continually decreases. And it is said to be uniformly retarded, when its decrease is continually proportional to the time, or by equal quantities in equal times; like that which is produced by the continual opposition of one and the same force; such as the force of gravity, in uniformly retarding the motion of a body that is thrown upwards.

The laws of motion, uniformly accelerated or retarded, are these: 1. In uniformly varied motions, the space, S or s, is as the square of the time, or as the square of the greatest velocity, or as the rectangle or product of the time and velocity.

That is, $S : s :: T^2 : t^2 :: V^2 : v^2 :: TV : tv$.

2. The velocity is as the time, or as the space divided by the time, or as the square root of the space.

That is, $V : v :: T : t :: \frac{S}{T} : \frac{s}{t} :: \sqrt{S} : \sqrt{s}$.

3. The time is as the velocity, or as the space divided by the velocity, or as the square root of the space.

That is, $T : t :: V : v :: \frac{S}{V} : \frac{s}{v} :: \sqrt{S} : \sqrt{s}$.

4. When a space is described, or passed over, by an uniformly varied motion, the velocity either beginning at nothing, and continually accelerated; or else beginning at some determinate velocity, and continually retarded till the velocity be reduced to nothing; then the space, so described by any body, is exactly equal to half the space that would be run over in the same time by the greatest velocity if uniformly continued for that time. So, for instance, if g denote the space run over in one second, or any other time, by such a variable motion; then $2g$ would be the space that would be run over in one second, or the

same time, by the greatest velocity uniformly continued for the same time; or $2g$ would be the greatest velocity per second which the moving body had. Consequently, if t be any other time, s the space run over in that time, and v the greatest velocity attained in it; then, from the foregoing articles, it will be

$$1'' : t'' :: 2g : 2gt = v \text{ the velocity,}$$
$$\text{and } 1^2 : t^2 :: g : gt^2 = s \text{ the space.}$$

And hence, for any such uniformly varied motions, the relations among the several quantities concerned, will be expressed by the following equations: viz,

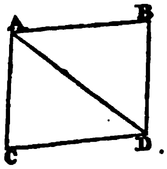
$$s = gt^2 = \tfrac{1}{2}tv = \frac{v^2}{4g}; \qquad t = \frac{v}{2g} = \frac{2s}{v} = \sqrt{\frac{s}{g}};$$
$$v = 2gt = \frac{2s}{t} = 2\sqrt{gs}; \; g = \frac{v}{2t} = \frac{s}{t^2} = \frac{v^2}{4s}.$$

And these equations will hold good in the motion either generated or destroyed by the force of gravity, or by any other uniform force whatever. See also the articles GRAVITY, ACCELERATION, RETARDATION, &c. Again,

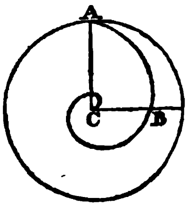
Simple MOTION, is that which is produced by some one power or force only, and is always rectilinear, or in one direction, whether the force be only momentary or continued. And

Compound MOTION, is that which is produced by two or more powers acting in different directions. See COMPOUND, and COMPOSITION *of Motion*.

If a moving body be acted on by a double power; the one according to the direction AB, the other according to AC; then, with the compound motion, or that which is compounded of these two together, it will describe the diagonal AD of the parallelogram, whose sides AB and AC it would have described in the same time with each of the respective powers separately applied.



And if the radius of a circle be made to revolve about the centre C, while a point in the radius sets off from A, and keeps moving along the radius towards the centre; then, by this compound motion, the path of the point will be a kind of spiral ABC.



For the particular laws of motion, arising from the collision of bodies, both elastic and non-elastic, and that where the directions are both perpendicular and oblique, see PERCUSSION.

For CIRCULAR *Motion*, and *the Laws of* PROJECTILES, see the respective words.

For the Motion of Pendulums, and the Laws of Oscillation, see PENDULUM.

Perpetual MOTION, is a motion which is supplied and renewed from itself, without the intervention of any external cause. The celebrated problem of a perpetual motion, consists in the inventing a machine, which has the principle of its motion within itself; and is a problem that has engaged the attention of certain mathematicians for 2000 years; though none perhaps have prosecuted it with attention and earnestness equal to those of the last century. Infinite are the schemes, designs, plans, engines, wheels, &c, to which this long-desired perpetual motion has given birth.

M. Lahire has proved the impossibility of any such machine, and finds that it amounts to this; viz, to find a body which is both heavier and lighter at the same time, or to find a body which is heavier than itself. Indeed there seems but little in nature to countenance all this assiduity and expectation: among all the laws of matter and motion, we know of none yet that seem likely to furnish any principle or foundation for such an effect.

Action and reaction it is allowed are always equal; and a body that gives any quantity of motion to another, always loses just so much of its own; but under the present state of things, the resistance of the air, the friction of the parts of machines, &c, do necessarily retard every motion. To continue the motion therefore—either, first, there must be a supply from some foreign cause; which in a perpetual motion is excluded. Or, 2dly, all resistance from the friction of the parts of matter must be removed; which necessarily implies a change in the nature of things. Or, 3dly and lastly, there must be some method of gaining a force equivalent to what is lost, by the artful disposition and combination of mechanic powers; to which last point then all endeavours are to be directed: but how, or by what means, such force should be gained, is still a mystery. The multiplication of powers or forces, it is certain, avails nothing; for what is gained in power is lost in time, so that the quantity of motion still remains the same. This is an invariable law of nature; by which nothing is left to art, but the choice of the several combinations that may produce the same effect.

There are various ways by which absolute force may be gained; but since there is always an equal gain in opposite directions, and no increase obtained in the same direction; in the circle of actions necessary to make a perpetual movement, this gain must be presently lost, and will not serve for the necessary expense of force employed in overcoming friction, and the resistance of the medium. And therefore, though it could be shown, that in an infinite number of bodies, or in an infinite machine, there could be a gain of force for ever, and a motion continued to infinity, it does not follow that a perpetual movement can be made. That which was proposed by M. Leibnitz in the Leipsic Acts of 1690, as a consequence of the common estimation of the forces of bodies in motion, is of this kind, and for this and other reasons ought to be rejected. See ORFFYREUS'S *Wheel*, &c, also my Recreations, vol. 2, prob. 52 on Mechanics.

Animal MOTION, is that by which the situation, figure, magnitude, &c, of the parts and members of animals are changed. Under these motions, are included all the animal functions; as respiration, circulation of the blood, excretion, walking, running, &c.

Animal motions are usually divided into two species; viz, Natural and Spontaneous.

Natural MOTION, is that involuntary one which is effected without the command of the will, by the mere mechanism of the parts. Such as the motion of the heart and pulse; the peristaltic motion of the intestines, &c. But

Spontaneous, or *Muscular* MOTION, is that which is performed by means of the muscles, at the command of the will; which is hence called voluntary motion. Borelli has a celebrated treatise on this subject, entitled De Motu Animalium.

Intestine MOTION, denotes an agitation of the particles of which a body consists. Some philosophers will have every body, and every particle of a body, in continual motion. As for fluids, it is the definition they give of them, that their parts are in continual motion. And as to solids, they infer the like motion from the effluvia continually emitted through their pores. Hence intestine motion is represented to be a motion of the internal and smaller parts of matter, continually excited by some external, latent agent, which of itself is insensible, and only discovers itself by its effects; appointed by nature to be the great instrument of the changes in bodies.

MOTION, in Astronomy, is peculiarly applied to the orderly courses of the heavenly bodies.

Mean MOTION. See MEAN.

The motions of the celestial luminaries are of two kinds: Diurnal, or Common; and Secondary, or Proper.

Diurnal, or *Primary* MOTION, is that with which all the heavenly bodies, and the whole mundane sphere, appear to revolve every day about the earth, from east to west. This is also called the motion of the primum mobile, and the common motion, to distinguish it from that rotation which is peculiar to each planet, &c.

Secondary, or *Proper* MOTION, is that with which a star, planet, or the like, advances a certain space every day from the west towards the east. See the several motions of each luminary, with the irregularities, &c, of them, under the proper articles, EARTH, MOON, STAR, &c.

Angular MOTION, is that by which the angular position of any thing varies. See ANGULAR.

Horary MOTION, is the motion during each hour. See HORARY.

Paracentric MOTION *of Impetus*. See PARACENTRIC.

MOTION *of Trepidation*, &c. See TREPIDATION and LIBRATION.

MOTIVE *Power*, or *Force*, is the whole power or force acting upon any body, or quantity of matter, to move it; and is proportional to the momentum or quantity of motion it can produce in a given time. And it is thus distinguished from the accelerative force, which is considered as affecting the celerity only.

MOTRIX, something that has the power or faculty of moving. See *Vis Motrix*, and MOTION.

MOVEABLE, something susceptible of motion, or that is disposed to be moved. A sphere is the most moveable of all bodies, or is the easiest to be moved on a plane. A door is moveable on its hinges; the magnetic needle on a pin or pivot, &c. Moveable is often used in contradistinction to fixed or fixt.

MOVEABLE *Feasts*, are such as are not always held on the same day of the year or month; though they may be on the same day of the week. Thus, Easter is a moveable feast; being always held on the Sunday which falls upon or next after the first full moon following the 21st of March. See Philos. Trans. No. 240, pa. 185. All the other moveable feasts follow Easter, keeping their constant distance from it; so that they are fixed with respect to this, though moveable through the course of the year. Such are Septuagesima, Sexagesima, Ash-Wednesday, Ascension-Day, Pentecost, Trinity-Sunday, &c.

MOVEMENT, a term often used in the same sense with automaton. The most usual movements for keeping time are clocks and watches: the latter are such as show the parts of time by inspection, and are portable in the pocket; the former such as publish it by sounds, and are fixed as furniture.

MOVEMENT, in its popular use, signifies all the inner works of a clock, watch, or other machine, that move, and by that motion carry on the design of the instrument. The movement of a clock, or watch, is the inside; or that part which measures the time, and strikes; exclusive of the frame, case, dial-plate, &c.

The parts common to both of these movements are, the main-spring with its appurtenances, lying in the spring box, and in the middle of it lapping about the spring-arbor, to which one end of it is fastened. On the upper part of the spring-arbor is the endless screw, and its wheel; but in spring clocks this is a ratchet-wheel with its click, that stops it. That part which the main-spring draws, and round which the chain or string is wrapped, is called the fusee, the proper curve for which is the hyperbola; in large works, going with weights, it is cylindrical, and is called the barrel. The small teeth at the bottom of the fusee or barrel, which stop it in winding up, is called the ratchet; and that which stops it when wound up, and is for that end driven up by the spring, the gardegut. The wheels are various: the parts of a wheel are, the hoop or rim; the teeth, the cross, and the collet, or piece of brass soldered on the arbor or spindle on which the wheel is riveted. The little wheels, playing in the teeth of the larger, are called pinions; and their teeth, which are 4, 5, 6, 8, &c, are called leves; the ends of the spindle are called pivots; and the guttured wheel, with iron spikes at bottom, in which the line of common clocks runs, the pulley.

Theory of Calculating the Numbers for MOVEMENTS.

1. It is first to be observed, that a wheel, divided by its pinion, shows how many turns the pinion has to one turn of the wheel.

2. That from the fusee to the balance the wheels drive the pinions, consequently the pinions run faster, or make more revolutions, than the wheel; but it is the contrary from the great wheel to the dial-wheel.

3. That the wheels and pinions are written down either as vulgar fractions, or in the way of division in common arithmetic: for example, a wheel of 60 teeth, moving a pinion of 5, is set down either thus $\frac{60}{5}$, or thus 5)60, which is better. And the number of turns the pinion has in one turn of the wheel, as a quotient, thus 5) 60 (12. A whole movement may be written as annexed: where the uppermost number expresses the pinion of report 4, the dial-wheel 36, and the turns of the pinion 9; the second, the pinion and great wheel; the third, the second wheel, &c; the fourth, the contrate wheel; and the last, 17, the crown-wheel.

$$\begin{array}{rrl} 4\,) & 36 & (\,9 \\ \hline 5\,) & 55 & (\,11 \\ 5\,) & 45 & (\,9 \\ 5\,) & 40 & (\,8 \\ \hline & & \;\;17 \end{array}$$

4. Hence, from the number of turns any pinion makes, in one turn of the wheel it works in, may be determined the number of turns a wheel or pinion has at any greater distance, viz, by multiplying the quotients together; the product being the number of turns. Thus, suppose the wheels and pinions as in the case above; the quotient 11 multiplied by 9, gives 99, the number of turns in the second pinion 5 to one turn of the wheel 55, which runs concentrical, or on the same spindle, with the pinion 5. Again, 99 multiplied by 8, gives 792, the number of turns the last pinion has to one turn of the first wheel 5. Hence we proceed to find, not only the turns, but the number of beats of the balance, in the time of those turns. For, having found the number of turns the crown-wheel has in one turn of the wheel proposed, those turns multiplied by its notches, give half the number of beats in that one

turn of the wheel. Suppose, for example, the crown-wheel to have 720 turns, to one of the first wheel; this number multiplied by 15, the notches in the crown-wheel, produces 10800, half the number of strokes of the balance in one turn of the first wheel of 80 teeth.—The general division of a movement is, into the clock, and watch parts.

MOULDINGS, in Architecture, are certain projections beyond the naked of a wall, column, wainscot, &c, the assemblage of which forms cornices, door-cases, and other decorations of architecture.

Mouldings, are annexed to great guns by way of ornament, or perhaps in some parts for strength; and probably are derived from the hoops or rings which bound the long iron bars together, anciently used in making cannon.

MOUNTAIN, a considerable eminence of land, elevated above every thing around it. The name is also given to a chain of such masses; as when we speak of Mount Atlas in Africa; or Mount Caucasus, extending from Colchis to the Caspian Sea; or the Pyrenean Mountains, which separate France from Spain; and the Apennine Mountains, traversing the whole of Italy.

Naturalists reckon several kinds of mountains; and conjecture that they have not all the same origin, nor the same date. As, 1st, Those mountains which form a chain, and are covered with snow, are considered as primitives or antediluvian. These greatly exceed other mountains in height; in general their elevation is very sudden, and their ascent very steep and difficult: their shape is pyramidical, crowned with sharp and prominent rocks. No shells, or other organized marine bodies, are found in the upper parts of these primitive mountains, except on the sides near the base. The stone of which they consist is an immense mass of quartz, which penetrates into the bowels of the earth in a direction almost vertical. Of this kind in Europe are the Pyrenees, the Alps, the Apennines, those in Tyrol, in Silesia, in Carpathia, Saxony, Norway, &c. In Asia are the Riphean Mountains, Mounts Caucasus, Taurus, and Libanus. In Africa, Atlas and the Mountains of the Moon; and in America the Apalachian Mountains, and the Andes or Cordilleras. Many of the latter have been the seats of volcanoes.—2d, Another kind of mountains are such as are either detached, or surrounded with groups of little hills, the crust of which is gravelly and confusedly arranged together. These are truncated, or have a wide mouth in the shape of a funnel in the summit, being composed of, or surrounded with heaps of calcined and half vitrified bodies, lava, &c. These appear to have been formed by different strata thrown up into the air, on the eruptions of subterraneous fire: such as the isles of Santorin, Monta-Nuovo, Ætna, Adam's Peak in the island of Ceylon, the Peak of Teneriffe, and many others, have been formed in this manner. —3d, Those mountains, whether arranged in a group or not, the earth or stone of which is disposed in strata, and of one or more colours and substances, are supposed to be produced by the substances deposited slowly and gradually by the waters, or by soil gained at the time of great floods. Though these mountains, formed by strata, sometimes degenerate into little hills, and even become almost flat, they always consist of an immense collection of fossils of different kinds, in good preservation, and which are pretty easily detached from their beds. These fossils, consisting of marine shells, intermixed and confounded with heaps of organised bodies of other species, have an appearance of great disorder, by means of some extraordinary and violent currents. All these phenomena seem to prove that most of these mountains chiefly owe their origin to the sea, which once covered some parts of our continents, now left dry by its retreat.

Of those mountains which extend in a direction north and south, it has been observed that their west side is usually much steeper than the east side; but, in such as extend east and west, the south sides are much steeper than the northern: that the Alps are steeper on their western and southern sides, than on the eastern and northern: that in America the Cordilleras are steepest on the western side. And so in like manner, in all continents, as well as hills and islands, the west and southern sides are commonly the steepest.

Mountains, *Attraction of.* As attraction is found to be a general property of all matter, evincing itself universally by the tendency of all bodies towards the centre of the globe; so particularly in hills, it is shown by their drawing the plumb-line aside from the perpendicular, sideways towards the hill, more or less according to its magnitude, density, and situation. And by the observed effect of these, compared with that of the whole earth, it has been determined that the medium density of this whole globe of earth, is about 5 times that of common water. See the articles Attraction, Density, and Earth, also my Tracts, vol. 2.

Mountains, *Height of.* The following is a list of the measured altitudes of the most remarkable mountains in most parts of the earth, in English feet.

Chimboraço	19595	Source of the Nile	8082
Cayambourou	19591	Monast. St. Bernard	7944
Antisana	19290	Pic de los Reyes	7620
Pichinho	15670	Puy de Domme	5088
Mont Blanc	15662	Mount Hecla	4887
Monte Rosa	15084	Mount Vesuvius	3938
Pic of Teneriffe	14026	Ben Laurs	3858
Aiguille d'Argenture	13402	Ben Moir	3723
Pic d'Ossano	11700	Snowdon	3555
Mount Ætna	10954	Ben Gloe	3472
City of Quito	9977	Schihallen	3461
Pic du Medi	9300	Table Hill, Good Hope	3454
Mount Cenis	9212	Ben Lomond	3180
Canegou	8544	Tinto	2342
Gondar, in Abyssinia	8440	Geneva Lake	1232

Caspian Sea below the ocean 306 feet.

MOYNEAU. See Moineau.

MULLER (John), commonly called Regiomontanus, from Mons Regius, or Koningsberg, a town in Franconia, where he was born in 1436, and he became the greatest astronomer and mathematician of his time. Having first acquired grammatical learning in his own country, he was admitted, while yet a boy, into the academy at Leipsic, where he formed a strong attachment to the mathematical sciences, arithmetic, geometry, astronomy, &c. But not finding proper assistance in these studies at this place, he removed, when only 15 years of age, to Vienna, to study under the celebrated Purbach, the professor there, who read lectures on those sciences with the highest reputation. A strong and affectionate friendship soon took place between them; and our author made such rapid improvement in the sciences, that he was soon able to be assisting to his master, and to become a companion in all his labours. In this manner they spent about ten years together; elucidating obscurities, observing the motions of the heavenly bodies, and comparing and correcting the tables of them; particularly those of Mars, which they found to disagree with the motions, sometimes as much as 2 degrees.

About this time there arrived at Vienna the cardinal Bessarion, who came to negotiate some affairs for the pope; who, being a lover of astronomy, soon formed an acquaintance with Purbach and Regiomontanus. He had begun to form a Latin version of Ptolemy's Almagest, or an epitome of it; but not having time to go on with it himself, he requested Purbach to complete the work, and for that purpose to return with him into Italy, to make himself master of the Greek tongue, which he was as yet unacquainted with. To these proposals Purbach only assented, on condition that Regiomontanus would accompany him, and share in all the labours. They first however, by means of an Arabic version of Ptolemy, made some progress in the work; but this was soon interrupted by the death of Purbach, which happened in 1461, in the 39th year of his age. The whole task then devolved on Regiomontanus, who finished the work, at the request of Purbach, made to him when on his death-bed. This work our author afterwards revised and perfected at Rome, when he had learned the Greek language, and consulted the commentator Theon, &c.

Regiomontanus accompanied the cardinal Bessarion in his return to Rome, being then near 30 years of age. Here he applied himself diligently to the study of the Greek language; not neglecting however to make astronomical observations and compose various works in that science; as his Dialogue against the Theories of Cremonensis. The cardinal going to Greece soon after, Regiomontanus went to Ferrara, where he continued the study of the Greek language under Theodore Gaza; who explained to him the text of Ptolemy, with the commentaries of Theon; till at length he became so perfect in it, that he could compose verses, and read it like a critic.—In 1463 he went to Padua, where he became a member of the university; and, at the request of the students, explained Alfraganus, an Arabian philosopher.—In 1464 he removed to Venice, to meet and attend his patron Bessarion. Here he wrote, with great accuracy, his Treatise on Triangles, and a Refutation of the Quadrature of the Circle, which cardinal Cusan pretended he had demonstrated. The same year he returned with Bessarion to Rome; where he made some stay, to procure the most curious books: those which he could not purchase, he took the pains to transcribe, for he wrote with great facility and elegance; and others he got copied at a great expense. For as he was certain that none of these books could be had in Germany, he thought on his return thither, he would at his leisure translate and publish some of the best of them. During this time too he had a severe contest with George Trabezonde, whom he had greatly offended by animadverting on some passages in his translation of Theon's Commentary.

Being now weary of rambling about, and having procured a great number of manuscripts, which was one great object of his travels, he returned to Vienna, and performed for some time the offices of his professorship, by reading of lectures &c. After being thus employed, he went to Buda, on the invitation of Matthias king of Hungary, who was a great lover of letters and the sciences, and had founded a rich and noble library there: for he had bought up all the Greek books that could be found on the sacking of Constantinople; also those that were brought from Athens, or wherever else they could be met with through the whole Turkish dominions, collecting them all together into a library at Buda. But a war breaking out in this country, he looked out for some other place to settle in, where he might pursue his studies, and for this purpose he retired to Noremberg. He tells us, that the reasons which induced him to desire to reside in this city the remainder of his life were, that the artists there were dextrous in fabricating his astronomical machines; and besides, he could from thence easily transmit his letters by the merchants into foreign countries. Being now well versed in all parts of learning, and having made the utmost proficiency in mathematics, he determined to occupy himself in publishing the best of the ancient authors, as well as his own lucubrations. For this purpose he set up a printing-house, and formed a nomenclature of the books he intended to publish, which still remains.

Here that excellent man, Bernard Walther, one of the principal citizens, who was well skilled in the sciences, especially astronomy, cultivated an intimacy with Regiomontanus; and as soon as he understood those laudable designs of his, he took upon himself the expense of constructing the astronomical instruments, and of erecting a printing-house. And first he ordered astronomical rules to be made of tin, for observing the altitudes of the sun, moon, and planets. He next constructed a rectangular, or astronomical radius, for taking the distance of those luminaries. Then an armillary astrolabe, such as was used by Ptolemy and Hipparchus, for observing the places and motions of the stars. Lastly, he made other smaller instruments, as the torquet, and Ptolemy's meteoroscope, with some others which had more of curiosity than utility in them. From this apparatus it evidently appears, that Regiomontanus was a most diligent observer of the laws and motions of the celestial bodies, if there were not still stronger evidences of it in the accounts of the observations themselves which he made with them.

With regard to the printing-house, which was the other part of his design in settling at Noremberg, as soon as he had completed it, he put to press two works of his own, and two others. The latter were, The New Theories of his master Purbach, and the Astronomicon of Manilius. And his own were, the New Calendar, in which were given (as he says in the index of the books which he intended to publish) the true conjunctions and oppositions of the luminaries, their eclipses, their true places every day, &c. His other work was his Ephemerides, of which he thus speaks in the said index: "The Ephemerides, which is vulgarly called an Almanac, for 30 years: where you may every day see the true motion of all the planets, of the moon's nodes, with the aspects of the moon to the sun and planets, the eclipses of the luminaries; and in the fronts of the pages are marked the latitudes." He published also most acute commentaries on Ptolemy's Almagest: a work which cardinal Bessarion so highly valued, that he scrupled not to esteem it worth a whole province. He prepared also new versions of Ptolemy's Cosmography; and at his leisure hours examined and explained works of another nature. He inquired how high the vapours are carried above the earth, which he fixed to be not more than 12 German miles. He set down observations of two comets that appeared in the years 1471 and 1472.

In 1474, pope Sixtus the 4th conceived a design of reforming the calendar; and sent for Regiomontanus to Rome, as the most proper and able person to accomplish his purpose. Regiomontanus was very unwilling to interrupt the studies, and printing of books, he was engaged in at Noremberg; but receiving great promises from the pope, who also for the present named him bishop of Ratisbon, he at length consented to go. He arrived at Rome in 1475, but died there the year after, at only 40 years of age; not without a suspicion of having been poisoned by the sons of George Trabezonde, in revenge for the death of their father, which was said to have been caused by the grief he felt on account of the criticisms made by Regiomontanus on his translation of Ptolemy's Almagest.

Purbach first of any reduced the trigonometrical tables of sines, from the old sexagesimal division of the radius, to the decimal scale. He supposed the radius to be divided into 600,000 equal parts, and computed the sines of the arcs to every ten minutes, in such equal parts of the radius, by the decimal notation. This project of Purbach was perfected by Regiomontanus; who not only extended the sines to every minute, the radius being 600,000, as designed by Purbach, but afterwards, disliking that scheme, as evidently imperfect, he computed them likewise to the radius 1,000,000, for every minute of the quadrant. Regiomontanus also introduced the tangents into trigonometry, the canon of which he called fœcundus, because of the many great advantages arising from them. Besides these things, he enriched trigonometry with many theorems and precepts. Indeed, excepting for the use of logarithms, the trigonometry of Regiomontanus is but little inferior to what ours was, before the improvements made in it by Euler. His Treatise, on both Plane and Spherical Trigonometry, is in 5 books; it was written about the year 1464, and printed in folio at Noremberg in 1533. In the 5th book are various problems concerning rectilinear triangles, some of which are resolved by means of algebra: a proof that this science was not wholly unknown in Europe before the treatise of Lucas De Burgo.

Regiomontanus was author of some other works besides those already mentioned. Peter Ramus, in the account he gives of the admirable works attempted and performed by Regiomontanus, tells us, that in his workshop at Noremberg there was an automaton in perpetual motion: that he made an artificial fly which, taking its flight from his hand, would fly round the room, and at last, as if weary, would return to his master again: that he fabricated an eagle, which, on the emperor's approach to the city, he sent out, high in the air, a great way to meet him, and that it kept him company to the gates of the city. Let us no more wonder, adds Ramus, at the dove of Archytas, since Noremberg can show a fly, and an eagle, armed with geometrical wings. Nor are those famous artificers, who were formerly in Greece and Egypt, any longer of such account, since Noremberg can boast of her Regiomontanuses. For Wernerus first, and then the Schoneri, father and son, afterwards, revived the spirit of Regiomontanus.

MULTANGULAR Figure, is one that has many angles, and consequently many sides also. These are otherwise called polygons.

MULTILATERAL Figures, are such as have many sides, or more than four sides.

MULTINOMIAL, or Multinomial *Roots*, are such as are composed of many names, parts, or members; as, $a + b + c + d$ &c.—For the raising an infinite multinomial to any power, or extracting any root out of such power, see a method by M. Demoivre, in the Philos. Trans. No. 230. See also Polynomial.

MULTIPLE, Multiplex, a number which comprehends some other number several times. Thus, 6 is a multiple of 2, this being contained in 6 just 3 times. Also 12 is a common multiple of 6, 4, and 3; comprehending the first twice, second thrice, and the third four times.

Multiple *Ratio* or *Proportion*, is that which is between multiple numbers &c. If the less term of a ratio be an aliquot part of the greater, the ratio of the greater to the less is called multiple; and that of the less to the greater submultiple.—A submultiple number, is that which is contained in the multiple. Thus, the numbers 2, 3, and 4 are submultiples of 12 and 24.—Duple, triple, &c ratios; as also subduples, subtriples, &c, are so many species of multiple and submultiple ratios.

Multiple *Superparticular Proportion*, is when one number or quantity contains another more than once, and a certain aliquot part; as 10 to 3, or $3\frac{1}{3}$ to 1.

Multiple *Superpartient Proportion*, is when one number or quantity contains another several times, and some parts besides; as 29 to 6, or $4\frac{5}{6}$ to 1.

MULTIPLICAND, is one of the two factors in the rule of multiplication, being that number given to be multiplied by the other, called the multiplicator, or multiplier.

MULTIPLICATION, is, in general, the taking or repeating of one number or quantity, called the multiplicand, as often as there are units in another number, called the multiplier; and the number or quantity resulting from the multiplication, is called the product of the two foregoing numbers or factors.—Multiplication is a compendious addition; performing at once, what in the usual way of addition would require many operations; for the multiplicand is only added to itself, or repeated, as often as is expressed by the units in the multiplier. Thus, if 6 were to be multiplied by 5, the product is 30, which is the sum arising from the addition of the number 6 five times to itself.—In every multiplication, 1 is in proportion to the multiplier, as the multiplicand is to the product.

Multiplication is of various kinds, in whole numbers, in fractions, decimals, algebra, &c.

1. Multiplication *of Whole Numbers*, is performed by the following rules: When the multiplier consists of only one figure, set it under the first, or right-hand figure, of the multiplicand; then, drawing a line under it, begin at the said first figure, and multiply every figure of the multiplicand by the multiplier; setting down the several products below the line, proceeding orderly from right to left. But if any of these products amount to 10, or several 10's, either with or without some overplus, then set down only the overplus, or set down 0 if there be none; and carry, to the next product, as many units as the former contained of tens. Thus, to multiply 35092 by 4.

Multiplicand	35092
Multiplier	4
Product	140368

When the multiplier consists of several figures, multiply the multiplicand by each figure of it, as before, and place the several lines of products below each other in such order, that the first figure of each line may fall straight under its respective multiplier, or multiplying figure; then add these several lines of products together, as they stand, and the sum of them all will be the product of the whole multiplication. Thus, to multiply 63017 by 236:

Multiplicand - -	63017
Multiplier - -	236
Product of 63017 by 6	378102
Product of 63017 by 30	189051
Product of 63017 by 200	126034
Whole product	14872012

The several lines of products may be set down in any order, or any of them first, and any other of them second, &c; for the order of placing them can make no difference in the sum total. There are many abbreviations, and peculiar cases, according to circumstances, which may be seen in most books of arithmetic. The mark or character now used for multiplication, is either the × cross, or a single point . ; the former being introduced by Oughtred, and the latter I think by Leibnitz.

To Prove MULTIPLICATION. This may be done various ways; either by dividing the product by the multiplier, then the quotient will be equal to the multiplicand; or divide the same product by the multiplicand, and the quotient will come out equal to the multiplier: or in general divide the product by either of the two factors, and the quotient will be equal to the other factor, when the operations are all right. But the more usual, and compendious way of proving multiplication, is by what is called the cross, by casting out the nines; which is performed thus: Add the figures of the multiplicand all together, and as often as the sum amounts to 9, reject it, and set down the last overplus as in the margin; this in the foregoing example is 8. Then do the same by the multiplier, setting down the last overplus, which is 2, on the right of the former remainder 8. Next multiply these two remainders, 2 and 8, together, and from their product 16, cast out the 9, and there remains 7, which set down over the two former. Lastly, add up, in the same manner, all the figures of the whole product of the multiplication, viz 14872012, casting out the 9's, and then there remains 7, to be set down under the two first remainders. Thus when the figure at top, is the same as that at bottom, as they are here both 7's, the work it may be presumed is right; but if these two figures should not be the same, it is certainly wrong.

$$\begin{array}{ccc} & 7 & \\ 8 & & 2 \\ & 7 & \end{array}$$

The above method of proving multiplication depends on a particular property of the number 9: which is this. If the sum of the digits of any number be divisible by 9, the number itself is also divisible by 9; and consequently the sum of the digits of any number being divided by 9, leaves the same remainder as the number itself when divided by 9. Another method is derived from a peculiar property of the number 11, which is this. When a number is divisible by 11, the sum of the 1st, 3d, &c, digits, is equal to the sum of the 2d, 4th, &c, digits, or the one exceeds the other by some exact multiple of 11. Consequently any number whatever when divided by 11, will leave the same remainder as the difference of the two sums when divided by that number; observing always to subtract the latter sum from the former, or from the former plus some multiple of 11, when the sum of the digits in the 2d, 4th, &c, places is the greatest. Whence the following rule. Cast all the 11's out of the sums of the digits, both in the even and odd places of the multiplicand, and subtract the former remainder from the latter, or from the latter plus 11, and reserve the difference; do exactly the same with the multiplier and product. Multiply the two first differences together, and cast all the 11's out of the result, so shall this last remainder be the same as that before found in the product, if the work be right. Thus in the above example:

	Multiplic.	Multiplier	Product			
Sum of odd digits	13	8	13		1	
Sum of even digits	4	3	12	9		5
Differences	9	5	1		1	

And the product 9 × 5 ÷ 11, leaving the remainder 1, which being the same as the remainder of the product, indicates that the work is right.

2. *To Multiply Money, or any other thing, consisting of different Denominations together, by any number, usually called Compound Multiplication.* Begin at the lowest denomination, and multiply the number of each name separately by the multiplier, setting down the products below them. But if any of these products amount to as much as 1 or more of the next higher denominations, carry so many to the next product, and set down only the overplus.

If the multiplier exceeds 12, then resolve it into its factors, if it be a compound number; and multiply successively by those factors; but if the given multiplier be not a compound number, then resolve the next greater or less compound number into its factors, and multiply with those factors as above; and from the result deduct, or add, so many times the multiplicand, as this last compounded number is greater or less than the given multiplier. Also if there are fractional parts in the given multiplier, take such parts of the multiplicand as these are of a unit, which added to the product will give the answer sought; as will appear in the following examples.

Ex. 1. Multiply 1*l.* 12*s.* 4½*d.*

	1*l.*	12*s.*	4½*d.*
by			9
	*L.*14	11	4½

Ex. 2. Multiply 12*s.* 7½*d.* by 24

		12*s.*	7½*d.*
24 = 6 × 4			4
	2	10	6
			6
	*L.*15	3	0

Multiply 1*l.* 13*s.* 8*d.* by 29½

	1*l.*	13*s.*	8*d.*
29½ = 4 × 7 + 1 + ½			4
4 =	6	14	8
			7
28 =	47	2	8
1 =	1	13	8
½ =	0	16	10
29½ =	*L.*49	13	2

3. *To Multiply Vulgar Fractions.*—Multiply all the given numerators together for the numerator, and all the denominators together for the denominator of the product sought.

Thus, $\frac{2}{3}$ multiplied by $\frac{4}{5}$, or $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$.

And $\frac{2}{5} \times \frac{3}{5} \times \frac{3}{7} = \frac{18}{175}$.

And here it may be noted that, when there are any common numbers in the numerators and denominators, these may be omitted in both, which will make the operation shorter, and bring out the whole product in a fraction much simpler or in lower terms. Thus,

$\frac{2}{3} \times \frac{3}{4} \times \frac{5}{6}$, become $\frac{2 \times 5}{4 \times 6} = \frac{10}{24}$ or $\frac{5}{12}$, by leaving out the two 3's.

Also, when any numerators and denominators will both abbreviate or divide by one and the same number, let them be divided, and the quotients used instead of them. So, in the above example, after omitting the two 3's, let the 2 and 6 be both divided by 2, and use the quotients 1 and 3 instead of them, so shall the expression become $\frac{1\times5}{4\times3}=\frac{5}{12}$, as before.

4. *To Multiply Decimals.*—Multiply the given numbers together the same as if they were whole numbers, and point off as many decimals in the whole product as there are in both factors; as in the annexed example, where the number of decimals is five, because there are three in the multiplicand, and two in the multiplier.—When it happens that there are not so many figures in the product as are equal to the number of decimals in both factors, then prefix as many ciphers as will supply the defect.

$$\begin{array}{r} 2\cdot305 \\ 21\cdot86 \\ \hline 13830 \\ 18440 \\ 2305 \\ 4610 \\ \hline 50\cdot38730 \end{array}$$

5. *Cross* **Multiplication**, otherwise called *Duodecimal Arithmetic*, is the multiplying of numbers together whose subdivisions proceed by 12's; as feet, inches, and parts, that is 12th-parts, &c; a rule of frequent use in squaring, or multiplying together the dimensions of the works of bricklayers, carpenters, and other artificers. *For Example.* To multiply 5 feet 3 inches by 2 feet 4 inches. Set them down as in the margin, and multiply all the parts of the multiplicand by each part of the multiplier; thus, 2 times 3 make 6 inches, and 2 times 5 make 10 feet; then 4 times 3 make 12 parts, or 1 inch to carry; and 4 times 5 make 20, and 1 to carry makes 21 inches, or 1 f. 9 inc. to set down below the former line: Lastly adding the two lines together, the whole sum or product amounts to 12 f. 3 inch.—See **Duodecimals**.

$$\begin{array}{rr} 5^{\text{f.}} & 3^{\text{in.}} \\ 2 & 4 \\ \hline 10 & 6 \\ 1 & 9 \\ \hline 12 & 3 \end{array}$$

6. **Multiplication** *in Algebra.* This is performed, 1. When the quantities are simple, by only joining the letters together like a word; and if the simple quantities have any coefficients or numbers joined with them, multiply the numbers together, and prefix the product of them to the letters so joined together. But, in algebra, we have not only to attend to the quantities themselves, but also to the signs of them; and the general rule for the signs is this: When the signs are alike, or the same, either both + or both −, then the sign of the product will always be +; but when the signs are different, or unlike, the one +, and the other −, then the sign of the product will be −. Hence these

Examples.

$$\begin{array}{llllll} \text{Mult.} & +a & -2a & +6x & -8x & -3ab \\ \text{By} & +b & -4b & -3a & +5a & -5ac \\ \hline \text{Products} & +ab & +8ab & -18ax & -40ax & +15a^2bc \\ \hline \end{array}$$

2. In compound quantities, multiply every term or part of the multiplicand by each term separately of the multiplier, and set down all the products with their signs, collecting always into one sum as many terms as are similar or like to one another. And it is usual in algebra, to begin to multiply on the left hand, and thence proceed towards the right; being directly contrary to the method in multiplication of numbers.

Examples.

$$\begin{array}{lll} a+b & a-b & a+b \\ a+b & a-b & a-b \\ \hline a^2+ab & a^2-ab & a^2+ab \\ \quad\;+ab+b^2 & \quad\;-ab+b^2 & \quad\;-ab-b^2 \\ \hline a^2+2ab+b^2 & a^2-2ab+b^2 & a^2\quad\cdot\quad-b^2 \end{array}$$

$$\begin{array}{lll} 2a-3b & 2a+4x & a^2-ax \\ 4a+5b & 2a-4x & 2a+2x \\ \hline 8a^2-12ab & 4a^2+8ax & 2a^3-2a^2x \\ \quad\;+10ab-15b^2 & \quad\;-8ax-16x^2 & \quad\;+2a^2x-2ax^2 \\ \hline 8a^2-2ab-15b^2 & 4a^2\quad\cdot\quad-16x^2 & 2a^3\quad\cdot\quad-2ax^2 \\ \hline \end{array}$$

3. In surd quantities, if the terms can be reduced to a common surd, the quantities under each may be multiplied together, and the mark of the same surd prefixed to the product; but if not, then the different surds may be set down with the mark of multiplication between them, to denote their product.

Examples.

$$\begin{array}{lllll} 7\sqrt{ax} & \sqrt{7} & \sqrt[3]{7ab} & \sqrt{12a} & 6a\sqrt{2cx} \\ 5\sqrt{cx} & \sqrt{5} & \sqrt[3]{4ac} & \sqrt{3a} & 2b\sqrt{3ax} \\ \hline 35\sqrt{acx^2} & \sqrt{35} & \sqrt[3]{28a^2bc} & \sqrt{36a^2}=6a & 12ab\sqrt{6acx^2} \\ \hline \end{array}$$

4. Powers or roots of the same quantity are multiplied together, by adding their exponents.
Thus, $a^2\times a^3=a^5$; and $(a+x)^3\times(a+x)^5=(a+x)^3$: also $x^2\times x^{\frac{1}{2}}=x^{\frac{5}{2}}$; and $a^{\frac{1}{2}}\times a^{\frac{1}{2}}=a^1$ or a.

To Multiply Numbers together by Logarithms.—This is performed by adding together the logarithms of the given numbers, and taking the number answering to that sum, which will be the product sought.

Descartes, at the beginning of his Geometry, performs multiplication (and indeed all the other common arithmetical rules) in geometry, or by lines; but this is no more than taking a 4th proportional to three given lines, of which the first represents unity, and the 2d and 3d the two factors or terms to be multiplied, the product being expressed by the 4th proportional; because, in every multiplication, unity or 1 is to either of the two factors, as the other factor is to the product.

MULTIPLIER, or **Multiplicator**, is the number or quantity which multiplies another, called the multiplicand, in any operation of multiplication.

MUNSTER (**Sebastian**), an eminent German divine and mathematician, was born at Ingelheim in 1489. At the age of 14 he was sent to Heidelberg to study. Two years after, he entered the convent of the Cordeliers; where he assiduously studied divinity, mathematics, and geography. He was the first who published a Chaldee Grammar and Lexicon; and he shortly after produced a Talmudic Dictionary. He afterwards became professor of the Hebrew language at Basil. He was one of the first who attached himself to Luther, and embraced Protestantism: yet behaved himself with great moderation; never concerning himself with their disputes; but shut himself up at home and pursued his favourite studies, which were mathematics, natural philosophy, with the Hebrew and other Oriental languages. He published a great number of books on these subjects; particularly, a Latin version, from the Hebrew, of all the books of the Old Testament, with learned notes, printed at Basil in 1534 and 1546; Josephus's History of the Jews in Latin; a Treatise of Dialling, in folio, 1536; Universal Cosmography, in 6 books folio, Basil 1550. For these works he was styled the German Strabo; as he was the German Esdras, for his Oriental writings.

Munster was a meek, pacific, studious, retired man, who wrote a great number of books, but never meddled in controversy.—He died of the plague at Basil, in 1552, at 63 years of age.

MURAL-*Arch*, or Instrument, or Quadrant, is one that is fixed against a wall or pillar, such as is employed in fixed observatories.

MURDERERS, a small species of ordnance once used on shipboard; but now out of use.

MUSIC, the science of sound, considered as capable of producing melody, or harmony. Among the ancients, music was taken in a much more extensive sense than among the moderns: what we call the science of music, was by the ancients rather called Harmonica.

Music is one of the seven sciences called liberal, and comprehended also among the mathematical sciences, as having for its object discrete quantity or number; not however considering it in the abstract, like arithmetic; but in relation to time and sound, with intent to constitute a delightful melody.

This science is also Theoretical and Practical. Theoretical, which examines the nature and properties of concords and discords, explaining the proportions between them by numbers. And Practical, which teaches not only composition, or the manner of composing tunes, or airs; but also the art of singing with the voice, and playing on musical instruments.

It appears that music was one of the most ancient of the arts; and, of all others, vocal music must doubtless have been the first kind. For man had not only the various tones of his own voice to make his observations on, before any other art or instrument was invented, but had the various natural strains of birds to give him occasion to improve his own voice, and the modulations of sounds it was capable of. The first invention of wind instruments Lucretius ascribes to the observation of the winds whistling in the hollow reeds. As for other kinds of instruments, there were so many occasions for cords or strings, that men could not be long in observing their various sounds; which might give rise to stringed instruments. And for the pulsative instruments, as drums and cymbals, they might arise from the observation of the naturally hollow noise of concave bodies.

As to the inventors and improvers of music, Plutarch, in one place, ascribes the first invention of it to Apollo; and in another place to Amphion, the son of Jupiter and Antiope. The latter indeed, it is generally allowed, first brought music into Greece, and invented the lyre. To him succeeded Chiron, the demigod; then Demodocus; Hermes Trismegistus; Olympus; and Orpheus, whom some make the first introducer of music into Greece, and the inventor of the lyre: to whom they add Phemius, Thales, and Thamgris, who, it has been said, was the first inventor of instrumental music without singing.

These were the eminent musicians before Homer's time: others of a later date were, Terpander, who was contemporary with Lycurgus, and set his laws to music; to whom also some attribute the first institution of musical modes, and the invention of the lyre: also, Lasus Hermionensis, Melanippides, Philoxenus, Timotheus, Phrynnis, Epigonius, Lysander, Simmicus, and Diodorus; who were all of them considerable improvers of music. Lasus, it is said, was the first author who wrote upon music, in the time of Darius Hystaspis; Epigonius invented an instrument of 40 strings, called the Epigonium. Simmicus also invented an instrument of 35 strings, called a Simmicium; Diodorus improved the tibia, by adding new holes; and Timotheus the lyre, by adding a new string; for which he was fined by the Lacedemonians.

As the accounts we have of the inventors of musical instruments among the ancients are very obscure, so also are the accounts of those instruments themselves; of most of them indeed we know little more than the bare names. The general division of instruments is, into stringed instruments, wind instruments, and those of the pulsatile kind. Of stringed instruments, mention is made of the lyra or cithara, the psalterium, trigonum, sambuca, pectis, magas, barbiton, testudo, epigonium, simmicium, and panderon; which were all struck with the hand, or a plectrum. Of wind instruments, were the tibia, fistula, hydraulic organs, tubæ, cornua, and lituus. And the pulsatile instruments were the tympanum, cymbalum, crepitaculum, tintinnabulum, crotalum, and sistrum.

Music has ever been in the highest esteem in all ages, and among all people; nor could authors express their opinion of it strongly enough, but by inculcating that it was used in heaven, and as one of the principal entertainments of the gods, and the souls of the blessed. The effects ascribed to it by the ancients are almost miraculous: by its means, it has been said, diseases have been cured, unchastity corrected, seditions quelled, passions raised and calmed, and even madness occasioned. Athenæus assures us, that anciently all laws, divine and civil, exhortations to virtue, the knowledge of divine and human things, with the lives and actions of illustrious men, were written in verse, and publicly sung by a chorus to the sound of instruments; which was found the most effectual means to impress morality on the minds of men, and a right sense of their duty.

Dr. Wallis has endeavoured to account for the surprising effects attributed to the ancient music; and ascribes them chiefly to the novelty of the art, and the hyperboles of the ancient writings: nor does he doubt, but the modern music, in like cases, would produce effects at least as considerable as the ancient. The truth is, we can match most of the ancient stories of this kind in the modern histories. If Timotheus could excite Alexander's fury with the Phrygian mode, and soothe him into indolence with the Lydian; a more modern musician has driven Eric, king of Denmark, into such a rage, as to kill his best servants. Dr. Niewentyt speaks of an Italian who, by varying his music from brisk to solemn, and the contrary, could so move the soul, as to cause distraction and madness; and Dr. South has founded his poem, called Musica Incantans, on an instance he knew of the same kind.

Music however is found not only to exert its force on the affections, but on the parts of the body also; witness the Gascon knight, mentioned by Mr. Boyle, who could not contain his water at the playing of a bagpipe; and the woman, mentioned by the same author, who would burst into tears at the hearing of a certain tune, with which other people were but a little affected. To say nothing of the trite story of the Tarantula, we have an instance, in the History of the Academy of Sciences, of a musician being cured of a violent fever, by a little concert occasionally played in his room.

Nor are our minds and bodies alone affected with sounds, but even inanimate bodies are so. Kircher speaks of a large stone, that would tremble at the sound of one particular organ pipe; and Morhoff mentions one Petter, a Dutchman, who could break rummer-glasses with the tone of his voice. Mersenne also mentions a particular part of a pavement, that would shake and tremble, as if the earth would open, when the organs played. Mr. Boyle adds, that seats will tremble at the sound of organs; that he has felt his hat do so under his hand, at certain notes both of organs and discourse; and that he was well informed every well-built vault would thus answer to some determinate note.

It has been disputed among the learned, whether the ancients or moderns best understood and practised music. Some maintain that the ancient art of music, by which such wonderful effects were performed, is quite lost; and others, that the true science of harmony is now arrived at much greater perfection than was known or practised among the ancients. This point seems no other way to be determinable but by comparing the principles and practice of the one with those of the other. As to the theory or principles of harmonics, it is certain we understand it better than the ancients; because we know all that they knew, and have improved considerably on their foundations. The great dispute then lies on the practice; with regard to which it may be observed, that among the ancients, music, in the most limited sense of the word, included harmony, rythmus, and verse; and consisted of verses sung by one or more voices alternately, or in choirs, sometimes with the sound of instruments, and sometimes by voices only. Their musical faculties, we have just observed, were melopœia, rythmopœia, and poesis; the first of which may be considered under two heads, melody and symphony. As to the latter, it seems to contain nothing but what relates to the conduct of a single voice, or making what we call melody. It does not appear that the ancients ever thought of the concert, or harmony of parts; which is a modern invention, for which we are beholden to Guido Aretine, a Benedictine friar.

Not that the ancients never joined more voices or instruments than one in the same symphony; but that they never joined several voices so as that each had a distinct and proper melody, which made among them a succession of various concords, and were not in every note unisons, or at the same distance from each other as octaves. This last indeed agrees to the general definition of the word symphonia; yet it is plain that in such cases there is but one song, and all the voices perform the same individual melody. But when the parts differ, not by the tension of the whole, but by the different relations of the successive notes, this is the modern art, which requires so peculiar a genius, and on which account the modern music seems to have much the advantage of the ancient. For further satisfaction on this head, see Kircher, Perrault, Wallis, Malcolm, Cerceau, and others; who unanimously agree, that after all the pains they have taken to know the true state of the music of the ancients, they could not find the least reason to think there was any such thing in their days as music in parts.

The ancient musical notes are very mysterious and perplexed: Boethius and Gregory the Great first put them into a more easy and obvious method. In the year 1204, Guido Aretine, a Benedictine of Arezzo in Tuscany, first introduced the use of a staff with five lines, on which, with the spaces, he marked his notes by setting a point up and down upon them, to denote the rise and fall of the voice: though Kircher says this artifice was in use before Guido's time.

Another contrivance of Guido's was to apply the six syllables, *ut, re, mi, fa, sol, la*, which he took out of the Latin hymn,

UT queant laxis REsonare fibris
MIra gestorum FAmuli tuorum,
SOLve polluti LAbii reatum,
O Pater Alme.

We find another application of them in the following lines.

UT RElevet MIserum FAtum, SOLitosque LAbores
Aevi, sit dulcis musica noster amor.

Besides his notes of music, by which, according to Kircher, he distinguished the tones, or modes, and the seats of the semitones, he also invented the scale, and several musical instruments, called polyplectra, as spinets and harpsichords.

The next considerable improvement was in 1330, when Joannes Muria, or de Muris, doctor at Paris (or as Bayle and Gesner make him, an Englishman), invented the different figures of notes, which express the times or length of every note, at least their true relative proportions to one another, now called longs, breves, semi-breves, crotchets, quavers, &c.

The most ancient writer on music was Lasus Hermionensis; but his works, as well as those of many others, both Greek and Roman, are lost. Aristoxenus, disciple of Aristotle, is the earliest author extant on the subject: after whom came Euclid, author of the Elements of Geometry; and Aristides Quintilianus wrote after Cicero's time. Alypius stands next; after him Gaudentius the philosopher, and Nicomachus the Pythagorean, and Bacchius. Of which seven Greek authors we have a fair copy, with a translation and notes, by Meibomius. Ptolemy, the celebrated astronomer, wrote in Greek on the principles of harmonics, about the time of the emperor Antoninus Pius. This author keeps a medium between the Pythagoreans and Aristoxenians. He was succeeded at a considerable distance by Manuel Bryennius.

Of the Latins, we have Boetius, who wrote in the time of Theodoric the Goth; and one Cassiodorus, about the same time; Martianus, and St. Augustine, not far remote. And of the moderns are Zarlin, Salinas, Vincenzo Galileo, Doni, Kircher, Mersenne, Paran, De Caux, Perrault, Descartes, Wallis, Holder, Malcolm, Rousseau, &c.

MUSICAL *Numbers*, are the numbers 2, 3, and 5, together with their composites. They are so called, because all the intervals of music may be expressed by such numbers. This is now generally admitted by musical theorists. Mr. Euler seems to suppose, that 7 or other primes might be introduced; but he speaks of this as a doubtful and difficult matter. Here 2 corresponds to the octave, 3 to the fifth, or rather to the 12th, and 5 to the third major, or rather the seventeenth. From these three may all other intervals be found.

MUSICAL *Proportion*, or Harmonical Proportion, is when, of four terms, the first is to the 4th, as the difference of the 1st and 2d is to the difference of the 3d and 4th: as 2, 3, 4, and 8 are in musical proportion, because 2 : 8 : : 1 : 4. And hence, if there be only three terms, the middle term supplying the place of both the 2d and 3d, the 1st is to the 3d, as the difference of the 1st and 2d, is to the difference of the 2d and 3d: as in these 2, 3, 6; where 2 : 6 : : 1 : 3. See HARMONICAL *Proportion*.

MUSSCHENBROEK (PETER), a very distinguished natural philosopher and mathematician, was born at Utrecht about the year 1700. He was first professor of these sciences in his own university, and was afterwards invited to the chair at Leyden, which he filled with reputation and honour till his death, which happened in 1761. He was a member of several academies, particularly the Academy of Sciences at Paris. He published several works in Latin, all of them displaying his great penetration and accuracy. As,

1. His Elements of Physico-Mathematics, in 1726.
2. Elements of Physics, in 1736.
3. Institutions of Physics; containing an abridgment of the new discoveries made by the moderns; in 1748.
4. Introduction to Natural Philosophy; which he began to print in 1760; and which was completed and published at Leyden, in 1762, by M. Lulofs, after the death of the author. It was translated into French by M. Sigaud Delafond, and published at Paris in 1769, in 3 vols. 4to; under the title of A Course of Experimental and Mathematical Physics.

He had also several papers, chiefly on meteorology, printed in the volumes of Memoirs of the Academy of Sciences, viz, in those of the years 1734, 1735, 1736, 1753, 1756, and 1760.

MUTULE, a kind of square modillion in the Doric frize.

MYOPS, one who is near-sighted, or purblind, from whatever cause it may happen; either from too great a convexity of the cornea, or from too great length of the bulb, &c, causing the adunation of the rays of light in a focus before the retina.

MYRIAD, the number of 10,000, or ten thousand.

N.

NABONASSAR, first king of the Chaldeans or Babylonians; memorable for the Jewish era which bears his name, which began on Wednesday February 26th in the 3967th year of the Julian period, or 747 years before Christ; the years of this epoch being Egyptian ones, of 365 days each. This is a remarkable era in chronology, because Ptolemy assures us there were astronomical observations made by the Chaldeans from Nabonassar to his time; also Ptolemy, and the other astronomers, account their years from that epoch.

The Babylonians having revolted from the Medes, who had overthrown the Assyrian monarchy, did, under Nabonassar, found a dominion, which was much increased under Nebuchadnezzar. It is probable this Nabonassar is that Baladan in the 2d Book of Kings, xx, 12, father of Merodach, who sent ambassadors to Hezekiah. See 2 Chron. xxii.

NADIR, that point of the heavens diametrically under our feet, or opposite to the zenith, which is directly over our heads. The zenith or nadir are the two poles of the horizon, each being 90° distant from it.

The Sun's NADIR, is the axis of the cone projected by the shadow of the earth: so called, because that axis being prolonged, gives a point in the ecliptic diametrically opposite to the sun.

NAKED, in Architecture, as the Naked of a Wall, &c. is the surface, or plane, from whence the projectures arise; or which serves as a ground to the projectures.

NAPIER or NEPER (JOHN), baron of Merchiston in Scotland, the inventor of logarithms, was the eldest son of sir Archibald Napier of Merchiston, and born in the year 1550. Having given early indications of great natural parts, his father was careful to have them cultivated by a liberal education. After going through the ordinary course of studies at the university of St. Andrews, he made the tour of France, Italy, and Germany. On his return to his native country, his literature and other fine accomplishments soon rendered him conspicuous; he however retired from the world to pursue literary researches, in which he made an uncommon progress, as appears by the several useful discoveries with which he afterwards favoured mankind. He chiefly applied himself to the study of mathematics; without however neglecting that of the Scriptures; in both of which he discovered the most extensive knowledge and profound penetration. His Essay on the Book of the Apocalypse indicates the most acute investigation; though time hath discovered that his calculations concerning particular events had proceeded on fallacious data. But what has chiefly rendered his name famous, was his great and fortunate discovery of logarithms in trigonometry, by which the ease and expedition in calculation have so wonderfully assisted the science of astronomy and the arts of practical geometry and navigation. Napier, having a great attachment to astronomy and spherical trigonometry, had occasion to make many numeral calculations of such triangles, with sines, tangents, &c; and these being expressed in large numbers, they hence occasioned a great deal of labour and trouble: to spare themselves part of this labour, Napier, and other authors about his time, set themselves to find out certain short modes of calculation, as is evident from many of their writings. To this necessity, and these endeavours it is, that we owe several ingenious contrivances; particularly the computation by Napier's Rods, and several other curious and short methods that are given in his Rabdologia; and at length, after trials of many other means, the most complete one of logarithms, in the actual construction of a large table of numbers in arithmetical progression, adapted to a set of as many others in geometrical progression. The property of such numbers had been long known, viz, that the addition of the former answered to the multiplication of the latter, &c; but it wanted the necessity of such very troublesome calculations as those above mentioned, joined to an ardent disposition, to make such a use of that property. Perhaps also this disposition was urged into action by certain attempts of this kind which it seems were made elsewhere; such as the following, related by Wood in his Athenæ Oxonienses, under the article Briggs, on the authority of Oughtred and Wingate, viz, "That one Dr. Craig, a Scotchman, coming out of Denmark into his own country, called upon John Neper baron of Marcheston near Edinburgh, and told him, among other discourses, of a new invention in Denmark (by Longomontanus as 'tis said) to save the tedious multiplication and division in astronomical calculations. Neper was very solicitous to know farther of him concerning this matter, but he could give no other account of it, than that it was by proportionable numbers. Which hint Neper taking, he desired him at his return to call upon him again. Craig, after some weeks had passed, did so, and Neper then showed him a rude draught of that he called Canon Mirabilis Logarithmorum. Which draught, with some alterations, he printed in 1614. It came forthwith into the hands of our author Briggs, and into those of William Oughtred, from whom the relation of this matter came."

Whatever might be the inducement however, Napier published his invention in 1614, under the title of Logarithmorum Canonis Descriptio, &c, containing the description and canon of his logarithms, which are those of the kind that is called hyperbolic. This work coming presently to the hands of Mr. Briggs, then professor of geometry at Gresham-college in London, he immediately gave it the greatest encouragement, teaching the nature of the logarithms in his public lectures, and at the same time recommending a change in the scale of them, by which they might be advantageously altered to the kind which he afterwards computed himself, which are thence called Briggs's Logarithms, and are those now in common use. Mr. Briggs also presently wrote to lord Napier upon this proposed change, and made journeys to Scotland the two following years, to visit Napier, and consult with him on the subject of this alteration, before he set about making it. Briggs, in a letter to archbishop Usher, March 10, 1615, writes thus: " Napier lord of Markinston hath set my head and hands at work with his new and admirable logarithms. I hope to see him this summer, if it please God; for I never saw a book which pleased me better, and made me more wonder." Briggs accordingly made lord Napier the visit, and staid a month with him.

The following passage, from the Life of Lilly the astrologer, contains a curious account of the meeting of those two illustrious men. " I will acquaint you (says Lilly) with one memorable story related unto me by John Marr, an excellent mathematician and geometrician, whom I conceive you remember. He was servant to King James and Charles the First. At first when the lord Napier, or Marchiston, made public his logarithms, Mr. Briggs, then reader of the astronomy lectures at Gresham-college in London, was so surprised with admiration of them, that he could have no quietness in himself until he had seen that noble person the lord Marchiston, whose only invention they were: he acquaints John Marr herewith, who went into Scotland before Mr. Briggs, purposely to be there when those two so learned persons should meet. Mr. Briggs appoints a certain day when to meet at Edinburgh; but failing thereof, the lord Napier was doubtful he would not come. It happened one day as John Marr and the lord Napier were speaking of Mr. Briggs; ' Ah, John (said Marchiston), Mr. Briggs will not now come.' At the very instant one knocks at the gate; John Marr hastens down, and it proved Mr. Briggs to his great contentment. He brings Mr. Briggs up into my lord's chamber, where almost one quarter of an hour was spent, each beholding other almost with admiration before one word was spoke. At last Mr. Briggs began: ' My lord, I have undertaken this long journey purposely to see your person, and to know by what engine of wit or ingenuity you came first to think of this most excellent help into astronomy, viz, the logarithms; but, my lord, being by you found out, I wonder nobody else found it out before, when now known it is so easy.' He was nobly entertained by the lord Napier; and every summer after that, during the lord's being alive, this venerable man Mr. Briggs went purposely into Scotland to visit him."

Napier made also considerable improvements in spherical trigonometry &c, particularly by his Catholic or Universal Rule, being a general theorem by which he resolves all the cases of right-angled spherical triangles in a manner very simple, and easy to be remembered, namely, by what he calls the Five Circular Parts. His Construction of Logarithms too, besides the labour of them, manifests the greatest ingenuity. Kepler dedicated his Ephemerides to Napier, which were published in the year 1617; and it appears from many passages in his letter about this time, that he accounted Napier to be the greatest man of his age in the particular department to which he applied his abilities.

The last literary exertion of this eminent person was the publication of his Rabdology and Promptuary, in the year 1617; soon after which he died at Marchiston, the 3d of April in the same year, and in the 68th year of his age.—The list of his works is as follows:

1. A Plain Discovery of the Revelation of St. John; 1593.
2. Logarithmorum Canonis Descriptio; 1614.
3. Mirifici Logarithmorum Canonis Constructio; &c. Quibus accessere propositiones ad triangula sphærica faciliore calculò resolvenda. Una cum Annotationibus aliquot doctissimi D. Henrici Briggii in eas, et memoratam appendicem. Published by the author's son in 1619.
4. Rabdologia, seu Numerationis per Virgulas, libri duo; 1617. This contains the description and use of the bones or rods; with several other short and ingenious modes of calculation.
5. His Letter to Anthony Bacon (the original of which is in the archbishop's library at Lambeth), entitled, Secret Inventions, Profitable and Necessary in these days for the Defence of this Island, and withstanding Strangers Enemies to God's Truth and Religion; dated June 2, 1596.

NAPIER'S *Bones*, or *Rods*, an instrument contrived by Lord Napier, for the more easy performing of the arithmetical operations of multiplication, division, &c. These rods are five in number, made of bone, ivory, horn, wood, or pasteboard, &c. Their faces are divided into nine little squares (fig. 7, pl. 20); each of which is parted into two triangles by diagonals. In these little squares are written the numbers of the multiplication-table; in such a manner that the units, or right-hand figures, are found in the right-hand triangle; and the tens, or the left-hand figures, in the left-hand triangle; as in the figure.

To Multiply Numbers by NAPIER'S *Bones.* Dispose the rods in such a manner, as that the top figures may exhibit the multiplicand; and to these, on the left-hand, join the rod of units: in which seek the right-hand figure of the multiplier; and take out the numbers corresponding to it, in the squares of the other rods, by adding the several numbers occurring in the same rhomb together, and their sums. After the same manner write out the numbers corresponding to the other figures of the multiplier; disposing them under one another as in the common multiplication; and lastly add the several numbers into one sum. For example, suppose the multiplicand 5978, and the multiplier 937. From the outermost triangle on the right-hand (fig. 8, pl. 20) which corresponds to the right-hand figure of the multiplier 7, take out the figure 6, placing it under the line. In the next rhomb towards the left,

$$\begin{array}{r} 5978 \\ 937 \\ \hline 41846 \\ 17934 \\ 53802 \\ \hline 5601386 \end{array}$$

add 9 and 5; their sum being 14, write the right-hand figure 4, against 6; carrying the left-hand figure 1 to 4 and 3, which are found in the next rhomb; and join the sum 8 to 46, already set down. After the same manner, in the last rhomb, add 6 and 5, and the latter figure of the sum 11, set down as before, and carry 1 to the 3 found in the left-hand triangle; the sum 4 join as before on the left-hand of 1846. Thus you will have 41846 for the product of 5978 by 7. And in the same manner are to be found the products for the other figures of the multiplier; after which the whole is to be added together as usual.

To perform Division by NAPIER's *Bones.* Dispose the rods so, that the uppermost figures may exhibit the divisor; to these on the left-hand, join the rod of units. Descend under the divisor, till you meet those figures of the dividend in which it is first required how oft the divisor is found, or at least the next less number, which is to be subtracted from the dividend; then the number corresponding to this, in the place of units, set down for a quotient. And by determining the other parts of the quotient after the same manner, the division will be completed.

For example; suppose the dividend 5601386, and the divisor 5978; since it is first inquired how often 5978 is found in 56013, descend under the divisor (fig. 8) till in the lowest series you find the number 53802, approaching nearest to 56013; the former of which is to be subtracted from the latter, and the figure 9 corresponding to it in the rod of units set down for the quotient. To the remainder 2211 join the following figure 8, of the dividend; and the number 17934 being found as before for the next less number to it, the corresponding number 3 in the rod of units is to be set down for the next figure of the quotient. After the same manner the third and last figure of the quotient will be found to be 7; and the whole quotient 937.

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5978 ) 5601386 ( 937
       53802
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        22118
        17934
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         41846
         41846
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NATURAL *Day, Year, &c.* See DAY, YEAR, &c.

NATURAL *Horizon,* is the sensible or physical horizon.

NATURAL *Magic,* is that which only makes use of natural causes; such as the treatise of J. Bapt. Porta, Magia Naturalia.

NATURAL *Philosophy,* otherwise called *Physics,* is that science which considers the powers of nature, the properties of natural bodies, and their actions on one another.

Laws of NATURE, are certain axioms, or general rules, of motion and rest, observed by natural bodies in their actions on one another. Of these laws, Sir I. Newton has established the three following.

1st LAW.—That every body perseveres in the same state, either of rest, or uniform rectilinear motion; unless it is compelled to change that state by the action of some foreign force or agent. Thus, projectiles persevere in their motions, except so far as they are retarded by the resistance of the air, and the action of gravity: and thus a top, once set in motion, only ceases to turn round, because it is resisted by the air, and by the friction of the plane upon which it moves. Thus also the larger bodies of the planets and comets preserve their progressive and circular motions a long time undiminished, in regions void of all sensible resistance.—As body is passive in receiving its motion, and the direction of its motion, so it retains them, or perseveres in them, without any change, till it be acted on by something external.

2d LAW.—The motion, or change of motion, is always proportional to the moving force by which it is produced, and in the direction of the right line in which that force is impressed. If a given force produce a certain motion, a double force will produce double the motion, a triple force triple the motion, and so on. And this motion, since it is always directed to the same point with the generating force, if the body were in motion before, is either to be added to it, as when the motions conspire: or subtracted from it, as when they are opposite; or combined obliquely, when oblique: being always compounded with it according to the determination of each.

3d LAW.—Re-action is always contrary, and equal to action; or the actions of two bodies upon each other, are always mutually equal, and directed contrary ways; and are to be estimated always in the same right line. Thus, if one body press or draw another, it is equally pressed or drawn by it. So, if I press a stone with my finger, the finger is equally pressed by the stone: if a horse draw a weight forward by a rope, the horse is equally opposed or drawn back towards the weight; the equal tension or stretch of the rope hindering the progress of the one, as it promotes that of the other. Again, if any body, by striking on another, do in any manner change its motion, it will itself, by means of the other, undergo also an equal change in its own motion, on account of the equality of the pressure. When two bodies meet, each endeavours to persevere in its state, and resists any change: and because the change which is produced in either may be equally measured by the action which it excites upon the other, or by the resistance which it meets with from it, it follows that the changes produced in the motions of each are equal, but are made in contrary directions: the one acquires no new force but what the other loses in the same direction; nor does this last lose any force but what the other acquires; and hence, though by their collisions, motion passes from the one to the other, yet the sum of their motions, estimated in a given direction, is preserved the same, and is unalterable by their mutual actions upon each other. In these actions the changes are equal; not those, we mean, of the velocities, but those of the motions, or momentums; the bodies being supposed free from any other impediments. For the changes of velocities, which are likewise made contrary ways, inasmuch as the motions are equally changed, are reciprocally proportional to the bodies or masses.—The same law obtains also in attractions.

NAVIGATION, is the art of conducting a ship at sea from one port or place to another. This is perhaps the most useful of all arts, and is of the highest antiquity. It is impossible to say who were the inventors of it; but it is probable that many people cultivated it, independent of each other, who inhabited the sea coasts, and had occasion, or found it convenient, to convey themselves upon the water from place to place; beginning from rafts and logs of wood, and gradually improving in the structure and management of their vessels, according to the length of time and extent of their voyages. Writers however ascribe the invention of this art to different persons, or nations, according to the different sources of their information. Thus,

The poets refer the invention of navigation to Neptune, some to Bacchus, others to Hercules, to Jason, or to

Janus, who it is said made the first ship. Historians ascribe it to the Æginetes, the Phœnicians, Tyrians, and the ancient inhabitants of Britain. Some are of opinion that the first hint was taken from the flight of the kite; and some, as Oppian (De Piscibus, lib. 1), from the fish called Nautilus; while some ascribe it to accident; and others again deriving the hint and invention from Noah's ark.

However, history represents the Phœnicians, especially those of the capital Tyre, as the first navigators that made any extensive progress in the art, so far as has come to our knowledge; and indeed it must have been this very art that made their city what it was. For this purpose, Lebanon, and the other neighbouring mountains, furnishing them with excellent wood for ship-building, they were speedily masters of a numerous fleet, with which constantly hazarding new navigations, and settling new trades, they soon arrived at a high pitch of opulence and population; so as to be in a condition to send out colonies, the principal of which was that of Carthage; which, keeping up their Phœnician spirit of commerce, in time far surpassed Tyre itself; sending their merchant-ships through Hercules's pillars, now the straits of Gibraltar, and thence along the western coasts of Africa and Europe; and even, according to some authors, to America itself. The city of Tyre being destroyed by Alexander the Great, its navigation and commerce were transferred by the conqueror to Alexandria, a new city, well situated for these purposes, and proposed for the capital of the empire of Asia, the conquest of which Alexander then meditated. And thus arose the navigation of the Egyptians; which was afterwards so cultivated by the Ptolemies, that Tyre and Carthage were quite forgotten.

Egypt being reduced to a Roman province after the battle of Actium, its trade and navigation fell into the hands of Augustus: in whose time Alexandria was only inferior to Rome; and the magazines of the capital of the world were wholly supplied with merchandises from the capital of Egyyt.

At length, Alexandria itself underwent the fate of Tyre and Carthage; being surprised by the Saracens, who, in spite of the Emperor Heraclius, overspread the northern coasts of Africa, &c; and the merchants being driven thence, Alexandria has ever since been in a languishing state, though still it has a considerable part of the commerce of the Christian merchants trading to the Levant.

The fall of Rome and its empire drew along with it, not only that of learning and the polite arts, but that of navigation also; the barbarians, into whose hands it fell, contenting themselves with the spoils of the industry of their predecessors.

But no sooner were the brave among those nations well settled in their new provinces; some in Gaul, as the Franks; others in Spain, as the Goths; and others in Italy, as the Lombards; than they began to learn the advantages of navigation and commerce, with the methods of managing them, from the people they subdued; and this with so much success, that in a little time some of them became able to give new lessons, and set on foot new institutions for its advantage. Thus it is to the Lombards we usually ascribe the invention and use of banks, book-keeping, exchanges, rechanges, &c.

It does not appear which of the European nations, after the settlement of their new masters, first engaged in navigation and commerce.—Some think it began with the French; though the Italians seem to have the juster title to it, and are usually considered as the restorers of both, as well as of the polite arts, which had been banished together from the time the empire was torn asunder. It is the people of Italy then, and particularly those of Venice and Genoa, who have the glory of this restoration; and it is to their advantageous situation for navigation that they in a great measure owe their glory. From about the time of the 6th century, when the inhabitants of the islands in the bottom of the Adriatic began to unite together, and by their union to form the Venetian state, their fleets of merchantmen were sent to all the parts of the Mediterranean; and at last to those of Egypt, particularly Cairo, a new city, built by the Saracen princes on the eastern banks of the Nile, where they traded for their spices and other products of the Indies. Thus they flourished, and increased their commerce, their navigation, and their conquests on the terra firma, till the league of Cambray in 1508, when a number of jealous princes conspired to their ruin; which was the more easily effected by the diminution of their East-India commerce, of which the Portuguese had got one part, and the French another. Genoa too, which had cultivated navigation at the same time with Venice, and that with equal success, was a long time its dangerous rival, disputed with it the empire of the sea, and shared with it the trade of Egypt, and other parts both of the east and west.

Jealousy soon began to break out; and the two republics coming to blows, there was almost continual war for three centuries, before the superiority was ascertained; when, towards the end of the 14th century, the battle of Chioza ended the strife: the Genoese, who till then had usually the advantage, having now lost all; and the Venetians, almost become desperate, at one happy blow, beyond all expectation, secured to themselves the empire of the sea, and the superiority in commerce.

About the same time that navigation was retrieved in the southern parts of Europe, a new society of merchants was formed in the North, which not only carried commerce to the greatest perfection it was capable of, till the discovery of the East and West Indies, but also formed a new scheme of laws for the regulation of it, which still obtain under the name of, Uses and Customs of the Sea. This society is that celebrated league of the Hanse-Towns, which was begun about the year 1164.

The art of navigation has been greatly improved in modern times, both in respect to the form of the vessels themselves, and the methods of working or conducting them. The use of rowers is now entirely superseded by the improvements made in the sails, rigging, &c. The ancients were neither so well skilled in finding the latitudes, nor in steering their vessels in places of difficult navigation, as the moderns. But the greatest advantage which these have over the ancients, is from the mariner's compass, by which they are enabled to find their way with as much facility in the midst of an immeasurable ocean, as the ancients could have done by creeping along the coast, and never going out of sight of land. Some people indeed contend, that this is no new invention, but that the ancients were acquainted with it. They say, it was impossible for Solomon's ships to go to Ophir, Tarshish, and Parvaim, which last they will have to be Peru, without this useful instrument. They insist, that it was impossible for the ancients to be acquainted with the attractive virtue of the magnet, without knowing its polarity. They even affirm, that this property of the magnet is plainly mentioned in the Book of Job, where the loadstone is called topaz, or the stone that turns itself. But, not to mention that Mr. Bruce has lately made it appear highly probable that Solomon's ships made no more than coasting voyages, it is certain that the Romans, who conquered Judea, were ignorant of this instrument; and it is very probable, that so useful an invention, if once it had been commonly known to a nation, would never have been forgotten, or perfectly concealed from so enterprising a people as the Romans, who were so much interested in the discovery of it.

Among those who do agree that the mariner's compass is a modern invention, it has been much disputed who was the inventor. Some give the honour of it to Flavio Gioia of Amalfi in Campania, about the beginning of the 14th century; while others say that it came from the East, and was earlier known in Europe. But, at whatever time it was invented, it is certain, that the mariner's compass was not commonly used in navigation before the year 1420. In that year, the science was considerably improved under the auspices of Henry duke of Visco, brother to the king of Portugal. In the year 1485, Roderic and Joseph, physicians to king John the 2d of Portugal, together with one Martin de Bohemia, a Portuguese native of the island of Fayal, and pupil to Regiomontanus, calculated tables of the sun's declination for the use of sailors, and recommended the astrolabe for taking observations at sea. The celebrated Columbus, it is said, availed himself of Martin's instructions, and improved the Spaniards in the knowledge of this art; for the farther progress of which, a lecture was afterwards founded at Seville by the emperor Charles the 5th.

The discovery of the variation of the compass, is claimed by Columbus, and by Sebastian Cabot. The former certainly did observe this variation without having heard of it from any other person, on the 14th of September 1492, and it is very probable that Cabot might do the same. At that time it was found that there was no variation at the Azores, for which reason some geographers made that the first meridian, though it has since been discovered that the variation alters in time. The use of the cross-staff now began to be introduced among sailors. This ancient instrument is described by John Werner of Nuremberg, in his annotations on the first book of Ptolemy's Geography, printed in 1514: he recommends it for observing the distance between the moon and some star, from which to determine the longitude.

At this time the art of navigation was very imperfect, from the use of the plane chart, which was the only one then known, and which, by its gross errors, must have greatly misled the mariner, especially in places far distant from the equator; and also from the want of books of instruction for seamen.

At length two Spanish treatises appeared on this subject, the one by Pedro de Medina, in 1545; and the other by Martin Cortes, or Curtis as it is printed in English, in 1556, though the author says he composed it at Cadiz in 1545, containing a complete system of the art as far as it was then known. Medina, in his dedication to Philip prince of Spain, laments that multitudes of ships daily perished at sea, because there were neither teachers of the art, nor books by which it might be learned; and Cortes, in his dedication, boasts to the emperor, that he was the first who had reduced navigation into a compendium, valuing himself much on what he had performed. Medina defended the plane chart; but he was opposed by Cortes, who showed its errors, and endeavoured to account for the variation of the compass, by supposing the needle was influenced by a magnetic pole, different from that of the world, and which he called the Point Attractive: which notion has been further prosecuted by others. Medina's book was soon translated into Italian, French, and Flemish, and served for a long time as a guide to foreign navigators. However, Cortes was the favourite author of the English nation, and was translated in 1561, by Richard Eden, while Medina's work was much neglected, though translated also within a short time of the other. At that time a system of navigation consisted of materials such as the following. An account of the Ptolemaic hypothesis, and the circles of the sphere; of the roundness of the earth, the longitudes, latitudes, climates, &c, and eclipses of the luminaries; a calendar; the method of finding the prime, epact, moon's age, and tides; a description of the compass, an account of its variation, for the discovering of which Cortes said an instrument might easily be contrived; tables of the sun's declination for 4 years, in order to find the latitude from his meridian altitude; directions to find the same by certain stars; of the course of the sun and moon; the length of the days; of time and its divisions; the method of finding the hour of the day and night; and lastly, a description of the sea-chart, on which to discover where the ship is; they made use also of a small table, that showed, on an alteration of one degree of the latitude, how many leagues were run on each rhumb, together with the departure from the meridian; which might be called a table of distance and departure, as we have now a table of difference of latitude and departure. Besides, some instruments were described, especially by Cortes; one of which was for finding the place and declination of the sun, with the age and place of the moon; certain dials, the astrolabe, and cross-staff; with a complex machine to discover the hour and latitude at once.

About the same time proposals were made for finding the longitude by observations of the moon. In 1530, Gemma Frisius advised the keeping of the time by means of small clocks or watches, then newly invented, as he says. He also contrived a new kind of cross-staff, and an instrument called the Nautical Quadrant; which last was much praised by William Cuningham, in his Cosmographical Glass, printed in the year 1559.

In the year 1537 Pedro Nunez, or Nonius, published a book in the Portuguese language, to explain a difficulty in navigation, proposed to him by the commander Don Martin Alphonso de Susa. In this work he exposes the errors of the plane chart, and gives the solution of several curious astronomical problems; among which, is that of determining the latitude from two observations of the sun's altitude and the intermediate azimuth being given. He observed, that though the rhumbs are spiral lines, yet the direct course of a ship will always be in the arch of a great circle, by which the angle with the meridians will continually change: all that the steersman can here do for preserving the original rhumb, is to correct these deviations as soon as they appear sensible. But thus the ship will in reality describe a course without the rhumb-line intended; and therefore his calculations for assigning the latitude, where any rhumb-line crosses the several meridians, will be in some measure erroneous. He invented a method of dividing a quadrant by means of concentric circles, which, after being much improved by Dr. Halley, is used at present, and is called a Nonius.

In 1577, Mr. William Bourne published a treatise of navigation, in which, by considering the irregularities in the moon's motion, he shows the errors of the sailors in finding her age by the epact, and also in determining the hour from observing on what point of the compass the sun and moon appeared. In sailing towards high latitudes, he advises to keep the reckoning by the globe, as the plane chart is most erroneous in such situations. He despairs of our ever being able to find the longitude, unless the variation of the compass should be occasioned by some such attractive point as Cortes had imagined; of which however he doubts: but as he had shown how to find the variation at all times, he advises to keep an account of the observations, as useful for finding the place of the ship; which advice was prosecuted at large by Simon Stevin in a treatise published at Leyden in 1599; the substance of which was the same year printed at London in English by Mr. Edward Wright, entitled the Haven-finding Art. In the same old tract also is described the method by which our sailors estimate the rate of a ship in her course, by the instrument called the Log. The author of this contrivance is not known; neither was it farther noticed till 1607, when it is mentioned in an East-India voyage published by Purchas: but from this time it became common, and is mentioned by all authors on navigation; and it still continues to be used as at first, though many attempts have been made to improve it, and contrivances proposed to supply its place; some of which have succeeded in still water, but proved useless in a stormy sea.

In 1581 Michael Coignet, a native of Antwerp, published a treatise, in which he animadverted on Medina. In this he showed, that as the rhumbs are spirals, making endless revolutions about the poles, numerous errors must arise from their being represented by straight lines on the sea-charts; but though he hoped to find a remedy for these errors, he was of opinion that the proposals of Nonius were scarcely practicable, and therefore in a great measure useless. In treating of the sun's declination, he took notice of the gradual decrease in the obliquity of the ecliptic; he also described the cross-staff with three transverse pieces, as it was then in common use among the sailors. He likewise gave some instruments of his own invention; but all of them are now laid aside, excepting perhaps his Nocturnal. He constructed a sea-table, to be used by such as sailed beyond the 60th degree of latitude; and at the end of the book is delivered a method of sailing on a parallel of latitude, by means of a ring-dial and a 24-hour-glass.

In the same year Mr. Robert Norman published his Discovery of the Dipping-needle, in a pamphlet called the New Attractive; to which is always subjoined Mr. William Burrough's Discourse of the Variation of the Compass.—In 1594, Captain John Davis published a small treatise, entitled The Seaman's Secrets, which was much esteemed in its time.

The writers of this period complained much of the errors of the plane chart, which continued still in use, though they were unable to discover a proper remedy: till Gerrard Mercator contrived his Universal Map, which he published in 1569, without clearly understanding the principles of its construction; these were first discovered by Mr. Edward Wright, who sent an account of the true method of dividing the meridian from Cambridge, where he was a fellow, to Mr. Blundeville, with a short table for that purpose, and a specimen of a chart so divided. These were published by Blundeville in 1594, among his Exercises; to the later editions of which was added his Discourse on Universal Maps, first printed in 1589. However, in 1599 Mr. Wright printed his Correction of certain Errors in Navigation, in which work he shows the reason of this division, the manner of constructing his table, and its uses in navigation. A second edition of this treatise, with further improvements, was printed in 1610, and a third edition by Mr. Moxon, in 1657.—The method of approximation, by what is called the middle latitude, now used by our sailors, occurs in Gunter's Works, first printed in 1623.—About this time logarithms began to be introduced, which were applied to navigation in a variety of ways by Mr. Edmund Gunter; though the first application of the logarithmic tables to the cases of sailing, was by Mr. Thomas Addison, in his Arithmetical Navigation, printed in 1625.—In 1635 Mr. Henry Gellibrand printed a Mathematical Discourse on the Variation of the Magnetical Needle, containing his discovery of the changes to which the variation is subject.—In 1631, Mr. Richard Norwood published an excellent Treatise of Trigonometry, adapted to the invention of logarithms, particularly in applying Napier's general canons; and for the farther improvement of navigation, he undertook the laborious work of measuring a degree of the meridian, for examining the divisions of the log-line. He has given a full and clear account of this operation in his Seaman's Practice, first published in 1637; where he also describes his own excellent method of setting down and perfecting a sea-reckoning, &c. This treatise, and that of trigonometry, were often reprinted, as the principal books for learning scientifically the art of navigation. What he had delivered, especially in the latter of them, concerning this subject, was contracted as a manual for sailors in a very small piece, called his Epitome, which has gone through a great number of editions.—About the year 1645, Mr. Bond published, in Norwood's Epitome, a very great improvement on Wright's method, by a property in his meridian line, by which its divisions are more scientifically assigned than the author was able to effect; which he deduced from this theorem, that these divisions are analogous to the excesses of the logarithmic tangents of half the respective latitudes increased by 45 degrees, above the logarithm of the radius: this he afterwards explained more fully in the 3d edition of Gunter's Works, printed in 1653; and the demonstration of the general theorem was supplied by Mr. James Gregory of Aberdeen, in his Exercitationes Geometricæ, printed at London in 1668, and afterwards by Dr. Halley, in the Philos. Trans. No. 219, as also by Mr. Cotes, No. 388.—In 1700, Mr. Bond, who imagined that he had discovered the longitude, by having discovered the true theory of the magnetic variation, published a general map, on which curve lines were drawn, expressing the paths or places where the magnetic needle had the same variation. The positions of these curves will indeed continually experience alterations; and therefore they should be corrected from time to time, as they have already been for the years 1744, and 1756, by Mr. William Mountaine, and Mr. James Dodson.—The allowances proper to be made for lee-way, are very particularly set down by Mr. John Buckler, and published in a small tract first printed in 1702, entitled a New Compendium of the whole Art of Navigation, written by Mr. William Jones.

As it is now generally agreed that the earth is a spheroid, whose axis or polar diameter is shorter than the equatorial diameter, Dr. Murdoch published a tract in 1741, in which he adapted Wright's, or Mercator's sailing to such a figure; and in the same year Mr. Maclaurin also, in the Philos. Trans. No. 461, for determining the meridional parts of a spheroid; and he has further prosecuted the same speculation in his Fluxions, printed in 1742.

The method of finding the longitude at sea, by the observed distances of the moon from the sun and stars, commonly called the lunar method, was proposed at an early stage in the art of navigation, (viz, in 1514, by John Werner of Nuremberg,) and has now been happily carried into effectual execution by the encouragement of the Board of Longitude, which was established in England in the year 1714, for rewarding any successful endeavours to keep the longitude at sea. In the year 1767, this board published a Nautical Almanac, which has been continued annually ever since, by the advice, and under the direction of the astronomer-royal at Greenwich: this work is purposely adapted to the use of navigators in long voyages, and, among a great many useful articles, it contains tables of the lunar distances accurately computed for every 3 hours in the year, for the purpose of comparing the distance thus known for any time, with the distance observed in an unknown place, from which to compute the longitude of that place. Under the auspices of this Board too, besides giving encouragement to the authors of many useful tables and other works, which would otherwise have been lost, time-keepers have been brought to a great degree of excellence, by Mr. Harrison, Mr. Arnold, and many other persons, which have proved highly advantageous in keeping the time during long voyages at sea, and thence giving the longitude to a good degree of accuracy.

Some of the other principal writers on navigation are, Stevin, before 1600, in his Hydrography; Bartholomew Crescenti, of Rome, in 1607; Willebrord Snell, at Leyden, in 1624, his Tiphys Batavus; Geo. Fournier, at Paris, 1633; John Baptist Riccioli, at Bologna, in 1661; Dechales, in 1674 and 1677; the Sieur Blondel St. Aubin, in 1671 and 1673; M. Dassier, in 1683; M. Sauveur, in 1692; M. John Bouguer, in 1698; F. Pezenas, in 1733 and 1741; and M. Peter Bouguer, who, in 1753, published a very elaborate treatise on this subject, entitled, Nouveau Traité de Navigation; in which he gives a variation compass of his own invention, and attempts to reform the log, as he had before done in the Memoirs of the Academy of Sciences for 1747. He is also very particular in determining the lunations more accurately than by the common methods, and in describing the corrections of the dead reckoning. This book was abridged and improved by M. Lacaille, in 1760. To these may be added the navigation of Don George Juan of Spain, in 1757. And, in our own nation, the several treatises of Messrs. Newhouse, Seller, Hodgson, Atkinson, Harris, Patoun, Hauxley, Wilson, Moore, Nicholson, &c; but, above all, The Elements of Navigation, in 2 vols, by Mr. John Robertson, first printed about the year 1750, and since often re-printed; which is the most complete work of the kind extant; and to which work is prefixed a Dissertation on the Rise and Progress of the modern Art of Navigation, by Dr. James Wilson, containing a very learned and elaborate history of the writings and improvements in this art.

For an account of the several instruments employed for the purposes of navigation, with the methods for the longitude, and the various kinds and methods of navigation, &c, see the respective articles themselves, as also the preface to Robertson's Navigation.

Navigation is either Proper or Common.

Navigation, *Common*, usually called coasting, in which the places are at no great distance from each other, and the ship sails usually in sight of land, and mostly within soundings. In this, little else is required besides an acquaintance with the lands, the compass, and sounding-line; each of which, see in its place.

Navigation, *Proper*, is where the voyage is long, and pursued through the main ocean. And here, besides the requisites in the former case, are also required the use of Mercator's Chart, the azimuth and amplitude compasses, the log-line, and other instruments for celestial observations; as forestaffs, quadrants, and other sectors, &c.

Navigation turns chiefly upon four things; two of which being given or known, the rest are thence easily found out. These four things are, the difference of latitude, difference of longitude, the reckoning or distance run, and the course or rhumb sailed on. The latitudes are easily found, and that with sufficient accuracy: the course and distance are had by the log-line, or dead reckoning, together with the compass. Nor is there any thing wanting to the perfection of navigation, but to determine the longitude. Mathematicians and astronomers for many ages have applied themselves, with great assiduity, to supply this grand desideratum, but not altogether with the success desired, considering the importance of the object, and the magnificent rewards offered by several states to the discoverer. See Longitude.

Sub-Marine Navigation, or the art of sailing under water, is mentioned by Mr. Boyle, as the desideratum of the art of navigation. This, he says, was successfully attempted, by Cornelius Drebbel; several persons who were in the boat breathing freely all the time. See Diving-*Bell*.

Inland Navigation, is that performed by small craft, upon canals, &c, cut through a country.

NAVIGATOR, a person capable of conducting a ship at sea to any place proposed.

NAUTICAL *Chart*, the same as Sea-Chart.

Nautical *Compass*, the same as Sea-Compass.

Nautical *Planisphere*, a projection or construction of the terrestrial globe on a plane, for the use of mariners; such as the plane chart, and Mercator's chart.

NEAP, or Neep-*Tides*, are those that happen at equal distances between the spring tides. The neap tides are the lowest, as the spring tides are the highest ones, being the opposites to them. And as the highest of the spring tides happens about 3 days after the full or change of the moon, so the lowest of the neap tides fall about 3 days after the quarters, or 4 days before the full and change; when the seamen say it is deep neap.

NEAPED. When a ship wants water, so that she cannot get out of the harbour, out of the dock, or off the ground, they say, she is neaped, or beneaped.

NEBULÆ, or Nebulous, or Cloudy, a term applied to certain fixed stars, which show a dim, hazy light; being less than those of the 6th magnitude, and therefore scarcely visible to the naked eye, to which at best they only appear like little dusky specks or clouds.—Through a moderate telescope, most of these nebulous stars plainly appear to be congeries or clusters of several little stars. In the nebulous star called Præsepe, in the breast of Cancer, there are reckoned 36 little stars, 3 of which Mr. Flamsteed sets down in his catalogue. In the nebulous star of Orion, are reckoned 21. F. le Compte adds, that there are 40 in the Pleiades; 12 in the star in the middle of Orion's sword; 500 in the extent of two degrees of the same constellation; and 2500 in the whole constellation. It may further be observed, that the galaxy, or milky-way, is a continued assemblage of nebulæ, or vast clusters of small stars.

Though some of these nebulous spots in the heavens consist of clusters of small stars, others appear as luminous spots of different forms. A remarkable one is in the midway between the two stars on the blade of Orion's sword, marked θ by Bayer, discovered in the year 1656 by Huygens: it contains only 7 stars, and the other part is a bright spot on a dark ground, appearing like an opening into brighter regions beyond.

Dr. Halley and others have discovered nebulæ in several parts of the heavens. In the Connoissance des Temps, for 1783 and 1784, there is a catalogue of 103 nebulæ, observed by Messier and Mechain. But to Dr. Herschel we owe catalogues of 2000 nebulæ, and clusters of stars, discovered by him. Some of these form a round compact system; others are more irregular, and of various forms, some being long and narrow. The globular systems of stars appear thicker in the middle, than they would do if the stars were all at equal distances from each other; they are therefore condensed toward the centre. These he supposes are brought together by their mutual attractions, and that the gradual condensation toward the centre is a proof of a central power of such a kind; and that though the forms are various, it is plain that there is always a tendency to sphericity. And granting that these nebulæ and clusters of stars are formed by mutual attraction, Dr. H. concludes, that we may judge of their relative age by the disposition of their component parts, those being the oldest that are most compressed. He supposes, and indeed offers powerful arguments to prove, that the milky-way is the nebula of which our sun is one of its component parts.

Dr. Herschel has also discovered other phenomena in the heavens, which he calls nebulous stars; that is, stars surrounded by a faint luminous atmosphere of large extent. Those which have been thus styled by other astronomers, he says, ought not to have been so called; for, on examination, they have proved to be either mere clusters of stars, plainly to be distinguished by his large telescopes, or such nebulous appearances as might be occasioned by a multitude of stars at a vast distance. The milky-way consists entirely of stars; and he says, "I have been led on by degrees from the most evident congeries of stars, to other groups in which the lucid points were smaller, but still very plainly to be seen; and from them to such in which they could barely be suspected, until I arrived at last to spots in which no trace of a star was to be discovered. But then the gradations to these latter were by such connected steps, as left no room for doubt, but that all these phenomena were equally occasioned by stars variously dispersed in the immense expanse of the universe."

In the same paper is given an account of some nebulous stars, one of which is thus described: "Nov. 13, 1790. A most singular phenomenon! A star of the 8th magnitude, with a faint luminous atmosphere, of a circular form, and of about 3′ in diameter. The star is perfectly in the centre, and the atmosphere is so diluted, faint, and equal throughout, that there can be no surmise of its consisting of stars, nor can there be a doubt of the evident connection between the atmosphere and the star. Another star, not much less in brightness, and in the same field of view with the above, was perfectly free from any such appearance. Hence Dr. H. draws the following consequences: granting the connection between the star and the surrounding nebulosity, if it consist of stars very remote, which gives the nebulous appearance, the central star, which is visible, must be immensely greater than the rest; or if the central star be no larger than common, how extremely small and compressed must be those other luminous points which occasion the nebulosity. As, by the former supposition, the luminous central point must far exceed the standard of what we call a star; so in the latter, the shining matter about the centre will be too small to come under the same denomination; we therefore either have a central body which is not a star, or a star which is involved in a shining fluid, of a nature totally unknown to us." This last opinion Dr. H. adopts.

Light reflected from the star could not be seen at this distance. Besides, the outer parts are nearly as bright as those near the star. Further, a cluster of stars will not so completely account for the mildness or soft tint of the light of these nebulæ, as a self-luminous fluid. What a field of novelty, says Dr. H., is here opened to our conceptions! A shining fluid, of a brightness sufficient to reach us from the regions of a star of the 8th, 9th, 10th, 11th, 12th magnitude; and of an extent so considerable as to take up 3, 4, 5, or 6 minutes in diameter. He conjectures that this shining fluid may be composed of the light perpetually emitted from millions of stars. See Philos. Trans. an. 1791, pa. 1, or my Abridg. vol. 17, pa. 18.

In the vol. for 1811, pa. 269, Dr. Herschel has much farther continued his observations into the nature, construction, and uses of nebulous matter. He shows that it is distributed through the immensity of space, in quantities inconceivably great, and in separate collections of all shapes and sizes, and of all degrees of brightness, between a mere milky appearance and that of a fixed star. He states substantial reasons for conceiving that the whole furniture of the universe is furnished and formed out of collections of it; that it is naturally opaque though self-shining; that by its central gravitation each collection gradually becomes more and more condensed, and more and more rounded in its form; that from the excentricity of its shape, and gravitation, it acquires gradually a rotary motion; that probably this condensation, and roundness, and rotation, go on continually increasing, till the mass come to a hard or firm consistence, and receive all the other characters of a comet or a planet; that by a still further process of condensation, the body becomes a real star self-shining; and that thus, the waste of the celestial bodies, by the perpetual diffusion of their light, is continually compensated and restored, by new formations of such bodies, to replenish for ever the universe with planets and stars!

Dr. H. has again returned to this prolific subject, in a paper communicated to the Royal Society, and read at their meetings Feb. 24, and March 3, 1814. He here relates his observations on the relative magnitudes of the stars, considering those of the first magnitude to be equal to our sun; determined the magnitudes and changes in the appearance of a great number of fixed stars; gave a history of the alterations which he has noticed in the aspect of the sidereal heavens, during the last 30 years; and described those stars which have increased in magnitude or brilliancy, have lost or acquired surrounding nebulæ, or have had wings, or tails, or other peculiarities. He seems of opinion that new sidereal bodies are in a constant and progressive state of formation; that nebulous appearances gradually assume a globular form. He considers the origin and progress of sidereal bodies to be nearly in the following order: first, vague and indistinct nebulæ, like the milky-way; 2d, detached or clustered nebulæ, which consolidate into clusters of stars; 3dly, these stars, becoming more definite, appear with nebulous appendages, in the different forms of wings, tails, &c; and lastly, that all are finally concentrated into one clear, bright, and large star.

NEEDHAM (John Tuberville), a respectable philosopher and catholic divine, was born at London December 10, 1713. His father possessed a considerable patrimony at Hilston, in the county of Monmouth, being of the catholic branch of the Needham family, and who died young, leaving but a small fortune to his four children. Our author, who was the eldest son, studied in the English college of Douai, where he took orders, taught rhetoric for several years, and surpassed all the other professors of that seminary in the knowledge of experimental philosophy.

In 1740, he was engaged by his superiors in the service of the English mission, and was intrusted with the direction of the school erected at Twyford, near Winchester, for the education of the Roman Catholic youth.—In 1744 he was appointed professor of philosophy in the English college at Lisbon, where, on account of his bad health, he remained only 15 months. After his return, he passed several years at London and Paris, which were chiefly employed in microscopical observations, and in other branches of experimental philosophy. The results of these observations and experiments were published in the Philos. Trans. in the year 1749, and in a volume in 12mo at Paris in 1750; and an account of them was also given by M. Buffon, in the first volumes of his natural history. An intimate connection subsisted for a long time between Mr. Needham and this illustrious French naturalist: they made their experiments and observations together; though the results and systems which they deduced from the same objects and operations were totally different.

Mr. Needham was elected a member of the Royal Society of London in the year 1747, and of the Antiquarian Society some time after.—From the year 1751 to 1767 he was chiefly employed in finishing the education of several English and Irish noblemen, by attending them as tutor in their travels through France, Italy, and other countries. He then retired from this wandering life to the English seminary at Paris, and in 1768 was chosen by the Royal Academy of Sciences in that city a corresponding member.

When the regency of the Austrian Netherlands, for the revival of philosophy and literature in that country, formed the project of an Imperial Academy, which was preceded by the erection of a small literary society to prepare the way for its execution, Mr. Needham was invited to Brussels, and was appointed successively chief director of both these foundations; an appointment which he held, together with some ecclesiastical preferments in the Low Countries, till his death, which happened December the 30th 1781.

Mr. Needham's papers inserted in the Philosophical Transactions, were the following, viz;

1. Account of Chalky Tubulous Concretions, called Malm: vol. 42.—2. Microscopical Observations on Worms in Smutty Corn: vol. 42.—3. Electrical Experiments lately made at Paris: vol. 44.—4. Account of M. Buffon's Mirror, which burns at 66 feet: ib.—5. Observations on the Generation, Composition, and Decomposition of Animal and Vegetable Substances: vol. 45.—6. On the Discovery of Asbestos in France: vol 51.

Other works printed at Paris, in French, are,

1. New Microscopical Discoveries: 1745.
2. The same enlarged: 1750.
3. On Microscopical, and the Generation of Organized Bodies: 2 vols, 1769.

NEEDLE, *Magnetical*, denotes a needle, or a slender piece of iron or steel, touched with a loadstone; which, when freely suspended on a pivot or centre, on which it plays, settles at length in a certain direction, either duly, or nearly north-and-south, and called the magnetic meridian. Magnetical needles are of two kinds; horizontal and inclinatory.

Horizontal Needles, are those equally balanced on each side of the pivot which sustains them; and which, playing horizontally, with their two extremes, point out the north and south parts of the horizon.

Construction of a Horizontal Needle. Having procured a thin light piece of pure steel, about 6 inches long, a perforation is made in the middle, over which a brass cap is soldered on, having its inner cavity conical, so as to play freely on the stile or pivot, which has a fine steel point. To give the needle its verticity, or directive faculty, it is rubbed or stroked leisurely on each pole of a magnet, from the south pole towards the north; first beginning with the northern end, and going back at each repeated stroke towards the south: being careful not to give a stroke in a contrary direction, which would counteract the power it had already obtained. Also the hand should not return directly back again the same way it came, but should return in a kind of oval figure, carrying it about 6 or 8 inches beyond the point where the touch ended, but not beyond on the side where the touch begins.

Before touching, the north end of the needle, in our hemisphere, is made a little lighter than the other end; because the touch always destroys an exact balance, and thus causing the needle to dip. And if, after touching, the needle be out of its equilibrium, something must be filed off from the heavier side, till it be found to balance evenly.

Needles may also acquire the magnetic virtue by means of artificial magnetic bars in the following manner: Lay two equal needles parallel, and about an inch asunder, with the north end of one and the south end of the other pointing the same way, and apply two conductors in contact with their ends: then, with two magnetic hard bars, one in each hand, and held as nearly horizontal as can be, with the upper ends, of contrary names, turned outwards to the right and left, let a needle be stroked or rubbed from the middle to both ends at the same time, for ten or twelve times, the north end of a bar going over the south end of a needle, and the south end of a bar going over the north end of a needle: then, without moving from the place, change hands with the bars, or in the same hands turn the other ends downwards, and stroke the other needle in the same manner; so will they both be magnetical. But to make them still stronger, repeat the operation three or four times from needle to needle, and lastly turn the lower side of each needle upwards, and repeat the operations of touching them, as on the former sides.

The needles that were formerly applied to the compass, on board merchant-ships, were formed of two pieces of steel wire, each being bent in the middle, so as to form an obtuse angle, while their ends, being applied together, make an acute one, so that the whole represented the form of a lozenge. Dr. Knight, who has so much improved the compass, found, by repeated experiments, that partly from the foregoing structure, and partly from the unequal hardening of the ends, these needles not only varied from the true direction, but from one another, and from themselves.—Also the needles formerly used on board the men of war, and some of the larger trading vessels, were made of one piece of steel, of a spring temper, and broad towards the ends, but tapering towards the middle. Every needle of this form is found to have six poles instead of two, one at each end, two where it becomes tapering, and two at the hole in the middle.

To remedy these errors and inconveniences, the needle which Dr. Knight contrived for his compass, is a slender parallelopipedon, being quite straight and square at the ends, and so has only two poles; but the curves are a little confused about the hole in the middle, though it is, upon the whole, the simplest and best.—Mr. Michell suggests, that it would be useful to increase the weight and length of magnetic needles, which would render them both more accurate and permanent; also to cover them with a coat of linseed oil, or varnish, to preserve them from rust.—A needle may be prepared occasionally without touching it on a loadstone: for a fine steel sewing-needle, gently laid on the water, or delicately suspended in the air, will take the north-and-south direction.—Thus also a needle heated in the fire, and cooled again in the direction of the meridian, or only in an erect position, acquires the same faculty.

Declination or Variation of the Needle, is the deviation of the horizontal needle from the meridian; or the angle it makes with the meridian, when freely suspended in an horizontal plane. A needle is always changing the line of its direction, traversing slowly to certain limits towards the east and west sides of the meridian. It was at first thought that the magnetic needle pointed due north; but it was observed by Cabot and Columbus that it had a deviation from the north, though they did not suspect that this deviation had itself a variation, and was continually changing. This change in the variation was first observed, according to Bond, by Mr. John Mair, secondly by Mr. Gunter, and thirdly by Mr. Gellibrand, by comparing together the observations made at different times near the same place by Mr. Burrowes, Mr. Gunter, and himself, on which subject he published a discourse in 1635. Soon after this, Mr. Bond ventured to deliver the rate at which the variation changes for several years; by which he foretold that at London in 1657 there would be no variation of the compass, and from that time it would gradually increase the other way, or towards the west, making apparent vibratory motions between certain limits; which happened accordingly: and upon this variation he proposed a method of finding the longitude, which has been further improved by many others since his time, though with very little success. See Variation.

The period of the variation, according to Mr. Henry Philips, is only 370 years, but Mr. Bond 600 years, and their yearly motion 36 minutes. The first good observations of the variation were made by Burrowes, about the year 1580, when the variation at London was 11° 15′ east; and since that time the needle has been moving to the westward at that place; also by the observations of different persons, it has been found to point, at different times, as below:

Years.	Observers.	Variat.		E. or W.
1580	Burrowes	11°	15′	East.
1622	Gunter	5	56	
1634	Gellibrand	4	3	
1640	Bond	3	7	
1657	Bond	0	0	
1665	Bond	1	23	West.
1666	Bond	1	36	
1672	-	2	30	
1683	-	4	30	
1692	-	6	00	
1723	Graham	14	17	
1747	-	17	40	
1774	Royal Society	21	16	
1775	Royal Society	21	43	
1776	Royal Society	21	47	
1777	Royal Society	22	12	
1778	Royal Society	22	20	
1779	Royal Society	22	28	
1780	Royal Society	22	41	
1804	Royal Society	24	10	
1805	Royal Society	24	8	
1806	Royal Society	24	9	
1807	Royal Society	24	10	
1808	Royal Society	24	12	
1812	Royal Society	24	16	

By this table it appears that, from the first observations in 1580 till 1657, the change in the variation was 11° 15′ in 77 years, which is at the rate nearly of 9′ a year; and from 1657 till 1780, or the space of 123 years, it changed 22° 41′, which is at the rate of 11′ a year nearly; which it may be presumed is very near the truth.

The variation and dip of the needle was for many years carefully observed by the Royal Society while they met at Crane-court; but were discontinued for many years after removing to their new apartments in Somerset-place; though they have lately been there renewed again.

Dipping, or *Inclinatory* Needle, is a needle to show the dip of the magnetic needle, or how far it points below the horizon.

The inclination or dip of the needle was first observed by Robert Norman, a compass-maker at Ratcliffe; and according to him, the dip at that place, in the year 1576, was 71° 50′; and at the Royal Society it was observed for some years lately as follows:

viz. in 1776	72°	30′
1778	72	25
1780	72	17
1805	70	25
1808	70	1.

Mr. Henry Bond makes the variation and dip of the needle depend on the same motion of the magnetic poles in their revolution, and upon it he founded a method of discovering the longitude at sea.

NEEP *Tides*. See NEAP *Tides*.

NEGATIVE, in Algebra, something marked with the sign −, or minus, as being contrary to such as are positive, or marked with the sign plus, + ; as negative powers and roots, negative quantities, &c. See POWER, ROOT, QUANTITY, &c.

NEGATIVE *Sign*, the sign of subtraction −, or that which denotes something in defect. Stifel is the first author I find who used this mark − for subtraction, or negation; before his time, the word minus itself was used, or else its initial *m*.

The use of the negative sign in algebra, is attended with several consequences which at first sight are not admitted without some difficulty, and has sometimes given occasion to notions that seem to have no real foundation. This sign implies, that the real value of the quantity represented by the letter to which it is prefixed, is to be subtracted; and it serves, with the positive sign, to keep in view what elements or parts enter into the composition of quantities, and in what manner, whether as increments or decrements, that is whether by addition or subtraction, which is of great use in algebra.

Hence it serves to express a quantity of an opposite quality to a positive; such as a line in a contrary position, a motion with opposite direction, or a centrifugal force in opposition to gravity; and thus it often saves the trouble of distinguishing, and demonstrating separately, the various cases of proportions, and preserves their analogy in view. But as the proportions of lines depend on their magnitude only, without regard to their position; and motions and forces are said to be equal or unequal, in any given ratio, without regard to their directions; and in general the proportion of quantities relates to their magnitude only, without determining whether they are to be considered as increments or decrements; so there is no ground to imagine any other proportion of $+a$ and $-b$, than that of the real magnitudes of the quantities represented by a and b, whether these quantities are, in any particular case, to be added or subtracted.

As to the usual arithmetical operations of addition, subtraction, &c, the case is different, as the effect of the negative sign is here to be carefully attended to, and is to be considered always as producing, in those operations, an effect directly opposite to the positive sign. Thus, it is the same thing to subtract a decrement, as to add an equal increment, or to subtract $-b$ from $a-b$, is to add $+b$ to it: and because multiplying a quantity by a negative number, implies only a repeated subtraction of it, the multiplying $-b$ by $-n$, is subtracting $-b$ as often as there are units in n, and is therefore equivalent to adding $+b$ so many times, or the same as adding $+nb$. But if we infer from this, that 1 is to $-n$ as $-b$ to nb, according to the rule, that unit is to one of the factors as the other factor is to the product, there is not ground to imagine that there is any mystery in this, or any other meaning than that the real quantities represented by 1, n, b, and nb are proportional. For that rule relates only to the magnitude of the factors and product, without determining whether any factor, or the product, is additive or subtractive. But this must be determined in algebraic computations; and this is the proper use concerning the signs, without which the operation could not proceed. Because a quantity to be subtracted is never produced, in composition, by any repeated addition of a positive, or repeated subtraction of a negative, a negative square number is never produced by composition from a root. Hence the $\sqrt{-1}$, or the square root of a negative, implies an imaginary quantity, and in resolution is a mark or character of the impossible cases of a problem, unless it is compensated by another imaginary symbol or supposition, for then the whole expression may have a real signification. Thus $1+\sqrt{-1}$, and $1-\sqrt{-1}$, taken separately, are both imaginary, but yet their sum is the number 2: as the conditions that separately would render the solution of a problem impossible, in some cases destroy each other's effect when conjoined. In the pursuit of general conclusions, and of simple forms for representing them, expressions of this kind must sometimes arise, where the imaginary symbol is compensated in a manner that is not always so obvious. By proper substitutions, however, the expression may be transformed into another, wherein each particular term may have a real signification, as well as the whole expression.

The theorems that are sometimes briefly discovered by the use of this symbol, may be demonstrated without it by the inverse operation, or some other way; and though such symbols are of great use in the computations in the method of fluxions, trigonometry, &c, their evidence cannot be said to depend upon any arts of this kind. See Maclaurin's Fluxions, book 2, chap. 1; Maseres's Use of the Negative Sign, Ludlam's Algebra, and Carnot's Geometrie de Position.

For the rules or ways of using the negative sign in the several rules of algebra, see those rules severally, viz, ADDITION, SUBTRACTION, MULTIPLICATION, &c. And for the method of managing the roots of negative quantities, see IMPOSSIBLES.

NEIL (WILLIAM), an ingenious mathematician, son of Sir Paul Neil, usher of the privy chamber to King Charles 1, and was grandson of Dr. Rd. Neil, archbishop of York. He was born Dec. 7, 1637, and was educated at Oxford, in Wadham-college, under Dr. Wilkins; by whose instructions, and those of Dr. Seth Ward, he greatly improved his genius in mathematics. His success in that study appeared as early as 1657, at 19 years of age, when he, first of any one, accurately rectified a curve line, the semicubical parabola, as was testified by the letters of Dr. Wallis, Lord Viscount Brouncker, and Sir Chr. Wren, printed in the Philos. Trans. an. 1673, and my Abridg. vol. 2, pa. 112.—Mr. Neil became an early member of the Royal Society, of which he was elected a fellow in 1663. And his Theory of Motion was communicated to the society in 1669. But the further expectations, which had been conceived of his genius in mathematical and philosophical subjects, were disappointed by his early death, which happened 1670, in the 33d year of his age.

NEPER. See NAPIER.

NEWEL, the upright post that stairs turn about; being that part of the staircase which sustains the steps.

NEWTON (DR. JOHN), an eminent English mathematician and divine, was the grandson of John Newton of Axmouth in Devonshire, and son of Humphrey Newton of Oundle in Northamptonshire, where he was born in 1622. After receiving the proper foundation of a grammatical education, he was sent to Oxford, where he was entered a commoner of St. Edmund's-hall in 1637. He took the degree of bachelor of arts in 1641; and the year following he was created master, in precedence to many students of quality, on account of his distinguished talents in the great branches of literature. His genius leading him strongly to astronomy and mathematics, he applied himself diligently to those sciences, as well as to divinity, and made a great proficiency in them, which he found of some service to him during Cromwell's government.

After the restoration of Charles the 2d, he reaped the fruits of his loyalty: being created doctor of divinity at Oxford, Sept. 1661, he was made one of the king's chaplains, and rector of Ross in Herefordshire, instead of Mr. John Toombes, ejected for nonconformity. He held this living till his death, which happened at Ross on Christmas day 1678, at 56 years of age.

Mr. Wood gave him the character of a capricious and humorsome person. However that be, his writings are a proof of his great application to study, and a sufficient monument of his genius and skill in the mathematical sciences. These are,

1. Institutio Mathematica: 1654, in 12mo.
2. Tabulæ Mathematicæ: 1654, in 12mo.
3. Astronomia Britannica, &c: 1656, in 4to.
4. Help to Calculation; with Tables of Declination, &c: 1657, 4to.
5. Trigonometria Britannica, in 2 books; the one composed by our author, and the other translated from the Latin of Henry Gellibrand: 1658, folio.
6. Chiliades Centum Logarithmorum, printed with,
7. Geometrical Trigonometry: 1659.
8. Mathematical Elements, 3 parts: 1660, 4to.
9. A Perpetual Diary, or Almanac: 1662.
10. On the Use of the Carpenter's Rule: 1667.
11. Ephemerides, showing the interest and rate of money at 6 per cent. &c: 1667.
12. Chiliades Centum Logarithmorum et Tabula Partium Proportionalium: 1667.
13. The Rule of Interest, or the Case of Decimal Fractions, &c, part 2: 1668, 8vo.
14. School-pastimes for young children, &c: 1669, 8vo.
15. Art of Practical Gauging, &c: 1669.
16. Introduction to the art of Rhetoric: 1671.
17. The Art of Natural Arithmetic in Whole Numbers, and Fractions Vulgar and Decimal: 1671, 8vo.
18. The English Academy: 1677, 8vo.
19. Cosmography.
20. Introduction to Astronomy.
21. Introduction to Geography: 1678, 8vo.

NEWTON (*Sir* **Isaac**), one of the greatest philosophers and mathematicians the world has produced, was born at Woolstrop in Lincolnshire on Christmas day 1642. He was descended from the eldest branch of the family of Sir John Newton, Bart. who were lords of the manor of Woolstrop, and had been possessed of the estate for about two centuries before; to which they had removed from Westley in the same county, but originally they came from the town of Newton in Lancashire. Other accounts say, I think more truly, that he was the only child of Mr. John Newton of Colesworth, near Grantham in Lincolnshire, who had there an estate of about 120*l.* a year, which he kept in his own hands. His mother was of the ancient and opulent family of the Ayscoughs, or Askews, of the same county. Our author losing his father while he was very young, the care of his education devolved on his mother, who, though she married again after his father's death, did not neglect to improve by a liberal education the promising genius that was observed in her son. At 12 years of age, by the advice of his maternal uncle, he was sent to the grammar school at Grantham, where he made a good proficiency in the languages, and laid the foundation of his future studies. Even here was observed in him a strong inclination to figures and philosophical subjects. One trait of this early disposition is told of him: he had then a rude method of measuring the force of the wind blowing against him, by observing how much farther he could leap in the direction of the wind, or blowing on his back, than he could leap the contrary way, or opposed to the wind: an early mark of his original infantine genius.

After a few years spent here, his mother took him home; intending, as she had no other child, to have the pleasure of his company; and that, after the manner of his father before him, he should occupy his own estate. But instead of attending to the markets, or the business of the farm, he was occupied in studying and poring over his books, even by stealth, from his mother's knowledge. On one of these occasions his uncle discovered him one day in a hay-loft at Grantham, whither he had been sent to the market, working a mathematical problem; and having otherwise observed the boy's mind to be uncommonly bent upon learning, he prevailed upon his sister to part with him; and he was accordingly sent, in 1660, to Trinity-college, in Cambridge, where his uncle, having himself been a member of it, had still many friends. Isaac was here noticed by Dr. Barrow, who was soon after appointed the first Lucasian professor of mathematics; and observing his bright genius, contracted a great friendship for him. At his outsetting here, Euclid was first put into his hands, as usual, but that author was soon dismissed; our author's genius and application soon rendering him master of the Elements: and as the analytical method of Descartes was then much in vogue, he particularly applied to it, and Kepler's Optics, &c, making several improvements on them, which he entered on the margins of the books as he went on, as his custom was in studying any author.

Thus he was employed till the year 1664, when he opened a way into his new method of Fluxions and Infinite Series; and the same year took the degree of bachelor of arts. In the mean time, observing that the mathematicians were much engaged in the business of improving telescopes, by grinding glasses into one of the figures made by the three sections of a cone, on the principle then generally entertained, that light was homogeneous, he set himself to grinding of optic glasses, of other figures than spherical, having as yet no distrust of the homogeneous nature of light: but not hitting presently on any thing in this attempt to satisfy his mind, he procured a glass prism, that he might try the celebrated phenomena of colours, discovered by Grimaldi not long before. He was much pleased at first with the vivid brightness of the colours produced by this experiment; but after a while, considering them in a philosophical way, with that circumspection which was natural to him, he was surprised to see them in an oblong form, which, according to the received rule of refractions, ought to be circular. At first he thought the irregularity might possibly be no more than accidental;

but this was what he could not leave without further inquiry: accordingly, he soon invented an infallible method of deciding the question; and the result was, his New Theory of Light and Colours.

However, the theory alone, unexpected and surprising as it was, did not satisfy him; he rather considered the proper use that might be made of it for improving telescopes, which was his first design. To this end, having now discovered that light was not homogeneous, but an heterogeneous mixture of differently refrangible rays, he computed the errors arising from this different refrangibility; and, finding them to exceed some hundreds of times those occasioned by the circular figure of the glasses, he threw aside his glass works, and took reflections into consideration. He was now sensible that optical instruments might be brought to any degree of perfection desired, in case there could be found a reflecting substance which would polish as finely as glass, and reflect as much light as glass transmits, and the art of giving it a parabolical figure be also attained: but these seemed to him very great difficulties; nay, he almost thought them insuperable, when he further considered, that every irregularity in a reflecting superficies makes the rays deviate 5 or 6 times more from their due course, than the like irregularities in a refracting one.

Amidst these speculations, he was forced from Cambridge, in 1666, by the plague; and it was more than two years before he made any further progress in the subject. However, he was far from passing his time idly in the country; on the contrary, it was here, at this time, that he first started the hint that gave rise to the system of the world, which is the main subject of the Principia. In his retirement, he was sitting alone in a garden, when some apples falling from a tree, led his thoughts upon the subject of gravity; and, reflecting on the power of that principle, he began to consider, that, as this power is not found to be sensibly diminished at the remotest distance from the centre of the earth to which we can rise, neither at the tops of the loftiest buildings, nor on the summits of the highest mountains, it appeared to him reasonable to conclude, that this power must extend much farther than is usually thought. "Why not as high as the moon?" said he to himself; "and if so, her motion must be influenced by it; perhaps she is retained in her orbit by it: however, though the power of gravity is not sensibly weakened in the little change of distance at which we can place ourselves from the centre of the earth, yet it is very possible that, at the height of the moon, this power may differ in strength much from what it is here." To make an estimate what might be the degree of this diminution, he considered, that if the moon be retained in her orbit by the force of gravity, no doubt the primary planets are carried about the sun by the like power; and, by comparing the periods of the several planets with their distances from the sun, he found, that if any power like gravity held them in their courses, its strength must decrease in the duplicate proportion of the increase of distance. This he concluded, by supposing them to move in perfect circles, concentric to the sun, from which the orbits of the greatest part of them do not much differ. Supposing therefore the force of gravity, when extended to the moon, to decrease in the same manner, he computed whether that force would be sufficient to keep the moon in her orbit.

In this computation, being absent from books, he took the common estimate in use among the geographers and our seamen, before Norwood had measured the earth, namely, that 60 miles make one degree of latitude; but as that is a very erroneous supposition, each degree containing about 69$\frac{1}{15}$ of our English miles, his computation upon it did not make the power of gravity, decreasing in a duplicate proportion to the distance, answerable to the power which retained the moon in her orbit: whence he concluded, that some other cause must at least join with the action of the power of gravity on the moon. For this reason he laid aside, for that time, any further thoughts on the matter. Mr. Whiston (in his Memoirs, pa. 33) says, he told him that he thought Descartes's vortices might concur with the action of gravity.

Nor did he resume this inquiry on his return to Cambridge, which was shortly after. The truth is, his thoughts were now engaged on his newly projected reflecting telescope, of which he made a small specimen, with a metallic reflector spherically concave. It was but a rude essay, chiefly defective from the want of a good polish for the metal; which instrument is now in the possession of the Royal Society. In 1667 he was chosen fellow of his college, and took the degree of master of arts. And in 1669, Dr. Barrow resigned to him the mathematical chair at Cambridge, the business of which appointment interrupted for a while his attention to the telescope: however, as his thoughts had been for some time chiefly employed upon optics, he made his discoveries in that science the subject of his lectures, for the first three years after he was appointed mathematical professor; and having now brought his Theory of Light and Colours to a considerable degree of perfection, and having been elected a fellow of the Royal Society in Jan. 1672, he communicated it to that body, to have their judgment upon it; and it was afterwards published in their Transactions, viz. of Feb. 19, 1672. This publication occasioned a dispute upon the truth of it, which gave him so much uneasiness, that he resolved not to publish any thing further for a while on the subject; and in that resolution, he laid up his Optical Lectures, though he had prepared them for the press. And the Analysis by Infinite Series, which he had intended to subjoin to them, unhappily for the world, underwent the same fate, and for the same reason.

In this temper he resumed his telescope; and observing that there was no absolute necessity for the parabolic figure of the glasses, since, if metals could be ground truly spherical, they would be able to bear as great apertures as men could give a polish to, he completed another instrument of the same kind. This answering the purpose so well, as, though only half a foot in length, to show the planet Jupiter distinctly round, with his four satellites, and also Venus horned, he sent it to the Royal Society, at their request, together with a description of it, with further particulars; which were published in the Philosophical Transactions for March 1672. Several attempts were also made by that society to bring it to perfection; but, for want of a proper composition of metal, and a good polish, nothing succeeded, and the invention lay dormant, till Hadley made his Newtonian telescope in 1723. At the request of Leibnitz, in 1676, he explained his invention of Infinite Series, and took notice how far he had improved it by his Method of Fluxions, which however he still concealed, and particularly on this occasion, by a transposition of the letters that make up the two fundamental propositions of it, into an alphabetical order; the letters concerning which are inserted in Collins's Commercium Epistolicum, printed 1712. In the winter between the years 1676 and 1677, he discovered the grand proposition, that, by a centripetal force acting reciprocally as the square of the distance, a planet must revolve in an ellipsis, about the centre of force placed in its lower focus, and, by a radius drawn to that centre, describe areas proportional to the times. In 1680 he made several astronomical observations on the comet that then appeared; which, for some considerable time, he took not to be one and the same, but two different comets; and on this occasion several letters passed between him and Mr. Flamsteed.

He was still under this mistake, when he received a letter from Dr. Hooke, explaining the nature of the line described by a falling body, supposed to be moved circularly by the diurnal motion of the earth, and perpendicularly by the power of gravity. This letter put him on inquiring anew what was the real figure in which such a body moved; and that inquiry, convincing him of another mistake which he had before fallen into concerning that figure, put him upon resuming his former thoughts with regard to the moon; and Picart having not long before, viz, in 1679, measured a degree of the earth with sufficient accuracy, by using his measures, that planet appeared to be retained in her orbit by the sole power of gravity; and consequently that this power decreases in the duplicate ratio of the distance; as he had formerly conjectured. On this principle, he found the line described by a falling body to be an ellipsis, having one focus in the centre of the earth. And finding by this means, that the primary planets really moved in such orbits as Kepler had supposed, he had the satisfaction to see that this inquiry, which he had undertaken at first out of mere curiosity, could be applied to the greatest purposes. Hereupon he drew up about a dozen propositions, relating to the motion of the primary planets round the sun, which were communicated to the Royal Society in the latter end of 1683. Becoming thus known to Dr. Halley, that gentleman, who had attempted the demonstration in vain, applied, in August 1684, to Newton, who assured him that he had absolutely completed the proof. This was also registered in the books of the Royal Society; at whose earnest solicitation Newton finished the work, which was printed under the care of Dr. Halley, and came out about midsummer 1687, under the title of, Philosophiæ Naturalis Principia Mathematica, containing in the third book, the Cometic Astronomy, which had been lately discovered by him, and now made its first appearance in the world: a work which may be considered as the production of a celestial intelligence, rather than of a man.

This work however, in which the great author has built a new system of natural philosophy on the most sublime geometry, did not meet at first with all the applause it deserved, and which it was destined one day to receive. Two reasons concurred in producing this effect: Descartes had then got full possession of the opinion of the scientific world. His philosophy was indeed the creature of a fine imagination, gaily dressed out: he had given her likewise some of nature's fine features, and painted the rest to a seeming likeness of her. On the other hand, Newton had with an unparalleled penetration, and force of genius, pursued nature up to her most secret abode, and was intent to demonstrate her residence to others, rather than anxious to describe particularly the way by which he arrived at it himself: he finished his piece in that elegant conciseness, which had justly gained the ancients universal esteem. In fact, the consequences flow with such rapidity from the principles, that the reader is often left to supply a long chain of reasoning to connect them: so that it required some time before the world could understand it. The best mathematicians were obliged to study it with care, before they could make themselves master of it; and those of a lower rank durst not venture upon it, till encouraged by the testimonies of the more learned. But at last, when its value became sufficiently known, the approbation which had been so slowly gained, became universal, and nothing was to be heard from all quarters, but one general burst of admiration. "Does Mr. Newton eat, drink, or sleep like other men?" says the marquis de l'Hospital, (one of the greatest mathematicians of the age,) to the English who visited him. "I represent him to myself as a celestial genius entirely disengaged from matter."

In the midst of these profound mathematical researches, just before his Principia went to the press in 1686, the privileges of the university being attacked by James the 2d, Newton appeared among its most strenuous defenders, and was on that occasion appointed one of their delegates to the high-commission court; where they made such a defence, that James thought proper to drop the affair. Our author was also chosen one of their members for the Convention-Parliament in 1688, in which he sat till it was dissolved.

Newton's merit was well known to Mr. Montague, then chancellor of the exchequer, and afterwards earl of Halifax, who had been educated at the same college with him; and when he undertook the great work of recoining the money, he fixed his eye upon Newton for an assistant in it; and accordingly, in 1696, he was appointed warden of the mint, in which employment, he rendered very signal service to the nation. And three years after he was promoted to be master of the mint, a place worth 12 or 15 hundred pounds per annum, which he held till his death. On this promotion, he appointed Mr. Whiston his deputy in the mathematical professorship at Cambridge, giving him the full profits of the place, which appointment itself he also procured for him in 1703. The same year our author was chosen president of the Royal Society, a situation which he held till his death, having then presided over it for 25 years; he had also been chosen a member of the Royal Academy of Sciences at Paris in 1699, as soon as the new regulation was made for admitting foreigners into that society.

From the first discovery of the heterogeneous mixture of light, and the production of colours thence arising, he had employed a great part of his time in bringing the experiment, on which the theory is founded, to a degree of exactness that might satisfy himself. The truth is, this seems to have been his favourite invention; 30 years he had spent in this arduous task, before he published it in 1704. In infinite series and fluxions, and in the power and rule of gravity in preserving the solar system, there had been some, though distant hints, given by others before him: whereas in dissecting a ray of light into its primary constituent particles, which then admitted of no further separation; in the discovery of the different refrangibility of these particles thus separated; and that these constituent rays had each its own peculiar colour inherent in it; that rays falling in the same angle of incidence have alternate fits of reflection and refraction; that bodies are rendered transparent by the minuteness of their pores, and become opaque by having them large; and that the most transparent body, by having an extreme thinness, will become less pervious to the light: in all these, which make up his new theory of light and colours, he was absolutely and entirely the first inventor; and as the subject is of the most subtle and delicate nature, he thought it necessary to be himself the last finisher of it.

In fact, the affair that chiefly employed his researches for so many years, was far from being confined to the subject of light alone. On the contrary, all that we know of natural bodies, seemed to be comprehended in it; he had found out, that there was a natural action at a distance between light and other bodies, by which both the reflections and refractions, as well as inflections, of the former, were constantly produced. To ascertain the force and extent of this principle of action, was what had all along engaged his thoughts, and what after all, by its extreme subtlety, escaped his most penetrating spirit. However, though he has not made so full a discovery of this principle, which directs the course of light, as he has in regard to the power by which the planets are kept in their courses; yet he gave the best possible directions for such as should be disposed to carry on the work, and furnished matter abundantly sufficient to animate them to the pursuit. He has indeed hereby opened a way of passing from optics to an entire system of physics; and, if we consider his queries as containing the history of a great man's first thoughts, even in that view they must be always at least entertaining and curious.

This same year, and in the same book with his Optics, he published, for the first time, his Method of Fluxions. It has been already observed, that these two inventions were intended for the public so long before as 1672; but were laid by then, in order to prevent his being engaged on that account in a dispute about them. And it is not a little remarkable, that even now this last piece proved the occasion of another dispute, which continued for many years. Ever since 1684, Leibnitz seems to have claimed the honour of having first invented this method.—Newton saw his design from the beginning, and had sufficiently obviated it in the first edition of the Principia, in 1687 (viz, in the Scholium to the 2d lemma of the 2d book): and with the same view, when he now published that method, he took occasion to acquaint the world, that he invented it in the years 1665 and 1666. In the Acta Eruditorum of Leipsic, where an account is given of this book, the author of that account ascribed the invention to Leibnitz, intimating that Newton borrowed it from him. Dr. Keill, the astronomical professor at Oxford, undertook Newton's defence; and after several answers on both sides, Leibnitz complaining to the Royal Society, this body appointed a committee of their members to examine the merits of the case. These, after considering all the papers and letters relating to the point in controversy, decided in favour of Newton and Keill; as is related at large in the life of this last-mentioned gentleman; and these papers themselves were published in 1712, under the title of Commercium Epistolicum Johannis Collins, 8vo.

In 1705, the honour of knighthood was conferred upon our author by queen Anne, in consideration of his great merit. And in 1714 he was applied to by the House of Commons, for his opinion on a new method of discovering the longitude at sea by signals, which had been laid before them by Ditton and Whiston, in order to procure their encouragement; but the petition was thrown aside on reading Newton's paper delivered to the committee.

The following year, 1715, Leibnitz, with the view of bringing the world more easily into the belief that Newton had taken the method of fluxions from his differential method, attempted to foil his mathematical skill by the famous problem of the trajectories, which he therefore proposed to the English by way of challenge; but the solution of this, though the most difficult proposition he was able to devise, and what might pass for an arduous affair to any other, yet was hardly any more than an amusement to Newton's penetrating genius: he received the problem at 4 o'clock in the afternoon, as he was returning from the Mint; and, though extremely fatigued with business, yet he finished the solution before he went to bed.

As Leibnitz was privy-counsellor of justice to the elector of Hanover, so when that prince was raised to the British throne, Newton came more under the notice of the court; and it was for the immediate satisfaction of George the First, that he was prevailed on to put the last hand to the dispute about the invention of fluxions. In this court, Caroline princess of Wales, afterwards queen-consort to George the Second, happened to have a curiosity for philosophical inquiries; no sooner therefore was she informed of our author's attachment to the house of Hanover, than she engaged his conversation, which soon endeared him to her. Here she found in every difficulty that full satisfaction, which she had in vain sought for elsewhere; and she was often heard to declare publicly, that she thought herself happy in coming into the world at a juncture of time, which put it in her power to converse with him. It was at this princess's solicitation, that he drew up an abstract of his Chronology; a copy of which was at her request communicated, about 1718, to signior Conti, a Venetian nobleman, then in England, on a promise to keep it secret. But notwithstanding this promise, the abbé, (who, while here, had also affected to show a particular friendship for Newton, though privately betraying him as much as lay in his power to Leibnitz,) was no sooner got across the water into France, than he dispersed copies of it, and procured an antiquary to translate it into French, as well as to write a confutation of it. This, being printed at Paris in 1725, was delivered as a present from the bookseller that printed it to our author, that he might obtain, as was said, his consent to the publication; but though he expressly refused such consent, yet the whole was published the same year. Hereupon Newton found it necessary to publish a Defence of himself, which was inserted in the Philosophical Transactions. Thus he, who had so much all his life long been studious to avoid disputes, was unavoidably all his lifetime, in a manner, involved in them; nor did this last dispute even finish at his death, which happened the year following. Newton's paper was republished in 1726 at Paris, in French, with a letter of the abbé Conti in answer to it; and the same year some dissertations were printed there by father Souciet against Newton's Chronological Index, an answer to which was inserted by Halley in the Philos. Trans. No. 397.

Some time before this business, in his 80th year, our author was seized with an incontinence of urine, thought to proceed from the stone in the bladder, and deemed to be incurable. However, by the help of a strict regimen and other precautions, which till then he never had occasion for, he procured considerable intervals of ease during the five remaining years of his life. Yet he was not free from some severe paroxysms, which even forced out large drops of sweat that ran down his face. In these circumstances he was never observed to utter the least complaint, nor express the least impatience; and as soon as he had a moment's ease, he would smile and talk with his usual cheerfulness. He was now obliged to rely upon Mr. Conduit, who had married his niece, for the discharge of his office in the mint. Saturday morning March 18, 1727, he read the newspapers, and discoursed a long time with Dr. Mead his physician, having then the perfect use of all his senses and his understanding; but that night he entirely lost them, and did not recover them afterwards; he died the Monday following, March 20, in the 85th year of his age. His corpse lay in state in the Jerusalem-chamber, and on the 28th was conveyed into Westminster-abbey, the pall being supported by the lord-chancellor, the dukes of Montrose and Roxburgh, and the earls of Pembroke, Sussex, and Macclesfield. He was interred near the entrance into the choir on the left hand, where a stately monument is erected to his memory, with a most elegant inscription upon it.

Newton's character has been attempted by M. Fontenelle and Dr. Pemberton, the substance of which is as follows. He was of a middle stature, and somewhat inclined to be fat in the latter part of his life. His countenance was pleasing and venerable at the same time; especially when he took off his peruke and showed his white hair, which was pretty thick. He never made use of spectacles, and lost but one tooth during his whole life. Bishop Atterbury says, that, in the whole air of Sir Isaac's face and make, there was nothing of that penetrating sagacity which appears in his compositions: that he had something rather languid in his look and manner, which did not raise any great expectation in those who did not know him.

His temper it is said was so equal and mild, that no accident could disturb it. A remarkable instance of which is related as follows. Sir Isaac had a favourite little dog, which he called Diamond. Being one day called out of his study into the next room, Diamond was left behind. When Sir Isaac returned, having been absent but a few minutes, he had the mortification to find, that Diamond having overset a lighted candle among some papers, the nearly finished labour of many years was in flames, and almost consumed to ashes. This loss, as Sir Isaac was then very far advanced in years, was irretrievable; yet, without once striking the dog, he only rebuked him with this exclamation, "Oh Diamond! Diamond! thou little knowest the mischief thou hast done!" Dr. Wallis (Algeb. p. 347) says, some papers on series and curves were ready for the press in 1671, but by mischance were burnt.

Newton was indeed of so meek and gentle a disposition, and so great a lover of peace, that he would rather have chosen to remain in obscurity, than to have the calm of life ruffled by those storms and disputes, which genius and learning always draw upon those that are most eminent for them.

From his love of peace, no doubt, arose that unusual kind of horror which he felt for all disputes: a steady unbroken attention, free from those frequent recoilings inseparably incident to others, was his peculiar felicity; he knew it, and he knew the value of it. No wonder then that controversy was looked on as his bane. When some objections, hastily made to his discoveries concerning light and colours, induced him to lay aside the design he had taken of publishing his Optical Lectures, we find him reflecting on that dispute, into which he had been unavoidably drawn, in these terms: "I blamed my own imprudence for parting with so real a blessing as my quiet, to run after a shadow." It is true this shadow, as Fontenelle observes, did not escape him afterwards, nor did it cost him that quiet which he so much valued, but proved as much a real happiness to him as his quiet itself; yet this was a happiness of his own making: he took a resolution from these disputes, not to publish any more concerning that theory, till he had put it above the reach of controversy, by the most exact experiments, and the strictest demonstrations; and accordingly it has never been called in question since. In the same temper, after he had sent the manuscript to the Royal Society, with his consent to the printing of it by them; yet upon Hooke's injuriously insisting that he himself had demonstrated Kepler's problem before our author, he determined, rather than be involved again in controversy, to suppress the third book; and he was very hardly prevailed on to alter that resolution. It is true, the public was thereby a gainer; that book, which is indeed no more than a corollary of some propositions in the first, being originally drawn up in the popular way, with a design to publish it in that form; whereas he was now convinced that it would be best not to let it go abroad without a strict demonstration.

In contemplating Newton's genius, it presently becomes a doubt, which of these endowments had the greatest share, sagacity, penetration, strength, or diligence; and, after all, the mark that seems most to distinguish it is, that he himself made the justest estimation of it, declaring, that if he had done the world any service, it was due to nothing but industry and patient thought; that he kept the subject of consideration constantly before him, and waited till the first dawning opened gradually, by little and little, into a full and clear light. It is said, that when he had any mathematical problems or solutions in his mind, he would never quit the subject on any account. And his servant has said, when he has been getting up in a morning, he has sometimes begun to dress, and with one leg in his breeches, sat down again on the bed, where he has remained for hours before he has got his clothes on: and that dinner has been often three hours ready for him before he could be brought to table.

After all, notwithstanding his anxious care to prevent interruption in his intense application to study, he could nevertheless, when occasion required it, lay aside his thoughts, though engaged in the most intricate researches, when his other affairs required his attention; and, as soon as he had leisure, resume the subject at the point where he had left off. This he seems to have done not so much by any extraordinary strength of memory, as by the force of his inventive faculty, to which every thing opened itself again with ease, if nothing intervened to ruffle him. The readiness of his invention made him not think of putting his memory much to the trial; but this was the offspring of a vigorous intenseness of thought, out of which he was but a common man. He spent therefore the prime of his age in those abstruse researches, when his situation in a college gave him leisure, and while study was his proper business. But as soon as he was removed to the mint, he applied himself chiefly to the duties of that office; and so far quitted mathematics and philosophy, as not to engage in any pursuits of either kind afterwards.

Dr. Pemberton observes, that though his memory was much decayed in the last years of his life, yet he perfectly understood his own writings, contrary to what I had formerly heard, says the doctor, in discourse from many persons. This opinion of theirs might arise perhaps from his not being always ready at speaking on these subjects, when it might be expected he should. But on this head it may be observed, that great geniuses are often liable to be absent, not only in relation to common life, but with regard to some of the parts of science that they are best informed of: inventors seem to treasure up in their minds what they have found out, after another manner than those do the same things who have not this inventive faculty. The former, when they have occasion to produce their knowledge, are in some measure obliged immediately to investigate part of what they want; and for this they are not equally fit at all times: from whence it has often happened, that such as retain things chiefly by means of a very strong memory, have appeared off-hand more expert than the discoverers themselves.

It was evidently owing to the same inventive faculty that Newton, as this writer found, had read fewer of the modern mathematicians than one could have expected; his own prodigious invention readily supplying him with what he might have occasion for in the pursuit of any subject he undertook. However, he often censured the handling of geometrical subjects by algebraic calculations; and his book of algebra he called by the name of Universal Arithmetic, in opposition to the injudicious title of Geometry, which Descartes had given to the treatise in which he shows how the geometrician may assist his invention by such kind of computations. He frequently praised Slusius, Barrow, and Huygens, for not being influenced by the false taste which then began to prevail. He used to commend the laudable attempt of Hugo d'Omerique to restore the ancient analysis; and very much esteemed Apollonius's book De Sectione Rationis, for giving us a clearer notion of that analysis than we had before. Dr. Barrow may be esteemed as having shown a compass of invention equal, if not superior, to any of the moderns, our author only excepted: but Newton particularly recommended Huygens's style and manner: he thought him the most elegant of any mathematical writer of modern times, and the truest imitator of the ancients. Of their taste and mode of demonstration our author always professed himself a great admirer; and even censured himself for not following them yet more closely than he did; and spoke with regret of his mistake at the beginning of his mathematical studies, in applying himself to the works of Descartes, and other algebraic writers, before he had considered the Elements of Euclid with that attention which so excellent a writer deserves.

But if this was a fault, it is certain it was a fault to which we owe both his great inventions in speculative mathematics, and the doctrine of fluxions and infinite series. And perhaps this might be one reason why his particular reverence for the ancients is omitted by Fontenelle, who however certainly makes some amends by that just elogium which he makes of our author's modesty, which amiable quality he represents as standing foremost in the character of this great man's mind and manners. It was in reality greater than can be easily imagined, or will be readily believed: yet it always continued so without any alteration; though the whole world, says Fontenelle, conspired against it; let us add, though he was thereby robbed of his invention of fluxions. Nicholas Mercator publishing his Logarithmotechnia in 1668, where he gave the quadrature of the hyperbola by an infinite series, which was the first appearance in the learned world of a series of this sort drawn from the particular nature of the curve, and that in a manner very new and abstracted; Dr. Barrow, then at Cambridge, where Newton, then about 26 years of age, resided, recollected, that he had met with the same thing in the writings of that young gentleman; and there not confined to the hyperbola only, but extended, by general forms, to all kinds of curves, even such as are mechanical; to their quadratures, their rectifications, and their centres of gravity; to the solids formed by their rotations, and to the superficies of those solids; so that, when their determinations were possible, the series stopped at a certain point, or at least their sums were given by stated rules: and if the absolute determinations were impossible, they could yet be infinitely approximated; which is the happiest and most refined method, says Fontenelle, of supplying the defects of human knowledge that man's imagination could possibly invent. To be master of so fruitful and general a theory was a mine of gold to a geometrician; but it was a greater glory to have been the discoverer of so surprising and ingenious a system. So that Newton, finding by Mercator's book, that he was in the way to it, and that others might follow in his track, should naturally have been forward to open his treasures, and secure the property, which consisted in making the discovery; but he contented himself with his treasure which he had found, without regarding the glory. What an idea does it give us of his unparalleled modesty, when we find him declaring, that he thought Mercator had entirely discovered his secret, or that others would, before he should become of a proper age for writing! His manuscript on infinite series was communicated to none but Mr. John Collins and the lord Brounker, then president of the Royal Society, who had also done something in this way himself; and even that had not been complied with, but for Dr. Barrow, who would not suffer him to indulge his modesty so much as he desired.

It is further observed, concerning this part of his character, that he never talked either of himself or others, nor ever behaved in such a manner, as to give the most malicious censurers the least occasion even to suspect him of vanity. He was candid and affable, and always put himself upon a level with his company. He never thought either his merit or his reputation sufficient to excuse him from any of the common offices of social life. No singularities, either natural or affected, distinguished him from other men. Though he was firmly attached to the church of England, he was averse to the persecution of the nonconformists. He judged of men by their manners; and the true schismatics, in his opinion, were the vicious and the wicked. Not that he confined his principles to natural religion, for it is said he was thoroughly persuaded of the truth of revelation; and amidst the great variety of books which he had constantly before him, that which he studied with the greatest application was the Bible, at least in the latter years of his life: and he understood the nature and force of moral certainty as well as he did that of a strict demonstration.

Sir Isaac did not neglect the opportunities of doing good, when the revenues of his patrimony and a profitable employment, improved by a prudent œconomy, put it in his power. We have two remarkable instances of his bounty and generosity; one to Mr. Maclaurin, extra professor of mathematics at Edinburgh, to encourage whose appointment he offered 20 pounds a-year to that office; and the other to his niece Barton, upon whom he had settled an annuity of 100 pounds per annum. When decency on any occasion required expense and show, he was magnificent without grudging it: at all other times, that pomp which seems great to low minds only, was utterly retrenched, and the expense reserved for better uses.—On this head it may be remarked however, as a curious fact, that by an order of council, dated Jan. 28, 1675, (which was 3 years after his election into the Royal Society,) it was ordered, that he should be excused from making the usual weekly payments (one shilling per week), on account of his low circumstances, as he represented.

Newton never married; and it has been said, that "perhaps he never had leisure to think of it; that, being immersed in profound studies during the prime of his age, and afterwards engaged in an employment of great importance, and even quite taken up with the company which his merit drew to him, he was not sensible of any vacancy in life, nor of the want of a companion at home." These however do not appear to be any sufficient reasons for his never marrying, if he had had an inclination so to do. It is much more likely that he had a constitutional indifference to the state, and even to the sex in general; and it has even been said of him, that he never once knew woman.—He left at his death, it seems, 32 thousand pounds; but he made no will, which, Fontenelle tells us, was because he thought a legacy was no gift.—As to his works, besides what were published in his life-time, there were found after his death, among his papers, several discourses on the subjects of antiquity, history, divinity, chemistry, and mathematics: several of which were published at different times, as appears from the following catalogue of all his works; where they are ranked in the order of time in which those upon the same subject were published.

1. Several Papers relating to his Telescope, and his Theory of Light and Colours, printed in the Philosophical Transactions, Nos. 80, 81, 82, 83, 84, 85, 88, 96, 97, 110, 121, 123, 128; or vols. 6, 7, 8, 9, 10, 11.

2. Optics, or a Treatise of the Reflections, Refractions, and Inflections, and the Colours of Light; 1704, 4to.—A Latin translation by Dr. Clarke; 1706, 4to.—And a French translation by Pet. Coste, Amst. 1729, 2vols 12mo.—Beside several English editions in 8vo.

3. Optical Lectures; 1728, 8vo.—Also in several Letters to Mr. Oldenburg, secretary of the Royal Society, inserted in the General Dictionary, under our author's article.

4. Lectiones Opticæ; 1729, 4to.

5. Naturalis Philosophiæ Principia Mathematica; 1687, 4to.—A second edition in 1713, with a Preface, by Roger Cotes.—The 3d edition in 1726, under the direction of Dr. Pemberton.—An English translation, by Motte, 1729, 2 volumes 8vo, printed in several editions of his works, in different nations, particularly an edition, with a large Commentary, by the two learned Jesuits, Le Seur and Jacquier, in 4 volumes 4to, in 1739, 1740, and 1742.

6. A System of the World, translated from the Latin original; 1727, 8vo.—This, as has been already observed, was at first intended to make the third book of his Principia, an English translation by Motte, 1729, 8vo.

7. Several Letters to Mr. Flamsteed, Dr. Halley, and Mr. Oldenburg.—See our author's article in the General Dictionary.

8. A Paper concerning the Longitude; drawn up by order of the House of Commons; ibid.

9. Abregé de Chronologie, &c; 1726, under the direction of the abbé Conti, together with some observations upon it.

10. Remarks on the Observations made upon a Chronological Index of Sir I. Newton, &c. Philos. Trans. vol. 33. See also the same, vol. 34 and 35, by Dr. Halley.

11. The Chronology of Ancient Kingdoms amended, &c; 1728, 4to.

12. Arithmetica Universalis, &c; under the inspection of Mr. Whiston, Cantab. 1707, and again in 1722, 8vo. Printed I think without the author's consent, and even against his will: an offence which it seems was never forgiven. There are also English editions of the same, particularly one by Wilder, with a Commentary, in 1769, 2 vols 8vo. And a Latin edition, with a Commentary, by Castilion, 2 vols 4to, Amst. &c.

13. Analysis per Quantitatum Series, Fluxiones, et Differentias, cum Enumeratione Linearum Tertii Ordinis; 1711, 4to; under the inspection of W. Jones, Esq. F. R. S.—The last tract had been published before, together with another on the Quadrature of Curves, by the Method of Fluxions, under the title of Tractatus duo de Speciebus et Magnitudine Figurarum Curvilinearum; subjoined to the first edition of his Optics in 1704; and other letters in the Appendix to Dr. Gregory's Catoptrics, &c, 1735, 8vo.—Under this head may be ranked Newtoni Genesis Curvarum per Umbras; Leyden, 1740.

14. Several Letters relating to his Dispute with Leibnitz, on his Right to the Invention of Fluxions; printed in the Commercium Epistolicum D. Johannis Collins et aliorum de Analysi Promota, jussu Societatis Regiæ editum; 1712, 8vo.

15. Postscript and Letter of M. Leibnitz to the Abbé Conti, with Remarks, and a Letter of his own to that Abbé; 1717, 8vo. To which was added, Raphson's History of Fluxions, as a Supplement.

16. The Method of Fluxions, and Analysis by Infinite Series, translated into English from the original Latin; to which is added, a Perpetual Commentary, by the translator Mr. John Colson; 1736, 4to.

17. Several Miscellaneous Pieces, and Letters, as follow:—(1) A Letter to Mr. Boyle on the subject of the Philosopher's Stone. Inserted in the General Dictionary, under the article Boyle.—(2) A Letter to Mr. Aston, containing directions for his travels; ibid. under our author's article.—(3) An English Translation of a Latin Dissertation on the Sacred Cubit of the Jews. Inserted among the miscellaneous works of Mr. John Greaves, vol. 2, published by Dr. Thomas Birch, in 1737, 2 vols. 8vo. This Dissertation was found subjoined to a work of Sir Isaac's, not finished, entitled Lexicon Propheticum.—(4) Four Letters from Sir Isaac Newton to Dr. Bentley, containing some arguments in proof of a Deity; 1756, 8vo.—(5) Two Letters to Mr. Clarke, &c.

18. Observations on the Prophecies of Daniel and the Apocalypse of St. John; 1733, 4to.

19. Tables for purchasing College Leases; 1742, 12mo.

20. Corollaries, by Whiston.

21. A Collection of several pieces of our author's, under the following title, Newtoni Is. Opuscula Mathematica

Philos. et Philol. collegit J. Castilioneus; Laus. 1744, 4to, 8 tomes.

22. Two Treatises on the Quadrature of Curves, and Analysis by Equations of an Infinite Number of Terms, explained: translated by John Stewart, with a large Commentary; 1745, 4to.

23. Description of an Instrument for observing the Moon's Distance from the Fixed Stars at Sea. Philos. Trans. vol. 42.

24. Newton also published Barrow's Optical Lectures, in 1699, 4to: and Bern. Varenii Geographia, &c; 1681, 8vo.

25. The whole works of Newton, published by Dr. Horsley; 1779, 4to, in 5 volumes.

The following is a list of the papers left by Newton at his death, as mentioned above.

A Catalogue of Sir Isaac Newton's Manuscripts and Papers, as annexed to a Bond, given by Mr. Conduit, to the Administrators of Sir Isaac; by which he obliges himself to account for any profit he shall make by publishing any of the papers.

Dr. Pellet, by agreement of the executors, entered into Acts of the Prerogative Court, being appointed to peruse all the papers, and judge which were proper for the press.

No.

1. Viaticum Nautarum; by Robert Wright.
2. Miscellanea; not in Sir Isaac's hand-writing.
3. Miscellanea; part in Sir Isaac's hand.
4. Trigonometria; about 5 sheets.
5. Definitions.
6. Miscellanea; part in Sir Isaac's hand.
7. 40 sheets in 4to, relating to Church History.
8. 126 sheets written on one side, being foul draughts of the Prophetic Stile.
9. 88 sheets relating to Church History.
10. About 70 loose sheets in small 4to, of Chemical papers; some of which are not in Sir Isaac's hand.
11. About 62 ditto, in folio.
12. About 15 large sheets, doubled into 4to; Chemical.
13. About 8 sheets ditto, written on one side.
14. About 5 sheets of foul papers, relating to Chemistry.
15. 12 half-sheets of ditto.
16. 104 half-sheets, in 4to, ditto.
17. About 22 sheets in 4to, ditto.
18. 24 sheets, in 4to, on the Prophecies.
19. 29 half-sheets; being an answer to Mr. Hooke, on Sir Isaac's Theory of Colours.
20. 87 half-sheets relating to the Optics, some of which are not in Sir Isaac's hand.

From No. 1 to No. 20 examined on the 20th of May 1727, and judged not fit to be printed.

T. Pellet.

Witness, *Tho. Pilkington.*

21. 328 half-sheets in folio, and 63 in small 4to; being loose and foul papers relating to the Revelations and Prophecies.
22. 8 half-sheets in small 4to, relating to Church Matters.
23. 24 half-sheets in small 4to; being a discourse relating to the 2d of Kings.
24. 353 half-sheets in folio, and 57 in small 4to; being foul and loose papers relating to Figures and Mathematics.
25. 201 half-sheets in folio, and 21 in small 4to; loose and foul papers relating to the Commercium Epistolicum.
26. 91 half-sheets in small 4to, in Latin, on the Temple of Solomon.
27. 37 half-sheets in folio, on the Host of Heaven, the Sanctuary, and other Church Matters.
28. 44 half-sheets in folio, on ditto.
29. 25 half-sheets in folio; being a farther account of the Host of Heaven.
30. 51 half-sheets in folio; being an Historical Account of two notable Corruptions of Scripture.
31. 88 half-sheets in small 4to; being Extracts of Church History.
32. 116 half-sheets in folio; being Paradoxical Questions concerning Athanasius, of which several leaves in the beginning are very much damaged.
33. 56 half-sheets in folio, De Motu Corporum; the greatest part not in Sir Isaac's hand.
34. 61 half sheets in small 4to; being various sections on the Apocalypse.
35. 25 half-sheets in folio, of the Working of the Mystery of Iniquity.
36. 20 half-sheets in folio, on the Theology of the Heathens.
37. 24 half-sheets in folio; being an Account of the Contest between the Host of Heaven, and the Transgressors of the Covenant.
38. 31 half-sheets in folio; being Paradoxical Questions concerning Athanasius.
39. 107 quarter-sheets in small 4to, on the Revelations.
40 74 half-sheets in folio; being loose papers relating to Church History.

May 22, 1727, examined from No. 21 to No. 40 inclusive, and judged them not fit to be printed; only No. 33 and No. 38 should be reconsidered.

T. Pellet.

Witness, *Tho. Pilkington.*

41. 167 half-sheets in folio; being loose and foul papers relating to the Commercium Epistolicum.
42. 21 half-sheets in folio; being the 3d letter on Texts of Scripture, very much damaged.
43. 31 half-sheets in folio; being foul papers relating to Church Matters.
44. 495 half-sheets in folio; being loose and foul papers relating to Calculations and Mathematics.
45. 335 half-sheets in folio; being loose and foul papers relating to the Chronology.
46. 112 sheets in small 4to, relating to the Revelations and other Church Matters.
47. 126 half-sheets in folio; being loose papers relating to the Chronology, part in English and part in Latin.
48. 400 half-sheets in folio; being loose mathematical papers.
49. 109 sheets in 4to, relating to the Prophecies, and Church Matters.
50. 127 half-sheets in folio, relating to the University; great part not in Sir Isaac's hand.
51. 18 sheets in 4to; being Chemical papers.
52. 255 quarter-sheets; being Chemical papers.

53. An Account of Corruptions of Scripture; not in Sir Isaac's hand.
54. 31 quarter-sheets; being Flammell's Explication of Hieroglyphical Figures.
55. About 350 half-sheets; being Miscellaneous papers.
56. 6 half-sheets; being An Account of the Empires, &c, represented by St. John.
57. 9 half-sheets folio, and 71 quarter-sheets 4to; being Mathematical papers.
58. 140 half-sheets, in 9 chapters, and 2 pieces in folio, titled, Concerning the Language of the Prophets.
59. 606 half-sheets folio, relating to the Chronology; 9 more in Latin.
60. 182 half-sheets folio; being loose papers relating to the Chronology and Prophecies.
61. 144 quarter-sheets, and 95 half-sheets folio; being loose Mathematical papers.
62. 137 half-sheets folio; being loose papers relating to the Dispute with Leibnitz.
63. A folio Common-place book; part in Sir Isaac's hand.
64. A bundle of English Letters to Sir Isaac, relating to Mathematics.
65. 54 half-sheets; being loose papers found in the Principia.
66. A bundle of loose Mathematical Papers; not Sir Isaac's.
67. A bundle of French and Latin Letters to Sir Isaac.
68. 136 sheets folio, relating to Optics.
69. 22 half-sheets folio, De Rationibus Motuum, &c; not in Sir Isaac's hand.
70. 70 half-sheets folio; being loose Mathematical Papers.
71. 38 half-sheets folio; being loose papers relating to Optics.
72. 47 half-sheets folio; being loose papers relating to Chronology and Prophecies.
73. 40 half-sheets folio; Procestus Mysterii Magni Philosophicus, by Wm. Yworth; not in Sir Isaac's hand.
74. 5 half-sheets; being a Letter from Rizzetto to Martine, in Sir Isaac's hand.
75. 41 half-sheets; being loose papers of several kinds, part in Sir Isaac's hand.
76. 40 half-sheets; being loose papers, foul and dirty, relating to Calculations.
77. 90 half-sheets folio; being loose Mathematical papers.
78. 176 half-sheets folio; being loose papers relating to Chronology.
79. 176 half-sheets folio; being loose papers relating to the Prophecies.
80. { 12 half-sheets folio; An Abstract of the Chronology.
92 half-sheets folio; The Chronology.
81. 40 half-sheets folio; The History of the Prophecies, in 10 chapters, and part of the 11th unfinished.
82. 5 small bound books in 12mo, the greatest part not in Sir Isaac's hand, being rough Calculations.

May 26th 1727, Examined from No. 41 to No. 82 inclusive, and judged not fit to be printed, except No. 80, which is agreed to be printed, and part of No. 61, and 81, which are to be reconsidered.

Th. Pellet.
Witness, *Tho. Pilkington.*

It is astonishing what care and industry Sir Isaac had employed about the papers relating to Chronology, Church History, &c; as, on examining the papers themselves, which are in the possession of the family of the earl of Portsmouth, it appears that many of them are copies over and over again, often with little or no variation; the whole number being upwards of 4000 sheets in folio, or 8 reams of folio paper; besides the bound books &c in this catalogue, of which the number of sheets is not mentioned. Of these there have been published only the Chronology, and Observations on the Prophecies of Daniel and the Apocalypse of St. John.

Many other curious particulars concerning Sir Isaac Newton, may be seen in Mr. Edmund Turnor's Collections relating to the town of Grantham, published in 1806.

NEWTONIAN *Philosophy*, the doctrine of the universe, or the properties, laws, affections, actions, forces, motions, &c, of bodies, both celestial and terrestrial, as delivered by Newton.

This term however is differently applied; which has given occasion to some confused notions relating to it. For some authors, under this term, include all the corpuscular philosophy, considered as it now stands reformed and corrected by the discoveries and improvements made in several parts of it by Newton. In which sense it is, that Gravesande calls his Elements of Physics, Introductio ad Philosophiam Newtonianam. And in this sense the Newtonian is the same as the new philosophy; and stands contradistinguished from the Cartesian, the Peripatetic, and the ancient Corpuscular.

Others, by Newtonian philosophy, mean the method or order used by Newton in philosophising; viz, the reasoning and inferences drawn directly from phenomena, exclusive of all previous hypotheses; the beginning from simple principles, and deducing the first powers and laws of nature from a few select phenomena, and then applying those laws &c to account for other things. In this sense, the Newtonian philosophy is the same with the experimental philosophy, or stands opposed to the ancient corpuscular, and to all hypothetical and fanciful systems.—Others again, by this term, mean that philosophy to which physical bodies are considered mathematically, and where geometry and mechanics are applied to the solution of phenomena. In which sense, the Newtonian is the same with the mechanical and mathematical philosophy.—Others, by Newtonian philosophy, understand that of physical knowledge which Newton has handled, improved, and demonstrated.—And lastly, others, by this philosophy, mean the new principles which Newton has brought into philosophy; with the new system founded upon them, and the new solutions of phenomena thence deduced; or that which characterizes and distinguishes his philosophy from all others. And this is the sense in which we shall here chiefly consider it.

This philosophy was first published in the year 1687, the author being then professor of mathematics in the university of Cambridge; a 2d edition, with considerable additions and improvements, appeared in 1713; and a 3d in 1726. An edition, with a very large commentary, was published in 1739, by Le Seur and Jacquier; besides the complete edition of all Newton's works, with notes, by

Dr. Horsley, in 1779 &c. Several authors have endeavoured to make it plainer; by setting aside many of the more sublime mathematical researches, and substituting either more obvious reasonings or experiments instead of them; particularly Whiston, in his Prælect. Phys. Mathem.; Gravesande, in Elem. et Inst.; Pemberton, in his View &c; and Maclaurin, in his Account of Newton's Philosophy.

The chief parts of the Newtonian philosophy, as delivered by the author, except his Optical Discoveries &c, are contained in his Principia, or Mathematical Principles of Natural Philosophy. He founds his system on the following definitions. 1. Quantity of Matter, is the measure of the same, arising from its density and bulk conjointly.—Thus, air of a double density, in the same space, is double in quantity; in a double space, is quadruple in quantity; in a triple space, is sextuple in quantity, &c.—2. Quantity of Motion, is the measure of the same, arising from the velocity and quantity of matter conjunctly.—This is evident, because the motion of the whole is the motion of all its parts; and therefore in a body double in quantity, with equal velocity, the Motion is double, &c.—3. The Vis Insita, Vis Inertiæ, or innate force of matter, is a power of resisting, by which every body, as much as in it lies, endeavours to persevere in its present state, whether it be of rest, or moving uniformly forward in a right line.—This definition is proved to be just by experience, from observing the difficulty with which any body is moved out of its place, upwards, or obliquely, or even downwards when acted on by a body endeavouring to urge it quicker than the velocity given it by gravity; and any how to change its state of motion or rest. And therefore this force is the same, whether the body have gravity or not; and a cannon-ball, void of gravity, if it could be, being discharged horizontally, will go the same distance in that direction, in the same time, as if it were endued with gravity.—4. An Impressed Force, is an action exerted on a body, in order to change its state, whether of rest or motion.—This force consists in the action only; and remains no longer in the body when the action is over. For a body maintains every new state it acquires, by its vis inertiæ only.—5. A Centripetal Force, is that by which bodies are drawn, impelled, or any way tend towards a point, as to a centre. This may be considered of three kinds, absolute, accelerative, and motive.—6. The Absolute quantity of the centripetal force, is a measure of the same, proportional to the efficacy of the cause that urges it to the centre.—7. The Accelerative quantity of a centripetal force, is the measure of the same, proportional to the velocity which it generates in a given time.—8. The Motive quantity of a centripetal force, is a measure of the same, proportional to the motion which it generates in a given time.—This is always known by the quantity of a force equal and contrary to it, that is just sufficient to hinder the descent of the body.

After these definitions, follow certain Scholia, treating of the nature and distinctions of Time, Space, Place, Motion, Absolute, Relative, Apparent, True, Real, &c. After which, the author proposes to show how we are to collect the true motions from their causes, effects, and apparent differences; and vice versa, how, from the motions, either true or apparent, we may arrive at the knowledge of their causes and effects. In order to this, he lays down the following axioms or laws of motion.

1st Law. Every body perseveres in its state of rest, or of uniform motion in a right line, unless it be compelled to change that state by forces impressed on it.—Thus, "Projectiles persevere in their motions, so far as they are not retarded by the resistance of the air, or impelled downwards by the force of gravity. A top, whose parts, by their cohesion, are perpetually drawn aside from rectilinear motions, does not cease its rotation otherwise than as it is retarded by the air or friction, &c. The greater bodies of the planets and comets, meeting with less resistance in more free spaces, preserve their motions, both progressive and circular, for a much longer time."

2d Law. The Alteration of motion is always proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed. Thus, if any force generate a certain quantity of motion, a double force will generate a double quantity, whether that force be impressed all at once, or in successive moments.

3d Law. To every action there is always opposed an equal re-action: or the mutual actions of two bodies upon each other, are always equal, and directed to contrary parts. Thus, whatever draws or presses another, is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the stone: &c.

From this axiom, or law, Newton deduces the following corollaries.—1. A body by two forces conjoined will describe the diagonal of a parallelogram, in the same time that it would describe the sides by those forces apart.—2. Hence is explained the composition of any one direct force out of any two oblique ones, viz, by making the two oblique forces the sides of a parallelogram, and the diagonal the direct one.—3. The quantity of motion, which is collected by taking the sum of the motions directed towards the same parts, and the difference of those that are directed to contrary parts, suffers no change from the action of bodies among themselves; because the motion which one body loses, is communicated to another.—4. The common centre of gravity of two or more bodies does not alter its state of motion or rest by the actions of the bodies among themselves; and therefore the common centre of gravity of all bodies, acting on each other, (excluding external actions and impediments,) is either at rest, or moves uniformly in a right line.—5. The motions of bodies included in a given space are the same among themselves, whether that space be at rest, or move uniformly forward in a right line without any circular motion. The truth of this is evident from the experiment of a ship; where all motions are just the same, whether the ship be at rest, or proceed uniformly forward in a straight line.—6. If bodies, any how moved among themselves, be urged in the direction of parallel lines by equal accelerative forces, they will all continue to move among themselves, after the same manner as if they had not been urged by such forces.

The mathematical part of the Newtonian Philosophy depends chiefly on the following lemmas; especially the first; containing the doctrine of prime and ultimate ratios.—Lem. 1. Quantities, and the ratios of quantities, which in any finite time converge continually to equality, and before the end of that time approach nearer the one to the other than by any given difference, become ultimately equal.—Lem. 2 shows, that in a space bounded by two right lines and a curve, if an infinite number of parallelograms be inscribed, all of equal breadth; then the ultimate ratio of the curve space and the sum of the parallelograms, will be a ratio of equality.—LEM. 3 shows, that the same thing is true when the breadths of the parallelograms are unequal.

In the succeeding lemmas it is shown, in like manner, that the ultimate ratios of the sine, chord, and tangent of arcs infinitely diminished, are ratios of equality, and therefore that in all our reasonings about these, we may safely use the one for the other:—that the ultimate form of evanescent triangles, made by the arc, chord, or tangent, is that of similitude, and their ultimate ratio is that of equality; and hence, in reasonings about ultimate ratios, these triangles may safely be used one for another, whether they are made with the sine, the arc, or the tangent.—The author then demonstrates some properties of the ordinates of curvilinear figures; and shows that the spaces which a body describes by any finite force urging it, whether that force is determined and immutable, or continually varied, are to each other, in the very beginning of the motion, in the duplicate ratio of the forces:—and lastly, having added some demonstrations concerning the evanescence of angles of contact, he proceeds to lay down the mathematical part of his system, which depends on the following theorems.

THEOR. 1. The areas which revolving bodies describe by radii drawn to an immoveable centre of force, lie in the same immoveable planes, and are proportional to the times in which they are described.—To this prop. are annexed several corollaries, respecting the velocities of bodies revolving by centripetal forces, the directions and proportions of those forces, &c; such as, that the velocity of such a revolving body, is reciprocally as the perpendicular let fall from the centre of force upon the line touching the orbit in the place of the body, &c.

THEOR. 2. Every body that moves in any curve line described in a plane, and, by a radius drawn to a point either immoveable or moving forward with a uniform rectilinear motion, describes about that point areas proportional to the times, is urged by a centripetal force directed to that point.—With corollaries relating to such motions in resisting mediums, and to the direction of the forces when the areas are not proportional to the times.

THEOR. 3. Every body that, by a radius drawn to the centre of another body, any how moved, describes areas about that centre proportional to the times, is urged by a force compounded of the centripetal forces tending to that other body, and of the whole accelerative force by which that other body is impelled.—With several corollaries.

THEOR. 4. The centripetal force of bodies, which by equal motions describe different circles, tend to the centres of the same circles; and are one to the other as the squares of the arcs described in equal times, applied to the radii of the circles.—With many corollaries, relating to the velocities, times, periodic forces, &c. And, in a scholium, the author further adds, Moreover, by means of the foregoing proposition and its corollaries, we may discover the proportion of a centripetal force to any other known force, such as that of gravity. For if a body by means of its gravity revolve in a circle, concentric to the earth, this gravity is the centripetal force of that body.—But from the descent of heavy bodies, the time of one entire revolution, as well as the arc described in any given time, is given by a corol. to this prop. And by such propositions, Mr. Huygens, in his excellent book De Horologio Oscillatorio, has compared the force of gravity with the centrifugal forces of revolving bodies.

On these, and such like principles, depends the Newtonian mathematical philosophy. The author further shows how to find the centre to which the forces impelling any body are directed, having the velocity of the body given: and finds that the centrifugal force is always as the versed sine of the nascent arc directly, and as the square of the time inversely; or directly as the square of the velocity, and inversely as the chord of the nascent arc. From these premises, he deduces the method of finding the centripetal force directed to any given point when the body revolves in a circle; and this, whether the central point be near, or at immense distance; so that all the lines drawn from it may be considered as parallels. And he shows the same thing with regard to bodies revolving in spirals, ellipses, hyperbolas, or parabolas. He shows also, having the figures of the orbits given, how to find the velocities and moving powers; and indeed resolves all the most difficult problems relating to the celestial bodies with a surprising degree of mathematical skill. These problems and demonstrations are all contained in the first book of the Principia: but an account of them here would neither be generally understood, nor easily comprised in the limits of this work.

In the second book, Newton treats of the properties and motion of fluids, and their powers of resistance, with the motion of bodies through such resisting mediums, those resistances being in the ratio of any powers of the velocities; and the motions being either made in right lines or curves, or vibrating like pendulums. And here he demonstrates such principles as entirely overthrow the doctrine of Descartes's vortices, which was the fashionable system in his time; concluding the book with these words: "So that the hypothesis of vortices is utterly irreconcileable with astronomical phenomena, and rather serves to perplex than explain the heavenly motions. How these motions are performed in free spaces without vortices, may be understood by the first book; and I shall now more fully treat of it in the following book Of the System of the World."—In this second book he makes great use of the doctrine of fluxions, then lately invented; for which purpose he lays down the principles of that doctrine in the 2d lemma, in these words: "The moment of any genitum, is equal to the moments of each of the generating sides drawn into the indices of the powers of those sides, and into their coefficients continually:" which rule he demonstrates, and then adds the following scholium concerning the invention of that doctrine: "In a letter of mine," says he, "to Mr. J. Collins, dated December 10, 1672, having described a method of tangents, which I suspected to be the same with Slusius's method, which at that time was not made public; I subjoined these words: 'This is one particular, or rather a corollary, of a general method which extends itself, without any troublesome calculation, not only to the drawing of tangents to any curve lines, whether geometrical or mechanical, or any how respecting right lines or other curves, but also to the resolving other abstruser kinds of problems about the curvature, areas, lengths, centres of gravity of curves, &c; nor is it (as Hudden's method de maximis et minimis) limited to equations which are free from surd quantities. This method I have interwoven with that other of working in equations, by reducing them to infinite series.' So far that letter. And these last words relate to a treatise I composed on that subject in the year 1671." Which, at least, is therefore the date of the invention of the doctrine of fluxions.

On entering upon the 3d book of the Principia, Newton briefly recapitulates the contents of the two former books in these words: "In the preceding books I have laid down the principles of philosophy; principles not philosophical, but mathematical; such, to wit, as we may build our reasonings upon in philosophical inquiries. These principles are, the laws and conditions of certain motions, and powers or forces, which chiefly have respect to philosophy. But lest they should have appeared of themselves dry and barren, I have illustrated them here and there with some philosophical scholiums, giving an account of such things as are of a more general nature, and which philosophy seems chiefly to be founded on; such as the density and the resistance of bodies, spaces void of all matter, and the motion of light and sounds. It remains, he adds, that from the same principles I now demonstrate the frame of the system of the world. Upon this subject, I had indeed composed the 3d book in a popular method, that it might be read by many. But afterwards considering that such as had not sufficiently entered into the principles could not easily discern the strength of the consequences, nor lay aside the prejudices to which they had been many years accustomed; therefore to prevent the disputes which might be raised on such accounts, I chose to reduce the substance of that book into the form of propositions, in the mathematical way, which should be read by those only, who had first made themselves masters of the principles established in the preceding books."

As a necessary preliminary to this 3d part, Newton lays down the following rules for reasoning in natural philosophy:—1. We are to admit no more causes of natural things, than such as are both true and sufficient to explain their natural appearances.—2. Therefore to the same natural effects we must always assign, as far as possible, the same causes.—3. The qualities of bodies which admit neither intension nor remission of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be esteemed the universal qualities of all bodies whatever.—4. In experimental philosophy, we are to consider propositions collected by general induction from phenomena, as accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined, till such time as other phenomena occur, by which they may either be made more accurate, or liable to exceptions.

The phenomena first considered are, 1. That the satellites of Jupiter, by radii drawn to his centre, describe areas proportional to the times of description; and that their periodic times, the fixed stars being at rest, are in the sesquiduplicate ratio of their distances from that centre. 2. The same thing is likewise observed of the phenomena of Saturn. 3. The five primary planets, Mercury, Venus, Mars, Jupiter, and Saturn, with their several orbits, encompass the sun. 4. The fixed stars being supposed at rest, the periodic times of the said five primary planets, and of the earth, about the sun, are in the sesquiduplicate proportion of their mean distances from the sun. 5. The primary planets, by radii drawn to the earth, describe areas no ways proportional to the times: but the areas which they describe by radii drawn to the sun are proportional to the times of description. 6. The moon, by a radius drawn to the centre of the earth, describes an area proportional to the time of description. All which phenomena are clearly evinced by astronomical observations. The mathematical demonstrations are next applied by Newton in the following propositions.

Prop. 1. The forces by which the satellites of Jupiter are continually drawn off from rectilinear motions, and retained in their proper orbits, tend to the centre of that planet; and are reciprocally as the squares of the distances of those satellites from that centre.—Prop. 2. The same thing is true of the primary planets, with respect to the sun's centre.—Prop. 3. The same thing is also true of the moon, in respect of the earth's centre.—Prop. 4. The moon gravitates towards the earth; and by the force of gravity is continually drawn off from a rectilinear motion, and retained in her orbit.—Prop. 5. The same thing is true of all the other planets, both primary and secondary, each with respect to the centre of its motion.—Prop. 6. All bodies gravitate towards every planet; and the weights of bodies towards any one and the same planet, at equal distances from its centre, are proportional to the quantities of matter they contain.—Prop. 7. There is a power of gravity tending to all bodies, proportional to the several quantities of matter which they contain.—Prop. 8. In two spheres mutually gravitating each towards the other, if the matter in places on all sides, round about and equidistant from the centres, be similar; the weight of either sphere towards the other, will be reciprocally as the square of the distance between their centres.—Hence are compared together the weights of bodies towards different planets: hence also are discovered the quantities of matter in the several planets: and hence likewise are found the densities of the planets.—Prop. 9. The force of gravity, in parts downwards from the surface of the planets towards their centres, decreases nearly in the proportion of the distances from those centres.

These, and many other propositions and corollaries, are proved or illustrated by a great variety of experiments, in all the great points of physical astronomy; such as, That the motions of the planets in the heavens may subsist an exceeding long time:—That the centre of the system of the world is immoveable:—That the common centre of gravity of the earth, the sun, and all the planets, is immoveable:—That the sun is agitated by a perpetual motion, but never recedes far from the common centre of gravity of all the planets:—That the planets move in ellipses which have their common focus in the centre of the sun; and, by radii drawn to that centre, they describe areas proportional to the times of description:—The aphelions and nodes of the orbits of the planets are fixt:—To find the aphelions, excentricities, and principal diameters of the orbits of the planets:—That the diurnal motions of the planets are uniform, and that the libration of the moon arises from her diurnal motion:—Of the proportion between the axes of the planets and the diameters perpendicular to those axes:—Of the weights of bodies in the different regions of our earth:—That the equinoctial points go backwards, and that the earth's axis, by a nutation in every annual revolution, twice vibrates towards the ecliptic, and as often returns to its former position:—That all the motions of the moon, and all the inequalities of those motions, follow from the principles above laid down:—Of the unequal motions of the satellites of Jupiter and Saturn:—Of the flux and reflux of the sea, as arising from the actions of the sun and moon:—Of the forces with which the sun disturbs the motions of the moon; of the various motions of the moon, of her orbit, variation, inclinations of her orbit, and the several motions of her nodes:—Of the tides, with the forces of the sun and moon to produce them:—Of the figure of the moon's body:—Of the precession of the equinoxes:—And of the motions and trajectory of comets. The great author then concludes with a general scholium, containing reflections on the principal parts of the great and beautiful system of the universe, and of the infinite, eternal Creator and Governor of it.

"The hypothesis of vortices," says he, "is pressed with many difficulties. That every planet by a radius drawn to the sun may describe areas proportional to the times of description, the periodic times of the several parts of the vortices should observe the duplicate proportion of their distances from the sun. But that the periodic times of the planets may obtain the sesquiduplicate proportion of their distances from the sun, the periodic times of the parts of the vortex ought to be in the sesquiduplicate proportion of their distances. That the smaller vortices may maintain their lesser revolutions about Saturn, Jupiter, and other planets, and swim quietly and undisturbed in the greater vortex of the sun, the periodic times of the parts of the sun's vortex should be equal. But the rotation of the sun and planets about their axes, which ought to correspond with the motions of their vortices, recede far from all these proportions. The motions of the comets are exceeding regular, are governed by the same laws with the motions of the planets, and can by no means be accounted for by the hypothesis of vortices. For comets are carried with very excentric motions through all parts of the heavens indifferently, with a freedom that is incompatible with the notion of a vortex.

"Bodies, projected in our air, suffer no resistance but from the air. Withdraw the air, as is done in Mr. Boyle's vacuum, and the resistance ceases. For in this void a bit of fine down and a piece of solid gold descend with equal velocity. And the parity of reason must take place in the celestial spaces above the earth's atmosphere; in which spaces, where there is no air to resist their motions, all bodies will move with the greatest freedom; and the planets and comets will constantly pursue their revolutions in orbits given in kind and position, according to the laws above explained. But though these bodies may indeed persevere in their orbits by the mere laws of gravity, yet they could by no means have at first derived the regular position of the orbits themselves from those laws.

"The six primary planets are revolved about the sun, in circles concentric with the sun, and with motions directed towards the same parts, and almost in the same plane. Ten moons are revolved about the earth, Jupiter and Saturn, in circles concentric with them, with the same direction of motion, and nearly in the planes of the orbits of those planets. But it is not to be conceived that mere mechanical causes could give birth to so many regular motions: since the comets range over all parts of the heavens, in very excentric orbits. For by that kind of motion they pass easily through the orbs of the planets, and with great rapidity; and in their aphelions, where they move the slowest, and are detained the longest, they recede to the greatest distances from each other, and thence suffer the least disturbance from their mutual attractions. This most beautiful system of the sun, planets, and comets, could only proceed from the counsel and dominion of an intelligent and powerful Being. And if the fixed stars are the centres of other like systems, these being formed by the like wise counsel, must be all subject to the dominion of one; especially, since the light of the fixed stars is of the same nature with the light of the sun, and from every system light passes into all the other systems. And lest the system of the fixed stars should, by their gravity, fall on each other mutually, he hath placed those systems at immense distances one from another."

Then, after a truly pious and philosophical descant on the attributes of the Being who could give existence and continuance to such prodigious mechanism, and with so much beautiful order and regularity, the great author proceeds; "Hitherto we have explained the phenomena of the heavens and of our sea, by the power of gravity, but have not yet assigned the cause of this power. This is certain, that it must proceed from a cause that penetrates to the very centres of the sun and planets, without suffering the least diminution of its force; that it operates, not according to the quantity of the surfaces of the particles upon which it acts (as mechanical causes do), but according to the quantity of the solid matter which they contain, and propagates its virtue on all sides, to immense distances, decreasing always in the duplicate proportion of the distances. Gravitation towards the sun, is made up out of the gravitations towards the several particles of which the body of the sun is composed; and in receding from the sun, decreases accurately in the duplicate proportion of the distances, as far as the orb of Saturn, as evidently appears from the quiescence of the aphelions of the planets; nay, and even to the remotest aphelions of the comets, if those aphelions are also quiescent. But hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses. For whatever is not deduced from the phenomena, is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy, particular propositions are inferred from the phenomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and of gravitation, were discovered. And to us it is enough, that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.

"And now we might add something concerning a certain most subtle spirit, which pervades and lies hid in all gross bodies, by the force and action of which spirit, the particles of bodies mutually attract one another at near distances, and cohere, if contiguous, and electric bodies operate to greater distances, as well repelling as attracting the neighbouring corpuscles; and light is emitted, reflected, refracted, inflected, and heats bodies; and all sensation is excited, and the members of animal bodies move at the command of the will, namely, by the vibrations of this spirit, mutually propagated along the solid filaments of the nerves, from the outward organs of sense to the brain, and from the brain into the muscles. But these are things that cannot be explained in few words, nor are we furnished with that sufficiency of experiments which is required to an accurate determination and demonstration of the laws by which this electric and elastic spirit operates."

NICHE, a cavity, or hollow part, in the thickness of a wall, to place a figure or statue in.

NICOLE (FRANCIS), a celebrated French mathematician, was born at Paris December the 23d, 1683. His early attachment to the mathematics induced M. Montmort to take the charge of his education; and he opened to him the way to the higher geometry. He first became publicly remarkable by detecting the fallacy of a pretended quadrature of the circle. This quadrature a M. Mathulon so assuredly thought he had discovered, that he deposited, in the hands of a public notary at Lyons, the sum of 3000 livres, to be paid to any person who, in the judgment of the Academy of Sciences, should demonstrate the falsity of his solution. M. Nicole, piqued at this challenge, undertook the task, and exposing the paralogism, the Academy's judgment was, that Nicole had plainly proved that the rectilineal figure which Mathulon had given as equal to the circle, was not only unequal to it, but that it was even greater than the polygon of 32 sides circumscribed about the circle.—The prize of 3000 livres, Nicole presented to the public hospital of Lyons.

The Academy named Nicole, Elève-Mechanician, March 12, 1707; Adjunct in 1716, Associate in 1718, and Pensioner in 1724; which he continued till his death, which happened the 18th of January 1758, at 75 years of age.

His works were all inserted in the different volumes of the Memoirs of the Academy of Sciences; and are as follow;

1. A General Method for determining the Nature of Curves formed by the Rolling of other Curves upon any Given Curve; in the volume for the year 1707.—2. A General Method for Rectifying all Roulets upon Right and Circular Bases; 1708.—3. General Method of determining the Nature of those Curves, which cut an Infinity of other Curves given in Position, cutting them always in a Constant Angle; 1715.—4. Solution of a Problem proposed by M. de Lagny; 1716.—5. Treatise of the Calculus of Finite Differences; 1717.—6. Second Part of the Calculus of Finite Differences; 1723.—7. Second Section of ditto; 1723.—8. Addition to the two foregoing papers; 1724.—9. New Proposition in Elementary Geometry; 1725.—10. New Solution of a Problem proposed to the English Mathematicians, by the late M. Leibnitz; 1725.—11. Method of Summing an Infinity of New Series, which are not summable by any other known method; 1727.—12. Treatise of the Lines of the Third Order, or the Curves of the Second Kind; 1729.—13. Examination and Resolution of some Questions relating to Play; 1730.—14. Method of determining the Chances at Play.—15. Observations on the Conic Sections; 1731.—16. Manner of generating, in a Solid Body, all the Lines of the Third Order; 1731.—17. Manner of determining the Nature of Roulets formed on the Convex Surface of a Sphere; and of determining which are Geometric, and which are Rectifiable; 1732.—18. Solution of a Problem in Geometry; 1732.—19. The Use of Series in resolving many Problems in the Inverse Method of Tangents; 1737.—20. Observations on the Irreducible Case in Cubic Equations; 1738.—21. Observations on Cubic Equations; 1738.—22. On the Trisection of an Angle; 1740.—23. On the Irreducible Case in Cubic Equations; 1741.—24. Addition to ditto; 1743.—25. His Last Paper on the same; 1744.—26. Determination, by Incommensurables and Decimals, the Values of the Sides and Areas of the Series in a Double Progression of Regular Polygons, inscribed in and circumscribed about a Circle; 1747.

NICOMEDES, an ancient mathematician, who flourished in the 2d century of the Christian æra, and was celebrated for his invention of the curve called the Conchoid.

NIEUWENTYT (BERNARD), an eminent Dutch philosopher and mathematician, was born on the 10th of August 1654, at Westgraafdyk in North Holland, where his father was minister. He discovered very early a good genius and a strong inclination for learning; which was carefully improved by a suitable education. He had also that prudence and sagacity, which led him to pursue literature by sure and proper steps, acquiring a kind of mastery in one science before he proceeded to another. His father had designed him for the ministry; but seeing his inclination did not lie that way, he prudently left him to pursue the bent of his genius. Accordingly young Nieuwentyt, apprehending that nothing was more useful than fixing his imagination and forming his judgment well, applied himself early to logic, and the art of reasoning justly, in which he grounded himself on the principles of Descartes, with whose philosophy he was greatly delighted. From thence he proceeded to the mathematics, in which he made a considerable proficiency; though the application he gave to that branch of learning did not prevent him from studying both law and physic. In fact he succeeded in all these sciences so well, as deservedly to acquire the character of a good philosopher, a great mathematician, an expert physician, and an able and just magistrate.

Though he was naturally of a grave and serious disposition, yet he was very affable and agreeable in conversation. His engaging manner procured the affection of every one; and by this means he often drew over to his opinion those who before differed very widely from him. Thus accomplished, he acquired a great esteem and credit in the council of the town of Puremerende, where he resided; as he did also in the states of that province, who respected him the more, inasmuch as he never engaged in any cabals or factions, in order to secure it; regarding in his conduct, an open, honest, upright behaviour, as the best source of satisfaction, and relying solely on his merit. In fact, he was more attentive to cultivate the sciences, than eager to obtain the honours of the government; contenting himself with being counsellor and burgomaster, without courting or accepting any other posts, which might interfere with his studies, and draw him too much out of his library.—Nieuwentyt died the 7th of March 1730, at 76 years of age, having been twice married.—He was author of several works, in the Latin, French, and Dutch languages, the principal of which are the following.

1. A Treatise in Dutch, proving the Existence of God by the Wonders of Nature; a much esteemed work, which went through many editions. It was translated also into several languages, as the French, and the English, under the title of, The Religious Philosopher, &c.

2. A Refutation of Spinosa, in the Dutch language.—3. Analysis Infinitorum; 1695, 4to.—4. Considerationes secundæ circa Calculi Differentialis Principia; 1696, 8vo.—In this work he attacked Leibnitz, and was answered by John Bernoulli and James Herman.—5. A Treatise on

the New Use of the Tables of Sines and Tangents.—6. A Letter to Bothnia or Burmania, on the Subject of Meteors.

NITROGEN, or NITROUS GAS, (the phlogisticated air of Priestley,) forms the unrespirable part of atmospheric air, and exists in it in the proportion of 78 per cent, estimated by bulk, or 74 per cent. by weight. See Aikin's Chemical Dictionary, article AZOTE.

NIGHT, that part of the natural day, during which the sun is below the horizon: though sometimes it is understood that the twilight is referred to the day, or the time the sun is above the horizon; the remainder only being the night. Under the equator, the nights, in the former sense, are always equal to the days; each being 12 hours long. But under the poles, the night continues half a year.—The ancient Gauls and Germans divided their time not by days, but nights; as appears from Cæsar and Tacitus; also the Arabs and the Icelanders do the same. The same may also be observed of our Saxon ancestors: whence our custom of saying, sevennight, fortnight, &c.

NOCTILUCA, a species of phosphorus, so called because it shines in the night, without any light being thrown on it: such is the phosphorus made of urine. By which it stands distinguished from some other species of phosphorus, which require to be exposed to the sun-beams before they will shine; as the Bononian-stone, &c. Mr. Boyle has a particular treatise on this subject.

NOCTURNAL *Arch*, is the arch of a circle described by the sun, or a star, in the night.

NOCTURNAL, or NOCTURLABIUM, denotes an instrument, chiefly used at sea, to take the altitude or depression of the pole-star, and some other stars about the pole, for finding the latitude, and the hour of the night.

There are several kinds of this instrument; some of which are projections of the sphere; such as the hemispheres, or planispheres, on the plane of the equinoctial. The seamen commonly use two kinds; the one adapted to the pole-star, and the first of the guards of the Little Bear; the other to the pole-star and the pointers of the Great Bear.

The nocturnal consists of two circular plates (fig. 15, pl. 17) applied over each other. The greater, which has a handle to hold the instrument, is about 2½ inches diameter, and is divided into 12 parts, answering to the 12 months; also each month subdivided into every 5th day; and in such manner, that the middle of the handle corresponds to that day of the year in which the star here respected has the same right ascension with the sun.

When the instrument is fitted for two stars, the handle is made moveable. The upper circle is divided into 24 equal parts, for the 24 hours of the day, and each hour subdivided into quarters, as in the figure. These 24 hours are denoted by 24 teeth; to be told in the night. In the centre of the two circular plates is adjusted a long index A, moveable on the upper plate. And the three pieces, viz, the two circles and index, are joined by a rivet which is pierced through the centre, with a hole 2 inches in diameter, for the star to be observed through.

To Use the NOCTURNAL. Turn the upper plate till the longest tooth, marked 12, be against the day of the month on the under plate; and bringing the instrument near the eye, suspend it by the handle, with the plane nearly parallel to the equinoctial; then viewing the pole-star through the hole in the centre, turn the index about till, by the edge coming from the centre, you see the bright star or guard of the Little Bear, if the instrument be fitted to that star: then that tooth of the upper circle, under the edge of the index, is at the hour of the night on the edge of the hour-circle, which may be known without a light, by counting the teeth from the longest, which is for the hour of 12.

NODATED *Hyperbola*, one, so called by Newton, which by turning round decussates or crosses itself: as in the 2d, and several other species, of his Enumeratio Linearum Tertii Ordinis.

NODES, the two opposite points where the orbit of a planet intersects the ecliptic. That, where the planet ascends from the south to the north side of the ecliptic, is called the ascending node, or the Dragon's head in the moon, and marked thus ☊: and the opposite point, where the planet descends from the north to the south side of the ecliptic, is called the descending node, or Dragon's tail in the moon, and is thus marked ☋. Also the right line drawn from the one node to the other, is called the line of the nodes.

By observation it appears that, in all the planets, the line of the nodes continually changes its place, its motion being in antecedentia; i. e. contrary to the order of the signs, or from east to west; with a peculiar degree of motion for each planet. Thus, by a retrograde motion, the line of the moon's nodes completes its circuit in 18 years and 225 days, in which time the node returns again to the same point of the ecliptic. Newton has not only shown, that this motion arises from the action of the sun, but, from its cause, he has with great skill calculated all the elements and varieties in this motion. See his Princip. lib. 3, prop. 30, 31, &c.—The moon must be in or near one of the nodes, to make an eclipse either of the sun or moon.

For a full treatise on the nodes of the planets, see Lalande's Astronomy, in many articles as shown by the index at the end of the 3d volume, and the result of the whole in vol. 2, pa. 124, where he gives a table of the nodes of the several planets, for the year 1750, and their annual variations, thus:

Planets' ascending Nodes in 1750.

Planets.	Node ☊ in 1750.				Annual decrease.
Mercury	1^{s}	15°	21′	15″	45′
Venus	2	14	26	18	31
Mars	1	17	36	30	39·8
Jupiter	3	8	16	0	60
Saturn	3	21	31	17	30
Herschel	2	11	49	30	

See also our article ORBIT.

NODUS, or *Node*, in Dialling, denotes a point or hole in the gnomon of a dial, by the shadow or light of which is shown, either the hour of the day in dials without furniture, or the parallels of the sun's declination, and his place in the ecliptic, &c, in dials with furniture.

NOLLET (*the Abbé* JOHN ANTHONY), a considerable French philosopher, and a member of most of the philosophical societies and academies of Europe, was born at Pimpré, in the district of Noyon, the 19th of November 1700. From the profound retreat, in which the mediocrity of his fortune obliged him to live, his reputation continually increased from day to day. M. Dufay associated him in his Electrical Researches; and M. de Reaumur resigned to him his laboratory. It was under these masters that he developed his talents. M. Dufay took him along with him in a journey he made into England; and Nollet profited so well of this opportunity, as to institute a friendly and literary correspondence with some of the most celebrated men in this country.

The king of Sardinia gave him an invitation to Turin, to perform a course of experimental philosophy to the duke of Savoy. From thence he travelled into Italy, where he collected some good observations concerning the natural history of the country.

In France he was master of philosophy and natural history to the royal family; and professor-royal of experimental philosophy to the college of Navarre, and to the schools of artillery and engineers. The Academy of Sciences appointed him adjunct-mechanician in 1739, associate in 1742, and pensioner in 1757. Nollet died the 24th of April 1770, regretted by all his friends, but especially by his relations, whom he always succoured with an affectionate attention. The works published by Nollet, are the following:

1. Recueils de Lettres sur l'Electricité; 1753, 3 vols in 12mo.—2. Essai sur l'Electricité des Corps; 1 vol. in 12mo.—3. Recherches sur les Causes particulieres des Phenomenes Electriques; 1 vol. in 12mo.—4. L'Art des Experiences; 1770, 3 vols in 12mo.

His papers printed in the different volumes of the Memoirs of the Academy of Sciences, are much too numerous to be particularized here; they are inserted in all or most of the volumes from the year 1740 to the year 1767 inclusive, and generally several papers in each volume.

NONAGESIMAL, or NONAGESIMAL *Degree*, called also the mid-heaven, is the highest point, or 90th degree of the ecliptic, reckoned from its intersection with the horizon at any time; and its altitude is equal to the angle that the ecliptic makes with the horizon at their intersection, or equal to the distance of the zenith from the pole of the ecliptic. It is much used in the calculation of solar eclipses.

NONAGON, a figure having nine sides and angles.—In a regular nonagon, or that whose angles, and sides, are all equal, if each side be 1, its area will be 6·1818242 $= \frac{9}{4}$ of the tangent of 70°, to the radius 1. See my Mensuration, pa. 85, 4th edit.

NONES, in the Roman Calendar, the 5th day of the months January, February, April, June, August, September, November, and December; and the 7th of the other months March, May, July, and October: these last four months having 6 days before the nones, and the others only four.—They had this name probably, because they were always 9 days inclusively, from the first of the nones to the ides, i. e. reckoning inclusively both those days.

NONIUS, or NUNEZ (PETER), an eminent Portuguese mathematician and physician, was born in 1497, at Alcazar in Portugal, anciently a remarkable city, known by the name of Salacia, whence he was surnamed Salaciensis. He was professor of mathematics in the university of Coimbra, where he published some pieces which procured him great reputation. He was mathematical preceptor to Don Henry, son to king Emanuel of Portugal, and principal cosmographer to the king. Nonius was very serviceable to the designs which this court entertained, of carrying on their maritime expeditions into the East, by the publication of his book On the Art of Navigation, and various other works. He died in 1577, at 80 years of age.

Nonius was the author of several ingenious works and inventions, and was justly esteemed one of the most eminent mathematicians of his age. Concerning his Art of Navigation, father Dechales says, "In the year 1530, Peter Nonius, a celebrated Portuguese mathematician, on occasion of some doubts proposed to him by Martinus Alphonsus Sofa, wrote a Treatise on Navigation, divided into two books; in the first, he answers some of those doubts, and explains the nature of loxodromic lines. In the second book, he treats of rules and instruments proper for navigation, particularly sea-charts, and instruments serving to find the elevation of the pole; but says he is rather obscure in his manner of writing."—Furetiere, in his Dictionary, takes notice that Peter Nonius was the first who, in 1530, invented the angles which the loxodromic curves make with each meridian, calling them in his language Rhumbs, and which he calculated by spherical triangles. Stevinus acknowledges, that Peter Nonius was scarce inferior to the very best mathematicians of the age. And Schottus says, he explained a great many problems, and particularly the mechanical problem of Aristotle on the motion of vessels by oars. His Notes upon Purbach's Theory of the Planets, are very much to be esteemed: he there explains several things, which had either not been noticed before, or not rightly understood.

In 1542, he published a Treatise on the Twilight, which he dedicated to John the 3d, king of Portugal; to which he added what Alhazen, an Arabian author, has composed on the same subject. In this work he describes the method or instrument called, from him, a Nonius; a particular account of which see in the following article.—He corrected several mathematical mistakes of Orontius Finæus.—But the most celebrated of all his works, or that at least he appeared most to value, was his Treatise of Algebra, which he had composed in Portuguese, but translated it into the Castilian tongue, when he resolved to make it public, which he thought would render his book more useful, as this language was more generally known than the Portuguese. The dedication, to his former pupil, prince Henry, was dated from Lisbon, Dec. 1, 1564. This work contains 341 leaves, or 682 pages, in the Antwerp edition of 1567, in 8vo; the folios being numbered only on one side.

The catalogue of his works, chiefly in Latin, is this:

1. De Arte Navigandi, libri duo; 1530.—2. De Crepusculis; 1542.—3. Annotationes in Aristotelem.—4. Problema Mechanicum de Motu Navigii ex Remis.—5. Annotationes in Planetarum Theorias Georgii Purbachii, &c.—6. Libro de Algebra en Arithmetica y Geometra; 1564.

All these pieces, the Algebra excepted, were collected and published, in a folio volume, at Basil, in 1566.

NONIUS, is a name also erroneously given to the method of graduation now generally used in the division of the scales of various instruments, and which should be called Vernier, from its real inventor. The method of Nonius, so called from its inventor Pedro Nunez, or Nonius, and described in his treatise De Crepusculis, printed at Lisbon in 1542, consists in describing within the same quadrant, 45 concentric circles, dividing the outermost into 90 equal parts, the next within into 89, the next into 88, and so on, till the innermost was divided into 46 only. By this means, in most observations, the plumb-line or index must cross one or other of those circles in or very near a point of division: whence, by calculation, the degrees and minutes of the arch might easily be obtained. This method is also described by Nunez, in his treatise De Arte et Ratione Navigandi, lib. 2. cap. 6, where he imagines it was not unknown to Ptolemy. But as the degrees are thus divided unequally, and it is very difficult to attain exactness in the division, especially when the numbers, into which the arches are to be divided, are incomposite, of which there are no less than nine, the method of diagonals first published by Thomas Digges, Esq. in his treatise Alæ seu Scalæ Mathematicæ, printed at Lond. in 1573, and said to be invented by one Richard Chanseler, a very skilful artist, was substituted in its stead. However, Nonius's method was improved at different times; but the commodious division now so much in use, is the most considerable improvement of it. See VERNIER; also Robertson's Navigat. Pref. p. iv; or Robins's Tracts, v. 2, p. 265, for a curious history of many other such contrivances.

NORMAL, is used sometimes for a perpendicular.

NORTH *Star*, called also the Pole-star, is the last in the tail of the Little Bear.

NORTHERN *Signs*, are those six that are on the north side of the equator; viz. Aries, Taurus, Gemini, Cancer, Leo, Virgo.

NORTHING, in Navigation, is the difference of latitude, which a ship makes in sailing northwards.

NORWOOD (RICHARD), a respectable teacher of mathematics, in London, especially navigation, in which it seems he had some practice. He published several useful books; as, 1. The Epitome and Doctrine of Triangles, 1673, in 8vo; 2. Trigonometry, 1685, in 4to; 3. The Seaman's Practice, 1697, in 4to; where we find that for which he has been chiefly noted, viz. his determination of the magnitude of the earth, and the degrees of the meridian, by means of the distance measured between London and York, in the year 1635. This measurement at so early a date, was ingeniously devised, and simply executed, reflecting on the author considerable credit, whose means and convenience for the performance were small and humble. The deviation from an accurate result is however not so considerable as might be expected from the rude manner of his measuring with a chain, along the high road in all directions, to the right and left, as well as up and down hills, and sometimes only by pacing or stepping the distances. It seems however he did not make a sufficient allowance for those zigzag directions and estimations, as his conclusion gives the mean length of a degree of latitude too great by almost half a mile, viz, $69\frac{8}{15}$ miles to a degree, instead of $69\frac{1}{15}$, as deduced from later and more accurate measurements.

Mr. N. had also an ingenious paper on the Tides, on Wells, on Salt and Fresh Water, and on Whale-fishing, inserted in the Philos. Trans. an. 1667, or in my Abridgement, vol. 1, pa. 206.

NOSTRADAMUS (MICHEL), an able physician and celebrated astrologer, was born at St. Remy in Provence, in the diocese of Avignon, December 14, 1503. His father was a notary-public, and his grandfather a physician, from whom he received some tincture of the mathematics. He afterwards completed his courses of languages and philosophy at Avignon. Hence, going to Montpelier, he applied himself to physic; but being forced away by the plague, he travelled through different places till he came to Bourdeaux, undertaking all such patients as were willing to put themselves under his care. This course occupied him five years; after which he returned to Montpelier, and was created doctor of his faculty in 1529; after which he revisited the same places where he had practised physic before. At Agen he formed an acquaintance with Julius Cæsar Scaliger; but quitted it after a residence of about 4 years. He next settled at Marseilles, but repaired to Salon about the year 1544.

In 1546, Aix being afflicted with the plague, he went thither at the solicitation of the inhabitants, to whom he rendered great service, particularly by a powder of his own invention: so that the town, in gratitude, gave him a considerable pension for several years after the contagion ceased. In 1547 the city of Lyons, being visited with the same distemper, had recourse to our physician, who attended them also. Afterwards returning to Salon, he began a more retired course of life, and in this time of leisure applied himself closely to his studies. He had for a long time followed the trade of a conjurer occasionally; and now he began to fancy himself inspired, and miraculously illuminated with a prospect into futurity. As fast as these illuminations had discovered to him any future event, he entered it in writing, in simple prose, though in enigmatical sentences: but revising them afterwards, he thought the sentences would appear more respectable, and savour more of a prophetic spirit, if they were expressed in verse. This opinion determined him to throw them all into quatrains, and he afterward ranged them into centuries. For some time he could not venture to publish a work of this nature; but afterwards perceiving that the time of many events foretold in his quatrains was very near at hand, he resolved to print them, as he did, with a dedication addressed to his son Cæsar, an infant only some months old, and dated March 1, 1555. To this first edition, which comprised but 7 centuries, he prefixed his name in Latin, but gave to his son Cæsar the name as it is pronounced in French, Notradame.

The public were divided in their sentiments of this work: many considered the author as a simple visionary; by others he was accused of magic or the black art, and treated as an impious person, who held a commerce with the devil; while great numbers believed him to be really endued with the supernatural gift of prophecy. However, Henry the 2d, and queen Catharine of Medicis, his mother, were resolved to see our prophet, who receiving orders to that effect, he presently repaired to Paris; where he was very graciously received at court, and received a present of 200 crowns. He was sent afterwards to Blois, to visit the king's children there, and report what he should be able to discover concerning their destinies. It is not known what his prediction was; however he returned to Salon loaded with honour, and good presents.

Animated with this success, he augmented his work to the number of 1000 quatrains, and published it with a dedication to the king in 1558. That prince dying the next year of a wound which he received at a tournament, our prophet's book was immediately consulted; and this unfortunate event was found in the 35th quatrain of the first century, which runs thus in the London edition of 1672:

> Le Lion jeune le vieux surmontera,
> En champ bellique, par singulier duelle,
> Dans cage d'or l'œil il lui crevera,
> Deux playes une, puis mourir mort cruelle.

In English thus, from the same edition:

The young Lion shall overcome the old one,
In martial field by a single duel,
In a golden cage he shall put out his eye,
Two wounds from one, then he shall die a cruel death.

So remarkable a prediction added new wings to his fame; and he was honoured soon after with a visit from Emanuel duke of Savoy, and the princess Margaret of France, his consort. From this time Nostradamus found himself even overburthened with visitors, and his fame made every day new acquisitions. Charles the 9th, coming to Salon, was eager above all things to have a sight of him: Nostradamus, who then was in waiting as one of the retinue of the magistrates, being instantly presented to the king, complained of the little esteem his countrymen had for him; upon which the monarch publicly declared that he should hold the enemies of Nostradamus to be his enemies, and desired to see his children. Nor did that prince's favour stop here; in passing, not long after, through the city of Arles, he sent for Nostradamus, and presented him with a purse of 200 crowns, together with a brevet, constituting him his physician in ordinary, with the same appointment as the rest. But our prophet enjoyed these honours only a short time, as he died 16 months after, viz, July 2, 1566, at Salon, being then in his grand climacteric, or 63d year.—He had published several other pieces, chiefly relating to medicine.

He left three sons and three daughters. Cæsar the eldest son was born at Salon in 1555, and died in 1629: he left a manuscript, giving an account of the most remarkable events in the history of Provence, from 1080 to 1494, in which he inserted the lives of the poets of that country. These memoirs falling into the hands of his nephew Cæsar Nostradamus, gentleman to the duke of Guise, he undertook to complete the work, and being encouraged by the estates of the country, he carried the account up to the Celtic Gauls: the impression was finished at Lyons in 1614, and published under the title of Chronique de l'Histoire de Provence.—The second son, John, exercised with reputation the business of a proctor in the parliament of Provence.—He wrote the Lives of the Ancient Provençal Poets, called Troubadours, and the work was printed at Lyons in 1575, 8vo.—The youngest son it is said undertook the trade of peeping into futurity like his father.

NOTATION, is the representing of any given number by means of certain significant characters, or numerical symbols; and thus stands in contradistinction to Numeration, which is the wording or expressing in words any number represented by those symbols.

It is highly probable, that in the early stages of society, every distinct number had a peculiar characteristic representative, which must however have led to great difficulty and embarrassment, on account of the number of different characters with which the memory must have been incumbered; at the same time it must also have been very limited in its application. Therefore, as soon as the state of society required the use of great numbers, which must have immediately followed the introduction of commerce, it became necessary to have a more concise notation; and the most proper method of accomplishing this, was that of giving to each symbol a local as well as a simple value. This however was a refinement that could hardly be expected in the first rude efforts of the human mind; and probably there are now no traces left of the first attempt of this kind.

We know now of only three different modes of notation, namely; the Roman, the Grecian, and the Indian; the latter of which is the only one at present in use, at least in arithmetical calculations: but each of these agree in one material point, which is that of dividing all numbers into periods of tens, a custom almost universally adopted by all nations; and as this is not the best number that might have been employed for forming the radix of a system of arithmetic, we must look to some general physical cause, for this singular coincidence of different people, many of whom had probably no communication with each other.

Mathematicians, so far back as the time of Aristotle, have noticed this singularity; and have endeavoured to account for it from different principles: it had, however, no doubt, its origin in the natural formation of man. Every one in the infancy of his reason makes the first efforts of calculation on his fingers, which being ten in number, evidently led to the separation of quantities of all kinds into periods of tens. For after having counted to this number, they were under the necessity of beginning again, and committing to their memory that they had already counted one period of ten: having then completed a second, third, &c, period, they still continued to count in the same manner, and still employed their fingers, as the proper instruments for assisting the memory in retaining the number of those periods, as well as for still pursuing their calculations: this therefore necessarily led to the second principal separation of number into hundreds; and so on for thousands, tens of thousands, &c.

Hence it appears, that the idea of our present scale of notation had, in reality, its foundation in the structure of the human frame: but to what nation we are indebted for the method of expressing numbers by means of ten simple characters, by giving to each a local, as well as a primitive value, is unknown: it is however pretty evident that it is only an improvement on the first rude attempts at numbering, above-mentioned.

The honour of this invention has been ascribed to different nations; some have attributed it to the Greeks, others to the Arabs, the Chaldeans, Indians, &c.

The first traces of it however, that have been discovered, are among the Arabs, who themselves attribute it to the Indians; but whether it had its origin with those people, or they derived it from any other nation, is a very doubtful question, which will perhaps ever remain undecided.

Montucla, in his Histoire des Mathematiques, book 2, vol. 1, has entered minutely into the subject, and has shown in the most unequivocal manner, not only that the Indians were in possession of this art before it was known to the Arabs, but also that the characters employed by them 2000 years back, did not very materially differ from those in present use; and it is to this work we are indebted for the specimen of the ancient and modern characters that we have given in plate 23, the last line of which is Al-Sephadi's expression for the number 18446744073709551615; and each of the other lines stands opposite the name of the author, or the nation, by whom they have been employed. From these specimens it will be readily perceived how our modern symbols have been derived, with some slight modifications, from those of the most ancient date.

It has been before observed, that almost all nations have adopted, as it were by common consent, the decimal scale of notation: this is not however without some exceptions. The ancient Chinese are said to have employed the binary scale (see **Binary**), and a nation of Thrace, mentioned by Aristotle, used the quaternary scale, counting by periods of fours; and another people bordering on Senegal, make all their calculations by periods of fives, which they designate as follows; one, two, three, four, five, they call *ben, niard, niet, guyanet, guiron;* and six, seven, eight, nine, by *guiron ben, guiron niard, guiron niet,* &c. and ten by *fouque,* and probably eleven by *fouque ben.*

But these exceptions to the general mode of notation are very inconsiderable, and none of those scales that we have mentioned are by any means so well adapted to arithmetical purposes as our own; but this it must be acknowledged is inferior in many respects to the duodenary scale; which, by the addition of two extra characters, would perform all arithmetical operations with greater ease and expedition; and with respect to decimals, or duodecimals, as they would be in that case, we should have a great many more finite expressions than we have at present. In the decimal scale, if we consider only the reciprocals of all numbers under 20, we find only the six following that give finite decimals:
$\frac{1}{2} = \cdot 5$; $\frac{1}{4} = \cdot 25$; $\frac{1}{5} = \cdot 2$; $\frac{1}{8} = \cdot 125$; $\frac{1}{10} = \cdot 1$; $\frac{1}{16} = \cdot 0625$; but in the duodenary scale, we have nine finite expressions with the same numbers, which are as follows;

$\frac{1}{2} = \cdot 6$; $\frac{1}{3} = \cdot 4$; $\frac{1}{4} = \cdot 3$; $\frac{1}{6} = \cdot 2$; $\frac{1}{8} = \cdot 16$;
$\frac{1}{9} = \cdot 14$; $\frac{1}{12} = \cdot 1$; $\frac{1}{16} = \cdot 09$; $\frac{1}{18} = \cdot 08$.

Hence it is evident that, with this scale of notation, we should have more finite fractional numbers than in our common arithmetic; and beside this convenience, all operations would be more readily performed, and larger numbers would be expressed with fewer digits. Still however the advantages of this system are not such as can lead us to expect, or even to wish, that it should ever be substituted for that, which long established practice has rendered so familiar to all our ideas of numbers.

Notation *of the Greeks.* The Grecian notation, though it approached in many respects very near to that of the moderns, still it wanted one principal and distinguishing feature of the present improved system, which is that of giving to every character, a local as well as a simple value, for want of which they were under the necessity of employing a great number of characters, which were chiefly derived from the letters of their alphabet.

Instead of the figures	1,	2,	3,	4,	5,	6,	7,	8,	9,
The Greeks made use of these letters -	α,	β,	γ,	δ,	ε,	ζ,	ϛ,	η,	θ.
And instead of -	10,	20,	30,	40,	50,	60,	70,	80,	90,
They employed the characters -	ι,	κ,	λ,	μ,	ν,	ξ,	ο,	π,	ϟ.
For expressing the hundreds they had	ρ,	σ,	τ,	υ,	φ,	χ,	ψ,	ω,	ϡ,
And for the thousands they employed -	͵α,	͵β,	͵γ,	͵δ,	͵ε,	͵ζ,	͵ϛ,	͵η,	͵θ,

That is, they had recourse to the characters of the simple units, but instead of giving to them a local value, as we do, and in which consists the superiority of the modern method, they distinguished them by means of a small dash placed at the bottom of the letters. And hence we see that the Greeks could express with these characters any number under 10,000; thus,

9999 they represented by ͵θϡϟθ
7382 - - by ͵ϛτπβ
8036 - - by ͵ηλζ
6420 - - by ͵ζυκ
4001 - - by ͵δα
3000 - - by ͵γ

10000 was sometimes expressed by placing a small dash over the iota; thus, ί; but mathematicians, in their calculations, generally employed for this purpose the compound character $\overset{\alpha}{\text{M}}$; and any number of times this period, by placing the letter M under the characters expressing the number of periods that they wished to indicate; thus,

10000 was represented by $\overset{\alpha}{\text{M}}$
20000 - - by $\overset{\beta}{\text{M}}$
30000 - - by $\overset{\gamma}{\text{M}}$
100000 - - by $\overset{\iota}{\text{M}}$
800000 - - by $\overset{\pi}{\text{M}}$
972000 - - by $\overset{\theta o \beta}{\text{M}}$

So that placing the letter M under any given number had the same effect as our annexing four ciphers.

Diophantus and Pappus deviated a little in this respect from their predecessors, by making Μυ the characteristic of 10000, and then distinguishing any number of those periods by prefixing the number to this character; thus, the foregoing numbers, according to these authors, were represented by αΜυ, βΜυ, γΜυ, ιΜυ, πΜυ, θοβΜυ, &c.

And where smaller numbers were mixed with those larger periods, they were annexed to the foregoing characters; hence,

1719999 was written ϱοαΜυ͵θϡϟθ
43728097 - ͵δτοβΜυ͵ηϟζ

Thus resembling what we make use of at this day for expressing compound numbers, as 7ft 6in 7pts.

The same authors also sometimes employed a still more simple method, which was by omitting the character Μυ, and only separating the two sets of symbols by a point.

Thus 99999999 was written ͵θϡϟθ.͵θϡϟθ, which was the largest number that the Greeks could express; and therefore, when they wanted to make use of larger numbers than this, they were under the necessity of assuming a larger unit; several examples of which are to be met with in the ancient Greek authors.

Apollonius at length conceived the idea of dividing all numbers into periods of four characters, the first of which represented units, the second the number of 10000, the third the square of 10000's, &c. This was a great step towards our present system, for here was evidently a local value given to the different periods, and the same only wanted to have been carried downwards to the units, to have completed the discovery.

In this manner the circumference of a circle whose diameter is 1, according to the notation of Apollonius, would be expressed by

γ . ͵αυιε . θσξε . ͵γρπθ . ͵ζϡλβ . ͵γωμς . ͵βχμγ.

3 . 1415 . 9265 . 3589 . 7932 . 3846 . 2643.

From what has been already observed, it appears, that the Greek notation resembled what we now employ for compound numbers, and in short their whole arithmetic differed from ours chiefly in this, that for want of giving a local value to their characters, all their operations were performed much in the same manner as we now perform ours in duodecimals, and compound multiplication, division, &c. None of the Greek writers whose works have come down to us, have attempted to teach the first fundamental rules of their arithmetic; we can therefore only judge from the disposition of their characters, the exact method of operation that they followed. It seems however probable, that they generally worked from the left hand towards the right; but in their additions and subtractions this was so manifestly disadvantageous and troublesome, compared with what it would have been to have performed the same operations in a contrary order, that one can hardly suppose they could have overlooked such an evident and advantageous proceeding.

As to their multiplications, it was not of so much importance, and there seems no doubt that in this rule, their operations were performed from left to right, as we do ours in algebra. In their divisions they approached nearer to our method for compound division, except that they generally found the whole quotient at one step, which must therefore have been the result of several tedious trials, or by means of a table for that purpose; that is, they found the greatest quotient in the first period, and then, having subtracted, they found the second period of the quotient, &c. The square root was extracted in a manner also much resembling ours, differing from it, only in finding each period of the root at one step, as they did the quotients in division.

It would be inconsistent with the nature of our work to enter upon this subject at any considerable length, we shall therefore confine ourselves to exhibiting a few examples, with the corresponding operations in our arithmetic, referring the curious reader for farther information on this head, to an ingenious and learned Essay, by Delambre, added to the French translation of the works of Archimedes, where he will find ample gratification.

Example in Addition.

Greek	ωμζ . ͵γϡκα	Modern	847 3921
	ξ . ͵ηυ		60 8400
	ϡη . ͵βτκα		908 2321

Example in Subtraction.

Greek	θ . ͵γχλς	Modern	9 3636
	β . ͵γυ θ		2 3409
	ζ . σκζ		7 0227

Example in Multiplication.

ρνγ	153
ϱνγ	153
͵α.ετ	15300 = 100 . 153
͵ε͵βφϱν	7650 = 50 . 153
τϱνθ	459 = 3 . 153
β.͵γυθ	23409

The above examples will convey a slight idea of the notation of the Greeks, and their method of performing the fundamental rules of their arithmetic, which is evidently, in every respect, very much inferior to that of the moderns.

Roman NOTATION. The Romans also employed some of the letters of their alphabet for designating different numbers, which, though sufficiently commodious in point of representation, was by no means adapted for arithmetical calculations. The simple characters were as follow:

1	was represented by	I
5	- - by	V
10	- - by	X
50	- - by	L
100	- - by	C
500	- - by	D
1000	- - by	M

And by means of these characters, and the various combination of them, they expressed any number whatever. These are still in use for representing dates, numbering of chapters, pages, &c.

NOTES, in Music, are characters which mark the tones, i. e. the elevations and fallings of the voice, or sound, and the swiftness or slowness of its motions, &c; and these have undergone various alterations and improvements, before they arrived at their present state of perfection.

NOVEMBER, the eleventh month in the Julian year, but the ninth in the year of Romulus, beginning with March; whence its name. In this month, which contains 30 days, the sun enters the sign ♐, viz, usually about the 21st day of the month.

NUCLEUS, the kernel, is used by Hevelius, and some other astronomers, for the body of a comet, which others call its head, as distinguished from its tail, or beard.

Nucleus, is also used by some writers for the central parts of the earth, and other planets, which they suppose firmer, and as it were separated from them, considered as a cortex or shell.

NUEL, the same as Newel of a Staircase.

NUMBER, a collection or assemblage of several units, or several things of the same kind; as 2, 3, 4, &c, exclusive of the number 1: which is Euclid's definition of number.—Stevinus defines number as that by which the quantity of any thing is expressed: agreeably to which Newton conceives a number to consist, not in a multitude of units, as Euclid defines it, but in the abstract ratio of a quantity of any kind to another quantity of the same kind, which is accounted as unity: and in this sense, including all these three species of number, viz, Integers, Fractions, and Surds.

Wolfius defines number to be something which refers to unity, as one right line refers to another. Thus, assuming a right line for unity, a number may likewise be expressed by a right line. And in this way also Descartes considers numbers as expressed by lines, where he treats of the arithmetical operations as performed by lines, in the beginning of his Geometry.

Mathematicians divide number into different classes; as,

Numbers, *Absolute, Abstract, Abundant, Amicable, Applicate, Circular, Concrete, Composite, Cubic, Defective, Fractional, Figurate, Polygonal, Perfect, Prime, Pyramidal, Rational, Similar, Square,* &c, for which see the respective adjectives.

Beside these divisions, which form the principal heads under which numbers are considered, they are also divided into *even* and *odd*, and formerly they were distinguished into *evenly-even, evenly odd,* &c: but these denominations are now disused, and the same is expressed by saying, numbers of the form $4n$, $4n + 1$, $4n - 1$, &c; by which is to be understood that, in the first place, the number is exactly divisible by 4; in the second, the number being divided by 4 it leaves a remainder 1; and in the third place, when divided by 4, it leaves a remainder 3 or -1; and the same is implied when numbers are said to be of any other form as $7n + 1$, $7n + 3$, $11n + 2$, &c: this is a much more simple and general method of classing numbers, and is that which is now commonly employed.

We find from the different fragments that have been transmitted to us, some of which are found in the Elements of Euclid, that the ancient mathematicians had made some considerable progress in the investigation of the properties of numbers; but they wanted two powerful instruments, in order to fathom this subject, of which the moderns have availed themselves; these are the present mode of notation, which expresses numbers with so much facility, and the science of algebra, which generalizes the results, and with which we can operate with the same ease on known and unknown quantities. These inventions could not but have a powerful influence in promoting the progress of the science of numbers; and accordingly, we find the work of Diophantus, the most ancient author on algebra that we know of, is entirely dedicated to the properties of numbers, and contains many difficult questions which required considerable address and sagacity to resolve.

From Diophantus, to the time of Vieta and Bachet, mathematicians continued to occupy themselves with the subject of numbers, but without much success; at length Vieta, by adding a new degree of excellence to algebra, resolved many difficult problems relating to numbers. Bachet, in his work entitled Problêmes Plaisans et Délectables, gave a solution to all indeterminate equations of the first degree, by a method as ingenious, as it was general in its application. To the same ingenious author we are indebted for an excellent commentary on Diophantus, which was afterwards enriched by the marginal notes of Fermat, who was one of those that most contributed to bring this science to perfection, by the great variety of elegant theorems that he proposed, though he left many of them without demonstrations; they, however, had the effect of calling into action the talents of many eminent mathematicians. It was the custom at that time to propose questions by way of challenge to each other, the solutions to which were accordingly concealed, in order to secure to themselves, or to their nation, the honour of solving them; this was at least the case with the English and French mathematicians, between whom there was much rivalry at that time.

We are however inclined to think, that many of the theorems of Fermat were only the result of observation and trials, and that he himself never arrived at their demonstrations; though he expressly says, in one of his notes on Diophantus, pa. 180, that he was engaged in writing a work on this subject, which would contain multa varia et abstrusissima numerorum mysteria: and it has long been regretted by mathematicians that this work never appeared. Some celebrated foreign authors seem to attribute the circumstance to the ignorance of the persons into whose hands Fermat's papers were consigned at his death; but we are rather inclined to ascribe it to a different cause; we suppose that at the time the note was written, Fermat was really engaged in such a work, and expected to be able to complete the undertaking; but probably failing in some of his most celebrated theorems, he suppressed the work entirely: and this idea receives considerable strength from the circumstance of Euler having shown, that one at least of his theorems, though true in a great many cases, is not generally so. Fermat had said, that $2^x + 1$ is always a prime, if x be taken any number in the series 2, 4, 8, 16, &c; but Euler found that $2^{32} + 1 = 641 \times 6700417$, and therefore is not a prime number. It should however be observed, that Fermat had made no mention of his having demonstrated this theorem.

But of all those mathematicians who have treated on the science of numbers, Euler claims the most distinguished situation: we are also much indebted to the labours and ingenuity of Lagrange, Legendre, and Gauss. The two latter have published works expressly on this subject, entirely independent of each other's method, both of which possess a very great degree of merit; but the latter has the greatest claim to originality. The former work is in French, entitled Essai sur la Theorie des Nombres, par Legendre; the first edition of which was published in 4to, at Paris, in the year , and a second edition, with considerable improvements, in 1808. The latter work is in Latin, under the title of Disquisitiones Arithmeticæ, and it has since been translated into French.

It is impossible for us, in the space to which we must confined this article, to enter at length on the subject of numbers; we shall therefore confine ourselves to the enumeration of some of the most curious and important properties.

Properties of Numbers.

1. Every even number is of the form $2n$, and every odd number of the form $2n \pm 1$.

2. Every prime number, greater than 3, is contained in one of the formulæ $6n + 1$, or $6n - 1$; and every prime number greater than 2, in one of the forms $4n + 1$, or $4n - 1$.

3. Every even square number is of the form $4n$, and every odd square number of the form $8n + 1$.

4. The following table exhibits the forms of all square numbers, with regard to every modulus from 1 to 12.

Moduli.	Formulæ.			
2	$2n$	$2n + 1$		
3	$3n$	$3n + 1$		
4	$4n$	$4n + 1$		
5	$5n$	$5n \pm 1$		
6	$6n$	$6n + 1$	$6n + 4$	
7	$7n$	$7n + 1$	$7n + 2$	
8	$8n$	$8n + 1$	$8n + 4$	
9	$9n$	$9n + 1$	$9n + 4$	$9n + 7$
10	$10n$	$10n \pm 1$	$10n \pm 4$	$10n + 5$
11	$\left\{\begin{array}{l}11n\\11n + 3\end{array}\right.$	$11n + 1$	$11n + 4$	$11n + 9$
12	$12n$	$12n + 1$	$12n \pm 3$	$12n + 4$

And consequently every number that does not fall under some one of the above forms is not a square.

5. The following tables exhibit all the impossible forms of square numbers, as referred to the moduli 3, 4, and 5; that is, no number that falls under any of the forms in the table can be a square number.

Modulus 3.	Modulus 4.	Modulus 5.
$2t^2 \pm 3qu^2$	$2t^2 \backsim 4qu^2$	$2t^2 \backsim 5qu^2$
$3t^2 \pm 3qu^2$	$3t^2 \pm 4qu^2$	$3t^2 \backsim 5qu^2$
$5t^2 \pm 3qu^2$	$6t^2 \backsim 4qu^2$	$7t^2 \backsim 5qu^2$
$8t^2 \pm 3qu^2$	$10t^2 \pm 4qu^2$	$8t^2 \backsim 5qu^2$
$11t^2 \pm 3qu^2$	$11t^2 \backsim 4qu^2$	$12t^2 \backsim 5qu^2$
$14t^2 \pm 3qu^2$	$14t^2 \pm 4qu^2$	$13t^2 \backsim 5qu^2$
General Forms.	General Forms.	General Form.
$(3p+2)\,t^2 \pm 3qu^2$ and $3pt^2 + 3qu^2$	$(4p \pm 2)\,t^2 \backsim 4qu^2$ and $(4p+3)\,t^2 \pm 4qu^2$	$(5p \pm 2)\,t^2 \backsim 5qu^2$

Where it is only necessary to observe, that q and its respective moduli must be prime to each other.

6. The powers of all numbers from the 2d to the 12th (the 7th excepted), are of the following specified forms.

a^2	is one of the forms	$3n$ or	$3n + 1$
a^3	- -	$7n$ or	$7n \pm 1$
a^4	- -	$5n$ or	$5n + 1$
a^5	- -	$11n$ or	$11n \pm 1$
a^6	- -	$7n$ or	$7n + 1$
a^8	- -	$17n$ or	$17n \pm 1$
a^9	- -	$19n$ or	$19n \pm 1$
a^{10}	- -	$11n$ or	$11n + 1$
a^{11}	- -	$23n$ or	$23n \pm 1$
a^{12}	- -	$13n$ or	$13n + 1$

The 7th power is not reducible to a similar form, because neither $7 + 1$ nor $2 \times 7 + 1$ is a prime number.

7. Every prime number $8n + 1$, $8n + 5$, is, exclusively of all others, of the form $x^2 + y^2$; or, which is the same thing, every prime number of the form $4n + 1$ is the sum of two squares.

8. Every prime number $8n + 1$, $8n + 3$, is, exclusively of all others, of the form $x^2 + 2y^2$.

9. Every prime number $8n + 1$, $8n + 7$, is, exclusively of all others, of the form $x^2 - 2y^2$.

10. Every prime number $8n - 1$ is of the form $p^2 + q^2 + 2r^2$.

11. Every number of the form $24n + 5$ is the sum of five, or a less number of squares, whose roots are of the form $6n - 1$.

12. Every number of the form $8n + 6$, is the sum of six, or a less number of squares, whose roots are of the form $4m - 1$.

13. Every odd number, except those of the form $8n + 7$, is the sum of three squares. And no numbers of this form can be the sum of three squares.

14. Every odd number, without exception, is of the form $p^2 + q^2 + 2r^2$. It should be observed that in this form, as also in all others we have mentioned, any one or more of those squares may become zero.

15. Every number of the form $p^2 + q^2 + r^2$, when multiplied by 2, gives a number of the form $p^2 + q^2 + 2r^2$; and the latter form being multiplied again by 2, reproduces the former.

16. If a number be the sum of two squares, its double is the sum of two squares.

17. A number that is the sum of two squares, being multiplied by a number of the same form, gives a product that is the sum of two squares: that is, $(p^2 + q^2) \times (r^2 + s^2) = (x^2 + y^2)$.

18. The product of the sum of four squares, by the sum of four squares, is itself the sum of four squares; or,

$$(p^2 + q^2 + r^2 + s^2) \times (\dot{p}^2 + \dot{q}^2 + \dot{r}^2 + \dot{s}^2) = (w^2 + x^2 + y^2 + z^2).$$

19. Every number is a triangular number, or the sum of two or three triangular numbers. A square, or the sum of two, three, or four squares. A pentagonal, or the sum of two, three, four, or five pentagonals, &c.—This is one of the celebrated theorems of Fermat; but it has never been demonstrated, except for the two first cases. The other part of it still remains, to exercise the ingenuity of mathematicians.

20. Every number is a cube, or the sum of 2, 3, 4, 5, 6, 7, 8, or 9 cubes.—This is one of Dr. Waring's theorems, but we believe it has never been demonstrated.

21. If p and q be any two numbers prime to each other, then $p^2 + q^2$ can only be divided by numbers of the same form. Or, which is the same thing, a number that is the sum of two squares, can only be divided by numbers that are the sum of two squares: the two given squares being prime to each other.

22. And, in general, all numbers comprised in any of the following forms; viz, $p^2 + q^2$, $p^2 + 2q^2$, $p^2 - 2q^2$, can have for divisors, only those numbers which fall under the same form as themselves.

23. Every prime number $4n + 1$, which divides the formula $p^2 + 2q^2$, will also divide the formula $p^2 - 2q^2$.

24. A prime number $4n - 1$, that divides the formula $p^2 + aq^2$, can not be a divisor of the formula $p^2 - aq^2$.

25. Every prime number $8n + 1$, or $8n + 7$, will divide, at the same time, the two formulæ $p^2 + aq^2$, and $p^2 + 2aq^2$, or it will divide neither of them.

26. Every prime number $8n + 3$, or $8n + 5$, will always divide one of the two formulæ $p^2 + aq^2$, or $p^2 + 2aq^2$, but it can divide only one of them.

27. If c be a prime number, and n any number not divisible by c, then n divided by c, will leave the same remainder as n^c divided by c. Hence is readily deduced the following theorem, which is of the greatest use in the theory of numbers.

28. If c be a prime number, and n any number not divisible by c, then will $n^{c-1} - 1$ always be divisible by c.

29. If n be a prime number, then will $(1 \,.\, 2 \,.\, 3 \,.\, 4$ &c. $(n - 1) + 1)$ always be divisible by n.

This theorem is given in a more general form by M. Gauss, in his Disquisitiones Arithmeticæ; thus:

30. If n be any number whatever, and a, b, c, &c, all those numbers less than n, and also prime to it, then will $(a \,.\, b \,.\, c \,.$ &c. $(n - 1) + 1)$ be divisible by n.

From the former of these two enumerations is readily deduced the following corollary; namely,

31. If n be a prime number, then will $(1^2 \,.\, 2^2 \,.\, 3^2 \,.\, 4^2 \,.$ &c. $(\frac{n-1}{2})^2 + 1)$ when divided by n, always have a remainder ± 1; that is $+ 1$, when n is of the form $4n - 1$, and $- 1$ when n is of the form $4n + 1$.

32. If sa be made to represent the sum of all the divisors of any number a, then will $sn = s\,(n - 1) +$

$s(n-2) - s(n-3) - s(n-4) + s(n-5) + s(n-6) -$ &c.

33. Neither the sum nor difference of two cubes can be a cube.

34. Neither the sum nor difference of two biquadratures, can be a square.

35. And, generally, the equation $x^n \pm y^n = z^n$ is always impossible when n exceeds 2.—This is one of Fermat's theorems, which has never been demonstrated, except for the first two cases. Euler has generalized those cases, by demonstrating that the equation $a^3bx^4 \pm b^3ay^4 = z^2$ is always impossible.

36. The sum of any number of consecutive cubes, beginning at unity (or at any other number that is both a square and a cube), is a square number.

37. The second differences of consecutive square numbers is 2; the third difference of cubes is 6: and generally, the nth differences of the nth powers of any numbers in arithmetical progression, is constant, and is equal to $1 \times 2 \times 3 \times 4 - - - - (n-1) \times nd^n$, when d is the common difference of the series.

38. The area of a rational right-angled triangle cannot be a square number.

39. In any rational right-angled triangle, one of the sides is divisible by 5.

40. There cannot be more than three square numbers in an arithmetical progression.

41. The difference between any number, and another number formed with the same digits any how transposed, is always divisible by 9.

42. The difference between any odd power and its root, is divisible by double the exponent of that power.

43. The formula $a^m - a^n$ is always divisible by $a - 1$.

44. If $m - n$ be even, then will $a^m - a^n$ be divisible by $a + 1$.

45. If $m - n$ be odd, then will $a^m + a^n$ be divisible by $a + 1$.

46. The formula $a^m - b^m$ is always divisible by $a - b$, whatever be the value of m.

47. If m be even, then will $a^m - b^m$ be divisible both by $a - b$ and $a + b$.

48. If m be odd, then will $a^m + b^m$ be divisible by $a + b$.

49. No algebraical formula can be found that will contain prime numbers only.—This theorem is demonstrated by Legendre in his Essai sur la Theorie des Nombres.

50. The greatest known prime number is 2147483647.

The above numerical properties have been chiefly selected from Waring's Meditationes Algebraicæ; Euler's Anal. Infin., and his Algebra; Legendre's Essai sur la Theorie des Nombres; and from Gauss's Disquisitiones Arithmeticæ: to which works the reader is referred for every information that can be desired on this interesting subject.

NUMBER, *Golden*. See GOLDEN *Number* and CYCLE.

NUMBER *of Direction*, in Chronology, some one of the 35 numbers between the Easter limits, or between the earliest and latest day on which it can fall, i. e. between March 22 and April 25, which are 35 days; being so called, because it serves as a direction for finding Easter for any year; being indeed the number that expresses how many days after March 21, Easter-day falls. Thus, Easter-day falling as in the first line below, the number of direction will be as on the lower line:

	March										April		
Easter-day,	22,	23,	24,	25,	26,	27,	28,	29,	30,	31,	1,	2,	&c.
No. of dir.	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12,	&c

and so on, till the number of direction on the lower line be 35, which will answer to April 25, being the latest that Easter can happen. Therefore add 21 to the number of direction, and the sum will be so many days in March for the Easter-day: if the sum exceed 31, the excess will be the day of April.

To find the NUMBER *of Direction*. Enter the following table (which is adapted to the new style), with the dominical letter on the left hand, and the golden number at the top, then where the columns meet is the number of direction for that year. See Ferguson's Astron. pa. 381, ed. 8vo.

G. N.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Dom. Let.																			
A	29	19	5	26	12	33	19	12	26	19	5	26	12	5	26	12	33	19	12
B	27	13	6	27	13	34	20	13	27	20	6	27	13	6	20	13	34	20	6
C	8	14	7	21	14	35	21	7	28	21	7	28	14	7	21	14	28	21	7
D	19	15	8	22	15	29	22	8	29	15	8	29	15	1	22	15	29	22	8
E	30	16	2	23	16	20	23	9	30	16	9	23	16	2	23	9	30	23	9
F	24	17	3	24	10	31	24	10	31	17	10	24	17	3	24	10	31	17	10
G	25	18	4	25	11	32	18	11	32	18	4	25	18	4	25	11	32	18	11

Thus, for the year 1814, the dominical letter being B, and the golden number 10; on the line of B, and below 10, is 20 for the number of direction. To this add 21, the sum is 41 days from the 1st of March, which, deducting the 31 days of March, leaves 10 for the day of April, for Easter-day that year.

NUMERAL *Characters*. See CHARACTERS.

NUMERAL *Figures*. The antiquity of these in England has, for several reasons, been supposed as high as the eleventh century; in France about the middle of the tenth century; having been introduced into both countries from Spain, where they had been brought by the Moors or Saracens. See Wallis's Algebra, pa. 9 &c, and pa. 153 of additions at the end of the same; also Philos. Trans. No. 439 and 475. See also NOTATION.

NUMERATION, in Arithmetic, the art of estimating or pronouncing any number, or series of numbers.

Numbers are usually expressed by the ten following characters, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0; the first nine denoting respectively the first nine ordinal numbers; and the last, or cipher 0, joined to any of the others, denotes so many tens. In like manner, two ciphers joined to any one of the first nine significant figures, make it become so many hundreds, three ciphers make it thousands, and so on.

Weigelius indeed shows how to number, without going beyond a quaternary; i. e. by beginning to repeat at each fourth. And Leibnitz and De Lagny, in what they call their binary arithmetic, begin to repeat at every 2d place; using only the two figures 1 and 0. But these are rather matters of curiosity than use. See NOTATION.

That the nine significant figures may express not only units, but also tens, hundreds, thousands, &c, they have a local value given them, as hinted above; so that, though when alone, or in a right-hand place, they denote only units or ones, yet in the 2d place they denote tens, in the 3d place hundreds, in the 4th place thousands, &c; as the number 5555 is five thousand five hundred fifty and five.

Hence then, to express any written number, or assign the proper value to each character; beginning at the right hand, divide the proposed number into classes, of three characters to each class; and consider two classes as making up a period of six figures or places. Then every period, of six figures, has a name common to all the figures in it; the 1st being primes or units; the 2d is millions; the 3d is millions-of-millions, or billions; the 4th is millions-of-millions-of-millions or trillions; and so on; also every class, or half-period, of three figures, is read separately by itself, so many hundreds, tens, and units; only, after the left-hand half of each period, the word thousands is added; and at the end of the 2d, 3d, 4th &c period, its common name millions, billions, &c, is expressed. Thus, the number 4,591, is 4 thousand 5 hundred and 91.—The number 210,463, is 2 hundred and 10 thousands, and 463.—The number 281,427,307, is 281 millions, 427 thousands, and 307.

NUMERATOR, of a Fraction, is the number which shows how many of those parts, which the integer is supposed to be divided into, are denoted by the fraction. And, in the notation the numerator is set over the denominator, or number that shows into how many parts the integer is divided, in the fraction. So, ex. gr. $\frac{3}{4}$ denotes three-fourths, or 3 parts out of 4; where 3 is the numerator, and 4 the denominator.

NUMERICAL, NUMEROUS, or *Numeral*, something that relates to number.

NUMERAL *Algebra*, is that which makes use of numbers, in contradistinction from literal algebra, or that in which the letters of the alphabet are used.

NUNEZ (PETER). See NONIUS.

NUTATION, in Astronomy, a kind of libratory motion of the earth's axis; by which its inclination to the plane of the ecliptic is continually varying, by a certain number of seconds, backwards and forwards. The whole extent of this change in the inclination of the earth's axis, or, which is the same thing, in the apparent declination of the stars, is about 19″, and the period of that change is little more than 9 years, or the space of time from its setting out from any point and returning to the same point again, about 18 years and 7 months, being the same as the period of the moon's motions, upon which it chiefly depends; being indeed the joint effect of the inequalities of the action of the sun and moon upon the spheroidal figure of the earth, by which its axis is made to revolve with a conical motion, so that the extremity of it describes a small circle, or rather an ellipse, of 19·1 seconds diameter, and 14″·2 conjugate, each revolution being made in the space of 18 years 7 months, according to the revolution of the moon's nodes.

This is a natural consequence of the Newtonian system of universal attraction; the first principle of which is, that all bodies mutually attract each other in the direct ratio of their masses, and in the inverse ratio of the squares of their distances. From this mutual attraction, combined with motion in a right line, Newton deduces the figure of the orbits of the planets, and particularly that of the earth. If this orbit were a circle, and if the earth's form were that of a perfect sphere, the attraction of the sun would have no other effect than to keep the earth in its orbit, without causing any irregularity in the position of its axis. But neither is the earth's orbit a circle, nor its body a sphere; for the earth is sensibly protuberant towards the equator, and its orbit is an ellipsis, which has the sun in its focus. Now when the position of the earth is such, that the plane of the equator passes through the centre of the sun, the attractive power of the sun acts only so as to draw the earth towards it, still parallel to itself, and without changing the position of its axis; a circumstance which happens only at the time of the equinoxes. In proportion as the earth recedes from those points, the sun also goes out of the plane of the equator, and approaches that of the one or other of the tropics; the semidiameter of the earth, then exposed to the sun, being unequal to what it was in the former case, the equator is more powerfully attracted than the rest of the globe, which causes some alteration in its position, and its inclination to the plane of the ecliptic: and as that part of the orbit, which is comprized between the autumnal and vernal equinox, is less than that which is comprized between the vernal and autumnal, it follows, that the irregularity caused by the sun, during his passage through the northern signs, is not entirely compensated by that which he causes during his passage through the southern signs; and that the parallelism of the terrestrial axis, and its inclination to the ecliptic, is thence a little altered.

The like effect which the sun produces on the earth, by his attraction, is also produced by the moon, which acts with greater force, in proportion as she is more distant from the equator. Now, at the time when her nodes agree with the equinoctial points, her greatest latitude is added to the greatest obliquity of the ecliptic. At this time therefore, the power which causes the irregularity in the position of the terrestrial axis, acts with the greatest force; and the revolution of the nodes of the moon being performed in 18 years 7 months, hence it happens that in this time the nodes will twice agree with the equinoctial points; and consequently, twice in that period, or once every 9 years, the earth's axis will be more influenced than at any other time.

That the moon has also a like motion, is shown by Newton, in the first book of the Principia; but he observes indeed that this motion must be very small, and scarcely sensible.

As to the history of the nutation, it seems there have been hints and suspicions of the existence of such a circumstance, ever since Newton's discovery of the system of the universal and mutual attraction of matter; some traces of which are found in his Principia, as above-mentioned.

We find too, that Flamsteed had hoped, about the year 1690, by means of the stars near his zenith, to determine the quantity of the nutation which ought to follow from the theory of Newton; but he gave up that project, because, says he, if this effect exists, it must remain insensible till we have instruments much longer than 7 feet, and more solid and better fixed than mine. Hist. Cœlest. vol. 3, pa. 113.

And Horrebow gives the following passage, extracted from the manuscripts of his master Roemer, who died in 1710, and whose observations he published in 1753, under the title of Basis Astronomiæ. By this paragraph it appears, that Roemer suspected also a nutation in the earth's axis, and had some hopes to give the theory of it: it runs thus; "Sed de altitudinibus non perinde certus reddebar, tàm ob refractionum varietatem quàm ob aliam nondum liquido perspectam causam; scilicet per hos duos annos, quemadmodum et alias, expertus sum esse quandam in declinationibus varietatem, quæ nec refractionibus nec parallaxibus tribui potest, sine dubio ad vacillationem aliquam poli terrestris referendam, cujus me verisimilem dare posse theoriam, observationibus munitam, spero." Basis Astronomiæ, 1735, pa. 66.

These ideas of a nutation would naturally present themselves to those who might perceive certain changes in the declinations of the stars; and we have seen that the first suspicions of Bradley in 1727, were that there was some nutation of the earth's axis which caused the star γ Draconis to appear at times more or less near the pole; but further observations obliged him to search another cause for the annual variations (art. ABERRATION); it was not till some years after that he discovered the second motion which we now treat of, properly called the nutation. See the art. STAR, where Bradley's discovery of it is given at length; to which may be further added the following summary.

For the better explaining the discovery of the nutation by Bradley, we must recur to the time when he observed the stars in discovering the aberration. He perceived in 1728, that the annual change of declination in the stars near the equinoctial colure, was greater than what ought to result from the annual precession of the equinoxes being supposed 50", and calculated in the usual way; the star η Ursæ Majoris was in the month of September 1728, 20" more south than the preceding year, which ought to have been only 18"; from whence it would follow that the precession of the equinoxes should be 55"¼ instead of 50", without ascribing the difference between the 18 and 20" to the instrument, because the stars about the solstitial colure did not give a like difference. Philos. Trans. vol. 35, pa. 659.

In general, the stars situated near the equinoctial colure had changed their declination about 2" more than they ought by the mean precession of the equinoxes, the quantity of which is very well known, and the stars near the solstitial colure the same quantity less than they ought; but, Bradley adds, whether these small variations arise from some regular cause, or are occasioned by some change in the sector, I am not yet able to determine. Bradley therefore ardently continued his observations for determining the period and the law of these variations; for which purpose he resided almost continually at Wansted till 1732, when he was obliged to repair to Oxford to succeed Dr. Halley; he still however continued to observe with the same exactness all the circumstances of the changes of declination in a great number of stars. Each year he saw the periods of the aberration confirmed according to the rules he had lately discovered; but from year to year he found also other differences; the stars situated between the vernal equinox and the winter solstice approached nearer to the north pole, while the opposite ones receded further from it: he began therefore to suspect that the action of the moon on the elevated equatorial parts of the earth might cause a variation or libration in the earth's axis: his sector having been left fixed at Wansted, he often went there to make observations for many years, till the year 1747, when he was fully satisfied of the cause and effects, an account of which he then communicated to the world. Philos. Trans. vol. 45, an. 1748.

"On account of the inclination of the moon's orbit to the ecliptic," says Dr. Maskelyne, (Astronomical Observations 1776, pa. 2), "and the revolution of the nodes in antecedentia, which is performed in 18 years and 7 months, the part of the precession of the equinoxes, owing to her action, is not uniform; but subject to an equation, whose maximum is 18": and the obliquity of the ecliptic is also subject to a periodical equation of 9"·55; being greater by 19·1" when the moon's ascending node is in Aries, than when it is in Libra. Both these effects are represented together, by supposing the pole of the earth to describe the periphery of an ellipsis, in a retrograde manner, during each period of the moon's nodes, the greater axis, lying in the solstitial colure, being 19·1", and the lesser axis, lying in the equinoctial colure, 14·2"; being to the greater, as the cosine of double the obliquity of the ecliptic to the cosine of the obliquity itself. This motion of the pole of the earth is called the nutation of the earth's axis, and was discovered by Dr. Bradley, by a series of observations of several stars made in the course of 20 years, from 1727 to 1747, being a continuation of those by which he had discovered the aberration of light. But the exact law of the motion of the earth's axis has been settled by the learned mathematicians Dalembert, Euler, and Simpson, from the principles of gravity. The equation hence arising in the place of a fixed star, whether in longitude, right ascension, or declination (for the latitudes are not affected by it) has been sometimes called nutation, and sometimes deviation." And again (says the doctor, pa. 8), "the above quantity 19·1", of the greatest nutation of the earth's axis in the solstitial colure, is what I found from a scrupulous calculation of all Dr. Bradley's observations of γ Draconis, which he was pleased to communicate to me for that purpose. From a like examination of his observation of η Ursæ Majoris, I found the lesser axis of the ellipsis of nutation to be 14·1", or only $\frac{1}{10}$th of a second less than what it should be from the observations of γ Draconis. But the result from the observations of γ Draconis is most to be depended on."

Mr. Machin, secretary of the Royal Society, to whom Bradley communicated his conjectures, soon perceived that it would be sufficient to explain, both the nutation and the change of the precession, to suppose that the pole of the earth described a small circle. He stated the diameter of this circle at 18", and he supposed that it was described by the pole in the space of one revolution of the moon's nodes. But later calculations, and theory, have shown that the pole describes a small ellipsis, whose axes are 19·1" and 14·2", as above mentioned.

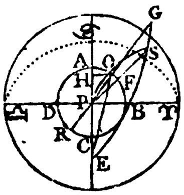
To show the agreement between the theory and observations, Bradley gives a great multitude of observations of a number of stars, taken in different positions; and out of more than 300 observations which he made, he found but 11 which were different from the mean by so much as 2". And by the supposition of the elliptic rotation, the agreement of the theory with observation comes out still nearer.

By the observations of 1740 and 1741, the star η Ursæ Majoris appeared to be 3" farther from the pole than it ought to be according to the observations of other years. Bradley thought this difference arose from some particular cause; which however was chiefly the fault of the circular hypothesis. He suspected also that the situation of the apogee of the moon might have some influence on the nutation. He invited therefore the mathematicians to calculate all these effects of attraction; which has been ably done by Dalembert, Euler, Walmesley, Simpson, and others; and the astronomers to continue to observe the positions of the smallest stars, as well as the largest, to discover the physical derangements which they may be subject to, and which had been observed in some of them.

Several effects arise from the nutation. The first of these, and that which is the most easily perceived, is the change in the obliquity of the ecliptic; the quantity of which ought to be varied from that cause by 18″ in about 9 years. Accordingly, the obliquity of the ecliptic was observed in 1764 to be 23° 28′ 15″, and in 1755 only 23° 28′ 5″: not only therefore had it not diminished by 8″, as it ought to have done according to the regular mean diminution of that obliquity; but it had been augmented by 10″; making together 18″, for the effect of the nutation in the 9 years.

The nutation changes equally the longitudes, the right-ascensions, and the declinations of the stars, as before observed; it is the latitudes only which it does not affect, because the ecliptic is immoveable in the theory of the nutation.

Dr. Bradley illustrates the foregoing theory of nutation in the following manner. Let P represent the mean place of the pole of the equator; about which point, as a centre, suppose the true pole to move in the small circle ABCD, whose diameter is 18″. Let E be the pole of the ecliptic, and EP be equal to the mean distance between the poles of the equator and ecliptic; and suppose the true pole of the equator to be at A, when the moon's ascending node is in the beginning of Aries; and at B, when the node gets back to Capricorn; and at C, when the same node is in Libra: at which time the north pole of the equator being nearer the north pole of the ecliptic, by the whole diameter of the little circle AC, equal to 18″; the obliquity of the ecliptic will then be so much less than it was when the moon's ascending node was in Aries. The point P is supposed to move round E, with an equal retrograde motion, answerable to the mean precession arising from the joint actions of the sun and moon: while the true pole of the equator moves round P, in the circumference ABCD, with a retrograde motion likewise, in a period of the moon's nodes, or of 18 years and 7 months. By this means, when the moon's ascending node is in Aries, and the true pole of the equator at A, is moving from A towards B; it will approach the stars that come to the meridian with the sun about the vernal equinox, and recede from those that come with the sun near the autumnal equinox, faster than the mean pole P does. So that, while the moon's node goes back from Aries to Capricorn, the apparent precession will seem so much greater than the mean, as to cause the stars that lie in the equinoctial colure to have altered their declination 9″, in about 4 years and 8 months, more than the mean precession would do; and in the same time, the north pole of the equator will seem to have approached the stars that come to the meridian with the sun of our winter solstice about 9″, and to have receded as much from those that come with the sun at the summer solstice.



Thus the phenomena before recited are in general conformable to this hypothesis. But to be more particular; let S be the place of a star, PS the circle of declination passing through it, representing its distance from the mean pole, and ♈ PS its mean right-ascension. Thus if O and R be the points where the circle of declination cuts the little circle ABCD, the true pole will be nearest that star at O, and furthest from it at R; the whole difference amounting to 18″, or to the diameter of the little circle. As the true pole of the equator is supposed to be at A, when the moon's ascending node is in Aries; and at B, when that node gets back to Capricorn; and the angular motion of the true pole about P, is likewise supposed equal to that of the moon's node about E, or the pole of the ecliptic; since in these cases the true pole of the equator is 90 degrees before the moon's ascending node, it must be so in all others.

When the true pole is at A, it will be at the same distance from the stars that lie in the equinoctial colure, as the mean pole P is; and as the true pole recedes back from A towards B, it will approach the stars which lie in that part of the colure represented by P ♈, and recede from those that lie in P ♎; not indeed with an equable motion, but in the ratio of the sine of the distance of the moon's node from the beginning of Aries. For if the node be supposed to have gone backwards from Aries 30°, or to the beginning of Pisces, the point which represents the place of the true pole will, in the mean time, have moved in the little circle through an arc, as AO, of 30° likewise; and would therefore in effect have approached the stars that lie in the equinoctial colure P ♈, and have receded from those that lie in P ♎ by 4½ seconds, which is the sine of 30° to the radius AP. For if a perpendicular fall from O on AP, it may be conceived as part of a great circle, passing through the true pole and any star lying in the equinoctial colure. Now the same proportion that holds in these stars, will obtain likewise in all others; and from hence we may collect a general rule for finding how much nearer, or farther, any star is to, or from, the mean pole, in any given position of the moon's node.

For, if from the right-ascension of the star, we subtract the distance of the moon's ascending node from Aries; then radius will be to the sine of the remainder, as 9″ is to the number of seconds that the star is nearer to, or farther from, the true, than the mean pole.

This motion of the true pole, about the mean at P, will also produce a change in the right-ascension of the stars, and in the places of the equinoctial points, as well as in the obliquity of the ecliptic; and the quantity of the equations, in either of these cases, may be easily computed for any given position of the moon's nodes.

Dr. Bradley then proceeds to find the exact quantity of the mean precession of the equinoctial points, by comparing his own observations made at Greenwich, with those of Tycho Brahé and others; the mean of all which he states at 1 degree in 71¼ years, or 50⅓″ per year; in order to show the agreement of the foregoing hypothesis with the phenomena themselves, of the alterations in the polar distances of the stars; the conclusions from which approach as near to a coincidence as could be expected on the foregoing circular hypothesis, the diameter of which is 18″; instead of the more accurate quantity 19·1″, as deduced by Dr. Maskelyne, and the elliptic theory as determined by the mathematicians, in which the greater axis (19·1″) is to the less axis (14·2″), as the cosine of the greatest declination is to the cosine of double the same.

To give an idea now of the nutation of the stars, in longitude, right ascension, and declination; suppose the pole of the equator to be at any time in the point O, also S the place of any star, and OH perpendicular to AE: then, like as AE is the solstitial colure when the pole of the equator was at A, and the longitude of the star S equal to the angle AES; so OE is the solstitial colure when that pole is at O, and the longitude is then only the angle OES; less than before by the angle AEO, which therefore is the nutation in longitude: counting the longitudes from the solstitial instead of the equinoctial colure, from which they differ equally by 90 degrees, and therefore have the same difference AEO. Now the angle AEO will be as the line HO = sin. AO to radius PB = sin. AO × PB = sin. AO × 9''; therefore as EO : HO :: radius 1 : $\frac{HO}{EO} = \frac{\text{sin. AO} \times 9''}{\text{sin. } 23^\circ\ 28'} = \frac{\text{sin. node} \times 9''}{\text{sin. } 23^\circ\ 28'}$, since AO is equal to longitude of the moon's node. This expression therefore gives the nutation in longitude, supposing the maximum of nutation, with Bradley, to be 18''; and it is negative, or must be subtracted from the mean longitude of the stars, when the moon's node is in the first 6 signs of its longitude; but additive in the latter 6, to give the true apparent longitude.

This equation of the nutation in longitude is the same for all the stars; but that for the declination and right ascension is various for the different stars. In the foregoing figure, PS is the mean polar distance, or mean codeclination of the star S, when the true place of the pole is O; and so the aparent codeclination; also, the angle SPE is the mean right ascension, and SOE the apparent one, counted from the solstitial colure; consequently OPS or OPF the difference between the right ascension of the star and that of the pole, which is equal to the longitude of the node increased by 3 signs or 90 degrees; supposing OF to be a small arc perpendicular to the circle of declination PFS; then is SF = SO, and PF the nutation in declination, or the quantity the declination of the star has increased; but radius 1 : 9'' :: cosin. OPF : PF = 9'' × cos. OPF; so that the equation of declination will be found by multiplying 9'' by the sine of the star's right ascension diminished by the longitude of the node; for that angle is the complement of the angle SPO. This nutation in declination is to be added to the mean declination to give the apparent, when its argument does not exceed 6 signs; and to be subtracted in the latter 6 signs. But the contrary for the stars having south declination.

To calculate the nutation in right-ascension, we must find the difference between the angle SOE the apparent, and SPE the mean right-ascension, counted from the solstitial colure EO. Now the true right ascension SOE is equal to the difference between the two variable angles GOE and GOS; the former of which arises from the change of one of the variable circles EO, and depends only on the situation of the node or of that of the pole O; the latter GOS depends on the angle GPS, which is the difference between the right ascension of the star and the place of the pole O. Now in the spherical triangle GPE, which changes into GOE, the side GE and angle G remain constant, and the other parts are variable; hence therefore the small variation PO of the side next the constant angle G, is to the small variation of the angle opposite to the constant side GE, as the tangent of the side PE opposite to the constant angle, is to the sine of the angle GPE opposite to the constant side; that is, as tang. 23° 28' : sin. OPE :: 9'' : $x = \frac{9'' \times \text{sin. OPE}}{\text{tang. } 23^\circ\ 28'}$; the difference between the angles GOE and GPE. This is the change which the nutation PO produces in the angle GPE, being the first part of the nutation sought, and is common to all the stars and planets. It is to be subtracted from the mean right-ascension in the first 6 signs of the longitude of the node, and added in the other six.

In like manner is found the change which the nutation produces in the other part of the right-ascension SPE, that is, in the angle SPG, which becomes SOG by the effect of the nutation. This small variation will be calculated from the same analogy, by means of the triangle SOG, in which the angle G is constant, as well as the side SG, while SP changes into SO. Hence therefore, tang. SP : sin. SPG :: 9'' : variation of SPG, that is, the cotangent of the declination is to the cosine of the distance between the star and the node, as 9'' are to the quantity the angle SPG varies in becoming the angle SOG, being the second part of the nutation in right-ascension; and if there be taken for the argument, the right-ascension of the star minus the longitude of the node, the equation will be subtractive in the first and last quadrant of the argument, and additive in the 2d and 3d, or from 3 to 9 signs. But the contrary for stars having south declination.

This second part of the nutation in right-ascension affects the return of the sun to the meridian, and therefore it must be taken into the account in computing the equation of time. But the former part of the nutation does not enter into that computation; because it only changes the place of the equinox, without changing the point of the equator to which a star corresponds, and consequently without altering the duration of the returns to the meridian.

All these calculations of the nutation, above explained, are upon Machin's hypothesis, that the pole describes a circle; however Bradley himself remarked that some of his observations differed too much from that theory, and that such observations were found to agree better with theory, by supposing that the pole, instead of the circle, describes an ellipse, having its less axis DB = 16'' in the equinoctial colure, and the greater axis AC = 18'', lying in the solstitial colure. But as even this correction was not sufficient to cause all the inequalities to disappear entirely, Dr. Bradley referred the determination of the point to theoretical and physical investigation. Accordingly several mathematicians undertook the task, and particularly Dalembert, in his Recherches sur la Précession des Equinoxes, where he determines that the pole really describes an ellipse, and that narrower than the one assumed above by Bradley, the greater axis being to the less, as the cosine of 23° 28' to the cosine of double the same. And as Dr. Maskelyne found, from a more accurate reduction of Bradley's observations, that the maximum of the nutation gives 19·1'' for the greater axis, therefore the above proportion gives 14·2'' for the less axis of it; and according to these data, the theory and observations are now found to agree very near together.

See Lalande's Astron. vol. 3, art. 2874 &c, where he makes the corrections for the ellipse. He observes however that by the circular hypothesis alone, the computations may be performed as accurately as the observations can be made; and he concludes with some corrections and rules for computing the nutation in the elliptic theory.

The following set of general tables very readily give the effect of nutation on the elliptical hypothesis; they were calculated by the late M. Lambert, and are taken from the Connoissance des Temps for the year 1788.

General Tables for Nutation in the Ellipse.

Table 1.

Degrees	0·6 + −	1·7 + −	2·8 + −	
	″	″	″	
0	0·00	3·93	6.80	30
1	0·14	4·04	6·86	29
2	0·27	4·16	6·93	28
3	0·41	4·28	6·99	27
4	0·55	4·39	7·06	26
5	0·68	4·50	7·11	25
6	0·82	4·61	7·17	24
7	0·95	4·72	7·23	23
8	1·11	4·83	7·28	22
9	1·23	4·94	7·33	21
10	1·36	5·05	7·38	20
11	1·50	5·15	7·42	19
12	1·63	5·25	7·47	18
13	1·77	5·35	7·51	17
14	1·90	5·45	7·55	16
15	2·03	5·55	7·58	15
16	2·16	5·65	7·62	14
17	2·30	5·74	7·65	13
18	2·43	5·83	7·68	12
19	2·56	5·92	7·71	11
20	2·68	6·01	7·73	10
21	2·81	6·10	7·75	9
22	2·94	6·19	7·76	8
23	3·07	6·27	7·77	7
24	3·19	6·35	7·79	6
25	3·32	6·43	7·80	5
26	3·44	6·51	7·82	4
27	3·56	6·58	7 83	3
28	3·69	6·66	7·84	2
29	3·81	6·73	7·85	1
30	3·93	6·80	7·85	0
	+ − 5·11	+ − 4·10	+ − 3·9	Degrees

Table 2.

Degrees	0·6 + −	1·7 + −	2·8 + −	
	″	″	″	
0	0·00	0·58	1·00	30
1	0·02	0·59	1·01	29
2	0·04	0·61	1·02	28
3	0·06	0·63	1·02	27
4	0·08	0.64	1·03	26
5	0·10	0·66	1·04	25
6	0·12	0·68	1·05	24
7	0·14	0·69	1·06	23
8	0·16	0·71	1·07	22
9	0·18	0·72	1·07	21
10	0·20	0·74	1·08	20
11	0·22	0·75	1·09	19
12	0·24	0·77	1·09	18
13	0·26	0·78	1·10	17
14	0·28	0·80	1·11	16
15	0·30	0·81	1·11	15
16	0·32	0·83	1·12	14
17	0·34	0·84	1·12	13
18	0·35	0·85	1·13	12
19	0·37	0·87	1·13	11
20	0.39	0·88	1·13	10
21	0·41	0·89	1·14	9
22	0·43	0·91	1·14	8
23	0·45	0·92	1·14	7
24	0·47	0·93	1·14	6
25	0·49	0·94	1·15	5
26	0·50	0·95	1·15	4
27	0·52	0·96	1·15	3
28	0·54	0·97	1·15	2
29	0·56	0·99	1·15	1
30	0·58	1·00	1·15	0
	+ − 5·11	+ − 4·10	+ − 3·9	Degrees

Table 3.

Degrees	0·6 − +	1·7 − +	2·8 − +	
	′	″	″	
0	0·00	7·71	13·36	30
1	0·27	7·95	13·50	29
2	0·54	8·18	13·62	28
3	0·81	8·40	13·75	27
4	1·08	8·63	13·87	26
5	1·35	8·85	13·98	25
6	1·61	9·07	14·10	24
7	1·88	9·29	14·20	23
8	2·15	9·50	14·31	22
9	2·41	9·71	14·41	21
10	2·68	9·92	14·50	20
11	2·94	10·12	14·59	19
12	3·21	10·32	14·67	18
13	3·47	10·52	14·76	17
14	3·73	10·72	14·83	16
15	3·99	10·91	14·90	15
16	4·25	11·10	14·97	14
17	4·51	11·28	15·03	13
18	4·77	11·47	15·09	12
19	5·02	11·65	15·15	11
20	5·28	11·82	15·20	10
21	5·53	11·99	15·24	9
22	5·78	12·16	15·28	8
23	6·03	12·32	15·32	7
24	6·28	12·48	15·35	6
25	6·52	12·64	15.37	5
26	6·76	12·79	15·39	4
27	7·01	12·94	15·41	3
28	7·25	13·09	15·42	2
29	7·48	13 23	15·43	1
30	7·71	13·36	15·43	0
	− + 5·11	− + 4·10	− + 3·9	Degrees

This table is constructed from Dalembert's ellipse, whose semi-axes are 9″ and 6·7″; half their sum and half their diff. are 7·85″ and 1·15″, as in the following formulæ. The number 15·43″, in the 2d formula, is = 6·7″ × cotangent of the ellipse's obliquity. If the semi-axes be 9·55″ and 4·1″, the formulæ will give the nutation as in Dr. Maskelyne's Tables, of 1776.

The Use of the Tables.

The right-ascension of a star minus the moon's mean longitude, gives the argument of the first of these three tables. The sum of the same two quantities gives the argument of the 2d table. Then the sum or the difference of the quantities found with these two arguments, will give the correction to be applied to the mean declination of the star, if it is north declination; but if it is southern, the signs + or − are to be changed into − and +.

From each of those two arguments for the declination subtracting 3 signs, or 90°, gives the arguments for correcting the right-ascension; the sum or difference of the quantities found, with these two arguments, in tables 1 and 2, is to be multiplied by the tangent of the star's declination, and to the product is to be added the quantity taken out of table 3, the argument of which is the mean longitude of the moon's ascending node: when the declination of the star is south, the tangent will be negative.

Example, To find the nutation in right-ascension and declin. for the star α Aquilæ, the 1st of July, 1788.

Right-ascension of the star	9ˢ	25°	7′	
Long. of the moon's node	8	15	40	
Diff. being argument 1,	1	9	27	+ 4·99″
Sum, argument 2, - -	6	10	47	− 0·22
Correction of the declination - - -				+ 4·77

The above two arguments being each diminished by 3 signs, give,

Argument 1 - - - - - - - -	10ˢ	9°	27′	− 6·06″
Argument 2 - - - - - - - -	3	10	47	+ 1·13
				− 4·93
Declin. of star north, its tangent - - - -				0·146
The product is - - - - - - - - -				− 0·72
Long. of the ☾'s node, argum. 3 - -				+ 14·94
Correction of right-ascension - - - -				+ 14·22

In general, let ☊ denote the longitude of the moon's ascending node; *r* the right-ascension of a star or planet; *d* its declination; the nutation in declination and right-ascension will be expressed by the two following formulæ; viz, the nutation in declination

$= 7''{\cdot}85 \times \text{sin.}\ (r - \text{☊}) + 1''{\cdot}15 \times \text{sin.}\ (r + \text{☊})$;

and the nutation in right-ascension

$= [7''{\cdot}85 \times \text{sin.}\ (r - \text{☊} - 90^\circ) + 1''{\cdot}15 \times \text{sin.}\ (r + \text{☊} - 90^\circ)] \times \text{tang.}\ d - 15''{\cdot}43 \times \text{sin.}\ \text{☊}$.

For the mathematical investigation of the effects of universal attraction, in producing the nutation, &c, see d'Alembert's Recherches sur la Precession des Equinoxes; Silvabelle's Treatise on the Precession of the Equinoxes &c, in the Philos. Trans. an. 1754, pa. 385; Walmesley's treatise de Præcessione Equinoctiorum et Axis Terræ Nutatione, in the Philos. Trans. an. 1756, pa. 700; Simpson's Miscellaneous Tracts, pa. 1; and other authors.

O.

OBELISK, a kind of quadrangular pyramid, very tall and slender, raised as an ornament in some public place, or to serve as a memorial of some remarkable transaction.

OBJECT, something presented to the mind, by sensation, or by imagination; or something that affects us by its presence, that affects the eye, ear, or some other of the organs of sense.—The objects of the eye, or vision, are painted on the retina; though not there erect, but inverted, according to the laws of optics. This is easily shown from Descartes's experiment, of laying bare the vitreous humour on the back part of the eye, and putting over it a bit of white paper, or the skin of an egg, and then placing the fore part of the eye to the hole of a darkened room. By this means there is obtained a pretty landscape of the external objects, painted invertedly on the back of the eye. In this case, how the objects thus painted invertedly should be seen erect, is matter of controversy.

OBJECT-*Glass*, of a telescope or microscope, is the glass placed at the end of the tube which is next or towards the object to be viewed.

To prove the goodness and regularity of an object-glass; describe two concentric circles on a piece of paper, the one having its diameter the same with the breadth of the object-glass, and the other half that diameter; divide the smaller circumference into 6 equal parts, pricking the points of division through with a fine needle; cover one side of the glass with this paper, and, exposing it to the sun, receive the rays through these 6 holes upon a plane; then by moving the plane nearer to or farther from the glass, it will be found whether the six rays unite exactly together at any distance from the glass; if they do, it is a proof of the regularity and just form of the glass; and the said distance is also its focal distance.—Another way of proving the accuracy of an object-glass, is by placing it in a tube, and trying it with small eye-glasses, at several distant objects; for that object-glass is always the best, which represents objects the brightest and most distinct, and which bears the greatest aperture, and the most convex and concave eye-glasses, without colouring or haziness.

A circular object-glass is said to be truly centred, when the centre of its circumference falls exactly in the axis of the glass; and to be ill centred, when it falls out of the axis. To prove whether object-glasses be well centred, hold the glass at a proper distance from the eye, and observe the two reflected images of a candle, varying the distance till the two images unite, which is the true centre point: then if this fall in the middle, or central point of the glass, it is a proof of its being truly centred.

As object-glasses are commonly included in cells that screw upon the end of the tube of a telescope, it may be proved whether they be well centred, by fixing the tube, and observing while the cell is unscrewed, whether the cross-hairs keep fixed upon the same lines of an object seen through the telescope.—For various methods of finding the true centre of an object-glass, see Smith's Optics, book 3, chap. 3; also the Philos. Trans. vol. 48, pa. 177.

OBJECTIVE *Line*, in Perspective, is any line drawn on the geometrical plane, whose representation is sought for in a draught or picture.

OBJECTIVE *Plane*, in Perspective, is any plane situated in the horizontal plane, whose perspective representation is required.

OBLATE, flatted, or shortened; as an oblate spheroid, having its axis shorter than its middle diameter; being formed by the rotation of an ellipse about the shorter axis.

OBLATENESS, of the earth, the flatness about the poles, or the diminution of the polar axis in respect of the equatorial. The ratio of these two axes has been determined in various ways; sometimes by the measures of different degrees of latitude, or of longitude, and sometimes by the length of pendulums vibrating seconds in different latitudes, &c; the results of all which, as well as accounts of the means of determining them, see under the articles EARTH and DEGREE. To what is there said, may be added the following, from An Account of the Experiments made in Russia concerning the Length of a Pendulum which swings Seconds, by Mr. Krafft, contained in the 6th and 7th volumes of the New Petersburg Transactions, for the years 1790 and 1793. These experiments were made at different times, and in various parts of the Russian empire: Mr. Krafft has collected and compared them, with a view to investigate the consequences that might thence be deduced; and from the whole he concludes, that the length *p* of a pendulum, which swings seconds in any given latitude *l*, and in a temperature of 10 degrees of Reaumur's thermometer, may be determined by the following equation, in lines of a French foot: viz,

$$p = 439{\cdot}178 + 2{\cdot}321 \text{ sine}^2 l.$$

This expression agrees, very nearly, not only with all the experiments made on the pendulum in Russia, but also with those of Mr. Graham, and those of Mr. Lyons in 79° 50′ north latitude, where he found its length to be 441·38 lines.—It also shows the augmentation of gravity from the equator to the parallel of a given latitude *l*: for, putting *g* for the gravity under the equator, G for that under the pole, and z for that under the latitude l; Mr. Krafft finds $z = (1 + 0{\cdot}0052848 \text{ sine}^2 l) \times g$; and consequently $G = 1{\cdot}0052848\, g$.

From this proportion of gravity under different latitudes, Mr. Krafft deduces, that on the hypothesis of the earth's being a homogeneous ellipsoid, its oblateness must be $\frac{1}{190}$; instead of $\frac{1}{230}$, which ought to be the result of this hypothesis; but on adopting the supposition that the earth is a heterogeneous ellipsoid, he finds its oblateness, as deduced from these experiments, to be $\frac{1}{297}$; which agrees nearly with that resulting from the measurement of degrees of the meridian.—This confirms an observation of M. Laplace, that, if the hypothesis of the earth's homogeneity be given up, then will theory, the measurement of degrees of latitude, and experiments with the pendulum, all agree in their result with respect to the oblateness of the earth.

OBLIQUE, aslant, indirect, or deviating from the perpendicular. As,

Oblique *Angle*, one that is not a right angle, but either greater or less than this, being either obtuse or acute.

Oblique-*angled Triangle*, that whose angles are all oblique.

Oblique *Ascension*, is that point of the equinoctial which rises with the centre of the sun, or star; or any other point of the heavens, in an oblique sphere.

Oblique *Circle*, in the stereographic projection, is any circle that is oblique to the plane of projection.

Oblique *Descension*, that point of the equinoctial which sets with the centre of the sun, or star, or other point of the heavens in an oblique sphere.

Oblique *Direction*, that which is not perpendicular to a line or plane.

Oblique *Force*, or Percussion, or Power, or Stroke, is that made in a direction oblique to a body or plane. It is demonstrated that the effect of such oblique force &c, upon the body, is to an equal perpendicular one, as the sine of the angle of incidence is to radius.

Oblique *Line*, that which makes an oblique angle with some other line.

Oblique *Planes*, in Dialling, are such as recline from the zenith, or incline towards the horizon.

Oblique *Projection*, is that where a body is projected or impelled in a line of direction that makes an oblique angle with the horizontal line.

Oblique *Sailing*, in Navigation, is that part which includes the application and calculation of oblique-angled triangles.

Oblique *Sphere*, in Geography, is that in which the axis is oblique to the horizon of a place.—In this sphere, the equator and parallels of declination cut the horizon obliquely. And it is this obliquity that occasions the inequality of days and nights, and the variation of the seasons. See Sphere.

OBLIQUITY, that which denotes a thing oblique.

Obliquity *of the Ecliptic*, is the angle which the ecliptic makes with the equator. See Ecliptic.

OBLONG, sometimes means any figure that is longer than it is broad; but more properly it denotes a rectangle, or a right-angled parallelogram, whose length exceeds its breadth.

Oblong, is also used for the quality or species of a figure that is longer than it is broad: as an oblong spheroid, formed by an ellipse revolved about its longer or transverse axis; in contradistinction from the oblate spheroid, or that which is flatted at its poles, being generated by the revolution of the ellipse about its conjugate or shorter axis.

OBSCURA *Camera*. See Camera *Obscura*.

Obscura *Clara*. See Clara *Obscura*.

OBSERVATION, in Astronomy and Navigation, is the observing with an instrument some celestial phenomenon; as, the altitude of the sun, moon, or stars, or their distances asunder, &c. But by this term the seamen commonly mean only the taking the meridian altitudes, in order to find the latitude. And the finding the latitude from such observed altitude, they call Working an Observation.

OBSERVATORY, a place destined for observing the heavenly bodies; or a building, usually erected on some eminence, for making astronomical observations.

Most nations, at almost all times, have had their observatories, either public or private ones, and in various degrees of perfection. A description of a great many of them may be seen in a dissertation of Weidler's, De præsenti Specularum Astronomicarum Statu, printed in 1727, and in different articles of his History of Astronomy, printed in 1741, viz, pa. 86 &c; as also in Lalande's Astronomy, the preface pa. 34; also in Bailly's astronomical works, and elsewhere.

As navigation essentially depends on the determinations made in observatories, such establishments have been considered of great national importance, especially in maritime states; and hence they have been liberally endowed by different governments. Even private observatories have been, in many places, erected at considerable expense; the number of which has been greatly increased of late years; a circumstance which, while it marks the progress of science, does honour to the age in which we live.

Fixed observatories are those where instruments are fixed in the meridian, by which, with the aid of astronomical clocks, the right-ascensions and declinations of the heavenly bodies are determined; and motion, time, and space made to measure each other. Such buildings and apparatus only are called *regular* observatories, though very useful operations are sometimes performed, and important discoveries made, where no instruments are fixed in the meridian.

History of Observatories.

All nations, where astronomy has been cultivated, boast of having had observatories at an early period; though ancient history affords but little information on the subject. It was not indeed, until considerable progress had been made, both in astronomy and the mechanical arts, that successful attempts were made, either in constructing instruments, or erecting edifices for astronomical purposes. The first observatories of man were the fields or hills, and his eyes his instruments; and his progress by these aids alone was astonishing. The instruments of ancient astronomers were very large, and of rude construction; mostly of wood, and some of stone. They consisted chiefly of gnomons, dials, and astrolabes; and long tubes, like telescopes, were used to assist the sight. For the same purpose deep wells were also sunk in dry places, from the bottom of which the stars might be seen in the day-time. Most buildings for astronomical observations were of great altitude, and chiefly erected in very high situations.

In Chaldæa, a country celebrated in the early annals of astronomy, the lofty temple of Belus, or tower of Babel, was used as an observatory. And in Egypt, the famous tomb of Osymandias was applied to the same purpose. This building, it has been said, contained a golden or gilded circle, for celestial observations, which was 365 cubits in circumference, and one cubit in thickness.

The pyramids of Egypt have also been supposed intended for observatories; and in support of this opinion it is argued, that they were built to face the four cardinal points. The great height of these pyramids is to be sure favourable for celestial observations, whether they be used as gnomons, or for the purposes of astrology, a favourite study in those times, and which chiefly required an accurate view of the rising and setting of stars. It is indeed certain that practical astronomy was much improved in Egypt, particularly in the famous school of Alexandria, where an observatory was built 300 years before the Christian era, and continued for more than five centuries under a succession of celebrated names, such as Aristellus, Hipparchus, Ptolemy, &c.

The Chinese and Gentoo nations appear to have made a very early progress, both in the theory and practice of astronomy. Those people have traditions and vestiges of ancient observatories, on which ingenious disquisitions may be seen in Bailly's History of Ancient Astronomy, and in the Asiatic Researches, by Sir Wm. Jones, by Messrs. Hunter, Bentley, Colebrooke, Sir Robert Barker, and others.

The Hindu institutions, five in number, were constructed nearly at the same period, about 250 years ago. They were built by order of the emperor Mahommed Shah, with a view to reform the calendar by means of astronomical observations; and he chose for his chief astronomer Jeysing, or Jayasinha, the rajah of Ambhere. These observatories were built at Delhi, Benares, Matra, Oujein, and Suvai Jeypoor, and all under the direction of Jeysing. The observatory at Benares may be seen minutely described, by Sir Rob. Barker, in the Philos. Trans. for 1777, where he has given several plates of views, both of the buildings and instruments. And as all the other observatories were built and furnished nearly on the same plan, his description may be deemed sufficient for the whole. The instruments are quadrants and gnomons of enormous size, built of stone, of most excellent masonry and construction, and very accurately divided and cut, into 90 degrees and other subdivisions. The quadrants are of different sizes, some as much as 20 feet radius. The account of the Benares observatory is furthur illustrated by Wm. Hunter, esq. in a very elaborate article in the Asiatic Researches, vol. 5; in which he gives a full and particular description of the other four Hindu observatories.

At Pekin, in China, an imperial observatory was built in the 13th century, on the city walls. And in 1669, father Verbiest, a missionary jesuit, having been made president of the tribunal of the mathematics there, and chief observer, obtained permission from the emperor Cam-hi, to furnish it with instruments, a catalogue of which may be seen in Duhalde's Description of China.

Other observatories were afterwards built in China by the French missionaries, and by the Portuguese jesuits, who very much distinguished themselves by their improvements in astronomy. The instruments of the Pekin observatory are described as very large; but the divisions less accurate, and the contrivance less commodious, than the instruments made at that period in Europe. The chief were, a sextant 8 feet radius, a quadrant 6 feet radius, an azimuthal horizon; also a celestial globe, an armillary zodiacal sphere, each 6 feet diameter.

It is said that Copernicus, in 1540, was the first European who set an instrument in the meridian. But it is stated by Weidler, Bailly, and Costard, that the first regular observatory in Europe was erected at Cassel in 1561, by William, landgrave of Hesse, who furnished it with the best instruments the age could afford; and where it is said he made very accurate observations, in concert with his friend and correspondent, Tycho Brahé, who was then rising into great fame.

The next observatory in Europe, that deserves particular notice, was that of Tycho Brahé himself, which it seems owed its origin to a very extraordinary cause, the appearance of a new star of the first magnitude, in the constellation Cassiopeia. It was seen by different astronomers about the 10th Nov. 1572, when it seemed to break forth instantaneously, which added to the great astonishment that universally prevailed on the occasion. It was brighter than Jupiter or Venus, when nearest to the earth, and was visible to the naked eye at mid-day. After a short time it gradually declined, and in 16 months totally disappeared. Tycho Brahé was so impressed with this phenomenon, that he formed the resolution of making a new and accurate catalogue of all the stars, as there had been nothing of the kind regularly performed since the days of Hipparchus, who, it is remarkable, had been stimulated to the like undertaking by a similar cause, that is, by the sudden appearance and disappearance of a new star.

For this purpose, Tycho Brahé first proposed to settle at Basle, which afforded at once a pure atmosphere, and a ready communication with the learned men of Germany, Italy, and France. But the landgrave of Hesse wrote to Frederick the 2nd, king of Denmark, entreating him to encourage the astronomer to remain in his own country. In consequence the king assigned to him the small but fruitful island Huen, or Heven, in the Sound, as a fit situation for an observatory, and conferred on him also other princely grants and immunities: his majesty besides undertook to defray the charge of building and furnishing the observatory there, without any limitation of expense; a munificence which has immortalized his name. The first stone of the observatory was laid the 8th of August 1576, and the place was called Uranibourg, or the Heavenly City. It was a building of 60 feet square, and 70 feet in height, with four towers, all contrived for astronomical purposes. It was furnished with a noble collection of instruments, many of them invented by the astronomer himself. He had numerous assistants, whom he supported and instructed. Among his instruments was a celestial globe, of 6 feet diameter, said to have cost 1000*l.* It was after his death carried to Prague; next to Neis, and lastly to Copenhagen, where it was burnt in the great conflagration which happened there in 1728. Many of the instruments of this great astronomer were long preserved, but have been gradually lost; and his favourite city Uranibourg, which, in his time, was visited by kings and princes, has been long a heap of ruins, though occasionally visited by the learned. His celebrated sextant has been consecrated in the heavens as a constellation, under the breast of the Lion: on large globes and atlases it is marked *Sextans Uraniæ*, but on common ones only *Sextans*.

We shall now proceed to notice some observatories of a more modern date, beginning with those of France.

French Observatories.

The Royal, or now Imperial, Observatory of Paris, was built in 1667. It is 160 feet in front by 120 feet in breadth, and 90 feet high. Its vaults are 90 feet deep; so that it is 180 feet from top to bottom. A particular description of the building is given by Blondel, and the arrangement and disposition of the instruments in Bernoulli's Lettres Astronomiques, also in Lalande's Astronomie, and in Monnier's Histoire Celeste.

Besides the above building, new rooms have been constructed, close by the side of the observatory, where a large transit instrument and circle, by Ramsden, have been set up. In 1788 new vaults were made, and also a small observatory erected on the top of the building, which commands an extensive view of the horizon; and the king (Louis the 16th) established three observers here, that the course of observations might as little as possible be interrupted.

The following account of other observatories at Paris, given by Lalande in 1792, is worthy of notice here, as interesting in the history of practical astronomy.

The astronomers of the Academy had besides several private observatories erected in different parts of Paris, as the royal observatory was not sufficient for all. That of Monnier has been, from the year 1742, in the garden of the Capuchins. That of the Marine, which Joseph de l'Isle used in 1748 at the Hotel de Clugny, occupied by M. Messier. That of Lacaille still exists in the Mazarin-college. That of the palace of Luxembourg is above the Port Royal. Joseph de Lisle observed there, and Lalande likewise occupied it for some time. That of M. Pingré at the abbey of St. Généviève was built in 1756. There is one of Cagnoli's rue de Richlieu, which this able astronomer built at his own expense in 1785, when he still resided at Paris.

The observatory of the military school, built for M. Jeaurat in 1768, was occupied afterwards by M. d'Agelet. The late M. Bergeret, receiver-general of finances, constructed in 1774 a large mural quadrant of 8 English feet radius, the last and the best instrument made by the celebrated Bird. This instrument was obtained by the military academy, as well as an excellent transit instrument, and a parallactic telescope. M. d'Agelet made a great number of observations there from 1778 to 1785, when he left it to make a voyage round the world with La Perouse. In 1788, the changes made in the military school occasioned the demolition of this observatory; but it has been rebuilt, a little more to the west, with all necessary attention and expense, so that it is the most complete observatory at Paris. Lalande, having received the direction of it, began in 1789 to make a series of observations. M. le François Lalande, his relation and pupil, has also made a prodigious number of observations; and they observed. in 1791, more than ten thousand northern stars, with excellent instruments. An observatory was built in 1775, at the Royal-college, for the use of the professor of astronomy of this celebrated school. M. Geoffroy d'Assy built, in 1788, an observatory at his house, rue de Paradis, which was used by M. Delambre.

Such was the state of observatories at Paris in 1792. At present (1813) Delambre is the chief of the imperial university. Messier and Biot succeeded him at the Royal-college, now the College de France. Burckhardt is astronomer at the military school; Lefrançois Lalande resides at the Place de Cambray; and Bouvard superintends the imperial observatory, assisted by Aragon.

It may be noticed here, that the famous mural quadrant, with which Lalande and his relation determined the position of a great number of stars, as above-mentioned, has been consecrated in the heavens as a constellation, and is placed between Hercules, the Serpent, and Boötes. It is marked Quadrans Muralis, and contains 40 stars.

The following were the other observatories established in different parts of France, as stated by Lalande.

The *Marseilles* observatory, which has been rendered famous by the observations of Sylvabelle.

At *Toulouse*, the observatory of M. Darquier has been made sacred by the zeal and abilities of this learned man. Observatories have also been built in the same city by M. Garipuy and M. Bonrepos. Here astronomy has been more successfully cultivated than in any other provincial city in France. The principal observatory is at present (1813) under the superintendence of M. Vidal.

At *Lyons*, the College observatory, which was built by father St. Bonnet, is a very fine edifice, on an elevated situation.

At *Dijon*, M. Necker, about the year 1780, converted the tower of the king's lodge to an observatory, and the abbé Bertrand has made very accurate observations there.

At *Montpellier* there has long been an observatory erected on one of the towers of the city; where M. Ratte and Poitevin have distinguished themselves as able astronomers.

At *Beziers*, the bishop's tower was converted to an observatory, where some interesting observations have been made by M. Bouillet, particularly on Saturn's ring.

At *Avignon*, an observatory was built by father Bonfa so early as 1683; and it has been since occupied by a succession of learned ecclesiastics, who have distinguished themselves in practical astronomy.

At *Strasburg*, Brackenhoffer, professor of mathematics, had an observatory over the gates of the city, and he has been succeeded by Herzenschneider in 1790.

At *Bourdeaux* is an observatory 75 feet high, and 20 feet square. It is situated in the finest part of Tournay, in latitude 45°. Here M. Turgot procured a complete set of observations to be made on the length of a pendulum vibrating seconds; upon which father Boscovich has made an interesting memoir.

At *Brest* a small observatory was built for the naval academy, and plans have been set on foot for erecting a more considerable edifice.

At *Rouen* there is an observatory belonging to M. Bouin, in which he has made many good observations.

At *Montauban* the duc de la Chapelle founded an observatory, where he himself has made many accurate and interesting observations, particularly of the transit of Venus over the sun in 1769.

German Observatories.

In Germany a great number of observatories have been established, and that country has produced also several very able astronomers.

At *Berlin*, Frederic the 1st, king of Prussia, founded an observatory in 1711, under the direction of Leibnitz, who was president of the Academy of Sciences there. It is a large square tower, very steady. Here Grischow and Kies made various observations: and Lalande also observed here about the year 1752, where, he says, he raised enormous pillars, to which he attached the mural quadrants, north and south. (Memoires de l'Academie, 1751 and 1752.) King Frederic the 2d added a very fine building to it, where the Academy of Sciences of Prussia has held its assemblies. M. Bode has been many years the astronomer-royal there, and has distinguished himself both as an accurate observer, and as the publisher of the most complete celestial atlas extant, entitled Uranographia, which is accompanied with a well arranged catalogue of the stars, and an interesting history of the constellations.

At *Vienna*, the empress Maria Theresa built an observatory in the year 1755 for the university, and furnished it with many superb instruments. There is also one belonging to the academical college, which was built and endowed by the Jesuits in 1735, and it is also furnished with very fine instruments, chiefly made by English artists, and a succession of very learned men have observed there. The reputation of the university observatory was maintained for many years by the abbé Maximilian Hell, who conducted the Vienna Ephemeris, and the work is now continued by M. Treisneckir, his successor.

At *Gottingen* there is an observatory memorable by the labours of Tobias Mayer, and by those more recently of Harding, who discovered the planet Juno in 1804.

At *Nuremberg* an observatory was built so early as the year 1678, and another in 1692. M. Zimmert and M. Wuzzelbau have distinguished themselves here both as able authors and accurate observers.

At *Cassel* an observatory was built, in 1714, by Charles I, landgrave of Hesse, heir to the territories and taste of the celebrated William, the early friend and fellow-labourer of Tycho Brahé.

In 1740 an observatory was built at *Griessen*; and in 1768 at Ourtsbourg, in Franconia. In 1788 one was built at Leipsic, on an old tower of great firmness. Observatories have been also erected and supported with great credit at Manheim, Cremsmunster, Lambach, Polling, Prague, and Gratz.

At *Bremen* there is an observatory belonging to Dr. Olbers, an eminent physician, who has rendered his name immortal by the discovery of the two new planets, Pallas and Vesta.

At *Lilienthal*, near Bremen, M. Schroeter, governor of the district, erected an observatory about the year 1786, and furnished it with excellent instruments. He is highly celebrated as an accurate and interesting observer, particularly of the surfaces and rotations of the planets and the moon. He approaches nearer than any other astronomer to Dr. Herschel in telescopic discoveries.

At *Seeberg*, near Gotha, a considerable observatory was built, in the year 1788, by the duke of Saxe Gotha, and he appointed M. Zach, now baron Zach, the superintendant, who has highly distinguished himself as a profound and accurate astronomer. In 1798 he was visited by Lalande, when, according to Voiron (Histoire de l'Astronomie, pa. 369), all the great astronomers of Germany met at Gotha, to see the patriarch of astronomy, and to pay him their homage. This observatory is reckoned one of the most beautiful and complete in Europe; it is situated on a fine elevation, about a league from the town. There is here a large transit, with two murals of 8 feet radius, and a circle of 8 feet diameter, all by Ramsden and his successor Berge.

At *Brunswick* there is an observatory belonging to Dr. Gauss, well known by his determinations of the orbits of the new planets, and other important labours.

In *Hungary* there are observatories at Buda, Tyrnau, and Erlau. Similar establishments are also at *Greiffswalde* in Pomerania, and at *Mittau* in Courland.

In *Poland* there is an observatory at *Cracow*, and another at *Wilna*: the latter was built and richly endowed by the countess Puzynina, a lady of fine genius as well as liberality. It was finished in 1753, and the instruments with which it is furnished were of great variety and value. In 1765 the king of Poland, by letters patent, gave it the title of Royal observatory, and appointed the learned Jesuit Poczobut astronomer-royal, who, in 1788, added another observatory, which he furnished with new instruments, chiefly made by Ramsden.

In *Sweden* observatories have been built at Stockholm and Upsal: that at Stockholm was founded in 1746, by the Academy of Sciences. In 1753, Wargentin was appointed astronomer to it, and in 1783 he was succeeded by Nicander. This observatory is situated on a hill north of the town, and contains a good collection of instruments, all made by English artists.

The observatory at *Upsal* was built and endowed in 1739 by the king of Sweden: it was first superintended by the celebrated Celsius, who has been followed by a succession of able astronomers, particularly Hooker and Wargentin. The latter is well known as the author of the tables of Jupiter's satellites.

At *Dantzic* there was an old observatory, celebrated as having been used by Hevelius, who has given a full description of it in his work, entitled Machina Cœlestis. A new observatory was also built in that city in the year 1778, which is at present superintended by Dr. Wolff.

At *Copenhagen* the famous astronomical tower was finished in 1656. It was built by King Christian IV, at the recommendation of Longomontanus, and has been for many years under the management of Mr. Bugge, who is celebrated as a very able astronomer. In his collection of observatories, he states that the kings of Denmark had established observatories in Norway, Iceland, and Greenland.

In *Holland* attention was paid to practical astronomy while it was a maritime state; but the science has of late been much neglected. In 1690 an observatory was erected on the college of the university; and at Utrecht an ancient tower was, in 1726, converted into an observatory. Here the celebrated Van Musschenbroek observed for many years with great accuracy, and he was succeeded by M. Hennert.

In *Spain* observatories have been built at Cadiz, Madrid, Seville, and Carthagena. The observations made at Cadiz (at the Marine academy) by Miguel and Varilla, have been published in two volumes, which also contain a catalogue of the instruments of the observatory, chiefly constructed by French artists; and hence the observatories of Spain differ very little from those of France. Of late years, however, English instruments have been introduced there.

At *Lisbon*, in 1728, King John the 5th had an observatory erected at his palace, which was well furnished, and accurate observations have been made there by the Jesuits, who also erected an observatory at their own college of St. Anthony, where father Carbon, in 1726, made good observations on the satellites of Jupiter. See Phil. Trans. vol. 35, pa. 408.

In 1787, a royal observatory was constructed at the Chateau de St. George, in Lisbon, which was superintended by M. Custodio Gomez. There is also one at *Coimbra*, which contains a fine equatorial by Troughton.

At *Petersburg* an observatory was built, in 1725, by the czar Peter, who showed great zeal for science in general, and particularly for astronomy. When he was in England, some years before that period, he visited the Royal Observatory at Greenwich, where he examined both the building and the instruments with very great attention. The observatory which he afterwards built is one of the most magnificent in Europe. It is 130 feet high, with three stories, all fit for astronomical purposes. M. Delisle has made, according to Lalande, a great number of excellent observations here, which are preserved in manuscript in the marine depôt.

At *Moscow* an observatory was built a few years ago, and furnished with some excellent English instruments, chiefly by Cary; but it is probable that they have been destroyed in the late conflagration of that city.

In *Italy*, practical astronomy has been cultivated with much assiduity and success during the last century, chiefly by ecclesiastics, and particularly by the Jesuits.

At *Rome*, cardinal Zelada constructed, at his own expense, on the southern part of the Roman college, a very fine observatory, with the large sector of father Boscovich, and other instruments by Ramsden and Dollond. The abbé Calandrelli observed here with great attention and accuracy for many years. Other buildings of a similar description have been erected in different parts of Rome.

At *Bologna* a magnificent observatory was built in 1714, in the palace of the Institute, by the munificence of the celebrated count Marsigli; and pope Benedict 14 gave afterwards a large sum of money towards the purchase of instruments. Here a succession of able astronomers have observed, among whom may be mentioned Manfredi, Zanotti, Canterzani, &c.

At *Pisa* the observatory is in the form of a tower. It was built in 1730, at the expense of the university, and supplied with superb apparatus made by Sisson, Short, Graham, &c. Perelli observed here for many years, and had for a successor M. Slope, who published an excellent collection of observations in 1789.

At *Milan* there is an observatory, which is reckoned one of the most useful in Italy. It was built in 1765, at the cost of the college of the Jesuits, chiefly through the zeal of father Pallavicini, and under the direction of father Boscovich, who also contributed liberally to the expense. The instruments have been made with great care by the principal French and English artists. Among the observers may be also mentioned Reggio, Oriani, and Cesaris.

At *Florence*, father Ximenes erected an observatory at the college of Jesuits, which contains a quadrant by Toscanelli, larger than any other known, with which he made observations to prove the secular diminution of the obliquity of the ecliptic. At his death he bequeathed the whole to the college. In 1772 the grand duke Leopold built an observatory, which M. Fontana superintended, and in 1786 several fine instruments by Ramsden were added to it.

At *Turin* father Beccaria erected a small observatory; but in 1790 a large one was built at a very considerable expense, by the king of Sardinia, at the Royal College of Nobles, and the direction of it given to the abbé Caluso.

At *Venice* an observatory was constructed by father Panigai, and a small one near the town by M. Miotti. One was also built at *Parma* by father Belgrado, and another at *Brescia* by father Cavalli.

At *Verona*, Cagnoli, eminent both as a mathematician and astronomer, erected an observatory at his own expense, in 1787, and placed in it the best instruments, with which he has made very accurate and important observations, particularly on the precession of the equinoxes, and on the places of 473 northern stars, and 28 southern, of which he has made a catalogue. In these determinations he has been perhaps more attentive than any other astronomer to the minute changes of refraction, and to the aberration of light.

At *Padua* there is an observatory, which in 1778 was furnished with instruments, chiefly made by Ramsden. It has been many years under the direction of M. Toaldo, who has published several useful works, especially a treatise on meteorology, which gained him the prize at the academy of Montpellier.

In some of the islands of the Mediterranean observatories have also been established. We shall, however, notice only those of *Malta* and *Sicily*.

In 1783, the grand-master Emmanuel de Rohan, an amateur and enlightened protector of science, invited to Malta chevalier d'Angos, a skilful astronomer, who converted a tower of the palace into an observatory, which was furnished with the finest instruments that could be procured. In a few years he made a great number of valuable observations, which he intended to publish, but in March 1789, the observatory having caught fire, the instruments were broken, and the papers burnt, a serious loss to astronomy, particularly as this was the most southern observatory of Europe, in latitude 36°.

At *Palermo* an observatory was constructed in the palace of the viceroy, under the direction of father Piazzi, who went to Paris in 1787 to study astronomy, and who afterwards visited England, in order to consult the principal artists on the construction of instruments. In 1789 he returned to Palermo, and added to the apparatus a fine transit instrument, and a complete circle, made by Ramsden. His first labours were directed to the formation of a correct catalogue of stars, and, as a foundation, he chose Wollaston's catalogue, and particularly, as his chief points of reference, Dr. Maskelyne's 36 stars. The positions of some of the larger stars he verified by nearly a hundred observations, and in the prosecution of this task, in 1801, he discovered a new planet, which he named Ceres, in honour of Sicily, as that island was, on account of its fertility, anciently consecrated to the goddess Ceres. This discovery was the more important, as it excited the curiosity and research of other astronomers, by which three more planets have been since discovered.

English Observatories.

The *Greenwich Observatory*, or the Royal Observatory of England, was built and endowed by King Charles II, who, to use the words of Bailly, "well knew how essential astronomy was to a maritime and commercial people like the English, who aspired to the empire of the seas." This building was erected on the site of the ancient moated tower of Duke Humphrey, uncle to Henry 6, and the first stone of it was laid Aug. 10, 1675, by Mr. Flamsteed, who had been appointed astronomer-royal. It is situated on the highest eminence of Greenwich park, about 160 feet above low-water mark. The soil here is particularly favourable for such an institution, being of a flinty gravel, through which the rain soon passes, and thus the atmosphere is generally dry, which contributes to the preservation of the instruments, as well as to the uniformity of refraction.

This establishment comprehends two principal buildings, one of which is the observatory, and the other the dwelling-house of the astronomer-royal. The observatory is an oblong edifice, running east and west, and containing four rooms or apartments on the ground-floor. The first, or most easterly room, has been lately erected for the reception and fitting up of a very fine transit circle, by Troughton, and a clock of great value by Hardy.

The next apartment is the transit room. It has a double sloping roof, with sliding shutters, which are opened both north and south, with great ease, by pulleys. The transit instrument, which is 8 feet long, and the axis 3 feet, is suspended on two stone pillars. This instrument is famous as having been used by Halley, Bradley, and Maskelyne. It was originally made by Bird, and has been successively improved by Dollond and Troughton. The astronomical or transit clock, which is attached to a stone pillar, was made by Graham, and has been rendered very accurate by Earnshaw.

The third apartment is the assistant observer's library and place for calculation; and the western apartment of the building is the quadrant room. Here is erected a stone pier, running north and south, to which are attached two mural quadrants, each of 8 feet radius. That on the eastern face, which observes the southern meridian, was made by Bird, and the other, which observes the northern, by Graham. Suspended to the western wall is the famous zenith sector, with which Bradley made the observations at Kew and Wanstead, that led to the discoveries of the aberration of light, and the nutation of the earth's axis.

South of the quadrant room is a small wooden building for making occasional observations in any direction, where only the use of a telescope, and an accurate knowledge of the time, are required. It is furnished with sliding shutters on the roof and sides, to view any point of the hemisphere, from the prime vertical down to the southern horizon. It contains some excellent telescopes, particularly a forty-inch achromatic, with a triple object-glass, and a five-feet achromatic, both by Dollond; with a six-feet reflector, by Dr. Herschel.

To the north of the observatory and east of the house are two small buildings, covered with hemispherical sliding domes, in each of which is an equatorial sector, by Sisson, and a clock, by Arnold. These are chiefly used for observing comets.

Of the dwelling-house, the lower apartments are occupied by the astronomer-royal, and over them is a large octagonal room, which contains a great variety of astronomical instruments, with a library, consisting chiefly of scientific and scarce works. On the top of the house is an excellent camera obscura, which could not be better placed for the exhibition of interesting objects.

In Flamsteed's time a well was sunk in the south-east corner of what is now the garden, behind the observatory, for the purpose of seeing the stars in the day-time, and observing the earth's annual parallax. It was a hundred feet deep, with stone stairs down to the bottom: but it has been long arched over, as the improvements in the telescope have rendered it unnecessary for astronomical purposes.

The observations made at the Royal Observatory are universally allowed to possess an unrivalled degree of accuracy. M. Delambre, in a paper composed by him on the life and labours of Dr. Maskelyne, and read before the National Institute, Jan. 4, 1813, makes the following remark. "He (Dr. Maskelyne) has given a catalogue of stars, not numerous, but so accurate, as to have served, almost solely for the last 30 years, as the foundation of all astronomical researches. In short, it may be said of the four volumes of Observations which he has published, that if, by a great revolution, the sciences should be lost, and that this collection only were saved, there would be found in it sufficient materials to construct almost an entire edifice of modern astronomy; which cannot be said of any other collection."

The following are the names of the astronomers who have officiated here in succession, with the times of their services respectively: Flamsteed, 43 years; Halley, 23 years; Bradley, 20 years; Bliss, 2 years; and Maskelyne, 46 years. Maskelyne has been succeeded by John Pond, Esq. F.R.S. who was appointed astronomer-royal in February 1812.

Dr. Herschel's Observatory at Slough, near Windsor, though not a fixed one, will ever claim a distinguished place in the history of astronomical institutions. It differs from all other observatories in plan and apparatus; and it exceeds all others in the number of its discoveries.

In describing this observatory, it should be premised, that Dr. Herschel's labours derive a peculiar character and interest from the circumstance, that his discoveries are the result of his own inventions. For to his profound knowledge of astronomy he unites that of optics, both in theory and practice, by which he has been enabled to cast and polish mirrors for reflecting telescopes, greatly superior to any others, not only in magnifying power, but in collecting, or, as it were, preserving light, by which vision is wonderfully extended, and which he very expressively denominates "the power of penetrating into space." The telescopes, which are all made under his direction, are of various sizes, from two feet in length up to forty feet, and the apparatus and machinery with which they are mounted are also of his invention, and exhibit a very ingenious display of mechanism.

As his larger telescopes could not be conveniently managed within the cover of a building, they are mounted in the open air, where they stand pointing to the heavens in different directions, and make a most magnificent and impressive appearance. Thus they are placed in what has been called the primitive observatory of man, "non sub tecto sed sub cœlo in puro dio."

His largest telescope is 40 feet long and 5 in diameter. It contains a mirror of about a ton weight; and this great instrument, with nearly an additional ton of cases, &c, is managed by a very slight force. It is placed on a large circular frame, which turns on rollers, and the top is suspended by ropes from very lofty ladder-work. Thus, by a system of wheels, pinions, racks, and pulleys, the motions, both horizontal and vertical, are given, and hence any celestial object is readily found and commodiously viewed. It was finished in 1787, and on the first trial a new satellite of Saturn was discovered by it, and a second soon after. A very full and accurate account of his inventions and discoveries, as well as a particular description of his telescopes and their apparatus (with plates), will be found in the Philosophical Transactions, to which he has been a most important contributor, having supplied that work with nearly 70 elaborate and ingenious communications.

Two of his telescopes, of smaller size, are famous in the annals of discovery. The first is a two-feet Newtonian reflector, with which his sister Miss Carolina Herschel, whose astronomical attainments do great honour to her sex, discovered six comets; and the other is his seven-feet reflector, by which he discovered the Georgian planet at Bath, in 1781. This telescope has, in consequence of the discovery, been made a constellation in the heavens with the universal approbation of astronomers. It is placed between Gemini, the Lynx, and Auriga, and contains 81 stars. In Bode's atlas it is engraved with its apparatus, and marked Telescopium Herschelii. Dr. Herschel, though in his 76th year, is still an active and indefatigable observer. He was born at Hanover, Nov. 15, 1738, a period which will be ever memorable in the history of astronomy.

The King's private Observatory in Richmond gardens is extremely beautiful in structure and apparatus, as well as in situation. It was built, in 1768, by order of his present majesty George 3, who, it is said, made several observations here, particularly of the transit of Venus in 1769. It contains a fine transit instrument, a zenith sector, and a mural arc, with several good telescopes, especially a ten-feet reflector of Dr. Herschel's. Here is a superb equatorial on the top of the building, which is covered with a moveable roof. There are also two fine orreries, with an excellent collection of philosophical instruments, and some cases of minerals and other natural curiosities. It was built under the direction of Dr. Demainbray, and has been, for some years, in the care of Mr. Rigaud.

Oxford Observatory is a most magnificent structure, and the instruments perfectly correspond with the building. It was begun in 1772, from very ample funds bequeathed by Dr. Radcliffe, and the land on which it stands was the gift of the duke of Marlborough. The transit instrument, which is 10 feet long, shows very small stars in the day-time. It is said to have cost 150 guineas, the zenith sector 200 guineas, and the two mural quadrants 600 guineas. There are also very excellent telescopes and clocks here, the former by Herschel and Dollond, and the latter by Shelton. It was built under the direction of Dr. Hornsby, professor of astronomy in the university, who observed here for many years, and he has been succeeded by Dr. Robertson, the present worthy professor of astronomy. The observations are all registered, and consist chiefly of the right ascensions and zenith distances of the sun, moon, planets, and fixed stars. In Dr. Hornsby's time, the registry was sometimes broken from ill health; for he had no assistant observer; but one has been of late added to the establishment, so that the observations will not, in future, be liable to the like interruptions.

At *Cambridge* there have been small observatories at Christchurch, Trinity, St. John's, &c, and a plan is said to be now on foot for erecting one upon a great scale, and worthy the scientific fame of that learned university.

Portsmouth Observatory.—At the Royal Marine Academy, Portsmouth, there is an observatory under the direction of Mr. Professor Inman, which is of peculiar utility, both in teaching the pupils practical astronomy, and in finding the rate of time-keepers for seamen.

At *Christ's Hospital*, Mr. Wales (who had served under Dr. Maskelyne and Capt. Cook) erected a small observatory at his own expense, when he became master of the royal mathematical school there.

The Royal Society have at Somerset House a small observatory, which is generally superintended by the secretary for the time being.

At *Highbury House*, near Islington, an observatory was built in the year 1787, by Alexander Aubert, Esq, which, for perfection of plan and splendour of apparatus, perhaps has never been equalled by any private individual. This gentleman, whose scientific and liberal pursuits deserve honourable mention, died in the year 1806, and his grand collection of instruments was disposed of by auction, and of course dispersed. Similar notice may be taken of other observatories contemporary with that of Highbury, particularly those of Count Bruhl at Harefield, Sir George Shuckburgh at Shuckburgh, William Larkins, Esq. at Blackheath, and the Hon. Charles Greville at Milford, all of which were on a great scale, and have been discontinued after the demise of the owners. Thus in private observatories, though the astronomers may bequeath their apparatus to their heirs, they cannot transfer either their taste or their science. It is only in public establishments that permanence can be expected.

Among the private observatories of the present day, the following alphabetical list may be also mentioned.

Blackheath -	Stephen Groombridge, Esq.
Blenheim -	Duke of Marlborough.
Cambridge -	Rev. Mr. Catton.
Chiselhurst -	Rev. Francis Wollaston.
Derby -	William Strutt, Esq.
East Sheen -	Rev. William Pearson.
Finsbury Square	Dr. Kelly.
Godwood -	The Duke of Richmond.
Gosport -	Dr. William Burney.
Hackney Wick -	Colonel Beaufoy.
Hayes -	William Walker, Esq.
Highbury Terrace	Capt. Huddart.
Hoddesdon -	William Hodgson, Esq.
Islington -	Gavin Lowe, Esq.
Paragon, Southwark	James Strode Butt, Esq.
Park-lane -	Sir Henry Englefield, Bart.
Rose Hill, Sussex	John Fuller, Esq.
Sherburn -	Earl of Macclesfield.
St. Ibbs, Hitchin -	Mr. Professor Lax.
Woolwich -	Royal Mil. Acad.

Scotch Observatories.—In the different universities of Scotland professorships of astronomy have been established; but it has been here, as in most other universities, the theory of the science has been more attended to than the practice. At Edinburgh and Aberdeen there have been, however, observatories; and at Glasgow there is also a small one belonging to the college, but of late a magnificent one has been erected by a society of gentlemen, which is likely, when finished, to be very useful as well as honourable to that commercial city.

Irish Observatories.—In Ireland two observatories have been established on a great scale, the one at Dublin, and the other at Armagh. The observatory belonging to Trinity-college, Dublin, commonly called the Dublin observatory, was begun in the year 1783. It was founded by Dr. Francis Andrews, provost of that college, who bequeathed a large income for this purpose. The apparatus are, a transit instrument of 6 feet focal length, with a 4-

feet axis, bearing 4 inches and a quarter aperture, with three different magnifying powers up to 600. An entire circle of 10 feet diameter on a horizontal axis for measuring meridian altitudes. An equatorial instrument, with circles of 5 feet in diameter: and an achromatic telescope, mounted on a polar axis, and carried by an heliostatic movement. Clocks were also ordered from Mr. Arnold, without any limitation of price.

The situation chosen for the observatory is on elevated ground, about four English miles N. W. of Dublin. The foundation is a solid rock of limestone, of several miles extent; and the soil is very favourable, being a calcareous substance called limestone gravel, which is remarkable for absorbing the rain, and thus contributing to a dry atmosphere. The plan of the building unites at once both elegance and convenience. In the centre is a magnificent dome of three stories high, with a moveable roof for the equatorial instrument, which is placed on a pillar of 16 feet square, of the most substantial masonry, and surrounded by a circular wall at a foot distance, that supports the moveable dome, and also the floors, which in no part touch the pillar: thus, no motion of the floor or wall can be communicated to the instrument. The aperture for observation in the dome is two feet and a half wide.

The most important erection belonging to this establishment is behind the main building, and at right angles to it, in order to obtain an uninterrupted view both north and south. This is the meridian or transit room, which contains both the transit instrument and the circle. It is 37 feet long, by 23 broad, and 21 high. Fine pillars of Portland stone are erected for both instruments on the most firm basis, and the floor is so framed as to let all the pillars rise totally detached from it. The clocks are attached to pillars of the greatest steadiness also: they were made by Arnold, who exerted his best skill, and are finished in a masterly manner; the pallets are of ruby; and all the last holes of the movement jewelled; the suspension springs are of gold, with Arnold's own five-barred pendulum, and cheeks capable of experimental adjustment, so as to make all vibrations isochronal, whatever may be the excursion of the pendulum.

The Rev. Dr. Usher, the first astronomy professor, did not long enjoy the pleasures of astronomy. He died in 1790, before the instruments had been all supplied. He was succeeded by the Rev. Dr. Brinkley, who was reared under Dr. Maskelyne, and had distinguished himself at Cambridge by profound analytical investigations, and who has since greatly enriched the Transactions of the Royal Irish Academy by mathematical and astronomical communications.

From a new 8 feet circle, by Berge, important results are expected, particularly on parallax, aberration of light, and refraction. Dr. B. has been for some time engaged in a series of observations with a view to explain the cause of variations which he has found in the zenith distances of certain stars at different times, which do not seem explicable by any cause at present generally allowed. He has found a difference between the zenith distances of α Lyræ, when in opposition and conjunction, which may be explained by a parallax of about 2 seconds. The new transit circle just erected at Greenwich possesses advantages for such purposes, and great hopes may therefore be formed from the concurrent operations of those two instruments.

Armagh Observatory.—At Armagh, the metropolitan city of Ireland, and anciently the seat of a large university, an observatory has been erected and endowed in 1793, by the most reverend Richard lord Rokeby, then primate of Ireland. It is erected on the summit of a gently rising hill, about 90 feet above the general level of the town. The tower, which joins the dwelling-house, contains a very fine equatorial by Troughton, fixed on a large pillar, which is raised so high that the instrument in the dome can overlook all the buildings. To the east of the house is a range of buildings for the transit room, and other astronomical purposes. The principal instruments, besides the equatorial and transit, are a ten-feet sextant by Troughton; a ten-feet reflecting telescope by Dr. Herschel; a five-feet triple object-glass achromatic telescope by Dollond; and also a fine night glass on an equatorial stand. The clocks are by Earnshaw of London, and Crossthwaite of Dublin.

In this establishment a liberal income is allowed to the principal astronomer, and a good salary to his assistant. It has been superintended from the beginning by the Rev. James Archibald Hamilton, D. D. dean of the cathedral church of St. Coleman, Cloyne. The registered observations here, are those made with the transit instrument and equatorial; and also an account of the temperature and weight of the atmosphere. Of these, a series of about 18 years is preserved. The right ascensions of the sun and moon, compared with the fixed stars, are regular and unbroken; but their north polar distances have not been so constantly taken, as they are only observed by the principal astronomer, whose pastoral duties must occasionally interfere with his astronomical labours.

A TABLE *of the Longitudes and Latitudes of the principal Observatories of Europe, as deduced from the most recent and accurate Determinations.*

Names of Places.	Latitude North.	Longitude from Greenwich in Time.	Names of Places.	Latitude North.	Longitude from Greenwich in Time.
Amsterdam -	$52^{\circ}\ 22'\ 17''$	$0^{h}\ 19^{m}\ 32^{s}$ E	Cambridge -	52 12 36	0 0 $17''$ E
Armagh - -	54 21 15	0 26 30 W	Cassel - -	51 19 20	0 38 7 E
Berlin - -	52 31 45	0 53 26 E	Coimbra - -	40 12 30	0 33 37 W
Blenheim -	51 50 28	0 5 25 W	Constantinople	41 1 27	1 55 41 E
Bologna - -	44 29 56	0 45 23 E	Copenhagen -	55 41 4	0 50 18 E
Bremen - -	53 4 46	0 35 12 E	Cracow - -	50 3 52	1 19 44 E
Breslaw - -	51 6 30	1 8 11 E	Cremsmunster -	48 3 29	0 56 32 E
Brunswick -	52 15 29	0 42 8 E	Dantzic - -	54 20 48	1 14 32 E
Buda - -	47 29 44	1 16 10 E	Dorpat - -	58 22 48	1 46 55 E
Cadiz - -	36 32 1	0 25 10 W	Dresden - -	51 3 9	0 54 50 E

Names of Places.	Latitude North.			Longitude from Greenwich in Time.		
Dublin - -	53°	23′	14″	0	25	20 w
Eisenberg -	50	57	58	0	39	50 E
Florence - -	43	46	41	0	45	3 E
Genoa - -	44	24	59	0	35	52 E
Glasgow - -	55	51	32	0	17	4 w
Gotha (Seeberg)	50	56	7	0	42	56 E
Gottingen -	51	31	54	0	39	42 E
Greenwich -	51	28	40	0	0	0
Highbury House	51	33	30	0	0	23 w
Hyeres - -	43	7	2	0	24	31 E
Leipsic - -	51	20	44	0	49	28 E
Leyden - -	52	9	30	0	17	55 E
Lilienthal -	53	8	25	0	35	35 E
Lisbon - -	38	42	50	0	36	34 w
London (Chr. Hos.)	51	30	57	0	0	24 w
Madrid - -	40	25	18	0	14	47 w
Manheim -	49	29	18	0	33	55 E
Marseilles -	43	17	50	0	21	29 E
Milan - -	45	28	2	0	36	45 E
Mirepoix - -	43	5	19	0	7	30 w
Mittau - -	56	39	6	1	34	51 E
Montauban -	40	0	55	0	13	19 E
Montpelier -	43	36	29	0	15	31 E
Moscow - -	55	45	45	2	30	12 E
Munich - -	48	8	20	0	46	20 E
Naples - -	40	50	15	0	57	5 E
Nuremberg -	49	26	55	0	41	17 E
Oxford - -	51	45	38	0	5	2 w
Padua - -	45	24	2	0	47	32 E
Palermo - -	38	6	44	0	53	21 E
Paris - -	48	50	14	0	9	21 E
Petersburg -	59	56	23	2	1	13 E
Pisa - -	43	43	11	0	41	26 E
Portsmouth Acad.	50	48	2	0	4	24 w
Prague - -	50	5	19	0	57	41 E
Ratisbon - -	49	0	58	0	48	26 E
Richmond -	51	28	8	0	1	15 w
Rome - -	41	54	1	0	49	51 E
Slough - -	51	30	20	0	2	24 w
Stockholm -	59	20	31	1	12	13 E
Strasburgh -	48	34	56	0	30	59 E
Toulouse - -	43	35	46	0	5	46 E
Turin - -	45	4	14	0	30	40 E
Upsal - -	59	51	50	1	10	36 E
Utrecht - -	52	5	12	0	20	27 E
Venice - -	45	25	54	0	49	24 E
Verona - -	45	26	6	0	44	1 E
Vienna - -	48	12	36	1	5	31 E
Viviers - -	44	29	13	0	18	41 E
Wilna - -	54	41	2	1	41	10 E

Observatory *Portable*. See Equatorial.

OBTUSE *Angle*, one that is greater than a right-angle.

Obtuse-*angled Triangle*, is a triangle that has one of its angles obtuse: and it can have only one such.

Obtuse *Cone*, or Obtuse-*Angled Cone*, one whose angle at the vertex, by a section through the axis, is obtuse.

Obtuse *Hyperbola*, one whose asymptotes form an obtuse angle.

Obtuse-*angular Section of a Cone*, a name given to the hyperbola by the ancient geometricians, because they considered this section only in the obtuse cone.

OCCIDENT, or Occidental, west, or westward, in Astronomy; a planet is said to be occident, when it sets after the sun.

Occident, in Geography, the westward quarter of the horizon, or that part of the horizon where the ecliptic, or the sun's place in it, descends into the lower hemisphere.

Occident *Equinoctial*, that point of the horizon where the sun sets, when he crosses the equinoctial, or enters the sign Aries or Libra.

Occident *Estival*, that point of the horizon where the sun sets at his entrance into the sign Cancer, or in our summer when the days are longest.

Occident *Hybernal*, that point of the horizon where the sun sets at midwinter, when entering the sign Capricorn.

Occidental *Horizon*. See Horizon.

OCCULT, in Geometry, is used for a line that is scarce perceivable, drawn with the point of the compasses, or a black-lead pencil. Occult or dry lines are used in several operations; as the raising of plans, designs of building, pieces of perspective, &c. They are to be effaced or rubbed out when the work is finished.

OCCULTATION, the obscuration of any star or planet, by the interposition of the body of the moon, or any other planet.—The occultation of a star by the moon, if observed in a place whose latitude and longitude are well determined, may be applied to the correction of the lunar tables; but if observed in a place whose latitude only is well known, it may be applied to the determining the longitude of the place.

Circle of Perpetual Occultation. See Circle.

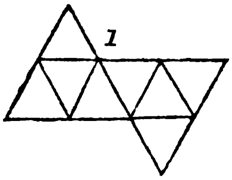
OCEAN, the vast collection of salt water, which encompasses most parts of the earth. By computation it appears that the ocean takes up considerably more of what we know of the terrestrial globe, than the dry land does. This is perhaps easiest known, by taking a good map of the world, and with a pair of scissars clipping out all the water from the land, and weighing the two parts separately: by which means it has been found, that the water occupies about two-thirds of the whole surface of the globe.

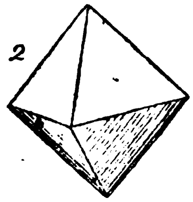
The great and universal ocean is sometimes, by geographers, divided into three parts. As, 1st, the Atlantic and European ocean, lying between part of Europe, Africa, and America; 2d, the Indian ocean, lying between Africa, the East-Indian islands, and New Holland; 3d, the Pacific ocean, or great south sea, which lies between the Philippine islands, China, Japan, and New Holland on the west, and the coast of America on the east. The ocean also takes divers other names, according to the different countries it borders on: as the British ocean, German ocean, &c. Also according to the position on the globe; as the northern, southern, eastern, and western oceans.

The ocean, penetrating the land at several straits, loses its name of ocean, and assumes that of sea or gulph; as the Mediterranean sea, the Persian gulph, &c. In very narrow places, it is called a strait, &c.

OCTAEDRON, or Octahedron, one of the five regular bodies; contained under 8 equal and equilateral triangles.—It may be conceived as consisting of two quadrilateral pyramids joined together at their bases.

To form an Octaedron. Join together 8 equal and equilateral triangles, as in fig. 1; then cut the lines half through,





and fold the figure up by these cut lines, till the extreme edges meet, and form the octaedron, as in figure 2.—In an octaedron, if

A be the linear edge or side,
B its whole surface,
C its solidity, or solid content,
R the radius of the circumscribed sphere, and
r the radius of the inscribed sphere: Then

$$A = r\sqrt{6} = R\sqrt{2} = \tfrac{1}{6}B\sqrt{3} = \sqrt[3]{\tfrac{3}{2}C\sqrt{2}}.$$
$$B = 12r^2\sqrt{3} = 4R^2\sqrt{3} = 2A^2\sqrt{3} = 6\sqrt[3]{\tfrac{1}{2}C^{2}}\sqrt{3}.$$
$$C = 4r^3\sqrt{3} = \tfrac{4}{3}R^3 = \tfrac{1}{3}A^3\sqrt{2} = \tfrac{1}{18}B\sqrt{(B\sqrt{3})}.$$
$$R = r\sqrt{3} = \tfrac{1}{2}A\sqrt{2} = \tfrac{1}{2}\sqrt{B\sqrt{\tfrac{1}{3}}} = \sqrt[3]{\tfrac{3}{4}C}.$$
$$r = \tfrac{1}{3}R\sqrt{3} = \tfrac{1}{6}A\sqrt{6} = \tfrac{1}{6}\sqrt{(B\sqrt{3})} = \tfrac{1}{2}\sqrt[3]{C\sqrt{\tfrac{4}{3}}}.$$

See my Mensuration, pa. 188, 4th edition.

OCTAGON, is a figure of 8 sides and angles; which, when these are all equal, is also called a regular one, or may be inscribed in a circle.

If the side of a regular octagon be s; then

Its area $= 2s^2 \times (1 + \sqrt{2}) = 4{\cdot}8284271s^2$; and the radius of its circumsc. circle $= \dfrac{s}{\sqrt{(2-\sqrt{2})}}$.

Octagon, in Fortification, denotes a place that has 8 sides, or 8 bastions.

OCTANT, the 8th part of a circle.

Octant, or Octile, means also an aspect, or position of two planets, when their places are distant by the 8th part of a circle, or 45 degrees.

OCTAVE, or 8th, in Music, is an interval of 8 sounds; every 8th note in the scale of the gamut being the same, as far as the compass of music requires.

Tones, or sounds, that are octaves to each other, or at an octave's distance, are alike, or the same nearly as the unison. In this case, the more acute of the two makes exactly two vibrations, while the deeper or graver makes but one; whence, they coincide at every two vibrations of the acuter, which, being more frequent, makes this concord more perfect than any other, and as it were an unison. Hence also, it happens, that two chords or strings, of the same matter, thickness, and tension, but the one double the length of the other, produce the octave.

The octave containing in it all the other simple concords, and the degrees being the differences of these concords; it is evident, that the division of the octave comprehends the division of all the rest. By joining therefore all the simple concords to a common fundamental, we have the following series

1 .	$\frac{5}{6}$.	$\frac{4}{5}$.	$\frac{3}{4}$.	$\frac{2}{3}$.	$\frac{5}{8}$.	$\frac{3}{5}$.	$\frac{1}{2}$
Fund.	3d l,	3d g,	4th,	5th,	6th l,	6th g,	8ve.

Mr. Malcolm observes, that any wind instrument being over-blown, the sound will rise to an octave, and no other concord; which he ascribes to the perfection of the octave, and its being next to unison.

Descartes, from an observation of the like kind, viz, that the sound of a whistle, or organ pipe, will rise to an octave, if forcibly blown, concludes, that no sound is heard, but its acute octave seems some way to echo or resound in the ear.

OCTILE. See Octant.

OCTOBER, the 8th month of the year, in Romulus's calendar; but the tenth in that of Numa, Julius Cæsar, &c, after the addition of January and February. This month contains 31 days; about the 22d of which, the sun enters the sign Scorpio ♏.

OCTOSTYLE, in Architecture, the face of a building adorned with 8 columns.

ODD *Number*, in Arithmetic, is any number in the series 1, 3, 5, 7, &c. An odd number, when divided by 2, always leaves the remainder 1; and hence all odd numbers are said to be of the form $2n + 1$, and all odd square numbers of the form $8n + 1$, that is, any odd square number, being divided by 8, always leaves the remainder 1: thus $9 = 8 + 1$, $25 = 3 \cdot 8 + 1$, $49 = 6 \cdot 8 + 1$, &c.

The difference of all the consecutive square numbers, beginning with unity, forms the series of odd numbers, 1, 3, 5, 7, 9, &c, as appears by subtracting each preceding square from the following one, in the series of squares.

Squares	1	4	9	16	25	36	49	&c.
Differences	1,	3,	5,	7,	9,	11,	13,	&c.

An odd number cannot be divided by an even number.

Every prime number except 2 is an odd number.

Any power of an odd number is an odd number.

If an odd number divide an even number, it will also divide half that even number.

Every odd number prime to 10, is a divisor of any repetend digit. See Number.

ODDLY-ODD. A number is said to be oddly-odd, when an odd number measures it by an odd number. So 15 is a number oddly-odd, because the odd number 3 measures it by the odd number 5.

OFFING, or Offin, in Navigation, that part of the sea which is at a considerable distance from shore; where there is deep water, and no need of a pilot to conduct the ship into port.

OFFSETS, in Surveying, are the perpendiculars let fall, and measured from the station lines, to the corners or bends in the hedge, fence, or boundary of any ground.

Offset-*Staff*, a slender rod or staff, of 10 links, or other convenient length. Its use is for measuring the offsetts, and other short lines and distances.

OFFWARD, in Navigation, the same with From the shore, &c.

OGEE, or OG, an ornamental moulding in the shape of an S; consisting of two members, the one concave, and the other convex.

OLBERS, is a name given by some astronomers to a new planet discovered by Dr. Olbers at Bremen, on the 28th of March 1802. It is thus named in honour of this indefatigable astronomer, who has since, viz, on the 29th of March 1807, discovered a second new planet, to which he has given the name of Vesta; the first having also received the appellation of Pallas. The former of these might be otherwise named Olbers[1], and the latter Olbers[2]. The elements of each may be found under their respective names. See Pallas and Vesta.

OLDENBURG (Henry), who wrote his name sometimes Grubendol, transposing the letters, was a learned German gentleman, and born in the duchy of Bremen in the Lower Saxony, about the year 1626, being descended from the counts of Aldenburg in Westphalia; whence his name. During the long English parliament in the time of Charles the 1st, he came to England as consul for his countrymen; in which capacity he remained at London in Cromwell's administration. But being discharged of that employment, he was engaged as tutor to lord Henry Obryan, an Irish nobleman, whom he attended to the university of Oxford; and in 1656 he entered himself a student in that university, chiefly to have the benefit of consulting the Bodleian library. He was afterwards appointed tutor to lord William Cavendish, and became intimately acquainted with Milton the poet. During his residence at Oxford, he became also acquainted with the members of that society there, which gave birth to the Royal Society; and on the establishing of this latter, he was elected a member of it: and when the Society found it necessary to have two secretaries, he was chosen assistant to Dr. Wilkins. He applied himself with extraordinary diligence to the duties of this office, and began the publication of the Philosophical Transactions with No. 1, in 1664. In order to discharge this task with more credit to himself and the Society, he held a correspondence with more than seventy learned persons, and others, on a great variety of subjects, in different parts of the world. This fatigue would have been insupportable, had he not, as he told Dr. Lister, managed it so as to make one letter answer another; and that, to be always fresh, he never read a letter before he was ready immediately to answer it: so that the multitude of his letters did not embarrass him, nor ever lie upon his hands. Among others, he was a constant correspondent of Mr. Robert Boyle, and he translated many of that ingenious gentleman's works into Latin.

About the year 1674 he was drawn into a dispute with Mr. Hooke, who complained, that the secretary had not done him justice, in the History of the Transactions, with respect to the invention of the spiral spring for pocket watches; the contest was carried on with some warmth on both sides, but was at length terminated to the honour of Mr. Oldenburg; for, pursuant to an open representation of the affair to the Royal Society, the council thought fit to declare, in behalf of their secretary, that they knew nothing of Mr. Hooke having printed a book entitled Lampas, &c; but that the publisher of the Transactions had conducted himself faithfully and honestly in managing the intelligence of the Royal Society, and that he had given no just cause for such reflections.

Mr. Oldenburg continued to publish the Transactions as before, to No. 136, June 25, 1677; after which the publication was discontinued till the January following; when they were again resumed by his successor in the secretary's office, Mr. Nehemiah Grew, who carried them on till the end of February 1678. Mr. Oldenburg died at his house at Charlton, between Greenwich and Woolwich, in Kent, August 1677, and was interred there, being near 52 years of age.

He published, besides what has been already mentioned, 20 tracts, chiefly on theological and political subjects; in which he principally aimed at reconciling differences, and promoting peace and unanimity.

OLYMPIAD, in Chronology, a revolution or period of 4 years, by which the Greeks reckoned their time: so called from the olympic games, which were celebrated every 4th year, during 5 days, near the summer solstice, on the banks of the river Alpheus, near Olympia, a town of Elis. As each olympiad consisted of 4 years, these were called the 1st, 2d, 3d, and 4th year of each olympiad; the first year commencing with the nearest new moon to the summer solstice.

The first olympiad began the 3938th year of the Julian period, the 3208th of the creation, 776 years before the birth of Christ, or 24 years before the foundation of Rome. And the computation by these ended with the 404th olympiad, being the 440th year of the present vulgar Christian era.

OMBROMETER, a name given by Mr. Roger Pickering (Philos. Trans. No. 473,) to what is more commonly, though less properly, called a pluviameter or rain-gage. See Pluviameter.

OMPHALOPTER, or Omphaloptic, in Optics, a glass that is convex on both sides, popularly called a convex lens.

OPACITY, a quality of bodies which renders them opake, or the contrary of transparency.

The Cartesians make opacity to consist in this, that the pores of the body are not all straight, or directly before each other; or rather not pervious every way. This doctrine however is deficient: for though, to have a body transparent, its pores must be straight, or rather open every way; yet it is inconceivable how it should happen, that not only glass and diamonds, but even water, whose parts are so very moveable, should have all their pores open and pervious every way; while the finest paper, or the thinnest gold leaf, should exclude the light, for want of such pores. So that another cause of opacity must be sought for.

Now all bodies have vastly more pores or vacuities than are necessary for an infinite number of rays to pass freely through them in right lines, without striking on any of the parts themselves. For since water is 19 times lighter or rarer than gold; and yet gold itself is so very rare, that magnetic effluvia pass freely through it, without any opposition; and quicksilver is readily received within its pores, and even water itself by compression; it must have much more pores than solid parts: consequently water must have at least 40 times as much vacuity as solidity.

The cause therefore, why some bodies are opake, does not consist in the want of rectilinear pores, pervious every way; but either in the unequal density of the parts, or in the magnitude of the pores; and to their being either empty, or filled with a different matter; by means of which, the rays of light, in their passage, are arrested by innumerable refractions and reflections, till at length falling on some solid part, they become quite extinct, and are utterly absorbed.

Hence cork, paper, wood, &c, are opake; while glass, diamonds, &c, are pellucid. For in the confines or joining of parts alike in density, such as those of glass, water, diamonds, &c, among themselves, no refraction or reflection takes place, because of the equal attraction every way; so that such of the rays of light as enter the first surface, pass straight through the body, excepting those that are lost and absorbed, by striking on solid parts: but in the bordering of parts of unequal density, such as those of wood and water, both with regard to themselves, and with regard to the air or empty space in their larger pores, the attraction being unequal, the reflections and refractions will be very great; and thus the rays will not be able to pass through such bodies, being continually driven about, till they become extinct.

That this interruption or discontinuity of parts is the chief cause of opacity, Sir Isaac Newton argues, appears from hence; that all opake bodies immediately begin to be transparent, when their pores become filled with a substance of nearly equal density with their parts. Thus, paper dipped in water or oil, some stones steeped in water, linen cloth dipped in oil or vinegar, &c, become more transparent than before.

OPAKE, not translucent, nor transparent, or not admitting a free passage to the rays of light.

OPEN *Flank*, in Fortification, is that part of the flank which is covered by the orillon or shoulder.

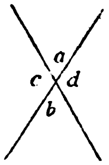
OPENING *of the Trenches*, is the first breaking of ground by the besiegers, in order to carry on their approaches towards a place.

OPERA-*Glass*, in Optics, is so called from its use in play-houses, and sometimes a Diagonal Perspective, from its construction, which is as follows. ABCD (fig. 5, pl. 21) represents a tube about 4 inches long; in each side of which there is a hole EF and GH, exactly against the middle of a plane mirror IK, which reflects the rays falling upon it to the convex glass LM; through which they are refracted to the concave eye-glass NO, whence they emerge parallel to the eye at the hole *rs*, in the end of the tube. Let PaQ be an object to be viewed, from which proceed the rays PC, *ab*, and Q*d*: these rays, being reflected by the plane mirror IK, will show the object in the direction *cp*, *ba*, *dq*, in the image *pq*, equal to the object PQ, and as far behind the mirror as the object is before it: the mirror being placed so as to make an angle of 45 degrees with the sides of the tube. And as, in viewing near objects, it is not necessary to magnify them, the focal distances of both the glasses may be nearly equal; or if that of LM be 3 inches, and that of NO one inch, the distance between them will be but 2 inches, and the object will be magnified 3 times, being sufficient for the purposes to which this glass is applied.

When the object is very near, as XY, it is viewed through a hole *xy*, at the other end of the tube AB, without an eye-glass; the upper part of the mirror being polished for that purpose, as well as the under. The tube unscrews near the object-glass LM, for taking out and cleansing the glasses and mirror. The position of the object will be erect through the concave eye-glass.

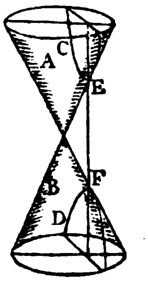
The peculiar artifice of this glass is to view a person at a small distance, so that no one shall know who is observed; for the instrument points to a different object from that which is viewed; and as there is a hole on each side, it is impossible to know on which hand the object is situated, which you are viewing.

OPHIUCUS, a constellation of the northern hemisphere; called also Serpentarius.



OPPOSITE *Angles*, or Vertical Angles, are those opposite to each other, made by two intersecting lines; as *a* and *b*, or *c* and *d*. —The opposite angles are equal to each other.

OPPOSITE *Cones*, denote two similar cones vertically opposite, having the same common vertex and axis, and the same sides produced; as the cones A and B.



OPPOSITE *Sections*, or *Hyperbolas*, are those made by cutting the opposite cones by the same plane; as the hyperbolas C and D.—These are always equal and similar, and have the same transverse axis EF, as also the same conjugate axis.

OPPOSITION, is that aspect or situation of two planets or stars, when they are diametrically opposite to each other; being 180°, or a semi-circle apart; and marked thus ☍. —The moon is in opposition to the sun when she is at the full.

OPTIC, or OPTICAL, something that relates to vision, or the sense of seeing, or the science of optics.

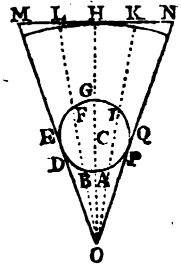
OPTIC *Angle*. See ANGLE.

OPTIC *Axis*. See AXIS.

OPTIC *Chamber*. See CAMERA *Obscura*.

OPTIC *Glasses*, are glasses ground either concave or convex; so as either to collect or disperse the rays of light; by which means vision is improved, and the eye strengthened, preserved, &c.—Among these, the principal are spectacles, reading glasses, telescopes, microscopes, magic lanterns, &c.

OPTIC *Inequality*, in Astronomy, is an apparent irregularity in the motions of very distant bodies; so called, because it is not really in the moving bodies, but arising from the situation of the observer's eye. For if the eye were in the centre, it would always see the motions as they really are.

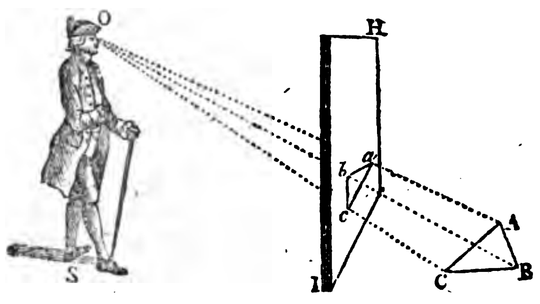


The optic inequality may be thus illustrated. Suppose a body revolving with a real uniform motion, in the periphery of a circle ABD &c; and suppose the eye in the plane of the same circle, but at a distance from it, viewing the motion of the body from O. Now when the body goes from A to B; its apparent motion is measured by the angle AOB or the arch or line HL, which it will appear to describe. But while it moves through the arch BD in an equal time, its apparent motion will be determined by the angle BOD, or the arch or line LM, which is less than the former LH. But it spends the same time in describing DE, as it does in AB or BD; during all which time of describing DE it appears stationary in the point M. When it really describes EFGIQ, it will appear to pass over MLHKN; so that it will seem to have gone retrograde. And lastly, from Q to P it will again appear stationary in the point N.

OPTIC *Nerves*, the second pair of nerves, springing from the crura of the medulla oblongata, and passing thence to the eye. These are covered with two coats, which they take from the dura and pia mater; and which, by their expansions, form the two membranes of the eye, called the uvea and cornea. And the retina, which is a third membrane, and the immediate organ of sight, is only an expansion of the fibrous, or inner, and medullary part of these nerves.

OPTIC *Pencil*. See PENCIL *of Rays*.

OPTIC *Pyramid*, in Perspective, is the pyramid ABCO, whose base is the visible object ABC, and the vertex is in the eye at O; being formed by rays drawn from the



several points of the perimeter to the eye. Hence may appear what is meant by optic triangle.

Optic *Place*, of a star, &c, is that point or part of its orbit, which is determined by our sight, when the star is seen there. This is either true or apparent; true when the observer's eye is supposed to be at the centre of the motion; or apparent, when his eye is at the circumference of the earth. See also Place.

Optic *Rays*, particularly means those by which an optic pyramid, or optic triangle, is terminated. As OA, OB, OC, &c.

OPTICIAN, a person skilled in optics.

OPTICS, the science of vision; including Catoptrics, and Dioptrics; and even Perspective; as also the whole doctrine of light and colours, and all the phenomena of visible objects.

Optics, in its more extensive acceptation, is a mixed mathematical science; which explains the manner in which vision is performed in the eye; treats of sight in general; gives the reasons of the several modifications or alterations, which the rays of light undergo in the eye; and shows why objects appear sometimes greater, sometimes smaller, sometimes more distinct, sometimes more confused, sometimes nearer and sometimes more remote. In this extensive signification it is considered by Newton, in his excellent work on this science. Indeed optics makes a considerable branch of natural philosophy; both as it explains the laws of nature, according to which vision is performed; and as it accounts for a variety of physical phenomena, otherwise inexplicable

The principal authors and discoveries in Optics, are the following:—Euclid seems to be the earliest author on optics that we have. He composed a treatise on optics and catoptrics; dioptrics being less known to the ancients; though it was not entirely unknown to them, for among the phenomena, at the beginning of that work, Euclid remarks the effect of bringing an object into view, by refraction, in the bottom of a vessel, by pouring water into it, which could not be seen over the edge of the vessel, before the water was poured in; and other authors speak of the then known effects of glass globes &c, both as burning glasses, and as to bodies seen through them. Euclid's work however is chiefly on catoptrics, or reflected rays; in which he shows, in 31 propositions, the chief properties of them, both in plane, convex, and concave surfaces, in his usual geometrical manner; beginning with that concerning the equality of the angles of incidence and reflection, which he demonstrates; and, in the last proposition, showing the effect of a concave speculum, as a burning glass, when exposed to the rays of the sun. The effects of burning glasses, both by refraction and reflection, are noticed by several others of the ancients; and it is probable that the Romans had a method of lighting their sacred fire by some such means. Aristophanes, in one of his comedies, introduces a person as making use of a globe filled with water to cancel a bond that was against him, by thus melting the wax of the seal. And if we give but a small degree of credit to what some ancient historians are said to have written concerning the exploits of Archimedes, we shall be induced to think that he constructed some very powerful burning mirrors. It is said that this eminent geometrician wrote a treatise on the subject of them, though it be not now extant; as also concerning the appearance of a ring or circle under water, and therefore could not have been ignorant of the common phenomena of refraction. We find many questions concerning such optical appearances in Aristotle. This author was also sensible, that it is the reflection of light from the atmosphere which prevents total darkness after the sun sets, and in places where he does not shine in the day-time. He was also of opinion, that rainbows, halos, and mock suns, were all occasioned by the reflection of the sunbeams in different circumstances, by which an imperfect image of his body was produced, the colour only being exhibited, and not his proper figure.

The ancients were not only acquainted with the more ordinary appearances of refraction, but knew also the production of colours by refracted light. Seneca says, that when the light of the sun shines through an angular piece of glass, it shows all the colours of the rainbow. These colours however, he says, are false, such as are seen in a pigeon's neck when it changes its position; and of the same nature he says is a speculum, which, without having any colour of its own, assumes that of any other body. It appears also, that the ancients were not unacquainted with the magnifying power of glass globes filled with water, though it does not appear that they knew any thing of the reason of this power: and it is supposed that the ancient engravers made use of a glass globe filled with water to magnify their figures, that they might work to more advantage.

Ptolemy, about the middle of the second century, wrote a considerable treatise on optics. The work is lost; but from the accounts given of it by others, it appears that he there treated of astronomical refractions. The first astronomers were not aware that the intervals between stars appear less when near the horizon than in the meridian; and on this account they must have been much embarrassed in their observations: but it is evident that Ptolemy was aware of this circumstance by the caution which he gives to allow something for it, whenever recourse is had to ancient observations. This philosopher also advances a very sensible hypothesis to account for the remarkably great apparent size of the sun and moon when seen near the horizon. The mind, he says, judges of the size of objects by means of a preconceived idea of their distance from us: and this distance is fancied to be greater when a number of objects are interposed between the eye and the body we are viewing: which is the case when we see the heavenly bodies near the horizon. In his Almagest, however, he ascribes this appearance to a refraction of the rays by vapours, which actually enlarge the angle under which the luminaries appear; just as the angle is enlarged by which an object is seen from under water.

Alhazen, an Arabian writer, was the next author of any celebrity, and wrote about the year 1100. Alhazen made many experiments on refraction, at the surface between air and water, air and glass, and water and glass; and hence he deduced several properties of atmospherical refraction; such as "that it increases the altitudes of all objects in the heavens;" and he first advanced that the stars are sometimes seen above the horizon by means of refraction, when they are really below it: which observation was confirmed by Vitello, Walther, and especially by the observations of Tycho Brahe. Alhazen observed, that refraction contracts the diameters and distances of the heavenly bodies, and that it is the cause of the twinkling of the stars. This refractive power he ascribed, not to the vapours contained in the air, but to its different degrees of transparency. And it was his opinion, that so far from vapour being the cause of the heavenly bodies appearing larger near the horizon, that it would make them appear less; observing that two stars appear nearer together in the horizon, than near the meridian. This phenomenon he ranks among optical deceptions. We judge of distance, he says, by comparing the angle under which objects appear, with their supposed distance; so that if these angles be nearly equal, and the distance of one object be conceived greater than that of the other, this will be imagined to be larger. And he further observes, that the sky near the horizon is always imagined to be farther from us than any other part of the concave surface.

In the writings of Alhazen, we also find the first distinct account of the magnifying power of glasses; and it is not improbable that his writings on this head gave rise to the useful invention of spectacles: for he says, that if an object be applied close to the base of the larger segment of a sphere of glass, it will appear magnified. He also treats of the appearance of an object through a globe, and says that he was the first who observed the refraction of rays into it.

In 1270, Vitello, a native of Poland, published a treatise on optics, containing all that was valuable in Alhazen, and digested in a better manner. He observes, that light is always lost by refraction, which makes objects appear less luminous. He gave a table of the results of his experiments on the refractive powers of air, water, and glass, corresponding to different angles of incidence. He ascribes the twinkling of the stars to the motion of the air in which the light is refracted; and he illustrates this hypothesis, by observing that they twinkle still more when viewed in water put in motion. He also shows, that refraction is necessary, as well as reflection, to form the rainbow; because the body which the rays fall upon is a transparent substance, at the surface of which one part of the light is always reflected, and another refracted. And he makes some ingenious attempts to explain refraction, or to ascertain the law of it. He also considers the foci of glass spheres, and the apparent size of objects seen through them; though with but little accuracy. To Vitello may be traced the idea of seeing images in the air. He endeavours to show, that it is possible, by means of a cylindrical convex speculum, to see the images of objects in the air, out of the speculum, when the objects themselves cannot be seen.—The Optics of Alhazen and Vitello were published at Basil in 1572, by Fred. Risner.

Contemporary with Vitello, was Roger Bacon, a man of very extensive genius, who wrote upon almost every branch of science; though it is thought his improvements in optics were not carried far beyond those of Alhazen and Vitello. He even assents to the absurd notion, held by all philosophers down to his time, that visible rays proceed from the eye, instead of towards it. From many stories related of him however, it would seem, that he made greater improvements than appear in his writings. It is said he had the use of spectacles; that he had contrivances, by reflection from glasses, to see what was doing at a great distance, as in an enemy's camp. And lord chancellor Bacon relates a story, of his having apparently walked in the air between two steeples, and which he supposed was effected by reflection from glasses, while he walked upon the ground. See the article BACON.

About 1279 was written a treatise on optics by Peccam, archbishop of Canterbury.

One of the next who distinguished himself as a theoretical optician, was Maurolyc, teacher of mathematics at Messina. In a treatise De Lumine et Umbra, published in 1575, he demonstrates, that the crystalline humour of the eye is a lens that collects the rays of light issuing from the objects, and throws them upon the retina, where the focus of each pencil is situated. From this principle he discovered the reason why some people are short-sighted, and others long-sighted; also why the former are relieved by concave glasses, and the others by convex ones.

Contemporary with Maurolyc, was John Baptista Porta, of Naples. He discovered the camera obscura, which throws considerable light on the nature of vision. His house was the constant resort of all the ingenious persons at Naples, whom he formed into what he called An Academy of Secrets; each member being obliged to contribute something that was not generally known, and might be useful. By this means he was furnished with materials for his Magia Naturalis, which contains his account of the camera obscura, and the first edition of which was published, as he informs us, when he was not quite 15 years old. He also gave the first hint of the magic lantern; which Kircher afterwards followed and improved. His experiments with the camera obscura convinced him, that vision is performed by the intromission of something into the eye, and not by visual rays proceeding from it, as had been formerly imagined; and he was the first who fully satisfied himself and others on this subject. He justly considered the eye as a camera obscura, and the pupil the hole in the window-shutter; but he was mistaken in supposing that the crystalline humour corresponds to the wall which receives the images; nor was it discovered till the year 1604, that this office is performed by the retina. He made a variety of just remarks concerning vision; and particularly explained several cases in which we imagine things to be without the eye, when the appearances are occasioned by some affection of the eye itself, or by some motion within the eye.—He remarked also that, in certain circumstances, vision will be assisted by convex or concave glasses; and he seems even to have made some small advances towards the discovery of telescopes.

Other treatises on optics, with various and gradual improvements, were afterwards successively published by several authors: as Aguilon, Opticorum libr. 6, Antv. 1613: L'Optique, Catoptrique, et Dioptrique of Herigone, in his Cursus Math. Paris 1637: the Dioptrics of Descartes, 1637: L'Optique et Catoptrique of Mersenne, Paris 1651: Scheiner, Optica, Lond. 1652: Manchini, Dioptrica Practica, Bologna, 1660: Barrow, Lectiones Opticæ, London 1663: James Gregory, Optica Promota, Lond. 1663: Grimaldi, Physico-mathesis de Lumine, Coloribus, et Iride, Bononia, 1665: Scaphusa, Cogitationes Physico-mechanicæ de Natura Visionis, Heidel. 1670: Kircher, Ars Magna Lucis et Umbræ, Rome 1671: Cherubin, Dioptrique Oculaire, Paris 1671: Leibnitz, Principe Generale de l'Optique, Leipsic Acts 1682: Newton's Optics and Lectiones Opticæ, 4to and 8vo, 1704 &c: Molyneux, Dioptrics, Lond. 1692: Dr. Jurin's Theory of Distinct and Indistinct Vision.—There is also a large and excellent work on optics, by Dr. Smith, 2 vols 4to; and an elaborate history of the present state of discoveries relating to vision, light, and colours, by Dr. Priestley, 4to, 1772; with a multitude of other authors of inferior note; besides lesser and occasional tracts and papers in the Memoirs of the several learned Academies and Societies of Europe; with improvements by many other persons, among whom are the respectable names of Snell, Fermat, Kepler, Huygens, Hortensius, Boyle, Hooke, Lahire, Lowthorp, Cassini, Halley, Delisle, Euler, Dollond, Clairaut, Dalembert, Zeiher, Bouguer, Buffon, Nollet, Baume; but the particular improvements by each author must be referred to the history of his life, under the articles of their names; while the history and improvements of the several branches are to be found under the various particular articles, as, Light, Colours, Reflection, Refraction, Inflection, Transmission, &c, Spectacles, Telescope, Microscope, &c, &c.

ORB, a spherical shell, hollow sphere, or space contained between two concentric spherical surfaces.—The ancient astronomers conceived the heavens as consisting of several vast azure transparent orbs or spheres, inclosing one another, and including the bodies of the planets.

The Orbis *Magnus*, or *Great* Orb, is that in which the sun is supposed to revolve; or rather it is that in which the earth makes its annual circuit.

ORBIT, is the path of a planet or comet; being the curve line described by its centre, in its proper motion in the heavens. So the earth's orbit, is the ecliptic, or the curve it describes in its annual revolution about the sun.

The ancient astronomers made the planets describe circular orbits, with a uniform velocity. Copernicus himself could not believe they should do otherwise; being unable to disentangle himself entirely from the excentrics and epicycles to which they had recourse, to account for the inequalities in their motions.

But Kepler found, from observations, that the orbit of the earth, and that of every primary planet, is an ellipsis, having the sun in one of its foci; and that they all move in these ellipses by this law, that a radius drawn from the centre of the sun to the centre of the planet, always describes equal areas in equal times; or, which is the same thing, in unequal times, it describes areas that are proportional to those times. And Newton has since demonstrated, from the nature of universal gravitation and projectile motion, that the orbits must of necessity be ellipses, and the motions are found to observe that law, both of the primary and secondary planets; excepting in so far as their motions and paths are disturbed by their mutual actions on one another; as the orbit of the earth by that of the moon; or that of Saturn by the action of Jupiter; &c.

Of these elliptic orbits, there have been two kinds assigned; the first that of Kepler and Newton, which is the common or conical ellipse; for which Seth Ward, though he himself employs it, thinks we might venture to substitute circular orbits, by using two points, taken at equal distances from the centre, on one of the diameters, as is done in the foci of the ellipsis, and which is called his Circular Hypothesis. The second is that of Cassini, of this nature, viz, that the products of the two lines drawn from the two foci, to any point in the circumference, are everywhere equal to the same constant quantity; whereas, in the common ellipse, it is the sum of those two lines that is always a constant quantity.

The orbits of the planets are not all in the same plane with the ecliptic, which is the earth's orbit round the sun, but are variously inclined to it, and to each other: but still the plane of the ecliptic, or earth's orbit, intersects the plane of the orbit of every other planet, in a right line which passes through the sun, called the line of the nodes, and the points of intersection of the orbits themselves are called the nodes.

The mean semidiameters of the several orbits, or the mean distances of the planets from the sun, with the excentricities of the orbits, their inclination to the ecliptic, and the places of their nodes, are as in the following table; where the 2d column contains the proportions of semidiameters of the orbits, the true semidiameter of that of the earth being 95 millions of miles; and the 3d column shows what part of the semidiameters the excentricities are equal to.

Planets.	Propor. semid. of orbit.	Excentr. in pts. of mean dist.	Inclina. of orbit.	Ascending node.
Mercury - - -	387	$\frac{4}{19}$	6° 54′	♉ 14° 43′
Venus - - - -	723	$\frac{1}{138}$	3 20	♊ 13 59
Earth - - - -	1000	$\frac{1}{59}$	0 0	- - -
Mars - - - -	1524	$\frac{1}{11}$	1 52	♉ 17 17
Vesta - - - -	2355	$\frac{1}{28}$	7 8⅓	♊ 13 18
Juno - - - -	2664	$\frac{1}{4}$	13 4	♍ 21 4
Pallas - - - -	2765	$\frac{1}{4}$	34 38	♍ 22 31
Ceres - - - -	2767	$\frac{1}{13}$	10 38	♊ 21 7
Jupiter - - - -	5201	$\frac{1}{21}$	1 20	♋ 7 29
Saturn - - - -	9539	$\frac{1}{18}$	2 30	♋ 21 13
Uranus - - - -	19034	$\frac{1}{21}$	0 46½	♊ 12 54

The orbits of comets are also very excentric ellipses.

ORDER, in Architecture, a system of the several members, ornaments, and proportions of columns and pilasters, or a regular arrangement of the projecting parts of a building, especially the column, so as to form one beautiful whole.

There are five orders of columns, of which three are Greek, viz, the Doric, Ionic, and Corinthian; and two Italic, viz, the Tuscan and Composite. The three Greek orders represent the three different manners of building, viz, the solid, the delicate, and the middling: the two Italic ones are imperfect productions of these.

Order, in Astronomy. A planet is said to move according to the order of the signs, when it is direct; proceeding from Aries to Taurus, thence to Gemini, &c. As, on the contrary, its motion is contrary to the order of the signs, when it is retrograde, or goes backward, from Pisces to Aquarius, &c.

Order, in the Geometry of Curve Lines, is denominated from the rank or order of the equation by which the geometrical line is expressed; so, the simple equation, or 1st power, denotes the 1st order of lines, which is the right line; the quadratic equation, or 2d power, defines the 2d order of lines, which are the conic sections and circle; the cubic equation, or 3d power, defines the 3d order of lines; and so on.

Or, the orders of lines are denominated from the number of points in which they may be cut by a right line. Thus, the right line is of the 1st order, because it can be cut only in one point by a right line; the circle and conic sections are of the 2d order, because they can be cut in two points by a right line; while those of the 3d order, are such as can be cut in three points by a right line; and so on.

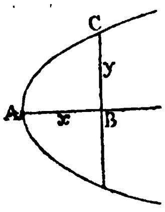
It is to be observed, that the order of curves is always one degree lower than the corresponding line; because the 1st order, or right line, is no curve; and the circle and conic sections, which are the 2d order of lines, are only the 1st order of curves; &c. See CURVES and LINES. Also Newton's Enumeratio Linearum Tertii Ordinis.

ORDINATES, in the Geometry of Curve Lines, are right lines drawn parallel to each other, and cutting the curve in a certain number of points.

The parallel ordinates are usually all cut by some other line, which is called the absciss, and commonly the ordinates are perpendicular to the abscissal line. When this line is a diameter of the curve, the property of the ordinates is then the most remarkable; for, in the curves of the first kind, or the conic sections and circle, the ordinates are all bisected by the diameter, making the part on one side of it equal to the part on the other; and in the curves of the 2d order, which may be cut by an ordinate in three points, then the three parts of the ordinate, lying between these three intersections of the curve and the intersection with the diameter, have the part on one side the diameter equal to both the two parts on the other side of it. And so for curves of any order, whatever the number of intersections may be, the sum of the parts of any ordinate, on one side of the diameter, being in all cases equal to the sum of the parts on the other side of it.

The use of ordinates in a curve, and their abscisses, is to define or express the nature of the curve, by means of the general relation or equation between them; and the greatest number of factors, or the dimensions of the highest term, in such equation, is always the same as the order of the line; that equation being a quadratic, or its highest term of two dimensions, in the lines of the 2d order, being the circle and conic sections; and a cubic equation, or its highest term containing 3 dimensions, in the lines of the 3d order; and so on.

Thus, y denoting an ordinate BC, and x its absciss AB; also a, b, c, &c, given quantities: then $y^2 = ax^2 + bx + c$ is the general equation for the lines of the 2d order; and $xy^2 - cy = ax^3 + bx^2 + cx + d$ is the equation for the lines of the 3d order; and so on.



ORDNANCE, are all sorts of great guns, used in war; such as cannons, mortars, howitzers, &c.

ORFFYREUS's *Wheel*, in Mechanics, is a machine so called from its inventor, which he asserted to be a perpetual motion. This machine, according to the account given of it by Gravesande, in his Œuvres Philosophiques, published by Allemand, Amst. 1774, consisted externally of a large circular wheel, or rather drum, 12 feet in diameter, and 14 inches deep; being very light, as it was formed of an assemblage of deals, having the intervals between them covered with waxed cloth, to conceal the interior parts of it. The two extremities of an iron axis, on which it turned, rested on two supports. On giving a slight impulse to the wheel, in either direction, its motion was gradually accelerated; so that after two or three revolutions it acquired so great a velocity as to make 25 or 26 turns in a minute. This rapid motion it actually preserved during the space of 2 months, in a chamber of the landgrave of Hesse, the door of which was kept locked, and sealed with the landgrave's own seal. At the end of that time it was stopped, to prevent the wear of the materials. The professor, who had been an eye-witness to these circumstances, examined all the external parts of the machine, and was convinced that there could not be any communication between it and any neighbouring room. Orffyreus however was so incensed, or pretended to be so, that he broke the machine in pieces, and wrote on the wall, that it was the impertinent curiosity of professor Gravesande which made him take this step. The prince of Hesse, who had seen the interior parts of this wheel, but sworn to secrecy, being asked by Gravesande, whether, after it had been in motion for some time, there was any change observable in it, and whether it contained any pieces that indicated fraud or deception, answered both questions in the negative, and declared that the machine was of a very simple construction.

ORGANICAL *Description of Curves*, is the description of them on a plane, by means of instruments, and commonly by a continued motion. The most simple construction of this kind, is that of a circle by means of a pair of compasses. The next is that of an ellipse by means of a thread and two pins in the foci, or the ellipse and hyperbola, by means of the elliptical and hyperbolic compasses. A great variety of descriptions of this sort are to be found in Schooten De Organica Conic. Sect. in Plano Descriptione; in Newton's Arithmetica Universalis, De Curvarum Descriptione Organica; Maclaurin's Geometria Organica; Brackenridge's Descriptio Linearum Curvarum; &c.

ORGUES, or ORGANS, in Fortification, long and thick pieces of wood, shod with pointed iron, and hung each by a separate rope over the gate-way of a town, ready on any surprise or attempt of the enemy to be let down to stop up the gate. The ends of the several ropes are wound about a windlass, so as to be let down all together.

ORGUES is also used for a machine composed of several harquebusses or musket-barrels, bound together; so as to make several explosions at the same time. They are used to defend breaches and other places attacked.

ORIENT, the east, or eastern point of the horizon.

ORIENT *Equinoctial*, is used for that point of the horizon where the sun rises when he is in the equinoctial, or when he enters the signs Aries and Libra.

ORIENT *Æstival*, is the point where the sun rises in the middle of summer, when the days are longest.

ORIENT *Hybernal*, is the point where the sun rises in the middle of winter, when the days are shortest.

ORIENTAL, situated towards the east with regard to us: in opposition to occidental or the west.

ORIENTAL *Astronomy, Philosophy,* &c, used for those of the east, or of the Arabians, Chaldeans, Persians, Indians, &c.

ORILLON, in Fortification, a small rounding of earth, lined with a wall, raised on the shoulder of those bastions that have casemates, to cover the cannon in the retired flank, and prevent their being dismounted by the enemy.—There are other sorts of orillons, properly called epaulements, or shoulderings, which are almost of a square figure.

ORION, a constellation of the southern hemisphere, with respect to the ecliptic, but half in the northern, and half on the southern side of the equinoctial, which runs across the middle of his body. The stars in this constellation are, 38 in Ptolemy's catalogue, 42 in Tycho's, 62 in Hevelius's, and 78 in Flamsteed's. But some telescopes have discovered several thousands of stars in this constellation, of which there are 2 of the first magnitude, and 4 of the second, besides a great many of the third and fourth. One of those two stars of the first magnitude is on the middle of the left foot, and is called Regel; the other is on the right shoulder, and called Betelguese; of the 4 of the second magnitude, one is on the left shoulder, and called Bellatrix, and the other three are in the belt, lying nearly in a right line and at equal distances from each other, forming what is popularly called the Yard-wand.

This constellation is one of the 48 old asterisms, and one of the most remarkable in the heavens. It is in the figure of a man, having a sword by his side, and seems attacking the bull with a club in his right hand, his left bearing a shield.

No constellation was so terrible to the mariners of the early periods, as this of Orion. He is mentioned in this way by all the Greek and Latin poets, and even by their historians; his rising and setting being attended by storms and tempests: and as the northern constellations are made the followers of the Pleiades; so are the southern ones made the attendants of Orion.

The name of this constellation is also met with in Scripture several times, viz, in the books of Job, Amos, and Isaiah. In Job it is asked, "Canst thou bind the sweet influence of the Pleiades, or loose the bands of Orion?" And Amos says, "Seek him that maketh the Seven Stars and Orion, and turneth the shadow of death into morning."

ORION'S *River*, the same as the constellation Eridanus.

ORLE, ORLET, or **ORLO,** in Architecture, a fillet under the ovolo, or quarter-round of a capital.—When it is at the top or bottom of the shaft, it is called the cincture.—Palladio also uses Orlo for the plinth of the bases of columns and pedestals.

ORRERY, an astronomical machine, for exhibiting the various motions and appearances of the sun and planets; hence often called a Planetarium. The term Orrery applied to this instrument, we are informed by Desaguliers, arose from the following circumstance:—Mr. Rowley, a mathematical instrument-maker, having got one from Mr. George Graham, the original inventor, to be sent abroad with some of his own instruments, he copied it, and afterwards constructed one for the earl of Orrery. Sir Richard Steele, who knew nothing of Mr. Graham's machine, thinking to do justice to the first encourager, as well as to the inventor of such a curious instrument, called it an orrery, and gave Rowley the praise due to Mr. Graham. Desaguliers's Experim. Philos. vol. 1, pa. 430. The figure of this grand orrery is exhibited at fig. 1, pl. 24. It is since made in various other figures.

ORTEIL, in Fortification. See BERME.

ORTELIUS (ABRAHAM), a celebrated geographer, was born at Antwerp, in 1527. He was well skilled in the languages and mathematics, and acquired such reputation by his skill in geography, that he was surnamed the Ptolemy of his time. Justus Lipsius, and most of the learned men of the 16th century, were our author's intimate friends. He passed some time at Oxford in the reign of Edward the 6th; and he visited England a second time in 1577.

His Theatrum Orbis Terræ was the completest work of the kind that had ever been published, and gained our author a reputation adequate to his immense labour in compiling it. He wrote also several other excellent geographical works; the principal of which are, his Thesaurus, and his Synonyma Geographica.—The learned world is also indebted to him for the Britannia, which was undertaken by Cambden at his request.—He died at Antwerp, 1598, at 71 years of age.

ORTHODROMICS, in Navigation, is Great-circle sailing, or the art of sailing in the arch of a great circle, which is the shortest course: for the arch of a great circle is orthodromia, or the shortest distance between two points or places.

ORTHOGONIAL, in Geometry, is the same as rectangular, or right-angled.—When the term refers to a plane figure, it supposes one leg or side to stand perpendicular to the other: when spoken of solids, it supposes their axes to be perpendicular to the plane of the horizon.

ORTHOGRAPHIC, or **ORTHOGRAPHICAL** *Projection of the Sphere*, is the projection of its surface or of the sphere on a plane, passing through the middle of it, by an eye vertically at an infinite distance. See PROJECTION.

ORTHOGRAPHY, in Geometry, is the drawing or delineating the front plan or side of any object, and of expressing the heights or elevations of every part: being so called from its delineating objects by perpendicular right lines falling on the geometrical plan; or rather, because all the horizontal lines are here straight and parallel, and not oblique as in representations of perspective.

ORTHOGRAPHY, in Architecture, is the profile or elevation of a building, showing all the parts in their true proportion. This is either external or internal.

External **ORTHOGRAPHY,** is a delineation of the outer face or front of a building; showing the principal wall with its apertures, roof, ornaments, and every thing visible to an eye placed before the building. And

Internal **ORTHOGRAPHY,** called also a Section, is a delineation or draught of a building, such as it would appear if the external wall were removed.

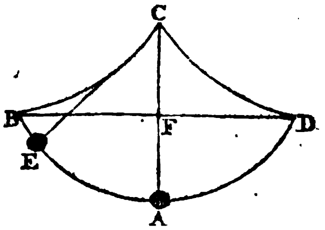
ORTHOGRAPHY, in Fortification, is the profile, or representation of a work; or a draught so conducted, as that the length, breadth, height, and thickness of the several parts are expressed, such as they would appear if it were perpendicularly cut from top to bottom.

ORTHOGRAPHY, in Perspective, is the front side of any place; that is, the side or plane that lies parallel to a straight line that may be imagined to pass through the outward convex points of the eyes, continued to a convenient length.

ORTIVE, or *Eastern Amplitude*, in Astronomy, is an arch of the horizon intercepted between the point where a star rises, and the east point of the horizon.

OSCILLATION, in Mechanics, denotes the vibration, or the reciprocal ascent and descent of a pendulum.

If a simple pendulum be suspended between two semicycloids, BC, CD, that have the diameter CF of the generating circle equal to half the length of the string, so that the string, as the body E oscillates, folds about them, then will the body oscillate in another cycloid BEAD, similar and equal to the former. And the time of the oscillation in any arc AE, measured from the lowest point A, is always the same constant quantity, whether that arc be larger or smaller. But the oscillations in a circle are unequal, those in the smaller arcs being less than those in the larger; and so always less and less as the arcs are smaller,



but still greater than the time of oscillation in a cycloidal arc; till the circular arc becomes very small, and then the time of oscillation in it is very nearly equal to the time in the cycloid, because the circle and cycloid have the same curvature at the vertex, the length of the string being the common radius of curvature to them both at that point.

The time of one whole oscillation in the cycloid, or of an ascent and descent in any arch of it, is to the time in which a heavy body would fall freely through CF or FA, the diameter of the generating circle, or through half the length of the pendulum string, as the circumference of a circle is to its diameter, that is as 3·1416 to 1. So that if l denote the length of the pendulum CA, and $g = 16\frac{1}{12}$ feet $= 193$ inches, the space through which a heavy body falls in the 1st second of time, and $p = 3{\cdot}1416$ the circumference of a circle whose diameter is 1: then by the laws of falling bodies, it is $\sqrt{g} : \sqrt{\frac{1}{2}l} :: 1'' : \sqrt{\frac{l}{2g}}$, the time of falling through CF or $\frac{1}{2}l$; therefore $1 : p :: \sqrt{\frac{l}{2g}} : p\sqrt{\frac{l}{2g}}$, which is the time of one vibration in any arch of the cycloid which has the diameter of its generating circle equal to $\frac{1}{2}l$. Or, by substituting the known numbers for p and q, the time of an oscillation becomes barely $\frac{4}{25}\sqrt{l}$ or $\frac{16}{100}\sqrt{l}$ very nearly, or more nearly $\frac{130}{813}l$, l being the length of the pendulum in inches. And therefore this is also very nearly the time of an oscillation in a small circular arc, whose radius is l inches.

Hence the times of the oscillation of pendulums of different lengths, are directly in the subduplicate ratio of their lengths, or as the square roots of their lengths.—The more exact time of oscillating in a circular arc, when this is of some finite small length, is $\frac{4}{25}\sqrt{l} \times (1 + \frac{h}{8l})$; where h is the height of the vibration, or the versed sine of the single arc of ascent or descent, to the radius l.

The celebrated Huygens first resolved the problem concerning the oscillations of pendulums, in his book De Horologio Oscillatorio, reducing compound pendulums to simple ones. And his doctrine is founded on this hypothesis, that the common centre of gravity of several bodies, connected together, must ascend exactly to the same height from which it fell, whether those bodies be united, or separated from one another in ascending again, provided that each begin to ascend with the velocity acquired by its descent.

This supposition was opposed by several persons, and very much suspected by others. And those even who believed the truth of it, yet thought it too daring to be admitted without proof into a science which demonstrates every thing.

At length James Bernoulli demonstrated it, from the nature of the lever; and published his solution in the Mem. Acad. des Scienc. of Paris, for the year 1703. After his death, which happened in 1705, his brother John Bernoulli gave a more easy and simple solution of the same problem, in the same Memoirs for 1714; and about the same time, Dr. Brook Taylor published a similar solution in his Methodus Incrementorum: which gave occasion to a dispute between these two mathematicians, who accused each other of having stolen their solutions. The particulars of which dispute may be seen in the Leipsic Acts for 1716, and in Bernoulli's works, printed in 1743.

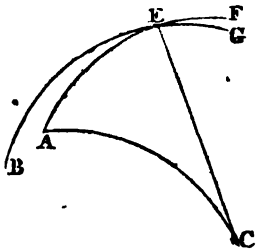
Axis of OSCILLATION, is a line parallel to the horizon, supposed to pass through the centre or fixed point about which the pendulum oscillates, and perpendicular to the plane in which the oscillation is made.

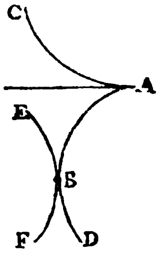
Centre of OSCILLATION, in a suspended body, is a certain point in it, such that the oscillation of the body will be made in the same time as if that point alone were suspended at that distance from the point of suspension. Or it is the point into which, if the whole weight of the body be collected, the several oscillations will be performed in the same time as before: the oscillations being made only by the force of gravity of the oscillating body. See CENTRE *of Oscillation*.

OSCULATION, in Geometry, denotes the contact between any curve and its osculatory circle, that is, the circle of the same curvature with the given curve, at the point of contact or of osculation. If AC be the evolute of the involute curve AEF, and the tangent CE the radius of curvature at the point E, with which, and the centre C, if the circle BEG be described; this circle is said to osculate the curve AEF in the point E, which point E M. Huygens calls the point of osculation, or kissing point.

The line CE is called the osculatory radius, or the radius of curvature; and the circle BEG the osculatory or kissing circle.

The evolute AC is the locus of the centres of all the circles that osculate the involute curve AEF.





OSCULATION also means the point of concourse of two branches of a curve which touch each other. For example, if the equation of a curve be $y = \sqrt{x} + \sqrt[4]{x^3}$, it is easy to see that the curve has two branches touching one another at the point where $x = 0$, because the roots have each the signs + and −.

The point of osculation differs from the cusp or point of retrocession (which is also a kind of point of contact of two branches) in this, that in this latter case the two branches terminate, and pass no farther, but in the former the two branches exist on both sides of the point of osculation. Thus, in the second figure above, the point B is the osculation of the two branches ABD, EBF; but A, though it is also a tangent point, is a cusp or the point of retrocession, of AC and AB, the branches not passing beyond the point A.

OSCULATORY *Circle*, is the same as the circle of curvature; that is, the circle having the same curvature with any curve at a given point. See the foregoing article, Osculation, where BEG, in the last figure but one, is the osculatory circle of the curve AEF at the point E; and CE the osculatory radius, or the radius of curvature.

This circle is called osculatory, because that, of all the circles that can touch the curve in the same point, that one touches it the closest, or in such manner that no other tangent circle can be drawn between it and the curve; so that, in touching the curve, it embraces it as it were, both touching and cutting it at the same time, being on one side at the convex part of the curve, and on the other at the concave part of it.

In a circle, all the osculatory radii are equal, being the common radius of the circle; the evolute of a circle being only a point, which is its centre. See some properties of the osculatory circle in Maclaurin's Algebra, Appendix De Linearum Geometricarum Proprietatibus generalibus Tractatus, Theor. 2, § 15, &c, treated in a pure geometrical manner.

Osculatory *Parabola*. See **Parabola**.

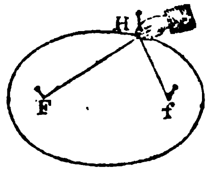
Osculatory *Point*, the osculation, or point of contact between a curve and its osculatory circle.

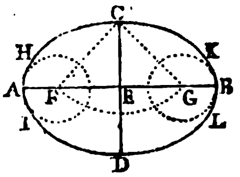
OSTENSIVE *Demonstrations*, such as plainly and directly demonstrate the truth of any proposition. In which they stand distinguished from apagogical ones, or reductions ad absurdum, or ad impossibile, which prove the truth proposed by demonstrating the absurdity or impossibility of the contrary.

OTACOUSTIC, an instrument that aids or improves the sense of hearing. See **Acoustics**.

OVAL, an oblong curvilinear figure, having two unequal diameters, and bounded by a curve line returning into itself. Or a figure contained by a single curve line, imperfectly round, its length being greater than its breadth, like an egg: whence its name. The proper oval, or egg-shape, is an irregular figure, being narrower at one end than at the other; in which it differs from the ellipse, which is the mathematical oval, and is equally broad at both ends. The common people confound the two together: but geometricians call the oval a false ellipse.

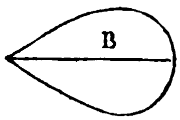
The method of describing an oval chiefly used among artificers, is by a cord or string, as FH*f*, whose length is equal to the greater diameter of the intended oval, and which is fastened by its extremes to two points or pins, F and *f*, planted in its longer diameter; then, holding it always stretched out as at H, with a pin or pencil carried round the inside, the oval is described: which will be so much the longer and narrower as the two fixed points are farther apart. This oval so described is the true mathematical ellipse, the points F and *f* being the two foci. But, in architectural designs, where great accuracy is required, the elliptic compasses are better employed. See **Compasses** *Elliptical*.

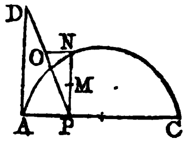




Another popular way to describe an oval of a given length and breadth, is thus:—Set the given length and breadth, AB and CD, to bisect each other perpendicularly at E; with the centre C, and radius AE, describe an arc to cross AB in F and G; then with these centres, F and G, and radii AF and BG, describe two little arcs HI and KL for the smaller ends of the oval; and lastly, with the centres C and D, and radius CD, describe the arcs HK and IL, for the flatter or longer sides of the oval. But this, it is evident, does not form a true ellipse. Sometimes other points, instead of C and D, are to be taken by trial, as centres in the line CD, produced if necessary, so as to make the two last arcs join best with the two former ones.

Oval denotes also certain roundish figures, of various and pleasant shapes, among curve lines of the higher kinds. These figures are expressed by equations of all dimensions above the 2d, and more especially the even dimensions, as the 4th, 6th, &c. Of this kind is the equation $a^2y^2 = -x^4 + ax^3$, which denotes the oval B, in shape of the section of a pear through the middle, and is easily described by means of points.





For, if a circle be described whose diameter AC is $= a$, and AD be perpendicular and equal to AC; then, taking any point P in AC, joining DP, and drawing PN parallel to AD, and NO parallel to AC; and lastly taking PM = NO, the point M will be one point of the oval sought.—In like manner the equation

$$y^4 - 4y^2 = -ax^4 + bx^3 + cx^2 + dx + e$$

expresses several very pretty ovals, among which the following 12 are some of the most remarkable. For when the equation $ax^4 = bx^3 + cx^2 + dx + e$ has four real unequal roots, the given equation denotes the three following species, in fig. 1, 2, 3:

Fig. 1.

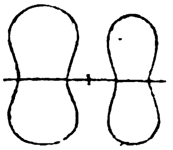


Fig. 2.

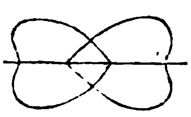
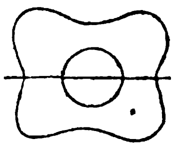


Fig. 3.



When the two less roots are equal, the three species will be expressed as in fig. 4, 5, 6, thus:

Fig. 4.

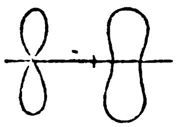


Fig. 5.

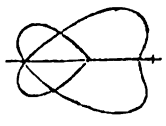
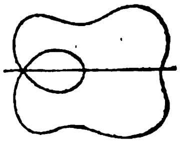


Fig. 6.



When the two less roots become imaginary, it will denote the three species exhibited in fig. 7, 8, 9:

Fig. 7.

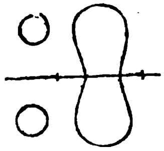


Fig. 8.

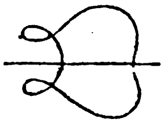
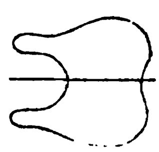


Fig. 9.



When the two middle roots are equal, the species will be as appears in fig. 10: when two pair of roots are equal, the species will be as in fig. 11: and when the two middle roots become imaginary, the species will be as appears in fig. 12:

Fig. 10.

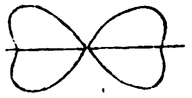


Fig. 11.

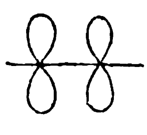
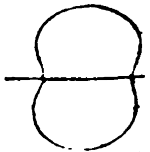


Fig. 12.



OUGHTRED (**William**), an eminent English mathematician and divine, was born at Eton in Buckinghamshire, 1573, and educated in the school there; whence he was elected to King's-college in Cambridge in 1592, where he continued about 12 years, and became a fellow; employing his time in close application to useful studies, particularly the mathematical sciences, which he contributed greatly, by his example and exhortation, to bring into vogue among his acquaintances there.

About 1603 he quitted the university, and was presented to the rectory of Aldbury, near Guildford in Surrey, where he lived a long retired and studious life, seldom travelling so far as London once a year; his recreation being a diversity of studies. "As often," says he, "as I was tired with the labours of my own profession, I have allayed that tediousness by walking in the pleasant, and more than Elysian Fields of the diverse and various parts of human learning, and not of the mathematics only." About the year 1628 he was appointed by the earl of Arundel tutor to his son lord William Howard, in the mathematics, and his Clavis was drawn up for the use of that young nobleman. He always held a correspondence by letters with many of the most eminent scholars of his time, on mathematical subjects: the originals of which were preserved, and communicated to the Royal Society, by William Jones, esq. The chief mathematicians of that age owed much of their skill to him; and his house was always full of young gentlemen who came from all parts to receive his instruction: nor was he without invitations to settle in France, Italy, and Holland. "He was as facetious," says Mr. David Lloyd, "in Greek and Latin, as solid in arithmetic, geometry, and the sphere, of all measures, music, &c; exact in his style as in his judgment; handling his tube and other instruments at 80 as steadily as others did at 30; owing this, as he said, to temperance and exercise; principling his people with plain and solid truths, as he did the world with great and useful arts; advancing new inventions in all things but religion, which he endeavoured to promote in its primitive purity, maintaining that prudence, meekness, and simplicity were the great ornaments of his life."

Notwithstanding Oughtred's great merit, being a strong royalist, he was in danger, in 1646, of a sequestration by the committee for plundering ministers; several articles being deposed and sworn against him: but, on his day of hearing, William Lilly, the famous astrologer, applied to sir Bulstrode Whitlocke and all his old friends; who appeared so numerous in his behalf, that though the chairman and many other presbyterian members were active against him, yet he was cleared by the majority. This is told us by Lilly himself, in the History of his own Life, where he styles Oughtred the most famous mathematician then of Europe.—He died in 1660, at 86 years of age, and was buried at Aldbury. It is said he died of a sudden ecstasy of joy, about the beginning of May, on hearing the news of the vote at Westminster, which passed for the restoration of Charles the 2d.—He left one son, whom he put apprentice to a watch-maker, and wrote a book of instructions in that art for his use.

He published several works in his life-time; the principal of which are the following:

1. Arithmeticæ in Numero et Speciebus Institutio, in 8vo, 1631. This treatise he intended should serve as a general key to the mathematics. It was afterwards reprinted, with considerable alterations and additions, in 1648, under the title of A Key to the Mathematics. It was also published in English, with several additional tracts; viz, one on the Resolution of all kinds of Affected Equations in Numbers; a second on Compound Interest; a third on the easy Art of Delineating all manner of Plain Sun-dials; also a Demonstration of the Rule of False-Position. A 3d edition of the same work was printed in 1652, in Latin, with the same additional tracts, together with some others, viz, On the Use of Logarithms; A Declaration of the 10th book of Euclid's Elements; a treatise on Regular Solids; and the Theorems contained in the books of Archimedes.

2. The Circles of Proportion, and a Horizontal Instrument; in 1633, 4to; published by his scholar Mr. William Foster.—3. Description and Use of the Double Horizontal Dial; 1636, 8vo.—4. Trigonometria: his treatise on Trigonometry, in Latin, in 4to, 1657: and another edition in English, together with Tables of Sines, Tangents, and Secants.

He left behind him a great number of papers on mathematical subjects; and in most of his Greek and Latin mathematical books, there were found notes in his own hand-writing, with an abridgment of almost every proposition and demonstration in the margin, which came into the museum of the late William Jones, esq. These books and manuscripts then passed into the hands of his friend sir Charles Scarborough the physician; the latter of which were carefully looked over, and all that were found fit for the press, printed at Oxford in 1676, in 8vo, under the title of

5. Opuscula Mathematica hactenus inedita. This collection contains the following pieces: (1) Institutiones Mechanicæ: (2) De Variis Corporum Generibus Gravitate et Magnitudine comparatis: (3) Automata: (4) Quæstiones Diophanti Alexandrini, libri tres: (5) De Triangulis Planis Rectangulis: (6) De Divisione Superficierum: (7) Musicæ Elementa: (8) De Propugnaculorum Munitionibus: (9) Sectiones Angulares.

6. In 1660, sir Jonas Moore annexed to his Arithmetic a treatise entitled, "Conical Sections; or, The several Sections of a Cone; being an Analysis or Methodical Contraction of the two first books of Mydorgius, and whereby the nature of the Parabola, Hyperbola, and Ellipsis, is very clearly laid down. Translated from the papers of the learned William Oughtred."

Oughtred, though undoubtedly a very great mathematician, was yet far from having the happiest method of treating the subjects he wrote upon. His style and manner were very concise, obscure, and dry; and his rules and precepts so involved in symbols and abbreviations, as rendered his mathematical writings very troublesome to read, and difficult to be understood. Besides the characters and abbreviations before made use of in algebra, he introduced several others; as

× to denote multiplication;
: : for proportion or similitude of ratios;
∺ for continued proportion;
$\left.\begin{array}{l}\sqsupset\\ \sqsupset\end{array}\right\}$ for greater and less;

OUNCE, a small weight, being the 16th part of a pound avoirdupois; and the 12th part of a pound troy.—The avoirdupois ounce is divided into 16 drachms or drams; also the ounce troy into 24 pennyweights, and the pennyweight into 24 grains.

OVOLO, in Architecture, a round moulding, whose profile or sweep, in the Ionic and composite capital, is usually a quadrant of a circle; whence it is also popularly called the quarter round.

OUTWARD *Flanking Angle*, or the *Angle of the Tenaille*, is that comprehended by the two flanking lines of defence.

OUTWORKS, in Fortification, all those works made on the outside of the ditch of a fortified place, to cover and defend it.—Outworks, called also advanced and detached works, are those which not only serve to cover the body of the place, but also to keep the enemy at a distance, and prevent them from taking advantage of the cavities and elevations usually found in the places about the counterscarp; which might serve them either as lodgments, or as rideaux, to facilitate the carrying on their trenches, and planting their batteries against the place. Such are ravelins, tenailles, hornworks, queue d'arondes, envelopes, and crownworks. Of these, the most usual are ravelins, or halfmoons, formed between the two bastions, on the flanking angle of the counterscarp, and before the curtain, to cover the gates and bridges.

It is a general rule in all outworks, that if there be several of them, one before another, to cover one and the same tenaille of a place, the nearer ones must gradually, and one after another, command those which are farthest advanced out into the campagne; that is, must have higher ramparts, that so they may overlook and fire upon the besiegers, when they are masters of the more outward works. The gorges also of all outworks should be plain, and without parapets; lest, when taken, they should serve to secure the besiegers against the fire of the retiring besieged; whence the gorges of outworks are only palisadoed, to prevent a surprise.

OX-EYE, in Optics. See SCIOPTIC, and CAMERA *Obscura*

OXGANG, or OXGATE, of land, is usually taken for 15 acres; being as much land as it is supposed one ox can plow in a year. In Lincolnshire they still corruptly call it oskin of land.—In Scotland, the term is used for a portion of arable land, containing 13 acres.

OXYDS, a compound of oxygen and some other body, in such proportion as not to produce an acid.

OXYGEN, a certain simple substance that enters into the composition of water and air; being that which generates or produces acids.

This, one of the most characteristic properties of this body, was discovered by Dr. Priestley in 1774. It was at first called dephlogisticated air, and afterwards successively known by the names of eminently respirable air, pure air, vital air, as long as it was not known that this aerial form is merely one of its states of combination. As soon as this truth was well proved, and clearly explained by Lavoisier, it appeared necessary to give it a new name, which might be applicable to all the states in which it could exist, as well that of gas as of the liquid or solid form; and it finally received the name of Oxygen.

Oxygen, like many other natural bodies, is found in three states, but in none of them is it alone or insulated. In the gaseous form it is dissolved in caloric; in the liquid and solid form it is combined with different substances. As oxygen is often contained, in a more or less solid form, in several natural fossils, which have undergone combustion, and as it has much attraction for caloric, it is only requisite that some one of those fossils should be heated more or less, in order to disengage this principle, and obtain it in the form of gas or air. Thus, the chemists expose certain substances, particularly metals burned by nature or art, to an active fire in close vessels, so disposed as to conduct and receive, under inverted jars, the gas or elastic fluid to be collected; which is thus the product of a true combustion.

The two chief sources from which oxygen is derived, (each of them immense in extent,) are water and air. In the former it is condensed into a liquid form, and combined with about a third of its weight of hydrogen; in the latter it is united with azot, and forms rather more than $\frac{1}{5}$ part of the atmosphere.—There are various other smaller sources of oxygen, such as many parts of the organized world, vegetable or animal (independently of the water they contain so abundantly), mineral acids, and metallic oxyds, &c; but the quantities from these last sources are exceedingly small, in comparison with the preceding.—Most of the green parts of vegetables, while living, yield oxygen gas when exposed to the sun's rays.—The purest possible oxygen gas is obtained by a higher degree of voltaic electricity, from such substances as it is capable of completely decomposing. One of the next purest oxygen gases is obtained by distilling, per se, the dry oxymuriat of potash.

The black oxyd of manganese contains a great quantity of oxygen so loosely combined, as to be expelled by a moderate red heat; and this is the method usually pursued: an earthen or iron retort is filled with the black oxyd of manganese in powder, and heated in a brisk fire. The first product of gas comes over when the manganese is faintly red, and consists chiefly of carbonic acid, so that a taper is immediately extinguished. After this, if small samples of the gas be examined as it comes over, by dipping a bit of kindled wood in it, the fire will soon be found to burn with increased flame and brightness, a sign of the presence of oxygen; soon after which it may be collected for use. If the manganese be very good, one pound of it will yield 1400 cubic inches of great purity; that is, containing no more than $\frac{1}{10}$ of carbonic acid or any other gas.—Manganese, if moistened with sulphuric acid, will also give out much oxygen, on applying no greater heat than that of a taper; and it may thus be obtained very expeditiously, with the simplest apparatus possible.

All the oxyds of mercury, when heated red hot, are decomposed, the metals return to the state of running mercury (which is driven up in vapour and soon condenses), and the oxygen which it contained appears in the gaseous form, mixed with any acid which may have existed in the oxyd.—Oxygen gas may also be obtained very cheap, and considerably pure, by the destructive distillation of nitre in a moderate red heat.—The burning of the several combustible bodies in oxygen gas, forms a number of most beautiful and instructive experiments, and has contributed more than any thing else to give accurate ideas on the nature of combustion in general.

The characters that peculiarly distinguish oxygen gas, are the eminent degree in which it supports combustion and respiration; it being proved that neither of these can continue without oxygen, and that it is solely owing to *its* presence that atmospheric air, and the other compound gasses, are fitted for maintaining those grand processes of the material world. If a small animal be immersed in oxygen gas, it will live much longer than in the same quantity of common air; and if the carbonic acid, generated in the process, be occasionally removed by alkalies, the animal will remain in the gas uninjured for a much longer time. In this, and in many other respects, the process of respiration and combustion agree; but still there are some circumstances which render it probable that the diluted state of oxygen (such as it exists in common air) is altogether fitter for animal respiration, than a purer oxygen.

OXYGONE, in Geometry, is acute-angled, meaning a figure consisting wholly of acute angles, or such as are less than 90 degrees each.—The term is chiefly applied to triangles, where the three angles are all acute.

OXYGONIAL, is acute-angular.

OXYMURIATIC ACID, is the same as dephlogisticated muriatic acid, or chlorine.

OZANAM (JAMES), an eminent French mathematician, was descended from a family of Jewish extraction, but which had long been converts to the Romish faith; and some of whom had held considerable places in the parliaments of Provence. He was born at Boligneux in Bressia, in the year 1640; and being a younger son, though his father had a good estate, it was thought proper to educate him for the church, that he might enjoy some small benefices which belonged to the family, to serve as a provision for him. Accordingly he studied divinity four years; but then, on the death of his father, he devoted himself entirely to the mathematics, to which he had always been strongly attached. Some mathematical books, which fell into his hands, first excited his curiosity; and by his extraordinary genius, without the aid of a master, he made so great a progress, that at the age of 15 he wrote a treatise on that subject.

For a maintenance he first went to Lyons to teach the mathematics, which answered very well; but his generous disposition procured him still better success elsewhere. Among his scholars were two foreigners, who expressing their uneasiness to him, at being disappointed of some bills of exchange for a journey to Paris; he asked them how much would do, and being told 50 pistoles, he lent them the money immediately, even without their note for it. On their arrival at Paris, mentioning this generous action to M. Daguesseau, father of the chancellor, this magistrate was so pleased with it, that he engaged them to invite Ozanam to Paris, with a promise of his favour. The opportunity was eagerly embraced; and the business of teaching the mathematics here soon brought him in a considerable income: but he wanted prudence for some time to make the best use of it. He was young, handsome, and sprightly; and much addicted both to gaming and gallantry, which continually drained his purse. However, this expense in time led him to think of matrimony, and he soon after married a young woman without a fortune. She made amends for this defect however by her modesty, virtue, and sweet temper; so that though the state of his purse was not amended, yet he had more real enjoyment than before, being indeed completely happy in her, as long as she lived. He had twelve children by this lady, though most of them died young; and he was lastly rendered quite unhappy by the death of his wife also, which happened in 1701. Neither did this misfortune come single: for the war breaking out about the same time, on account of the Spanish succession, it swept away all his scholars, who being foreigners, were obliged to leave Paris. Thus he sunk into a very melancholy state; under which however he received some relief, and amusement, from the honour of being admitted this same year an elève of the Royal Academy of Sciences. But he never recovered his wonted health and spirits; so that, though he lingered through a few dull years, with a strong presentiment of his approaching dissolution, he might rather be said to exist than to live, until the year 1717, when he was seized with an apoplexy, which terminated his existence on the 3d of April, at 77 years of age.

Ozanam possessed a mild and calm disposition, a cheerful and pleasant temper, an inventive genius, and a generosity almost unparalleled. After marriage his conduct was irreproachable; and at the same time that he was sincerely pious, he had a great aversion to disputes about theology. On this subject he used to say, that it was the business of the Sorbonne doctors to discuss, of the pope to decide, and of a mathematician to go straight to heaven in a perpendicular line.—He wrote a great number of useful books; a list of which is as follows:

1. A treatise of Practical Geometry; 12mo, 1684.—2. Tables of Sines, Tangents, and Secants; with a treatise on Trigonometry; 8vo, 1685.—3. A treatise of Lines of the First Order; of the Construction of Equations; and of Geometric Lines, &c; 4to, 1687.—4. The Use of the Compasses of Proportion, &c; with a treatise on the Division of Lands; 8vo, 1688.—5. An Universal Instrument for readily resolving Geometrical Problems without calculation; 12mo, 1688.—6. A Mathematical Dictionary; 4to, 1690.—7. A General Method for drawing Dials, &c, 12mo, 1693.—8. A Course of Mathematics, in 5 volumes, 8vo, 1693.—9. A treatise on Fortification, Ancient and Modern; 4to, 1693.—10. Mathematical and Philosophical Recreations; 2 vols 8vo, 1694; and again with additions in 4 vols, 1724.—11. New Treatise on Trigonometry; 12mo, 1699.—12. Surveying and Measuring all Sorts of Artificers' Works; 12mo, 1699.—13. New Elements of Algebra; 2 vols 8vo, 1702.—14. Theory and Practice of Perspective; 8vo, 1711.—15. Treatise of Cosmography and Geography; 8vo, 1711.—16. Euclid's Elements, by Dechales, corrected and enlarged; 12mo, 1709.—17. Boulanger's Practical Geometry enlarged, &c; 12mo, 1691.—18. Boulanger's treatise on the Sphere corrected and enlarged; 12mo.

Ozanam has also the following pieces in the Journal des Sçavans: viz, (1) Demonstration of this theorem, that neither the Sum nor the Difference of two Fourth Powers, can be a Fourth Power; journal of May 1680.—(2) Answer to a Problem proposed by M. Comiers; journal of Nov. 17, 1681.—(3) Demonstration of a Problem concerning False and Imaginary Roots; journal of April 2 and 9, 1685.—(4) Method of finding in Numbers the Cubic and Sursolid Roots of a Binomial, when it has one; journal of April 9, 1691.

Also in the Memoires de Trevoux, of December 1703, he has this piece, viz, Answer to certain articles of Objection to the first part of his Algebra. And lastly, in the Memoirs of the Academy of Sciences, of 1707, he has Observations on a Problem of Spherical Trigonometry.

P.

PACE, or *Geometrical Pace*, an uncertain lineal measure, by some supposed to be equal to 5 feet, by others 4·4, &c.

PAGAN (BLAISE FRANÇOIS Comte de), an eminent French mathematician and engineer, was born at Avignon in Provence, 1604; and entered on the profession of a soldier at 14 years of age. In 1620 he was employed at the siege of Caen, in the battle of Pont de Cé, with the reduction of the Navareins, and the rest of Béarn; where he signalized himself, and acquired a reputation far above his years. He was present, in 1621, at the siege of St. John d'Angeli, as also that of Clarac and Montauban, where he lost an eye by a musket-shot. After this time, there happened neither siege, battle, nor any other occasion, in which he did not signalize himself by some effort of courage and conduct. At the passage of the Alps, and the barricade of Suza, he put himself at the head of the forlorn hope, composed of the bravest youths among the guards; and undertook to arrive the first at the attack, by a private way which was extremely dangerous; when, having gained the top of a very steep mountain, he cried out to his followers, "There lies the way to glory!" On which, sliding along this mountain, they came first to the attack; when immediately commencing a furious onset, and the army coming to their assistance, they forced the barricades. When the king laid siege to Nancy in 1633, Pagan attended him, in drawing the lines and forts of circumvallation.—In 1642 he was sent to the service in Portugal, as field-marshal; and the same year he unfortunately lost the sight of his other eye by a distemper, and thus became totally blind.

But though he was thus prevented from serving his country with his conduct and courage in the field, he resumed the vigorous study of fortification and the mathematics; and in 1645 he gave the public a treatise on the former subject, which was esteemed the best extant.—In 1651 he published his Geometrical Theorems, which showed an extensive and critical knowledge of his subject.—In 1655 he printed a Paraphrase of the Account of the River of Amazons, by father de Rennes; and, though blind, it is said he drew the chart of the river and the adjacent parts of the country, as in that work.—In 1657 he published The Theory of the Planets, cleared from that multiplicity of excentric cycles and epicycles, which the astronomers had invented to explain their motions. This work distinguished him among astronomers, as much as that of Fortification had among engineers. And in 1658 he printed his Astronomical Tables, which are plain and succinct.

Few great men are without some foible: Pagan's was that of a prejudice in favour of judicial astrology; and though he is more reserved than most others on that head, yet we cannot place what he did on that subject among those productions which do honour to his understanding. He was beloved and respected by all persons illustrious for rank as well as science; and his house was the rendezvous of all the polite and learned both in city and court.—He died at Paris, universally regretted, Nov. 18, 1665, at 61 years of age.

Pagan had a universal genius; and, having turned his attention chiefly to the art of war, and particularly to the branch of fortification, he made extraordinary progress and improvements in it. He understood mathematics not only better than is usual for a gentleman whose view is to push his fortune in the army, but even to a degree superior to that of the ordinary masters who teach that science. He had so particular a genius for this kind of learning, that he acquired it more readily by meditation than by reading authors upon it; and accordingly he spent less time in such books than he did in those of history and geography. He had also made morality and politics his particular study; so that he may be said to have drawn his own character in his Homme Heroïque, and to have been one of the completest gentlemen of his time. Having never married, that branch of his family, which removed from Naples to France in 1552, became extinct in his person.

PALILICUM, the same as Aldebaran, a fixed star of the first magnitude, in the eye of Taurus, the Bull.

PALISADES, or PALISADOES, in Fortification, stakes or small piles driven into the ground, in various situations, as some defence against the surprise of an enemy. They are usually about 6 or 7 inches square, and 9 or 10 feet long, driven about 3 feet into the ground, and 6 inches apart from each other, being braced together by pieces nailed across them near the tops; and secured by thick posts at the distance of every 4 or 5 yards.

PALISADES are placed in the covert-way, parallel to and at 3 feet distance from the parapet or ridge of the glacis, to secure it against a surprise. They are also used to fortify the avenues of open forts, gorges, half-moons, the bottoms of ditches, the parapets of covert-ways; and in general all places liable to surprise, and easy of access.

PALISADOES are usually planted perpendicularly; though some make an angle inclining out towards the enemy, that the ropes cast over them, to tear them up, may slip.

PALLADIO (ANDREW), a celebrated Italian architect in the 16th century, was a native of Vicenza in Lombardy, and the disciple of Triffin, a learned Patrician, or Roman nobleman of that town. Palladio was one of those who laboured particularly to restore the ancient beauties of architecture, and contributed greatly to revive a true taste in that art. Having learned the principles of it, he went to Rome; where, applying himself with great diligence to study the ancient monuments, he entered into the spirit of their architects, and possessed himself of all their beautiful ideas. This enabled him to restore their rules, which had been corrupted by the barbarous Goths. He made exact drawings of the principal works of antiquity which were to be met with at Rome; to which he added Commentaries, which went through several impressions, with the figures. This, though a very useful work, is greatly exceeded by the 4 books of architecture which he published in 1570. The last book treats of the Roman temples, and is executed in such a manner, as gives him the preference to all his predecessors on that subject. It was tranlated into French by Roland Friatt, and into

English by several authors. Inigo Jones wrote some excellent remarks upon it, which were published in an edition of Palladio by Leoni, 1742, in 2 volumes folio. Palladio died in 1580.

PALLAS, is the name given by Dr. Olbers to a new planet discovered by him at Bremen, March 28, 1802, being now the 7th in order from the sun, and distant from him about 263 million miles: it performs its periodic revolution in 1682 days, or 4 years 7 months 11 days; but it is too small to be perceived by the naked eye, or even with the assistance of a telescope of an inferior kind. Its elements, as far as they have been at present ascertained, are stated below, but it is probable that future observations may show them to stand in need of some corrections.

Revolution in its orbit 4 years 7 months 11 days.

Mean longitude Jan. 1st, 1804 -	9^s	29°	53′
Annual motion - - -	2	18	11
Aphelion - - - -	10	1	7
Node - - - -	5	22	28
Excentricity - - -		0·2463	
Inclination - - -		34	39

PALLETS, in Clock and Watch Work, are those pieces or levers which are connected with the pendulum or balance, and receive the immediate impulse of the swing-wheel, or balance-wheel, so as to maintain the vibrations of the pendulum in clocks, and of the balance in watches.—The pallets in all the ordinary constructions of clocks and watches, are formed on the verge or axis of the pendulum or balance, and are of various lengths and shapes, according to the construction of the piece, or the fancy of the artist.

PALLIFICATION, or PILING, in Architecture, denotes the piling of the ground-work, or the strengthening it with piles, or timber driven into the ground; which is practised when buildings are erected on a moist or marshy soil.

PALLISADES. See PALISADES.

PALM, an ancient long-measure, taken from the extent of the hand.—The Roman palm was of two kinds: the great palm, taken from the length of the hand, answered to our span, and contained 12 fingers, digits, or fingers breadths, or 9 Roman inches, equal to about 8¾ English inches. The small palm, taken from the breadth of the hand, contained 4 digits or fingers, equal to about 3 English inches.—The Greek palm, or doron, was also of two kinds. The small contained 4 fingers, equal to little more than 3 inches. The great palm contained 5 fingers. The Greek double palm, called dichas, contained also in proportion.

The modern palm is different in different places where it is used. It contains,

	Inc.	Lines
At Rome - - - -	8	3½
At Naples, according to Riccioli, -	8	0
Ditto, according to others, - -	8	7
At Genoa - - - -	9	9
At Morocco and Fez - -	7	2
Languedoc, and some other parts of France,	9	9
The English palm is - - -	3	0

PALM-SUNDAY, the last sunday in lent, or the sunday next before easter day. So called, from the primitive days, on account of a pious ceremony then in use, of bearing palms, in memory of the triumphant entry of Jesus Christ into Jerusalem, 8 days before the feast of the passover.

PAPPUS, a very eminent Greek mathematician of Alexandria towards the latter part of the 4th century, particularly mentioned by Suidas, who says he flourished under the emperor Theodosius the Great, who reigned from the year 379 to 395 of Christ. His writings indicate him to have been a consummate mathematician. Many of his works are lost, or at least have not yet been discovered. Suidas mentions several of his works, as also Vossius De Scientiis Mathematicis. The principal of these are, his Mathematical Collections, in 8 books, the first and part of the second being lost. He wrote also a Commentary on Ptolemy's Almagest; a Universal Chorography; A Description of the Rivers of Libya; A Treatise of Military Engines; Commentaries on Aristarchus of Samos, concerning the Magnitude and Distance of the Sun and Moon; &c. Of these, there have been published, The Mathematical Collections, in a Latin translation, with a large Commentary, by Commandine, in folio, 1588; and a second edition of the same in 1660. In 1644, Mersenne exhibited a kind of abridgment of them in his Synopsis Mathematica, in 4to: but this contains only such propositions as could be understood without figures. In 1655, Meibomius gave some of the Lemmata of the 7th book, in his Dialogue on Proportions. In 1688, Dr. Wallis printed the last 12 propositions of the 2d book, at the end of his Aristarchus Samius. In 1703, Dr. David Gregory gave part of the preface of the 7th book, in the Prolegomena to his Euclid. In 1706, Dr. Halley gave that Preface entire, in the beginning of his Apollonius. And lastly, the reverend and learned Dr. Trail (in an appendix to his Account of the Life and Writings of Rob. Simson, M. D.) has added a critical account of the Mathematical Collections of this author.

As the contents of the principal work, the Mathematical Collections, are exceedingly curious, and no account of them having ever appeared in English when this was written, I shall here give a very brief analysis of those books, extracted from my notes on this author.

Of the Third Book—The subjects of the third book consist chiefly of three principal problems; for the solution of which, a great many other problems are resolved, and theorems demonstrated. The first of these three problems is, To find Two Mean Proportionals between two given lines—The 2d problem is, To find, what are called, three Medietates in a semicircle; where, by a Medietas is meant a set of three lines in continued proportion, whether arithmetical, or geometrical, or harmonical; so that to find three medietates, is to find an arithmetical, a geometrical, and an harmonical set of three terms each. And the third problem is, From some points in the base of a triangle, to draw two lines to meet in a point within the triangle, so that their sum shall be greater than the sum of the other two sides that are without them. A great many curious properties are premised to each of these problems; then their solutions are given according to the methods of several ancient mathematicians, with an historical account of them, and his own demonstrations; and lastly, their applications to various matters of great importance. In his historical anecdotes, many curious things are preserved concerning mathematicians that were ancient even in his time, which we should otherwise have known nothing at all about.

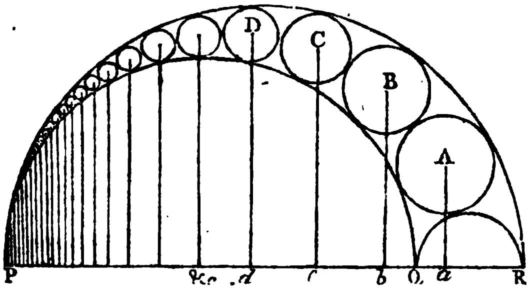
In order to the solution of the first of the three problems above mentioned, he begins by premising four general theorems concerning proportions. Then follows a dissertation on the nature and division of problems by the ancients, into Plane, Solid, and Linear, with examples of them, taken from the writings of Eratosthenes, Philo, and Hero. A solution is then given to the problem concerning two mean proportionals, by four different ways, namely, according to Eratosthenes, Nicomedes, Hero, and after a way of his own, in which he not only doubles the cube, but also finds another cube in any proportion whatever to a given cube.

For the solution of the second problem, he lays down very curious definitions and properties of Medietates of all kinds, and shows how to find all in a great variety of cases, both as to what the ancients had done in them, and what was done by others whom he calls the moderns. Medietas seems to have been a general term invented to express three lines, having either an arithmetical, or a geometrical, or an harmonical relation; for the words proportion (or ratio), and analogy (or similar proportions), are restricted to a geometrical relation only. But he shows how all the medietates may be expressed by analogies.

The solution of the 3d problem leads Pappus to the consideration of a number of admirable and seemingly paradoxical problems, concerning the inflecting of lines to a point within triangles, quadrangles, and other figures, the sum of which shall exceed the sum of the surrounding exterior lines.

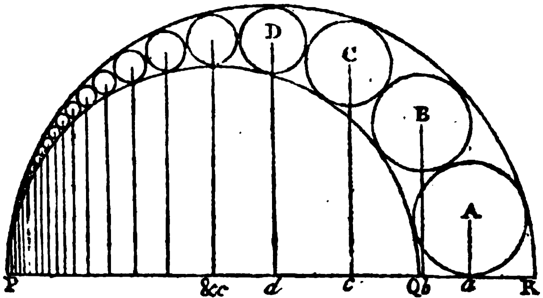
Finally, a number of other problems are added, concerning the inscription of all the regular bodies within a sphere. The whole being effected in a very general and purely mathematical way; making all together 58 propositions, viz, 44 problems and 14 theorems.

Of the 4th Book of Pappus.—In the 4th book are first premised a number of theorems relating to triangles, parallelograms, circles, with lines in and about circles, and the tangencies of various circles: all preparatory to this curious and general problem, viz, relative to an infinite series of circles inscribed in the space, called *αρϐελον*, Arbelon, contained between the circumferences of two circles touching inwardly. Where it is shown, that if the infinite series of circles be inscribed in the manner of this first figure, where three semicircles are described on the lines PR, PQ, QR, and the perpendiculars A*a*, B*b*, C*c*, &c, let fall from the centres of the series of inscribed circles; then the property of these perpendiculars is this, viz, that the first perpendicular A*a* is equal to the diameter or double the radius of the circle A; the second perpendicular B*b* equal to double the diameter or 4 times the radius of the second circle B; the third perpendicular C*c* equal to 3 times the diameter or 6 times the radius of the third circle C; and so on, the series of perpendiculars being



to the series of the diameters, as 1, 2, 3, 4, &c, to 1,
or to the series of radii, as 2, 4, 6, 8, &c, to 1.

But if the several small circles be inscribed in the manner of this second figure, the first circle of the series touching the part of the line QR; then the series of perdendiculars A*a*, B*b*, C*c*, &c, will be 1, 3, 5, 7, &c, times the radii of the circles A, B, C, D, &c; viz, according to the series of odd numbers; the former proceeding by the series of even numbers.



Pappus next treats of the Helix, or Spiral, proposed by Conon, and resolved by Archimedes, demonstrating its principal properties: in the demonstration of some of which, he makes use of the same principles as Cavallerius did lately, adding together an infinite number of infinitely short parallelograms and cylinders, which he imagines a triangle and cone to be composed of.—He next treats of the properties of the Conchoid, which Nicomedes invented for doubling the cube; applying it to the solution of certain problems concerning Inclinations, with the finding of two mean proportionals, and cubes in any proportion whatever.—Then of the *τετϱαγωνιζουσα*, or Quadratrix, so called from its use in squaring the circle, for which purpose it was invented and employed by Dinostratus, Nicomedes, and others: the use of which however he disapproves, as it requires postulates equally hard to be granted, as the problem itself to be demonstrated by it. —Next he treats of Spirals, described on planes, and on the convex surfaces of various bodies.—From another problem, concerning Inclinations, he shows, how to trisect a given angle; to describe an hyperbola, to two given asymptotes, and passing through a given point; to divide a given arc or angle in any given ratio; to cut off arcs of equal lengths from unequal circles; to take arcs and angles in any proportion, and arcs equal to right lines; with parabolic and hyperbolic loci, which last is one of the inclinations of Archimedes.

Of the 5th Book of Pappus.—This book opens with reflections on the different natures of men and brutes, the former acting by reason and demonstration, the latter by instinct, yet some of them with a certain portion of reason or foresight, as bees, in the curious structure of their cells, which he observes are of such a form as to complete the space quite around a point, and yet require the least materials to build them, to contain the same quantity of honey. He shows that the triangle, square, and hexagon, are the only regular polygons capable of filling the whole space round a point; and remarks that the bees have chosen the fittest of these; proving afterwards, in the propositions, that of all regular figures of the same perimeter, that is of the largest capacity which has the greatest number of sides or angles, and consequently that the circle is the most capacious of all figures whatever.

And thus he finishes this curious book on Isoperimetrical figures, both plane and solid; in which many curious and important properties are strictly demonstrated, both of planes and solids, some of them being old in his time, and many new ones of his own. In fact, it seems he has here brought together into this book, all the properties relating to isoperimetrical figures then known, and their different degrees of capacity. In the last theorem of the book, he has a dissertation to show, that there can be no more regular bodies besides the five Platonic ones, or, that only the regular triangles, squares, and pentagons, will form regular solid angles.

Of the 6th Book of Pappus.—In this book he treats of certain spherical properties, which had been either neglected, or improperly and imperfectly treated by some celebrated authors before his time.—Such are some things in the 3d book of Theodosius's Spherics, and in his book on Days and Nights, as also some in Euclid's Phenomena. For the sake of these, Pappus premises and intermixes many curious geometrical properties, especially of circles of the sphere, and spherical triangles. He adverts to some curious cases of variable quantities; showing how some increase and decrease both ways to infinity; while others proceed only one way by increase or decrease, to infinity, and the other way to a certain magnitude; and others again both ways to a certain magnitude, giving a maximum and minimum.—Here are also some curious properties concerning the perspective of the circles of the sphere, and of other lines. Also the locus is determined of all the points from whence a circle may be viewed, so as to appear an ellipse, whose centre is a given point within the circle; which locus is shown to be a semicircle passing through that point.

Of the 7th Book of Pappus.—In the introduction to this book, he describes very particularly the nature of the mathematical composition and resolution of the ancients, distinguishing the particular process and uses of them, in the demonstration of theorems and solution of problems. He then enumerates all the analytical books of the ancients, or those proceeding by resolution, which he does in the following order, viz, 1st, Euclid's Data, in one book: 2d, Apollonius's Section of a Ratio, 2 books: 3d, his Section of a Space, 2 books: 4th, his Tangencies, 2 books: 5th, Euclid's Porisms, 3 books: 6th, Apollonius's Inclinations, 2 books: 7th, his Plane Loci, 2 books: 8th, his Conics, 8 books: 9th, Aristæus's Solid Loci, 5 books: 10th, Euclid's Loci in Superficies, 2 books; and 11th, Eratosthenes's Medietates, 2 books. So that all the books are 31, the arguments or contents of which he exhibits, with the number of the loci, determinations, and cases, &c; with a multitude of lemmas and propositions laid down and demonstrated; the whole making 238 propositions, of the most curious geometrical principles and properties, relating to those books.

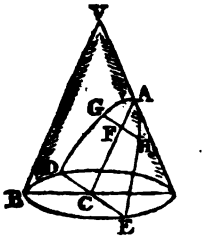
Of the 8th Book of Pappus.—The 8th book is altogether on Mechanics. It opens with a general oration on the subject of mechanics; defining the science, enumerating the different kinds and branches of it, and giving an account of the chief authors and writings on it. After an account of the centre of gravity, on which the science of mechanics so greatly depends, he shows, in the first proposition, that such a point really exists in all bodies. Some of the following propositions are also concerning the properties of the centre of gravity. He next comes to the inclined plane, and in prop. 9, shows what power will draw a given weight up a given inclined plane, when the power is given which can draw the weight along a horizontal plane. In the 10th prop. concerning the moving a given weight with a given power, he treats of what the ancients called a Glossocomum, which is nothing more than a series of wheels-and-axles, in any proportions, turning each other, till we arrive at the given power. In this proposition, as well as in several other places, he refers to some books that are now lost; as Archimedes on the Balance, and the Mechanics of Hero and of Philo. Then, from prop. 11 to prop. 19, he treats on various miscellaneous subjects, as, the organical construction of solid problems; the diminution of an architectural column; to describe an ellipse through five given points; to find the axis of an ellipse organically; to find also organically, the inclination of one plane to another, the nearest point of a sphere to a plane, the points in a spherical surface cut by lines joining certain points, and to inscribe seven hexagons in a given circle. Prop. 20, 21, 22, 23, teach how to construct and adapt the Tympani, or wheels of the glossocomum, to one another, showing the proportions of their diameters, the number of their teeth, &c. And prop. 24 shows how to construct the spiral threads of a screw.

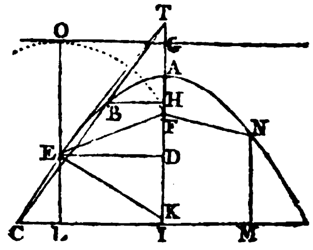
He comes then to the five mechanical powers, by which a given weight is moved by a given power. He here proposes briefly to show what has been said of these powers by Hero and Philo, adding also some things of his own. Their names are, the axis-in-peritrochio, the lever, pulley, wedge, and screw; and he observes, those authors showed how they are all reduced to one principle, though their figures be very different. He then treats of each of these powers separately, giving their figures and properties, their construction and uses.

He next describes the manner of drawing very heavy weights along the ground, by the machine termed Chelone, which is a kind of sledge placed upon two loose rollers, and drawn forward by any power whatever, a third roller being always laid under the fore part of the chelone, as one of the other two is quitted and left behind by the motion of the machine. In fact, this is the same machine as has always been employed on many occasions, in moving very great weights to moderate distances.

Finally, Pappus describes the manner of raising great weights to any proposed height by the combination of mechanic powers, as, by cranes and other machines; illustrating this, and the former parts, by drawings of the machines that are described.

PARABOLA, in Geometry, a figure arising from the section of a cone, when cut by a plane parallel to one of its sides, as the section ADE parallel to the side VB of the cone. See CONIC *Sections*, where some general properties are given.





Some other Properties of the Parabola.—1. From the same point of a cone only one parabola can be drawn; all the other sections between the parabola and the parallel side of the cone being ellipses, and all without them hyperbolas. The parabola has but one focus, through which the axis AC passes; all the other diameters being parallel to this, and also infinite in length.

2. The parameter of the axis is a third proportional to any absciss and its ordinate; viz, AC : CD :: CD : p

the parameter. And therefore if x denote any absciss AC, and y the ordinate CD, it will be $x : y :: y : p = \frac{y^2}{x}$ the parameter; or, by multiplying extremes and means $px = y^2$, which is the equation of the parabola.

3. The focus F is the point in the axis where the double ordinate GH is equal to the parameter. Therefore, in the equation of the curve $px = y^2$, taking $p = 2y$, it becomes $2yx = y^2$, or $2x = y$, that is $2AF = FH$, or $AF = \frac{1}{2}FH$, or the focal distance from a vertex AF is equal to half the ordinate there, or $= \frac{1}{4}p$, one-fourth of the parameter.

4. The abscisses of a parabola are to one another, as the squares of their corresponding ordinates. This is evident from the general equation of the curve $px = y^2$, where, p being constant, x is as y^2.

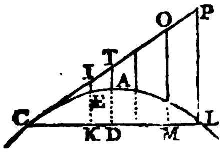
5. The line FE (fig. 2 above) drawn from the focus to any point of the curve, is equal to the sum of the focal distance and the absciss of the ordinate to that point; that is $FE = FA + AD = GD$, taking $AG = AF = \frac{1}{4}p$. Or EF is always $=$ EO, drawn parallel to DG, to meet the perpendicular GO, called the directrix.

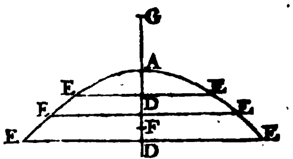
6. If a line TBC cut the curve of a parabola in two points, and the axis produced in T, and BH and CI be ordinates at those two points; then is AT a mean proportional between the abscisses AH and AI, or $AT^2 = AH \cdot AI$. —And if TE touch the curve in E, then is $AT = AD =$ the mean between AH and AI.

7. If FE be drawn from the focus to the point of contact of the tangent TE, and EK perpendicular to the same tangent; then is $FT = FE = FK$; and the subnormal DK equal to the constant quantity $2AF$ or $\frac{1}{2}p$.

8. The diameter EL being parallel to the axis AK, the perpendicular EK, to the curve or tangent at E, bisects the angle LEF. And therefore all rays of light LE, MN, &c, coming parallel to the axis, will be reflected into the point F, which is therefore called the focus, or burning point; for the angle of incidence LEK is $=$ the angle of reflection KEF.

9. If IEK (next fig. below) be any line parallel to the axis, limited by the tangent TC and ordinate CKL to the point of contact; then shall $IE : EK :: CK : KL$. And the same thing holds true when CL is also in any oblique position.





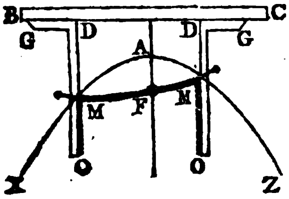
10. The external parts of the parallels IE, TA, ON, PL, &c, are always proportional to the squares of their intercepted parts of the tangent; that is,
the external parts IE, TA, ON, PL,
are proportional to CI^2, CT^2, CO^2, CP^2,
or to the squares CK^2, CD^2, CM^2, CL^2.
A property from which is immediately derived the common theory of projectiles.

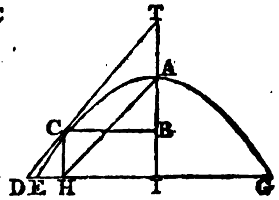
And as this property is common to every position of the tangent, if the lines IE, TA, ON, &c, be appended to the points I, T, O, &c, of the tangent, and moveable about them, and of such lengths as that their extremities E, A, N, &c, &c, be in the curve of a parabola in any one position of the tangent; then making the tangent revolve about the point C, the extremities E, A, N, &c, will always form the curve of some parabola, in every position of the tangent.

The same properties too that have been shown of the axis and its abscisses and ordinates, &c, are true of those of any other diameter. All which, besides many other curious properties of the parabola, may be seen demonstrated in my Treatise on Conic Sections, and in the Course of Mathematics.

11. *To Construct a Parabola by Points.*—In the axis produced take $AG = AF$ (last fig. above) the focal distance, and draw a number of lines EE, EE, &c, perpendicular to the axis AD; then with the distances GD, GD, &c, as radii, and the centre F, describe arcs crossing the parallel ordinates in E, E, &c. Then with a steady hand, or by the side of a slip of bent whale-bole, draw the curve through all the points E, E, E, &c.

12. *To describe a Parabola by a continued Motion.*—If the rule or the directrix BC be laid upon a plane, (first fig. below) with the square GDO, in such manner that one of its sides DG lies along the edge of that rule; and if the thread FMO equal in length to DO, the other side of the square, have one end fixed in the extremity of the rule at O, and the other end in some point F: then slide the side of the square DG along the rule BC, and at the same time keep the thread continually tight by means of the pin M, with its part MO close to the side of the square DO; so shall the curve AMX, which the pin describes by this motion, be one part of a parabola.—And if the square be turned over, and moved on the other side of the fixed point F, the other part of the same parabola AMZ will be described.





13. *To draw Tangents to the Parabola.*—If the point of contact C be given: (last fig. above) draw the ordinate CB, and produce the axis till AT be $=$ AB; then join TC, which will be the tangent.

14. Or if the point be given in the axis produced: take $AB = AT$, and draw the ordinate BC, which will give C the point of contact; to which draw the line TC as before.

15. If D be any other point, neither in the curve nor in the axis produced, through which the tangent is to pass: draw DEG perpendicular to the axis, and take DH a mean proportional between DE and DG, and draw HC parallel to the axis; so shall C be the point of contact, through which and the given point D the tangent DCT is to be drawn.

16. When the tangent is to make a given angle with the ordinate at the point of contact: take the absciss AI equal to half the paramater, or to double the focal distance, and draw the ordinate IE: also draw AH to make with AI the angle HAI equal to the given angle; then draw HC parallel to the axis, and it will cut the curve in C the point of contact, where a line drawn to make the given angle with CB will be the tangent required.

17. *To find the Area of a Parabola.* Multiply the base EG by the perpendicular height AI, and $\frac{2}{3}$ of the product will be the area of the space AEGA; because the parabolic space is $\frac{2}{3}$ of its circumscribing parallelogram.

18. *To find the Length of the Curve* AC, commencing at

the vertex.—Let $y =$ the ordinate BC, $p =$ the parameter, $q = \frac{2y}{p}$, and $s = \surd(1 + q^2)$; then shall $\frac{1}{2}p \times (qs +$ hyp. log, of $q + s)$ be the length of the curve AC.

See various other rules for the areas, and lengths of the curve, &c, in my Treatise on Mensuration, sec 6, pa. 271, &c, 4th edition.

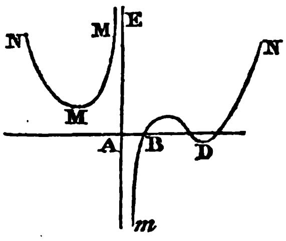
Parabolas *of the Higher Kinds*, are algebraic curves, defined by the general equation $a^{n-1}x = y^n$; that is, either $a^2x = y^3$, or $a^3x = y^4$, or $a^4x = y^5$, &c.

Some call these by the name of paraboloids: and in particular, if $a^2x = y^3$, it is called a cubical paraboloid; if $a^3x = y^4$, it is a biquadratical paraboloid, or a sursolid paraboloid. In respect of these, the parabola of the first kind, above explained, is called the Apollonian, or quadratic parabola.

Those curves are also to be referred to parabolas, that are expressed by the general equation $ax^{n-1} = y^n$, where the indices of the quantities on each side are equal, as before; and these are called semi-parabolas: as $ax^2 = y^3$ the semi-cubical parabola; or $ax^3 = y^4$ the semi-biquadratical parabola; &c.

They are all comprehended under the more general equation $a^m x^n = y^{m+n}$, where the two indices on one side are still equal to the index on the other side of the equation; which include both the former kinds of equations, as well as such as these following ones, viz, $a^2x^2 = y^4$, or $a^2x^3 = y^5$, or $a^4x^3 = y^7$, &c.

Cartesian **Parabola**, is a curve of the 2d order expressed by the equation, $xy = ax^3 + bx^2 + cx + d$, containing four infinite legs, viz, two hyperbolic ones, MM and Bm, to the common asymptote AE, tending contrary ways, and two parabolic legs MN and DN joining them, being Newton's 66th species of lines of the 3d order, and called by him a Trident. It is used by Descartes, in the 3d book of his Geometry, for finding the roots of equations of 6 dimensions, by means of its intersections with a circle. Its most simple equation is $xy = x^3 + g^3$. And points through which it is to pass may be easily found by means of a common parabola whose absciss is $ax^2 + bx + c$, and an hyperbola whose absciss is $\frac{d}{x}$; for y will be equal to the sum or difference of the corresponding ordinates of this parabola and hyperbola.



Descartes, in the place abovementioned, shows how to describe this curve by a continued motion. And Maclaurin does the same thing in a different way, in his Organica Geometria.

Diverging **Parabola**, is a name given by Newton to a species of five different lines of the third order, expressed by the equation $y^2 = ax^3 + bx^2 + cx + d$.

The first is a bell-form parabola, with an oval at its head (fig. 1); which is the case when the equation $0 = ax^3 + bx^2 + cx + d$, has three real and unequal roots; so that one of the most simple equations of a curve of this kind is $py^2 = x^3 + ax^2 + a^2x$.

Fig. 1.

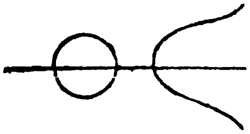


Fig. 2

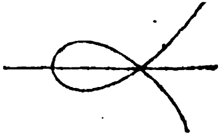


Fig. 3.

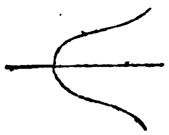
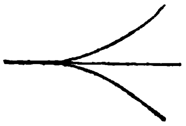


Fig. 4.



The 2d is also a bell-form parabola, with a conjugate point, or infinitely small oval, at the head (fig. 1); being the case when the equation $0 = ax^3 + bx^2 + cx + d$ has its two less roots equal; the most simple equation of which is $py^2 = x^3 + ax^2$.

The 3d is a parabola, with two diverging legs, crossing one another like a knot (fig. 2); which happens when the equation $0 = ax^3 + bx^2 + cx + d$ has its two greater roots equal; the more simple equation being $py^2 = x^3 + ax^2$.

The 4th is a pure bell-form parabola (fig. 3); being the case when $0 = ax^3 + bx^2 + cx + d$ has two imaginary roots; and its most simple equation is $py^2 = x^3 + a^3$, or $py^2 = x^3 + a^2x$.

The 5th a parabola with two diverging legs, forming at their meeting a cusp or double point (fig. 4); being the case when the equation $0 = ax^3 + bx^2 + cx + d$ has three equal roots; so that $py^2 = x^3$ is the most simple equation of this curve, which indeed is the semi-cubical, or Neilian parabola.

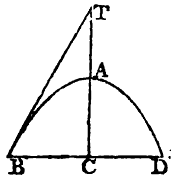
If a solid generated by the rotation of a semi-cubical parabola, about its axis, be cut by a plane, each of these five parabolas will be exhibited by its sections. For, when the cutting plane is oblique to the axis, but falls below it, the section is a diverging parabola, with an oval at its head. When it is oblique to the axis, but passes through the vertex, the section is a diverging parabola, having an infinitely small oval at its head. When the cutting plane is oblique to the axis, it falls below it, and at the same time touches the curve surface of the solid, as well as cuts it, the section is a diverging parabola, with a nodus or knot. When the cutting plane falls above the vertex, either parallel or oblique to the axis, the section is a pure diverging parabola. And lastly, when the cutting plane passes through the axis, the section is the semi-cubical parabola from which the solid was generated.

PARABOLIC *Asymptote*, is used for a parabolic line approaching to a curve, so that they never meet; yet by producing both indefinitely, their distance from each other becomes less than any given line.

There may be as many different kinds of these asymptotes as there are parabolas of different orders. When a curve has a common parabola for its asymptote, the ratio of the subtangent to the absciss approaches continually to the ratio of 2 to 1, when the axis of the parabola coincides with the base; but this ratio of the subtangent to the absciss approaches to that of 1 to 2, when the axis is perpendicular to the base. And by observing the limit to which the ratio of the subtangent and absciss approaches, parabolic asymptotes of various kinds may be discovered. See Maclaurin's Fluxions, art. 337.

Parabolic *Conoid*, is a solid generated by the rotation of a parabola about its axis.—This solid is equal to half its circumscribed cylinder; and therefore if the base be multiplied by the height, half the product will be the solid content.

To find the Curve Surface of a Paraboloid.



Let BAD be the generating parabola, AC = AT, and BT a tangent at B. Put $p = 3·1416$, $y =$ BC, $x =$ AC = AT, and $t =$ BT $= \sqrt{(4x^2 + y^2)}$; then is the curve surface $= \frac{2}{3} ay \times (y + \frac{tt}{t+y})$.

See various other rules and geometrical constructions for the surfaces and solidities of parabolic conoids, in my Mensuration, part 3, sec. 6, 4th edition.

Parabolic *Pyramidoid*, is a solid figure thus named by Dr. Wallis, from its genesis, or formation, which is thus: Let all the squares of the ordinates of a parabola be conceived to be so placed, that the axis shall pass perpendicularly through all their centres; then the aggregate of all these planes will form the parabolic pyramidoid.—This figure is equal to half its circumscribed parallelopipedon. And therefore the solid content is found by multiplying the base by the altitude, and taking half the product; or the one of these by half the other.

Parabolic *Space*, is the space or area included by the curve line and base or double ordinate of the parabola. The area of this space, it has been shown under the article Parabola, is $\frac{2}{3}$ of its circumscribed parallelogram; which is its quadrature, and which was first found out by Archimedes, though some say by Pythagoras.

Parabolic *Spindle*, is a solid figure conceived to be formed by the rotation of a parabola about its base or double ordinate.—This solid is equal to $\frac{8}{15}$ of its circumscribed cylinder. See my Mensuration, prob. 15, pa. 296, &c, 4th edition.

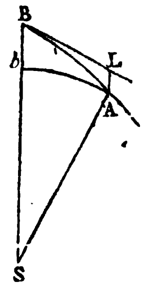
Parabolic *Spiral*. See **Helicoid** *Parabola*.

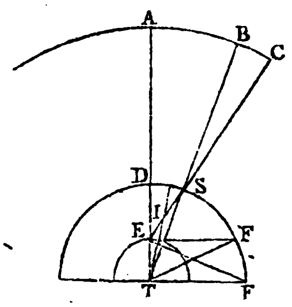
Paraboliform *Curves*, a name sometimes given to the parabolas of the higher orders.

PARABOLOIDES, parabolas of the higher orders.—The equation for all curves of this kind being $a^{m} - {}^{n}x^{n} = y^{m}$, the proportion of the area of any one, to the complement of it to the circumscribing parallelogram, will be as *m* to *n*.

PARACENTRIC *Motion*, denotes the space by which a revolving planet approaches nearer to, or recedes farther from, the sun, or centre of attraction.

Thus, if a planet in A move towards B; then is SB − SA = *b*B the paracentric motion of that planet: where S denotes the place of the sun.





Paracentric *Solicitation of Gravity*, is the same as the Vis Centripeta; and is expressed by the line AL drawn from the point A, parallel to the ray SB (infinitely near SA), till it intersect the tangent BL.

PARACHUTE, or *Fall-breaker*, an instrument in form of a large umbrella, by means of which a person may safely descend to the ground with a small velocity, from a balloon, or from any great height in the air.—This is effected by the great resistance of the air against the descending machine; which, being resisted by a force increasing as the square of the velocity, soon comes to descend with a uniform motion. And, to determine what size it is necessary the parachute ought to have, in order that the velocity may be at any given moderate rate, see the solution of prob. 1, tract 38, vol. 3, of my Mathematical and Philosophical Tracts.

PARALLACTIC *Angle*, called also simply **Parallax**, is the angle EST (last fig. above) made at the centre of a star, &c, by two lines, drawn, the one from the centre of the earth at T, and the other from its surface at E.—Or, which amounts to the same thing, the parallactic angle is the difference of the two angles CEA and BTA, under which the real and apparent distances from the zenith are seen.—The sines of the parallactic angles ELT, EST, at the same or equal distances DS from the zenith, are in the reciprocal ratio of the distances, TL, and TS, from the centre of the earth.

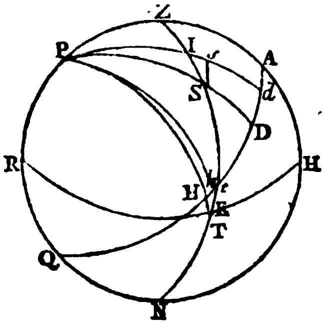
PARALLAX, is an arch of the heavens intercepted between the true place of a star, and its apparent place. The true place of a star S, is that point of the heavens B, in which it would be seen by an eye placed in the centre of the earth at T. And the apparent place, is that point of the heavens C, where a star appears to an eye on the surface of the earth at E. This difference of places, is what is called absolutely the parallax, or the parallax of altitude; which Copernicus calls the commutation; and which therefore is an angle formed by two visual rays, drawn, the one from the centre, the other from the circumference of the earth, and traversing the body of the star; being measured by an arch of a great circle intercepted between the two points of true and apparent places, B and C.

The **Parallax** *of Altitude* CB is properly the difference between the true distance from the zenith AB, and the apparent distance AC. Hence the parallax diminishes the altitude of a star, or increases its distance from the zenith; and it has therefore a contrary effect to the refraction.—The parallax is greatest in the horizon, called the horizontal parallax EFT. From hence it decreases all the way to the zenith D or A, where it is nothing; the real and apparent places there coinciding.

The horizontal parallax is the same, whether the star be in the true or apparent horizon.—The fixed stars have no sensible parallax, on account of their immense distance, to which the semidiameter of the earth is but a mere point: and therefore lines drawn from any two parts of the earth, to the stars, may be considered as parallel. Hence also, the nearer a star is to the earth, the greater is its parallax; and on the contrary, the farther it is off, the less is the parallax, at an equal elevation above the horizon. So the star at S has a less parallax than the star at I. Saturn is so high, that it is difficult to observe in him any parallax at all.

Parallax increases the right and oblique ascension, and diminishes the descension; it diminishes the northern declination and latitude in the eastern part, and increases them in the western; but it increases the southern declination in the eastern and western part; it diminishes the longitude in the western part, and increases it in the eastern. Parallax therefore has just opposite effects to refraction.

The doctrine of parallaxes is of the greatest importance in astronomy, for determining the distances of the planets, comets, and other phenomena of the heavens; for the calculation of eclipses, and for finding the longitude.



Parallax *of Right Ascension and Descension*, is an arch of the equinoctial D*d*, by which the parallax of altitude increases the ascension, and diminishes the descension.

Parallax *of Declination*, is an arch of a circle of declination SI, by which the parallax of altitude increases or diminishes the declination of a star.

Parallax *of Latitude*, is an arch of a circle of latitude SI, by which the parallax of altitude increases or diminishes the latitude.

Menstrual Parallax *of the Sun*, is an angle formed by two right lines; one drawn from the earth to the sun, and another from the sun to the moon, at either of their quadratures.

Parallax *of the Annual Orbit of the Earth*, is the difference between the heliocentric and geocentric place of a planet, or the angle at any planet, subtended by the distance between the earth and sun. There are various methods for finding the parallaxes of the celestial bodies; some of the principal and easier of which are as follow:

To observe the Parallax *of a Celestial Body.*—Observe when the body is in the same vertical with a fixed star which is near it, and in that position measure its apparent distance from the star. Observe again when the body and star are at equal altitudes from the horizon; and there measure their distance again. Then the difference of these distances will be the parallax very nearly.

To observe the Moon's Parallax.—Observe very accurately the moon's meridian altitude, and note the moment of time. To this time, equated, compute her true latitude and longitude, and from these find her declination; also from her declination, and the elevation of the equator, find her true meridian altitude. Subtract the refraction from the observed altitude: then the difference between the remainder and the true altitude, will be the parallax sought. If the observed altitude be not meridional, reduce it to the true altitude for the time of observation. By this means, in 1583, Oct. 12 day 5 h. 19 m. from the moon's meridian altitude observed at 13° 38′, Tycho found her parallax to be 54 minutes.

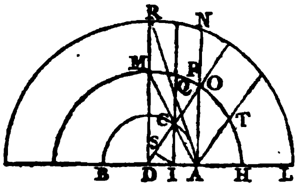
To observe the Moon's Parallax *in an Eclipse.*—In an eclipse of the moon observe when both horns are in the same vertical circle, and at that moment take the altitudes of both horns; then half their sum will be nearly the apparent altitude of the moon's centre; from which subtract the refraction, which gives the apparent altitude freed from refraction. But the true altitude is nearly equal to the altitude of the centre of the shadow at that time: now the altitude of the centre of the shadow is known, because we know the sun's place in the ecliptic, and his depression below the horizon, which is equal to the altitude of the opposite point of the ecliptic, in which the centre of the shadow is. Having thus the true and apparent altitudes, their difference is the parallax sought. Lahire makes the greatest horizontal parallax 1° 1′ 25″, and the least 54′ 5″. M. le Monnier determined the mean parallax of the moon to be 57′ 12″. Others have made it 57′ 18″.

From the Moon's Parallax EST, *and altitude* SF (last fig. but one); *to find her distance from the Earth.*—From her apparent altitude given, there is given her apparent zenith distance, i. e. the angle AES; or by her true altitude, the complement angle ATS. Therefore, since at the same time, the parallactic angle S is known, the 3d or supplemental angle TES is also known. Then, considering the earth's semidiameter TE as 1, in the triangle TES are given all the angles and the side TE, to find ES the moon's distance from the surface of the earth, or TS her distance from the centre.

Thus Tycho, by the observation above mentioned, found the moon's distance at that time from the earth, was 62 of the earth's semidiameters. According to Lahire's determination, her distance when in the perigee is near 56 semidiameters, but in her apogee near 63¼; and therefore the mean nearly 59¾, or in round numbers 60 semidiameters.

Hence also, since, from the moon's theory, there is given the ratio of her distances from the earth in the several degrees of her anomaly; those distances being found, by the rule of three, in semidiameters of the earth, the parallax is thence determined to the several degrees of the true anomaly.

To observe the Parallax *of Mars.*— 1. Suppose Mars to be in the meridian and equator at H: and that the observer, under the equator in A, observes him culminating with some fixed star. 2. If now the observer were in the centre of the earth, he would see Mars constantly in the same point of the heavens with the star; and therefore, together with it, in the plane of the horizon, or of the 6th horary: but since Mars here has some sensible parallax, and the fixed star has none, Mars will be seen in the horizon, when in P, the plane of the sensible horizon; and the star, when in R, the plane of the true horizon: therefore observe the time between the transit of Mars and of the star through the plane of the 6th hour.—3. Convert this time into minutes of the equator, at the rate of 15 degrees to the hour; by which means there will be obtained the arch PM, to which the angle PAM, and consequently the angle AMD, is nearly equal; which is the horizontal parallax of Mars.



If the observer be not under the equator, but in a parallel IQ, that difference will be a less arch QM; therefore, since the small arches QM and PM are nearly as their sines AD and ID; and since ADG is equal to the distance of the place from the equator, i. e. to the elevation of the pole, or the latitude; therefore AD is to ID, as radius to the cosine of the latitude; hence we have this proportion, as the cosine of the latitude ID is to radius, so is the parallax observed in I, to the parallax under the equator.

Since Mars and the fixed star cannot be commodiously observed in the horizon; let them be observed in the circle of the 3d hour: and since the parallax observed there TO, is to the horizontal one PM, as IS to ID; say, as the sine of the angle IDS, or 45° (since the plane DO is in the middle between the meridian DH and the true horizon DM), is to radius, so is the parallax TO to the horizontal parallax PM.

If Mars be likewise out of the plane of the equator, the parallax found will be an arch of a parallel; which must therefore be reduced, as above, to an arch of the equator.—Lastly, if Mars be not stationary, but either direct or retrograde, by observations for several days find out what his motion is every hour, that his true place from the centre may be assigned for any given time.

By this method Cassini, who was the author of it, observed the greatest horizontal parallax of Mars to be 25″; but Mr. Flamsteed found it near 30″. Cassini observed also the parallax of Venus by the same method.

To find the Sun's PARALLAX.—The great distance of the sun renders his parallax too small to fall under even the nicest immediate observation. Many attempts have indeed been made, both by the ancients and moderns, and many methods invented for that purpose. The first was that of Hipparchus, which was followed by Ptolemy, &c, and was founded on the observation of lunar eclipses. The second was that of Aristarchus, in which the angle subtended by the semidiameter of the moon's orbit, seen from the sun, was sought from the lunar phases. But these both proving deficient, astronomers now have recourse to the parallaxes of the nearer planets, Mars and Venus. Now from the theory of the motions of the earth and planets, there is known at any time the proportion of the distances of the sun and planets from us; and the horizontal parallaxes being reciprocally proportional to those distances; by knowing the parallax of a planet, that of the sun may be thence found.

Thus Mars, when opposite to the sun, is only half the distance of the sun from us, and therefore his parallax will be twice as great as that of the sun. And Venus, when in her inferior conjunction with the sun, is sometimes nearer us than he is; and therefore her parallax is greater in the same proportion. Thus, from the parallaxes of Mars and Venus, Cassini found the sun's parallax to be 10″; whence his distance comes out 22000 semidiameters of the earth.

But the most accurate method of determining the parallaxes of these planets, and thence the parallax of the sun, is that of observing their transit. However, Mercury, though frequently to be seen on the sun, is not fit for this purpose; because he is so near that luminary, that the difference of their parallaxes is always less than the solar parallax required. But the parallax of Venus, being almost 4 times as great as the solar parallax, will cause very sensible differences between the times in which she will seem to be passing over the sun at different parts of the earth. This method of determining the sun's parallax appears to have been first proposed by Mr. James Gregory, viz, in his Optica Promota, Schol. pa. 130, published in 1663.

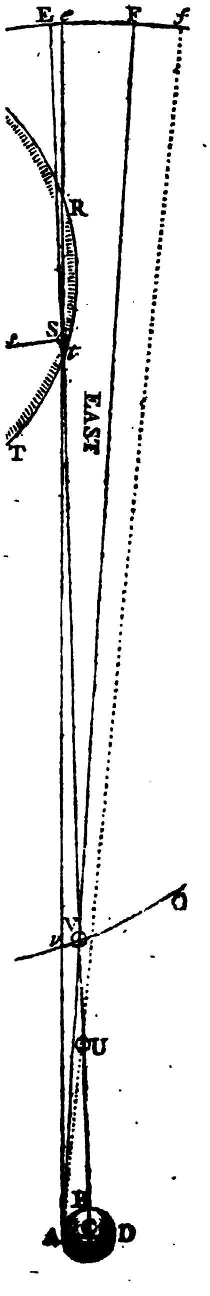
With the view of engaging the attention of astronomers to this method of determining the sun's parallax, Dr. Halley communicated to the Royal Society, in 1691, a paper, containing an account of the several years in which such a transit may happen, computed from the tables which were then in use: those at the ascending node occur in the month of November O. S. in the years 918, 1161, 1396, 1631, 1639, 1874, 2109, 2117; and at the descending node in May O. S. in the years 1048, 1283, 1291, 1518, 1526, 1761, 1769, 1996, 2004. Philos. Trans. Abr. vol. 3, pa. 448, &c.

Dr. Halley even then concluded, that if the interval of time between the two interior contacts of Venus with the sun, could be measured to the exactness of a second, in two places properly situated, the sun's parallax might be determined within its 500th part. And this conclusion was more fully explained in a subsequent paper, concerning the transit of Venus in the year 1761, in the Philos. Trans. No. 348, or Abr. vol. 11, pa. 553.

It does not appear that any of the preceding transits had been observed; except that of 1639, by our ingenious countryman Mr. Horrox, and his friend Mr. Crabtree, of Manchester. But Mr. Horrox died on the 3d of January, 1641, at the age of 25, just after he had finished his treatise, Venus in Sole visa, in which he discovers a more accurate knowledge of the dimensions of the solar system, than his learned commentator Hevelius.

To give a general idea of this method of determining the horizontal parallax of Venus, and thence, by analogy, the parallax and distance of the sun, and of all the planets from him; let DBA be the earth, V Venus, and TSR the eastern limb of the sun. Now, to an observer at B, the point *t* of that limb will be on the meridian, and its place as referred to the heavens will be at E, and Venus will appear just within it at S. But to an observer at A, at the same instant, Venus is east of the sun, in the right line AVF; the point *t* of the sun's limb appears at *e* in the heavens, and if Venus were then visible she would appear at F. The angle CVA is the horizontal parallax of Venus; which is equal to the opposite angle FVE, measured by the arc FE. ASC is the sun's horizontal parallax, equal to the opposite angle *e*SE, measured by the arc *e*E; and FA*e* or VA*e* is Venus's horizontal parallax from the sun, which may be found by observing how much later in absolute time her total ingress on the sun, is, as seen from A, than as seen from B, which is the time she takes to move from V to *v*, in her orbit OV*v*.

If Venus were nearer the earth, as at U, her horizontal parallax from the sun would be the arc *fe*, which measures the angle FA*e*; and this angle is greater than the angle FA*e*, by the difference of their measures F*f*. So that, as the distance of the celestial object from the earth is less, its parallax is the greater.



Now it has been already observed, that the horizontal parallaxes of the planets are inversely as their distances from the earth's centre; and consequently, as the sun's distance at the time of the transit, is to Venus's distance, so is the parallax of Venus to that of the sun: and as the sun's mean distance from the earth's centre, is to his distance on the day of the transit, so is his horizontal parallax on that day, to his horizontal parallax at the time of his mean distance from the earth's centre. Hence his true distance in semidiameters of the earth may be obtained by the following analogy, viz, as the sine of the sun's parallax is to radius, so is unity or the earth's semidiameter, to the number of semidiameters of the earth in the sun's distance from the centre; which number multiplied by the number of miles in the earth's semidiameter, will give the number of miles in the sun's distance. Then from the proportional distances of the planets, determined by the theory of gravity, their true distances may be found. And from their apparent diameters at these known distances, their real diameters and bulks may be found.

Mr. Short, with great labour, deduced the quantity of the sun's parallax from the best observations that were made of the transit of Venus, on the 6th of June, 1761 (for which see Philos. Trans. vol. 51 and 52) both in Britain and in foreign parts, and found it to have been 8″·52 on the day of the transit, when the sun was very nearly at his greatest distance from the earth; and consequently 8″·65 when the sun is at his mean distance from the earth. See Philos. Trans. vol. 52, pa. 611, &c. Whence,

As sin. 8″·65	-	-	log.	5·6219140
to radius	-	-	-	10·0000000
So is 1 semidiameter		-	-	0·0000000
to 23822·84 semidiameters		-	-	4·3780860

that is, $23882\frac{84}{100}$ is the number of the earth's semidiameters contained in its distance from the sun; and this number of semidiameters being multiplied by 3985, the number of English miles contained in the earth's semidiameter (though later observations make this semidiameter only 395·7 miles), there is obtained 95,173,127 miles for the earth's mean distance from the sun. And hence, from the analogies under the article DISTANCE, the mean distances of all the rest of the planets from the sun, in miles, are found as follow, viz,

Mercury's distance	-	-	36,841,468
Venus's distance	-	-	68,891,486
Mars's distance	-	-	145,014,148
Vesta's distance	-	-	224,145,086
Juno's distance	-	-	253,541,210
Pallas' distance	-	-	263,153,691
Ceres' distance	-	-	263,344,042
Jupiter's distance	-	-	494,990,976
Saturn's distance	-	-	907,956,130
Uranus' distance	-	-	1,816,074,574

In another paper (Philos. Trans. vol. 53, pa. 169), Mr. Short states the mean horizontal parallax of the sun at 8″·69. And Mr. Hornsby, from several observations of the transit of June 3, 1769 (for which see the Philos. Trans. vol. 59), deduces the sun's parallax for that day equal to 8″·65, and the mean parallax 8″·78; whence he makes the mean distance of the earth from the sun to be 93,726,900 English miles, and the distances of the other planets will be

Mercury's distance	-	-	36,281,700
Venus's distance	-	-	67,795,500
Mars's distance	-	-	142,818,000
Vesta's distance	-	-	220,739,033
Juno's distance	-	-	249,688,461
Pallas' distance	-	-	259,154,878
Ceres' distance	-	-	259,342,332
Jupiter's distance	-	-	487,472,000
Saturn's distance	-	-	894,162,000
Uranus' distance	-	-	1,788,477,960

See the Philos. Trans. vol. 61, pa. 572.

But others, by taking the results of those observations that are most to be depended on, have made the sun's parallax at his mean distance from the earth to be 8″·6045; and some make it only 8″·54. According to the former of these, the sun's mean distance from the earth is 95,109,736 miles; and according to the latter it is 95,834,742 miles. On the whole there seems reason to conclude that the sun's horizontal parallax may be stated at 8″·6, and his distance near 95 millions of miles. Hence, the following horizontal parallaxes:

Mean parallax of the sun		-	0′	8″·6
Moon's greatest	-	-	61	32
Moon's least	-	-	54	4
Moon's mean	-	-	57	48
Mars's		-	0	25

Of the PARALLAX *of the Fixed Stars.* As to the fixed stars, their distance is so great, that it has never been found that they have any sensible parallax, either with respect to the earth's diameter, or even with regard to the diameter of the earth's annual orbit round the sun, though this diameter is about 190 millions of miles. For, any of those stars being observed from opposite ends of this diameter, or at the interval of half a year between the observations, when the earth is in opposite points of her orbit, yet still the star appears in the same place and situation in the heavens, without any change that is sensible, or measurable with the very best instruments, not amounting to a single second of a degree. That is, the diameter of the earth's annual orbit, at the nearest of the fixed stars, does not subtend an angle of a single second; or, in comparison of the distance of the fixed stars, the extent of 190 millions of millions is but as a point!

The parallax of the fixed stars is a subject which has engaged the attention of many able astronomers, but hitherto their labours have been unsuccessful. Dr. Herschel, to whom astronomy is so much indebted for his ingenious labours and accurate observations, has proposed, in the Philosophical Transactions, a method for determining the annual parallax by means of double stars, by which it would become sensible, and might be ascertained at least to a greater degree of accuracy than could be effected by any other method, though it should not exceed the 10th part of a second. See STAR. This problem is highly interesting, as it seems to offer the only rational data for determining the distances of the fixed stars; and if this could be ascertained with any tolerable degree of probability, it could not fail of being very gratifying to astronomers, and all those who contemplate with admiration the magnificent works of the Deity.

PARALLAX is also used, in Levelling, for the angle contained between the line of true level, and that of apparent level. And, in other branches of science, for the difference between the true and apparent places.

PARALLEL, in Geometry, is applied to lines, figures, and bodies, which are every-where equidistant from each other; or which, though infinitely produced, would never either approach nearer, or recede farther from, each other;

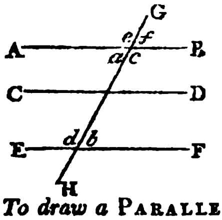
their distance being every-where measured by a perpendicular line between them. Hence,

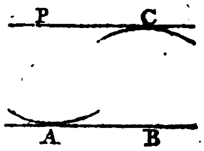
Parallel *right lines* are those which, though infinitely produced, would never meet: which is Euclid's definition of them.—Newton, in lemma 22, book 1, of his Principia, defines parallels to be such lines as tend to a point infinitely distant.—Parallel lines stand opposed to lines converging, and diverging.

Some define an inclining or converging line, to be that which will meet another at a finite distance, and a parallel line, that which will only meet at an infinite distance.

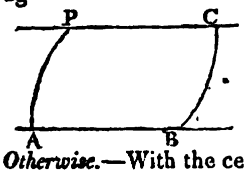
As a perpendicular is by some said to be the shortest of all lines that can be drawn to another; so a parallel is said to be the longest.

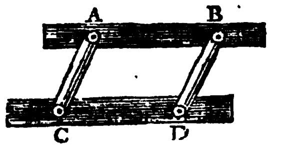
It is demonstrated by geometricians, that two lines, AB and CD, that are each parallel to one and the same right line EF, are also parallel to each other. And that if two parallel lines AB and EF be cut by any other line GH; then 1st, the alternate angles are equal; viz the angle $a = \angle b$, and $\angle c = \angle d$. 2d, The external angle is equal to the internal one on the same side of the cutting line; viz, the $\angle e = \angle d$, and the $\angle f = \angle b$. 3d, That the two internal angles on the same side are, taken together, equal to two right angles; viz, $\angle a + \angle d = 180°$, or $\angle c + \angle b = 180°$.





To draw a **Parallel** *Line.*—If the line to be parallel to AB must pass through a given point P: Take the nearest distance between the point P and the given line AB, by setting one foot of the compasses in P, and with the other describe an arc just to touch the line in A; then with that distance as a radius, and a centre B taken any where in the line, describe another arc C; lastly, through P draw a line PC to touch the arc C, and that will be the parallel sought.





Otherwise.—With the centre P, and a convenient radius, describe an arc BC, cutting the given line in B. Next, with the same radius, and centre B, describe another arc PA, cutting also the given line in A. Lastly, take AP between the compasses, and apply it from B to C; and through P and C draw the parallel PC required. Or, draw the line with the parallel ruler, described below, by laying one edge of the ruler along AB, and extending the other to the given point or distance. When the one line is to be at a given distance from the other; take that distance between the compasses as a radius, and with two centres, taken any where in the given line, describe two arcs; then lay a ruler just to touch the arcs, and by it draw the parallel.

Parallel *Planes*, are every-where equidistant, or have all the perpendiculars that are drawn between them, every-where equal.

Parallel *Rays*, in Optics, are those which keep always at an equal distance in respect to each other, from the visual object to the eye, from which the object is supposed to be infinitely distant.

Parallel *Ruler*, is a mathematical instrument, consisting of two equal rulers, AB and CD, either of wood or metal, connected together by two slender cross bars or blades AC and BD, moveable about the points or joints A, B, C, D.—There are other forms of this instrument, a little varied from the above; some having the two blades crossing in the middle, and fixed only at one end of them, the other two ends sliding in grooves along the two rulers; &c.

The use of this instrument is obvious. For the edge of one of the rulers being applied to any line, the other opened to any extent will be always parallel to the former; and consequently any parallels to this may be drawn by the edge of the ruler, opened to any extent.

Parallel *Sailing*, in Navigation, is the sailing on or under a parallel of latitude, or parallel to the equator.—Of this there are three cases.

1. Given the Distance and Difference of Longitude; to find the Latitude.—Rule. As the diff. of longitude is to the distance, so is radius to the cosine of the latitude. 2. Given the Lat. and Diff. of Longitude; to find the Distance.—Rule. As radius is to the cosine of the lat. so is the diff. of longitude to the distance. 3. Given the Latitude and Distance; to find the difference of longitude.—Rule. As the cosine of lat. is to radius, so is the distance to the diff. of longitude.

Parallel *Sphere*, is that situation of the sphere where the equator coincides with the horizon, and the poles with the zenith and nadir.—In this sphere, all the parallels of the equator become parallels of the horizon; consequently no stars ever rise or set, but all turn round in circles parallel to the horizon, as well as the sun himself, which when in the equinoctial wheels round the horizon the whole day. Also, After the sun rises to the elevated pole, he never sets for 6 months; and after his entering again on the other side of the line, he never rises for 6 months longer.

This position of the sphere can only happen to those who live at the poles of the earth, if any such there be. The greatest height the sun can rise to them, is 23½ degrees. They have but one day and one night, each being half a year long. See Sphere.

Parallels, or *Places of Arms*, in a Siege, are deep trenches, 15 or 18 feet wide, joining the several attacks together; and serving to place the guard of the trenches in, to be at hand to support the workmen when attacked.—There are usually three in an attack: the first is about 600 yards from the covert-way, the second between 3 and 400, and the third near or on the glacis.—It is said they were first invented or used by Vauban.

Parallels *of Altitude*, or Almacantars, are circles parallel to the horizon, conceived to pass through every degree and minute of the meridian between the horizon and zenith; having their poles in the zenith.

Parallels, or **Parallel** *Circles*, called also Parallels of Latitude, and Circles of Lat. are lesser circles of the sphere, parallel to the equinoctial or equator.

Parallels *of Declination*, are lesser circles parallel to the equinoctial.

Parallels *of Latitude*, in Geography, are lesser circles parallel to the equator. But in Astronomy they are parallel to the ecliptic.

PARALLELISM, the quality of a parallel, or that which denominates it such. Or it is that by which two things, as lines, rays, or the like, become equidistant from each other.

Parallelism *of the Earth's Axis*, is that invariable situation of the axis, in the progress of the earth through the annual orbit, by which it always keeps parallel to itself; so that if a line be drawn parallel to its axis, while in any one position; the axis, in all other positions or parts of the orbit, will always be parallel to the same line.

In consequence of this parallelism, the axis of the earth points always, as to sense, to the same place or point in the heavens, viz, to the poles. Because, though really the axis, in the annual motion, describes the surface of a cylinder, whose base is the circle of the earth's annual orbit, yet this whole circle is but as a point in comparison with the distance of the fixed stars; and therefore all the sides of the cylinder seem to tend to the same point, which is the celestial pole.—To this parallelism is owing the change and variety of seasons, with the inequality of days and nights.

This parallelism is the necessary consequence of the earth's double motion; the one round the sun, the other round its own axis. Nor is there any necessity to imagine a third motion, as some have done, to account for this parallelism.

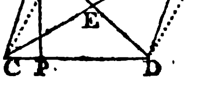
Parallelism *of Rows of Trees*. The eye placed at the end of an alley bounded by two rows of trees, planted in parallel lines, never sees them parallel, but always inclining to each other, towards the farther end.

Hence mathematicians have taken occasion to inquire, in what lines the trees must be disposed, to correct this effect of the perspective, and make the rows still appear parallel. And, to produce this effect, it is evident that the unequal intervals of any two opposite or corresponding trees may be seen under equal visual angles. For this purpose, M. Fabry, Tacquet, and Varignon observe, that the rows must be opposite semi-hyperbolas. See the Mem. Acad. Sciences, an. 1717. But notwithstanding the ingenuity of their speculations, it has been proved by Dalembert, and Bouguer, that to produce the effect proposed, the trees are to be ranged merely in two diverging right lines.

PARALLELOGRAM, in Geometry, is a quadrilateral right-lined figure, whose opposite sides are parallel to each other.—A parallelogram may be conceived as generated by the motion of a right line, along a plane, always parallel to itself.—Parallelograms have several particular denominations, and are of several species, according to certain particular circumstances, as follow:

When the angles of the parallelogram are right ones, it is called a rectangle.—When the angles are right, and all its sides equal, it is a square.—When the sides are equal, but the angles oblique ones, the figure is a rhombus or lozenge. And when both the sides and angles are unequal, it is a rhomboides. Every other quadrilateral whose opposite sides are neither parallel nor equal, is called a trapezium.

Properties of the **Parallelogram**.—1. In every parallelogram ABDC, the diagonal divides the figure into two equal triangles, ABD, ACD. Also the opposite angles and sides are equal, viz, the side AB = CD, and AC = BD, also the angle A = ∠ D, and the ∠ B = ∠ C. And the sum of any two succeeding angles, or next the same side, is equal to two right angles, or 180 degrees, as $\angle A + \angle C = \angle C + \angle D = \angle D + \angle B = \angle B + \angle A =$ two right-angles.



2. All parallelograms, as ABDC and *ab*DC, are equal, that are on the same base CD, and between the same parallels A*b*, CD; or that have either the same or equal bases and altitudes; and each is double a triangle of the same or equal base and altitude.

3. The areas of parallelograms are to one another in the compound ratio of their bases and altitudes. If their bases be equal, the areas are as their altitudes; and if the altitudes be equal, the areas are as the bases. And when the angles of the one parallelogram are equal to those of another, the areas are as the rectangles of the sides about the equal angles.

4. In every parallelogram, the sum of the squares of the two diagonals, is equal to the sum of the squares of all the four sides of the figure, viz,

$AD^2 + BC^2 = AB^2 + BD^2 + DC^2 + CA^2$. Also the two diagonals bisect each other; so that $AE = ED$, and $BE = EC$.

5. *To find the Area of a* **Parallelogram**.—Multiply any one side, as a base, by the height, or perpendicular let fall upon it from the opposite side. Or, multiply any two adjacent sides together, and the product by the sine of their contained angle, the radius being 1: viz,

The area is $= CD \times AP = AC \times CD \times \sin. \angle C$.

Complement of a **Parallelogram**. See **Complement**.

Centre of Gravity of a **Parallelogram**. See **Centre** *of Gravity*, and **Centrobaric** *Method*.

Parallelogram, or **Parallelism**, or **Pentagraph**, also denotes a machine used for the ready and exact reduction or copying of designs, schemes, plans, prints, &c. in any proportion. See **Pentagraph**.

Parallelogram *of Forces*. See **Forces**, *Parallelogram of*.

Parallelogram *of the Hyperbola*, is the parallelogram formed by the two asymptotes of an hyperbola, and the parallels to them, drawn from any point of the curve. This term was first used by Huygens, at the end of his Dissertatio de Causa Gravitatis. This parallelogram, so formed, is of an invariable magnitude in the same hyperbola; and the rectangle of its sides is equal to the power of the hyperbola.

This parallelogram is also the modulus of the logarithmic system; and if it be taken as unity or 1, the hyperbolic sectors and segments will correspond to Napier's or the natural logarithms; for which reason these have been called the hyperbolic logarithms. If the parallelogram be taken = ·43429448190 &c, these sectors and segments will represent Briggs's logarithms; in which case the two asymptotes of the hyperbola make between them an angle of 25° 44′ 25″½.

Newtonian or Analytic **Parallelogram**, a term used for an invention of Sir Isaac Newton, to find the first term of an infinite converging series. It is sometimes called the Method of the Parallelogram and Ruler; because a ruler or right line is also used in it. This analytical parallelogram is formed by dividing any geometrical parallelogram into equal small squares or parallelograms, by lines drawn horizontally and perpendicularly through the equal divisions of the sides of the parallelogram. The small cells, thus formed, are filled with the dimensions or powers of the species *x* and *y*, and their products.

For instance, the powers of y, as y^0 or $1, y, y^2, y^3, y^4$, &c, being placed in the lowest horizontal range of cells; and the powers of x, as $x^0 = 1, x, x^2, x^3$, &c, in the vertical column to the left; or vice versa; these powers and their products will stand as in this figure:

A D

x^4	x^4y	x^4y^2	x^4y^3	x^4y^4
x^3	x^3y	x^3y^2	x^3y^3	x^3y^4
x^2	x^2y	x^2y^2	x^2y^3	x^2y^4
x	xy	xy^2	xy^3	xy^4
1	y	y^2	y^3	y^4

B C

Now when any literal equation is proposed, involving various powers of the two unknown quantities x and y, to find the value of one of these in an infinite series of the powers of the other; mark such of the cells as correspond to all its terms, or that contain the same powers and products of x and y; then let a ruler be applied to two, or perhaps more, of the parallelograms so marked, of which let one be the lowest in the left-hand column at AB, the other touching the ruler towards the right hand; and let all the rest, not touching the ruler, lie above it. Then select those terms of the equation which are represented by the cells that touch the ruler, and from them find the first term or quantity to be put in the quotient.

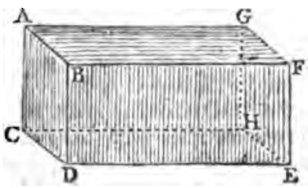
Of the application of this rule, Newton has given several examples in his Method of Fluxions and Infinite Series, pa. 9 and 10, but without demonstration; which has been supplied by others. See Colson's Comment on that treatise, pa. 192 et seq. Also Newton's Letter to Oldenburg, Oct. 24, 1676. Maclaurin's Algebra, pa. 251. And especially Cramer's Analyses des Lignes Courbes, pa. 148.—This author observes, that this invention, which is the true foundation of the method of series, was but imperfectly understood, and not valued as it deserved, for a long time. He thinks it however more convenient in practice to use the Analytical Triangle of the abbé de Gua, which takes in no more than the diagonal cells lying between A and C, and those which lie between them and B.

Parallelogram *Protractor*, a mathematical instrument, consisting of a semicircle of brass, with four rulers in form of a parallelogram, made to move to any angle. One of these rulers is an index, which shows on the semicircle the quantity of any inward and outward angle.

PARALLELOPIPED, or **Parallelopipedon**, is a solid figure contained under six parallelograms, the opposites of which are equal and parallel. Or, it is a prism whose base is a parallelogram.

Properties of the Parallelopipedon.—All parallelopipedons, whether right or oblique, that have their bases and altitudes equal, are equal; and each equal to triple a pyramid of an equal base and altitude.—A diagonal plane divides the parallelopipedon into two equal triangular prisms.—See other properties under the general term Prism, of which this is only a particular species.

To measure the Surface and Solidity of a Parallelopipedon.—Find the areas of the three parallelograms AD, BE, and BG, which add into one sum; and double that sum will be the whole surface of the parallelopipedon. Or,



For the solidity; multiply the base by the altitude; that is, any one face or side by its distance from the opposite side; as AD × DE, or AB × BE, or BG × BD.

PARAMETER, a certain constant right line in each of the three conic sections; otherwise called also latus rectum.—This line is called parameter, or equal measurer, because it measures the conjugate axis by the same ratio which is between the two axes themselves; being indeed a third proportional to them; viz, a third proportional to the transverse and conjugate axes, in the ellipse and hyperbola; and, which is the same thing, a third proportional to any absciss and its ordinate in the parabola. So if t and c be the two axes in the ellipse and hyperbola, and x and y an absciss and its ordinate in the parabola;

then $t : c :: c : p = \frac{c^2}{t}$ the param. in the former,

and $x : y :: y : p = \frac{y^2}{x}$ the param. in the last.

The parameter is equal to the double ordinate drawn through the focus of any of the three conic sections.

PARAPET, or *Breastwork*, in Fortification, is a defence or screen, on the extreme edge of a rampart, or other work, serving to cover the soldiers and the cannon from the enemy's fire.—The thickness of the parapet is 18 or 20 feet, commonly lined with masonry; and 7 or 8 feet high, when the enemy has no command above the battery; otherwise, it should be raised higher, to cover the men while they load the guns. There are certain openings, called Embrasures, cut in the parapet, from the top downwards, to within about 2½ or 3 feet of the bottom of it, for the cannon to fire through; the solid pieces of it between one embrasure and another, being called Merlons.

Parapet is also a little breast-wall, raised on the brinks of bridges, quays, or high buildings; to serve as a stay, and prevent people from falling over.

PARDIES (Ignatius Gaston), an ingenious French mathematician and philosopher, was born at Pau, in the province of Gascony, in 1636, his father being a counsellor of the parliament of that city.—At the age of 16 he entered into the order of Jesuits, and made so great a proficiency in his studies, that he taught polite literature, and composed many pieces in prose and verse with a distinguished delicacy of thought and style, before he was well arrived at the age of manhood. Propriety and elegance of language appear to have been his first pursuits; for which purpose he studied the Belles Lettres, and other learned productions. But afterwards he devoted himself to mathematical and philosophical studies, and read, with due attention, the most valuable authors, ancient and modern, in those sciences: so that, in a short time he made himself master of the Peripatetic and Cartesian philosophy, and taught them both with great reputation. Notwithstanding he embraced Cartesianism, yet he affected to be rather an inventor in philosophy himself. In this spirit he sometimes advanced very bold opinions, which met with opposers, who charged him with starting absurdities: but he was ingenious enough to give his notions a plausible turn, so as to clear them seemingly from contradictions. His reputation procured him a call to Paris, as professor of rhetoric in the college of Lewis the Great. He also taught the mathematics in that city, as he had before done in other places. He had from his youth a happy genius for that science, and made a great progress in it; and the glory which his writings acquired him, raised the highest expectations from his future labours; but these were all blasted by his early death, in 1673, at 37 years of age; falling a victim to his zeal, he having caught a contagious disorder by preaching to the prisoners in the Bicetre.

Pardies wrote with great neatness and elegance. His principal works are as follow:

1. Horologium Thaumaticum duplex; 1662, in 4to.—2. Dissertatio de Motu et Natura Cometarum; 1665, 8vo.—3. Discours du Mouvement Local; 1670, 12mo.—4. Elemens de Geometrie; 1670, 12mo.—This has been translated into several languages; in English by Dr. Harris, in 1711.—5. Discours de la Connoissance des Bêtes; 1672, 12mo.—6. Lettre d'un Philosophe à un Cartesien de ses amis; 1672, 12mo.—7. La Statique ou la Science des Forces Mouvantes; 1673, 12mo.—8. Description et Explication de deux Machines propres à faire des Cadrans avec une grande facilité; 1673, 12mo.—9. Remarques du Mouvement de la Lumiere.—10. Globi Cœlestis in tabula plana redacti descriptio; 1675, folio.

Part of his works were printed together, at the Hague, 1691, in 12mo; and again at Lyons, 1725.—Pardies had a dispute also with Sir Isaac Newton, about his new theory of light and colours, in 1672. His Letters are inserted in the Philosophical Transactions for that year.

PARENT (Anthony), a respectable French mathematician, was born at Paris in 1666. He showed an early propensity to the mathematics, eagerly perusing such books in that science as fell in his way. His custom was to write remarks in the margins of the books he read; and in this way he had filled a number of books with a kind of commentary by the time he was 13 years of age; and not many years after a treatise on gnomonics, and another on geometry.

His friends then sent for him to Paris to study the law; and in obedience to them he went through a course in that faculty: which was no sooner finished than, urged by his passion for mathematics, he shut himself up in the college of Dormans, that no avocation might take him from his beloved study: and, with an allowance of less than 200 livres a-year, he lived content in this retreat, from which he never stirred but to the Royal College, to hear the lectures of M. Lahire or M. de Sauveur; adding to his small income by teaching some pupils. M. Parent made two campaigns with the marquis d'Aligre, by which he instructed himself sufficiently in viewing fortified places; of which he drew a number of plans, though he had never learned the art of drawing.

From this period he spent his time in a continual application to the study of natural philosophy, and mathematics in all its branches, both speculative and practical; to which he also added anatomy, botany, and chemistry; his genius and indefatigable application overcoming every obstacle to these pursuits.

M. de Billettes being admitted into the Academy of Sciences at Paris in 1699, with the title of their mechanician, he named M. Parent for his elève or disciple, a branch of mathematics in which he chiefly excelled. It was soon discovered in this society, that he engaged in all the different subjects which were brought before them; and indeed that he had a hand in every thing. In his productions he was charged with obscurity; a fault for which he was indeed justly blamed.

By a regulation of the academy in 1716, the class of elèves was suppressed, as that distinction seemed to put too great an inequality between the members. M. Parent was made an adjunct or assistant member for the class of geometry: though he enjoyed this promotion but a very short time; being cut off by the small-pox the same year, at 50 years of age.

M. Parent, besides leaving many pieces in manuscript, published the following works:

1. Elemens de Mecanique et de Physique, 12mo, 1700.
2. Recherches de Mathematiques et de Physique; 3 vols 4to, 1714.
3. Arithmetique theorico-pratique; in 8vo, 1714.
4. A great many papers in the volumes of the Memoirs of the Academy of Sciences, from the year 1700 to 1714, several papers in almost every volume, on a variety of branches in the mathematics.

PARGETING, in Building, is used for the plastering of walls; sometimes for plaster itself.

PARHELION, or Parhelium, denotes a mock-sun, or meteor, appearing as a very bright light by the side of the sun; being formed by the reflection of his beams in a cloud properly situated.

Parhelia usually accompany the coronæ, or luminous circles, and are placed in the same circumference, and at the same height. Their colours resemble those of the rainbow; the red and yellow are on that side towards the sun, and the blue and violet on the other. Though coronæ are sometimes seen entire, without any parhelia; and sometimes parhelia without coronæ.

The apparent size of parhelia is the same as that of the true sun; but they are not always round, nor so bright as the sun; and when several appear, some are brighter than others. They are tinged externally with colours like the rainbow, and many of them have a long fiery tail opposite to the sun, but paler towards the extremity. Some parhelia have been observed with two tails and others with three. These tails mostly appear in a white horizontal circle, commonly passing through all the parhelia, and would go through the centre of the sun if it were entire. Sometimes there are arcs of lesser circles, concentric to this, touching those coloured circles which surround the sun: these are also tinged with colours, and contain other parhelia.

Parhelia are generally situated in the intersections of circles; but Cassini says, those which he saw in 1683, were on the outside of the coloured circle, though the tails were in the circle that was parallel to the horizon. M. Aepinus apprehends, that parhelia with elliptical coronæ are more frequent in the northern regions, and those with circular ones in the southern. They have been visible for one, two, three, or four hours together; and it is said that in North America they continue several days, and are visible from sun-rise to sun-set. When the parhelia disappear, it sometimes rains, or there falls snow in the form of oblong spiculæ. And Mariotte accounts for the appearance of parhelia from an infinity of small particles of ice floating in the air, which multiply the image of the sun, either by refracting or breaking his rays, and thus making him appear where he is not; or by reflecting them, and serving as mirrors.

Many philosophers have written on parhelia; as Aristotle, Pliny, Scheiner, Gassendi, Descartes, Huygens, Hevelius, Lahire, Cassini, Grey, Halley, Maraldi, Musschenbroek, &c. See Smith's Optics, book 1, chap. 11; Priestley's Hist. of Light, &c, pa. 613; Musschenbroek's Introduction, &c, vol. 2, pa. 1038, 4to; and Dr. Thomas Young's Philosophy.

PARODICAL *Degrees*, in an equation, a term that has been sometimes used to denote the several regular terms, or lower powers of the unknown quantity x, in an equation, when the indices of the powers ascend or descend orderly in an arithmetical progression. Thus $x^3 + mx^2 + nx = p$ is a cubic equation where no term is wanting, but having all its parodic degrees; the indices of the terms regularly descending thus, 3, 2, 1, 0.

PART, *Aliquant*, *Aliquot*, *Circular*, *Proportional*, *Similar*, &c. See the respective adjectives.

PARTICLE, the minute part of a body, or an assemblage of several of the atoms of which natural bodies are composed. Particle is sometimes considered as synonymous with atom, and corpuscle; and sometimes they are distinguished. Particles are, as it were, the elements of bodies; by the various arrangement and texture of which, with the difference of the cohesion, &c, are constituted the several kinds of bodies, hard, soft, liquid, dry, heavy, light, &c. The smallest particles or corpuscles cohere with the strongest attractions, and always compose larger particles of weaker cohesion: and many of these, cohering, compose still larger particles, whose vigour is still weaker; and so on for divers successions, till the progression end in the largest particles, upon which the operations in chemistry, and the colours of natural bodies, depend; and which, by cohering, compose bodies of sensible magnitude.

PARTY *Arches*, in Architecture, are arches built between separate tenures, where the property is intermixed, and apartments over each other do not belong to the same estate.

PARTY *Walls*, are partitions of brick made between buildings in separate occupations, for preventing the spread of fire. These are made thicker than the external walls; and their thickness in London is regulated by act of parliament of the 14th of George the Third.

PASCAL (BLAISE), a respectable French mathematician and philosopher. He was born at Clermont, in Auvergne, in the year 1623. His father, Stephen Pascal, was president of the Court of Aids in his province: he was also a very learned man, an able mathematician, and a friend of Descartes. Having an extraordinary tenderness for this child, his only son, he quitted his office in his province, and settled at Paris in 1631, that he might be quite at leisure to attend his son's education, which he conducted himself, and young Pascal never had any other master.

From his infancy Blaise gave proofs of a very extraordinary capacity. He was extremely inquisitive; desiring to know the reason of every thing; and when good reasons were not given him, he would seek for better; nor would he ever yield his assent but on such as appeared to him well grounded. What is told of his manner of learning the mathematics, as well as the progress he quickly made in that science, seems almost miraculous. From a simple mathematical definition, he discovered by degrees, and by the unaided force of his mind, that the three angles of every triangle are together equal to two right angles, as well as several of the other theorems of Euclid. At 16 years of age Pascal composed a tract on the Conic Sections, which was considered as a prodigy of sagacity. Scarcely had he attained his 19th year, when he invented the famous arithmetical machine which bears his name, and by which all kinds of operations in numbers may be performed, by the use of the eyes and hands only. Soon afterwards his experiments decided the opinions of philosophers respecting the weight of the air. He invented the arithmetical triangle, and the elements of the arithmetic of probabilities.

All these labours ruined the health of Pascal. Bodily weakness obliged him to suspend all mental exertions, and to commence a course of moderate exercise. One day in 1654, as he was riding to the bridge of Neuilly, in a chariot-and-four, the two foremost horses ran away close to a precipice, where there was no parapet, down which they rushed into the Seine. Fortunately they broke the traces by their first effort, and left the chariot standing on the very brink of the precipice. This accident so much disturbed the brain of Pascal, that ever after he imagined there was an abyss on his left hand. He afterwards wholly renounced the world, and retired to the abbey of Port-Royal, where the regular life which he led, procured him very long intervals of health, during which he wrote the celebrated Provincial Letters, one of the most perfect works in the French language. For many years Pascal relinquished all purely human sciences. But having been tormented by a most severe tooth-ache, which almost wholly deprived him of rest, he sought by intense application the means of mitigating his pain; and the discoveries which he then made on the cycloidal curve are, even at the present day, reckoned among the greatest efforts of the human mind. The first idea of that remarkable curve seemed to have occurred to Galileo, and several other mathematicians had successively developed its properties. Pascal, having attentively considered that curve, wished to make a trial of the talents of his cotemporary geometricians. With this view he proposed to them some new problems on the cycloid, promising 40 pistoles to the first person, and 20 to the second, who should solve these problems. The only person who returned answers to all the problems, and claimed the prizes, were Dr. Wallis and father Laloubere the Jesuit. Huygens squared the segment comprehended between the vertex of the cycloid and that of the diameter of the generating circle. Slusius measured the arc of that curve in a very elegant manner; and Wren found its rectification. But all these researches did not entirely answer the questions in the programma circulated by Pascal, under the name of A. Detonville. He affirmed that Wallis and Laloubere were mistaken in several particulars, and therefore he withheld the promised rewards. He himself however gave perfect solutions of all the problems which he had proposed, and of several others, which were necessary to complete the theory of the cycloid. After languishing for several years in a very imbecile state of body and mind, M. Pascal died at Paris the 19th of August 1662, at 39 years of age.

Towards the close of his life, he employed himself wholly in devout and moral reflections, writing down those which he deemed worthy of being preserved. The first bit of paper he could find was employed for this purpose; and he commonly set down only a few words of each sentence, as he wrote them merely for his own use. The scraps of paper on which he had written these thoughts, were found after his death filed upon different pieces of string, without any order or connection; and being copied exactly as they were written, they were afterwards arranged and published, under the title of Pensées, &c, or Thoughts upon Religion and other Subjects; being parts of a work he had intended against atheists and infidels, which has been much admired. After his death appeared also two other little tracts; the one entitled, The Equilibrium of Fluids; and the other, The Weight of the Mass of Air. The works of Pascal were collected in 5 volumes, 8vo, and published at the Hague, and at Paris, in 1779. This edition of Pascal's works may be considered as the first published; at least the greater part of them were not before collected into one body, and some of them had remained only in manuscript. For this collection, the public were indebted to the abbé Bossu, and Pascal was deserving of such an editor.

PATE, in Fortification, a kind of platform, like what is called a horse-shoe; not always regular, but commonly oval, encompassed only with a parapet, and having nothing to flank it. It is usually erected in marshy grounds, to cover a gate of a town, or the like.

PATH *of the Vertex*, a term frequently used by Mr. Flamsteed, in his Doctrine of the Sphere, denoting a circle, described by any point of the earth's surface, as the earth turns round its axis. This point is considered as vertical to the earth's centre; and is the same with what is called the vertex or zenith in the Ptolemaic projection. The semidiameter of this path of the vertex, is always equal to the complement of the latitude of the point or place that describes it; that is, to the place's distance from the pole of the world.

PAVILION, in Architecture, is a kind of turret, or building usually insulated, and contained under a single roof; sometimes square and sometimes in form of a dome: thus called from the resemblance of its roof to a tent.

PAVO, *Peacock*, a new constellation, in the southern hemisphere, added by the modern astronomers. It contains 14 stars.

PAUSE, or Rest, in Music, a character of silence and rest; called also by some a mute figure; because it shows that some part or person is to be silent, while the others continue the song.

PECK, a measure or vessel used in measuring grain, pulse, and the like dry substances. It contains 2 gallons, or the 4th part of a bushel.

PEDESTAL, in Architecture, the lowest part of an order of columns; being that which sustains the column, and serves it as a foot to stand upon. It is a square body or die, with a cornice and base. The proportions and ornaments of the pedestal are different in the different orders. Vignola indeed, and most of the moderns, make the pedestal, and its ornaments, in all the orders, one third of the height of the column, including the base and capital. But some deviate from this rule.

Perrault makes the proportions of the three constituent parts of pedestals, the same in all the orders; viz, the base one fourth of the pedestal; the cornice an eighth part; and the socle or plinth of the base, two thirds of the base itself. The height of the die is what remains of the whole height of the pedestal.

The *Tuscan* Pedestal is the simplest and lowest of all; from 3 to 5 modules high. It has only a plinth for its base, and an astragal crowned for its cornice.

The *Doric* Pedestal is made 4 or 5 modules in height, by the moderns; for no ancient columns, of this order, are found with any pedestal, or even with any base.

The *Ionic* Pedestal is from 5 to 7 modules high.

The *Corinthian* Pedestal is the richest and most delicate of all, and is from 4 to 7 modules high.

The *Composite* Pedestal is of 6 or 7 modules in height.

Square Pedestal has its breadth and height equal.

Double Pedestal, is that which supports two columns, being broader than it is high.

Continued Pedestal, is that which supports a row of columns without any break or interruption.

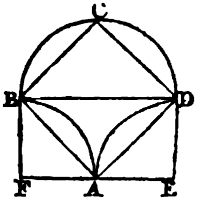
Pedestals *of Statues*, are those serving to support figures or statues.

PEDIMENT, in Architecture, a kind of low pinnacle; serving to crown porticos, or finish a frontispiece; and placed as an ornament over gates, doors, windows, niches, altars, &c; being usually of a triangular form, but sometimes an arch of a circle. Its height is various, but it is thought most beautiful when the height is one fifth of the length of its base.

PEDOMETER, or Podometer, foot-measurer, or way-wiser; a mechanical instrument, in form of a watch, and consisting of various wheels and teeth; which, by means of a chain, or string, fastened to a man's foot, or to the wheel of a chariot, advance a notch each step, or each revolution of the wheel: by which it numbers the paces or revolutions, and so the distance from one place to another.

Pedometer is also sometimes used for the common surveying wheel, an instrument chiefly used in measuring roads; popularly called the way-wiser. See Perambulator.

PEGASUS, the Horse, a constellation of the northern hemisphere, figured in the form of a flying horse; being one of the 48 ancient constellations. It is fabled, by the Greeks, to have been the offspring of an amour between Neptune and the Gorgon Medusa; and to have been that on which Bellerophon rode when he overcame the Chimera; and that flying from mount Helicon to heaven, he there became a constellation; having thrown his rider in the flight; and that the stroke of his hoof on the mount opened the sacred fountain Hippocrene.—The stars in this constellation, in Ptolemy's catalogue, are 20, in Tycho's 19, in Hevelius's 38, and in the Britannic catalogue 89.



PELECOIDES, or *Hatchet-form*, in Geometry, a figure in form of a hatchet. As the figure ABCDA, contained under the semicircle BCD and the two quadrantal arcs AB and AD. The area of the pelecoides is equal to the square AC, and this again is equal to the rectangle BE. It is equal to the square, because the two segments AB and AD, which it wants of the square on the lower part, are compensated by the two equal segments BC and CD, by which it exceeds on the upper part. And the square is equal to the rectangle BE, because the triangle ABD, which is half the square, is also half the rectangle BE of the same base and height with it.

PELL (Dr. John), an eminent English mathematician, descended from an ancient family in Lincolnshire, was born at Southwick in Sussex, March 1, 1610, where his father was minister. He received his grammar education at the free-school at Stenning in that county. At the age of 13 he was sent to Trinity-college in Cambridge, though at that time as good a scholar as most masters of arts in that university; but though he was eminently skilled in the Greek and Hebrew languages, he never offered himself a candidate at the election of scholars or fellows of his college.

In 1629 he drew up the "Description and Use of the Quadrant, written for the Use of a Friend," in two books; the original manuscript of which is still extant among his papers in the Royal Society. And the same year he held a correspondence with Mr. Briggs on the subject of logarithms.

In 1630 he wrote, "Modus supputandi Ephemerides Astronomicas, &c, ad an. 1630 accommodatus;" and, "A Key to unlock the meaning of Johannes Trithemius, in his Discourse of Steganography," which Key he imparted to Mr. Samuel Hartlib and Mr. Jacob Homedæ. The same year he took the degree of master of arts at Cambridge. And the year following he was incorporated in the university of Oxford. June the 7th, he wrote "A Letter to Mr. Edmund Wingate, on Logarithms," and Oct. 5, 1631, "Commentationes in Cosmographiam Alstedii."

March 6, 1634, he finished his "Astronomical History of Observations of Heavenly Motions and Appearances;" and April the 10th, his "Ecliptica Prognostica, or Foreknower of the Eclipses, &c."—In 1634 he translated "The Everlasting Tables of Heavenly Motions," grounded on the Observations of all Times, and agreeing with them all, by Philip Lansberg, of Ghent in Flanders. And June the 12th, the same year, he committed to writing, "The Manner of deducing his Astronomical Tables out of the Tables and Axioms of Philip Lansberg."—March the 9th, 1635, he wrote "A Letter of Remarks on Gellibrand's Mathematical Discourse on the Variation of the Magnetic Needle." And the 3d of June following, another on the same subject.

His eminence in mathematical knowledge was now so great, that he was thought worthy of a professor's chair in that science; and, on the vacancy of one at Amsterdam in 1639, Sir William Boswell, the English resident with the States-General, used his interest, that he might succeed in that professorship: it was not filled however till 1643, when Pell was chosen to it; and he read with great applause public lectures on Diophantus.—In 1644 he printed at Amsterdam, in two pages 4to, "A Refutation of Longomontanus's Discourse, De Vera Circuli Mensura."

In 1646, on the invitation of the Prince of Orange, he removed to the new college at Breda, as professor of mathematics, with a salary of 1000 guilders a year.—His "Idea Matheseos," which he had addressed to Mr. Hartlib, who in 1639 had sent it to Descartes and Mersenne, was printed 1650 at London, in 12mo, in English, with the title of "An Idea of Mathematics," at the end of Mr. John Durie's Reformed Library-keeper. It is also printed by Mr. Hooke, in his Philosophical Collections, No. 5, pa. 127; and is esteemed our author's principal work.

In 1652 Pell returned to England: and in 1654 he was sent by the protector Cromwell agent to the protestant cantons in Switzerland; where he continued till June 23, 1658, when he set out for England, where he arrived about the time of Cromwell's death. His negociations abroad gave afterwards a general satisfaction, as it appeared he had done no small service to the interest of king Charles the 2d, and of the church of England; so that he was encouraged to enter into holy orders; and in the year 1661 he was instituted to the rectory of Fobbing in Essex, given him by the king. In December that year he brought into the upper house of convocation the calendar reformed by him, assisted by Sancroft, afterwards archbishop of Canterbury.—In 1673 he was presented by Sheldon, bishop of London, to the rectory of Laingdon in Essex; and, on the promotion of that bishop to the see of Canterbury soon after, became one of his domestic chaplains. He was then doctor of divinity, and expected to be made a dean; but he attended so much to his improvement in the philosophical and mathematical sciences that he lost sight of his private advantage. The truth is, he was a helpless man, as to worldly affairs, and his tenants and relations imposed on him, cozened him of the profits of his parsonage, and kept him so indigent, that he wanted necessaries, even ink and paper, to his dying day. He was for some time confined to the King's-bench prison for debt; but, in March 1682, was invited by Dr. Whitler to live in the college of physicians. Here he continued till June following; when he was obliged, by his ill state of health, to remove to the house of a grandchild of his in St. Margaret's Church-yard, Westminster. But he died at the house of Mr. Cotborne, reader of the church of St. Giles's in the Fields, December the 12th, 1685, in the 74th year of his age, and was interred at the expense of Dr. Busby, master of Westminster-school, and Mr. Sharp, rector of St. Giles's, in the rector's vault under that church.—Dr. Pell published some other things not yet mentioned, a list of which is as follows: viz,

1. An Exercitation concerning Easter; 1644, in 4to.

2. A Table of 10,000 square numbers, &c; 1672, folio.

3. An Inaugural Oration at his entering on the Professorship at Breda.

4. He made great alterations and additions to Rhonius's Algebra, printed at London 1668, 4to, under the title of, An Introduction to Algebra; translated out of the High Dutch into English by Thomas Branker, much altered and augmented by D. P. (Dr. Pell.) Also a Table of Odd Numbers, less than 100,000, showing those that are incomposite, &c, supputated by the same Thomas Branker. See this table under the article PRIME *Numbers*.

5. His Controversy with Longomontanus concerning the Quadrature of the Circle; Amsterdam, 1646, 4to.

He also wrote a Demonstration of the 2d and 10th books of Euclid; which piece was in MS. in the library of lord Brereton in Cheshire: as also Archimedes's Arenarius; and the greatest part of Diophantus's 6 books of Arithmetic; of which author he was preparing a new edition, in which he intended to have corrected the translation, and made new illustrations. He designed also to publish an edition of Apollonius, but laid it aside, in May 1645, at the desire of Golius, who was engaged in an edition of that author from an Arabic manuscript given him at Aleppo 18 years before. Letters of Dr. Pell to Sir Charles Cavendish, in the Royal Society.

Some of his manuscripts he left at Brereton in Cheshire, where he resided some years, being the seat of William lord Brereton, who had been his pupil at Breda. A great many others came into the hands of Dr. Busby; which Mr. Hooke was desired to use his endeavours to obtain for the Royal Society. But they continued buried under dust, and mixed with the papers and pamphlets of Dr. Busby, in 4 large boxes, till 1755; when Dr. Birch, secretary to the Royal Society, procured them for that body, from the trustees of Dr. Busby. The collection contains not only Pell's mathematical papers, letters to him, and copies of those from him, &c, but also several manuscripts of Walter Warner, the mathematician and philosopher, who lived in the reigns of James the First and Charles the First.

Dr. Pell invented the method of ranging the several steps of an algebraical calculus, in a proper order, in so many distinct lines, with the number affixed to each step, and a short description of the operation or process in the line. He also invented the character ÷ for division, ● for involution, and ɯ for evolution.

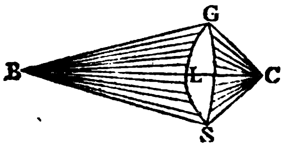
He was also the first who discovered the method of solving the equation $ax^2 - y^2 = 1$, being the same as that given by Euler in the second volume of his algebra. This problem was proposed by Fermat as a challenge to all the English mathematicians, though it is probable (as he never gave a solution to it himself) that he was unacquainted with the true mode of operation at the time he proposed it.

PEMBERTON (HENRY), M. D. & F. R. S. born at London in 1694, was a learned physician and mathematician; as well as an expert mechanist, readily performing with his own hand several mechanical operations. After studying grammar at a school, and the higher classics under Mr. John Ward, afterwards professor of rhetoric at Gresham-college, he went to Leyden, to attend the lectures of the celebrated Boerhaave, to qualify himself for the profession of medicine. Here also, as well as in England, he constantly mixed, with his professional studies, those of the best mathematical authors, whom he contemplated with great effect. From hence he went to Paris, to perfect himself in the practice of anatomy, to which he readily attained, being naturally dextrous in all manual operations. Having obtained his main object, he returned to London, enriched also with other branches of scientific knowledge, and a choice collection of mathematical books, both ancient and modern, from the sale of the valuable library of the abbé Gallois, which took place during his stay in Paris. After his return he assiduously attended St. Thomas's Hospital, to acquire the London practice of physic, though he seldom afterwards practised, owing to his delicate state of health. In 1719 he returned to Leyden, to take his degree of M. D. where he was kindly entertained by his friend Dr. Boerhaave. After his return to London, he became more intimately acquainted with Dr. Mead, Sir I. Newton, and other eminent men, with whom he afterwards cultivated the most friendly connexions. Hence he was useful in assisting Sir I. Newton in preparing a new edition of his Principia, in writing an account of his philosophical discoveries, in bringing forward Mr. Robins, and writing some pieces printed in the 2d volume of that gentleman's collection of tracts, in Dr. Mead's Treatise on the Plague, and in his edition of Cowper on the Muscles, &c. Being chosen professor of physic in Gresham-college, he undertook to give a course of lectures on chemistry, which was improved every time he exhibited it, and was published in 1771, by his friend Dr. James Wilson. In this situation too, at the request of the college of physicians, he revised and reformed their Pharmacopœia, in a new and much improved edition. After a long and laborious life spent in improving science, and assisting its cultivators, Dr. Pemberton died in 1771, at 77 years of age.

Besides the doctor's writings above-mentioned, he wrote numerous other pieces; as, 1. Epistola ad Amicum de Cotesii inventis; demonstrating Cotes's celebrated theorem, and showing how his theorems by ratios and logarithms may be done by the circle and hyperbola.—2. Observations on Poetry, especially the Epic, occasioned by Glover's Leonidas.—3. A Plan of a Free State, with a King at the Head; not published.—4. Account of the Ancient Ode; printed in the preface to West's Pindar.—5. On the Dispute about Fluxions, in the 2d vol. of Robins's works.—6. On the Alteration of the Style and Calendar.—7. On reducing the Weights and Measures to one Standard.—8. A Dissertation on Eclipses.—9. On the Loci Plani, &c. His numerous communications to the Royal Society, on a variety of interesting subjects, extend from the 32d to the 62d vol. of the Philos. Trans.

After his death many valuable pieces were found among his papers, viz. A short History of Trigonometry, from Menelaus to Napier. A comment on an English Translation of Newton's Principia. Demonstrations of the Spherics and spherical Projections, enough to compose a treatise on those subjects. A Dissertation on Archimedes's Screw. Improvements in Guaging. In a given latitude, to find the point of the ecliptic that ascends the slowest. To find when the oblique ascension differs most from the arch to which it belongs. On the principles of Mercator's and middle-latitude sailing. To find the heliacal rising of a star. To compute the moon's parallax. To determine the course of a comet in a parabolic orbit. And others, all neatly performed. On the whole, Dr. Pemberton appears to have been a clear and industrious author, but his writings are too diffuse and laboured.

PENCIL *of Rays*, in Optics, is a double cone, or pyramid of rays, joined together at the base; as BGSC: the one cone having its vertex in some point of the object at B, and the crystalline humour, or the glass GLS for its base; and the other having its base on the same glass, or crystalline, but its vertex in the point of convergence, as at C.



PENDULUM, in Mechanics, any heavy body, so suspended as that it may swing backwards and forwards, about some fixed point, by the force of gravity. These alternate ascents and descents of the pendulum, are called its oscillations, or vibrations; each complete oscillation being the descent from the highest point on one side, down to the lowest point of the arch, and so on up to the highest point on the other side. The point round which the pendulum moves, or vibrates, is called its centre of motion, or point of suspension; and a right line drawn through the centre of motion, parallel to the horizon, and perpendicular to the plane in which the pendulum moves, is called the axis of oscillation. There is also a certain point within every pendulum, into which, if all the matter that composes the pendulum were collected, or condensed as into a point, the times in which the vibrations would be performed, would not be altered by such condensation; and this point is called centre of oscillation. The length of the pendulum is always estimated by the distance of this point below the centre of motion; being usually near the bottom of the pendulum; but in a cylinder, or any other uniform prism or rod, it is at the distance of one third from the bottom, or two-thirds from and below the centre of motion.

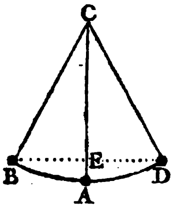
The length of a pendulum, so measured to its centre of oscillation, that it will perform each vibration in a second of time, thence called the seconds pendulum, has, in the latitude of London, been generally taken at $39\frac{2}{10}$ or $39\frac{1}{5}$ inches; but by some very ingenious and accurate experiments, the late celebrated Mr. George Graham found the true length to be $39\frac{128}{1000}$, inches, or $39\frac{1}{8}$ inches very nearly.

The length of the pendulum vibrating seconds at Paris, was found by Varin, Deshays, Deglos, and Godin, to be

$440\frac{5}{9}$ lines; by Picard $440\frac{1}{2}$ lines; and by Mairan $440\frac{17}{30}$ lines.

Galileo was the first who made use of a heavy body annexed to a thread, and suspended by it, for measuring time, in his experiments and observations. But according to Sturmius, it was Riccioli who first observed the isochronism of pendulums, and made use of them in measuring time. After him, Tycho, Langrene, Wendeline, Mersenne, Kircher, and others, observed the same thing; though, it is said, without any intimation of what had been done by Riccioli. But it was the celebrated Huygens who first demonstrated the principles and properties of pendulums, and probably the first who applied them to clocks. He demonstrated, that if the centre of motion were perfectly fixed and immoveable, and all manner of friction, and resistance of the air, &c, removed, that a pendulum, once set in motion, would for ever continue to vibrate without any decrease of motion, and that all its vibrations would be perfectly isochronal, or performed in the same time, the arc of vibration remaining constant. Hence the pendulum has universally been considered as the best chronometer or measurer of time. And as all pendulums of the same length perform their vibrations in the same time, the arc of vibration being the same, without regard to their different weights, it has been suggested, by means of them, to establish an universal standard for all countries. On this principle Mouton, canon of Lyons, has a treatise, De Mensura posteris transmittenda; and several others since, as Whitehurst, &c. See *Universal* MEASURE.

Pendulums are either simple or compound, and each of these may be considered either in theory, or as in practical mechanics among artisans.



A Simple PENDULUM, in Theory, consists of a single weight, as A, considered as a point, and an inflexible right line AC, supposed void of gravity or weight, and suspended from a fixed point or centre C, about which it moves.

A Compound PENDULUM, in Theory, is a pendulum consisting of several weights moveable about one common centre of motion, but connected together so as to retain the same distance both from one another, and from the centre about which they vibrate.

The Doctrine and Laws of Pendulums.—1. A pendulum raised to B, through the arc of the circle AB, will fall, and rise again, through an equal arc, to a point equally high, as D; and thence will fall to A, and again rise to B; and thus continue rising and falling perpetually, supposing neither friction nor resistance. For it is the same thing, whether the body fall down the inside of the curve BAD, by the force of gravity, or be retained in it by the action of the string; as they will both have the same effect; and it is otherwise known, from the oblique descents of bodies, that the body will descend and ascend along the curve in the manner above described.

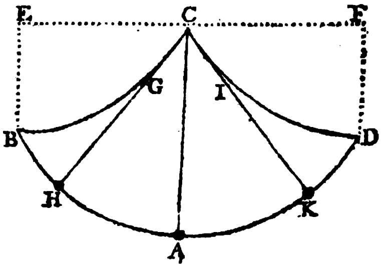
Experience also confirms this theory, in any finite number of oscillations. But if they be supposed infinitely continued, a difference will arise. For the resistance of the air, and the friction and rigidity of the string about the centre C, will take off part of the force acquired in falling; hence it happens that it does not rise precisely to the same point from whence it fell. Thus, the ascent continually diminishing the oscillation, this will be at last stopped, and the pendulum will hang at rest in its natural direction, which is perpendicular to the horizon.

Now as to the real time of oscillation in a circular arc BAD: it is demonstrated by mathematicians, that if $p = 3{\cdot}1416$, denote the circumference of a circle whose diameter is 1; $g = 16\frac{1}{12}$ feet or 193 inches, the space a heavy body falls in the first second of time, in our latitude; and $r =$ CA the length of the pendulum; also $a =$ AE the height of the arch of vibration; then the time of each oscillation in the arc BAD will be equal to $p\sqrt{\frac{r}{2g}} \times$ into the series $1 + \frac{1^2 a}{2^2 d} + \frac{1^2.3^2 a^2}{2^2.4^2 d^2} + \frac{1^2.3^2.5^2 a^3}{2^2.4^2.6^2 d^3}$ &c, where $d = 2r$ is the diameter of the arc described, or twice the length of the pendulum.

And here, when the arc is a small one, as in the case of the vibrating pendulum of a clock, all the terms of this series after the 2d may be omitted, on account of their smallness; and then the time of a whole vibration will be nearly equal to $p\sqrt{\frac{r}{2g}} \times (1 + \frac{a}{8r})$.

So that the times of vibration of a pendulum in different small arcs of the same circle, are as $8r + a$, or 8 times the radius, added to the versed sine of the semiarc.

And farther, if D denote the number of degrees in the semiarc AB, whose versed sine is a, then the quantity last mentioned, for the time of a whole vibration, is changed to $p\sqrt{\frac{r}{2g}} \times (1 + \frac{D^2}{52524})$. And therefore the times of vibration in different small arcs, are as $52524 + D^2$, or as the number 52524 added to the square of the number of degrees in the semiarc AB. See my Tracts, vol. 3, prob. 15, pa. 338.



2. Let CB be a semicycloid, having its base EC parallel to the horizon, and its vertex B downwards; and let CD be the other half of the cycloid, in a similar position to the former. Now suppose a pendulum string, of the same length with the curve of each semicycloid BC, or CD, having its end fixed in C, and the thread applied all the way close to the cycloidal curve BC, and consequently the body or pendulum weight coinciding with the point B. If now the body be let go from B, it will descend by its own gravity, and in descending it will unwind the string from off the arch BC, as at the position C*d*H; and the ball H will describe a semicycloid BHA, equal and similar to BGC, when it has arrived at the lowest point A; after which, it will continue its motion, and ascend, by another equal and similar semicycloid AKD, to the same height D, as it fell from at B, the string now wrapping itself upon the other arch CID. From D it will descend again, and pass along the whole cycloid DAB, to the point B; and thus perform continual successive oscillations between B and D, in the curve of a cycloid; as it before oscillated in the curve of a circle, in the former case.

This contrivance, to make the pendulum oscillate in the curve of a cycloid, is the invention of the celebrated Huygens, to make the pendulum perform all its vibrations in equal times, whether the arch, or extent of the vibration be great or small; which is not the case in a circle, where the larger arcs take a longer time to run through them, than the smaller ones do, as is well known both from theory and practice.

It should be observed however, that in speaking of the equal times of vibrations in cycloidal arcs, the string by which the body is suspended is considered void of gravity and resistance, and as this is not absolutely true, it follows that theory and practice will be a little at variance on this head.

The chief properties of the cycloidal pendulum then, as demonstrated by Huygens, are the following. 1st, That the time of an oscillation in all arcs, whether larger or smaller, is always the same quantity, viz, whether the body begin to descend from the point B, and describe the semiarc BA; or that it begins at H, and describes the arc HA; or that it sets out from any other point; as it will still descend to the lowest point A in exactly the same time. And it is farther proved, that the time of a whole vibration through any double arc BAD, or HAK, &c, is in proportion to the time in which a heavy body will fall freely, by the force of gravity, through a space equal to $\frac{1}{2}$AC, half the length of the pendulum, as the circumference of a circle is to its diameter. So that, if $g = 16\frac{1}{12}$ feet denote the space a heavy body falls in the first second of time, $p = 3{\cdot}1416$ the circumference of a circle whose diameter is 1, and $r =$ AC the length of the pendulum; then, because, by the nature of descents by gravity, $\sqrt{g} : \sqrt{\frac{1}{2}r} :: 1'' : \sqrt{\frac{r}{2g}}$, that is the time in which a body will fall through $\frac{1}{2}r$, or half the length of the pendulum; therefore, by the above proportion, as $1 : p :: \sqrt{\frac{r}{2g}} : p\sqrt{\frac{r}{2g}}$, which is the time of an entire oscillation in the cycloid.

And this conclusion is confirmed by experience. For example, if it were required to find the length of the pendulum that will so oscillate in one second; this will give the equation $p\sqrt{\frac{r}{2g}} = 1$; which reduced, gives $r = \frac{2g}{p^2} = \frac{386}{3{\cdot}1416^2}$ inches $= 39{\cdot}11$ or $39\frac{1}{9}$ inches, for the length of the seconds pendulum; which the best experiments show to be about $39\frac{1}{8}$ inches.

3. Hence also, we have a method of determining, from the experimented length of a pendulum, the space a heavy body will fall perpendicularly through in a given time: for, since $p\sqrt{\frac{r}{2g}} = 1$, therefore, by reduction, $g = \frac{1}{2}p^2r$ is the space a body will fall through in the first second of time, when r denotes the length of the seconds pendulum; and as constant experience shows that this length is nearly $39\frac{1}{8}$ inches, in the latitude of London, in this case g or $\frac{1}{2}p^2r$ becomes $\frac{1}{2} \times 3{\cdot}1416^2 \times 39\frac{1}{8} = 193{\cdot}07$ inches $= 16\frac{1}{12}$ feet, very nearly, for the space a body will fall in the first second of time, in the latitude of London: a fact which has been abundantly confirmed by experiments made there. And in the same manner, Mr. Huygens found the same space fallen through at Paris, to be 15 French feet.

The whole doctrine of pendulums, oscillating between two semicycloids, both in theory and practice, was delivered by that author, in his Horologium Oscillatorium, sive Demonstrationes de Motu Pendulorum. And every thing that regards the motion of pendulums has since been demonstrated in different ways, and particularly by Newton, who has given an admirable theory on the subject, in his Principia, where he has extended to epicycloids the properties demonstrated by Huygens of the cycloids.

4. As the cycloid may be considered as coinciding, in A, with any small arc of a circle described from the centre C, passing through A, where it is known the two curves have the same radius and curvature; therefore the time in the small arc of such a circle, will be nearly equal to the time in the cycloid; so that the times in very small circular arcs are equal, because these small arcs may be considered as portions of the cycloid, as well as of the circle. And this is one great reason why the pendulums of clocks are made to oscillate in as small arcs as possible, viz, that their oscillations may be the nearer to a constant equality.

This may also be deduced from a comparison of the times of vibration in the circle, and in the cycloid, as laid down in the foregoing articles. It has there been shown, that the times of vibration in the circle and cycloid are thus, viz,

time in the circle nearly $p\sqrt{\frac{r}{2g}} \times (1 + \frac{a}{8r})$,

time in the cycloidal arc $p\sqrt{\frac{r}{2g}}$;

where it is evident, that the former always exceeds the latter in the ratio of $1 + \frac{a}{8r}$ to 1; but this ratio always approaches nearer to an equality, as the arc, or as its versed sine a, is smaller; till at length, when it is very small, the term $\frac{a}{8r}$ may be omitted, and then the times of vibration become both the same quantity, viz, $p\sqrt{\frac{r}{2g}}$.

Farther, by the same comparison, it appears, that the time lost in each second, or in each vibration of the seconds pendulums, by vibrating in a circle, instead of a cycloid, is $\frac{a}{8r}$, or $\frac{D^2}{52524}$; and consequently the time lost in a whole day of 24 hours, is $\frac{5}{3}D^2$ nearly. In like manner, the seconds lost per day by vibrating in the arc of d degrees, is $\frac{5}{3}d^2$. Therefore if the pendulum keep true time in one of these arcs, the seconds lost or gained per day, by vibrating in the other, will be $\frac{5}{3}(D^2 - d^2)$. So, for example, if a pendulum measure true time in an arc of 3 degrees, on each side of the lowest point, it will lose $11\frac{2}{3}$ seconds a day by vibrating 4 degrees; and $26\frac{2}{3}$ seconds a day by vibrating 5 degrees; and so on.

5. The action of gravity is less in those parts of the earth where the oscillations of the same pendulum are slower, and greater where these are swifter; for the time of oscillation is reciprocally proportional to $\sqrt{g}$. And it being found by experiment, that the oscillations of the same pendulum are slower near the equator, than in places farther from it; it follows that the force of gravity is less there; and consequently the parts about the equator are higher or farther from the centre, than the other parts; and the shape of the earth is not a true sphere, but somewhat like an oblate spheroid, flatted at the poles, and raised gradually towards the equator. And hence also the times of the vibration of the same pendulum, in different latitudes, afford a method of determining the true figure of the earth, and the proportion between its axis and the equatorial diameter.

Thus, M. Richer found by an experiment made in the island of Cayenne, about 4 degrees from the equator, where a pendulum 3 feet $8\frac{3}{5}$ lines long, which at Paris vibrated seconds, required to be shortened a line and a quarter to make it vibrate seconds. And many other observations have confirmed the same principle. See Newton's Principia, lib. 3, prop. 20. By comparing the different observations of the French astronomers, Newton apprehends that 2 lines may be considered as the length a seconds pendulum ought to be decreased at the equator.

From some observations made by Mr. Campbell, in 1731, in Black-river, in Jamaica, 18° north latitude, it is collected, that if the length of a simple pendulum that swings seconds in London, be 39·126 English inches, the length of one at the equator would be 39·00, and at the poles 39·206. Philos. Trans. numb. 432.

And hence Mr. Emerson has computed the following Table, showing the length of a pendulum that swings seconds at every 5th degree of latitude, as also the length of the degree of latitude there, in English miles.

Degrees of Lat.	Length of Pendulum.	Length of the Degree.
	inches.	miles.
0	39·027	68·723
5	39·029	68·730
10	39·032	68·750
15	39·036	68·783
20	39·044	68·830
25	39·057	68·882
30	39·070	68·950
35	39·084	69·020
40	39·097	69·097
45	39·111	69·176
50	39·126	69·256
55	39·142	69·330
60	39·158	69·401
65	39·168	69·467
70	39·177	69·522
75	39·185	69·568
80	39·191	69·601
85	39·195	69·620
90	39·197	69·628

Capt. John Warren has lately made experiments on pendulums, at Madras, latitude 13° 4′ 12″, for which place he concludes the length of the seconds pendulum to be 39·02627 inches. He farther deduces the length at the equator to be 38·987 or 39 nearly, and that at the pole 39·207; hence he deduces the effect of gravity, in one second of time, to be at the equator 16·0328 feet, and at the poles 16·1233 feet; and hence also he deduces the ellipticity of the earth's figure to be $\frac{1}{325}$ nearly. See the Asiatic Researches, vol. xi, art. 5.

6. If two pendulums vibrate in similar arcs, the times of vibration are in the sub-duplicate ratio of their lengths. And the lengths of pendulums vibrating in similar arcs, are in the duplicate ratio of the times of a vibration directly; or in the reciprocal duplicate ratio of the number of oscillations made in any one and the same time. For, the time of vibration t being as $p\sqrt{\frac{r}{2g}}$, where p and g are constant or given, therefore t is as $\sqrt{r}$, and r as t^2. Hence therefore the length of a half-second pendulum will be $\frac{1}{4}r$ or $\frac{39\frac{1}{8}}{4} = 9·781$ inches; and the length of the quarter-second pendulum will be $\frac{1}{16}r = \frac{39\frac{1}{8}}{16} = 2·445$ inches; and so of others.

7. The foregoing laws, &c, of the motion of pendulums cannot strictly hold good, unless the thread that sustains the ball be void of weight, and the gravity of the whole ball be collected into a point. In practice therefore, a very fine thread, and a small ball, but of a very heavy matter, should be used. But a thick thread, and a bulky ball, disturb the motion very much; for in that case the simple pendulum becomes a compound one; it being much the same thing, as if several weights were applied to the same inflexible rod in several places.

8. M. Krafft, in the new Petersburg Memoirs, vols 6 and 7, has given the result of many experiments on pendulums, made in different parts of Russia, with deductions from them, from which he derives this theorem: If x be the length of a pendulum that swings seconds in any given latitude l, and in a temperature of 10 degrees of Reaumur's thermometer; then will the length of that pendulum, for that latitude, be thus expressed, viz,

$x = (439·178 + 2·321 \times \sin.^2 l)$ lines of a French foot.

And this expression agrees very nearly, not only with all the experiments made on the pendulum in Russia, but also with those of Mr. Graham, and those of Mr. Lyons in 79° 50′ north latitude, where he found its length to be 441·38 lines. See OBLATENESS.

Simple PENDULUM, in Mechanics, an expression commonly used among artists, to distinguish such pendulums as have no provision for correcting the effects of heat and cold, from those that have such provision. Also Simple Pendulum, and Detached Pendulum, are terms sometimes used to denote such pendulums as are not connected with any clock, or clock-work.

Compound PENDULUM, in Mechanics, is a pendulum whose rod is composed of two or more wires or bars of metal. These, by undergoing different degrees of expansion and contraction, when exposed to the same heat or cold, have the difference of expansion or contraction made to act in such manner as to preserve constantly the same distance between the point of suspension, and centre of oscillation, though exposed to very different and various degrees of heat or cold. There are a great variety of constructions for this purpose; but they may be all reduced to the Gridiron, the Mercurial, and the Lever Pendulum.

It may be just observed by the way, that the vulgar method of remedying the inconvenience arising from the extension and contraction of the rods of common pendulums, is by applying the bob, or small ball, with a screw, at the lower end; by which means the pendulum is at any time made longer or shorter, as the ball is screwed downwards or upwards, and thus the time of its vibration is kept continually the same.

Angular PENDULUM, is formed of two pieces or legs, like a sector, and suspended by the angular point. This form has been invented to diminish the length of the common pendulum, and at the same time to preserve, or even increase the time of vibration. In this pendulum, the time of vibration depends on the length of the legs, and on the angle contained between them conjointly, the duration of the time of vibration increasing with the angle. For, the wider the opening between the two legs, the higher, it is evident, the centre of gravity ascends, as the shorter its distance below the point of suspension, and consequently the longer the distance of the centre of oscillation, or the slower the vibration, since the distance of the latter is reciprocally proportional to that of the former, by my Course of Mathematics, vol. 2, prop. 56. Hence, a pendulum of this construction may be made to oscillate in any given time whatever; for the distance of the centre of oscillation may be computed by that prop. and the time of vibration by prop. 30, or by the preceding parts of this article.—It may be easily shown, that in this kind of pendulum, the squares of the times of vibration, are directly as the secants of half the angles contained by the legs, or reciprocally as the cosines of the same. Hence, if a pendulum of this construction vibrates half seconds when its legs are close, it will vibrate whole seconds, when the legs are opened to an angle of 151° $2\frac{1}{2}'$. If the two legs, for instance, be 15 inches long, and they make an angle of 150° 23′, the time of vibration will be 1 second; if the angle be increased to 178° $49\frac{1}{2}'$, the time of vibration would be 5 seconds. See Gregory's Mechanics, vol. 1, pa. 269.

If an isosceles right-angled triangle be suspended at its vertex, the centre of oscillation will be in the middle of its base. And if a right-angled cone be suspended at its vertex, the centre of oscillation will be in the centre of its base. In either case therefore, the time of vibration will be the same as that of a simple pendulum whose length is equal to the altitude of the triangle or of the cone. Other pendulums, whose lengths shall be equal to the distance of the centre of oscillation, may be readily found from the known rules for the centre of oscillation. Thus, in a parabola vibrating in its own plane, and suspended at its vertex, the distance of the centre of oscillation below the vertex, is $\frac{5}{7}$ axis + $\frac{1}{3}$ parameter: and when this quantity is equal to the axis, the base of the parabola is to its axis, as 1·85164 to 1. For several other cases, see the part above quoted of Dr. Gregory's Mechanics.

The Conical or *Circular* PENDULUM. This is so called from the figure described by the string of the pendulum. This kind of pendulum was invented by M. Huygens, and is also claimed by Dr. Hooke. Its time of vibration depends both on the length of the string and on the magnitude of the circle described by the ball of the pendulum, or only on the altitude of the cone described, by the pendulum; for if a denote the altitude of the cone described, and $p = 3·1416$, also $g = 16\frac{1}{12}$ feet, the distance freely fallen by a heavy body in 1 second of time; then it is well known that the time of each revolution of the pendulum, is $t = p\sqrt{\frac{2a}{g}} = 1·108\sqrt{a}$ seconds nearly; and is therefore equal to double the time of vibration of a common simple pendulum, whose length is equal to the height of the cone.

Several other ingenious contrivances, by means of different rods and levers, as also hollow pendulums, &c, have been devised by several artists; as, for instance, by a Mr. Chandler, by Mr. Troughton, and by Mr. Adam Reid, an ingenious mechanist at Woolwich. This last contrivance is by a long steel rod, which passes easy through a hollow shorter rod of zinc, only connected together at their bottoms; the pendulum ball or weight being connected to the upper end of the zinc rod. As the long steel rod lengthens by heat, and lowers the ball, the zinc does the same, and raises the ball as much, by which the pendulum is preserved, of the same length in all temperatures, when once the rods have been adjusted together of proper lengths.

The *Gridiron* PENDULUM was the invention of Mr. John Harrison, a very ingenious artist, and celebrated for his invention of the watch for finding the difference of longitude at sea, about the year 1725; and of several other time-keepers and watches since that time; for all which he received the parliamentary reward of between 20 and 30 thousand pounds. It consists of 5 rods of steel, and 4 of brass, placed in an alternate order, the middle rod being of steel, by which the pendulum ball is suspended; these rods of brass and steel, thus placed in an alternate order, and so connected with each other at their ends, that while the expansion of the steel rods has a tendency to lengthen the pendulum, the expansion of the brass rods, acting upwards, tends to shorten it. And thus, when the lengths of the brass and steel rods are duly proportioned, their expansions and contractions will exactly balance and correct each other, and so preserve the pendulum invariably of the same length. The simplicity of this ingenious contrivance is much in its favour; and the difficulty of adjustment seems the only objection to it. Mr. Harrison in his first machine for measuring time at sea, applied this combination of wires of brass and steel, to prevent any alterations by heat or cold, and in the machines or clocks he has made for this purpose, a like method of guarding against the irregularities arising from this cause is used.

The *Mercurial* PENDULUM was the invention of the ingenious Mr. Graham, in consequence of several experiments relating to the materials of which pendulums might be formed, in 1715. Its rod is made of brass, and branched towards its lower end, so as to embrace a cylindric glass vessel 13 or 14 inches long, and about 2 inches diameter; which being filled about 12 inches deep with mercury, forms the weight or ball of the pendulum. If upon trial the expansion of the rod be found too great for that of the mercury, more mercury must be poured into the vessel: if the expansion of the mercury exceeds that of the rod, so as to occasion the clock to go fast with heat, some mercury must be taken out of the vessel, so as to shorten the column. And thus may the expansion and contraction of the quicksilver in the glass be made exactly to balance the expansion and contraction of the pendulum rod, so as to preserve the distance of the centre of oscillation from the point of suspension invariably the same. Mr. Graham made a clock of this kind, and compared it with one of the best of the common sort, for 3 years together; when he found the errors of his to be only about one-eighth part of those of the latter. Philos. Trans. No. 392.

The *Lever* PENDULUM. From all that appears concerning this construction of a pendulum, we are inclined to believe that the idea of making the difference of the expansion of different metals operate by means of a lever, originated with Mr. Graham, who in the year 1737 constructed a pendulum, having its rod composed of one bar of steel between two of brass, which acted on the short end of a lever, to the other end of which, the ball or weight of the pendulum was suspended. This pendulum however was, upon trial, found to move by jerks; and therefore laid aside by the inventor, to make way for the mercurial pendulum, just mentioned.

Mr. Short informs us in the Philos. Trans. vol. 47, art. 88, that a Mr. Frotheringham, a quaker in Lincolnshire, caused a pendulum of this kind to be made: it consisted of two bars, one of brass, and the other of steel, fastened together by screws, with levers to raise or let down the bulb; above which these levers were placed. M. Cassini too, in the History of the Royal Academy of Sciences at Paris, for 1741, describes two kinds of pendulums for clocks, compounded of bars of brass and steel, and in which he applies a lever to raise or let down the bulb of the pendulum, by the expansion or contraction of the bar of brass.

Mr. John Ellicott also, in the year 1738, constructed a pendulum on the same principle, but differing from Mr. Graham's in many particulars. The rod of Mr. Ellicott's pendulum was composed of two bars only; the one of brass, and the other of steel. It had two levers, each sustaining its half of the ball or weight; with a spring under the lower part of the ball to relieve the levers from a considerable part of its weight, and so to render their motion more smooth and easy. The one lever in Mr. Graham's construction was above the ball: whereas both the levers in Mr. Ellicott's were within the ball; and each lever had an adjusting screw, to lengthen or shorten the lever, so as to render the adjustment the more perfect. See the Philos. Trans. vol. 47, pa. 479; where Mr. Ellicott's methods of construction are described, and illustrated by figures.

Notwithstanding the great ingenuity displayed by these very eminent artists on this construction, it must further be observed, in the history of improvements of this nature, that Mr. Cumming, another eminent artist, has given, in his Essays on the Principles of Clock and Watch-work, Lond. 1766, an ample description, with plates, of a construction of a pendulum with levers, in which it seems he has united the properties of Mr. Graham's and Mr. Ellicott's, without being liable to any of the defects of either. The rod of this pendulum is composed of one flat bar of brass, and two of steel; he uses three levers within the ball of the pendulum; and, among many other ingenious contrivances, for the more accurate adjusting of this pendulum to mean time, it is provided with a small ball and screw below the principal ball or weight, one entire revolution of which on its screw will only alter the rate of the clock's going one second per day; and its circumference is divided into 30, one of which divisions will therefore alter its rate of going one second in a month.

Pendulum *Clock*, is a clock having its motion regulated by the vibration of a pendulum. It is controverted between Galileo and Huygens, which of the two first applied the pendulum to a clock. For the pretensions of each, see Clock. After Huygens had discovered, that the vibration made in arcs of a cycloid, however unequal they might be in extent, were all equal in time; he soon perceived, that a pendulum applied to a clock, so as to make it describe arcs of a cycloid, would rectify the otherwise unavoidable irregularities of the motion of the clock; since, though the several causes of those irregularities should occasion the pendulum to make greater or smaller vibrations, yet, by virtue of the cycloid, it would still make them perfectly equal in point of time; and the motion of the clock governed by it, would therefore be preserved perfectly equable. But the difficulty was, how to make the pendulum describe arcs of a cycloid; for naturally the pendulum, being suspended by a fixed point, can only describe circular arcs about it.

Here M. Huygens contrived to fix the iron rod or wire, which bears the ball or weight, at the top to a silken thread, placed between two cycloidal cheeks, or two little arcs of a cycloid, made of metal. Hence the motion of vibration, applying successively from one of those arcs to the other, the thread, which is extremely flexible, easily assumes the figure of them, and by that means causes the ball or weight at the bottom to describe a just cycloidal arc.

This is doubtless one of the most ingenious and useful inventions many ages have produced: by means of which it has been asserted there have been clocks that would not vary a single second in several days: and the same invention also gave rise to the whole doctrine of involute and evolute curves, with the radius and degree of curvature, &c.

It is true, the pendulum is still liable to its irregularities, how minute soever they may be. The silken thread by which it was suspended, shortens in moist weather, and lengthens in dry; by which means the length of the whole pendulum, and consequently the times of the vibrations, are somewhat varied.

To obviate this inconvenience, M. Lahire, instead of a silken thread, used a little fine spring; which was not indeed subject to shorten and lengthen from those causes; yet he found it grew stiffer in cold weather, and then made its vibrations faster than in warm; to which also we may add its expansion and contraction by heat and cold. He therefore had recourse to a stiff wire or rod, firm from one end to the other. Indeed by this means he renounced the advantages of the cycloid; but he found, as he says, by experience, that the vibrations in circular arcs are performed in times as equal, provided they be not of too great extent, as those in cycloids. But the experiments of Sir Jonas Moore, and others, have demonstrated the contrary.

The ordinary causes of the irregularities of pendulums Dr. Derham ascribes to the alterations in the gravity and temperature of the air, which increase and diminish the weight of the ball, and by that means make the vibrations greater and less; an accession of weight in the ball being found by experiment to accelerate the motion of the pendulum; for a weight of 6 pounds added to the ball, Dr. Derham found made his clock gain 13 seconds every day.

A general remedy against the inconveniences of pendulums, is to make them long, the ball heavy, and to vibrate but in small arcs. These are the usual means employed in England; the cycloidal cheeks being generally neglected. See the forogoing article.

Pendulum clocks resting against the same rail have been found to influence each other's motion. See the Philos. Trans. No. 453, sect. 5 and 6, where Mr. Ellicott has given a curious and exact account of this phenomenon.

Pendulum *Royal*, a name used among us for a clock, whose pendulum swings seconds, and goes 8 days without winding up; showing the hour, minute, and second. The numbers in such a piece are thus calculated. First cast up the seconds in 12 hours, which are the beats in one turn of the great wheel; and they will be found to be 43200 = 12 × 60 × 60. The swing-wheel must be 30, to swing 60 seconds in one of its revolutions; now let the half of 43200, viz 21600, be divided by 30, and the quotient will be 720, which must be separated into quotients. The first of these must be 12, for the great wheel, which moves round once in 12 hours. Now 720 divided by 12, gives 60, which may also be conveniently broken into two quotients, as 10 and 6, or 12 and 5, or 8 and $7\frac{1}{2}$; which last is most convenient: and if the pinions be all taken 8, the work will stand as above.

$$\begin{array}{r} 8\,)\ 96\ (\ 12 \\ 8\,)\ 64\ (\ 8 \\ \underline{8\,)\ 60\ (\ 7\tfrac{1}{2}} \\ 30 \end{array}$$

According to this computation, the great wheel will go round once in 12 hours, to show the hour; the next wheel once in an hour, to show the minutes; and the swing-wheel once in a minute, to show the seconds. See Clockwork.

Ballistic Pendulum. See Ballistic *Pendulum.*

Level Pendulum. See Level.

Pendulum *Watch.* See Watch.

PENETRABILITY, capability of being penetrated. See Impenetrability.

PENETRATION, the act by which one thing enters another, or takes up the place already possessed by another. The schoolmen define penetration the co-existence of two or more bodies, so that one is present, or has its extension in the same place as the other. Most philosophers hold the penetration of bodies absurd, i. e. that two bodies should be at the same time in the same place; and accordingly impenetrability is laid down as one of the essential properties of matter.—What is popularly called penetration, only amounts to the matter of one body's being admitted into the vacuity or pores of another. Such is the penetration of water through the substance of gold.

PENINSULA, in Geography, is a portion or extent of land which is nearly surrounded with water, being joined to the continent only by an isthmus, or narrow neck. Such is Africa, the greatest peninsula in the world, which is joined to Asia by an isthmus at the extremity of the Red Sea; such also is Peloponnesus, or the Morea, joined to Greece: and Jutland, &c. Peninsula is the same with what is otherwise called Chersonesus.

PENNY, a well-known copper coin, being the 12th part of a shilling. The penny was formerly a silver coin first struck in England by our Saxon ancestors, being the 240th part of their pound, and its true weight was about 22½ grains troy.

In Etheldred's time, the penny was the 20th part of the troyounce, and equal in weight to our three pence; which value it retained till the time of Edward III.

Till the time of King Edward the first, the penny was struck with a cross so deeply sunk in it, that it might, on occasion, be easily broken, and parted into two halves, thence called halfpennies; or into four, thence called fourthings, or farthings. But that prince coined it without the cross; instead of which he struck round halfpence and farthings. Though there are said to be instances of such round halfpence having been made in the reign of Henry the first, if not also in that of the two Williams.

Edward the first also reduced the weight of the penny to a standard; ordering that it should weigh 32 grains of wheat, taken out of the middle of the ear. This penny was called the penny sterling; and 20 of them were to weigh an ounce; whence the penny became a weight as well as a coin.

By the 9th of Edward the 3d, it was diminished to the 26th part of the troy ounce; by the 2d of Henry the 6th it was the 32d part; by the 5th of Edward the 4th, it became the 40th, and also by the 36th of Henry the 8th, and afterwards, the 45th; but by the 2d of Elizabeth, 60 pence were coined out of the ounce, and during her reign 62, which last proportion is still observed in our times.

The French penny, or denier, is of two kinds; the Paris penny, called denier Parisis; and the penny of Tours, called denier Tournois.

The Dutch penny, called pennink, or pening, is a real money, worth about one-fifth more than the French penny Tournois. The pennink is also used as a money of account, in keeping books by pounds, florins, and patards; 12 penninks make the patard, and 20 patards the florin.

At Hamburg, Nuremberg, &c, the penny, or pfennig of account, is equal to the French penny Tournois. Of these, 8 make the krieuk; and 60 the florin of those cities; also 90 the French crown, or 4s. 6d. sterling.

Penny-*Weight,* a troy weight, being the 20th part of an ounce, containing 24 grains; each grain weighing a grain of wheat gathered out of the middle of the ear, well dried. The name took its rise from its being actually the weight of one of our ancient silver pennies. See the foregoing article.

PENTAGON, in Geometry, a plane figure consisting of five angles, and consequently five sides also. If the angles be all equal, it is a regular pentagon. It is a remarkable property of the pentagon, that its side is equal in power to the sides of a hexagon and a decagon inscribed in the same circle; that is, the square of the side of the pentagon, is equal to both the squares taken together of the sides of the other two figures; and consequently those three sides will constitute a right-angled triangle. Euclid, 1.13, prop. 10.—Pappus has also demonstrated, that 12 regular pentagons contain more than 20 triangles inscribed in the same circle; lib. 5, prop. 45.—The dodecahedron, which is the fourth regular body or solid, is contained under 12 equal and regular pentagons.

To find the Area of a Regular Pentagon. Multiply the square of its side by 1·7204774, or by $\frac{5}{4}$ of the tangent of 54°, or by $\frac{5}{4}\sqrt{(1 + \frac{2}{5}\sqrt{5})}$. Hence, if s denote the side of the pentagon, its area will be $1{\cdot}7204774s^2 = \frac{5}{4}s^2\sqrt{(1 + \frac{2}{5}\sqrt{5})} = \frac{5}{4}s^2 \times \text{tang. } 54°$.

PENTAGRAPH, otherwise called a parallelogram, a mathematical instrument for copying designs, prints, plans, &c, in any proportion. The common pentagraph (plate 24, fig. 2) consists of four rulers or bars, of metal or wood, two of them from 15 to 18 inches long, the other two half that length. At the ends, and in the middle, of the long rulers, as also at the ends of the shorter ones, are holes, on the exact fixing of which the perfection of the instrument chiefly depends. Those in the middle of the long rulers, are to be at the same distance from those at the end of the long ones, and those of the short ones; so that, when put together, they may always make a parallelogram.

The instrument is fitted together for use, by several little pieces, particularly a little pillar, No. 1, having at one end a nut and screw, joining the two long rulers together; and at the other end a small knot for the instrument to slide on. The piece No. 2 is a rivet with a screw and nut, by which each short ruler is fastened to the middle of each long one. The piece No. 3 is a pillar, one end of which, being hollowed into a screw, has a nut fitted to it; and at the other end is a worm to screw into the table; when the instrument is to be used, it joins the ends of the two short rulers. The piece No. 4 is a pen, or pencil, or portcrayon, screwed into a little pillar. Lastly, the piece No. 5 is a brass point, moderately blunt, screwed likewise into a little pillar.

Use of the Pentagraph.—1. To copy a design in the same size or scale as the original. Screw the worm No. 3 into the table; lay a paper under the pencil No. 4, and the design under the point No. 5. This done, conduct the point over the several lines and parts of the design, and the pencil will draw or repeat the same on the paper.

2. When the design is to be reduced, for example to half the scale; the worm must be placed at the end of the long ruler No. 4, and the paper and pencil in the middle. In this situation conduct the brass point over the several lines

of the design, as before; and the pencil at the same time will draw its copy in the proportion required; the pencil here only moving half the lengths that the point moves.

3. On the contrary, when the design is to be enlarged to a double size; the brass point, with the design, must be placed in the middle at No. 3, the pencil and paper at the end of the long ruler, and the worm at the other end.

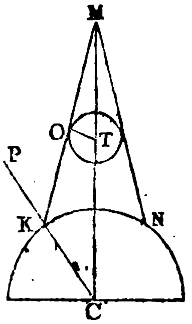
4. To reduce or enlarge in other proportions, there are holes drilled at equal distances on each ruler; viz, all along the short ones, and half way of the long ones, for placing the brass point, pencil, and worm, in a right line in them; i. e. if the piece carrying the point be put in the third hole, the other two pieces must be put each in its third hole; &c.

PENTANGLE, a plane figure of five angles, or the same as the PENTAGON.

PENUMBRA, in Astronomy, a faint or partial shade, in an eclipse, observed between the perfect shadow, and the full light. The penumbra arises from the magnitude of the sun's body: were he only a luminous point, the shadow would be all perfect; but by reason of the diameter of the sun it happens, that a place which is not illuminated by the whole body of the sun, may yet receive rays from some part of it. Thus, suppose S the sun, and T the moon, and the shadow of the latter projected on a plane, as GH (plate 24, fig. 3). The true proper shadow of T, viz GH, will be encompassed with an imperfect shadow, or penumbra, HL and GE, each portion of which is illuminated by an entire hemisphere of the sun.

The degree of light or shade of the penumbra, will be more or less in different parts, as those parts lie open to the rays of a greater or less part of the sun's body: thus, from L to H, and from E to G, the light continually diminishes; in the confines of G and H, the penumbra is darkest, and becomes lost and confounded with the total shade; as near E and L it is thin and confounded with the total light.

A penumbra must be found in all eclipses, whether of the sun, the moon, or the other planets, primary or secondary; but it is most considerable with us in eclipses of the sun; which is the case here referred to.



To determine how much of the surface of the earth can be involved in the penumbra, let the apparent semidiameter of the sun be supposed the greatest, or about 16′ 20″, which is when the earth is in her perihelion; also let the moon be in her apogee, and therefore at her greatest distance from the earth, or about 64 of the earth's semidiameters. Let KNC be the earth, T the moon, and MKN the penumbra, involving the part of the earth from K to N, which it is required to find. Here then are given the angle KMC = 16′ 20″, TC = 64, KC = 1, and OT = $\frac{11}{40}$ of KC. Hence, in the right-angled triangle OTM, as sin. OMT : radius : : OT : TM = 210½OT = 58KC nearly. Therefore MC = MT + TC = 58 + 64 = 122 semidiameters of the earth. Then, in the triangle KMC, there are given KC = 1, and MC = 122, also the angle KMC = 16′ 20″, to find the angle C; thus, as

KC : MC : : sin. ∠KMC : sin. ∠MKP = 35° 25′ 35″;
from this take the angle ∠KMC - 0 16 20,
leaves the ∠C - - - 35 9 11,
the double of which is the arc KN 70° 18′ 22″,
or nearly a space of 4866 miles in diameter.

PERAMBULATOR, an instrument for measuring distances; called also pedometer, way-wiser, and surveying wheel. This wheel is contrived to measure out a pole, or 16½ feet, in making two revolutions; consequently its circumference is 8¼ feet, and its diameter 2·626 feet, or 2 feet 5½ inches and $\frac{12}{1000}$ parts, very nearly. It is either driven forward by two handles, by a person walking; or is drawn by a coach wheel, &c, to which it is attached by a pole. It contains various movements, by wheels, or clock-work, with indices on its face, which is like that of a clock, to point out the distance passed over, in miles, furlongs, poles, yards, &c. Its advantages are, its readiness and expedition; being very useful for measuring roads and great distances on level ground. See the fig. plate 21, fig. 6.

PERCH, in Surveying, a square measure, being the 40th part of a rood, or the 160th part of an acre; that is, the square of a pole or rod, of the length of 5½ yards, or 16½ feet.

PERCH is by some also made to mean a measure of length; being the same as the rod or pole of 5½ yards or 16½ feet long. But it is better, for preventing confusion, to distinguish them.

PERCUSSION, in Physics, the impression a body makes in falling or striking upon another; or the shock or collision of two bodies, which meeting alter each other's motion.—Percussion is either direct or oblique. It is also either between elastic or nonelastic bodies, which have each their different laws. It is true, we know of no bodies in nature that are either perfectly elastic or the contrary; but all partaking of that property in different degrees; even the hardest and the softest being not entirely divested of it. But, for the sake of perspicuity, it is usual, and proper, to treat of these under two distinct heads.

Direct PERCUSSION is that in which the impulse is made in the direction of a line perpendicular at the place of impact, and which also passes through the common centre of gravity of the two striking bodies. As is the case in two spheres, when the line of the direction of the stroke passes through the centres of both spheres; for then the same line, joining their centres, passes perpendicularly through the point of impact. And

Oblique PERCUSSION, is that in which the impulse is made in the direction of a line that does not pass through the common centre of gravity of the striking bodies; whether that line of direction is perpendicular to the place of impact, or not. The force of percussion is the same as the momentum, or quantity of motion, and is represented by the product arising from the mass or quantity of matter moved, multiplied by the velocity of its motion; and that without any regard to the time or duration of action; for its action is considered totally independent of time, or but as for an instant, or an infinitely small time.

This leads us to consider a question that has been greatly canvassed among philosophers and mathematicians, viz, what is the relation between the force of percussion and mere pressure or weight? Now let M denote any mass, body, or weight, having no motion or velocity, but simply its pressure; then will that pressure or force be denoted by M itself, if it be considered as acting for some certain finite assignable time; but, considered as a force of percussion, that is, as acting but for an infinitely small time, its velocity being 0, or nothing, its percussive force will be 0 × M, that is 0, or nothing; and is therefore incomparable with any percussive force whatever. But if we consider the two forces, viz, of percussion and pressure, with respect to the effects they produce: we shall find that there are instances in which they appear to resemble each other. Those who argue that the two forces are totally incomparable both in their nature and effects, support their hypothesis in the following manner. The intensity of any force is very properly estimated by the effect it produces in a given time: but the effect of the pressure M, in 0 time, or an infinitely small time, is nothing at all; that is, it will not, in an infinitely small time, produce, for example, any motion, either in itself, or in any other body: its intensity therefore, as its effect, is infinitely less than any the smallest force of percussion. It is true, indeed, that we see motion and other considerable effects produced by mere pressure, and to counteract which requires the opposition of some considerable percussive force: but then it must be observed, that the former has been an infinitely longer time than the latter in producing its effect; and it is no wonder in mathematics that an infinite number of infinitely small quantities makes up a finite one. It has therefore only been for want of considering the circumstance of time, that any question could have arisen on this head. Hence it is said that these two forces are related to each other, only as a surface is to a solid or body: by the motion of the surface through an infinite number of points, or through a finite right line, a solid or body is generated: and by the action of the pressure for an infinite number of moments, or for some finite time, a quantity equal to a given percussive force is generated: but the surface itself is infinitely less than any solid, and the pressure infinitely less than any percussive force. This point, say they, may be easily illustrated by some familiar instances, which prove at least the enormous disproportion between the two forces, if not also their absolute incomparability. And first, the blow of a small hammer, upon the head of a nail, will drive the nail into a board; when it is hard to conceive any weight so great as will produce a like effect, i. e. that will sink the nail as far into the board, at least unless it is left to act for a very considerable time: and even after the greatest weight has been laid as a pressure on the head of the nail, and has sunk it as far as it can as to sense, by remaining for a long time there without producing any farther sensible effect; let the weight be removed from the head of the nail, and instead of it, let it be struck a small blow with a hammer, and the nail will immediately sink farther into the wood. Again, it is also well known, that a ship-carpenter, with a blow of his mallet, will drive a wedge in below the greatest ship, lying aground, and so overcome her weight, and lift her up. Lastly, let us consider a man with a club to strike a small ball, upwards or in any other direction; it is evident that the ball will acquire a certain determinate velocity by the blow, suppose that of 10 feet per second, or minute, or any other time whatever: now it is a law, universally allowed in the communication of motion, that when different bodies are struck with equal forces, the velocities communicated are reciprocally as the weights of the bodies that are struck; that is, that a double body, or weight, will acquire half the velocity from an equal blow; a body 10 times as great, one 10th of the velocity; a body 100 times as great, the 100th part of the velocity; a body a million times as great, the millionth part of the velocity; and so on without end: whence it follows, that there is no body or weight, how great soever, but will acquire some finite degree of velocity, and be overcome, by any given small finite blow, or percussion.

Those however that take the contrary side of the question; in answer to what is above stated, make the following reply. We do not, say they, contend for the absolute comparability of percussion and pressure; all we assert is, that there are instances in which they produce similar effects, and as forces can only be compared by their effects, it is improper to consider them as absolutely incongruous: it is true that percussion is a momentary cause, but the effect it produces is not instantaneous; thus the blow of a hammer may be considered as acting for an indefinitely small portion of time, but the effect it produces can only be complete after a certain finite time; and if, in this time, a certain quantity of pressure will produce the same motion in any body, then it follows that there may be instances in which the effects of these two forces are equal, and consequently that they are comparable with each other: and to this it may be farther added, that though we allow percussion to be an instantaneous force, yet, must that body by which it is communicated have been in motion for a certain time, in order to have attained the velocity with which it strikes, and to which alone we attribute its superior force; there seems therefore no reason why pressure should not be allowed to act for the same time in any cases when we are comparing the effect of the two forces with each other. In fact, the difference between percussion and pressure seems to consist in this, that, in the latter force, the whole mass of the body is acting by continual and successive impulses, whereas in the former, these efforts, as it were, are all collected into one sum, and then instantaneously applied. And the reason why they do not produce the same effect is, that when any resistance to motion is made, either by friction or otherwise, a certain force is necessary to overcome the opposing force, before any absolute motion can ensue, and no force short of that by which the body is opposed will produce any effect whatever, however often it may be repeated; and therefore, when the opposing force is greater than the pressing force, no motion can ensue; but the momentum of the moving body, or the percussion, being as it were the accumulated sum of all the successive efforts of the pressing body, there is a sufficient quantity of action, first to overcome the opposing force, and the overplus is then employed in the generation of motion. It is therefore to this circumstance that we must attribute the apparent incongruity of percussion and pressure, and not to any existing difference in the nature of the two forces; for the very same cause will also prevent the comparison of a greater and less percussive force. For example, after a pile-engine has been employed for a certain time in driving a pile, until its action upon it is very trifling, it would be in vain for a man with a mallet, to endeavour to drive the pile an inch lower, because he could not produce such a momentum as is equivalent to the resisting force acting against the pile, and consequently it would remain in its place, however long his efforts may continue, for the effect of each blow ceases with it, and none of these, taken singly, having any motion, the same is true whatever may be the number of times that they are repeated; but if the ram of the pile-engine be again employed, the desired effect may be produced: the effect of this last then, in this case, is infinitely greater than the former, yet no one will be bold enough to assert that these two percussive forces are absolutely incomparable on this account.

Much more might be advanced in support of the hypothesis, that pressure and percussion are not incongruous in their nature, and that we are only prevented from comparing them, or their effects, by certain circumstances that arise in the application of the two forces to practical purposes.

The nature and laws of percussion have been investigated by Aristotle, Galileo, Descartes, Huygens, and others. Aristotle started the idea that percussion and weight are not comparable; and most moderns have acquiesced in that opinion.

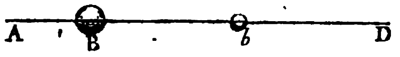
It appears that Descartes had some ideas of the laws of percussion; though it must be acknowledged, in some cases perhaps wide of the truth. The first who gave the true laws of motion in non-elastic bodies, was Doctor Wallis, in the Philos. Trans. numb. 43, where he also shows the true cause of reflections in other bodies, and proves that they proceed from their elasticity. Not long after, the celebrated sir Christopher Wren and Mr. Huygens imparted to the Royal Society the laws that are observed by perfectly elastic bodies, and gave exactly the same construction, though each was ignorant of what the other had done. And all those laws, thus published in the Philos. Trans. without demonstration, were afterwards demonstrated by Dr. Keill, in his Philos. Lect. in 1700; and they have since been followed by a number of other authors.

We have before observed that in percussion, we distinguish at least three several kinds of bodies; the perfectly hard, the perfectly soft, and the perfectly elastic. The two former are considered as utterly void of elasticity; having no force to separate them, or throw them off from each other again, after collision; and therefore either remaining at rest, or else proceeding uniformly forward together as one body or mass of matter. The laws of percussion therefore to be considered, are of two kinds: those for elastic, and those for non-elastic bodies.

The one only general principle, for determining the motions of bodies from percussion, and which belongs equally to both the kinds of bodies, i. e. both the elastic and nonelastic, is this: viz, that there exists in the bodies the same momentum, or quantity of motion, estimated in any one and the same direction, both before the stroke and after it. And this principle is the immediate result of the third law of nature or motion, that reaction is equal to action, and in a contrary direction; whence it happens that whatever motion is communicated to one body by the action of another, exactly the same motion doth this latter lose in the same direction, or exactly the same does the former communicate to the latter in the contrary direction.— From this general principle too it results, that no alteration takes place in the common centre of gravity of bodies by their actions on one another; but that the said common centre of gravity perseveres in the same state, whether of rest or of uniform motion, both before and after the shock of the bodies.

Now, from either of these two laws, viz, that of the preservation of the same quantity of motion, in one and the same direction, and that of the preservation of the same state of the centre of gravity, both before and after the shock, all the circumstances of the motions of both the kinds of bodies after collision may be estimated; in conjunction with their own peculiar and separate constitutions, namely, that of the one sort being elastic, and the other nonelastic.

The effects of these different constitutions, here alluded to, are these; that nonelastic bodies, on their shock, will adhere together, and either remain at rest, or else move together as one mass with a common velocity; or if elastic, they will separate after the shock with the very same relative velocity with which they met each other. The former of these consequences is evident, viz, that nonelastic bodies keep together as one mass after they meet; because there exists no power to separate them; and without a cause there can be no effect. And the latter consequence results immediately from the very definition and essence of elasticity itself, being a power always equal to the force of compression, or shock; and which restoring force therefore, acting the contrary way, will generate the same relative velocity between the bodies, or the same quantity of motion, as before the shock, and the same motion also of their common centre of gravity.



To apply now the general principle to the determination of the motions of bodies after their shock; let B and b be any two bodies, and V and v their respective velocities, estimated in the direction AD; which quantities V and v will be both positive if the bodies both move towards D, but one of them as v will be negative if the body b move towards A, and v will be $= 0$ if the body b be at rest. Hence then BV is the momentum of B towards D, and bv is the momentum of b towards D, whose sum is $BV + bv$, which is the whole quantity of motion in the direction AD, and which momentum must also be preserved after the impact.

Now if the bodies have no elasticity, they will move together as one mass $B + b$ after they meet, with some common velocity, which call y, in the direction AD; therefore the momentum in that direction after the shock, being the product of the mass and velocity, will be $(B + b) \times y$. But the momenta, in the same direction, before and after the impact, are equal, that is $BV + bv = (B + b)y$; from which equation any one of the quantities may be determined, when the rest are given. So, if we would find the common velocity after the stroke, it will be $y = \frac{BV + bv}{B + b}$, equal to the sum of the momenta divided by the sum of the bodies; which is also equal to the velocity of the common centre of gravity of the two bodies, both before and after the collision. The signs of the terms, in this value of y, will be all positive, as observed above, when the bodies move both the same way AD; but one term bv must be made negative when the motion of b is in the contrary direction; and that term will be absent or nothing, when b is at rest, before the shock.

Again, for the case of elastic bodies, which will separate after the stroke, with certain velocities, x and z, viz, x the velocity of B, and z the velocity of b after the collision, both estimated in the direction AD, which quantities will be either positive, or negative, or nothing, according to the circumstances of the masses B and b, with those of their celerities before the stroke. Hence then Bx and bz are the separate momenta after the shock, and $Bx + bz$ their sum, which must be equal to the sum $BV + bv$ in the same direction before the stroke: also $z - x$ is the relative velocity with which the bodies separate after the blow, and which must be equal to $V - v$ the same with which they meet; or, which is the same thing, that $V + x = v + z$; that is, the sum of the two velocities of the one body, is equal to the sum of the velocities of the other, taken before and after the stroke; which is another remarkable theorem. Hence then, for determining the two unknown quantities x and z, there are these two equations,

$$\text{viz, } BV + bv = Bx + bz,$$
$$\text{and } V - v = z - x;$$
$$\text{or } V + x = v + z;$$

the resolution of which equations gives those two velocities, as below,

$$\text{viz, } x = \frac{2bv + (B - b)V}{B + b},$$
$$\text{and } z = \frac{2BV - (B - b)v}{B + b}.$$

From these general values of the velocities, which are to be understood in the direction AD, any particular cases may easily be drawn. As, if the two bodies B and b be equal, then $B - b = 0$, and $B + b = 2B$, and the two velocities in that case become, after impulse, $x = v$, and $z = V$, the very same as they were before, but changed to the contrary bodies, i. e. the bodies have taken each other's velocity that it had before, and with the same sign also. So that, if the equal bodies were before both moving the same way, or towards D, they will do the same after, but with interchanged velocities. But if they before moved contrary ways, B towards D, and b towards A, they will rebound contrary ways, B back towards A, and b towards D, each with the other's velocity. And, lastly, if one body, as b, were at rest before the stroke, then the other B will be at rest after it, and b will go on with the motion that B had before. And thus may any other particular cases be deduced from the first general values of x and z.

We may now conclude this article with some remarks on these motions, and the mistakes of some authors concerning them. And first, we observe this striking difference between the motions that are communicated by elastic and by nonelastic bodies, viz, that a nonelastic body, by striking, communicates or continues exactly its whole momentum in the direction of its motion; as is evident. But the stroke of an elastic body may either communicate its whole motion to the body it strikes, or it may communicate only a part of it, or it may even communicate more than it had, so to speak. For, if the striking body remain at rest after the stroke, it has just lost all its motion, and therefore has communicated all it had; and if it still move forward in the same direction, it has still some motion left in that direction, and therefore has only communicated a part of what motion it had; but if the striking body rebound back, and move in the contrary direction, the other body has received not only the whole of the motion that the first had, but also as much more as the first has acquired in the contrary direction.

It has been denied by some authors, and in the Encyclopédie, that the same quantity of motion remains after the shock, as before it; and hence they seize an opportunity to reprehend the Cartesians for making that assertion, which they do, not only with respect to the case of two bodies, but also of all the bodies in the whole universe. And yet nothing is more true, if the motion be considered as estimated always in one and the same direcion, accounting that as negative, which is in the contrary or opposite direction. For it is a general law of nature, that no motion, nor force, can be generated, nor destroyed, nor changed, but by some cause which must produce an equal quantity in the opposite direction. And this being the case in one body, or two bodies, it must necessarily be the case in all bodies, and in the whole solar system, since all bodies act upon one another. And hence also it is manifest, that the common centre of gravity of the whole solar system must always preserve its original condition, whether it be of rest or of uniform motion; since the state of that centre is not changed by the mutual actions of bodies on each other, any more than their quantity of motion, in one and the same direction.

What may have led authors into the mistake above alluded to, which they bring no proof of, seems to be the discovery of M. Huygens, that the sums of the two products are equal, both before and after the shock, that are made by multiplying each body by the square of its velocity, viz, that $BV^2 + bv^2 = Bx^2 + bz^2$, where V and v are the velocities before the shock, and x and z the velocities after it. Such an expression, namely the product of the mass by the square of the velocity, is called the vis viva, or living force; and hence it has been inferred that the whole vis viva before the shock, or $BV^2 + bv^2$, is equal to that after the stroke, or $Bx^2 + bz^2$; which is indeed very true, as will be shown presently. But when they hence infer, that therefore the forces of bodies in motion are as the squares of the velocities, and that there is not the same quantity of motion between the two striking bodies, both before and after the shock, they are grossly mistaken, and thereby show that they are ignorant of the true derivation of the equation $BV^2 + bv^2 = Bx^2 + bz^2$. For this equation is only a consequence of the very principle above laid down, and which is not acceded to by those authors, viz, that the quantity of motion is the same before and after the shock, or that $BV + bv = Bx + bz$, the truth of which last equation they deny, because they think the former one is true, never considering that they may be both true, and much less that the one is a consequence of the other, and derived from it; which however is now found to be the case, as is proved in this manner:

It has been shown that the sum of the two momenta, in the same direction, before and after the stroke, are equal, or that $BV + bv = Bx + bz$; and also that the sum of the two velocities of the one body, is equal to the sum of those of the other, or that $V + x = v + z$; and it is now proposed to show that from these two equations there results the third equation $BV^2 + bv^2 = Bx^2 + bz^2$, or the equation of the living forces.

Now because $BV + bv = Bx + bz$, by transposition it is - - $BV - Bx = bz - bv$; which shows that the difference between the two momenta of the one body, before and after the stroke, is equal to the difference between those of the other body; which is another important theorem. But now, to derive the equation of the vis viva, set down the two foregoing equations, and multiply them together, so shall the products give the said equation required; thus,

Mult. $BV - Bx = bz - bv$, the equat. of the momenta,
by $V + x = z + v$, the equat. of the velocities,
produc. $BV^2 - Bx^2 = bz^2 - bv^2$,
or $BV^2 + bv^2 = Bx^2 + bz^2$,

the very equation of the vis viva required. See Keill's Lect. Philos. sect. 14, theor. 29, at the end. And for the geometrical determinations after impact, see the article COLLISION.

When the elasticity of the bodies is not perfect, but only partially so, as is the case with all the bodies we know of, the determination of the motions after collision may be determined in a similar manner. In this case also the sum of the momenta will still be the same, both before and after collision, but the velocities after will be less than in the case of perfect elasticity, in the ratio of the imperfection. Hence, with the same notation as before, the two equations will now be $\mathrm{BV} + bv = \mathrm{B}x + by$, and $\mathrm{V} - v = \frac{m}{n}(y - x)$, where m to n denotes the ratio of perfect to imperfect elasticity. And the resolution of these two equations, give the following values of x and y, viz, $x = \mathrm{V} - \frac{m+n}{m} \cdot \frac{b}{\mathrm{B}+b}(\mathrm{V} - v)$, $y = v + \frac{m+n}{m} \cdot \frac{\mathrm{B}}{\mathrm{B}+b}(\mathrm{V} - v)$, for the velocities of the two bodies after impact, in the case of imperfect elasticity; which would become the same as the former if n were $= m$.

Hence, if the two bodies B and b be equal, then $x = \mathrm{V} - \frac{m+n}{2m}(\mathrm{V} - v)$, and $y = v + \frac{m+n}{2m}(\mathrm{V} - v)$, where the velocity lost by B is just equal to that gained by b. And if in this case b was at rest before the impact, viz $v = 0$, then the resulting motions would be $x = \frac{m-n}{2m}\mathrm{V}$, and $y = \frac{m+n}{2m}\mathrm{V}$, which are in the ratio of $m - n$ to $m + n$.

Also, if $m = n$, or the bodies perfectly elastic, then $x = 0$, and $y = \mathrm{V}$; that is, B would be at rest, and b go on with the first motion of B.—Further, in this case also, the velocity of B before the impact, is to that of b after it, as V to $\frac{m+n}{2m}\mathrm{V}$, or as $2m$ to $m + n$. But if the bodies be now supposed to vibrate in circles, as pendulums, in which case the chords (C and c) of the arcs described, are known to be proportional to' the velocities; then it will be $2m : m + n :: \mathrm{C} : c$; hence $m : n :: \mathrm{C} : 2c - \mathrm{C}$. So that, by measuring these chords, of the arcs thus experimentally described, the ratio of m to n, or the degree of elasticity in the bodies, may be determined.

Centre of **Percussion**, is the point in which the shock or impulse of a body which strikes another is the greatest that it can be. See Centre.—The centre of percussion is the same as the centre of oscillation, when the striking body moves round a fixed axis. See Oscillation.—But if all the parts of the striking body move with a parallel motion, and with the same velocity, then the centre of percussion is the same as the centre of gravity.

PERFECT Number, is one that is equal to the sum of all its aliquot parts, when added together. Eucl. lib. 7, def. 22. As the number 6, which is $= 1 + 2 + 3$, the sum of all its aliquot parts; also 28, for $28 = 1 + 2 + 4 + 7 + 14$, the sum of all its aliquot parts. —It is proved by Euclid, in the last prop. of book the 9th, that if the common geometrical series of numbers 1, 2, 4, 8, 16, 32, &c, be continued to such a number of terms, as that the sum of the said series of terms shall be a prime number, then the product of this sum by the last term of the series will be a perfect number. The same rule may be otherwise expressed thus: If n denote the number of terms in the given series 1, 2, 4, 8, &c; then it is well known that the sum of all the terms of the series is $2^n - 1$, and it is evident that the last term is 2^{n-1}: consequently the rule becomes thus, viz, $2^{n-1} \times (2^n - 1) =$ a perfect number, whenever $2^n - 1$ is a prime number.

Now the sums of one, two, three, four, &c, terms of the series 1, 2, 4, 8, &c, form the series 1, 3, 7, 15, 31, &c; so that the number will be found perfect whenever the corresponding term of this series is a prime, as 1, 3, 7, 31 &c. Whence the table of perfect numbers may be found and exhibited as follows; where the 1st column shows the number of terms, or the value of n; the 2d column is the last term of the series 1, 2, 4, 8, &c, and is expressed by 2^{n-1}; the 3d column contains the corresponding sums of the said series, or the values of the quantity $2^n - 1$; which numbers in this 3d column are easily constructed by adding always the last number in this column to the next following number in the 2d column: and lastly, the 4th column shows the correspondent perfect numbers, or the values of $2^{n-1} \times (2^n - 1)$, the product of the numbers in the 2d and 3d columns, when $2^n - 1$, or the number in the 3d column, is a prime number; the products in the other cases being omitted, as not perfect numbers.

Values of n	Values of 2^{n-1}	Values of $2^n - 1$	Perf. Numbers, or $2^{n-1} \times (2^n - 1)$
1	1	1	1
2	2	3	6
3	4	7	28
4	8	15	—
5	16	31	496
6	32	63	—
7	64	127	8128

Hence the first four perfect numbers are found to be 6, 28, 496, 8128; and thus the table might be continued to find others; but the trouble would be very great, for want of a general method to distinguish which numbers are primes, as the case requires. Several learned mathematicians have endeavoured to facilitate this business, but hitherto with only a small degree of effect. After the foregoing four perfect numbers, there is a long interval before any more occur. The first eight are as follow, with the factors and products which produce them; being all the primes that are yet known.

The first perfect numbers.	Their values.
6	$= (2^2 - 1)\, 2$
28	$= (2^3 - 1)\, 2^2$
496	$= (2^5 - 1)\, 2^4$
8128	$= (2^7 - 1)\, 2^6$
33550336	$= (2^{13} - 1)\, 2^{12}$
8589869056	$= (2^{17} - 1)\, 2^{16}$
137438691328	$= (2^{19} - 1)\, 2^{18}$
2305843008139952128	$= (2^{31} - 1)\, 2^{30}$

See several considerable tracts on the subject of perfect numbers in the Memoirs of the Petersburg Academy, vol. 2 of the new vols, and in several other volumes.

PERIÆCI. See Periœci.

PERIGÆUM, or Perigee, is that point of the orbit of the sun or moon, which is the nearest to the earth. In which sense it stands opposed to apogee, which is the most distant point from the earth.

Perigee, in the Ancient Astronomy, denotes a point in a planet's orbit, where the centre of its epicycle is at the least distance from the earth.

PERIHELION, Perihelium, that point in the orbit of a planet or comet which is nearest to the sun. In which sense it stands opposed to aphelion, or aphelium, which is the highest or most distant point from the sun. Instead of this term, the ancients used perigeum; because they placed the earth in the centre.

PERIMETER, in Geometry, the ambit, limit, or outer bounds of a plane figure; being the sum of all the lines by which it is inclosed or formed. In circular figures, &c, instead of this term, the word circumference or periphery is used.

PERIOD, in Astronomy, the time in which a star or planet makes one revolution, or returns again to the same point in the heavens. The sun's, or properly the earth's tropical period, is 365 days 5 hours 48 minutes 45 seconds 30 thirds. That of the moon is 27 days 7 hours 43 minutes. That of the other planets as below.—There is a remarkable harmony between the distances of the planets from the sun, and their periods round him; the great law of which is, that the squares of the periodic times are always proportional to the cubes of their mean distances from the sun.

The periods, both tropical and sidereal, with the proportions of the mean distances of the several planets, are as follow:

Planets.	Tropical Periods.			Sidereal Periods.			Proport. Dists.
Mercury - -	87^d	23^h	$14'$	87^d	23^h	$16'$	36710
Venus - - -	224	16	42	224	16	49	72333
Earth - - -	365	5	49	365	6	9	100000
Mars - - -	686	22	18	686	23	31	152369
Vesta - - -	-	-	-	-	-	-	235513
Juno - - -	-	-	-	-	-	-	266400
Pallas - - -	1682			-	-	-	276500
Ceres - - -	1681			-	-	-	276700
Jupiter - - -	4330	8	58	4332	8	51	520110
Saturn - - -	10749	7	22	10761	14	37	953800
Uranus - - -	30456	1	41	-	-	-	1908180

As to the comets, the periods of very few of them are known. There is one however of between 75 and 76 years, which appeared for the last time in 1759; another was supposed to have its period of 129 years, which was expected to appear in 1789 or 1790, but it did not; and the comet which appeared in 1680 it is thought has its period of 575 years.

PERIOD, in Chronology, denotes an epoch, or interval of time, by which the years are reckoned; or a series of years by which time is measured, in different nations. Such are the Calippic and Metonic Periods, two different corrections of the Greek calendar, the Julian Period, invented by Joseph Scaliger; the Victorian Period, &c.

Calippic PERIOD. See CALIPPIC *Period*.

Constantinopolitan PERIOD, is that used by the Greeks, and is the same as the *Julian* PERIOD, which see.

Chaldaic PERIOD. See SAROS.

Dionysian PERIOD. See *Victorian* PERIOD.

Hipparchus's PERIOD, is a series or cycle of 304 solar years, returning in a constant round, and restoring the new and full moons to the same day of the solar year; as Hipparchus thought.—This period arises by multiplying the Calippic period by 4. Hipparchus assumed the quantity of the solar year to be 365^d 5^h 55^m 12^s; and hence he concluded, that in 304 years Calippus's period would err a whole day. He therefore multiplied the period by 4, and from the product cast away an entire day. But even this does not restore the new and full moons to the same day throughout the whole period: they are sometimes anticipated 1^d 8^h 23^m 29^s 20 thirds.

Julian PERIOD, so called as being adapted to the Julian year, is a series of 7980 Julian years; arising from the multiplications of the cycles of the sun, moon, and indiction together, or the numbers 28, 19, 15; commencing on the 1st day of January in the 764th Julian year before the creation, and therefore is not yet completed. This comprehends all other cycles, periods, and epochs, with the times of all memorable actions and histories; and therefore it is not only the most general, but the most useful of all periods in chronology.

As every year of the Julian period has its particular solar, lunar, and indiction cycles, and no two years in it can have all these three cycles the same, every year of this period becomes accurately distinguished from another.—This period was invented by Joseph Scaliger, as containing all the other epochs, to facilitate the reduction of the years of one given epoch to those of another. It agrees with the Constantinopolitan period, used by the Greeks, except in this, that the cycles of the sun, moon, and indiction, are reckoned differently; and that the first year of the Constantinopolitan period differs from that of the Julian period. To find the year answering to any given year of the Julian period, and vice versa; see EPOCH.

Metonic PERIOD. See CYCLE *of the Moon*.

Victorian PERIOD, an interval of 532 Julian years; at the end of which, the new and full moons return again on the same day of the Julian year, according to the opinion of the inventor Victorinus, or Victorius, who lived in the time of pope Hilary. Some ascribe this period to Dionysius Exiguus, and hence they call it the Dionysian period: others again call it the Great Paschal Cycle, because it was invented for computing the time of Easter.

The Victorian period is produced by multiplying the solar cycle 28 by the lunar cycle 19, the product being 532. But neither does this restore the new and full moons to the same day throughout its whole duration, by 1^d 16^h 58^m 59^s 40 thirds.

PERIOD, in Arithmetic, is a distinction made by a point, or a comma, after every 6th place, or figure; and is used in numeration, for the readier distinguishing and naming the several figures or places, which are thus distinguished into periods of six figures each. See NUMERATION.

PERIOD is also used in arithmetic, in the extraction of roots, to point off, or separate the figures of the given number into periods, or parcels, of as many figures each as are expressed by the degree of the root to be extracted, viz, of two places each for the square root, three for the cube root, and so on.

PERIODIC, or PERIODICAL, appertaining to period, or going by periods. Thus, the periodical motion of the moon, is that of her monthly period or course about the earth, called her periodical month, containing 27 days 7 hours 45 minutes.

PERIODICAL *Month*. See MONTH.

PERIŒCI, or PERIŒCIANS, in Geography, are such as live in opposite points of the same parallel of latitude. Hence they have the same seasons at the same time, with the same phenomena of the heavenly bodies; but their times of the day are opposite, or differ by 12 hours, being noon with the one when it is midnight with the other.

PERIPATETIC *Philosophy*, the system of philosophy taught and established by Aristotle, and maintained by his followers, the Peripatetics. See ARISTOTLE.

PERIPATETICS, the followers of Aristotle. Though some derive their establishment from Plato himself, the master of both Xenocrates and Aristotle.

PERIPHERY, in Geometry, is the circumference, or bounding line, of a circle, ellipse, or other regular curvilineal figure. See CIRCUMFERENCE, and CIRCLE.

PERISCII, or PERISCIANS, those inhabitants of the earth, whose shadows do, in one and the same day, turn quite round to all the points of the compass, without disappearing.—Such are the inhabitants of the two frozen zones, or who live within the compass of the arctic and antarctic circles; for as the sun never sets to them, after he is once up, but moves quite round about, so do their shadows also.

PERISTYLE, in the ancient Architecture, a place or building encompassed with a row of columns on the inside; by which it is distinguished from the periptere, where the columns are disposed on the outside.

PERISTYLE is also used, by modern writers, for a range of columns, either within or without a building.

PERITROCHIUM, in Mechanics, is a wheel or circle, concentric with the base of a cylinder, and moveable together with it, about an axis. The axis, with the wheel, and levers fixed in it to move it, make that mechanical power, called Axis *in Peritrochio.*

PERMUTATIONS *of Quantities*, in Algebra, denotes the different orders in which any quantities may be arranged; thus, the permutation of the three quantities, *a*, *b*, *c*, taken two and two together, are six; as, *ab*, *ba*, *ac*, *ca*, *bc*, *cb*; being thus distinguished from combinations, which only relate to the different collection of quantities without regard to their order, so that the combinations of the above three quantities are only three, as *ab*, *ac*, *bc*. Therefore, having found the number of combinations of any number of things, we must then find the number of permutations that any one combination will admit of, and the product of the two will be the number of permutation. Or the number of permutation (P) of any number (n) of things, taken any number (r) at a time, may be obtained from the following general formula, or theorem, $\text{P} = n \times (n-1) \times (n-2) \times (n-3) \text{ - - - - - } (n - r - 1)$, while the numbers of combinations (C) of the same things, taken the same number at a time, will be represented by

$$\text{C} = \frac{n \times (n-1) \times (n-2) \times (n-3) \text{ - - - - } (n - \overline{r - 1})}{1 \times 2 \times 3 \times 4 \text{ - - - - } r}.$$

From the foregoing theorem it appears, that the number of permutations or changes that can be made upon any number (n) of things taken together, that is, without considering them as taken a certain number at a time, as the number of changes that may be made on a given number of bells, &c, will be expressed by the continued product $n \times (n-1) \times (n-2) \times (n-3)$ &c. - - - - - $(n - n + 1)$; and thus, the number of changes that may be rung on 12 bells will be found to be expressed by the number 479001600.

When there are a certain number of things of one sort, and a certain number of another, &c, to find the number of changes that can be made out of them all.

Take the series $1 \times 2 \times 3 \times 4$ &c, up to the number of things given. Also the series $1 \times 2 \times 3$ &c, up to the number of things given of the first sort, and the same again for the number of the second sort, &c; then the first product divided by the joint products of the last series will give the answer. Or calling n the whole number of things; r, s, t, &c, the number of each sort, and P the number of permutations required, we shall have

$$\text{P} = \frac{1 \times 2 \times 3 \times 4 \times 5 \text{ - - - - - - - - } n}{(1 \times 2 \times 3 \,..\, r) \times (1 \times 2 \times 3 \,..\, s) \times (1 \times 2 \times 3 \,..\, t)}.$$

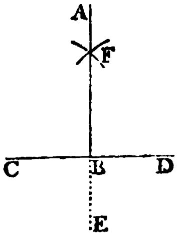
But if in this last problem, instead of supposing the permutations to take place among the number of things taken collectively, it was required to find the number of permutations of the same things, taken any given number of things at a time, the operation is more tedious; and indeed the best rule that has yet been given for it, is little better than mere trial, being as follows:—Find all the different forms of combination of all the given things, taken as many at a time as in the question; then find the number of permutations in any form, and multiply it by the number of combinations in that form. Do the same for every distinct form, and the sum of all the products will give the whole number of permutations required.

But when only the number of combinations are required as in the following question: To find the number of combinations that can be formed out of a given number of things, in which there are m things of one sort, n things of another sort, p things of another sort, &c, by taken 1 at a time, 2 at a time, &c, to any given number of things at a time. Then we have a very simple rule which was given in No. 103 of Nicholson's Philosophical Journal; as follows,

Place in a horizontal row $m + 1$ units, annexing ciphers on the right hand, till the whole number of units and ciphers exceeds the greatest number of things to be taken at a time by unity.

Under each of these terms write the sum of the $n + 1$ left-hand terms, including that as one of them, under which the number is placed; and under each of these write the sum of the $p + 1$ left-hand terms of the last line: and under each of these last the $q + 1$ left-hand terms, and so on through all the number of different things; then the last line will be the answer: that is, the second term shows the number of combinations taken 1 at a time, the third term the number of combinations taken 2 at a time, &c.

PERPENDICULAR, in Geometry, or NORMAL. One line is perpendicular to another, when the former meets the latter so as to make the angles on both sides of it equal to each other. And those angles are called right angles. And hence, to be perpendicular to, or to make right-angles with, means one and the same thing. So, when the angle ABC is equal to the angle ABD, the line AB is said to be perpendicular, or normal, or at right angles to the line CD.



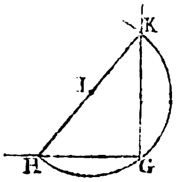
A line is perpendicular to a curve, when it is perpendicular to the tangent of the curve at the point of contact.

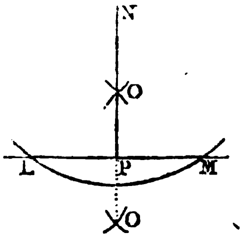
A line is perpendicular to a plane, when it is perpendicular to every line drawn in the plane through the bottom of the perpendicular. And one plane is perpendicular to another, when every line in the one plane which is perpendicular to the line of their common section, is perpendicular to the other plane.

From the very principle and notion of a perpendicular, it follows, 1. That the perpendicularity is mutual, that is, if the first AB is perpendicular to the second CD, then is the second perpendicular to the first.—2. That only one perpendicular can be drawn from one point in the same plane.—3. That if a perpendicular be continued through the line it was drawn perpendicular to; the continuation BE will also be perpendicular to the same.—4. That a line which is perpendicular to another line, is also perpendicular to all the parallels of the other.—5. That a perpendicular is the shortest of all those lines which can be drawn from the same point to the same right line. Hence the distance of a point from a line or plane, is a line drawn from the point perpendicular to the line or plane: and hence also the altitude of a figure is a perpendicular let fall from the vertex to the base.

To Erect a Perpendicular from a given point in a line. —1. When the given point B is near the middle of the line; with any interval in the compasses take the two equal parts BC, BD: and from the two centres C and D, with any radius greater than BC or BD, strike two arcs intersecting in F; then draw BFA, which will be the perpendicular required.

2. When the given point G is at or near the end of the line; with any centre I and radius IG describe an arc HGK through G; then a ruler laid by H and I will cut the arc in the point K, through which the perpendicular GK must be drawn.





To let fall a Perpendicular upon a given line LM from a given point N. With the centre N, and a convenient radius, describe an arc cutting the given line in L and M; with these two centres, and any other convenient radius, strike two other arcs intersecting in O, the point through which the perpendicular NOP must be drawn.

Perpendiculars are best drawn, in practice, by means of a square, laying one side of it along the given line, and the other to pass through the given point.

PERPENDICULAR, in Gunnery, is a small instrument used for finding the centre line of a piece, in the operation of pointing it to a given object. See *Pointing of a* GUN.

PERPETUAL *Motion*. See MOTION.

Circle of PERPETUAL *Occultation and Apparition*. See CIRCLE.

PERPETUAL, or *Endless, Screw*. See SCREW.

PERPETUITY, in the Doctrine of Annuities, is the number of years in which the simple interest of any principal sum will amount to the same as the principal itself. Or it is the quotient arising by dividing 100, or any other principal, by its interest for one year. Thus, the perpetuity, at the rate of 5 per cent. interest, is $\frac{100}{5} = 20$; at 4 per cent. $\frac{100}{4} = 25$; &c.

PERRY (Captain JOHN), was a celebrated English engineer. After acquiring great reputation for his skill in this country, he resided many years in Russia, having been recommended to the czar Peter while in England, as a person capable of serving him on a variety of occasions, relating to his new design of establishing a fleet, making his rivers navigable, &c. His salary in this service was to be 300l. per annum, besides travelling expenses and subsistence money on whatever service he should be employed, with a further reward to his satisfaction at the conclusion of any work he should finish.

After some conversation with the czar himself, particularly respecting a communication between the rivers Volga and Don, he was employed on that work for three summers successively; but not being well supplied with men, partly on account of the ill success of Peter's arms against the Swedes at the battle of Narva, and partly by the discouragement of the governor of Astracan, he was ordered at the end of 1707 to stop, and next year was employed in refitting the ships at Veronise, and in 1709 in making the river of that name navigable. But after repeated disappointments, and fruitless applications for his salary, he at length quitted the kingdom, under the protection of Mr. Whitworth, the English ambassador, in 1712. (See his Narrative in the Preface to The State of Russia.)

In 1721 he was employed in stopping the breach at Dagenham, made in the bank of the river Thames, near the village of that name in Essex, and about 3 miles below Woolwich, in which he happily succeeded, after several other persons had failed in that undertaking. He was also employed, the same year, about the harbour at Dublin, and published at that time an Answer to the objections made against it.—Besides this piece, Captain Perry was author of The State of Russia, 1716, 8vo; and An Account of the Stopping of Dagenham Breach, 1721, 8vo.—He died Feb. 11th, 1733.

PERSEUS, a constellation of the northern hemisphere, being one of the 48 ancient asterisms.—The Greeks fabled that this is Perseus, whom they make the son of Jupiter by Danae. The father of that lady had been told, that he should be killed by his grandchild, and having only Danae to take care of, he locked her up; but Jupiter found his way to her in a shower of gold, and Perseus verified the oracle. He cut off also the head of the gorgon, and affixed it to his shield; and after many other great exploits he rescued Andromeda, the daughter of Cassiopeia, whom the sea-nymphs, in revenge for that lady's boasting of superior beauty, had fastened to a rock to be devoured by a monster. Jupiter his father in honour of the exploit, they say, afterwards took up the hero, and the whole family with him, into the skies.—The number of stars in this constellation, in Ptolemy's catalogue, are 29; in Tycho's 29, in Hevelius's 46, and in the Britannic catalogue 59.

PERSIAN *Wheel*, in Mechanics, a machine for raising a quantity of water, to serve for various purposes. Such a wheel is represented in plate 25, *fig.* 1; with which water may be raised by means of a stream AB turning a wheel CDE, according to the order of the letters, with buckets *a, a, a, a*, &c, hung upon the wheel by strong pins *b, b, b, b*, &c, fixed in the side of the rim; which must be made as high as the water is intended to be raised above the level of that part of the stream in which the wheel is placed. As the wheel turns, the buckets on the right hand go down into the water, where they are filled, and return up full on the left hand, till they come to the top at K; where they strike against the end *n* of the fixed trough M, by which they are overset, and so empty the water into the trough; whence it is to be conveyed in pipes to any place it is intended for: and as each bucket gets over the trough, it falls into a perpendicular position again, and so goes down empty till it comes to the water at A, where it is filled as before. On each bucket is a spring *r*, which going over the top or crown of the bar *m* (fixed to the trough M) raises the bottom of the bucket above the level of its mouth, and so causes it to empty all its water into the trough.

Sometimes this wheel is made to raise water no higher than its axis; and then instead of buckets hung upon it, its spokes C, *d*, *e*, *f*, *g*, *h*, are made of a bent form, and hollow within; these hollows opening into the holes C, D, E, F, in the outside of the wheel, and also into those at O in the box N upon the axis. So that, as the holes C, D, &c, dip into the water, it runs into them; and as the wheel turns, the water rises in the hollow spokes, *c*, *d*, &c, and runs out in a stream P from the holes at O, and falls into the trough Q, whence it is conveyed by pipes.

PERSIAN, or PERSIC, in Architecture, a name common to all statues of men; serving instead of columns to support entablatures.

PERSIAN *Era* and *Year*. See EPOCH and YEAR.

PERSPECTIVE, the art of delineating visible objects on a plane surface, such as they appear at a given distance, or height, on a transparent plane, placed commonly perpendicular to the horizon, between the eye and the object. This is particularly called

Linear PERSPECTIVE, as regarding the position, magnitude, form, &c, of the several lines, or contours of objects, and expressing their diminution.

Some make this a branch of Optics; others an art and science derived from it: its operations however are all geometrical.

History of PERSPECTIVE. This art derives its origin from painting, and particularly from that branch of it which was employed in the decorations of the theatre, where landscapes were chiefly introduced. Vitruvius, in the proem to his 7th book, says that Agatharchus, at Athens, was the first author who wrote upon this subject, on occasion of a play exhibited by Æschylus, for which he prepared a tragic scene; and that afterwards the principles of the art were more distinctly taught in the writings of Democritus and Anaxagoras, the disciples of Agatharchus, which are not now extant.

The perspective of Euclid and of Heliodorus Larisseus contains only some general elements of optics, that are by no means adapted to any particular practice; though they furnish some materials that might be of service even in the linear perspective of painters.

Geminus, of Rhodes, a celebrated mathematician, in Cicero's time, also wrote upon this science. It is also evident that the Roman artists were acquainted with the rules of perspective, from the account which Pliny (Nat. Hist. lib. 35, cap. 4) gives of the representation on the scene of those plays given by Claudius Pulcher; by the appearance of which the crows were so deceived, that they endeavoured to settle on the fictitious roofs. However, of the theory of this art among the ancients we know nothing; as none of their writings have escaped the general wreck of ancient literature in the dark ages of Europe. Doubtless this art must have been lost, when painting and sculpture no longer existed. However, there is reason to believe that it was practised much later in the eastern empire.

John Tzetzes, in the 12th century, speaks of it as well acquainted with its importance in painting and statuary. And the Greek painters, who were employed by the Venetians and Florentines, in the 13th century, it seems brought some optical knowledge along with them into Italy: for the disciples of Giotto are commended for observing perspective more regularly than any of their predecessors in the art had done; and he lived in the beginning of the 14th century.

The Arabians were not ignorant of this science; as may be presumed from the optical writings of Alhazen, about the year 1100. And Vitellus, a Pole, about the year 1270, wrote largely and learnedly on optics. And, of our own nation, friar Bacon, as well as John Peckham, archbishop of Canterbury, treated this subject with great accuracy, considering the times in which they lived.

The first authors who professedly laid down rules of perspective, were Bartolomeo Bramantino, of Milan, whose book, Régole di Perspectiva, e Misure delle Antichita di Lombardia, is dated 1440; and Pietro del Borgo, likewise an Italian, who was the most ancient author met with by Ignatius Danti, and who it is supposed died in 1443. This last writer supposed objects placed beyond a transparent tablet, and so to trace the images, which rays of light, emitted from them, would make upon it. Albert Durer constructed a machine on the principles of Borgo, by which he could trace the perspective appearance of objects.

Leon Battista Alberti, in 1450, wrote his treatise De Pictura, in which he treats chiefly of perspective.

Balthazar Peruzzi, of Siena, who died in 1536, had diligently studied the writings of Borgo; and his method of perspective was published by Serlio in 1540. To him it is said we owe the discovery of points of distance, to which are drawn all lines that make an angle of 45° with the ground line.

Guido Ubaldi, another Italian, soon after discovered, that all lines that are parallel to one another, if they be inclined to the ground line, converge to some point in the horizontal line; and that through this point also will pass a line drawn from the eye parallel to them. His Perspective was printed at Pisaro in 1600, and contained the first principles of the method afterwards discovered by Dr. Brook Taylor.

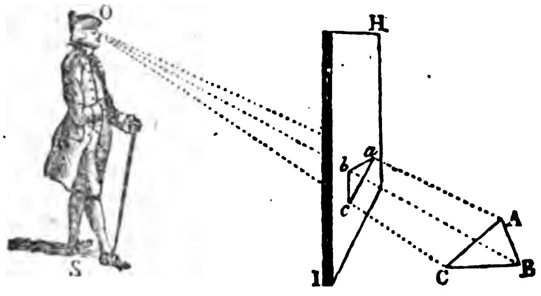
In 1583 was published the work of Giacomo Barozzi, of Vignola, entitled, The two Rules of Perspective, with a learned commentary by Ignatius Danti. In 1615 Marolois' work was printed at the Hague, and engraved and published by Hondius. And in 1625, Sirigatti published his treatise of perspective, which is little more than an abstract of Vignola's.

Since that time the art of perspective has been gradually improved by subsequent geometricians, particularly by professor Gravesande, and still more by Dr. Brook Taylor, whose principles are in a great measure new, and far more general than those of any of his predecessors. He did not confine his rules, as they had done, to the horizontal plane only, but made them general, so as to affect every species of lines and planes, whether they were parallel to the horizon or not; and thus his principles were made universal. Besides, from the simplicity of his rules, the tedious progress of drawing out plans and elevations for any object, is rendered useless, and therefore avoided; for by this method, not only the fewest lines possible are required to produce any perspective representation, but every figure thus drawn will bear the nicest mathematical examination. Further, his system is the only one calculated for answering every purpose of those who are practitioners in the art of design; for by it they may produce either the whole, or only so much of an object as is wanted; and by fixing it in its proper place, its apparent magnitude may be determined in an instant. It explains also the perspective of shadows, the reflection of objects from polished planes, and the inverse practice of perspective. His Linear Perspective was first published in 1715; and his New Principles of Linear Perspective in 1719, which he intended as an explanation of his first treatise. And his method has been chiefly followed by all others since.

In 1738, Mr. Hamilton published his Stereography, in 2 vols folio, after the manner of Dr. Taylor. But the neatest system of perspective, both as to theory and practice, on the same principles, is that of Mr. Kirby. There are also good treatises on the subject, by Desargues, Debosse, Albertus, Lamy, Niceron, Pozzo the Jesuit, Ware, Cowley, Priestley, Ferguson, Emerson, Malton, Henry Clarke, &c, &c.

Of the Principles of PERSPECTIVE. To give an idea of the first principles and nature of this art; suppose a transparent plane, as of glass, &c, HI raised perpendicularly on a horizontal plane; and the spectator S directing his eye O to the triangle ABC: if now we conceive the rays AO, BO, CO, &c, in their passage through the plane, to leave their traces or vestiges in *a*, *b*, *c*, &c, on the plane; there will appear the triangle *abc*; which, as it strikes the eye by the same rays *ao*, *bo*, *co*, by which the reflected particles of light from the triangle are transmitted to the same, it will exhibit the true appearance of the triangle ABC, though the object should be removed, the same distance and height of the eye being preserved.

The business of perspective then, is to show, by what certain rules the points *a*, *b*, *c*, &c, may be found geometrically: and hence also we have a mechanical method of delineating any object very accurately.



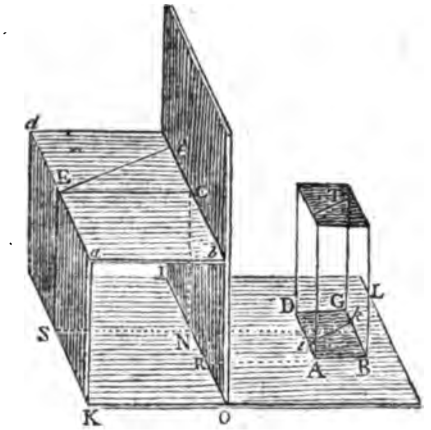
Hence it appears that *abc* is the section of the plane of the picture with the rays, which proceed from the original object to the eye: and therefore, when this is parallel to the picture, its representation will be both parallel to the original, and similar to it, though smaller in proportion as the original object is farther from the picture. When the original object is brought to coincide with the picture, the representation is equal to the original; but as the object is removed farther and farther from the picture, its image becomes smaller and smaller, and also rises higher and higher in the picture, till at last, when the object is supposed to be at an infinite distance, its image vanishes in an imaginary point, exactly as high above the bottom of the picture as the eye is above the ground plane, on which the spectator, the picture, and the original object are supposed to stand.

This may be familiarly illustrated in the following manner: Suppose a person at a window looks through an upright pane of glass at any object beyond; and, keeping his head steady, draws the figure of the object upon the glass, with a black-lead pencil, as if the point of the pencil touched the object itself; he would then have a true representation of the object in perspective; as it appears to his eye. For properly drawing on the glass, it is necessary to lay it over with strong gum water, which will be fit for drawing upon when dry, and will then retain the traces of the pencil. The person should also look through a small hole in a thin plate of metal, fixed about a foot from the glass, between it and his eye; keeping his eye close to the hole, otherwise he might shift the position of his head, and so make a false delineation of the object.

Having traced out the figure of the object, he may go over it again, with pen and ink; and when that is dry, cover it with a sheet of paper, tracing the image upon this with a pencil; then taking away the paper, and laying it upon a table, he may finish the picture, by giving it the colours, lights, and shades, as he sees them in the object itself; and thus he will have a true resemblance of the object on the paper.

Of certain Definitions in PERSPECTIVE.

The point of sight, in perspective, is the point E, where the spectator's eye should be placed to view the picture. And the *point of sight*, in the picture, called also the *centre of the picture*, is the point C directly opposite to the eye, where a perpendicular from the eye at E meets the picture. Also this perpendicular EC is the *distance of the picture*: and if this distance be transferred to the horizontal line on each side of the point C, as is sometimes done, the extremes are called the points of distance.



The *original plane*, or *geometrical plane*, is the plane KL upon which the real or original object ABGD is situated. The line OI, where the ground plane cuts the bottom of the picture, is called the *section* of the original plane, the *ground-line*, the *line of the base*, or the *fundamental line*. If an original line AB be continued, so as to intersect the picture, the point of intersection R is called the intersection of that original line, or its *intersecting point*. The *horizontal plane* is the plane *abgd*, which passes through the eye, parallel to the horizon, and cuts the perspective plane or picture at right angles; and the *horizontal line bg* is the common intersection of the horizontal plane with the picture.

The *vertical plane* is that which passes through the eye at right angles both to the ground plane and to the picture, as ECSN. And the *vertical line* is the common section of the vertical plane and the picture, as CN.

The *line of station* SN is the common section of the vertical plane with the ground plane, and perpendicular to the ground line OI.

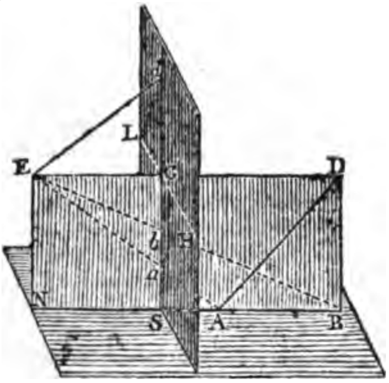
The *line of the height of the eye* is a perpendicular, as ES, let fall from the eye upon the ground plane.

The *vanishing line* of the original plane, is that line where a plane passing through the eye, parallel to the original picture, cuts the picture: thus *bg* is the vanishing line of ABGD, being the greatest height to which the image can rise, when the original object is infinitely distant.

The *vanishing point* of the original line, is that point where a line drawn from the eye, parallel to that original line, intersects the picture: thus C and *g* are the vanishing points of the lines AB and *ki*. All lines parallel to each other have the same vanishing point. If from the point of sight a line be drawn perpendicular to any vanishing line, the point where that line intersects the vanishing line, is called the centre of that vanishing line: and the *distance of a vanishing line* is the length of the line which is drawn from the eye, perpendicular to the said line.

Measuring points are points from which any lines in the perspective plane are measured, by laying a ruler from them to the divisions laid down upon the ground line. The measuring point of all lines parallel to the ground line, is either of the points of distance on the horizontal line, or point of sight. The measuring point of any line perpendicular to the ground line, is in the point of distance on the horizontal line; and the measuring point of a line oblique to the ground line is found by extending the compasses from the vanishing point of that line to the point of distance on the perpendicular, and setting off on the horizontal line.

Some general Maxims or Theorems in PERSPECTIVE.

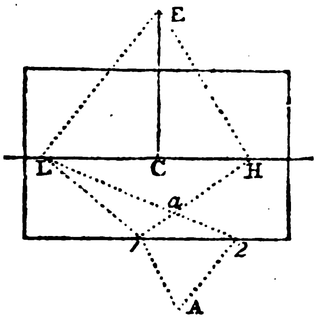


1. The representation *ab*, of a line AB, is part of a line SC, which passes through the intersecting point S, and the vanishing point C, of the original line AB.

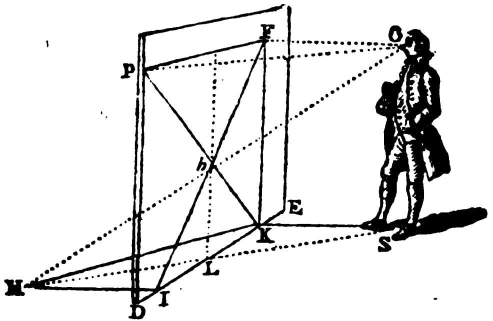
2. If the original plane be parallel to the picture, it can have no vanishing line upon it; and consequently the representation will be parallel. When the original is perpendicular to the ground line, as AB, then its vanishing point is in C, the centre of the picture, or point of sight; because EC is perpendicular to the picture, and therefore parallel to AB.

3. The image of a line bears a certain proportion to its original. And the image may be determined by transferring the length or distance of the given line to the intersecting line; and the distance of the vanishing point to the horizontal line; i. e. by bringing both into the plane of the picture.

PROB. *To find the representation of an Objective point* A.—Draw A1 and A2 at pleasure, intersecting the bottom of the picture in 1 and 2; and from the eye E draw EH parallel to A1, and EL parallel to A2; then draw H1 and L2, which will intersect each other in *a*, the representation of the point A.



Otherwise. Let H be the given objective point,



from which draw HI perpendicular to the fundamental line DE. From the fundamental line DE cut off IK = IH: through the point of sight F draw a horizontal line FP, and make FP equal to the distance of the eye SK: lastly, join FI and PK, and their intersection *h* will be the appearance of the given objective point H, as required. And thus, by finding the representations of the two points, which are the extremes of a line, and connecting them together, there will be formed the representation of the line itself. In like manner, the representations of all the lines or sides of any figure or solid, determine those of the solid itself; which therefore are thus put into perspective.

Aerial PERSPECTIVE, is the art of giving a due diminution or gradation to the strength of light, shade, and colours of objects, according to their different distances, the quantity of light which falls upon them, and the medium through which they are seen.

PERSPECTIVE *Machine*, is a machine for readily and easily making the perspective drawing and appearance of any object, which requires little or no skill in the art. There have been invented various machines of this kind. One of which may even be seen in the works of Albert Durer. A very convenient one was invented by Dr. Bevis, and is described by Mr. Ferguson, in his Perspective, pa. 113. And another is described in Kirby's Perspective, pa. 65.

PERSPECTIVE *Plan*, or *Plane*, is a glass or other transparent surface supposed to be placed between the eye and the object, and usually perpendicular to the horizon.

Scenographic PERSPECTIVE. See SCENOGRAPHY.

PERSPECTIVE *of Shadows*. See SHADOW.

Specular PERSPECTIVE, is that which represents the objects in cylindrical, conical, spherical, or other mirrors.

PERTICA, a kind of comet, the same with VERU.

PETARD, a military engine, somewhat resembling in shape a high-crowned hat; serving formerly to break down gates, barricades, draw-bridges, or the like works intended to be surprised. It is about 8 or 9 inches wide, and weighs from 55 to 70 pounds. Its use was chiefly in a clandestine or private attack, to break down the gates &c. It has also been used in countermines, to break through the enemies' galleries, and give vent to their mines: but the use of petards is now discontinued.—Their invention is ascribed to the French Hugonots in the year 1579. The most signal exploit performed with them was the taking the city Cahors, as we are told by d'Aubigné.

PETIT (PETER), a considerable mathematician and philosopher of France, was born at Montluçon in the diocese of Bourges, in the year 1589 according to some, but in 1600 according to others.—He first cultivated the mathematics and philosophy in the place of his nativity; but in 1633 he repaired to Paris, to which place his reputation had procured him an invitation. Here he became highly celebrated for his ingenious writings, and for his connections with Pascal, Descartes, Mersenue, and the other great men of that time. He was employed on several occasions by cardinal Richelieu; he was commissioned by this minister to visit the sea-ports, with the title of the king's engineer; and was also sent into Italy on the king's business. He was at Tours in 1640, where he married; and was afterwards made intendant of the fortifications. Baillet, in his Life of Descartes, says, that Petit had a great genius for mathematics; that he excelled particularly in astronomy; and had a singular passion for experimental philosophy. About 1637 he returned to Paris from Italy, when the Dioptrics of Descartes were much spoken of. He read them, and communicated his objections to Mersenne, with whom he was intimately acquainted. And yet he soon after embraced the principles of Descartes, becoming not only his friend, but his partisan and defender also. He was intimately connected with Pascal, with whom he made at Rouen the same experiments concerning the vacuum, which Torricelli had before made in Italy; and was assured of their truth by frequent repetitions. This was in 1646 and 1647; and though there appears to be a long interval from this date to the time of his death, we meet with no other memoirs of his life. He died August the 20th 1667, at Lagny, near Paris, whither he had retired for some time before his decease.

Petit was the author of several works on physical and astronomical subjects; the chief of which are,

1. Chronological Discourse, &c, 1636, 4to. In defence of Scaliger.—2. Treatise on the Proportional Compasses.—3. On the Weight and Magnitude of Metals.—4. Construction and Use of the Artillery Calipers.—5. On a vacuum.—6. On Eclipses.—7. On Remedies against the Inundations of the Seine at Paris.—8. On the Junction of the Ocean with the Mediterranean Sea, by means of the rivers Aude and Garonne.—9. On Comets.—10. On the proper day for celebrating Easter.—11. On the Nature of Heat and Cold, &c.

PETTY (Sir WILLIAM), a singular instance of a universal genius, was the elder son of Anthony Petty, a clothier at Rumsey in Hampshire, where he was born May the 16th, 1623. While a boy he took great delight in spending his time among the artificers there, whose trades he could work at when but 12 years of age. He then went to the grammar-school in that place, where at 15 he became master of the Latin, Greek, and French languages, with arithmetic and those parts of practical geometry and astronomy useful in navigation. Soon after, he went to the university of Caen in Normandy; and after some stay there he returned to England, where he was promoted in the king's navy. In 1643, when the civil war began, and the times became troublesome, he went into the Netherlands and France for three years; and having vigorously prosecuted his studies, especially in physic, at Utrecht, Leyden, Amsterdam, and Paris, he returned home to Rumsey. In 1647 he obtained a patent to teach the art of double writing for 17 years. In 1648 he published at London, "Advice to Mr. Samuel Hartlib, for the advancement of some particular parts of learning." At this time he adhered to the prevailing party of the nation; and went to Oxford, where he taught anatomy and chemistry, and was created a doctor of physic, and rose into such repute, that the philosophical meetings, which preceded and laid the foundation of the Royal Society, were first held at his house. In 1650 he was made professor of anatomy there; and soon after a member of the college of physicians in London, as also professor of music at Gresham-college, London. In 1652 he was appointed physician to the army in Ireland; as also to three lord lieutenants successively, Lambert, Fleetwood, and Henry Cromwell. After the rebellion was over in Ireland, he was appointed one of the commissioners for dividing the forfeited lands to the army who suppressed it; where he acquired a great fortune. When Henry Cromwell became lieutenant of that kingdom, in 1655, he appointed Dr. Petty his secretary, and clerk of the council: he likewise procured him to be elected a burgess for Westloo in Cornwall, in Richard Cromwell's parliament, which met in January 1658. But, in March following, Sir Hierom Sankey, member for Woodstock in Oxfordshire, impeached him of high crimes and misdemeanors in the execution of his office. This gave the doctor a great deal of trouble, as he was summoned before the house of commons; and notwithstanding the strenuous endeavours of his friends, in their recommendations of him to secretary Thurloe, and the defence he made before the house, his enemies procured his dismission from his public employments, in 1659. He then retired to Ireland, till the restoration of king Charles the Second; soon after which he came into England, where he was very graciously received by the king, resigned his professorship at Gresham-college, and was appointed one of the commissioners of the Court of Claims. Likewise, April the 11th, 1661, he received the honour of knighthood, and the grant of a new patent, constituting him surveyor-general of Ireland, and was there chosen a member of parliament.

On the incorporating of the Royal Society, he was one of the first members, and of its first council. And though he had left off the practice of physic, his name was continued as an honorary member of the college of physicians in 1663.

About this time he invented his double-bottomed ship, to sail against wind and tide, and afterwards presented a model of the vessel to the Royal Society; to whom also, in 1665, he communicated "A Discourse about the Building of Ships," containing some curious secrets in that art. But, upon trial, finding his ship failed in some respects, he at length gave up that project.

In 1666 sir William drew up a treatise, called Verbum

Sapienti, containing an account of the wealth and expenses of England, and the method of raising taxes in the most equal manner.—The same year, 1666, he suffered a considerable loss by the fire of London.—The year following he married Elizabeth, daughter of sir Hardresse Waller; and afterwards set up iron-works and pilchard-fishing, opened lead mines and a timber trade in Kerry, which turned to very good account. But all these concerns did not hinder him from the pursuit of both political and philosophical speculations, which he thought of public utility, publishing them either separately or by communication to the Royal Society, particularly on finances, taxes, political arithmetic, land carriage, guns, pumps, &c.

At the first meeting of the Philosophical Society at Dublin, on the plan of that at London, every thing was submitted to his direction: and when it was formed into a regular society, he was chosen president in Nov. 1684. On this occasion he drew up a "Catalogue of mean, vulgar, cheap, and simple Experiments," proper for the infant state of the society, and presented it to them; as he did also his Supellex Philosophica, consisting of 45 instruments requisite to carry on the design of their institution. In 1685 he made his will; in which he declares, that being then about 60, his views were fixed upon improving his lands in Ireland, and to promote the trade of iron, lead, marble, fish, and timber, which his estate was capable of. And as for studies and experiments, "I think now," says he, "to confine the same to the anatomy of the people, and political arithmetic; as also the improvement of ships, land-carriages, guns, and pumps, as of most use to mankind, not blaming the study of other men." But a few years after, all his pursuits were stopped by the effects of a gangrene in his foot, occasioned by the swelling of the gout, which put a period to his life, at his house, in Piccadilly, Westminster, Dec. 16, 1687, in the 65th year of his age. His corpse was carried to Rumsey, and there interred, near those of his parents.

Sir William Petty died possessed of a very large fortune, as appears by his will; where he makes his real estate about 6,500*l.* per annum, his personal estate about 45,000*l.* his bad and desperate debts 30,000*l.* and the demonstrable improvements of his Irish estate, 4000*l.* per annum; in all, at 6 per cent. interest, 15,000*l.* per annum. This estate came to his family, which consisted of his widow and three children, Charles, Henry, and Anne: of whom Charles was created baron of Shelbourne, in the county of Waterford in Ireland, by king William the Third; but dying without issue, was succeeded by his younger brother Henry, who was created viscount Dunkeron, in the county of Kerry, and earl of Shelbourne Feb. 11, 1718. He married the lady Arabella Boyle, sister of Charles earl of Cork, who brought him several children. He was member of parliament for Great Marlow in Buckinghamshire, and a fellow of the Royal Society: he died April 17, 1751. Anne was married to Thomas Fitzmorris, baron of Kerry and Lixnaw, and died in Ireland in 1737.

The variety of pursuits, in which Sir William Petty was engaged, shows him to have had a genius capable of any thing to which he chose to apply it: and it is very extraordinary, that a man of so active and busy a spirit could find time to write so many things, as it appears he did, by the following catalogue.

1. Advice to Mr. S. Hartlib &c; 1648, 4to.—2. A Brief of Proceedings between sir Hierom Sankey and the author &c; 1659, folio.—3. Reflections upon some persons and things in Ireland, &c; 1660, 8vo.—4. A Treatise of Taxes and Contribution, &c; 1662, 1667, 1685, 4to, all without the author's name. This last was re-published in 1690, with two other anonymous pieces, "The Privileges and Practice of Parliaments," and "The Politician Discovered;" with a new title-page, where it is said they were all written by sir William, which, as to the first, is a mistake.—5. Apparatus to the History of the Common Practice of Dyeing;" printed in Sprat's History of the Royal Society, 1667, 4to.—6. A Discourse concerning the Use of Duplicate Proportion, together with a New Hypothesis of Springing or Elastic Motions; 1674, 12mo.—7. Colloquium Davidis cum Anima sua, &c; 1679, folio.—8. The Politician Discovered, &c; 1681, 4to.—9. An Essay in Political Arithmetic; 1682, 8vo.—10. Observations upon the Dublin Bills of Mortality in 1681, &c; 1683, 8vo.—11. An Account of some Experiments relating to Land-carriage, Philos. Trans. No. 161.—12. Some Queries for examining Mineral Waters, ibid. No. 166.—13. A Catalogue of Mean, Vulgar, Cheap, and Simple Experiments, &c; ibid. No. 167.—14. Maps of Ireland, being an Actual Survey of the whole Kingdom, &c; 1685, folio.—15. An Essay concerning the Multiplication of Mankind; 1686, 8vo.—16. A further Assertion concerning the magnitude of London, vindicating it, &c; Philos. Trans. No. 185.—17. Two Essays in Political Arithmetic; 1687, 8vo.—18. Five Essays in Political Arithmetic; 1687, 8vo.—19. Observations upon London and Rome; 1687, 8vo.

His posthumous pieces are, (1) Political Arithmetic; 1690, 8vo, and 1755, with his life prefixed.—(2) The Political Anatomy of Ireland, with Verbum Sapienti, 1691, 1719.—(3) A Treatise of Naval Philosophy; 1691, 12mo.—(4) What a complete Treatise of Navigation should contain; Philos. Trans. No. 198.—(5) A Discourse of making Cloth with Sheep's Wool; in Birch's Hist. of the Roy. Soc.—(6) Supellex Philosophica; ibid.

PHANTASMAGORIA, a new optical instrument, which has within a few years afforded much entertainment by exhibiting, in theatres and other places of amusement, the representation of spectres and other figures on a transparent screen placed between the instrument and the spectators, and no light being suffered to appear, but that in which the images are enveloped, which renders the effect very singular; and this is still farther strengthened by the operator increasing or diminishing the size of the shadows at pleasure, by which the spectators, under the influence of an optical illusion, fancy that the figures are approaching or receding from them.

The first exhibition of this kind, (at least of late years,) was made by one Philidor at Vienna in 1790, and which was afterwards repeated by him at Paris in 1792 with very great success. And a similar spectacle was opened in that metropolis by M. Robertson in 1798; since which time they have become very common in all the countries of Europe. It seems however that something of a similar kind was exhibited so far back as the 17th century, being mentioned by Patin, in his "Relations Historiques," published at Amsterdam in 1695, though the instrument itself is not there described.

The phantasmagoria does not differ much in its construction from the magic lantern; indeed, it is now so constructed that it answers either purpose, the principal difference being, that in the phantasmagoria, the glass sliders on which the figures are painted, are rendered perfectly opake, except in the figures themselves, by which means all light is excluded except that in which the images are involved, and also the spectators are placed on the contrary side of the screen, which is made of some transparent thin substance, as muslin, or such like, that the figures may be seen through it: and the instrument is fixed on rollers or wheels, by which the operator can move it nearer to or farther from the screen, and thus give to the figures any size at pleasure: there are also other contrivances for giving the figures or any parts of them motion, as the arms, legs, eyes, &c, which have a very singular effect.

The greatest imperfection of this instrument is, that as the figures become smaller, which gives them the appearance of being at a greater distance, they become brighter, which is contrary to the natural order of things, as distance always decreases both the apparent magnitude and distinctness of objects. This defect however may be considerably lessened by the following construction, which is suggested by Dr. Young in his Lectures on Natural Philosophy.

The light of the lamp A (fig. 1. plate 29) is thrown by the mirror B, and the lenses C and D, on the painted slider at E, and the magnifier F forms the image on the screen at G. This lens is fixed to a slider, which may be drawn out of the principal support, or box H: and when the box is drawn back on its wheels, the rod IK lowers the point K, and by means of the rod KL adjusts the slider in such a manner, that the image is always distinctly painted on the screen G. When the box advances towards the screen, in order that the images may be diminished and appear to vanish, the support of the lens F suffers the screen M to fall and intercept a part of the light; thus taking off from the natural brightness of the object. The rod KN must be equal to KI, and the point I must be twice the focal length of the lens F, before the object, L being immediately under the focus of the lens. The screen M may have a triangular opening, so as to uncover the middle of the lens only, or the light may be intercepted in any other manner. See Dr. Young's Lectures of Natural Philosophy.

PHARON, the name of a game of chance. See Demoivre's Doctrine of Chances, pa. 77 and 105.

PHASES, in Astronomy, the various appearances, or quantities of illumination of the moon, Venus, Mercury, and the other planets, by the sun. These phases are very observable in the moon with the naked eye; by which she sometimes increases, sometimes wanes, is now bent into horns, and again appears a half-circle; at other times she is gibbous, and again a full circular face. And by help of the telescope, the like variety of phases is observed in Venus, Mars, &c. Copernicus, a little before the use of telescopes, foretold, that after-ages would find that Venus underwent all the changes of the moon; which prophecy was first fulfilled by Galileo, who, directing his telescope to Venus, observed her phases to resemble those of the moon; being sometimes full, sometimes horned, and sometimes gibbous.

PHASES *of an Eclipse.* To determine these for any time: Find the moon's place in her visible way for that moment; and from that point as a centre, with the interval of the moon's semidiameter, describe a circle: In like manner find the sun's place in the ecliptic, from which, with the semidiameter of the sun, describe another circle: the intersection of the two circles shows the phases of the eclipse, the quantity of obscuration, and the position of the cusps or horns.

PHENOMENON, or PHÆNOMENON, an appearance in physics, an extraordinary appearance in the heavens, or on earth; either discovered by observation of the celestial bodies, or by physical experiments, the cause of which is not obvious. Such are meteors, comets, uncommon appearance of stars and planets, earthquakes, &c. Such also are the effects of the magnet, phosphorus, &c.

PHILOLAUS, of Crotona, was a celebrated philosopher among the ancients. He was of the school of Pythagoras, to whom that philosopher's Golden Verses have been ascribed. He made the heavens his chief object of contemplation; and has been said to be the author of that true system of the world which Copernicus afterwards revived; but erroneously, because there is undoubted evidence that Pythagoras learned that system in Egypt. On that erroneous supposition however it was, that Bulliald placed the name of Philolaus at the head of two works, written to illustrate and confirm that system.

"He was (says Dr. Enfield, in his History of Philosophy) a disciple of Archytas, and flourished in the time of Plato. It was from him that Plato purchased the written records of the Pythagorean system, contrary to an express oath taken by the society of Pythagoreans, pledging themselves to keep secret the mysteries of their sect. It is probable that among these books were the writings of Timæus, on which Plato formed the dialogue which bore his name. Plutarch relates, that Philolaus was one of the persons who escaped from the house which was burned by Cylon, during the life of Pythagoras; but this account cannot be correct. Philolaus was contemporary with Plato, and therefore certainly not with Pythagoras. Interfering in affairs of state, he fell a sacrifice to political jealousy.

"Philolaus treated the doctrine of nature with great subtlety, but at the same time with great obscurity; referring every thing that exists to mathematical principles. He taught, that reason, improved by mathematical learning, is alone capable of judging concerning the nature of things: that the whole world consists of infinite and finite; that number subsists by itself, and is the chain by which its power sustains the eternal frame of things; that the Monad is not the sole principle of things, but that the Binary is necessary to furnish materials from which all subsequent numbers may be produced; that the world is one whole, which has a fiery centre, about which the ten celestial spheres revolve, heaven, the sun, the planets, the earth, and the moon; that the sun has a vitreous surface, whence the fire diffused through the world is reflected, rendering the mirror from which it is reflected visible; that all things are preserved in harmony by the law of necessity; and that the world is liable to destruction both by fire and by water. From this summary of the doctrine of Philolaus it appears probable that, following Timæus, whose writings he possessed, he so far departed from the Pythagorean system as to conceive two independent principles in nature, God and matter, and that it was from the same source that Plato derived his doctrine upon this subject."

PHILOSOPHER, a person well versed in philosophy; or who makes a profession of, or applies himself to, the study of nature or of morality.

PHILOSOPHER'S *Stone*, a long-sought-for preparation, which was to transmute or exalt impure metals, such as tin, lead, copper, &c, into gold. There are three methods by which the alchemists have attempted to arrive at the art of making gold; the first by separation, the second by maturation, and the third by transmutation, or turning all metals readily into pure gold, by melting them in the fire, and casting a little quantity of a certain preparation into the fused matter, upon which the fæces are volatilized and burnt, and the rest of the mass turned into pure gold. Many thousands of receipts have been given for conducting the experiments in this art, and many persons have ruined their fortunes in the pursuit of it; but repeated failures have at last put an end to this hopeless speculation.

PHILOSOPHICAL TRANSACTIONS, those of the Royal Society. See TRANSACTIONS.

PHILOSOPHIZING, the act of considering some object of our knowledge, examining its properties, with the phenomena it exhibits, and inquiring into their causes or effects, and the laws of them; the whole conducted according to the nature and reason of things, and directed to the improvement of knowledge.

The Rules of PHILOSOPHIZING, as established by sir Isaac Newton, are, 1. That no more causes of a natural effect be admitted than are true, and suffice to account for its phenomena. This agrees with the sentiments of most philosophers, who hold that nature does nothing in vain; and that it were vain to do that by many means, which might be done by fewer.

2. That natural effects of the same kind, proceed from the same causes. Thus, for instance, the cause of respiration is one and the same in man and brute; the cause of the descent of a stone, the same in Europe as in America; the cause of light, the same in the sun and in culinary fire; and the cause of reflection, the same in the planets as the earth.

3. Those qualities of bodies which are not capable of being heightened, and remitted, and which are found in all bodies on which experiments can be made, must be considered as universal qualities of all bodies. Thus, the extension of body is only perceived by our senses, nor is it perceivable in all bodies: but since it is found in all that we have perception of, it may be affirmed of all. So we find that several bodies are hard; and argue that the hardness of the whole only arises from the hardness of the parts: whence we infer that the particles, not only of those bodies which are sensible, but of all others, are likewise hard. Lastly, if all the bodies about the earth gravitate towards the earth, and this according to the quantity of matter in each; and if the moon gravitate towards the earth also, according to its quantity of matter; and the sea again gravitate towards the moon; and all the planets and comets gravitate towards each other: it may be affirmed universally, that all bodies in the creation gravitate towards each other. This rule is the foundation of all natural philosophy.

PHILOSOPHY, the knowledge or study of nature or morality, founded on reason and experience. Literally and originally, the word signified a love of wisdom. But by philosophy is now meant the knowledge of the nature and reasons of things; as distinguished from history, which is the bare knowledge of facts; and from mathematics, which is the knowledge of the quantity and measures of things. These three kinds of knowledge ought to be joined as much as possible. History furnishes matter, principles, and practical examinations; and mathematics completes the evidence.

Philosophy being the knowledge of the reasons of things, all arts must have their peculiar philosophy which constitutes their theory: not only law and physic, but the lowest and most abject arts are not without their reasons. It is to be observed that the bare intelligence and memory of philosophical propositions, without any ability to demonstrate them, is not philosophy, but history only. However, where such propositions are determinate and true, they may be usefully applied in practice, even by those who are ignorant of their demonstrations. Of this we see daily instances in the rules of arithmetic, practical geometry, and navigation; the reasons of which are often not understood by those who practise them with success. And this success in the application produces a conviction of mind, which is a kind of medium between philosophical or scientific knowledge, and that which is historical only.

If we consider the difference there is between natural philosophers, and other men, with regard to their knowledge of phenomena, we shall find it consists not in an exacter knowledge of the efficient cause that produces them, for that can be no other than the will of the Deity; but only in a greater and more enlarged comprehension, by which analogies, harmonies, and agreements are described in the works of nature, and the particular effects explained; that is, reduced to general rules, which rules, grounded on the analogy and uniformness observed in the production of natural effects, are more agreeable, and sought after by the mind; for that they extend our prospect beyond what is present, and near to us, and enable us to make very probable conjectures, concerning things that may have happened at very great distances of time and place, as well as to predict things to come; which sort of endeavour towards omniscience is much affected by the mind. Berkeley, Princip. of Hum. Knowledge, sect. 104, 105.

From the first broachers of new opinions, and the first founders of schools, philosophy is become divided into several sects, some ancient, others modern; such are the Platonists, Peripatetics, Epicureans, Stoics, Pyrrhonians, and Academics; also the Cartesians, Newtonians, &c. See the particular articles for each. Philosophy may be divided into two branches, or it may be considered under two circumstances, theoretical and practical.

Theoretical or *Speculative* PHILOSOPHY, is employed in mere contemplation. Such is physics, which is a bare contemplation of nature, and natural things.

Philosophy may be divided into three parts; intellectual, moral, and physical: the intellectual part comprises logic and metaphysics; the moral part contains the laws of nature and nations, ethics and politics; and lastly the physical part comprehends the doctrine of bodies, animate or inanimate: these, with their various subdivisions, will comprise the whole of philosophy.

Practical PHILOSOPHY, is that which lays down the rules of a virtuous and happy life; and excites us to the practice of them. Most authors divide it into two kinds, answerable to the two sorts of human actions to be directed by it; viz, logic, which governs the operations of the understanding; and ethics, properly so called, which direct those of the will.

For the several particular kinds of philosophy, see the articles, Arabian, Aristotelian, Atomical, Cartesian, Corpuscular, Epicurean, Experimental, Hermetical, Leibnitzian, Mechanical, Moral, Natural, Newtonian, Oriental, Platonic, Scholastic, Socratic, &c, &c.

PHLOGISTON, in Chemistry, a term that seems to be almost banished from our language. It was invented by Stahl, according to whom there is only one substance in nature capable of combustion, this he called phlogiston, and all those bodies which can be inflamed contain more or less of it. Combustion by his theory is merely the separation of this substance. Those bodies which contain some of it are incombustibles. All combustibles are composed of an incombustible body and phlogiston united; and during the combustion the phlogiston flies off, and the incombustible body is left behind. Thus when sulphur is burnt, the substance that remains is sulphuric acid, an incombustible body. Sulphur therefore is said to be composed of sulphuric acid and phlogiston. This theory has long since given place to that established by Lavoisier, and so much improved by Dr. Thomson of Edinburgh. See the article Combustion.

PHŒNIX, a constellation of the southern hemisphere; being one of the new-added asterisms, unknown to the ancients, and is not visible in our northern parts of the globe. There are 13 stars in this constellation.

PHONICS, otherwise called Acoustics, is the doctrine or science of sounds. Phonics may be considered as an art analogous to optics; and may be divided, like that, into direct, refracted, and reflected. These branches, the bishop of Ferns, in allusion to the parts of optics, denominates phonics, diaphonics, and cataphonics. See Acoustics.

PHOSPHORUS, a matter which shines, or even burns spontaneously, and without the application of any sensible fire. Phosphori are either natural or artificial.

Natural Phosphori, are matters which become luminous at certain times, without the assistance of any art or preparation. Such are the glow-worms, frequent in our colder countries; lantern-flies, and other shining insects, in hot countries; rotten-wood; the eyes, blood, scales, flesh, sweat, feathers, &c, of several animals; diamonds, when rubbed after a certain manner, or after having been exposed to the sun or light; sugar and sulphur, when pounded in a dark place; sea-water, and some mineral waters, when briskly agitated; a cat's or horse's back, duly rubbed with the hand, &c, in the dark; nay Dr. Croon tells us, that on rubbing his own body briskly with a well-warmed shirt, he has frequently made both to shine; and Dr. Sloane adds, that he knew a gentleman of Bristol, and his son, both whose stockings would shine much after walking. All natural phosphori have this in common, that they do not shine always, and that they never give any heat. Of all the natural phosphori, that which has occasioned the greatest speculation, is the

Barometrical or *Mercurial* Phosphorus. M. Picard first observed, that the mercury of his barometer, when shaken in a dark place, emitted light. And many fanciful explanations have been given of this phenomenon, which however is now found to be a mere electrical effect. Mr. Hawksbee has several experiments on this appearance. Passing air forcibly through the body of quicksilver, placed in an exhausted receiver, the parts were violently driven against the side of the receiver, and gave all around the appearance of fire; continuing thus till the receiver was half full again of air.

From other experiments he found, that though the appearance of light was not producible by agitating the mercury in the same manner in the common air, yet that a very fine medium, nearly approaching to a vacuum, was not at all necessary. And lastly, from other experiments he found that mercury inclosed in water, which communicated with the open air, by a violent shaking of the vessel in which it was inclosed, emitted particles of light in great plenty, like little stars. By including the vessel of mercury, &c, in a receiver, and exhausting the air, the phenomenon was changed; and on shaking the vessel, instead of sparks of light, the whole mass appeared one continued circle of light.

Further, if mercury be inclosed in a glass tube, close stopped, that tube is found, on being rubbed, to give much more light, than when it had no mercury in it. When this tube has been rubbed, after raising successively its extremities, that the mercury might flow from one end to the other, a light is seen creeping in a serpentine manner all along the tube, the mercury being all luminous. By making the mercury run along the tube afterwards without rubbing it, it emitted some light, though much less than before; this proves that the friction of the mercury against the glass, in running along, does in some measure electrify the glass, as the rubbing it with the hand does, only in a much less degree. This is more plainly proved by laying some very light down near the tube, for this will be attracted by the electricity raised by the running of the mercury, and will rise to that part of the glass along which the mercury runs; from which it is evident, that what has been long known in the world under the name of the phosphorus of the barometer, is not a phosphorus, but merely a light raised by electricity, the mercury electrifying the tube. Philos. Trans. No. 484.

Artificial Phosphori, are such as owe their luminous quality to some art or preparation. Some of these are made by the maceration of plants alone, and without any fire; such as thread, linen cloth, but above all paper: the luminous appearance of this last, which it is now known is an electrical phenomenon, is greatly increased by heat. Almost all bodies, by a proper treatment, have that power of shining in the dark, which, at first was supposed to be the property of one, and afterwards only of a few. See Philos. Trans. No. 478, in vol. 44, pa. 83.

The discovery of phosphorus was made in 1677 by one Brandt, a citizen of Hamburgh, in his researches for the philosopher's stone, and the preparation was long kept a lucrative secret in the hands of a few persons; but as it was generally known to have been prepared from human urine, and as the method then employed, though tedious and disgusting, was extremely simple, it was detected by several chemists, but first by Kunckel and Mr. Boyle, and the real nature of phosphorus has been gradually explained by a vast number of ingenious and elaborate researches. Kunckel having first discovered the method of preparing this substance, it is generally called Kunckel's phosphorus.

The earliest method of preparing phosphorus was in the following manner. A large quantity of human urine was collected, and after remaining for a time to become putrid, it was evaporated to dryness in any suitable vessel. The residue was then mixed with charcoal in powder, and heated gradually to low redness in an iron pot, till the mass began to send forth blue luminous vapours. It was then removed into a coated earthen retort with a receiver, and heat applied gradually till it reached the utmost intensity; during which the phosphorus distilled over, and partly concreted in the neck of the retort, and partly fell in drops into the receiver. This, which was at first black and foul, was purified by melting, and was formed into sticks, which were long sold at a very high price, as a great philosophical curiosity.

This disgusting process is now completely laid aside, and phosphorus is obtained in a much more certain manner, from the white earth left after the calcination of bones; but for the process of which operation we must refer the reader to Murray's and Parkinson's Chemistries, and to the article Phosphorus in Aikin's Chemical Dictionary.

Many curious and amusing experiments are made with phosphorus; as by writing with it, when the letters will appear like flame in the dark, though in the light nothing appears but a dim smoke; also a little bit of it rubbed between two papers, presently takes fire, and burns vehemently; &c. By washing the face, or hands, &c, with liquid phosphorus, they will shine very considerably in the dark, and the lustre will be communicated to adjacent objects, yet, without hurting the skin: and on bringing in the candle, the shining disappears, and no change is perceivable.

Phosphorus, in Astronomy, is the morning star, or the planet Venus, when she rises before the sun. The Latins call it Lucifer, the French Etoile de berger, and the Greeks Phosphorus.

PHOSPHURETS, in Chemistry, are substances formed by an union with phosphorus: thus, we have the phosphuret of carbon, which is a compound of carbon with phosphorus; we have likewise the phosphuret of lime, hydrogen, &c.

PHOSPHURETTED *Hydrogen*, phosphorus dissolved in hydrogen gas; which may be done by introducing phosphorus into a glass jar of hydrogen gas standing over mercury, and then melting it by means of a burning glass; the gas dissolves a large proportion of it. The compound has a very fetid odour, something like that from putrid fish. When it comes into contact with common air, it burns with great rapidity, and if mixed with that air it detonates violently. Oxygen gas produces a still more rapid and brilliant combustion than common air. When bubbles of it are made to pass up through water, they explode in succession as they reach the surface of the liquid; a beautiful column of white smoke is formed. This gas is the most combustible substance known. Its combustion is the combination of its phosphorus and hydrogen with the oxygen of the atmosphere, and the products are phosphoric acid and water. These substances, mixed or combined, constitute the white smoke.

PHYSICAL. Something belonging to nature, or existing in it. Thus, we say a physical point, in opposition to a mathematical one, which last only exists in the imagination. Or a physical substance or body, in opposition to spirit, or metaphysical substance, &c.

Physical, or *Sensible Horizon*. See Horizon.

Physico-*Mathematics*, or *Mixed Mathematics*, includes those branches of physics which, uniting observation and experiment to mathematical calculation, explain mathematically the phenomena of nature.

PHYSICS, called also *Physiology*, and *Natural Philosophy*, is the doctrine of natural bodies, their phenomena, causes, and effects, with their various affections, motions, operations, &c. So that the immediate and proper objects of physics, are body, space, and motion. The origin of physics is referred, by the Greeks, to the Barbarians, viz, the brachmans, the magi, and the Hebrew and Egyptian priests. From these it passed to the Greek sages or sophi, particularly to Thales, who it is said first professed the study of nature in Greece. Hence it descended into the schools of the Pythagoreans, the Platonists, and the Peripatetics; whence it passed into Italy, and thence through the rest of Europe: though the druids, bards, &c, had a kind of system of physics of their own.—Physics may be divided, with regard to the manner in which it has been treated, into the following kinds.

Symbolical Physics, or such as was couched under symbols: such was that of the old Egyptians, Pythagoreans, and Platonists; who delivered the properties of natural bodies under arithmetical and geometrical characters, and hieroglyphics.

Peripatetical Physics, or that of the Aristotelians, who explained the nature of things by matter, form, and privation, elementary and occult qualities, sympathies, antipathies, attractions, &c.

Experimental Physics, which inquires into the reasons and natures of things from experiments: such as those in chemistry, hydrostatics, pneumatics, optics, &c.

Mechanical or *Corpuscular* Physics, which explains the appearances of nature from the matter, motion, structure, and figure of bodies and their parts; all according to the settled laws of nature and mechanics. See each of these articles under its proper head.

PIASTER, a Spanish money, more usually called Piece of Eight, about the value of 4*s.* 6*d.*

PIAZZA, popularly called Piache, an Italian name for a portico, or covered-walk, supported by arches.

PIAZZI, a small new primary planet, discovered Jan. 1, 1800, by the astronomer Piazzi of Palermo. It is also called Ceres; which see.

PICARD (John,) an able mathematician of France, and one of the most learned astronomers of the 17th century, was born at Fleche, and became priest and prior of Rille in Anjou. Coming afterwards to Paris, his talents for mathematics and astronomy soon made him known and respected. In 1666 he was appointed astronomer in the Academy of Sciences. And five years after, he was sent, by order of the king, to the castle of Uraniburg, built by Tycho Brahé in Denmark, to make astronomical observations there, and from thence he brought the original manuscripts, written by Tycho Brahé; which are the more valuable, as they differ in many places from the printed copies, and contain a book more than has yet appeared. These discoveries were followed by many others, particularly in astronomy. He was one of the first, who applied the telescope to astronomical quadrants: he first executed the work called, La Connoissance des Temps, which he calculated from 1679 to 1683 inclusively: he first observed the light in the vacuum of the barometer, or the mercurial phosphorus: he also first of any went through several parts of France, to measure the degrees of the French meridian, and first gave a chart of the country, which the Cassinis afterwards carried to a great degree of perfection. He died in 1682 or 1683, leaving a name dear to his friends, and respectable to his contemporaries and to posterity. His works are,

1. A treatise on Levelling.
2. Practical Dialling by calculation.
3. Fragments of Dioptrics.
4. Experiments on Running Water.
5. Of Measurements.
6. Mensuration of Fluids and Solids.

7. Abridgment of the Measure of the Earth.

8. Journey to Uraniburg, or Astronomical Observations made in Denmark.

9. Astronomical Observations made in France.

10. La Connoissance des Temps, from 1679 to 1683.

All these, and some other of his works, which are much esteemed, are given in the 6th and 7th volumes of the Memoirs of the Academy of Sciences.

PICKET, *Picquet*, or *Piquet*, in Fortification &c, a stake sharp at one end, and usually shod with iron, used in laying out ground, to mark its several bounds and angles. There are also larger pickets, driven into the earth, to hold together fascines or faggots, in works that are thrown up in haste. As also various kinds of smaller pickets for divers other uses.

PIECES, in Artillery, include all kinds of great guns and mortars, ; meaning pieces of ordnance, or of artillery.

PIEDOUCHE, in Architecture, a little stand, or pedestal, either oblong or square, enriched with mouldings; serving to support a bust, or other little figure; and is more usually called a bracket pedestal.

PIEDROIT, in Architecture, a kind of square pillar, or pier, partly hid within a wall. Differing from the pilaster by having no regular base nor capital.

PIEDROIT is also used for a part of the solid wall annexed to a door or window; comprehending the door-post, chambranle, tableau, leaf, &c.

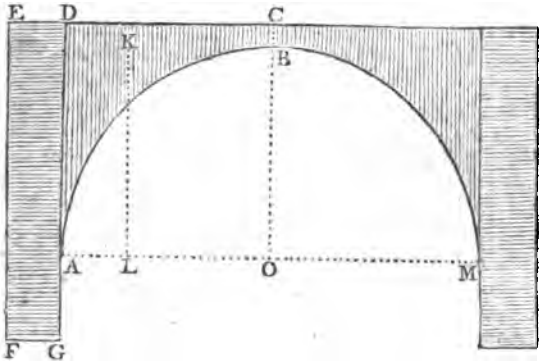
PIER, in Building, denotes a mass of stone, &c, opposed by way of fortress, against the force of the sea, or a great river, for the security of ships lying in any harbour or haven. Such are the piers at Dover, or Ramsgate, or Yarmouth, &c.

PIERS are also used in Architecture for a kind of pilasters, or buttresses, raised for support, strength, and sometimes for ornament.

Circular PIERS, are called Massive Columns, and are either with or without caps. These are often seen in Saracenic architecture.

PIERS, of a Bridge, are the walls built to support the arches, and from which they spring as bases, to stand upon. Piers should be built of large blocks of stone, solid throughout, and cramped together with iron, which will make the whole as one solid stone. Their extremities, or ends, from the bottom, or base, up to high-water mark, ought to project sharp out with a saliant angle, to divide the stream. Or perhaps the bottom part of the pier should be built flat or square up to about half the height of low-water mark, to encourage a lodgment against it for the sand and mud, to cover the foundation; lest, being left bare, the water should in time undermine and ruin it. The best form of the projection for dividing the stream, is the triangle; and the longer it is, or the more acute the saliant angle, the better it will divide it, and the less will the force of the water be against the pier; but it may be sufficient to make that angle a right one, as it will render the masonry stronger, and in that case the perpendicular projection will be equal to half the breadth or thickness of the pier. In rivers where large heavy craft navigate, and pass the arches, it may perhaps be better to make the ends semicircular; for though this figure does not divide the water so well as the triangle, it will better turn off, and bear the shock of the craft.

The thickness of the piers ought to be such as will make them of weight, or strength, sufficient to support their interjacent arch, independent of the assistance of any other arches. And then, if the middle of the pier be run up to its full height, the centring may be struck, to be used in another arch, before the hanches or spandrels are filled up. They ought also to be made with a broad bottom on the foundation, and gradually diminished in thickness by offsets up to low-water mark.



To find the thickness FG *of the Piers*, necessary to support an arch ABM, this is a general rule. Let K be the centre of gravity of the half arch ADCB, A = its area; KL perpendicular to AM the span of the arch, OB its height, and BC its thickness at the crown: then is the thickness of the pier $FG = \sqrt{(\frac{GA \times AL}{EF \times KL} \times 2A)}$.

The investigations of this rule, and other methods for this purpose, may be seen in my Tracts, vol. 1, pa. 72, &c.

PIKE, an offensive weapon, consisting of a shaft of wood, 12 or 14 feet long, headed with a flat-pointed steel, called the spear. Pliny says the Lacedemonians were the inventors of the pike. The Macedonian phalanx was evidently a battalion of pikemen. The pike was long used by the infantry, to enable them to sustain the attack of the cavalry; but it is now taken from them, and the bayonet, fixed to the muzzle of the firelock, is given instead of it. It is still used by some officers of infantry, under the name of spontoon.

Half PIKE is the weapon carried by an officer of foot; being only 8 or 9 feet long.

PILASTER, in Architecture, a square column, sometimes insulated, but more frequently let within a wall, and only projecting by a 4th or 5th part of its thickness. The pilaster is different in the different orders; borrowing the name of each order, and having the same proportions, and the same capitals, members, and ornaments, with the columns themselves.

Demi PILASTER, called also *Membretto*, is a pilaster that supports an arch; and it generally stands against a pier or column.

PILES, in Building, are large stakes, or beams, sharpened at the end, and shod with iron, to be driven into the ground, for a foundation to build upon in marshy places. Amsterdam, and some other cities, are wholly built upon piles. The stoppage of Dagenham-breach was effected by dove-tail piles, that is by piles mortised into one another by a dovetail joint. Piles are driven down by blows of a large iron weight, ram, or hammer, dropped continually upon them from a height, till the pile is sunk deep enough into the ground.

Notwithstanding the momentum, or force of a body in motion, is as the weight multiplied by the velocity, or simply as its velocity, the weight being given, or constant; yet the effect of the blow will be nearly as the square of that velocity, the effect being the quantity the pile sinks in the ground by the stroke. For the force of the blow, which is transferred to the pile, being destroyed, in some certain definite time, by the friction of the part which is within the earth, and which is nearly a constant quantity; and the spaces, in constant forces, being as the squares of the velocities; therefore the effects, which are those spaces sunk, are nearly as the square of the velocities; or, which is the same thing, nearly as the heights fallen by the ram or hammer, to the head of the pile. See, upon this subject, Leopold Belidor, also Desaguliers's Exper. Philos. vol. 1, pa. 336, and vol. 2, pa. 417: and Philos. Trans. 1779, pa. 120: also my Tracts, vol. 3, prob. 2, pa. 317.

There have been various contrivances for raising and dropping the hammer, for driving down the piles; some simple and moved by strength of men, and some complex and by machinery; but the completest pile-driver is esteemed that which was employed in driving the piles in the foundation of Westminster bridge. This machine was the invention of Mr. Vauloue, and the description of it is as follows.

Description of Vauloue's PILE-*Driver*. See fig. 2, pl. 25. A is the great upright shaft or axle, carrying the great wheel B and drum C, and turned by horses attached to the bars S, S. The wheel B turns the trundle X, having a fly O at the top, to regulate the motion, and to act against the horses, and keep them from falling when the heavy ram Q is disengaged to drive the pile P down into the mud &c, in the bottom of the river. The drum C is loose upon the shaft A, but is locked to the wheel B by the bolt Y. On this drum the great rope HH is wound; one end of it being fixed to the drum, and the other to the follower G, passing over the pulleys I and K. In the follower G are contained the tongs F, which take hold of the ram Q, by the staple R for drawing it up. D is a spiral or fusee fixed to the drum, on which winds the small rope T, which goes over the pulley U, under the pulley V, and is fastened to the top of the frame at 7. To the pulley-block V is hung the counterpoise W, which hinders the follower from accelerating as it goes down to take hold of the ram: for, as the follower tends to acquire velocity in its descent, the line T winds downwards upon the fusee, on a larger and larger radius, by which means the counterpoise W acts stronger and stronger against it; and so allows it to come down with only a moderate and uniform velocity. The bolt Y locks the drum to the great wheel, being pushed upward by the small lever 2, which goes through a mortise in the shaft A, turns upon a pin in the bar 3 fixed into the great wheel B, and has a weight 4, which always tends to push up the bolt Y through the wheel into the drum. L is the great lever turning on the axis *m*, and resting upon the forcing bar 5, 5, which goes down through a hollow in the shaft A, and bears upon the little lever 2.

By the horses going round, the great rope H is wound about the drum C, and the ram Q is drawn up by the tongs F in the follower G, till they come between the inclined planes E; which, by shutting the tongs at the top, open them below, and so discharge the ram, which falls down between the guide posts *bb* upon the pile P, and drives it by a few strokes as far into the ground as it can go, or as is desired; after which, the top part is sawed off close to the mud, by an engine for that purpose. Immediately after the ram is discharged, the piece 6 upon the follower G takes hold of the ropes *aa*, which raise the end of the lever L, and cause its end N to descend and press down the forcing bar 5 upon the little lever 2, which, by drawing down the bolt Y, unlocks the drum C from the great wheel B; and then the follower, being at liberty, comes down by its own weight to the ram; and the lower ends of the tongs slip over the staple R, and the weight of their heads causes them to fall outward, and shut upon it. Then the weight 4 pushes up the bolt Y into the drum, which locks it to the great wheel, and so the ram is drawn up as before.

As the follower comes down, it causes the drum to turn backward, and unwinds the rope from it, while the horses, the great wheel, trundle, and fly, go on with an uninterrupted motion: and as the drum is turning backward, the counterpoise W is drawn up, and its rope T wound upon the spiral fusee D. There are several holes in the under side of the drum, and the bolt Y always takes the first one that it finds when the drum stops by the falling of the follower upon the ram; till which stoppage, the bolt has not time to slip into any of the holes.

The peculiar advantages of this engine are, that the weight, called the ram, or hammer, may be raised with the least force; that, when it is raised to a proper height, it readily disengages itself and falls with the utmost freedom; that the forceps or tongs are lowered down speedily, and instantly of themselves again lay hold of the ram, and lift it up; on which account this machine will drive the greatest number of piles in the least time, and with the fewest labourers.

This engine was placed upon a barge on the water, and so was easily conveyed to any place desired. The ram was a ton weight; and the guides *b*, *b*, by which it was let fall, were 30 feet high.

A new machine for driving piles has been invented lately by Mr. S. Bunce of Kirby-street, Hatton-street, London. This, it is said, will drive a greater number of piles in a given time than any other; and that it can be constructed more simply to work by horses than Vauloue's engine above described.

Fig. 3 and 4, plate 25, represent a side and front section of the machine. The chief parts are, A, fig. 3, which are two endless ropes or chains, connected by cross pieces of iron B (fig. 4) corresponding with two cross grooves cut diametrically opposite in the wheel C (fig. 3) into which they are received; and by which means the rope or chain A is carried round. FHK is a side-view of a strong wooden frame moveable on the axis H. D is a wheel, over which the chain passes and turns within at the top of the frame. It moves occasionally from F to G upon the centre H, and is kept in the position F by the weight I fixed to the end K. In fig. 5, L is the iron ram, which is connected with the cross pieces by the hook M. N is a cylindrical piece of wood suspended at the hook at O, which by sliding freely upon the bar that connects the hook to the ram, always brings the hook upright upon the chain when at the bottom of the machine, in the position of OP. See fig. 3.

When the man at S turns the usual crane-work, the ram being connected to the chain, and passing between the guides, it is drawn up in a perpendicular direction; and when it is near the top of the machine, the projecting bar Q of the hook strikes against a cross piece of wood at R (fig. 3); and consequently discharges the ram, while the weight I of the moveable frame instantly draws the upper wheel into the position shown at F, and keeps the chain free of the ram in its descent. The hook, while descending, is prevented from catching the chain by the wooden piece N: for that piece being specifically lighter than the iron weight below, and moving with a less degree of velocity, cannot come into contact with the iron, till it is at the bottom, and the ram stops. It then falls, and again connects the hook with the chain, which draws up the ram, as before.

Mr. Bunce has made a model of this machine, which performs perfectly well: and he observes, that, as the motion of the wheel C is uninterrupted, there appears to be the least possible time lost in the operation.

Pile is also used among Architects, for a mass or body of building.

Pile, in Artillery, denotes a collection or heap of shot or shells, piled up by horizontal courses into either a pyramidal or else a wedge-like form; the base being an equilateral triangle, a square, or a rectangle. In the triangle and square, the pile terminates in a single ball or point, and forms a pyramid, as in plate 24, fig. 4 and 5, but with the rectangular base, it finishes at top in a row of balls, or an edge, forming a wedge, as in fig. 6.

In the triangular and square piles, the number of horizontal rows, or courses, or the number counted on one of the angles from the bottom to the top, is always equal to the number counted on one side, in the bottom row. And in rectangular piles, the number of rows, or courses, is equal to the number of balls in the breadth of the bottom row, or shorter side of the base: also, in this case, the number in the top row, or edge, is one more than the difference between the length and breadth of the base. All which is evident from the inspection of the figures, as above.

The courses in these piles are figurate numbers.

In a triangular pile, each horizontal course is a triangular number, produced by taking the successive sums of the ordinate numbers, viz,

$$\begin{aligned}1 &= 1\\ 1+2 &= 3\\ 1+2+3 &= 6\\ 1+2+3+4 &= 10, \text{\&c.}\end{aligned}$$

And the number of shot in the triangular pile, is the sum of all these triangular numbers, taken as far, or to as many terms, as the number in one side of the base. Therefore, to find this sum, or the number of all the shot in the pile, multiply continually together, the number in one side of the base row, and that number increased by 1, and the same number increased by 2; then $\frac{1}{6}$ of the last product will be the answer, or number of all the shot in the pile. That is, $\frac{1}{6}n \,.\, n+1 \,.\, n+2$ is the sum; where n is the number in the bottom row.

Again, in square piles, each horizontal course is a square number, produced by taking the square of the number in its side, or the successive sums of the odd numbers, thus,

$$\begin{aligned}1 &= 1\\ 1+3 &= 4\\ 1+3+5 &= 9\\ 1+3+5+7 &= 16, \text{\&c.}\end{aligned}$$

And the number of shot in the square pile is the sum of all these square numbers, continued so far, or to as many terms, as the number in one side of the base. Therefore, to find this sum, multiply continually together, the number in one side of the bottom course, and that number increased by 1; and double the same number increased by 1; then $\frac{1}{6}$ of the last product will be the sum or answer. That is, $\frac{1}{6}n \,.\, n+1 \,.\, 2n+1$ is the sum.

In a rectangular pile, each horizontal course is a rectangle, whose two sides have always the same difference as those of the base course, and the breadth of the top row, or edge, being only 1: because each course in ascending has its length and breadth always less by 1 than the course next below it. And these rectangular courses are found by multiplying successively the terms or breadths 1, 2, 3, 4, &c, by the same terms added to the constant difference of the two sides d; thus,

$$\begin{aligned}1 \,.\, 1+d &= 1+d\\ 2 \,.\, 2+d &= 4+2d\\ 3 \,.\, 3+d &= 9+3d\\ 4 \,.\, 4+d &= 16+4d, \text{\&c.}\end{aligned}$$

And the number of shot in the rectangular pile is the sum of all these rectangles, which evidently consist of the sum of the squares, together with the sum of an arithmetical progression, continued till the number of terms be the difference between the length and breadth of the base, and 1 less than the edge or top row. Therefore, to find this sum, multiply continually together, the number in the breadth of the base row, the same number increased by 1, and double the same number increased by 1, and also increased by triple the difference between the length and breadth of the base; then $\frac{1}{6}$ of the last product will be the answer. That is, $\frac{1}{6}b \,.\, b+1 \,.\, 2b+3d+1$ is the sum: where b is the breadth of the base, and d the difference between the length and breadth of the bottom course.

As to incomplete piles, which are only frustums, as wanting a similar small pile at the top; it is evident that the number in them will be found, by first computing the number in the whole pile, as if it were complete, and also the number in the small pile wanting at top, both by their proper rule; then subtracting the one number from the other.

In piling of shot, when room is an object, it may be observed that the square pile is the least eligible of any, as it takes up more room, in proportion to the number of shot contained in it, than either of the other two forms; and that the rectangular pile is the most eligible, as taking up the least room in proportion to the number it contains.

PILLAR, a kind of irregular column, round, and insulated, or detached from the wall. Pillars are not restricted to any rules, their parts and proportions being arbitrary; such for example as those that support Saracenic vaults, and other buildings, &c.

PINGRE' (Alexander Guy), a French astronomer, was born at Paris in 1711; and died in 1796, at 85 years of age. He applied with great assiduity to scientific pursuits, and became librarian of St. Geneviève at Paris. In 1760 he was sent to the South sea, to observe the approaching transit of Venus over the sun's disk. He was afterwards employed in proving the going of the timepieces of M. Leroy. He was first admitted a member of the Academy of Sciences; and afterwards of the National Institute. M. Pingré's works chiefly are; 1. State of the Heavens from 1754 to 1757. 2. Memoirs of Discoveries made in the South sea, 4to. 3. Historical and Theoretical Treatise on Comets, 2 vols. 4to. 4. Translation of Manilius's Astronomics, 8vo. 5 History of Astronomy in the 17th century.

PINION, in Mechanics, is an arbor, or spindle, in the body of which are several notches, which are catched by the teeth of a wheel that serves to turn it round. Or a pinion is any lesser wheel that plays in the teeth of a larger.

In a watch, &c, the notches of a pinion are called leaves, and not teeth, as in other wheels; and their number is commonly 4, 5, 6, 8, &c.

Pinion *of Report*, is that pinion, in a watch, commonly fixed on the arbor of a great wheel; and which used to have but four leaves in old watches: it drives the dial-wheel, and carries about the hand. The number of turns to be laid upon the pinion of report, is found by this proportion: as the beats in one turn of the great wheel, are to the beats in an hour, so are the hours on the face of the clock (viz 12 or 24), to the quotient of the hour-wheel or dial-wheel divided by the pinion of report, that is, by the number of turns which the pinion of report makes in one turn of the dial-wheel: which in numbers is 26928 : 20196 : : 12 : 9.—Or thus; as the hours of the watch's going, are to the numbers of the turns of the fusee, so are the hours of the face, to the quotient of the pinion of report. So, if the hours be 12, then as 16 : 12 : : 12 : 9; but if 24, then as 16 : 12 : : 24 : 18.

This rule may serve to lay the pinion of report on any other wheel, thus: as the beats in one turn of any wheel, are to the beats in an hour, so are the hours of the face, or dial-plate, of the watch, to the quotient of the dial-wheel divided by the pinion of report, fixed on the spindle of the aforesaid wheel.

PINT, a measure of capacity, being the 8th part of a gallon, both in ale and wine measure, &c. The wine pint contains 29 cubic inches; and the ale pint 35¼ cubic inches. The wine pint of pure spring water, weighs near 17 ounces avoirdupois, and the ale pint a little above 20 ounces.—The Paris pint contains about 2 pounds of common water. And the Scotch pint contains 108⅔ cubic inches, and therefore contains 3 English pints.

PISCES, the 12th sign or constellation in the zodiac; in the form of two fishes tied together by the tails. The Greeks, who have some fable to account for the origin of every constellation, tell us, that when Venus and Cupid were one time on the banks of the Euphrates, there appeared before them that terrible giant Typhon, who was so long a terror to all the gods. These deities immediately, they say, threw themselves into the water, and were there changed into these two fishes, the Pisces, by which they escaped the danger. But the Egyptians used the signs of the zodiac as part of their hieroglyphic language, and by the 12 they conveyed an idea of the proper employment during the 12 months of the year. The Ram and the Bull had, at that time, taken to the increase of their flock, the young of those animals being then growing up; the maid Virgo, a reaper in the field, spoke the approach of harvest; Sagittary declared autumn the time for hunting; and the Pisces, or fishes tied together, in token of their being taken, reminded men that the approach of spring was the time for fishing.

The ancients, as they gave one of the 12 months of the year to the patronage of each of the 12 superior deities, so they also dedicated to, or put under the tutelage of each, one of the 12 signs of the zodiac. In this division, the fishes naturally fell to the share of Neptune; and hence arises that rule of the astrologers, which throws every thing that regards the fate of fleets and merchandise, under the more immediate patronage and protection of this constellation.—The stars in the sign Pisces are, in Ptolemy's catalogue 38, in Tycho's 36, in Hevelius's 39, and in the Britannic catalogue 113.

PISCIS *Australis*, the Southern Fish, is a constellation of the southern hemisphere, being one of the old 48 constellations mentioned by the ancients. The Greeks have here again the fable of Venus and her son throwing themselves into the sea, to escape from the terrible Typhon. This fable is probably borrowed from the hieroglyphics of the Egyptians. With them, a fish represented the sea, its element; and Typhon was probably a land flood, perhaps represented by the sign Aquarius, or water-pourer, whose stream or river is represented as swallowed up by this fish, as the land floods and rivers are by the sea. And Venus was some queen, perhaps Semiramis, otherwise called Hamamah, who took to the river or the sea with her son, in a vessel, to avoid the flood, &c. The remarkable star Fomahaut, of the 1st magnitude, is just in the mouth of this fish. The stars of this constellation are, in Ptolemy's catalogue 18, and in Flamsteed's 24.

Piscis *Volans*, the Flying Fish, is a small constellation of the southern hemisphere, unknown to the ancients, being added by the moderns. It is not visible in our latitude, and contains only 8 stars.

PISTOLE, a gold coin in Spain, Italy, Switzerland, &c, of the value of about 16*s*. 6*d*.

PISTON, a part or member in several machines, particularly pumps, air-pumps, syringes, &c; called also the embolus, and popularly the sucker. The piston of a pump is a short cylinder of wood or metal, fitted exactly to the cavity of the barrel, or body; which, being worked up and down alternately, raises the water; and when raised, presses it again, so as to make it force up a valve with which it is furnished, and so escape through the spout of the pump. There are two sorts of pistons used in pumps; the one with a valve, called a bucket; and the other without a valve, called a forcer.

PITCH, in Music, is the acuteness or graveness of any particular sound, or of the tuning of any instrument. A sound less acute than some other sound with which it is compared, is said to be of a lower pitch than that other sound; and vice versa.

PITISCUS (Bartholomew), a German mathematician, who died in 1613. He was author of two respectable mathematical works: 1. Trigonometria first, published at Frankfort, in 1599, a large vol. in 4to, being a very complete work on that science, with very large tables of sines, tangents, and secants: it afterwards went through several editions, and was translated into English by Handson, in 1614. See my Tracts, vol. 1, pa. 294.—2. Thesaurus Mathematicus, in folio, 1613, being an edition of the large tables of Rheticus, with all the numerous errors corrected.

PITOT (Henry), a French mathematician, was born at Aramont in Languedoc, 1695, and died there in 1771, in his 77th year. Pitot learned the mathematics without a master, and repaired to Paris in 1728, where he was admitted a member of the Academy of Sciences in 1724. Besides a vast number of his memoirs printed in the Academy's collection, he published in 1731 the Theory of the Working of Ships, in 1 vol. 4to; a work of considerable merit, which was translated into English, and procured the author's admission into the Royal Society of London. In 1740, the states-general of Languedoc appointed him their chief engineer, with the office of inspector-general of the canal which joins the two seas. That province is indebted to him for several valuable monuments of his genius; and he conducted to Montpellier a copious supply of water, from a distance of 9 miles, a work which is the admiration of all strangers.

PLACE, in Philosophy, that part of infinite space which any body possesses. Aristotle and his followers divide place into External and Internal.

Internal PLACE, is that space or room which the body contains. And

External PLACE, is that which includes or contains the body; which is by Aristotle called the first or concave and immoveable surface of the ambient body.

Newton, better, and more intelligibly, distinguishes place into absolute and relative.

Absolute and *Primary* PLACE, is that part of infinite and immoveable space which a body possesses. And

Relative, or *Secondary* PLACE, is the space it possesses considered with regard to other adjacent objects.

Dr. Clark adds another kind of relative place, which he calls relatively common place; and defines it, that part of any moveable or measurable space which a body possesses; which place moves together with the body.

PLACE, Mr. Locke observes, is sometimes likewise taken for that portion of infinite space possessed by the material world; though this, he adds, were more properly called extension. The proper idea of place, according to him; is the relative position of any thing, with regard to its distance from certain fixed points; whence it is said a thing has or has not changed place, when its distance is or is not altered with respect to those bodies.

PLACE, in Optics, or *Optical* PLACE, is the point to which the eye refers an object.

Optic PLACE of a star, is a point in the surface of the mundane sphere in which a spectator sees the centre of the star, &c.—This is divided into True and Apparent.

True, or *Real Optic* PLACE, is that point of the surface of the sphere, where a spectator at the centre of the earth would see the star, &c.

Apparent, or *Visible Optic* PLACE, is that point of the surface of the sphere, where a spectator at the surface of the earth sees the star, &c.—The distance between these two optic places makes what is called the parallax.

PLACE *of the Sun*, or *Moon*, or *Star*, or *Planet*, in Astronomy, simply denotes the sign and degree of the zodiac which the luminary is in; and is usually expressed either by its latitude and longitude, or by its right ascension and declination.

PLACE *of Radiation*, in Optics, is the interval or space in a medium, or transparent body, through which any visible object radiates.

PLACE, in Geometry, usually called *Locus*, is a line used in the solution of problems, being that in which the determination of every case of the problem lies. See LOCUS, *Plane, Simple, Solid*, &c.

PLACE, in War and Fortification, a general name for all kinds of fortresses, where a party may defend themselves.

PLACE *of Arms*, a strong part where the arms &c, are deposited, and where usually the soldiers assemble and are drawn up.

PLAFOND, or PLATFOND, in Architecture, the ceiling of a room.

PLAIN &c. See PLANE.

PLAN, a representation of something, drawn on a plane. Such as maps, charts, and ichnographies.

PLAN, in Architecture, is particularly used for a draught of a building; such as it appears, or is intended to appear, on the ground; showing the extent, division, and distribution of its area into apartments, rooms, passages, &c. It is also called the ground plot, platform, and ichnography of the building; and is the first device or sketch the architect makes.

Geometrical PLAN, is that in which the solid and vacant parts are represented in their natural proportion.

Raised PLAN, is that where the elevation, or upright, is shown upon the geometrical plan, so as to hide the distribution.

Perspective PLAN, is that which is conducted and exhibited by degradations, or diminutions, according to the rules of perspective.

PLANE, or PLAIN, in Geometry, denotes a plane figure, or a surface lying evenly between its bounding lines: Euclid. Some define a plane, a surface, from every point of whose perimeter a right line may be drawn to every other point in the same, and always coinciding with it.—As the right line is the shortest extent from one point to another, so is a plane the shortest extension between one line and another.

PLANES are much used in astronomy, conic sections, spherics, &c, for imaginary surfaces, supposed to cut and pass through solid bodies. When a plane cuts a cone parallel to one side, it makes a parabola; when it cuts the cone obliquely, an ellipse or hyperbola; and when parallel to its base, a circle. Every section of a sphere is a circle. The sphere is wholly explained by planes, conceived to cut the celestial bodies, and to fill the areas or circumferences of the orbits: and in estimating their inclination, they are all referred to the plane of the earth's orbit, or plane of the ecliptic.

PLANE *of a Dial*, is the surface on which a dial is supposed to be described.

PLANE, in Mechanics. A *Horizontal* PLANE, is a plane that is level, or parallel to the horizon.

Inclined PLANE, is one that makes an oblique angle with a horizontal plane. The doctrine of the motion of bodies on inclined planes, makes a very considerable article in mechanics, and has been fully explained under the articles, MECHANICAL *Powers*, and INCLINED *Plane*.

PLANE *of Gravity*, or *Gravitation*, is a plane supposed to pass through the centre of gravity of the body, and in the direction of its tendency; that is, perpendicular to the horizon.

PLANE *of Reflection*, in Catoptrics, is a plane which passes through the point of reflection; and is perpendicular to the plane of the glass, or reflecting body.

PLANE *of Refraction*, is a plane passing through the incident and refracted ray.

Perspective PLANE, is a plane transparent surface, usually perpendicular to the horizon, and supposed to be placed between the spectator's eye and the object he views; through which the optic rays, emitted from the several points of the object, are supposed to pass to the eye, and in their passage to leave marks that represent them on the said plane.—Some call this the table, or picture, because the draught or perspective of the object is supposed to be upon it. Others call it the section, from its cutting the visual rays; and others again the glass, from its supposed transparency.

Geometrical PLANE, in Perspective, is a plane parallel to the horizon, upon which the object is supposed to be placed that is to be drawn.

Horizontal PLANE, in Perspective, is a plane passing through the spectator's eye, parallel to the horizon.

Vertical PLANE, in Perspective, is a plane passing through the spectator's eye, perpendicular to the geometrical plane, and usually at right angles to the perspective plane.

Objective PLANE, in Perspective, is any plane situate in the horizontal plane, of which the representation in perspective is required.

PLANE *of the Horopter*, in Optics, is a plane passing through the horopter AB, and perpendicular to a plane passing through the two optic axes CH and CI. See the fig. to the article HOROPTER.

PLANE *of the Projection*, is the plane upon which the sphere is projected.

PLANE *Angle*, is an angle contained under two right lines or surfaces.—It is so called in contradistinction to a solid angle, which is formed by three or more planes; and to a spherical angle, contained between two arcs of great circles on a sphere.

PLANE *Triangle*, is a triangle formed by three right lines; in opposition to a spherical and a mixt triangle.

PLANE *Trigonometry* is the doctrine of plane triangles, their measures, proportions, &c.

PLANE *Glass*, or *Mirror*, in Optics, is a glass or mirror having a flat or even surface.

PLANE *Chart*, in Navigation, is a sea-chart, having the meridians and parallels represented by parallel straight lines; and consequently having the degrees of longitude the same in every part. See CHART.

PLANE *Number*, is that which may be produced by the multiplication of two numbers, the one by the other. Thus, 6 is a plane number, being produced by the multiplication of the two numbers 2 and 3; also 15 is a plane number, being produced by the multiplication of the numbers 3 and 5. See NUMBER.

PLANE *Place*, *Locus Planus*, or *Locus ad Planum*, is a term used by the ancient geometricians, for a geometrical locus, when it was a right line or a circle, in opposition to a solid place, which was one of the conic sections. These plane loci are distinguished by the moderns into loci ad rectum, and loci ad circulum. See LOCUS.

PLANE *Problem*, is such a one as cannot be resolved geometrically, but by the intersection either of a right line and a circle, or of the circumferences of two circles. Such as the following: viz, Given the hypothenuse, and the sum of the other two sides, of a right-angled triangle; to find the triangle. Or this: Of four given lines to form a trapezium of a given area.

PLANE *Sailing*, in Navigation, is the art of working the several cases and varieties in a ship's motion on a plane chart; or of navigating a ship upon principles deduced from the notion of the earth's being an extended plane. This principle, though notoriously false, yet places being laid down accordingly, and a long voyage broken into many short ones, the voyage may be performed tolerably well by it, especially near the same meridian.

In plain sailing, it is supposed that these three, the rhumb line, the meridian, and parallel of latitude, will always form a right-angled triangle; and so posited, as that the perpendicular side will represent part of the meridian, or north and south line, containing the difference of latitude; the base of the triangle, the departure, or east and west line; and the hypothenuse the distance sailed. The angle at the vertex is the course; and the angle at the base, the complement of the course; any two of which, besides the right angle, being given, the triangle may be protracted, and the other three parts found.—For the doctrine of plane sailing, see SAILING.

PLANE *Scale*, is a thin ruler, on which are graduated the lines of chords, sines, tangents, secants, leagues, rhumbs, &c; being of great use in most parts of the mathematics, but especially in navigation. See its description and use under SCALE.

PLANE *Table*, an instrument much used in land-surveying; by which the draught or plan is taken upon the spot, as the survey or measurement goes on, without any future protraction, or plotting. This instrument consists of a plane rectangular board, of any convenient size, the centre of which, when used, is fixed by means of screws to a three-legged stand, having a ball and socket, or universal joint, at the top, by means of which, when the legs are fixed on the ground, the table is inclined in any direction. To the table belongs,

1. A frame of wood, made to fit round its edges, for the purpose of fixing a sheet of paper upon the table. The one side of this frame is usually divided into equal parts, by which to draw lines across the table, parallel or perpendicular to the sides; and the other side of the frame is divided into 360 degrees, from a centre which is in the middle of the table; by means of which the table is to be used as a theodolite, &c.

2. A magnetic needle and compass screwed into the side of the table, to point out directions and be a check upon the sights.

3. An index, which is a brass two-foot scale, either with a small telescope, or open sights erected perpendicularly upon the ends. These sights and the fiducial edge of the index are parallel, or in the same plane.

General Use of the PLANE *Table.*

To use this instrument properly, take a sheet of writing or drawing paper, and wet it to make it expand; then spread it flat upon the table, pressing down the frame upon the edges, to stretch it, and keep it fixed there; and when the paper is become dry, it will, by shrinking again, stretch itself smooth and flat from any cramps or unevenness. Upon this paper is to be drawn the plan or form of the thing measured.

The general use of this instrument, in land-surveying, is to begin by setting up the table at any part of the ground you think the most proper, and make a point upon a convenient part of the paper or table, to represent that point of the ground; then fix in that point of the paper one leg of the compasses, or a fine steel pin, and apply to it the fiducial edge of the index, moving it round the table, close by the pin, till through the sights you perceive some point or remarkable object, as the corner of a field, or a picket set up, &c; and from the station point draw a dry or obscure line along the fiducial edge of the index. Then turn the index to another object, and draw a line on the paper towards it. Do the same by another; and so on till as many objects are set as may be thought necessary. Then measure from your station towards as many of the objects as may be necessary, and no more, taking the requisite offsets to corners or crooks in the hedges, &c; laying the measured distances, from a proper scale, down upon the respective lines on the paper. Then move the table to any of the proper places measured to, for a second station, fixing it there in the original position, turning it about its centre for that purpose, both till the magnetic needle point to the same degree of the compass as at first, and also by laying the fiducial edge of the index along the line between the two stations, and turning the table till through the index the former station can be seen; and then fix the table there: from this new station repeat the same operations as at the former; setting several objects, that is, drawing lines towards them, on the paper, by the edge of the index, measuring and laying off the distances. And thus proceed from station to station; measuring only such lines as are necessary, and determining as many as you can by intersecting lines of direction drawn from different stations.

Of Shifting the Paper on the PLANE *Table.* When one paper is full of the lines &c measured, and the survey is not yet completed; draw a line in any manner through the farthest point of the last station line to which the work can be conveniently laid down; then take the sheet off the table, and fix another fair sheet in its place, drawing a line upon it, in a part of it the most convenient for the rest of the work, to represent the line drawn at the end of the work on the former paper. Then fold or cut the old sheet by the line drawn upon it; apply it so to the line on the new sheet, and, as they lie together in that position, continue or produce the last station line of the old sheet upon the new one; and place upon it the remainder of the measurement of that line, beginning at where the work left off on the old sheet. And so on, from one sheet to another, till the whole work is completed.

But it is to be noted, that if the said joining lines, upon the old and new sheet, have not the same inclination to the side of the table, the needle will not respect or point to the original degree of the compass, when the table is rectified. But if the needle be required to respect still the same degree of the compass, the easiest way then of drawing the lines in the same position, is to draw them both parallel to the same sides of the table, by means of the equal parallel divisions marked on the other two sides of the frame.

When the work of surveying is done, and you would fasten all the sheets together into one piece, or rough plan, the aforesaid lines are to be accurately joined together, in the same manner as when the lines were transferred from the old sheets to the new ones.

PLANET, or *Wandering Star*, in Astronomy, is a celestial body revolving about the sun, or some other planet, as a centre, or focus, in nearly a circular orbit, or in an ellipse of small excentricity.

The planets are usually distinguished into primary and secondary.

Primary PLANETS, are those that revolve about the sun as a centre, or focus; such as Mercury, Venus, the earth, &c.

Secondary PLANETS, are such as revolve about a primary planet as a centre, as the primary ones do about the sun; being more commonly called satellites; such is our moon, and the satellites of Jupiter, Saturn, and Uranus. See SATELLITE.

The primary planets are again distinguished into Superior and Inferior.

The *Superior* Planets are those which are above the earth, or farther from the sun; as Mars, Vesta, Juno, &c.

The *Inferior* Planets are those that are below the earth, as Mercury and Venus.

Till very lately the number of the primary planets was esteemed only six, which it was thought constituted the whole of our planetary system: these were Mercury, Venus, Earth, Mars, Jupiter, and Saturn; all of which it appears have been known from the highest antiquity.

But the great perfection to which telescopes have been brought, has, within a few years, nearly doubled the number of the planets. Dr. Herschel discovered Uranus at Bath, March 13, 1781. This planet was first named, in honour of his present majesty, the Georgium Sidus, while some astronomers called it Herschel, from its discoverer; but both these names have now given way to that of Uranus.

An eighth planet, Ceres, was discovered by Piazzi, at Palermo in Italy, January 1st, 1801.

A ninth, Pallas, was discovered by Dr. Olbers, at Bremen, on March 28th, 1802.

A tenth, Juno, was first observed by Mr. Harding, at Lilienthal near Bremen, on the 1st of September 1804.

And finally, another new planet, Vesta, making the number of planets in our system eleven, was discovered by Dr. Olbers, at Bremen, March 29th, 1807; being the second that this celebrated astronomer had discovered in 5 years. Four out of the five new planets have their orbits between those of Mars and Jupiter; these are Vesta, Juno, Pallas, and Ceres; but the other, Uranus, is the highest in our system. The order of the planets is therefore as follows; Mercury, Venus, Earth, Mars, Vesta, Juno, Pallas, Ceres, Jupiter, Saturn, and Uranus.

The planets were represented by the same characters as the chemists use to represent their metals by, on account of some supposed analogy between those celestial and the subterraneous bodies. Thus,

Mercury, the messenger of the Gods, represented by ☿, the same as that metal, imitating a man with wings on his head and feet, is a small bright planet, with a light tinct of blue, the sun's constant attendant, from whose side it never departs above 28 degrees, and by that means is usually hid in his splendour. It performs its course around him in about 3 months.

Venus, the goddess of love, marked ♀, from the figure of a woman, the same as denotes copper, from a slight tinge of that colour, or verging to a light straw colour. She is a very bright planet, revolving next above Mercury, and never appears above 48 degrees from the sun, finishing her course about him in about 7 months. When this planet goes before the sun, or is a morning star, it has been called Phosphorus, and also Lucifer; and when following him, or when it shines in the evening as an evening star, it is called Hesperus.

Tellus, the Earth, next above Venus, is denoted by ⊖, and performs its course about the sun in the space of a year.

Mars, the god of war, characterized ♂, a man holding out a spear, the same as iron, is a ruddy fiery-coloured planet, and finishes his course about the sun in about 2 years.

Vesta, Juno, Pallas, Ceres, are the planets next in order, and their periods of revolution about the sun are as below: Vesta in   days; Juno in 2007½ days; Pallas in 1682 days; and Ceres in 1681 days. These four planets are too small to be distinguished by the naked eye.

Jupiter, the chief god, or thunderer, marked ♃, to represent the thunderbolts, denoting the same as tin, from his pure white brightness. This planet is next above Mars, and completes its course round the sun in about 12 years.

Saturn, the father of the Gods, is expressed by ♄, to imitate an old man supporting himself with a staff, and is the same as denotes lead, from his feeble light and dusky colour. He revolves next above Jupiter, and performs his course in about 30 years.

Lastly, Uranus, the Georgian, or Herschel, is denoted by ♅, the initial of his name, with a cross for the Christian planet, or that discovered by the Christians. This is the highest, or outermost, of the known planets, and revolves around the sun in the space of about 90 years.

It is to be regretted that all the new planets have not been called by the names of their respective discoverers, instead of the fanciful and unmeaning names that have been imposed on them by the continental astronomers.

From these descriptions a person may easily distinguish all the old planets. For if, after sun-set, he sees a planet nearer the east than the west, he may conclude it is neither Venus nor Mercury; and he may determine whether it is Saturn, Jupiter, or Mars, by the colour, light, and magnitude: by which also he may distinguish between Venus and Mercury.

It is probable that all the planets are dark opake bodies, similar to the earth, and for the following reasons.

1. Because, in Mercury, Venus, and Mars, only that part of the disk is found to shine which is illuminated by the sun; and again, Venus and Mercury, when between the sun and the earth, appear like maculæ or dark spots on the sun's face: from which it is evident, that those three planets are opake bodies, illuminated by the borrowed light of the sun. And the same appears of Jupiter, from his being void of light in that part to which the shadow of his satellites reaches, as well as in that part turned from the sun: and that his satellites are opake, and reflect the sun's light, like the moon, is abundantly shown. Moreover, since Saturn, with his ring and satellites, and also Herschel, with his satellites, only yield a pale light, considerably fainter than that of the rest of the planets, and than that of the fixed stars, though these be vastly more remote; it is past a doubt that these planets too, with their attendants, are opake bodies.

2. Since the sun's light is not transmitted through Mercury or Venus, when placed against him, it is plain they are dense opake bodies; which is likewise evident of Jupiter, from his hiding the satellites in his shadow; and therefore, by analogy, the same may be concluded of Saturn, and all the rest.

3. From the variable spots of Venus, Mars, and Jupiter, it is evident that these planets have a changeable atmosphere; which kind of atmosphere, by a like argument, may be inferred of the satellites of Jupiter; and therefore, by similitude, the same may be concluded of the other planets.

4. In like manner, from the mountains observed in the moon and Venus, the same may be supposed in the other planets.

5. Lastly, since all these planets are opake bodies, shining with the sun's borrowed light, are furnished with mountains, and are encompassed with a changeable atmosphere; we may infer that they have waters, seas &c, as well as dry land, and are bodies like the moon, and therefore like the earth. And hence, it seems also probable, that the other planets have their animal inhabitants, as well as our earth has.

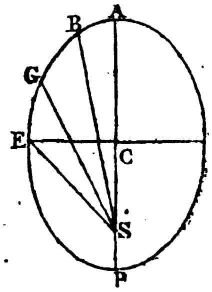
Of the Orbits of the Planets.

Though all the primary planets revolve about the sun, their orbits are not circles, but ellipses, having the sun in one of the foci. This circumstance was first discovered by Kepler, from the observations of Tycho Brahé: before that, all astronomers took the planetary orbits for excentric circles. All the planes of these orbits intersect in the sun; and the line in which the plane of each orbit cuts that of the earth, is called the Line of the nodes; and the two points in which the orbits themselves touch that plane, are the Nodes; also the angle in which each plane cuts that of the ecliptic, is called the Inclination of the plane or orbit.—The distance between the centre of the sun, and the centre of each orbit, is called the excentricity of the planet, or of its orbit.

The Motions of the Planets.

The motions of the primary planets are very simple, and tolerably uniform, as being compounded only of a projectile motion, forward in a right line, which is a tangent to the orbit, and a gravitation towards the sun at the centre. Besides, being at such vast distances from each other, the effects of their mutual gravitation towards one another are in a considerable degree, though not altogether, insensible: for the action of Jupiter upon Saturn, for ex. is found to be $\frac{1}{104}$ of the action of the sun upon Saturn, by comparing the matter of Jupiter with that of the sun, and the square of the distance of each from Saturn. So that the elliptic orbit of Saturn will be found more just, if its focus be supposed not in the centre of the sun, but in the common centre of gravity of the sun and Jupiter, or rather in the common centre of gravity of the sun and all the other planets. In like manner, the elliptic orbit of any other planet will be found more accurate, by supposing its focus to be in the common centre of gravity of the sun and all the planets that are below it. But the matter is far otherwise, in respect of the secondary planets: for every one of these, though it chiefly gravitates towards its respective primary one, as its centre, yet at equal distances from the sun, it is also attracted towards him with an equally accelerated gravity, as the primary one is towards him; but at a greater distance with less, and at a nearer distance with greater: from which double tendency towards the sun, and towards their own primary planets, it happens, that the motion of the satellites, or secondary planets, is very much compounded, and affected with various inequalities.

The motions even of the primary planets, in their elliptic orbits, are not equable, because the sun is not in their centre, but their focus. Hence they move, sometimes faster and sometimes slower, as they are nearer to or farther from the sun; but yet these irregularities are all certain, and follow according to an immutable law. Thus, the ellipsis PEA &c, representing the orbit of a planet, and the focus S the sun's place: the axis of the ellipse AP, is the line of the apses; the point A, the higher apsis or aphelion; P the lower apsis or perihelion; CS the excentricity; and ES the planet's mean distance from the sun. Now the motion of the planet in its perihelion P is swiftest, but in its aphelion A it is slowest; and at E the motion as well as the distance is a mean, being there such as would describe the whole orbit in the same time it is really described in. And the law by which the motion in every point is regulated, is this, that a line or radius drawn from the centre of the sun to the centre of the planet, and thus carried along with an angular motion, always describes an elliptic area proportional to the time; that is, the trilineal area ASB, is to the area ASG, as the time the planet is in moving over AB, to the time it is in moving over AG. This law was first discovered by Kepler, from observations; and



has since been accounted for and demonstrated by Sir Isaac Newton, from the general laws of attraction and projectile motion.

As to the periods and velocities of the planets, or the times in which they perform their courses, they are found to have a wonderful harmony with their distances from the sun, and with one another: the nearer each planet being to the sun, the quicker still is its motion, and its period the shorter, according to this general and regular law; viz, that the squares of their periodical times are as the cubes of their mean distances from the sun or focus of their orbits. The knowledge of this law we owe also to the sagacity of Kepler, who found that it obtained in all the primary planets; as astronomers have since found it also to hold good in the secondary ones. Kepler indeed deduced this law merely from observation, by a comparison of the several distances of the planets with their periods or times: the glory of investigating it from physical principles is due to Sir Isaac Newton, who has demonstrated that, in the present state of nature, such a law was inevitable.

The phenomena of the planets are, their Conjunctions, Oppositions, Elongations, Stations, Retrogradations, Phases, and Eclipses; for which see the respective articles. For a view of the comparative magnitudes of the planets, and of their several distances, &c; see the articles **Orbit** and **Solar System**, as also Plate 26, fig. 1.—The following Table contains a synopsis of the distances, magnitudes, periods, &c, of the several planets, according to the latest observations and improvements.

TABLE *of the* PLANETARY MOTIONS, DISTANCES, &c.

Anno 1815	MERCURY.	VENUS.	EARTH.	MARS.	VESTA, or OLBERS[2].	JUNO, or HARDING.	PALLAS, or OLBERS[1].	CERES, or PIAZZI.	JUPITER.	SATURN.	URANUS, or HERSCHEL, 1782.
Greatest elongation of inferior, and parallax of superior planets.	28° 20′	47° 48′	* *	47° 24′					11° 51′	6° 29′	3° 4¼′
Periodical revolutions round the sun.	d h m 87 23 16	d h m 224 16 49	d h m 365 6 9¼	d h m 686 23 31		2007½ da	1682 da.	1681 da.	d h m 4332 14 27	d h m 10759 1 51	d h 30689 0
Diurnal rotations upon their axes.	* * *	$23^h\ 20^m$	$23^h\ 56^m\ 4^s$	$24^h\ 40^m\ 22^s$					$9^h\ 56^m$	$10^h\ 17^m$	* *
Inclinations of their orbits to the ecliptic.	7° 0′	3° 23½′	* *	1° 51′	7° 8⅕′	13° 4′	34° 38′	10° 38′	1° 19′	2° 30′	0° 46½′
Place of the ascending node.	1^s 15° 47′	2^s 14° 44′	* * *	1^s 17° 59′	3^s 13° 18′	5^s 21° 4′	5° 22′ 31″	2^s 21° 7′	3^s 8° 50′	3^s 21° 48′	3^s 13° 1′
Place of the aphelion, or point farthest from the sun.	8^s 14° 13′	10^s 9° 38′	9^s 9° 15¼′	5^s 2° 6¼′		7^s 22° 49′	10° 4′ 36″	10^s 26° 9′	6^s 10° 57½′	9^s 0° 45½′	11^s 23° 23′
Greatest apparent diameters, seen from the earth.	11″	58″	*	27″			0·5″	1″	39″	18″	4″
Diameter in English miles; that of the sun being 883217.	3224	7867	7914	4189			80	165	89170	79042	35109
Proportional mean distances from the sun.	38710	72333	100000	152369	235513	266400	276700	276500	520259	954072	1918362
Mean distances from the sun in semidiameters of the earth.	9210	17210	23799	36262	56049	63400	65804	65851	123778	227028	452000
Mean distances from the sun in English miles.	37 mills.	68 mills.	95 mills.	144 mills.	less than Ceres.	290 mills.	265 mills.	260 mills.	490 mills.	900 mills.	1800 mills.
Excentricities in parts of mean distances.	$\frac{4}{19}$	$\frac{1}{138}$	$\frac{1}{59}$	$\frac{1}{11}$	$\frac{1}{28}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{13}$	$\frac{1}{21}$	$\frac{1}{18}$	$\frac{1}{21}$
Proportion of light and heat; that of the earth being 100.	668	191	100	43	18	14	13	13	3·7	1·1	0·276
Proportion of bulk; that of the sun being 1380000.	$\frac{1}{15}$	$\frac{8}{9}$	1	$\frac{7}{24}$			$\frac{1}{375036}$	$\frac{1}{118576}$	1424	1000	96
Proportion of density; that of the sun being ¼.	2	1¼	1	·7					·23	·02	*

A planet's motion, or distance from its apogee, is called the mean anomaly of the planet, and is measured by the area it describes in the given time: when the planet arrives at the middle of its orbit, or the point E, the area or time is called the true anomaly. When the planet's motion is reckoned from the first point of Aries, it is called its motion in longitude; which is either mean or true; viz, mean, which is such as it would have were it to move uniformly in a circle; and true, which is that with which the planet actually describes its orbit, and is measured by the arc of the ecliptic it describes. And hence may be found the planet's place in its orbit for any given time after it has left the aphelion: for suppose the area of the ellipsis be so divided by the line SG, that the whole elliptic area may have the same proportion to the part ASG, as the whole periodical time in which the planet describes its whole orbit, has to the given time; then will G be the planet's place in its orbit sought.

PLANETARIUM, an astronomical machine, contrived to represent the motions, orbits, &c, of the planets, as they really are in nature, or according to the Copernican system. The larger kind of them are called Orreries. See Orrery.

A remarkable machine of this sort was invented by Huygens, and described in his Opusc. Posth. tom. 2. p. 157, edit. Amst. 1728, which is still preserved among the curiosities of the university at Leyden. In this planetarium, the five primary planets perform their revolutions about the sun, and the moon performs her revolution about the earth, in the same time that they are really performed in the heavens. Also the orbits of the moon and planets are represented with their true proportions, excentricity, position, and declination from the ecliptic or orbit of the earth. So that by this machine the situation of the planets, with the conjunctions, oppositions, &c, may be known, not only for the present time, but for any other time, either past or yet to come; as in a perpetual ephemeris.

There was exhibited in London, viz, in the year 1791, a still much more complete planetarium of this kind; called "a planetarium or astronomical machine, which exhibited the most remarkable phenomena, motions, and revolutions of the universe. Invented, and partly executed, by the celebrated M. Phil. Matthew Hahn, member of the academy of sciences at Erfurt. But finished and completed by Mr. Albert de Mylius." This is a most stupendous and elaborate machine; consisting of the solar system in general, with all the orbits and planets in their due proportions and positions; as also the several particular planetary systems of such as have satellites, as of the earth, Jupiter, &c; the whole kept in continual motion by a chronometer, or grand eight-day clock; by which all these systems are made perpetually to perform all their motions exactly as in nature, exhibiting at all times the true and real motions, positions, aspects, phenomena, &c, of all the celestial bodies, even to the very diurnal rotation of the planets, and the unequal motions in their elliptic orbits. A description was published of this most superb machine; and it was purchased and sent as one of the presents to the emperor of China, in the embassy of Lord Macartney, in the year 1793.

But the planetariums or orreries now most commonly used, do not represent the true times of the celestial motions, but only their proportions; and are not kept in continual motion by a clock, but are only turned round occasionally with the hand, in order to give young beginners an idea of the planetary system; as also, if constructed with sufficient accuracy, to resolve problems, in a coarse way, relating to the motions of the planets, and of the earth and moon, &c.

Dr. Desaguliers (Exp. Philos. vol. 1, p. 430,) describes a planetarium of his own contrivance, which is one of the best of the common sort. The machine is contrived to be rectified or set to any latitude; and then by turning the handle of the planetarium, all the planets perform their revolutions round the sun in proportion to their periodical times, and they carry indices which show the longitudes of the planets, by pointing to the divisions graduated on circles for that purpose.

The planetarium represented in fig. 1, plate 22, is an instrument contrived by Mr. Wm. Jones, of Holborn, London, mathematical instrument maker, who has paid considerable attention to such machines, to bring them to a great degree of simplicity and perfection. It represents in a general manner, by various parts of its machinery, all the motions and phenomena of the planetary system. This machine consists of, the Sun in the centre, with the Planets in the order of their distance from him, viz, Mercury, Venus, the Earth and Moon, Mars, Jupiter with his moons, and Saturn with his ring and moons; and to it is also occasionally applied an extra long arm for the Georgian Planet and his moons. To the earth and moon is applied a frame CD, containing only four wheels and two pinions, which serve to preserve the earth's axis in its due parallelism in its motion round the sun, and to give the moon at the same time her due revolution about the earth. These wheels are connected with the wheel-work in the round box below, and the whole is set in motion by the winch H. The arm M that carries round the moon, points out on the plate C her age and phases for any situation in her orbit, upon which they are engraved. In like manner the arm points out her place in the ecliptic B, in signs and degrees, called her geocentric place, that is, as seen from the earth. The moon's orbit is represented by the flat rim A; the two joints of it, upon which it turns, denoting her nodes; and the orbit being made to incline to any required angle. The terrella, or little earth, of this machine, is usually made of a three-inch globe papered, &c, for the purpose; and by means of the terminating wire that goes over it, points out the changes of the seasons, and the different lengths of days and nights more conspicuously. By this machine are seen at once all the planets in motion about the Sun, with the same respective velocities and periods of revolution which they have in the heavens; the wheelwork being calculated to a minute of time, from the latest discoveries. See Mr. Jones's Description of his new portable Orrery.

PLANETARY, something that relates to the planets. Thus we say, planetary worlds, planetary inhabitants, planetary motions, &c. Huygens and Fontenelle bring several probable arguments for the reality of planetary worlds, animals, plants, men, &c.

Planetary *System*, is the system or assemblage of the planets, primary and secondary, moving in their respective orbits, round their common centre the sun. See *Solar* System.

Planetary *Days*. With the ancients, the week was shared among the seven planets, each planet having its day. This we learn from Dion Cassius and Plutarch, Sympos. lib. 4. q. 7. Herodotus adds, that it was the Egyptians who first discovered what god, that is what planet, presides over each day; for that among this people the planets were directors. And hence it is, that in most European languages the days of the week are still denominated from the planets; as sunday, monday, &c.

PLANETARY *Dials*, are such as have the planetary hours inscribed on them.

PLANETARY *Hours*, are the 12th parts of the artificial day and night. See *Planetary* HOUR.

PLANETARY *Squares*, are the squares of the seven numbers from 3 to 9, disposed magically. Cornelius Agrippa, in his book of magic, has given the construction of the seven planetary squares. And M. Poignard, canon of Brussels, in his treatise on sublime squares, gives new, general, and easy methods, for making the seven planetary squares, and all others to infinity, by numbers in all kinds of progressions. See MAGIC *Square*.

PLANETARY *Years*, the periods of time in which the several planets make their revolutions round the sun, or earth.—As from the proper revolution of the earth, or the apparent revolution of the sun, the solar year takes its original; so from the proper revolutions of the rest of the planets about the earth, as many kinds of years arise; viz, the Saturnian year, which is defined by 29 Egyptian years 174 days 58 minutes, equivalent in a round number to 30 solar years. The Jovial year, containing 11 years 317 days 14 hours 27 minutes. The Martial year, containing 1 year 321 days 23 hours 31 minutes. For Venus and Mercury, as their years, when judged of with regard to the earth, are almost equal to the solar year; they are more usually estimated from the sun, the true centre of their motions: in which case the former is equal to 224 days 16 hours 49 minutes; and the latter to 87 days 23 hours 16 minutes.

PLANIMETRY, that part of geometry which considers lines and plane figures, without any regard to heights or depths.—Planimetry is particularly restricted to the mensuration of planes and other surfaces; as contradistinguished from stereometry, or the mensuration of solids, or capacities of length, breadth, and depth.—Planimetry is performed by means of the squares of long measures, as square inches, square feet, square yards, &c; that is, by squares whose side is an inch, a foot, a yard, &c. So that the area or content of any surface is said to be found, when it is known how many such square inches, feet, yards, &c, it contains. See MENSURATION and SURVEYING.

PLANISPHERE, a projection of the sphere, and its various circles, on a plane; as upon paper or the like. In this sense, maps of the heavens and the earth, exhibiting the meridians and other circles of the sphere, may be called planispheres.

Planisphere is sometimes also considered as an astronomical instrument, used in observing the motions of the heavenly bodies; being a projection of the celestial sphere upon a plane, representing the stars, constellations, &c, in their proper situations, distances, &c. As the Astrolabe, which is a common name for all such projections.

In all planispheres, the eye is supposed to be in a point, viewing all the circles of the sphere, and referring them to a plane beyond them, against which the sphere is as it were flattened: and this plane is called the Plane of Projection, which is always some one of the circles of the sphere itself, or parallel to some one.

Among the infinite number of planispheres which may be furnished by the different planes of projection, and the different positions of the eye, there are two or three that have been preferred to the rest. Such as that of Ptolemy, where the plane of projection is parallel to the equator: that of Gemma Frisius, where the plane of projection is the colure, or solstitial meridian, and the eye the pole of the meridian, being a stereographical projection: or that of John de Royas, a Spaniard, whose plane of projection is a meridian, and the eye placed in the axis of that meridian, at an infinite distance; being an orthographical projection, and called the Analemma.

PLANO-*Concave* glass or lens, is that which is plane on one side, and concave on the other. And

PLANO-*Convex* glass or lens, is that which is plane on one side, and convex on the other. See LENS.

PLAT-BAND, in Architecture, is any flat square moulding, whose height much exceeds its projecture. Such are the faces of an architrave, and the plat-bands of the modillions of a cornice.

PLATFORM, in Artillery and Gunnery, a small elevation, or a floor of wood, stone, or the like, on which cannon, &c, are placed, for more conveniently working and firing them.

PLATFORM, in Architecture, a row of beams that support the timber-work of a roof, lying on the top of the walls, where the entablature ought to be raised. Also a kind of flat walk, or plane floor, on the top of a building; whence a fair view may be taken of the adjacent grounds. So, an edifice is said to be covered with a platform, when it has no arched roof.

PLATO, one of the most celebrated among the ancient philosophers, being the founder of the sect of the Academics, was the son of Aristo, and born at Athens, about 429 years before Christ. He was of a royal and illustrious family, being descended by his father from Codrus, and by his mother from Solon. The name given him by his parents was Aristocles; but being of a robust make, and remarkably broad-shouldered, from this circumstance he was nick-named Plato by his wrestling-master, which name he retained ever after.

From his infancy, Plato distinguished himself by his lively and brilliant imagination. He eagerly imbibed the principles of poetry, music, and painting. The charms of philosophy however prevailing, drew him from those of the fine arts; and at the age of twenty he attached himself to Socrates only, who called him the Swan of the Academy. The disciple profited so well of his master's lessons, that at twenty-five years of age he had the reputation of a consummate sage. He lived with Socrates for eight years, in which time he committed to writing, according to the custom of the students, the purport of a great number of his master's excellent lectures, which he digested by way of philosophical conversations; but made so many judicious additions and improvements of his own, that Socrates, hearing him one day recite his Lysis, cried out, O Hercules! how many fine sentiments does this young man ascribe to me, that I never thought of! And Laertius assures us, that he composed several discourses which Socrates had no manner of hand in. At the time when Socrates was first arraigned, Plato was a junior senator, and he assumed the orator's chair to plead his master's cause, but was interrupted in that design, and the judges passed sentence of condemnation upon Socrates. Upon this occasion Plato begged him to accept from him a sum of money sufficient to purchase his enlargement, but Socrates peremptorily refused the generous offer, and suffered himself to be put to death.

The philosophers who were at Athens were so alarmed at the death of Socrates, that most of them fled, to avoid the cruelty and injustice of the government. Plato retired, till the storm should be over, to Megara, where he was kindly entertained by Euclid the philosopher, who had been one of the first scholars of Socrates. Afterwards he determined to travel in pursuit of knowledge; and from Megara he went to Italy, where he conferred with Eurytus, Philolaus, and Archytas, the most celebrated of the Pythagoreans, from whom he learned all his natural philosophy, diving into the most profound and mysterious secrets of the Pythagoric doctrines. But perceiving other knowledge to be connected with them, he went to Cyrene, where he studied geometry and other branches of mathematics under Theodorus, a celebrated master.

Hence he travelled into Egypt, to learn the theology of their priests, with the sciences of arithmetic, astronomy, and the nicer parts of geometry. Having taken also a survey of the country, with the course of the Nile and the canals, he settled some time in the province of Sais, learning of the sages there their opinions concerning the universe, whether it had a beginning, whether it moved wholly or in part, &c; also concerning the immortality and transmigration of souls: and here it is also thought he had some communication with the books of Moses.

Plato's curiosity was not yet satisfied. He travelled into Persia, to consult the magi as to the religion of that country. He designed also to have penetrated into India, to learn of the Brachmans their manners and customs; but was prevented by the wars in Asia.

Afterwards, returning to Athens, he applied himself to the teaching of philosophy, opening his school in the Academia, a place of exercise in the suburbs of the city; whence it was that his followers took the name of Academics.

Yet, settled as he was, he made several excursions abroad: one in particular to Sicily, to view the fiery ebullitions of Mount Etna. Dionysius the tyrant then reigned at Syracuse, where Plato went to visit him; but, instead of flattering him like a courtier, he reproved him for the disorders of his court, and the injustice of his government. The tyrant, not used to disagreeable truths, was enraged at Plato, and would have put him to death, if Dion and Aristomenes, formerly his scholars, and then favourites of that prince, had not powerfully interceded for him. Dionysius however delivered him into the hands of an envoy of the Lacedemonians, who were then at war with the Athenians: and this envoy, touching on the coast of Ægina, sold him for a slave to a merchant of Cyrene; who, as soon as he had bought him, liberated him, and sent him home to Athens.

Some time after, he made a second voyage into Sicily, in the reign of Dionysius the younger; who sent Dion, his minister and favourite, to invite him to court, that he might learn from him the art of governing his people well. Plato accepted the invitation, and went; but the intimacy between Dion and Plato raising jealousy in the tyrant, the former was disgraced, and the latter sent back to Athens. But Dion, being taken into favour again, persuaded Dionysius to recall Plato, which he did, and received him with all the marks of goodwill and friendship that a great prince could bestow. He sent out a fine galley to meet him, and went himself in a magnificent chariot, attended by all his court, to receive him. But this prince's uneven temper hurried him into new suspicions. It seems indeed that these apprehensions were not altogether groundless: for Ælian says, and Cicero was of the same opinion, that Plato taught Dion how to dispatch the tyrant, and to deliver the people from oppression. However this may be, Plato was offended and complained; and Dionysius, incensed at these complaints, resolved to put him to death: but Archytas, who had great interest with the tyrant, being informed of it by Dion, interceded for the philosopher, and obtained leave for him to retire.

The Athenians received him joyfully at his return, and offered him the administration of the government; but he declined that honour, choosing rather to live quietly in the Academy, in the peaceable contemplation and study of philosophy; being indeed so desirous of a private retirement that he never married. His fame drew disciples from all parts, when he would admit them, as well as invitations to come to reside in many of the other Grecian states; but the three of his pupils that most distinguished themselves, were Speusippus his nephew, who continued the Academy after him, Xenocrates the Caledonian, and the celebrated Aristotle. It is said also that Theophrastus and Demosthenes were two of his disciples. He had it seems so great a respect for the science of geometry and the mathematics, that he had the following inscription painted in large letters over the door of his academy; LET NO ONE ENTER HERE, UNLESS HE HAS A TASTE FOR GEOMETRY AND THE MATHEMATICS!

But as his great reputation gained him on the one hand many disciples and admirers, so on the other it raised him some emulators, especially among his fellow-disciples, the followers of Socrates. Xenophon and he were particularly disaffected to each other. Plato was of so quiet and even a temper of mind, even in his youth, that he never was known to express a pleasure with any greater emotion than that of a smile; and he had such a perfect command of his passions, that nothing could provoke his anger or resentment; from hence, and the subject and style of his writings, he acquired the appellation of the Divine Plato. But though he was naturally of a reserved and very pensive disposition; yet, according to Aristotle, he was affable, courteous, and perfectly good-natured; and sometimes would condescend to crack little innocent jokes on his intimate acquaintances. Of his affability there needs no greater proof than his civil manner of conversing with the philosophers of his own times, when pride and envy were at their height. His behaviour to Diogenes is always mentioned in his history. This Cynic was greatly offended, it seems, at the politeness and fine taste of Plato, and used to catch all opportunities of snarling at him. Dining one day at his table with other company, when trampling upon the tapestry with his dirty feet, he uttered this brutish sarcasm, "I trample upon the pride of Plato:" to which the latter wisely and calmly replied, "with a greater pride."

This extraordinary man, being arrived at 81 years of age, died on his birth-day a very easy and peaceable death, in the midst of an entertainment, according to some; but, according to Cicero, as he was writing. Both the life and death of this philosopher were calm and undisturbed; and indeed he was finely composed for happiness. Besides the advantages of a noble birth, he had a large and comprehensive understanding, a vast fund of wit and good taste, great evenness and sweetness of temper, cultivated and refined by education and travel; so that it is no wonder he was honoured by his countrymen, esteemed by strangers, and adored by his scholars. Tully perfectly adored him: he tells us that he was justly called by Panætius, the divine, the most wise, the most sacred, the Homer of philosophers; thinks, that if Jupiter had spoken Greek, he would have done it in Plato's style, &c. But, panegyric aside, Plato was certainly a very wonderful man, of a large and comprehensive mind, an imagination infinitely fertile, and of a most flowing and copious eloquence. However, the strength and heat of fancy prevailing over judgment in his composition, he was too apt to soar beyond the limits of earthly things, to range in the imaginary regions of general and abstracted ideas; on which account, though there is always a greatness and sublimity in his manner, he did not philosophize so much according to truth and nature as Aristotle, though Cicero did not scruple to give him the preference.

The writings of Plato are all in the way of dialogue, where he seems to deliver nothing from himself, but every thing as the sentiments and opinions of others, of Socrates chiefly, of Timæus, &c. His style, as Aristotle observed, is between prose and verse: on which account some have not scrupled to rank him among the poets: and indeed, besides the elevation and grandeur of his style, his matter is frequently the offspring of imagination, instead of doctrines or truths deduced from nature. The first edition of Plato's works in Greek, was printed by Aldus at Venice in 1513: but a Latin version of them by Marsilius Ficinus had been printed there in 1491. They were reprinted together at Lyons in 1588, and at Francfort in 1602. The famous printer Henry Stephens, in 1578, gave a beautiful and correct edition of Plato's works at Paris, with a new Latin version by Serranus, in three volumes folio.—And the industrious Thomas Taylor has lately given us several of Plato's works in an English translation.

PLATONIC, something that relates to Plato, his school, philosophy, opinions, or the like.

PLATONIC *Bodies*, so called from Plato who treated of them, are what are otherwise called the regular bodies. They are five in number; the tetraedron, the hexaedron, the octaedron, the dodecaedron, and the icosaedron. See each of these articles, as also REGULAR BODIES.

PLATONIC *Year*, or the *Great Year*, is a period of time determined by the revolution of the equinoxes, or the time in which the stars and constellations return to their former places, in respect of the equinoxes.—The Platonic year, according to Tycho Brahé, is 25816 solar years, according to Riccioli 25920, and according to Cassini 24800 years.—This period being once accomplished, it was an opinion among the ancients, that the world was to begin anew, and the same series of things to return over again.

PLATONISM, the doctrine and sentiments of Plato and his followers, with regard to philosophy, &c. His disciples were called Academics, from Academia, the name of a villa in the suburbs of Athens where he opened his school. Among these were Xenocrates, Aristotle, Lycurgus, Demosthenes, and Isocrates. In physics, he chiefly followed Heraclitus; in ethics and politics, Socrates; and in metaphysics, Pythagoras.

After his death, two of the principal of his disciples, Xenocrates and Aristotle, continuing his office, and teaching, the one in the Academy, the other in the Lycæum, formed two sects, under different names, though in other respects the same; the one retaining the denomination of ACADEMICS, the other assuming that of PERIPATETICS. See these two articles.

Afterwards, about the time of the first ages of Christianity, the followers of Plato quitted the title of Academists, and took that of Platonists. It is supposed to have been at Alexandria, in Egypt, that they first assumed this new title, after having restored the ancient academy, and re-established Plato's sentiments; which had many of them been gradually dropped and laid aside. Porphyry, Plotin, Iamblichus, Proclus, and Plutarch, are those who acquired the greatest reputation among the Greek Platonists; Apuleius and Chalcidius, among the Latins; and Philo Judæus, among the Hebrews. The modern Platonists own Plotin the founder, or at least the reformer, of their sect.

The Platonic philosophy appears very consistent with the Mosaic; and many of the primitive fathers follow the opinions of that philosopher, as being favourable to Christianity. Justin is of opinion that there are many things in the works of Plato which this philosopher could not learn from mere natural reason; but thinks he must have learnt them from the books of Moses, which he might have read when in Egypt. Hence Numenius the Pythagorean expressly calls Plato the Attic Moses, and upbraids him with plagiarism; because he stole his doctrine concerning God and the world from the books of Moses. Theodoret says expressly, that he has nothing good and commendable concerning the Deity and his worship, but what he took from the Hebrew theology; and Clemens Alexandrinus calls him the Hebrew Philosopher. Gale is very particular in his proof of the point, that Plato borrowed his philosophy from the Scriptures, either immediately, or by means of tradition; and, besides the authority of the ancient writers, he brings some arguments from the thing itself. For example, Plato's confession, that the Greeks borrowed their knowledge of the one infinite God, from an ancient people, better and nearer to God than they; by which people, our author makes no doubt, he meant the Jews, from his account of the state of innocence; as, that man was born of the earth, that he was naked, that he enjoyed a truly happy state, that he conversed with brutes, &c. In fact, from an examination of all the parts of Plato's philosophy, physical, metaphysical, and ethical, this author finds, in every one, evident marks of its sacred original.

As to the manner of the creation, Plato teaches, that the world was made according to a certain exemplar, or idea, in the divine architect's mind. And all things, in the universe, in like manner, he shows, depend on the efficacy of internal ideas. This ideal world is thus explained by Didymus: "Plato supposes certain patterns, or exemplars, of all sensible things, which he calls ideas; and as there may be various impressions taken off from the same seal, so he says are there a vast number of natures existing from each idea." This idea he supposes to be an eternal essence, and to occasion the several things in nature to be such as itself is. And that most beautiful and perfect idea, which comprehends all the rest, he maintains to be the world.

Farther, Plato teaches that the universe is an intelligent animal, consisting of a body and a soul, which he calls the generated God, by way of distinction from what he calls the immutable essence, who was the cause of the generated God, or the universe.

According to Plato, there were two kinds of inferior and derivative gods; the mundane gods, all of which had a temporary generation with the world; and the supramundane eternal gods, which were all of them, one excepted, produced from that one, and dependent on it as their cause. Dr. Cudworth says, that Plato asserted a plurality of gods, meaning animated or intellectual beings, or dæmons, superior to men, to whom honour and worship are due; and applying the appellation to the sun, moon, and stars, and also to the earth. He asserts however, at the same time, that there was one supreme God, the self-originated being, the maker of the heaven and earth, and of all those other gods. He also maintains, that the Psyche, or universal mundane soul, which is a self-moving principle, and the immediate cause of all the motion in the world, was neither eternal nor self-existent, but made or produced by God in time; and above this self-moving Psyche, but subordinate to the Supreme Being, and derived by emanation from him, he supposes an immoveable Nous or intellect, which was properly the Demiurgus, or framer of the world.

The first matter of which this body of the universe was formed, he observes, was a rude indigested heap, or chaos: Now, adds he, the creation was a mixed production; and the world is the result of a combination of necessity and understanding, that is, of matter, which he calls necessity, and the divine wisdom: yet so that mind rules over necessity; and to this necessity he ascribes the introduction and prevalence both of moral and natural evil.

The principles, or elements, which Plato lays down, are fire, air, water, and earth. He supposes two heavens, the Empyrean, which he takes to be of a fiery nature, and to be inhabited by angels, &c; and the Starry heaven, which he teaches is not adamantine, or solid, but liquid and spirable.

With regard to the human soul, Plato maintained its transmigration, and consequently its future immortality and pre-existence. He asserted, that human souls are here in a lapsed state, and that souls sinning should fall down into these earthly bodies. Eusebius expressly says, that Plato held the soul to be ungenerated, and to be derived by emanation from the first cause.

His physics, or doctrine De Corpore, is chiefly laid down in his Timæus, where he argues on the properties of body in a geometrical manner; which Aristotle takes occasion to reprehend in him. His doctrine De Mente is delivered in his 10th book of laws, and his Parmenides.

St. Augustine commends the Platonic philosophy; and even says, that the Platonists were not far from Christianity. It is also certain that most of the celebrated fathers were Platonists, and borrowed many of their explanations of Scripture from the Platonic system. To account for this fact, it may be observed, that towards the end of the second century, a new sect of philosophers, called the modern, or later Platonics, arose of a sudden, spread with amazing rapidity through the greatest part of the Roman empire, swallowed up almost all the other sects, and proved very detrimental to Christianity.

The school of Alexandria in Egypt, instituted by Ptolemy Philadelphus, renewed and reformed the Platonic philosophy. The votaries of this system distinguished themselves by the title of Platonics, because they thought that the sentiments of Plato concerning the Deity and invisible things, were much more rational and sublime than those of the other philosophers. This new species of Platonism was embraced by such of the Alexandrian Christians as were desirous to retain, with the profession of the Gospel, the title, the dignity, and the habit of philosophers. Ammonius Saccas was its principal founder, who was succeeded by his disciple Plotinus, as this latter was by Porphyry, the chief of those formed in his school. From the time of Ammonius until the sixth century, this was almost the only system of philosophy publicly taught at Alexandria. It was brought into Greece by Plutarch, who renewed at Athens the celebrated academy, from whence issued many illustrious philosophers. The general principle on which this sect was founded, was, that truth was to be pursued with the utmost liberty, and to be collected from all the different systems in which it lay dispersed. But none that were desirous of being ranked among these new Platonists, called in question the main doctrines; those, for example, which regarded the existence of one God, the fountain of all things; the eternity of the world; the dependence of matter upon the Supreme Being; the nature of souls; the plurality of gods, &c.

In the fourth century, under the reign of Valentinian, a dreadful storm of persecution arose against the Platonists; many of whom, being accused of magical practices, and other crimes, were capitally convicted.

In the fifth century Proclus gave new life to the doctrine of Plato, and restored it to its former credit in Greece; with whom concurred many of the Christian doctors, who adopted the Platonic system. The Platonic philosophers were generally opposers of Christianity; but in the sixth century, Chalcidius gave the Pagan system an evangelical aspect; and those who, before it became the religion of the state, ranged themselves under the standard of Plato, now repaired to that of Christ, without any great change of their system.

Under the emperor Justinian, who issued a particular edict, prohibiting the teaching of philosophy at Athens, which edict seems to have been levelled at modern Platonism, all the celebrated philosophers of this sect took refuge among the Persians, who were at that time the enemies of Rome; and though they returned from their voluntary exile, when the peace was concluded between the Persians and Romans, in 533, they could never recover their former credit, nor obtain the direction of the public schools.

Platonism however prevailed among the Greeks, and was by them, and particularly by Gemistius Pletho, introduced into Italy, and established, under the auspices of Cosmo de Medicis, about the year 1439, who ordered Marsilius Ficinus to translate into Latin the works of the most renowned Platonists.

PLATONISTS, the followers of Plato; otherwise called Academics, from Academia, the name of the place that philosopher chose for his residence at Athens.

PLEIADES, an assemblage of seven stars in the neck of the constellation Taurus, the bull; though there are now only six of them visible to the naked eye. The largest of these is of the third magnitude, and called Lucido Pleiadum.—The Greeks fabled, that the name Pleiades was given to these stars from seven daughters of Atlas and Pleione, one of the daughters of Oceanus, who having been the nurses of Bacchus, were for their services taken up to heaven and placed there as stars, where they still shine. The meaning of which fable may be, that Atlas first observed these stars, and called them by the names of the daughters of his wife Pleione.

PLENILUNIUM, the full-moon.

PLENUM, in Physics, signifies that state of things, in which every part of space, or extension, is supposed to be full of matter: in opposition to a vacuum, which is a space devoid of all matter.—The Cartesians held the doctrine of an absolute plenum; namely on this principle, that the essence of matter consists in extension; and consequently, there being every where extension or space, there is every where matter: which is little better than begging the question.

PLINTH, in Architecture, a flat square member in form of a brick or tile; used as the foot or foundation of columns and pillars, &c.

PLOT, in Surveying, the plan or draught of any parcel of ground; as a field, farm, or manor, &c.

PLOTTING, in Surveying, the describing or laying down on paper, the several angles and lines, &c, of a tract of land, that has been surveyed and measured. Plotting is usually performed by two instruments, the protractor and plotting-scale; the former serving to lay off all the angles that have been measured and set down, and the latter all the measured lines. See these two instruments under their respective names.

Plotting *Scale*, a mathematical instrument chiefly used for the plotting of grounds in surveying, or setting off the lengths of the lines. It is either 6, 9, or 12 inches in length, and about an inch and half broad; being made of box-wood, brass, ivory, or silver; those of ivory are the neatest.

This instrument contains various scales or divided lines, on both sides of it. On the one side are a number of plane scales, or scales of equal divisions, each of a different number to the inch; as also scales of chords, for laying down angles; and sometimes even the degrees of a circle marked on one edge, answering to a centre marked on the opposite edge, by which means it serves also as a protractor. On the other side are several diagonal scales, of different sizes, or different divisions to the inch; serving to take off lines expressed by numbers to three dimensions, as units, tens, and hundreds; as also a scale of divisions which are the 100th parts of a foot. But the most useful of all the lines that can be laid upon this instrument, though not always done, is a line or plane scale upon the two opposite edges, made thin for that purpose. This is a very useful line in surveying; for by laying the instrument down upon the paper, with its divided edge along a line upon which are to be laid off several distances, for the places of off-sets, &c; these distances are all transferred at once from the instrument to the line on the paper, by making small marks or points against the respective divisions on the edge of the scale. See fig. 2 and 3, plates 26 and 27.

Plotting-*Table*, in Surveying, is used for a plane table, as improved by Mr. Beighton, who has obviated many inconveniences attending the use of the common plane table. See Philos. Trans. numb. 461.

PLOUGH, or Plow, in Navigation, an ancient mathematical instrument, made of box or pear-tree, and used to take the height of the sun or stars, in order to find the latitude. This instrument admits of the degrees to be very large, and has been much esteemed by many artists; though now quite out of use.

Plough-*Monday*, the first after Epiphany, or Jan. 6.

PLUMB-Line, a term among artificers for a line perpendicular to the horizon.

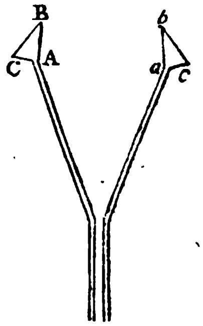
PLUMMET, Plumb-rule, or Plumb-line, an instrument used by masons, carpenters, &c, to draw perpendiculars; in order to judge whether walls, &c, be upright, or planes horizontal, and the like.

PLUNGER, in Mechanics, a solid brass cylinder, used as a forcer in forcing-pumps.

PLUS, in Algebra, the affirmative or positive sign +, signifying more or addition, or that the quantity following it is either to be considered as a positive or affirmative quantity, or that it is to be added to the other quantities; so 4 + 6 = 10, is read thus, 4 plus 6 is equal to 10. See Affirmative *Sign*. The more early writers of Algebra, as Lucas de Burgo, Cardan, Tartaglia, &c, wrote the word mostly at full length: afterwards this was contracted or abbreviated, using one or two of its first letters; which initial was, by the Germans I think, corrupted to the present character +; which I find first used by Stifelius, printed in his Arithmetic.

PLUVIAMETER, a machine for measuring the quantity of rain that falls. There is described in the Philos. Trans. (numb. 473), by Robert Pickering, under the name of an Ombrameter, an instrument of this kind. It consists of a tin funnel *d*, whose surface is an inch square (fig. 6, plate 25); a flat board *aa*; and a glass tube *bb*, set into the middle of it in a groove; and an index with divisions *cc*; the board and tube being of any length at pleasure. The bore of the tube is about half an inch, which Mr. Pickering says is the best size. The machine is fixed in some free and open place, as the top of the house, &c.

The rain-gauge employed at the house of the Royal Society is described by Mr. Cavendish, in the Philos. Trans. for 1776, p. 384. The vessel which receives the rain is a conical funnel, strengthened at the top by a brass ring, 12 inches in diameter. The sides of the funnel and inner lip of the brass ring are inclined to the horizon, in an angle of above 65°; and the outer lip in an angle of above 50°; which are such degrees of steepness, that there seems no probability either that any rain which falls within the funnel, or on the inner lip of the ring, shall dash out, or that any which falls on the outer lip shall dash into the funnel. The annexed figure is a vertical section of the funnel, ABC and *abc* being the brass ring, BA and *ba* the inner lip, and BC and *bc* the outer.



In fixing pluviameters, care should be taken that the rain may have free access to them, without being impeded or overshaded by buildings, &c; and therefore the tops of houses are mostly to be preferred. Also when the quantities of rain collected in them, at different places, are compared together, the instruments ought to be fixed at the same height above the ground at both places; because at different heights the quantities are always different, even in the same place. And hence also, any register or account of rain in the pluviameter, ought to be accompanied with a note of the height above the ground the instrument is placed at. See *Quantity of* Rain.

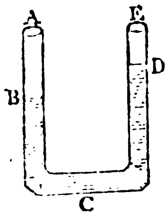
PNEUMATICS, that branch of natural philosophy which treats of the weight, pressure, and elasticity of the air, or elastic fluids, with the effects arising from them. Wolfius, instead of pneumatics, uses the term Aerometry.—This is a sister science to Hydrostatics; the one considering the air in the same manner as the other does water. And some consider pneumatics as a branch of mechanics; because it considers the air in motion, with the consequent effects.—For the nature and properties of air, see the article Air, where they are pretty largely treated of.

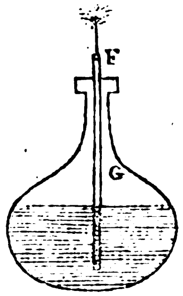
To which may be added the following, which respects more particularly the science of pneumatics, as contained in a few propositions, and their corollaries.

PROP. I. *The Air is a heavy fluid body, which surrounds and gravitates on all parts of the earth's surface.*

These properties of air are proved by experience. That it is a fluid, is evident from its easily yielding to any the least force impressed upon it, with little or no sensible resistance.—But when it is moved briskly, by any means, as by a fan, or a pair of bellows; or when any body is moved swiftly through it; in these cases we become sensible of it as a body, by the resistance it makes in such motions, and also by its impelling or blowing away any light substances. So that, being capable of resisting, or moving other bodies by its impulse, it must itself be a body, and be heavy, like all other bodies, in proportion to the matter it contains; and therefore it will press upon all bodies that are placed under it.—And being a fluid, it will spread itself over the entire surface of the earth; also like other fluids it will gravitate upon, and press every where upon the earth's surface.

The gravity and pressure of the air is also evident from many experiments. Thus, for instance, if water, or quicksilver, be poured into the tube ACE, and the air be suffered to press upon it, in both ends of the tube; the fluid will rest at the same height in both the legs: but if the air be drawn out of one end as E, by any means; then the air pressing on the other end A, will press down the fluid in this leg at B, and raise it up in the other to D, as much higher than at B, as the pressure of the air is equal to. By which it appears, not only that the air does really press, but also what the quantity of that pressure is equal to. And this is the principle of the barometer.



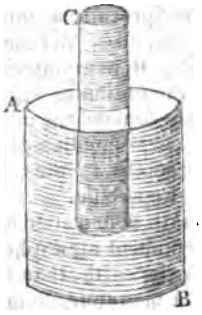


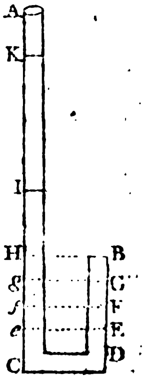
PROP. II. *The air is also an elastic fluid, being condensible and expansible. And the law it observes in this respect is this, that its density is always proportional to the force by which it is compressed.*

This property of the air is proved by many experiments. Thus, if the handle of a syringe be pushed inwards, it will condense the inclosed air into a less space; by which it is shown to be condensible. But the included air, thus condensed, will be felt to act strongly against the hand, and to resist the force compressing it more and more; and on withdrawing the hand, the handle is pushed back again to where it was at first. Which shows that the air is elastic.

Again, fill a strong bottle half full with water, and then insert a pipe into it, putting its lower end down near to the bottom, and cementing it very close round the mouth of the bottle. Then if air be strongly injected through the pipe, as by blowing with the mouth or otherwise, it will pass through the water from the lower end, and ascend up into the part before occupied by the air at G, and the whole mass of air become there condensed, because the water is not easily compressed into a less space. But on removing the force which injected the air at F, the water will begin to rise from thence in a jet, being pushed up the pipe by the increased elasticity of the air G, by which it presses on the surface of the water, and forces it through the pipe, till as much be expelled as there was air forced in; when the air at G will be reduced to the same density as at first, and, the balance being restored, the jet ceases.

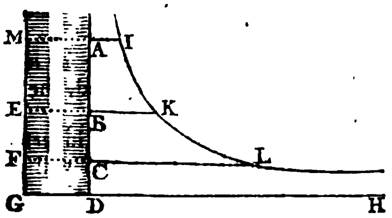
Likewise, if into a jar of water AB, be inverted an empty glass tumbler C, or such like; the water will enter it, and partly fill it, but not near so high as the water in the jar, compressing and condensing the air into a less space in the upper part C, and causing the glass to make a sensible resistance to the hand in pushing it down. But on removing the hand, the elasticity of the internal condensed air throws the glass up again.—All these showing that the air is condensible and elastic.





Again, to show the rate or proportion of the elasticity to the condensation; take a long slender glass tube, open at the top A, bent near the bottom or close end B, and equally wide throughout, or at least in the part BD (2d fig. above). Pour in a little quicksilver at A, just to cover the bottom to the bend at CD, and to stop the communication between the external air and the air in BD. Then pour in more quicksilver, and mark the corresponding heights at which it stands in the two legs: so, when it rises to H in the open leg AC, let it rise to E in the close one, reducing its included air from the natural bulk BD to the contracted space BE, by the pressure of the column He; and when the quicksilver stands at I and K, in the open leg, let it rise to F and G in the other, reducing the air to the respective spaces BF, BG, by the weights of the columns If, Kg. Then it is always found, that the condensations and elasticities are as the compressing weights, or columns of the quicksilver and the atmosphere together. So, if the natural bulk of the air BD be compressed into the spaces BE, BF, BG, or reduced by the spaces DE, DF, DG, which are $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ of BD, or as the numbers 1, 2, 3; then the atmosphere, together with the corresponding column He, If, Kg, will also be found to be in the same proportion, or as the numbers 1, 2, 3: and then the weights of the quicksilver are thus, viz, $He = \frac{1}{3}A$, $If = A$, and $Kg = 3A$; where A denotes the weight of the atmosphere. Which shows that the condensations are directly as the compressing forces. And the elasticities are also in the same proportion, since the pressures in AC are sustained by the elasticities in BD.—From the foregoing principles may be deduced many useful remarks, as in the following corollaries.

Corol. 1. The space that any quantity of air is confined in, is reciprocally as the force that compresses it. So, the forces which confine a quantity of air in the cylindrical spaces AG, BG, CG, are reciprocally as the same, or reciprocally as the heights AD, BD, CD. And therefore, if to the two perpendicular lines AD, DH, as asymptotes, the hyperbola IKL be described, and the ordinates AI, BK, CL be drawn; then the forces which confine the air in the spaces AG, BG, CG, will be as the corresponding ordinates AI, BK, CL, since these are reciprocally as the abscisses AD, BD, CD, by the nature of the hyperbola.



Corol. 2. All the air near the earth is in a state of compression, by the weight of the incumbent atmosphere.

Corol. 3. The air is denser near the earth, than in high places; or denser at the foot of a mountain, than at the top of it. And the higher above the earth, the rarer it is.

Corol. 4. The spring or elasticity of the air, is equal to the weight of the atmosphere above it; and they will produce the same effects; since they are always sustained and balanced by each other.

Corol. 5. If the density of the air be increased, preserving the same heat or temperature; its spring or elasticity will also be increased, and in the same proportion.

Corol. 6. By the gravity and pressure of the atmosphere upon the surfaces of fluids, the fluids are made to rise in any pipes or vessels, when the spring or pressure within is diminished or taken off.

PROP. III. *Heat increases the elasticity of the air, and cold diminishes it. Or heat expands, and cold contracts and condenses the air.*

This property is also proved by experience.—Thus, tie a bladder very close, with some air in it; and lay it before the fire; then as it warms, it will more and more distend the bladder, and at last burst it, if the heat be continued and increased high enough. But if the bladder be removed from the fire, it will contract again to its former state by cooling.—It was on this principle that the first air-balloons were made by Montgolfier: for by heating the air within them, by a fire underneath, the hot air distends them to a size which occupies a space in the atmosphere whose weight of common air exceeds that of the balloon.

Also, if a cup or glass, with a little air in it, be inverted into a vessel of water; and the whole be heated over the fire, or otherwise: the air in the top will expand till it fill the glass, and expel the water out of it; and part of the air itself will follow, by continuing or increasing the heat. —Many other experiments to the same effect might be adduced, all proving the properties mentioned in the proposition.

Schol. Hence, when the force of the elasticity of the air is considered, regard must be had to its heat or temperature; the same quantity of air being more or less elastic, as its heat is more or less. And it has been found by experiment that its elasticity is increased at the following rate, viz, by the 435th part, by each degree of heat expressed by Fahrenheit's thermometer, of which there are 180 between the freezing and boiling point. It has also been found (Philos. Trans. 1777, pa. 560 &c), that water expands the 6666th part, with each degree of heat; and mercury the 9600th part by each degree. Moreover, the relative or specific gravities of these three substances, are as follow:

Air	1·232	when the barom. is at 30, and the thermom. at 55.
Water	1000	
Mercury	13600	

Also these numbers are the weights of a cubic foot of each, in the same circumstances of the barometer and thermometer.

PROP. IV. *The weight or pressure of the atmosphere, upon any base at the surface of the earth, is equal to the weight of a column of quicksilver of the same base, and its height between* 28 *and* 31 *inches.*

This is proved by the barometer, an instrument which measures the pressure of the air; the description of which see under its proper article. For at some seasons, and in some places, the air sustains and balances a column of mercury of about 28 inches; but at others, it balances a column of 29, or 30, or near 31 inches high; seldom in the extremes 28 or 31, but commonly about the means 29 or 30, and indeed mostly near 30. A variation which depends partly on the different degrees of heat in the air near the surface of the earth, and partly on the commotions and changes in the atmosphere, from winds and other causes, by which it is accumulated in some places, and depressed in others, being thereby rendered denser and heavier, or rarer and lighter; which changes in its state are almost continually happening in any one place. But the medium state is from 29½ to 30 inches.

Corol. 1. Hence the pressure of the atmosphere on every square inch at the earth's surface, at a medium, is very near 15 pounds avoirdupois. For, a cubic foot of mercury weighing nearly 13600 ounces, a cubic inch of it will weigh the 1728th part of it, or almost 8 ounces, or half a pound, which is the weight of the atmosphere for every inch of the barometer on a base of a square inch; and therefore 29¾ inches, the medium height of the barometer, weighs almost 15 pounds, or rather 14¾lb very nearly.

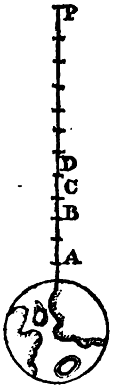
Corol. 2. Hence also the weight or pressure of the atmosphere, is equal to that of a column of water from 32 to 35 feet high, or on a medium 33 or 34 feet high. For water and quickislver are in weight nearly as 1 to 13·6; so that the atmosphere will balance a column of water 13·6 times higher than one of quicksilver; consequently 13·6 × 30 inches = 408 inches or 34 feet, is near the medium height of water, or it is more nearly 33¾ feet. And hence it appears that a common sucking pump will not raise water higher than about 34 feet. And that a syphon will not run if the perpendicular height of the top of it be more than 33 or 34 feet high.

Corol. 3. If the air were of the same uniform density, at every height, to the top of the atmosphere, as at the surface of the earth; its altitude would be about 5¼ miles at a medium. For the weights of the same volume of air and water, are nearly as 1·232 to 1000; therefore as 1·232 : 1000 : : 34 feet : 27600 feet, or 5¼ miles very nearly. And so high the atmosphere would be, if it were all of uniform density, like water. But, from its expansive and elastic quality, it becomes continually more and more rare the farther above the earth, in a certain proportion which will be treated of below.

Corol. 4. From this prop. and the last, it follows that the height is always the same, of a uniform atmosphere above any place, which shall be all of the uniform density with the air there, and of equal weight or pressure with the real height of the atmosphere above that place, whether it be at the same place at different times, or at any different places or heights above the earth; and that height is always about 27600 feet, or $5\frac{1}{4}$ miles, as found above in the 3d corollary. For, as the density varies in exact proportion to the weight of the column, it therefore requires a column of the same height in all cases, to make the respective weights or pressures. Thus, if W and w be the weights of atmosphere above any places, D and d their densities, and H and h the heights of the uniform columns, of the same densities and weights: Then $H \times D = W$, and $h \times d = w$; therefore $\frac{W}{D}$ or H is equal to $\frac{w}{d}$ or h; the temperature being the same.

PROP. V. *The density of the atmosphere, at different heights above the earth, decreases in such a proportion, that when the heights increase in arithmetical progression, the densities decrease in geometrical progression.*

Let the perpendicular line AP, erected on the earth, be conceived to be divided into a great number of very small parts A, B, C, D, &c, forming so many thin strata of air in the atmosphere, all of different density, gradually decreasing from the greatest at A: then the density of the several strata A, B, C, D, &c, will be in geometrical progression decreasing.



For, as the strata A, B, C, &c, are all of equal thickness, the quantity of matter in each of them, is as the density there; but the density in any one, being as the compressing force, is as the weight or quantity of matter from that place upward to the top of the atmosphere; therefore the quantity of matter in each stratum, is also as the whole quantity from that place upwards. Now if from the whole weight at any place as B, the weight or quantity in the stratum B be subtracted, the remainder will be the weight at the next higher stratum C; that is, from each weight subtracting a part which is proportional to itself, leaves the next weight; or, which is the same thing, from each density subtracting a part which is always proportional to itself, leaves the next density. But when any quantities are continually diminished by parts which are proportional to themselves, the remainders then form a series of continued proportionals; and consequently these densities are in geometrical progression. Thus, if the first density be D, and from each there be taken its nth part; then there remains its $\frac{n-1}{n}$ part, or the $\frac{m}{n}$ part, putting m for $n - 1$; and therefore the series of densities will be D, $\frac{m}{n}D$, $\frac{m^2}{n^2}D$, $\frac{m^3}{n^3}D$, &c, $\frac{m}{n}$ being the common ratio of the series.

Schol. Because the terms of an arithmetical series are proportional to the logarithms of the terms of a geometrical series; therefore different altitudes above the earth's surface, are as the logarithms of the densities, or weights of air, at those altitudes. So that,

if D denote the density at the altitude A,
and d the density at the altitude a;
then A being as the logarithm of D,
and a as the logarithm of d,
the dif. of altitude $A - a$ will be as
the log. of D − log. of d, or as log. of $\frac{D}{d}$.

And if $A = 0$, or D the density at the surface of the earth, then any altitude above the surface a, is as the log. of $\frac{D}{d}$. Or, in general, the log. of $\frac{D}{d}$ is as the altitude of the one place above the other, whether the lower place be at the surface of the earth, or any where else.

And from this property is derived the method of determining the heights of mountains, and other eminences, by the barometer, which is an instrument that measures the weight or density of the air at any place. See BAROMETER. For by taking with this instrument, the pressure or density at the foot of a hill for instance, and again at the top of it, the difference of the logarithms of these two pressures, or the logarithms of their quotient, will be as the difference of altitude, or as the height of the hill; supposing the temperatures of the air to be the same at both places, and the gravity of air not altered by the different distances from the earth's centre.

See more on this head under the articles ATMOSPHERE and BAROMETER.

By the weight and pressure of the atmosphere, the effect and operations of pneumatic engines may be accounted for, and explained; such as syphons, pumps, barometers, &c. See each of these articles, also AIR.

PNEUMATIC *Engine*, the same as the AIR PUMP.

POCKET *Electrical Apparatus.*—This is a contrivance of Mr. William Jones, in Holborn, the form of which is represented in plate 28, fig. 4.

This small machine is capable of a tolerably strong charge, or accumulation of electricity, and will give a small shock to one, two, three, or a greater number of persons. A is the Leyden phial or jar that holds the charge. B is the discharger to discharge the jar when required without electrifying the person that holds it. C is a ribbon prepared in a peculiar manner so as to be excited, and communicate its electricity to the jar. D are two hair, &c, skin rubbers, which are to be placed on the first and middle fingers of the left hand.

To charge the Jar. Place the two finger-caps D on the first and middle finger of the left hand; hold the jar A at the same time, at the joining of the red and black on the outside between the thumb and first finger of the same hand; then take the ribbon in your right hand, and steadily and gently draw it upwards between the two rubbers D, on the two fingers; taking care at the same time, that the brass ball of the jar is kept nearly close to the ribbon, while it is passing through the fingers. By repeating this operation twelve or fourteen times, the electrical fire will pass into the jar which will become charged, and by placing the discharger C against it, as in the plate, you will see a sensible spark pass from the ball of the jar to that of the discharger. If the apparatus be dry and in good order, you will hear the crackling of the fire when the ribbon is passing through the fingers, and the jar will discharge at the distance represented in the figure.

To electrify a Person. You must desire him to take the jar in one hand, and with the other touch the knob of it: or, if diversion is intended, desire the person to smell at the knob of it, in expectation of smelling the scent of a rose or a pink; this last mode has occasioned it to be sometimes called the Magic Smelling Bottle.

POETICAL *Rising* and *Setting*. See RISING and SETTING.—The ancient poets, referring the rising and setting of the stars to that of the sun, make three kinds of rising and setting, viz, Cosmical, Acronical, and Heliacal. See each of these words in its place.

POINT, a term used in various arts and sciences.

POINT, in Architecture. Arches of the third Point, and Arches of the fourth Point. See ARCHES.

POINT, in Astronomy, is a term applied to certain parts or places marked in the heavens, and distinguished by proper terms. The four grand points or divisions of the horizon, viz, the east, west, north, and south, are called the Cardinal Points.—The zenith and nadir are the Vertical Points.—The points where the orbits of the planets cut the plane of the ecliptic, are called the Nodes.—The points where the ecliptic and equator intersect, are called the Equinoctial Points. In particular, that where the sun ascends towards the north pole is called the Vernal Point; and that where he descends towards the south, the Autumnal Point.—The highest and lowest points of the ecliptic are called the Solstitial Points. Particularly, the former of them the Estival or Summer Point; the latter, the Brumal or Winter Point.

POINTS, in Electricity, are those acute terminations of bodies which facilitate the passage of the electrical fluid either *from* or *to* such bodies. Mr. Jallabert was probably the first person who observed that a body pointed at one end, and round at the other, produced different appearances on the same body, according as the pointed or round end was presented to it. But Dr. Franklin first observed and evinced the whole effect of pointed bodies, both in drawing and throwing off electricity at greater distances than other bodies could do it; though he candidly acknowledges, that the power of points to throw off the electric fire was communicated to him by his friend Mr. Thomas Hopkinson.

Dr. Franklin electrified an iron shot, 3 or 4 inches in diameter, and observed that it would not attract a thread when the point of a needle, communicating with the earth, was presented to it; and he found it even impossible to electrify an iron shot when a sharp needle lay upon it. This remarkable property, possessed by pointed bodies, of gradually and silently receiving or throwing off the electric fluid, has been evinced by a variety of other familiar experiments. Thus, if one hand be applied to the outside coating of a large jar fully charged, and the point of a needle, held in the other, be directed towards the knob of the jar, and moved gradually near it, till the point of the needle touch the knob or ball, the jar will be entirely discharged, so as to give no shock at all, or one that is hardly sensible. In this case the point of the needle has gradually and silently drawn away the superabundant electricity from the electrified jar.

Further, if the knob of a brass rod be held at such a distance from the prime conductor, that sparks may easily escape from the latter to the former, while the machine is in motion; then if the point of a needle be presented, though at twice the distance of the rod from the conductor, no more sparks will be seen passing to the rod. When the needle is removed, the sparks will be seen; but on presenting it again, they will again disappear. So that the point of the needle draws off silently almost all the fluid, which is thrown by the cylinder or globe of the machine upon the prime conductor. This experiment may be varied, by fixing the needle upon the prime conductor with the point upward; and then, though the knob of a discharging rod, or the knuckle of the finger, be brought very near the prime conductor, and the excitation be very strong, little or no spark will be perceived.—The influence of points is also evinced in the amusing experiment, commonly called the electrical horse-race, and many others. See THUNDER-*house*.

The late Mr. Henly exhibited the efficacy of pointed bodies, by suspending a large bladder, well blown, and covered with gold, silver, or brass leaf, by means of gum-water, at the end of a silken thread 6 or 7 feet long, hanging from the ceiling of a room, and electrifying the bladder by giving it a strong spark with the knob of a charged bottle: on presenting to it the knob of a wire, it caused the bladder to move towards the knob, and when nearly in contact gave it a spark, thus discharging its electricity. By giving the bladder another charge, and presenting the point of a needle to it, the bladder was not attracted by the point, but rather receded from it, especially when the needle was suddenly presented towards it.

But experiments evincing the efficacy of pointed bodies for silently receiving or throwing off the electric fluid, may be infinitely diversified, according to the fancy or convenience of the electrician. It may be observed, that in the case of points throwing off or receiving electricity, a current of air is sensible at an electrified point, which is always in the direction of the point, whether the electricity be positive or negative. A fact which has been well ascertained by many electricians, and particularly by Dr. Priestley and Sig. Beccaria. The former contrived to exhibit the influence of this current on the flame of a candle, presented to a pointed wire, electrified negatively, as well as positively. The blast was in both cases alike, and so strong as to lay bare the greatest part of the wick, the flame being driven from the point; and the effect was the same whether the electric fluid issued out of the point or entered into it. He farther evinced this phenomenon by means of thin light vanes; and he found, as Mr. Wilson had before observed, that the vanes would not turn in vacuo, nor in a close unexhausted receiver where the air had no free circulation. And in much the same manner, Beccaria exhibited to sense the influence of the wind or current of air driven from points.

As to the *Theory* of the phenomena of points, these are accounted for in a variety of ways, by different authors, though perhaps by none with perfect satisfaction. See Franklin's writings on Electricity; Lord Mahon's Principles of Electricity, 1779; Beccaria's Artificial Electricity, 1776, pa. 331; and Priestley's History of Electricity, vol. 2, pa. 191, edit. 1775.

As to the *Application* of the doctrine of points; it may be observed that there is not a more important fact in the history of electricity, than the use to which the discovery of the efficacy of pointed bodies has been applied. Dr. Franklin, having ascertained the identity of electricity and lightning, was presently led to propose a cheap and easy method of securing buildings from the damage of lightning, by fixing a pointed metal rod higher than any part of the building, and communicating with the ground, or with the nearest water. And this contrivance was actually executed in a variety of cases; and has usually been thought an excellent preservative against the terrible effects of lightning.

Some few instances however having occurred, in which buildings have been struck and damaged, though provided with these conductors; a controversy arose with regard to their expediency and utility. In this controversy Mr. Benjamin Wilson took the lead, and Dr. Musgrave, and some few other electricians, the least acquainted with the subject, concurred with him in their opposition to pointed elevated conductors. These gentlemen allege, that every point, as such, solicits the lightning, and thus contributes not only to increase the quantity of every actual discharge, but also frequently to occasion a discharge when it might not otherwise have happened: whereas, say they, if instead of pointed conductors, those with blunted terminations were used, they would as effectually answer the purpose of conveying away the lightning safely, without the same tendency to increase or invite it. Accordingly Mr. Wilson, in a letter to the marquis of Rockingham (Philos. Trans. vol. 54, art. 44), expresses his opinion, that, in order to prevent lightning from doing mischief to high buildings, large magazines, and the like, instead of the elevated external conductors, that, on the inside of the highest part of such building, and within a foot or two of the top, it may be proper to fix a rounded bar of metal, and to continue it down along the side of the wall to any kind of moisture in the ground.

On the other hand, it is urged by the advocates for pointed conductors, that points, instead of increasing an actual discharge, really prevent a discharge where it would otherwise happen, and that blunted conductors tend to invite the clouds charged with lightning. And it seems to be a certain fact, that though a sharp point will draw off a charge of electricity silently at a much greater distance than a knob, yet a knob will be struck with a full explosion or shock, the charge being the same in both cases, at a greater distance than a sharp point.

The efficacy of pointed bodies for preventing a stroke of lightning, is ingeniously explained by Dr. Franklin in the following manner:—An eye, he says, so situated as to view horizontally the underside of a thunder-cloud, will see it very ragged; with a number of separate fragments or small clouds one under another; the lowest sometimes not far from the earth. These, as so many stepping-stones, assist in conducting a stroke between a cloud and a building. To represent these by an experiment, he takes two or three locks of fine loose cotton, and connects one of them with the prime conductor by a fine thread of 2 inches, another to that, and a third to the second, by like threads, which may be spun out of the same cotton. Then by turning the globe, all these locks will extend themselves towards the table, as the lower small clouds do towards the earth; but on presenting a sharp point, erect under the lowest, it will shrink up to the second, the second up to the first, and all together to the prime conductor; where they will continue as long as the point continues under them. May not, he adds, in like manner, the small electrified clouds, whose equilibrium with the earth is soon restored by the point, rise up to the main body, and by that means occasion so large a vacancy, as that the grand cloud cannot strike in that place? Letters, pa. 121.

Mr. Henly too, as well as several other persons, with a view of determining the question, whether points or knobs are to be preferred for the terminations of conductors, made several experiments, showing in a variety of instances, the efficacy of points in silently drawing off the electricity, and preventing strokes which would happen to knobs in the same situation. Philos. Trans. vol. 4, part 2, art. 18. See also THUNDER-*House*.

Indeed it has been universally allowed, that in cases where the quantity of electricity, with which thunder-clouds are charged, is small, or when they move slowly in their passage to and over a building, pointed conductors, which draw off the electrical fluid silently, within the distance at which rounded ends will explode, will gradually exhaust them, and thus contribute to prevent a stroke and preserve the buildings to which they are annexed.

But it has been said by those who are averse to the use of such conductors, that if clouds, of great extent, and highly electrified, should be driven directly over them with great velocity, or if a cloud hanging directly over buildings to which they are annexed, suddenly receives a charge by explosion from another cloud at a distance, so as to enable it instantly to strike into the earth, these pointed conductors must take the explosion; on account of their greater readiness to admit electricity at a much greater distance than those that are blunted, and in proportion to the difference of that striking distance, do mischief instead of good: and therefore, they add, that such pointed conductors, though they may be sometimes advantageous, are yet at other times prejudicial; and that, as the purpose for which conductors are fixed upon buildings, is not to protect them from one particular kind of clouds only, but if possible from all, it cannot be advisable to use that kind of conductors which, if they diminish danger on the one hand, will increase it on the other. Besides, it is alleged, that if pointed conductors are attended with any the slightest degree of danger, that danger must be considerably augmented by carrying them high up into the air, and by fixing them upon every angle of a building, and by making them project in every direction. Such is the reasoning of Dr. Musgrave: see his paper in the Philos. Trans. vol. 68, part 2, art 36.

Mr. Wilson too, dissenting from the report of a committee of the Royal Society, appointed to inspect the damage done by lightning to the house of the Board of Ordnance, at Purfleet, in 1777, was led to justify his dissent, and to disparage the use of pointed and elevated conductors, by means of a magnificent apparatus which he constructed, and with which he might produce effects similar to those that had happened in the case referred to the consideration and decision of the committee. With this view he procured a model of the Board-house at Purfleet, resembling it as nearly as possible in every essential appendage, and furnished with conductors of different lengths and terminations. And to construct a substitute for a cloud, he joined together the broad rims of 120 drums, forming together a cylinder of 155 feet in length, and above 16 inches in diameter; and this immense cylinder, of about 600 square feet of coated surface, was connected occasionally with one end of a wire 4800 feet long. As this bulky apparatus, representing the thunder-cloud, could not conveniently be put in motion, he contrived to accomplish the same end by moving the model of the building, with a velocity answering to that of the cloud, which he states, at a moderate computation, to be about 4 or 5 miles an hour. This apparatus was charged by a machine with one glass cylinder, about 10 or 11 feet from its nearest end; and the whole of the apparatus was disposed in the great room of the Pantheon, and applied to use in a variety of experiments. But it is impossible within the limits of this article to do justice to Mr. Wilson's experiments, or to the inferences which he deduced from them: we can only observe, that most of his experiments, in which the model of the house, which was passed swiftly under the artificial cloud, and having annexed to it either the pointed or blunt conductors at the same or different heights, were intended to show, that pointed conductors are struck at a greater distance, and with a higher elevation, than the blunted ones: and from all his experiments made with pointed and rounded conductors, provided the circumstances be the same in both, he infers, that the rounded ones are much the safer of the two; whether the lightning proceeds from one cloud or from several; that those are still safer which rise little or nothing above the highest part of the building; and that this safety arises from the greatest resistance exerted at the larger surface. See Philos. Trans. for 1778, pa. 232.

The committee of the Royal Society however, which was composed of nine of the most distinguished electricians in the kingdom, and to whom was referred the consideration of the most effectual method of securing the powder-magazines at Purfleet against the effects of lightning, express their united opinion, that elevated sharp rods, constructed and disposed in the manner which they direct, are preferable to low conductors terminated in rounded ends, knobs, or balls of metal; and that the experiments and reasonings, made and alleged to the contrary by Mr. Wilson, are inconclusive.

Mr. Nairne also, in order to obviate the objections of Mr. Wilson and others, and to vindicate the preference generally given to high and pointed conductors, constructed a much more simple apparatus than that of Mr. Wilson, with which he made a number of well-designed and well-conducted experiments, which appear to prove the superiority of the pointed conductor as far as it is capable of being proved by an artificial electrical apparatus. From these last experiments it appears, that though the point was struck by means of a swift motion of the artificial cloud, yet a small ball of 3 tenths of an inch diameter was struck further off than the point, and a larger ball at a much greater distance than either, even with the swiftest motion. Upon the whole, Mr. Nairne seems to be justified in preferring elevated pointed conductors; next to them, those that are pointed, though they rise but little above the highest part of a building; and after them, those that are terminated in a ball, and placed even with the highest part of the building. See Philos. Trans. 1778, pa. 823.

On the other part, Dr. Musgrave, not yet satisfied, gave in another paper, being "Reasons for dissenting from the Report of the Committee appointed to consider of Mr. Wilson's Experiments; including Remarks on some Experiments exhibited by Mr. Nairne;" which is inserted, by mistake, before Mr. Nairne's paper, being at pa. 801 of the same volume.

And further, Mr. Wilson has another paper, on the same subject, at pa. 999 of the same vol. of Philos. Trans. for 1778, entitled, "New Experiments upon the Leyden Phial, respecting the termination of conductors;" repeating and asserting his former objections and reasonings.

In the Philos. Trans. too, for 1779, pa. 454, Mr. William Swift has a paper, further prosecuting this subject; making various experiments with simple and ingenious machinery, with models of houses and clouds, and with various kinds of conductors. From the experiments he infers in general, that "the whole current of these experiments tends to show the preference of points to balls, in order to diminish and draw off the electric matter when excited, or to prevent it from accumulating; and consequently the propriety or even necessity of terminating all conductors with points, to make them useful to prevent damage to buildings from lightning. Nay the very construction of all electrical machines, in which it is necessary to round all the parts, and to avoid making edges and points which would hinder the matter from being excited, will, I imagine, on reflection, be another corroborating proof of the result of the experiments themselves."

There were other communications made to the Royal Society on the important subject of conductors, some of which were received, and others rejected. On the whole, this contest turned out one of the most extraordinary that ever was agitated in the Society; producing the most remarkable disputes, differences, and strange consequences, that ever the Society experienced since it had existence; consequences which manifested themselves in various instances for many years after, and which continue to this very day. All which, with the various secret springs and astonishing intrigues, may probably be given to the public on some other occasion.

Point, in Geometry, according to Euclid, is that which has no parts, or is indivisible; being void of all extension, both as to length, breadth, and depth.

This is what is otherwise called the Mathematical Point, being the intersection of two lines, and is only conceived by the imagination; yet it is in this that all magnitude begins and ends; the extremes of a line being points; the extremes of a surface, lines; and the extremes of a solid, surfaces. And hence some define a point, the inceptive of magnitude.

Proportion of Mathematical **Points**. It is a popular maxim, that all infinites are equal; yet is the maxim false, whether of quantities infinitely great, or infinitely little. Dr. Halley, and others have shown that there are infinite quantities which are in a finite proportion to each other; and some that are infinitely greater than others. See **Infinite** *Quantity*.

And the same is shown by Mr. Robarts, of infinitely small quantities, or mathematical Points. He demonstrates, for instance, that the points of contact between circles and their tangents, are in the subduplicate ratio of the diameters of the circles; that the point of contact between a sphere and a plane is infinitely greater than between a circle and a line; and that the points of contact in spheres of different magnitudes, are to each other as the diameters of the spheres. Philos. Trans. vol. 27, pa. 470.

Conjugate **Point**, is used for that point into which the conjugate oval, belonging to some kind of curves, vanishes. Maclaurin's Algebra, pa. 308.

Point *of Contrary Flexure*, &c. See **Inflexion**, **Retrogradation** or **Retrogression**, &c, of curves.

Points *of the Compass*, or *Horizon*, &c, in Geography and Navigation, are the points of division when the whole circle, quite around, is divided into 32 equal parts. These points are therefore at the distance of the 32d part of the circle, or 11°15′, from each other; hence 5° 37′½ is the distance of the half points, and $2^{\circ}\ 48'\frac{3}{4}$ is the distance of the quarter points. See Compass. The principal of these are the four cardinal points, east, west, north, and south.

Point is also used for a cape or headland, jutting out into the sea.—The seamen say two points of land are one in another, when they are in a right line, the one behind the other.

Point, in Optics. As the

Point *of Concourse* or *Concurrence*, is that in which converging rays meet; and is usually called focus.

Point of *Dispersion*, *Incidence*, *Reflection*, *Refraction*, and *Radiant* Point. See these several articles.

Point, in Perspective, is a term used for various parts or places, with regard to the perspective plane. As, the

Point *of Sight*, or *of the Eye*, called also the Principal Point, is the point on a plane where a perpendicular from the eye meets it. See Perspective. Some authors, however, by the Point of Sight, or Vision, mean the point where the eye is actually placed, and where all the rays terminate. See Perspective.

Point *of Distance*, is a point in a horizontal line, at the same distance from the principal point as the eye is from the same. See Perspective.

Third Point, is a point taken at discretion in the line of distance, where all the diagonals meet that are drawn from the divisions of the geometrical plane.

Objective Point, is a point on a geometrical plane, whose representation on the perspective plane is required.

Accidental Point, and *Visual* Point. See Accidental and Visual.

Point *of View*, with regard to Building, Painting, &c, is a point at a certain distance from a building, or other object, where the eye has the most advantageous view or prospect of the same. And this point is usually at a distance equal to the height of the building.

Point, in Physics, is the smallest or least sensible object of sight, marked with a pen, or point of a compass, or the like. This is popularly called a Physical point; and of such does all physical magnitude consist.

Point-Blanc, *Point-Blank*, in Gunnery, denotes the horizontal or level position of a gun, or having its muzzle neither elevated nor depressed. And the point-blanc range, is the distance the shot goes, before it strikes the level ground, when discharged in the horizontal or point-blanc direction. Or sometimes this means the distance the ball goes horizontally in a straight-lined direction.

POINTING, in Artillery and Gunnery, is the laying a piece of ordnance in any proposed direction, either horizontal, or elevated, or depressed, to any angle. This is usually effected by means of the gunner's quadrant, which, being applied to, or in, the muzzle of the piece, shows by a plummet the degree of elevation or depression.

Pointing, in Navigation, is the marking on the chart in what point, or place, the vessel is.—This is done by means of the latitude and longitude, after these are known, or found by observation or computation. Thus, draw a line, with a pencil, across the chart according to the latitude; and another across the other way according to the longitude; then the intersection of these two lines, is the point or place on the chart where the ship is; which is then marked black with a pen, and the pencil lines rubbed out. From the point or place, thus found, the chart readily shows the direct distance and course run, as also that still to run to the intended port, &c.

POLAR, something that relates to the poles of the world: as polar virtue, polar tendency.

Polar *Circles*, are two lesser circles of the sphere, or globe, one about each pole, and at the same distance from it as is equal to the sun's greatest declination or the obliquity of the ecliptic; that is, at present 23° 28'.—The space included within each polar circle, is the frigid zone; and to every part of this space, the sun never sets at some time of the year, and never rises at another time; each of these being a longer duration as the place is nearer the pole.

Polar *Dials*, are such as have their planes parallel to some great circle passing through the poles, or to some one of the hour-circles; so that the pole is neither elevated above the plane, nor depressed below it.—This dial, therefore, can have no centre; and consequently its style, substyle, and hour-lines, are parallel.—This will therefore be an horizontal dial to those who live at the equator.

Polar *Projection*, is a representation of the earth, or heavens, projected on the plane of one of the polar circles.

Polar *Regions*, are those parts of the earth which lie near the north and south poles.

POLARITY, the quality of a thing having poles, or pointing to, or respecting some pole: as the magnetic needle, &c.—By heating an iron bar, and letting it cool again in a vertical position, it acquires a polarity, or magnetic virtue: the lower end becoming the north pole, and the upper end the south pole. But iron bars acquire a polarity by barely continuing a long time in an erect position, even without heating them. Thus, the upright iron bars of some windows, &c, are often found to have poles: Nay, an iron rod acquires a polarity, by the mere holding it erect; the lower end, in that case, attracting the south end of a magnetic needle; and the upper, the north end. But these poles are mutable, and shift with the situation of the rod.—Some modern writers, particularly Dr. Higgins, in his Philosophical Essay concerning Light, have maintained the polarity of the parts of matter, or that their simple attractions are more forcible in one direction, or axis of each atom, than in any other.

POLES, in Astronomy, the extremities of the axis on which the whole sphere of the world revolves; or the points on the surface of the sphere through which the axis passes. These are on every side at the distance of a quadrant, or 90°, from every point of the equinoctial, and are called, by way of eminence, the poles of the world. That which is visible to us in Europe, or raised above our horizon, is called the Arctic or North Pole; and its opposite one, the Antarctic or South Pole.

Poles, in Geography, are the extremities of the earth's axis; or the points on the surface of the earth through which the axis passes. Of which, that elevated above our horizon is called the Arctic or North Pole; and the opposite one, the Antarctic or South Pole.

In consequence of the situation of the poles, with the inclination of the earth's axis, and its parallelism during the annual motion of our globe round the sun, the poles have only one day and one night throughout the year, each being half a year in length. And because of the obliquity with which the rays of the sun fall upon the polar regions, and the great length of the night in the winter season, it is commonly supposed the cold is so intense, that those parts of the globe which lie near the poles have never been fully explored, though the attempt has been repeatedly made by the most celebrated navigators. And yet Dr. Halley was of opinion, that the solstitial day, at the pole, is as hot as at the equator when the sun is in the zenith; because all the 24 hours of that day under the pole the sun-beams are inclined to the horizon in an angle of 23° 28′; whereas at the equator, though the sun becomes vertical, yet he shines no more than 12 hours, being absent the other 12 hours: and besides, that during 3 hours 8 minutes of the 12 hours which he is above the horizon there, he is not so much elevated as at the pole. Experience however seems to show that this opinion and reasoning of Dr. Halley are not well founded: for in all the parts of the earth that we know, the middle of summer is always the less hot the farther the place is from the equator, or the nearer it is to the pole.

The great object for which navigators have ventured themselves in the frozen seas about the north pole, was to find out a more quick and ready passage to the East Indies. And this has been attempted three several ways: one by coasting along the northern parts of Europe and Asia, called the north-east passage; another, by sailing round the northern part of the American continent, called the north-west passage; and the third, by sailing directly over the pole itself.

The possibility of succeeding in the north-east was for a long time believed; and in the last century many navigators, particularly the Hollanders, attempted it with great fortitude and perseverance. But it was always found impossible to surmount the obstacles which nature had thrown in the way; and subsequent attempts have in a manner demonstrated the impossibility of ever sailing eastward along the northern coast of Asia. The reason of this impossibility is, that in proportion to the extent of land, the cold is always greater in winter, and *vice versa.* This is the case even in temperate climates; but much more so in those frozen regions when the sun's influence, even in summer, is but small. Hence, as the continent of Asia extends a vast way from west to east, and has besides the continent of Europe joined to it on the west, it follows, that about the middle part of that tract of land the cold should be greater than any where else. Experience has determined this to be fact; and it now appears, that about the middle of the northern part of Asia, the ice never thaws; neither have even the hardy Russians and Siberians themselves been able to overcome the difficulties they meet with in that part of their voyages.

With regard to the north-west passage, the same difficulties occur as in the other. According to Captain Cook's voyage, it appears that if there is any strait which divides the continent of America into two, it must lie in a higher latitude than 70°, and consequently be perpetually frozen up. And therefore if a north-west passage can be found, it must be by sailing round the whole American continent, instead of seeking a passage through it, which some have supposed to exist in the bottom of Baffin's Bay. But the extent of the American continent to the northward is yet unknown; and there is a possibility of its being joined to that part of Asia between the Piasida and Chatanga, which has never yet been circumnavigated. Indeed a rumour has lately gone abroad of some remarkable inlet being observed on the western coast of North America, which it is guessed may possibly lead to some communication with the eastern side, by the lakes, or a passage into Hudson's Bay: but there seems little or no probability of any success this way, in which many fruitless attempts have been made at various times. It remains therefore to consider, whether there is any probability of attaining the wished-for passage by sailing directly north, between the eastern and western continents.

The late celebrated mathematician, Mr. Maclaurin, was so fully persuaded of the practicability of passing by this way to the South and Indian seas, that he used to say, if his other avocations would permit, he would undertake the voyage of trial, even at his own expense. The practicability of this method, which would lead directly to the pole itself, has also been ingeniously supported by Mr. Daines Barrington, in some tracts published in the years 1775 and 1776, in consequence of the unsuccessful attempt made by captain Phipps in the year 1773, to reach a higher northern latitude than 81°. Mr. Barrington instances a great number of navigators who have reached very high northern latitudes; nay, some who have been at the pole itself, or gone beyond it. From all which he concludes, that if the voyage be attempted at a proper time of the year, there would not be any great difficulty in reaching the pole. Those vast pieces of ice which commonly obstruct the navigators, he thinks, proceed from the mouths of the great Asiatic rivers which run northward into the frozen ocean, and are driven eastward and westward by the currents. But, though we should suppose them to come directly from the pole, still our author thinks that this affords an undeniable proof that the pole itself is free from ice; because, when the pieces leave it, and come to the southward, it is impossible that they can at the same time accumulate at the pole.

The Altitude or Elevation of the POLE, is an arch of the meridian intercepted between the pole and the horizon of any place, and is equal to the latitude of the place.

To observe the Altitude of the POLE. With a quadrant, observe both the greatest and least meridian altitude of the pole star. Then half the sum of the two altitudes, will be the height of the pole, or the latitude of the place; and half the difference of the same will be the distance of the star from the pole. But, for accuracy, the observed altitudes should be corrected on account of refraction, before their sum or difference is taken. See REFRACTION.

POLE, in Spherics, or the pole of a great circle, is a point on the sphere equally distant from every part of the circumference of the great circle; or a point 90° distant from the circumference of any part of it.—The zenith and nadir are the poles of the horizon; and the poles of the equator are the same with those of the sphere or globe.

POLES, in Magnetism, are two points in a loadstone, corresponding to the poles of the world; one pointing to the north, and the other to the south. If the stone be broken in ever so many pieces, every fragment will still have its two poles. And if a magnet be bisected by a plane perpendicular to the axis; the two points before joined will become opposite poles, one in each segment.—In touching a needle, &c, with a magnet, that part intended for the north end must be touched with the south pole of the magnet; and that intended for the south end, with the north pole; for the poles of the needle become contrary to those of the magnet.—A piece of iron acquires a polarity by only holding it upright; though its poles are not fixed, but shift, and are inverted as the iron is. Fire destroys all fixed poles; but it strengthens the mutable ones.

Dr. Gilbert says, the end of a rod being heated, and left to cool pointing northward, it becomes a fixed north pole;

if southward, a fixed south pole. When the end is cooled while held downward, it acquires rather more magnetism than if cooled horizontally towards the north. But the best way is to cool it a little inclined to the north. Repeating the operations of heating and cooling does not increase the effect.

Dr. Power says, if a rod be held northwards, and the north end be hammered in that position, it will become a fixed north pole; and contrarily if the south end be hammered. The heavier the blows are, cæteris paribus, the stronger will the magnetism be; and a few hard blows have as much effect as a great number. And what is said of hammering, is to be likewise understood of filing, grinding, sawing, &c; nay, a gentle rubbing, when long continued, will produce poles.

Old punches and drills have all fixed north poles; because they are almost constantly used downwards. New drills have either mutable poles, or weak north ones. Drilling with such a one southward horizontally, it is a chance if you produce a fixed south pole; much less if you drill south downwards; but by drilling south upwards, you always make a fixed south pole. Mr. Ballard says, that in 6 or 7 drills, made in his presence, the bit of each became a north pole, merely by hardening.—A weak fixed pole may degenerate into a mutable one in a day, or even in a few minutes, by holding it in a position contrary to its pole. The loadstone itself will not make a fixed pole in every piece of iron: if the iron be thick, it is necessary that it have some considerable length.

Pole *of a Glass*, in Optics, is the thickest part of a convex glass, or the thinnest part of a concave one; being the same as what is otherwise called the vertex of the glass; and which, when truly ground, is exactly in the middle of its surface.

Pole, or *Rod*, in Surveying, is a lineal measure containing 5½ yards, or 16½ feet.—The square of it is called a square pole; but more usually a perch, or a rod.

Pole-Star, is a star of the 2d magnitude near the north pole, in the end of the tail of ursa minor, or the Little Bear. Its mean place in the heavens for the beginning of 1810, was as follows: viz,

Right ascension - -	13°	36′	15″
Annual variat. in ditto -	0	3	8
Declination - -	88	17	41
Annual variat. in ditto -	0	0	19$\frac{6}{10}$

The proximity of this star to the pole, on which account it is always above the horizon in these northern latitudes, makes it very useful in navigation, &c, for determining the meridian line, the elevation of the pole, and consequently the latitude of the place, &c.

POLEMOSCOPE, in Optics, an oblique kind of prospective glass, contrived for the seeing of objects that do not lie directly before the eye. It was invented by Hevelius, in 1637. See Opera *Glass*.

POLITICAL *Arithmetic*, the application of arithmetical calculations to political uses and subjects; such as the public revenues, the number of people, the extent and value of lands, taxes, trade, commerce, or whatever relates to the power, strength, riches, &c, of a nation or commonwealth. Or, as Davenant concisely defines it, the art of reasoning by figures, upon things relating to government. The chief authors who have attempted calculations of this kind, are, Sir William Petty, Major Graunt, Dr. Halley, Dr. Davenant, Mr. King, Dr. Price, M. Kerseboom, and M. de Parcieux.

Sir William Petty, among many other articles, states that, in his time, the people in England were about 6 millions, and their annual expense about 7*l.* each; that the rent of the lands was about 8 millions, and the interests and profits of the personal estates as much; that the rent of the houses in England was 4 millions, and the profits of the labour of all the people 26 millions yearly; that the corn used in England, at 5*s.* the bushel for wheat, and 2*s.* 6*d.* for barley, amounts to 10 millions per annum; that the navy of England required 36,000 men to man it, and the trade and other shipping about 48,000; that the whole number of people in England, Scotland, and Ireland, together, were about 9 millions and a half; and those in France about 13 millions and a half; and in the whole world about 350 millions; also that the whole cash of England, in current money, was then about 6 millions sterling. See his Political Arith. p. 74, &c.

Dr. Davenant gives some good reasons why many of Sir W. Petty's numbers are not to be entirely depended on; and advances others of his own, founded on the observations of Mr. Greg. King. Some of the particulars are, that the land of England is 39 millions of acres; that the number of people in London was about 530,000, and in all England five millions and a half, increasing 9000 annually, or about the 600th part; the yearly rent of the lands 10 millions, and that of the houses 2 millions; the produce of all kinds of grain 9 millions. Davenant's Essay on the probable methods, &c, in his works, vol. 6.

Major Graunt, in his observations on the bills of mortality, computes, that there are 39,000 square miles of land in England, or 25 million acres in England and Wales, and 4,600,000 persons, making about 5 acres and a half to each person; that the people of London were 640,000; and states the several numbers of persons living at the different ages.

Sir William Petty, in his discourse about duplicate proportion, further states, that it is found by experience, that there are more persons living between 16 and 26 than of any other age; and from thence he infers, that the square roots of every number of men's ages under 16, whose root is 4, show the proportion of the probability of such persons reaching the age of 70 years: thus, the probability of reaching that age by persons of the

ages of 16, 9, 4, and 1,
are as 4, 3, 2, 1, respectively.

Also that the probabilities of their order of dying, at ages above that, are as the square roots of the ages: thus, the probabilities of the order of dying first,

of the ages 16, 25, 36, &c,
are as the roots 4, 5, 6, &c,

that is, the odds are 5 to 4 that a person of 25 dies before one of 16, and so on, declining up to 70 years of age.

Dr. Halley has made a very exact estimation of the degrees of mortality of mankind, from a curious table of the births and burials, at the city of Breslau, in Silesia; with an attempt to ascertain the price of annuities upon lives, and many other curious particulars. See the Philos. Trans. vol. 17, pa. 596. Another table of this kind is given by Simpson, for the city of London; and several by Price, Morgan and Baily, for many different places.

Mr. Kerseboom, of Holland, has many and curious calculations and tables of the same kind. From his observations on the births of the people in England, it appears, that the number of males born, is in proportion to that of the females, as 18 to 17; and that of the inhabitants living in Holland are in the same proportion.

Dr. Brackenridge has given an estimate of the number of people in England, formed both from the number of houses, and also from the quantity of bread consumed. On the former principle, he finds the number of houses in England and Wales to be about 900,000; and, allowing 6 persons to each house, the number of people near 5 millions and a half. And on the latter principle, estimating the quantity of corn consumed at home at 2 millions of quarters, and 3 persons to every quarter of corn, makes the number of people 6 millions. See Philos. Trans. vol. 49, art. 45 and 113.

Dr. Derham, from a great number of registers of places, finds the proportions of the marriages to the births and burials; and Dr. Price has done the same for still more places; the mediums of all which are,

	Marriages to Births, as
Dr. Derham - -	1 to 4·7
Dr. Price - -	1 to 3·9

See Philos. Trans. No. 480; also Dr. Price's Observations on Reversionary Payments; and the articles of this Dictionary, EXPECTATION *of Life*, LIFE-*Annuities*, MORTALITY, POPULATION, &c.

POLLUX, in Astronomy, the hind twin, or the posterior part of the constellation Gemini.

POLLUX is also a fixed star of the second magnitude, in the constellation Gemini, or the Twins. See CASTOR and *Pollux*, also GEMINI.

POLYACOUSTICS, instruments contrived to multiply sounds, as polyscopes or multiplying glasses do the images of objects.

POLYEDRON. See POLYHEDRON.

POLYGON, in Geometry, a figure of many sides; and consequently of many angles also; for every figure has as many sides as angles. If the angles be all equal among themselves, the polygon is said to be a regular one; otherwise, it is irregular. Polygons also take particular names according to the number of their sides; thus a Polygon of

3 sides is called a trigon,
4 sides - a tetragon,
5 sides - a pentagon,
6 sides - a hexagon, &c;

and a circle may be considered as a polygon of an infinite number of small sides, or as the limit of the polygons.

Polygons have various properties, as below:

1. Every polygon may be divided into as many triangles as it hath sides.

2. The angles of any polygon taken together, make twice as many right angles, wanting 4, as the figure hath sides. Thus, if the polygon has 5 sides; the double of that is 10, from which subtracting 4, leaves 6 right angles, or 540 degrees, which is the sum of the 5 angles of the pentagon. And this property, as well as the former, belongs to both regular and irregular polygons.

3. Every regular polygon may be either inscribed in a circle, or described about it. But not so of the irregular ones, except the triangle, and another particular case as in the following property: An equilateral figure inscribed in a circle, is always equiangular.—But an equiangular figure inscribed in a circle is not always equilateral, but only when the number of sides is odd. For if the sides be of an even number, then they may either be all equal; or else half of them may be equal, and the other half equal to each other, but different from the former half, the equals being placed alternately.

4. Every polygon, circumscribed about a circle, is equal to a right-angled triangle, of which one leg is the radius of the circle, and the other the perimeter or sum of all the sides of the polygon. Or the polygon is equal to half the rectangle under its perimeter and the radius of its inscribed circle, or the perpendicular from its centre upon one side of the polygon. Hence, the area of a circle being less than that of its circumscribing polygon, and greater than that of its inscribed one, the circle is the limit of the inscribed and circumscribed polygons: in like manner the circumference of the circle is the limit between the perimeters of the said polygons; consequently the circle is equal to a right-angled triangle, having one leg equal to the radius, and the other leg equal to the circumference; and therefore its area is found by multiplying half the circumference by half the diameter. In like manner, the area of any polygon is found by multiplying half its perimeter by the perpendicular demitted from the centre upon one side.

5. In my Mensuration, pa. 15 &c, is given the geometrical construction of several polygons; by which it appears that, as the regular trigon, square, and pentagon, can be inscribed geometrically in a circle; and as an arc may be always bisected geometrically; therefore any polygon whose numbers of sides is expressed by 2^n, 3.2^n, or 5.2^n, may be inscribed in a given circle by the scale and compasses only. And it has lately been shown that a polygon, the number of whose sides is a prime number of the form $2^n + 1$, may also be inscribed geometrically in a circle, a problem that was far from being thought possible, till M. Gauss published his celebrated work entitled Disquisitiones Arithmeticæ, in which he has given a complete solution of this problem; it is however too complex to introduce in this place, and little suited to practical purposes. See PRIME NUMBER.

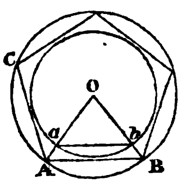
6. But though we cannot inscribe geometrically any regular polygon whatever in a circle, we have a practical method of performing it, by means of the known measure of the angles, some examples of which may be seen in the following table, which exhibits the most remarkable particulars in all the polygons, up to the dodecagon of 12 sides; viz, the angle at the centre AOB, the angle of the polygon C or CAB or double of OAB, and the area of the polygon when each side AB is 1. (See the following figure.)

No. of sides.	Name of polygon.	Ang. O. at cent.	Ang. C. of polygon.	Area.
3	Trigon	120	60	0·4330127
4	Tetragon	90	90	1·0000000
5	Pentagon	72	108	1·7204774
6	Hexagon	60	120	2·5980762
7	Heptagon	$51\frac{3}{7}$	$128\frac{4}{7}$	3·6339124
8	Octagon	45	135	4·8284271
9	Nonagon	40	140	6·1818242
10	Decagon	36	144	7·6942088
11	Undecagon	$32\frac{8}{11}$	$147\frac{3}{11}$	9·3656399
12	Dodecagon	30	150	11·1961524

By means of the numbers in this table, any polygons may be constructed, or their areas found: thus, (1st) *To inscribe a Polygon in a given Circle.* At the centre make the angle O equal to the angle at the centre of the proposed polygon, found in the 3d column of the table, the legs cutting the circle in A and B; and join A and B which will be one side of the polygon. Then take AB between the compasses, and apply it continually round the circumference, to complete the polygon.

(2d) *Upon the given Line* AB *to describe a regular Polygon.* From the extremities draw the two lines AO and BO, making the angles A and B each equal to half the angle of the polygon, found in the 4th column of the table, and their intersection O will be the centre of the circumscribed circle: then apply AB continually round the circumference as before.

(3d) *To describe a Polygon about a given Circle.*—At the centre O make the angle of the centre as in the 1st art. its legs cutting the circle in *a* and *b*: join *ab*, and parallel to it draw AB to touch the circle: and meeting O*a* and O*b* produced in A and B: with the radius OA, or OB, describe a circle, and around its circumference apply continually AB, which will complete the polygon as before.



(4th) *To find the Area of any regular Polygon*—Multiply the square of its side by the tabular area, found on the line of its name in the last column of the table, and the product will be the area. Thus, to find the area of the trigon, or equilateral triangle, whose side is 20. The square of 20 being 400 multiply the tabular area ·4330127 by 400, and the product 173·20508 will be the area.

7. There are also several curious algebraical theorems for inscribing polygons in circles, or finding the chord of any proposed part of the circumference, which is the same as angular sections. These kinds of sections, or parts and multiples of arcs, were first treated of by Vieta, as shown in the Introduction to my Log. pa. 9, and since pursued by several other mathematicians, in whose works they are usually to be found.

POLYGON, in Fortification, denotes the figure or perimeter of a fortress, or fortified place. This is either Exterior or Interior.

Exterior POLYGON is the perimeter or figure formed by lines connecting the points of the bastions to one another, quite round the work. And

Interior POLYGON, is the perimeter or figure formed by lines connecting the centres of the bastions, quite around.

Line of POLYGONS, is a line on some sectors, containing the homologous sides of the first nine regular polygons inscribed in the same circle; viz, from an equilateral triangle to a dodecagon.

POLYGONAL *Numbers*, are the continual or successive sums of numbers in arithmetical progression, beginning at 1, and regularly increasing; being called polygonals, because the number of points in them may be arranged in the form of the several polygonal figures in geometry, as is illustrated under the article FIGURATE *Numbers*; which see.

The several kinds of polygonal numbers, viz, the triangles, squares, pentagons, hexagons, &c, are formed from the addition of the terms of the arithmetical series, having respectively their common difference 1, 2, 3, 4, &c; viz, if the common difference of the arithmeticals be 1, the sums of their terms will form the triangles; if 2, the squares; if 3, the pentagons; if 4, the hexagons, &c. Thus:

Arith. Progres.	1,	2,	3,	4,	5,	6,	7.
Triang. Nos.	1,	3,	6,	10,	15,	21,	28.
Arith. Progres.	1,	3,	5,	7,	9,	11,	13.
Square Numbers	1,	4,	9,	16,	25,	36,	49.
Arith. Progres.	1,	4,	7,	10,	13,	16,	19.
Pentagonal Nos.	1,	5,	12,	22,	35,	51,	70.
Arith. Progres.	1,	5,	9,	13,	17,	21,	25.
Hexagonal Nos.	1,	6,	15,	28,	45,	66,	91.

The *Side* of a polygonal number is the number of points in each side of the polygonal figure when the points in the number are ranged in that form. And this is also the same as the number of terms of the arithmeticals that are added together in composing the polygonal number.

The *Angles*, or Numbers of Angles, are the same as those of the figure from which the number takes its name. So the angles of the triangular numbers are 3, of the square ones 4, of the pentagonals 5, of the hexagonals 6, and so on. Hence, the angles are 2 more than the common difference of the arithmetical series from which any rank of polygonals is formed: so the arithmetical series has for its common difference the number 1 or 2 or 3 &c as follows, viz, 1 in the triangles, 2 in the squares, 3 in the pentagons, &c; and, in general, if *a* be the number of angles in the polygon, then $a - 2 = d$ the common difference of the arithmetical series, or $d + 2 = a$ the number of angles.

PROB. 1. *To find any Polygonal Number proposed*; having given its side *n* and angles *a*. The polygonal number being evidently the sum of the arithmetical progression whose number of terms is *n* and common difference $a - 2$; and the sum of an arithmetical progression being equal to half the product of the extremes by the number of terms, the extremes being 1 and $1 + d(n - 1) = 1 + (a - 2) . (n - 1)$; therefore that number, or this sum, will be

$\frac{n^2d - n(d - 2)}{2}$ or $\frac{n^2(a - 2) - n(a - 4)}{2}$, where *d* is the common difference of the arithmeticals that form the polygonal number, and is always 2 less than the number of angles *a*.

Hence, for the several kinds of polygons, any particular number whose side is *n*, will be found from either of these two formulæ, by using for *d* its values 1, 2, 3, 4, &c; which gives these following formulæ for the polygonal number in each sort, viz, the

Triangular - $\frac{1}{2}(n^2 + n)$,
Square - $\frac{1}{2}(2n^2 - 0n) = n^2$,
Pentagonal - $\frac{1}{2}(3n^2 - n)$,
Hexagonal - $\frac{1}{2}(4n^2 - 2n)$,
Heptagonal - $\frac{1}{2}(5n^2 - 3n)$,
*m*gonal - $\frac{1}{2}[(m - 2)n^2 - (m - 4)n]$.

PROB. 2. *To find Sum of any Number of Polygonal Numbers of any order.*—Let the angles of the polygon be *a*, or the common difference of the arithmeticals that form the polygonals, *d*; and *n* the number of terms in the polygonal series, whose sum is sought: then is $\frac{1}{6}(n^2 - 1)\,dn + \frac{1}{2}(n + 1)n$ or $\frac{1}{6}(n^2 - 1) . (a - 2)n + \frac{1}{2}(n + 1)n$ the sum of the *n* terms sought.

Hence, substituting successively the numbers 1, 2, 3, 4, &c, for *d*, there is obtained the following particular cases, or formulæ, for the sums of *n* terms of the several ranks of polygonal numbers, viz, the sum of the

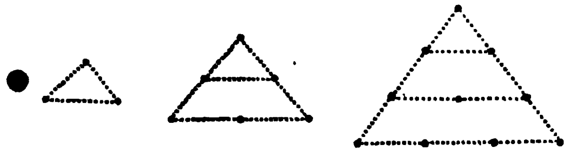
Triangulars	$\frac{1}{6}(n^2 + 3n + 2)n$,
Squares	$\frac{1}{6}(2n^2 + 3n + 1)n$,
Pentagonals	$\frac{1}{6}(3n^2 + 3n + 0)n$,
Hexagonals	$\frac{1}{6}(4n^2 + 3n - 1)n$,
Heptagonals	$\frac{1}{6}(5n^2 + 3n - 2)n$.

&c, which may be illustrated as follows:

Triangles.

Side.	1	2	3	4
Triangle.	1	3	6	10

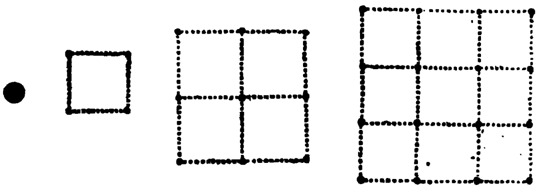
Figure.



Squares.

Side.	1	2	3	4
Square.	1	4	9	16

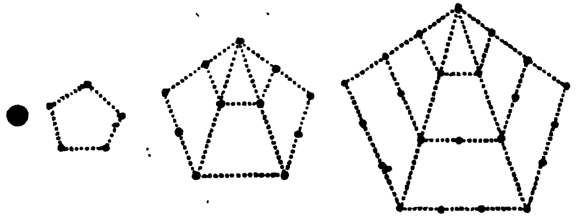
Figure.



Pentagons.

Side.	1	2	3	4
Pentagon.	1	5	12	22

Figure.



And thus may any polygonal number be represented by points in the figure whose name it bears.

Thus, in the above figures, begin with drawing a small polygon that has the number of sides required (as 5 or 6); this number remains constant for one and the same series of polygon numbers, and is always equal to 2 plus the difference of the arithmetical progression from which the series is produced. Then choose one of its angles, in order to draw from the angular point all the diagonals of this polygon, which, with the two sides containing the angle that has been taken, are to be indefinitely produced; after that has been done, take these two sides, and the diagonals of the first polygon on the indefinite lines, each as often as you choose; and draw, from the corresponding points marked by the compasses, lines parallel to the first polygon; and divide them into as many equal parts, or by as many points, as there are actually in the diagonals and the two sides produced.

This rule is general, from the triangle up to the polygon of an infinite number of sides.

Fermat discovered a very curious and general property of polygonal numbers, which is this: That every number is the sum of one, two, or three triangular numbers; the sum of one, two, three, or four squares; the sum of one, two, three, four, or five pentagonal numbers; and so on; that is generally: If *m* denote any order of polygonal numbers, then any number whatever may be resolved into *m* polygonal numbers of this order, or a less number.

This curious property has not however been demonstrated, except for the cases of triangles and squares, the other cases seeming to bid defiance to the efforts of those mathematicians who have attempted them. A demonstration for the squares may be seen in Leybourn's Mathematical Repository; and for both the squares and triangles, in Legendre's Essai sur la Theorie des Nombres.

POLYGONOMETRY, the science and principles of polygons. For which, see my Course of Mathematics, last volume.

POLYGRAM, in Geometry, a figure consisting of many lines.

POLYHEDRON, or POLYEDRON, a body or solid contained by many rectilinear planes or sides. When the sides of the polyhedron are regular polygons, all similar and equal, then the polyhedron becomes a regular body, and may be inscribed in a sphere; that is, a sphere may be described about it, so that its surface shall touch all the angles or corners of the solid. There are but five of these regular bodies, viz, the tetraedron, the hexaedron or cube, the octaedron, the dodecaedron, and the icosaedron. See REGULAR *Body*, and each of those five bodies severally.

Gnomonical POLYHEDRON, is a stone with several faces, on which are projected various kinds of dials. Of this sort, that in the Privy-garden, London, now gone to ruin, was esteemed the finest in the world.

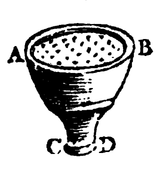
POLYHEDRON, in Optics. See POLYSCOPE.

POLYHEDROUS *Figure*, in Geometry, a solid contained under many sides or planes. See POLYHEDRON.

POLYNOMIAL, in Algebra, a quantity of many names or terms, and is otherwise called a Multinomial. As $a + 3b - 2c + 4d$, &c. See MULTINOMIAL.

POLYOPTRUM, in Optics, a glass through which objects appear multiplied, but diminished. This differs both in structure and phenomena from the common multiplying glasses called polyhedra or polyscopes.

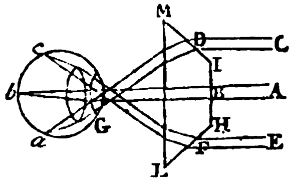
To construct the Polyoptrum.—From a glass AB, plane on both sides, and about 3 fingers thick, cut out spherical segments, scarce a 5th part of a digit in diameter.—If then the glass be removed to such a distance from the eye, that you can take in all the cavities at one view, you will see the same object, as if through so many several concave glasses as there are cavities, and all exceeding small.—Fit this, as an object-glass, in a tube ABCD, whose aperture AB is equal to the diameter of the glass, and the other CD is equal to that of an eye-glass, as for instance about a finger's breadth. The length of the tube AC is to be accommodated to the object-glass and eye-glass, by trial. In CD fit a convex eye-glass, or in its stead a meniscus having the distance of its principal focus a little larger than



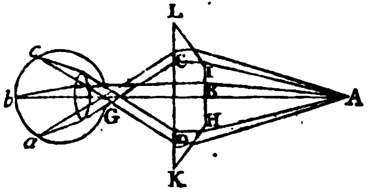
the length of the tube; so that the point from which the rays diverge after refraction in the object-glass, may be in the focus. If then the eye be applied near the eye-glass, a single object will be seen repeated as often as there are cavities in the object-glass, but still diminished.

POLYSCOPE, or POLYHEDRON, in Optics, is a multiplying glass, being a glass or lens which represents a single object to the eye as if it were many. It consists of several plane surfaces, disposed into a convex form, through every one of which the object is seen.

Phenomena of the Polyscope.—1. If several rays, as EF, AB, CD, fall parallel on the surface of a polyscope, they will continue parallel after refraction. If then the polyscope be supposed regular, LH, HI, IM will be as tangents cutting the spherical convex lens in F, B, and D; and consequently, rays falling on the points of contact, intersect the axis. Therefore, since the rest are parallel to these, they will also mutually intersect each other in G.—Hence, if the eye be placed where parallel rays decussate, rays of the same object will be propagated to it still parallel from the several sides of the glass. Therefore, since the crystalline humour, by its convexity, unites parallel rays, the rays will be united in as many different points of the retina, *a*, *b*, *c*, as the glass has sides. Consequently the eye, through a polyscope, sees the object repeated as many times as there are sides. And hence, since rays coming from very remote objects are parallel, a remote object is seen through a polyscope as often repeated as that has sides.



2. If rays AB, AC, AD, coming from a radiant point A, fall on several sides of a regular polyscope; after refraction they will decussate in G, and proceed on a little diverging.—Hence, if the eye be placed where the rays decussate after coming from the several planes, the rays will be propagated to it from the several planes a little diverging, or as if they proceeded from different points. But since the crystalline humour, by its convexity, collects rays from several points into the same point; the rays will be united in as many different points of the retina, *a*, *b*, *c*, as the glass has sides; and consequently the eye, being placed in the focus G, will see even a near object through the polyscope as often repeated as that has sides.



Thus may the images of objects be multiplied in a camera obscura, by placing a polyscope at its aperture, and adding a convex lens at a due distance from it. And it makes a very pleasant appearance, if a prism be applied so that the coloured rays of the sun refracted from it be received on the polyscope: for by this means they will be thrown on a paper or wall near at hand in little lucid specks, much exceeding the brightness of any precious stone; and in the focus of the polyscope, where the rays decussate (for in this experiment they are received on the convex side), will be a star of surprising lustre.

Farther, if images be painted in water-colours in the areolæ or little squares of a polyscope, and the glass be applied to the aperture of a camera obscura; the sun's rays, passing through it, will carry with them the images, and project them on the opposite wall.—This artifice bears a resemblance to that other, by which an image on paper is projected on the camera; viz, by wetting the paper with oil, and straining it tight in a frame; then applying it to the aperture of the camera obscura, so that the rays of a candle may pass through it upon the polyscope.

To make an Anamorphosis, or Deformed Image, which shall appear regular and beautiful through a Polyscope, or Multiplying Glass.—At one end of a horizontal table erect another perpendicularly, on which a figure may be designed; and on the other end erect another, to serve as a fulcrum or support, moveable on the horizontal one. To the fulcrum apply a plano-convex polyscope, consisting, for example, of 24 plane triangles; and let the polyscope be fitted in a draw-tube, of which that end towards the eye may have only a very small aperture, and a little farther off than the focus. Remove the fulcrum from the other perpendicular table, till it be out of the distance of the focus; and the more so, as the image is to be greater. Before the little aperture place a lamp; and trace the luminous areolæ projected from the sides of the polyscope, with a black lead pencil, on the vertical plane, or a paper applied upon it.

In the several areolæ, design the different parts of an image, in such a manner as that, when joined together, they may make one whole, looking every now and then through the tube to guide and correct the colours, and to see that the several parts match and fit well together. As to the intermediate space, it may be filled up with any figures or designs at pleasure, contriving it so, as that to the naked eye the whole may exhibit some appearance very different from that intended to appear through the polyscope.—The eye, now looking through the small aperture of the tube, will see the several parts and members dispersed among the areolæ to exhibit one continued image, all the intermediate parts disappearing. See ANAMORPHOSIS.

POLYSPASTON, in Mechanics, a machine so called by Vitruvius, consisting of an assemblage of several pulleys, used for raising heavy weights.

PONTON, or PONTOON, a kind of flat-bottomed boat, whose carcase of wood is lined within and without with tin. Some nations line them on the outside only, and that with plates of copper, which is better. Our pontoons are 21 feet long, nearly 5 feet broad, and 2 feet 1½ inch deep within. They are carried along with an army upon carriages, to make temporary bridges, called pontoon-bridges.

PONTOON-*Bridge*, a bridge made of pontoons slipped into the water, and moored by anchors and otherwise fastened together by ropes, at small distances from one another; then covered by beams of timber passing over them; upon which is laid a flooring of boards. By this means, whole armies of infantry, cavalry, and artillery are quickly passed over rivers.—For want of pontoons, &c, bridges are sometimes formed of empty powder-casks, or powder-barrels, which support the beams and flooring. Julius Cæsar and Aulus Gellius both mention pontoons (pontones); but theirs were no more than a kind of square flat vessels, proper for carrying over horse, &c.

PONT-VOLANT, or *Flying-bridge*, is a kind of bridge used in sieges, for surprising a post or outwork that has but narrow moats. It is made of two small bridges laid over each other, and so contrived that, by means of cords and pulleys placed along the sides of the under bridge, the upper may be pushed forwards, till it join the place where it is designed to be fixed. The whole length of both ought not to be above 5 fathoms, lest it should break with the weight of the men.

POPULATION *of the World.*—From Le Sage's Atlas, 1814, is in

	Millions.
Europe - - -	170
Asia . - -	380
Africa - - -	90
America, North - -	30
——— South - -	20
The Oceanic Islands -	20
Total population of the globe -	710

PORCH, in Architecture, a kind of vestibule supported by columns; much used at the entrance of the ancient temples, halls, churches, &c. Such is that before the door of St. Paul's, Covent Garden.

When a porch had four columns in front, it was called a tetrastyle; when six, hexastyle; when eight, octostyle, &c. See *Tetrastyle*, &c.

PORES, are the small interstices between the particles of matter which compose bodies; and are either empty, or filled with some insensible medium.—Condensation and rarefaction are only performed by closing and opening the pores. Also the transparency of bodies is supposed to arise from their pores being directly opposite to one another. And the matter of insensible perspiration is conveyed through the pores of the cutis.—Mr. Boyle has a particular essay on the porosity of bodies, in which he proves that the most solid bodies have some kind of pores: and indeed if they had not, all bodies would be alike specifically heavy.

Sir Isaac Newton shows, that bodies are much more rare and porous than is commonly believed. Water, for example, is 19 times lighter and rarer than gold; and gold itself is so rare, as very readily, and without the least opposition, to transmit magnetic effluvia, and easily to admit even quicksilver into its pores, and to let water pass through it: for a concave sphere of gold hath, when filled with water, and soldered up, upon pressing it with a great force, suffered the water to squeeze through it, and stand all over its outside, in multitudes of small drops like dew, without bursting or cracking the gold. Whence it may be concluded, that gold has more pores than solid parts, and consequently that water has above 40 times more pores than parts. Hence it is that the magnetic effluvia passes freely through all cold bodies that are not magnetic; and that the rays of light pass, in right lines, to the greatest distances through pellucid bodies.

PORIME, *Porima*, in Geometry, a kind of easy lemma, or theorem so easily demonstrated, that it is almost self-evident: such, for example, as that a chord is wholly within the circle.—Porime stands opposed to aporime, which denotes a proposition so difficult, as to be almost impossible to be demonstrated, or effected. Such as the quadrature of the circle, &c.

PORISM, *Porisma*, in Geometry, has by some been defined a general theorem, or canon, deduced from a geometrical locus, and serving for the solution of other general and difficult problems. Proclus derives the word from the Greek *πορίζω*, *I establish*, and conclude from something already done and demonstrated: and accordingly he defines porisma a theorem drawn occasionally from some other theorem already proved: in which sense it agrees with what is otherwise called corollary, and was much used as such even by the geometers two or three centuries ago.

Pappus says, A porism is that in which something was proposed to be investigated, or something between a theorem and a problem. Others derive it from *πόρος*, *a passage*, and make it of the nature of a lemma, or a proposition necessary for passing to another more important one.

But Dr. Simson, rejecting the accounts that have been given of a porism, defines it a proposition, either in the form of a problem or a theorem, in which it is proposed either to investigate, or demonstrate. And Mr. Playfair says, A porism is nothing else than that particular case, when the data of a problem are so related to one another as to render it indefinite, or capable of innumerable solutions. Edinburgh Philos. Trans. vol. 1, pa. 60.

Euclid wrote three books of porisms, being a curious collection of various things relating to the analysis of the more difficult and general problems. Those books however are lost; and nothing remains in the works of the ancient geometricians concerning this subject, besides what Pappus has preserved, in a very imperfect and obscure state, in his Mathematical Collections, viz, in the introduction to the 7th book.

Several attempts have been made to restore these writings in some degree, besides that which Pappus has left upon the subject. Thus, Fermat has given a few propositions of this kind; which are to be found in the collection of his works, in folio, 1679, pa. 116. The like was done by Bulliald, in his Exercitationes Geometricæ, 4to, 1657. Dr. Robert Simson gave also a specimen, in two propositions, in the Philos. Trans. vol. 32, pa. 330; and besides left behind him a considerable treatise on the subject of porisms, which has been printed in an edition of his works, at the expense of the earl of Stanhope, in 4to, 1776; an English translation of a part of which was published by Mr. Lawson in the year following.

The whole three books of Euclid were also restored by that ingenious mathematician Albert Girard, as appears by two notices that he gave, first in his Trigonometry, printed in French, at the Hague, in 1629, and also in his edition of the works of Stevinus, printed at Leyden in 1634, pa. 459; but whether his intention of publishing them was ever carried into execution, I have not been able to learn.

A learned paper on the subject of porisms, by the very ingenious professor Playfair, was inserted in the 3d volume of the Transactions of the Royal Society of Edinburgh. And as this paper contains a number of curious observations on the geometry of the ancients in general, as well as forms a complete treatise as it were on porism in particular, a pretty considerable abstract of it cannot but be deemed in this place very useful and important.

"The restoration of the ancient books of geometry (says the learned professor) would have been impossible, without the coincidence of two circumstances, of which, though the one is purely accidental, the other is essentially connected with the nature of the mathematical sciences. The first of these circumstances is the preservation of a short abstract of those books, drawn up by Pappus Alexandrinus, together with a series of such lemmata, as he judged useful to facilitate the study of them. The second is, the necessary connection that takes place among the objects of every mathematical work, which, by excluding whatever is arbitrary, makes it possible to determine the whole course of an investigation, when only a few points in it are known. From the union of these circumstances, mathematics has enjoyed an advantage of which no other branch of knowledge can partake; and while the critic or the historian has only been able to lament the fate of those books of Livy and Tacitus which are lost, the geometer has had the high satisfaction to behold the works of Euclid and Apollonius reviving under his hands.

" The first restorers of the ancient books were not, however, aware of the full extent of the work which they had undertaken. They thought it sufficient to demonstrate the propositions, which they knew from Pappus, to have been contained in those books; but they did not follow the ancient method of investigation, and few of them appear to have had any idea of the elegant and simple analysis by which these propositions were originally discovered, and by which the Greek geometry was peculiarly distinguished.

" Among these few, Fermat and Halley are to be particularly remarked. The former, one of the greatest mathematicians of the last age, and a man in all respects of superior abilities, had very just notions of the geometrical analysis, and appears often abundantly skilful in the use of it; yet in his restoration of the Loci Plani, it is remarkable, that in the most difficult propositions, he lays aside the analytical method, and contents himself with giving the synthetical demonstration. The latter, among the great number and variety of his literary occupations, found time for a most attentive study of the ancient mathematicians, and was an instance of, what experience shows to be much rarer than might be expected, a man equally well acquainted with the ancient and the modern geometry, and equally disposed to do justice to the merit of both. He restored the books of Apollonius, on the problem De Sectione Spatii, according to the true principles of the ancient analysis.

" These books however are but short, so that the first restoration of considerable extent that can be reckoned complete, is that of the Loci Plani by Dr. Simson, published in 1749; which, if it differs at all from the work it is intended to replace, seems to do so only by its greater excellence. This much at least is certain, that the method of the ancient geometers does not appear to greater advantage in the most entire of their writings, than in the restoration above mentioned: and that Dr. Simson has often sacrificed the elegance to which his own analysis would have led, in order to tread more exactly in what the lemmata of Pappus pointed out to him, as the track which Apollonius had pursued.

" There was another subject, that of porisms, the most intricate and enigmatical of any thing in the ancient geometry, which was still reserved to exercise the genius of Dr. Simson, and to call forth that enthusiastic admiration of antiquity, and that unwearied perseverance in research, for which he was so peculiarly distinguished. A treatise in three books, which Euclid had composed on porisms, was lost, and all that remained concerning them was an abstract of that treatise, inserted by Pappus Alexandrinus in his Mathematical Collections, in which, had it been entire, the geometers of later times would doubtless have found wherewithal to console themselves for the loss of the original work. But unfortunately it has suffered so much from the injuries of time, that all which we can immediately learn from it is, that the ancients put a high value on the propositions which they called porisms, and regarded them as a very important part of their analysis. The porisms of Euclid are said to be, " Collectio artificiosissima multarum rerum quæ spectant ad analysin difficiliorum et generalium problematum.' The curiosity, however, which is excited by this encomium is quickly disappointed; for when Pappus proceeds to explain what a porism is, he lays down two definitions of it, one of which is rejected by him as imperfect, while the other, which is stated as correct, is too vague and indefinite to convey any useful information.

" These defects might nevertheless have been supplied, if the enumeration which he next gives of Euclid's Propositions had been entire; but on account of the extreme brevity of his enunciations, and their reference to a diagram which is lost, and for the constructing of which no directions are given, they are all, except one, perfectly unintelligible. For these reasons, the fragment in question is so obscure, that even to the learning and penetration of Dr. Halley it seemed impossible that it could ever be explained; and he therefore concluded, after giving the Greek text with all possible correctness, and adding the Latin translation, ' Hactenus Porismatum descriptio nec mihi intellecta, nec lectori profutura. Neque aliter fieri potuit, tam ob defectum schematis cujus fit mentio, quam ob omissa quædam et transposita, vel aliter vitiata in propositionis generalis expositione, unde quid sibi velit Pappus haud mihi datum est conjicere. His adde dictionis modum nimis contractum, ac in re difficili, qualis hæc est, minime usurpandum.'

" It is true, however, that before this time, Fermat had attempted to explain the nature of porisms, and not altogether without success. Guiding his conjectures by the definition which Pappus censures as imperfect, because it defined porisms only ' ab accidente,' viz, ' porisma est quod deficit hypothesi a Theoremate Locali,' he formed to himself a tolerably just notion of these propositions, and illustrated his general description by examples that are in effect porisms. But he was able to proceed no farther; and he neither proved, that his notion of a porism was the same with Euclid's, nor attempted to restore, or explain any one of Euclid's propositions; much less did he suppose, that they were to be investigated by an analysis peculiar to themselves. And so imperfect indeed was this attempt, that the complete restoration of the porisms was necessary to prove, that Fermat had even approximated to the truth.

" All this did not however deter Dr. Simson from turning his thoughts to the same subject, which he appears to have done very early, and long before the publication of the Loci Plani in 1749.

" The account he gives of his progress, and of the obstacles he encountered, will be always interesting to mathematicians. ' Postquam vero apud Pappum legeram; porismata Euclidis collectionem fuisse artificiosissimam multarum rerum, quæ spectant ad analysin difficiliorum et generalium problematum, magno desiderio tenebar, aliquid de iis cognoscendi; quare sæpius et multis variisque viis tum Pappi propositionem generalem, mancam et imperfectam, tum primum lib. i.

" ' Porisma, quod solum ex omnibus in tribus libris integrum adhuc manet, intelligere et restituere conabar; frustra tamen, nihil enim proficiebam. Cumque cogitationes de hac re multum mihi temporis consumpserint, atque molestæ admodum evaserint, firmiter animum induxi hæc nunquam in posterum investigare; præsertim cum optimus geometra Halleius spem omnem de iis intelligendis abjecisset. Unde quoties menti occurrebant, toties eas arcebam. Postea tamen accidit, ut improvidum et propositi immemorem invaserint, meque detinuerint donec tandem lux quædam effulserit, quæ spem mihi faciebat inveniendi saltem Pappi propositionem generalem, quam quidem multa investigatione tandem restitui. Hæc autem paulo post una cum Porismate primo lib. i. impressa est inter Transactiones Phil. anni 1723, num. 177.'

"The propositions mentioned, as inserted in the Philosophical Transactions for 1723, are all that Dr. Simson published on the subject of porisms during his life, though he continued his investigations concerning them, and succeeded in restoring a great number of Euclid's propositions, together with their analysis. The propositions thus restored form a part of that valuable edition of the posthumous works of this geometer which the mathematical world owes to the munificence of the late earl Stanhope.

"The subject of porisms is not however exhausted, nor is it yet placed in so clear a light as to need no further illustration. It yet remains to enquire into the probable origin of these propositions, that is to say, into the steps by which the ancient geometers appear to have been led to the discovery of them.

"It remains also to point out the relations in which they stand to the other classes of geometrical truths; to consider the species of analysis, whether geometrical or algebraical, that belongs to them: and, if possible, to assign the reason why they have so long escaped the notice of modern mathematicans. It is to these points that the following observations are chiefly directed.

"I begin with describing the steps that appear to have led the ancient geometers to the discovery of porisms; and must here supply the want of express testimony by probable reasonings, such as are necessary, whenever we would trace remote discoveries to their sources, and which have more weight in mathematics than in any other of the sciences.

"It cannot be doubted, that it has been the solution of problems, which, in all states of the mathematical sciences, has led to the discovery of most geometrical truths. The first mathematical enquiries, in particular, must have occurred in the form of questions, where something was given, and something required to be done; and by the reasonings necessary to answer these questions, or to discover the relation between the things that were given and those that were to be found, many truths were suggested, which came afterwards to be the subjects of separate demonstration. The number of these was the greater, as the ancient geometers always undertook the solution of problems with a scrupulous and minute attention, which would scarcely suffer any of the collateral truths to escape their observation. We know from the examples which they have left us, that they never considered a problem as resolved, till they had distinguished all its varieties, and evolved separately every different case that could occur, carefully remarking whatever change might arise in the construction, from any change that was supposed to take place among the magnitudes which were given.

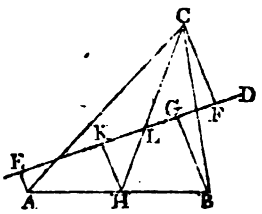
"Now as this cautious method of proceeding was not better calculated to avoid error, than to lay hold of every truth that was connected with the main object of enquiry, these geometers soon observed, that there were many problems which, in certain circumstances, would admit of no solution whatever, and that the general construction by which they were resolved would fail, in consequence of a particular relation being supposed among the quantities which were given.

"Such problems were then said to become impossible; and it was readily perceived, that this always happened, when one of the conditions prescribed was inconsistent with the rest, so that the supposition of their being united in the same subject, involved a contradiction. Thus, when it was required to divide a given line, so that the rectangle under its segments should be equal to a given space, it was evident, that if this space was greater than the square of half the given line, the thing required could not possibly be done; the two conditions, the one defining the magnitude of the line, and the other that of the rectangle under its segments, being then inconsistent with one another. Hence an infinity of beautiful propositions concerning the maxima and the minima of quantities, or the limits of the possible relations which quantities may stand in to one another.

"Such cases as these would occur even in the solution of the simplest problems; but when geometers proceeded to the analysis of such as were more complicated, they must have remarked, that their constructions would sometimes fail, for a reason directly contrary to that which has now been assigned. Instances would be found where the lines that, by their intersection, were to determine the thing sought, instead of intersecting one another, as they did in general, or of not meeting at all, as in the abovementioned case of impossibility, would coincide with one another entirely, and leave the question of consequence unresolved. But though this circumstance must have created considerable embarrassment to the geometers who first observed it, as being perhaps the only instance in which the language of their own science had yet appeared to them ambiguous or obscure, it would not probably be long till they found out the true interpretation to be put on it. After a little reflection, they would conclude, that since, in the general problem, the magnitude required was determined by the intersection of the two lines above mentioned, that is to say, by the points common to them both; so, in the case of their coincidence, as all their points were in common, every one of these points must afford a solution; which solutions therefore must be infinite in number; and also, though infinite in number, they must all be related to one another, and to the things given, by certain laws, which the position of the two coinciding lines must necessarily determine.

"On enquiring farther into the peculiarity in the state of the data which had produced this unexpected result, it might likewise be remarked, that the whole proceeded from one of the conditions of the problem involving another, or necessarily including it; so that they both together made in fact but one, and did not leave a sufficient number of independent conditions, to confine the problem to a single solution, or to any determinate number of solutions. It was not difficult afterwards to perceive, that these cases of problems formed very curious propositions, of an intermediate nature between problems and theorems, and that they admitted of being enunciated separately, in a manner peculiarly elegant and concise. It was to such propositions, so enunciated, that the ancient geometers gave the name of porisms.

"This deduction requires to be illustrated by examples." Mr. Playfair then gives several problems by way of illustration; one of which, which may here suffice to show the method, is as follows:



"A triangle ABC being given, and also a point D; to draw through D a straight line DG, such, that, perpendiculars being drawn to it from the three angles of the triangle, viz, AE, BG, CF, the sum of the two perpendiculars on the same side of DG, shall be equal to the remaining perpendicular: or, that AE and BG together, may be equal to CF.

"Suppose it done: Bisect AB in H, join CH, and draw HK perpendicular to DG.—Because AB is bisected in H, the two perpendiculars AE and BG are together double of HK; and as they are also equal to CF by hypothesis, CF must be double of HK; and CL of LH. Now, CH is given in position, and magnitude; therefore the point L is given; and the point D being also given, the line DL is given in position, which was to be found.

"The construction was obvious. Bisect AB in H, join CH, and take HL equal to one third of CH; the straight line which joins the points D and L is the line required.

"Now, it is plain, that while the triangle ABC remains the same, the point L also remains the same, wherever the point D may be. The point D may therefore coincide with L; and when this happens, the position of the line to be drawn is left undetermined; that is to say, any line whatever drawn through L will satisfy the conditions of the problem. Here therefore we have another indefinite case of a problem, and of consequence another porism, which may be thus enunciated: "A triangle being given in position, a point in it may be found, such, that any straight line whatever being drawn through that point, the perpendiculars drawn to this straight line from the two angles of the triangle which are on one side of it, will be together equal to the perpendicular that is drawn to the same line from the angle on the other side of it.

"This porism may be made much more general; for if, instead of the angles of a triangle, we suppose ever so many points to be given in a plane, a point may be found such, that any straight line being drawn through it, the sum of all the perpendiculars that fall on that line from the given points on one side of it, is equal to the sum of the perpendiculars that fall on it from all the points on the other side of it.

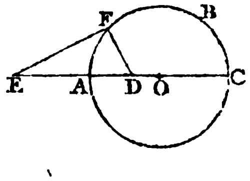
"Or still more generally, any number of points being given not in the same plane, a point may be found, through which if any plane be supposed to pass, the sum of all the perpendiculars which fall on that plane from the points on one side of it, is equal to the sum of all the perpendiculars that fall on the same plane from the points on the other side of it. It is unnecessary to observe, that the point to be found in these propositions, is no other than the centre of gravity of the given points; and that therefore we have here an example of a porism very well known to the modern geometers, though not distinguished by them from other theorems."

After some examples of other porisms, and remarks upon them, the author then adds,

"From this account of the origin of porisms, it follows, that a porism may be defined, *A proposition affirming the possibility of finding such conditions as will render a certain problem indeterminate, or capable of innumerable solutions.*

"To this definition, the different characters which Pappus has given will apply without difficulty. The propositions described in it, like those which he mentions, are, strictly speaking, neither theorems nor problems, but of an intermediate nature between both; for they neither simply enunciate a truth to be demonstrated, nor propose a question to be solved, but are affirmations of a truth, in which the determination of an unknown quantity is involved. In as far therefore as they assert, that a certain problem may become indeterminate, they are of the nature of theorems; and in as far as they seek to discover the conditions by which that is brought about, they are of the nature of problems.

"In the preceding definition also, and the instances from which it is deduced, we may trace that imperfect description of porisms which Pappus ascribes to the later geometers, viz, 'Porisma est quod deficit hypothesi a theoremate locali.' Now, to understand this, it must be observed, that if we take the converse of one of the propositions called Loci, and make the construction of the figure a part of the hypothesis, we have what was called by the ancients a Local Theorem. And again, if, in enunciating this theorem, that part of the hypothesis which contains the construction be suppressed, the proposition arising from thence will be a porism; for it will enunciate a truth, and will also require, to the full understanding and investigation of that truth, that something should be found, viz, the circumstance in the construction, supposed to be omitted.



"Thus when we say; If from two given points E and D, two lines EF and FD are inflected to a third point F, so as to be to one another in a given ratio, the point F is in the circumference of a circle given in position: we have a Locus.

"But when conversely it is said; If a circle ABC, of which the centre is O, be given in position, as also a point E, and if D be taken in the line EO, so that the rectangle EO, OD be equal to the square of AO, the semidiameter of the circle; and if from E and D, the lines EF and DF be inflected to any point whatever in the circumference ABC; the ratio of EF to DF will be a given ratio, and the same with that of EA to AD: we have a local theorem.

"And, lastly, when it is said; If a circle ABC be given in position, and also a point E, a point D may be found, such, that if the two lines EF and FD be inflected from E and D, to any point whatever F, in the circumference, these lines shall have a given ratio to one another: the proposition becomes a porism.

"Here it is evident, that the local theorem is changed into a porism, by leaving out what relates to the determination of the point D, and of the given ratio. But though all propositions formed in this way, from the conversion of Loci, be porisms, yet all porisms are not formed from the conversion of Loci. The first and second of the preceding, for instance, cannot by conversion be changed into Loci; and therefore the definition which describes all porisms as being so convertible, is not sufficiently comprehensive. Fermat's idea of porisms, as has been already observed, was founded wholly on this definition, and therefore could not fail to be imperfect.

" It appears therefore, that the definition of porisms given above, agrees with Pappus's idea of these propositions, as far at least as can be collected from the imperfect fragments which contain his general description of them. It agrees also with Dr. Simson's definition, which is this: ' Porisma est propositio in qua proponitur demonstrare rem aliquam, vel plures datas esse, cui, vel quibus, ut et cuilibet ex rebus innumeris, non quidem datis, sed quæ ad ea quæ data sunt eandem habent relationem, convenire ostendendum est affectionem quandam communem in propositione descriptam.'

" It cannot be denied, that there is a considerable degree of obscurity in this definition; notwithstanding which it is certain, that every proposition to which it applies must contain a problematical part, viz, ' in qua proponitur demonstrare rem aliquam, vel plures datas esse,' and also a theoretical part, which contains the property, or communis affectio, affirmed of certain things which have been previously described.

" It is also evident, that the subject of every such proposition, is the relation between magnitudes of three different kinds; determinate magnitudes which are given; determinate magnitudes which are to be found; and indeterminate magnitudes which, though unlimited in number, are connected with the others by some common property. Now, these are exactly the conditions contained in the definitions that have been given here.

" To confirm the truth of this theory of the origin of porisms, or at least the justness of the notions founded on it, I must add a quotation from an essay on the same subject, by a member of this society, the extent and correctness of whose views make every coincidence with his opinions peculiarly flattering. In a paper read several years ago before the Philosophical Society, Professor Dugald Stewart defined a porism to be ' A proposition affirming the possibility of finding one or more of the conditions of an indeterminate theorem.' Where, by an indeterminate theorem, as he had previously explained it, is meant one which expresses a relation between certain quantities that are indeterminate, both in magnitude and in number. The near agreement of this with the definition and explanations which have been given above, is too obvious to require to be pointed out; and I have only to observe, that it was not long after the publication of Simson's posthumous works, when, being both of us occupied in speculations concerning porisms, we were led separately to the conclusions which I have now stated.

" In an enquiry into the origin of porisms, the etymology of the term ought not to be forgotten. The question indeed is not about the derivation of the word Πορισμα, for concerning that there is no doubt; but about the reason why this term was applied to the class of propositions above described. Two opinions may be formed on this subject, and each of them with considerable probability; 1mo. One of the significations of ποριζω, is *to acquire* or *obtain*; and hence Πορισμα, the *thing obtained* or *gained*.

" Accordingly, Scapula says, ' Est vox a geometris desumpta qui theorema aliquid ex demonstrativo syllogismo necessario sequens inferentes, illud quasi lucrari dicuntur, quod non ex professo quidem theorematis hujus instituta sit demonstratio, sed tamen ex demonstratis recte sequatur.' In this sense Euclid uses the word in his Elements of Geometry, where he calls the corollaries of his proposition, porismata. This circumstance creates a presumption, that when the word was applied to a particular class of propositions, it was meant, in both cases, to convey nearly the same idea; as it is not at all probable, that so correct a writer as Euclid, and so scrupulous in his use of words, should employ the same term to express two ideas which are perfectly different. May we not therefore conjecture, that these propositions got the name of porisms, entirely with a reference to their origin? According to the idea explained above, they would in general occur to mathematicians when engaged in the solution of the more difficult problems, and would arise from those particular cases, where one of the conditions of the data involved in it some one of the rest. Thus a particular kind of theorem would be obtained, following as a corollary from the solution of the problem: and to this theorem the term Πορισμα might be very properly applied, since, in the words of Scapula, already quoted, ' Non ex professo theorematis hujus instituta sit demonstratio, sed tamen ex demonstratis recte sequatur.'

" 2do. But though this interpretation agrees so well with the supposed origin of porisms, it is not free from difficulty. The verb ποριζω has another signification, *to find out, to discover, to devise*; and is used in this sense by Pappus, when he says that the propositions called porisms, afford great delight, τοις δυναμενοις ορᾳν και ποριζειν, *to those who are able to understand and investigate.* Hence comes πορισμος, the *act of finding out* or *discovering*, and from πορισμος, in this sense, the same author evidently considers Πορισμα as being derived. His words are, Εφασαν δε (ὁι αρχαιοι) Πορισμα ειναι το προτεινομενον εις Πορισμον αυτȣ τȣ προτεινομενȣ, *The ancients said, that a Porism is something proposed for the* finding out, *or* discovering *of the very thing proposed*. It seems singular, however, that porisms should have taken their name from a circumstance common to them with so many other geometrical truths; and if this was really the case, it must have been on account of the enigmatical form of their enunciations, which required, that in the analysis of these propositions, a sort of double discovery should be made, not only of the truth, but also of the meaning of the very thing which was proposed. They may therefore have been called porismata, or investigations, by way of eminence.

" We might next proceed to consider the particular porisms which Dr. Simson has restored, and to show, that every one of them is the indeterminate case of some problem. But of this it is so easy for any one, who has attended to the preceding remarks, to satisfy himself, by barely examining the enunciations of those propositions, that the detail, unto which it would lead, seems to be unnecessary. I shall therefore go on to make some observations on that kind of analysis which is particularly adapted to the investigation of porisms.

" If the idea which we have given of these propositions be just, it follows, that they are always to be discovered by considering the cases in which the construction of a problem fails in consequence of the lines which, by their intersection, or the points which, by their position, were to determine the magnitude required, happening to coincide with one another—a porism may therefore be deduced from the problem it belongs to, in the same manner that the propositions concerning the maxima and minima of quantities are deduced from the problems of which they form the limitations; and such no doubt is the most natural and most obvious analysis of which this class of propositions will admit.

"It is not, however, the only one that they will admit of; and there are good reasons for wishing to be provided with another, by means of which, a porism that is any how suspected to exist, may be found out, independently of the general solution of the problem to which it belongs. Of these reasons, one is, that the porism may perhaps admit of being investigated more easily than the general problem admits of being resolved; and another is, that the former, in almost every case, helps to discover the simplest and most elegant solution that can be given of the latter.

"It is desirable to have a method of investigating porisms, which does not require that we should previously resolve the problems they are connected with, and which may always serve to determine, whether to any given problem there be attached a porism, or not. Dr. Simson's Analysis may be considered as answering to this description; for as that geometer did not regard these propositions at all in the light that is done here, nor in relation to their origin, an independent analysis of this kind was the only one that could occur to him; and he has accordingly given one which is extremely ingenious, and by no means easy to be invented, but which he uses with great skilfulness and dexterity throughout the whole of his Restoration.

"It is not easy to ascertain whether this be the precise method used by the ancients. Dr. Simson had here nothing to direct him but his genius, and has the full merit of the first inventor. It seems probable, however, that there is at least a great affinity between the methods, since the lemmata given by Pappus as necessary to Euclid's demonstrations, are subservient also to those of our modern geometer.

"It is, as we have seen, a general principle, that a problem is converted into a porism, when one, or when two, of the conditions of it, necessarily involve in them some one of the rest. Suppose then that two of the conditions are exactly in that state which determines the third; then, while they remain fixed or given, should that third one be supposed to vary, or differ, ever so little, from the state required by the other two, a contradiction will ensue. Therefore if, in the hypothesis of a problem, the conditions be so related to one another, as to render it indeterminate, a porism is produced; but if, of the conditions thus related to one another, some one be supposed to vary, while the others continue the same, an absurdity follows, and the problem becomes impossible. Wherever therefore any problem admits both of an indeterminate, and an impossible case, it is certain, that these cases are nearly related to one another, and that some of the conditions by which they are produced, are common to both.

"It is supposed above, that two of the conditions of a problem involve in them a third; and wherever that happens, the conclusion which has been deduced will invariably take place.

"But a porism may sometimes be so simple, as to arise from the mere coincidence of one condition of a problem with another, though in no case whatever, any inconsistency can take place between them. Thus, in the second of the foregoing propositions, the coincidence of the point given in the problem with another point, viz, the centre of gravity of the given triangle, renders the problem indeterminate; but as there is no relation of distance, or position, between these points, that may not exist, so the problem has no impossible case belonging to it. There are, however, comparatively but few porisms so simple in their origin as this, or that arise from problems in which the conditions are so little complicated; for it usually happens, that a problem which can become indefinite, may also become impossible; and if so, the connection between these cases, which has been already explained, never fails to take place.

"Another species of impossibility may frequently arise from the porismatic case of a problem, which will very much affect the application of geometry to astronomy, or any of the sciences of experiment or observation. For when a problem is to be resolved by help of data furnished by experiment or observation, the first thing to be considered is, whether the data so obtained, be sufficient for determining the thing sought; and in this a very erroneous judgment may be formed, if we rest satisfied with a general view of the subject. For though the problem may in general be resolved from the data that we are provided with, yet these data may be so related to one another in the case before us, that the problem will become indeterminate, and instead of one solution, will admit of an infinite number.

"Suppose, for instance, that it were required to determine the position of a point F from knowing that it was situated in the circumference of a given circle ABC, and also from knowing the ratio of its distances from two given points E and D; it is certain that in general these data would be sufficient for determining the situation of F. But nevertheless, if E and D should be so situated, that they were in the same straight line with the centre of the given circle; and if the rectangle under their distances from that centre, were also equal to the square of the radius of the circle, then, the position of F could not be determined.

"This particular instance may not indeed occur in any of the practical applications of geometry; but there is one of the same kind which has actually occurred in astronomy. And as the history of it is not a little singular, affording besides an excellent illustration of the nature of porisms, I hope to be excused for entering into the following detail concerning it.

"Sir Isaac Newton having demonstrated, that the trajectory of a comet is a parabola, reduced the actual determination of the orbit of any particular comet to the solution of a geometrical problem, depending on the properties of the parabola, but of such considerable difficulty, that it is necessary to take the assistance of a more elementary problem, in order to find, at least nearly, the distance of the comet from the earth, at the times when it was observed. The expedient for this purpose, suggested by Newton himself, was to consider a small part of the comet's path as rectilineal, and described with an uniform motion, so that four observations of the comet being made at moderate intervals of time from one another, four straight lines would be determined, viz, the four lines joining the places of the earth and the comet, at the times of observation, across which if a straight line were drawn, so as to be cut by them in three parts, in the same ratios with the intervals of time above-mentioned; the line so drawn would nearly represent the comet's path, and by its intersection with the given lines, would determine, at least nearly, the distances of the comet from the earth at the time of observation.

"The geometrical problem here employed, of drawing a line to be divided by four other lines given in position, into parts having given ratios to one another, had been already resolved by Dr. Wallis and Sir Christopher Wren, and to their solutions Sir Isaac Newton added three others of his own, in different parts of his works. Yet none of all these geometers observed that peculiarity in the problem which rendered it inapplicable to astronomy. This was first done by M. Boscovich, but not till after many trials, when, on its application to the motion of comets, it had never led to any satisfactory result. The errors it produced in some instances were so considerable, that Zanotti, seeking to determine by it the orbit of the comet of 1739, found, that his construction threw the comet on the side of the sun opposite to that on which he had actually observed it. This gave occasion to Boscovich, some years afterwards, to examine the different cases of the problem, and to remark that, in one of them, it became indeterminate, and that, by a curious coincidence, this happened in the only case which could be supposed applicable to the astronomical problem above-mentioned; in other words, he found, that in the state of the data, which must there always take place, innumerable lines might be drawn, that would be all cut in the same ratio, by the four lines given in position. This he demonstrated in a dissertation published at Rome in 1749, and since that time in the third volume of his Opuscula. A demonstration of it, by the same author, is also inserted at the end of Castillon's Commentary on the Arithmetica Universalis, where it is deduced from a construction of the general problem, given by Mr. Thomas Simpson, at the end of his Elements of Geometry. The proposition, in Boscovich's words, is this: 'Problema quo quæritur recta linea quæ quatuor rectas positione datas ita secet, ut tria ejus segmenta sint invicem in ratione data, evadit aliquando indeterminatum, ita ut per quodvis punctum cujusvis ex iis quatuor rectis duci possit recta linea, quæ ei conditioni faciat satis.'

" It is needless, I believe, to remark, that the proposition thus enunciated is a porism, and that it was discovered by Boscovich, in the same way in which I have supposed porisms to have been first discovered by the geometers of antiquity.

" A question nearly connected with the origin of porisms still remains to be solved, namely, from what cause has it arisen that propositions which are in themselves so important, and that actually occupied so considerable a place in the ancient geometry, have been so little remarked in the modern? It cannot indeed be said, that propositions of this kind were wholly unknown to the moderns before the restoration of what Euclid had written concerning them; for besides Boscovich's proposition, of which so much has been already said, the theorem which asserts, that in every system of points there is a centre of gravity, has been shown above to be a porism; and we shall see hereafter, that many of the theorems in the higher geometry belong to the same class of propositions. We may add, that some of the elementary propositions of geometry want only the proper form of enunciation to be perfect porisms. It is not therefore strictly true, that none of the propositions called porisms have been known to the moderns; but it is certain, that they have not met, from them, with the attention they met with from the ancients, and that they have not been distinguished as a separate class of propositions. The cause of this difference is undoubtedly to be sought for in a comparison of the methods employed for the solution of geometrical problems in ancient and modern times.

" In the solution of such problems, the geometers of antiquity proceeded with the utmost caution, and were careful to remark every particular case, that is to say, every change in the construction, which any change in the state of the data could produce. The different conditions from which the solutions were derived, were supposed to vary one by one, while the others remained the same; and all their possible combinations being thus enumerated, a separate solution was given, wherever any considerable change was observed to have taken place.

" This was so much the case, that the Sectio Rationis, a geometrical problem of no great difficulty, and one of which the solution would be dispatched, according to the methods of the modern geometry, in a single page, was made by Apollonius, the subject of a treatise consisting of two books. The first book has 7 general divisions, and 24 cases; the second, 14 general divisions, and 73 cases, each of which cases is separately considered. Nothing, it is evident, that was any way connected with the problem, could escape a geometer, who proceeded with such minuteness of investigation.

" The same scrupulous exactness may be remarked in all the other mathematical researches of the ancients; and the reason doubtless is, that the geometers of those ages, however expert they were in the use of their analysis, had not sufficient experience in its powers, to trust to the more general applications of it. That principle which we call the law of continuity, and which connects the whole system of mathematical truths by a chain of insensible gradations, was scarcely known to them, and has been unfolded to us, only by a more extensive knowledge of the mathematical sciences, and by that most perfect mode of expressing the relations of quantity, which forms the language of algebra; and it is this principle alone which has taught us, that though in the solution of a problem, it may be impossible to conduct the investigation without assuming the data in a particular state, yet the result may be perfectly general, and will accommodate itself to every case with such wonderful versatility, as is scarcely credible to the most experienced mathematician, and such as often forces him to stop, in the midst of his calculus, and look back, with a mixture of diffidence and admiration, on the unforeseen harmony of his conclusions. All this was unknown to the ancients; and therefore they had no resource, but to apply their analysis separately to each particular case, with that extreme caution which has just been described; and in doing so, they were likely to remark many peculiarities, which more extensive views, and more expeditious methods of investigation, might perhaps have induced them to overlook.

" To rest satisfied, indeed, with too general results, and not to descend sufficiently into particular details, may be considered as a vice that naturally arises out of the excellence of the modern analysis. The effect which this has had, in concealing from us the class of propositions we are now considering, cannot be better illustrated than by the example of the porism discovered by Boscovich, in the manner related above. Though the problem from which that porism is derived, was resolved by several mathematicians of the first eminence, among whom also was sir Isaac Newton, yet the porism which, as it happens, is the most important case of it, was not observed by any of them. This is the more remarkable, as Sir Isaac Newton takes notice of the two most simple cases, in which the problem obviously admits of innumerable solutions, viz, when the lines given in position are either all parallel, or all meeting in a point, and these two hypotheses he therefore expressly excepts. Yet he did not remark, that there are other circumstances which may render the solution of the problem indeterminate as well as these; so that the porismatic case considered above, escaped his observation: and if it escaped the observation of one who was accustomed to penetrate so far into matters infinitely more obscure, it was because he satisfied himself with a general construction, without pursuing it into its particular cases. Had the solution been conducted after the manner of Euclid or Apollonius, the porism in question must infallibly have been discovered."

In the "Account of the Life and Writings of Rob. Simson, M. D." published in 1813, by the Rev. Dr. Wm. Trail, we find many learned observations on the subject of porisms. After a particular account of the labours of many authors on this subject, from Euclid and others among the ancients down to Pappus and Proclus, and the attempts at restoration by many of the moderns, but chiefly by Dr. Simson, Dr. Trail says,

"After a certain progress in the prosecution of this subject, it became an important object to ascertain a just definition of the porism. The definition given by the later mathematicians, as stated by Pappus, but censured by him, 'quod deficit hypothesi a theoremate locali,' clearly implies that a porism had an immediate reference to a locus; though it is not less clear that Pappus considered loci as only one class of porisms, (a large one no doubt,) but that of course many porisms have no connexion whatever with loci.

"But the definition which Pappus quotes from the ancients (viz, that it is something proposed to be investigated), as more characteristic of porisms, is too general for any useful purpose; and though it does correspond to the nature of these propositions, yet it is deficient in discrimination, and of itself neither conveys any precise notion of Euclid's porisms, nor gives assistance to the investigation of any individual proposition.

"After much consideration of various forms of a definition which had occurred to him, the doctor finally settled the following: 'A porism is a proposition in which it is proposed to demonstrate that some one or more things are given, to which, as also to every one of innumerable other things, not indeed given, but having the same relation to those that are given, it is to be shown that there belongs some common affection described in the proposition.'

"The doctor illustrates the propriety and accuracy of this definition by many examples; and shows particularly wherein the definition blamed by Pappus coincides with his, and wherein it is deficient, by excluding many genuine porisms. The definition indeed, with much address, is so framed as to correspond with all the intimations of Pappus respecting porisms, and also with the character of the individual porisms of Euclid, which Dr. Simson had discovered; and therefore may justly be considered as expressive of the notions on this subject entertained by the ancients. It is not pretended that this was a definition of the ancients; for probably no precise definition was given by them, of either theorem, problem or porism. None appears in the works of the more early geometers, which are still preserved in a considerable degree of purity, and where such definitions would naturally have had a place. And we may affirm with much probability, that if any useful and characteristic definition of a porism had reached the times of Pappus, he would not have neglected so valuable a remnant of ancient mathematical science, in a work obviously designed for the preservation of the more curious portions of it. He does not omit a definition, which probably was only a traditional and pointed observation of some ancient geometer, and though of no use in explaining the character of a porism, yet it in some degree fortified his objection to the definition of the later mathematicians, who, he states, from inability, could not accomplish the investigation of porisms; but satisfied themselves with assuming the constructions as they found them in Euclid, or other geometers, and adding the demonstrations.

"It is observed by Pappus, that a porism is neither a problem nor a theorem, but something of an intermediate nature; and that it might be proposed either as a problem or as a theorem; some geometers contending for the one, and some for the other. Dr. Simson has given a form to the enunciation of a porism, implying this intermediate character between a problem and a theorem. In his enunciation it is affirmed that certain things *may* be found, which shall have the relations or properties therein described. Perhaps this form resembles more that of a theorem, than of a problem; but at the same time, the things, of which it is said that they may be found, must be actually investigated by analysis, as if the proposition were a problem. Were it simply proposed te investigate certain things which would have the proporoties expressed in the porism, it may be regarded as a problem; but if these things are found by a construction described in the enunciation, the proposition becomes a theorem, affirming the truth of the properties asserted; and then a demonstration only is required, without any investigation; in the manner which appears to have been practised by the later mathematicians, alluded to by Pappus.

"I cannot omit adverting in this place to a very ingenious theory of porisms proposed by Mr. professor Playfair of Edinburgh, first briefly in his account of the life of Dr. Stewart, and afterwards more fully explained in a memoir on that subject in the 3d volume of the Transactions of the Royal Society of Edinburgh. The result of his investigation is, that a porism is the case of a problem which becomes indeterminate; or more particularly 'a porism is a proposition affirming the possibility of finding such conditions as will render a certain problem indeterminate, or capable of innumerable solutions.' But though I admire the ingenuity, and fully admit the soundness, of this definition, and also the utility of the principle on which it is founded, in the discovery of porisms, I must acknowledge my doubt of that particular notion of a porism having ever been adopted, or even proposed, among the ancient geometricians. The circumstance of its being so satisfactory as a definition, is to me a proof that it was never generally known or embraced: for had it ever been approved and established, it seems scarce possible that it should afterwards have been neglected and lost. That, among the ancients, the consideration of the relations subsisting among the data, in some problems, might have occasionally suggested the particular case in which these problems would become indeterminate, is very probable. It might also have often occurred to them, that this indeterminate case involved an important general proposition, which might be separately stated as such, and preserved. Many porisms of Euclid may possibly have been invented in that way; but still I entertain a doubt, if ever the ancients were in possession of this notion as a principle, and as the proper ground of the definition of a porism. Pappus mentions the definition of the ancients, and apparently as the only one which they were known to possess, though, as has been remarked, it be of no particular use. He mentions also a definition of the later mathematicians, which he censures as erroneous: but, if such a complete and satisfactory definition, which not only accurately distinguishes that class of propositions, but points out an obvious source of the discovery of them, had ever been generally understood among the ancients, it is difficult to suppose that it could ever have been lost; and had it reached the time of Pappus, it is most improbable that he should neglect the recording of it in his detailed account of Euclid's treatise on this subject. With these strong internal probabilities, and the total want of external evidence, I must (with deference, however, to the opinion of those who may think differently) adhere to the judgment which I have already expressed, concerning the recent origin of this excellent definition, proposed by Mr. Playfair."

On this subject, see also several other places in Dr. Trail's works, particularly the note D, pa. 88.

Porism was also used in another sense, by the ancient geometricians, and even down to near the 17th century, to denote the same thing as the common corollary.

PORISTIC *Method*, is that which determines when, and by what means, and how many different ways, a problem may be resolved.

PORTA (John Baptista), called also in Italy Giovan Batista de la Porta, of Naples, flourished about the end of the 16th century, and was famous for his skill in philosophy, mathematics, medicine, natural history, &c, as well as for his indefatigable endeavours to improve and propagate the knowledge of those sciences. With this view, he not only established private schools for particular sciences, but to the utmost of his power promoted public academies. He had no small share in establishing the academy at Gli Ozioni, at Naples, and had one in his own house, called de Secreti, into which none were admitted members, but such as had made some new discoveries in nature. He invented the camera obscura, improved afterwards by Gravesande, and formed the plan of an encyclopædia. He died at Pisa, in the kingdom of Naples, in the year 1615. Porta gave the fullest proof of an extensive genius, and wrote a great many works; the principal of which are as follow:

1. His Natural Magic; a book abounding with curious experiments; but containing nothing of magic, in the common acceptation of the word, as he pretends to nothing above the power of nature.
2. Elements of Curve Lines.
3. A Treatise of Distillation.
4. A Treatise of Arithmetic.
5. Concerning Secret Letter-writing.
6. Of Optical Refractions.
7. A Treatise of Fortification.
8. A Treatise of Physiognomy.

Beside some Plays and other pieces of less note.

PORTAIL, in Architecture, the face or frontispiece of a church, viewed on the side in which the great door is placed. It means also the great door or gate itself of a palace, castle, &c.

PORTAL, in Architecture, a term used for a little square corner of a room, cut off from the rest of the room by the wainscot; frequent in the ancient buildings, but now disused.

Portal is sometimes also used for a little gate, portella; where there are two gates, a large and a small one.

Portal is sometimes also used for a kind of arch of joiner's work before a door.

PORTCULLICE, called also *Herse*, and *Sarrasin*, in Fortification, an assemblage of several large pieces of wood laid or joined across one another, like a harrow, and each pointed at the bottom with iron. These were formerly used to be hung over the gateways of fortified places, to be ready to let down in case of a surprise, when the enemy should come so quick, as not to allow time to shut the gates. But the orgues are now more generally used, being found to answer the purpose better.

PORT-Fire, in Gunnery, a paper tube, about 10 inches long, filled with a composition of meal-powder, sulphur, and nitre, rammed moderately hard; used to fire guns and mortars, instead of a match.

PORTICO, in Architecture, is a kind of gallery, raised upon arches, under which people walk for shelter.

POSITION, or *Site*, or *Situation*, in Physics, is an affection of place, expressing the manner of a body's being in it.

Position, in Architecture, denotes the situation of a building, with respect to the points of the horizon. The best it is thought is when the four sides point directly to the four winds, or cardinal points.

Position, in Astronomy, relates to the sphere. The position of the sphere is either right, parallel, or oblique; whence arise the inequality of days, the difference of seasons, &c.

Circles of Position, are circles passing through the common intersections of the horizon and meridian, and through any degree of the ecliptic, or the centre of any star, or other point in the heavens; used for finding out the position or situation of any star. These are usually counted six in number, cutting the equator into twelve equal parts, which the astrologers call the celestial houses.

Position, in Arithmetic, called also False Position, or Supposition, or Rule-of-False, is a rule so called, because it consists in calculating by false numbers, supposed or taken at random, according to the process described in any question or problem proposed, as if they were the true numbers, and then from the results, compared with that given in the question, the true numbers are found. It is sometimes also called Trial-and-Error, because it proceeds by trials of false numbers, and thence finds out the true ones by a comparison of the errors.—Position is either single or double.

Single Position is when only one supposition is employed in the calculation. And *Double* Position is that in which two suppositions are employed.—To the rule of position properly belong such questions as cannot be resolved from a direct process by any of the other usual rules in arithmetic, and in which the required numbers do not ascend above the first power: such, for example, as most of the questions usually brought to exercise the reduction of simple equations in algebra. But it will not bring out true answers when the numbers sought ascend above the first power; for then the results are not proportional to the positions, or supposed numbers, as in the single rule; nor yet the errors to the difference of the true number and each position, as in the double rule. Yet in all such cases, it is a very good approximation, and in exponential equations, as well as in many other things, it succeeds better than perhaps any other method whatever.

Those questions, in which the results are proportional to their suppositions, belong to single position: such are those which require the multiplication or division of the number sought by any number; or in which it is to be increased or diminished by itself any number of times, or by any part or parts of it. But those in which the results are not proportional to their positions, belong to the double rule: such are those, in which the numbers sought, or their multiples or parts, are increased or diminished by some given absolute number, which is no known part of the number sought.

In Single Position. Suppose, or assume any number at pleasure, for the number sought, and proceed with it as if it were the true number, that is, perform the same operations with it as, in the question, are described to be performed with the number sought: then if the result of those operations be the same with that mentioned or given in the question, the supposed number is the same as the true one that was required; but if it be not, make this proportion, viz, as the result is to that in the question, so is the supposed false number, to the true one required.

Example. Suppose that a person, after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, has yet remaining 60*l.*; what sum had he at first?

Suppose he had at first 120*l.*
Now $\frac{1}{3}$ of 120 is 40
and $\frac{1}{4}$ of it is 30
their sum is 70
which taken from 120
leaves remaining 50, instead of 60.
Therefore as 50 : 60 : : 120 : 144 the sum at first.
Proof. $\frac{1}{3}$ of 144 is 48
$\frac{1}{4}$ of it is 36
their sum 84
taken from 144
leaves just 60 as per quest.

To work by the Double Rule of Position.

In this rule, make two different suppositions, or assumptions, and work or perform the operations with each, described in the question, exactly as in the single rule: and if neither of the supposed numbers solve the question, that is, produce a result agreeing with that in the question; then observe the errors, or how much each of the false results differs from the true one, and also whether they are too great or too little; marking them with + when too great, and with − when too little. Next multiply, crosswise, each position by the error of the other; and if the errors be of the same affection, that is both +, or both −, subtract the one product from the other, as also the one error from the other, and divide the former of these two remainders by the latter, for the answer, or number sought. But if the errors be unlike, that is, the one +, and the other −, add the two products together, and also the two errors together, and divide the former sum by the latter, for the answer.

Rule 2. Multiply the difference of the two assumed numbers by one of the errors, and divide the product by the difference of the results, the quotient will be the correction of the assumed number belonging to that error: Then add this quotient or correction to the said assumed number when it is too small, but subtract it when too great, to give the answer.

This rule of position, or trial-and-error, is a good general way of approximating to the roots of the higher equations, to which it may be applied even before the equation is reduced to a final or simple state, by which it often saves much trouble in such reductions. It is also eminently useful in resolving exponential equations, and equations involving arcs, or sines, &c, or logarithms, and in short in any equations that are very intricate and difficult. And even in the extraction of the higher roots of common numbers, it may be very usefully applied. For examples, and the demonstration of the rules, see the 1st vol. of my Course of Mathematics.

The rule of position passed from the Moors into Europe, by Spain and Italy, along with their algebra, or method of equations, which was probably derived from the former.

Position, in Geometry, respects the situation, bearing, or direction of one thing, with regard to another. And Euclid says, "Points, lines, and angles, which have and keep always one and the same place and situation, are said to be given by position or situation." Data, def. 4.

POSITIVE *Quantities*, in Algebra, such as are of a real, affirmative, or additive nature; and which either have, or are supposed to have, the affirmative or positive sign + before them; as *a* or + *a*, or *bc*, &c. It is used in contradistinction from negative quantities, which are defective or subductive ones, and marked by the sign −; as − *a*, or − *ab*.

Positive *Electricity*. In the Franklinian system, all bodies supposed to contain more than their natural quantity of electric matter, are said to be positively electrified; and those which have less than that quantity, are said to be electrified negatively. These two electricities being at first produced, the one from glass, the other from amber or rosin, the former was called vitreous, the other resinous electricity.

POSTERN, or *Sally-port*, in Fortification, a small gate, usually made in the angle of the flank of a bastion, or in that of the curtain, or near the orillon, descending into the ditch; by which the garrison can march in and out, unperceived by the enemy, either to relieve the works, or to make private sallies, &c.—It means also any private or back door.

POSTICUM, in Architecture, the postern gate, or back-door of any fabric.

POSTULATE, a demand, petition, or an assertion of so obvious a nature, as to need neither demonstration nor explication, to render it either more plain or certain. This definition will nearly agree also to an axiom, which is a self-evident theorem, as a postulate is a self-evident problem.—Euclid lays down these three postulates in his Elements; viz, 1st, That from one point to another a line can be drawn. 2d, That a right line can be produced out at at pleasure. 3d, That with any centre and radius a circle may be described.—As to axioms, he has a great number; as, That two things which are equal to one and the same thing, are equal to each other, &c.

POTASH, in chemistry, one of the three fixed alkalies, procured from the burnt ashes of vegetables, by combustion in iron or other pots; whence the compound pot-ash.

POTASSIUM, a recently discovered and very singular metal, obtained by peculiar management, from pot-ash, which in modern chemistry can only be regarded as its oxyd.

POUND, a certain weight; which is of two kinds, viz, the pound troy, and the pound avoirdupois; the former consisting of 12 ounces troy, and the latter of 16 ounces avoirdupois. The pound troy is to the pound avoirdupois as 5760 to 6999½, or nearly 576 to 700.

POUND also is an imaginary money used in accounting, in several countries. Thus, in England there is the pound sterling, containing in value 20 shillings; in France the pound or livre Tournois and Parisis; in Holland and Flanders, a pound or livre de gros, &c.—The term arose from hence, that the ancient pound sterling, though it only contained 240 pence, as ours does; yet each penny being equal to five of ours, the pound of silver weighed a pound troy.

POUNDER, in Artillery, a term used to express a certain weight of shot or ball, or how many pounds weight the proper ball is for any cannon: as a 24 pounder, a 12 pounder, &c.

POWDER, *Gun*. See GUNPOWDER.

POWDER-*Triers*. See EPROUVETTE.

POWER, in Mechanics, denotes some force which, being applied to a machine, tends to produce motion; whether it does actually produce it or not. In the former case, it is called a moving power; in the latter, a sustaining power.

POWER is also used in Mechanics, for any of the six simple machines, viz, the lever, the balance, the screw, the wheel and axle, the wedge, and the pulley.

POWER *of a Glass*, in Optics, is used for the distance between the convexity and the solar focus.

POWER, in Arithmetic, the produce of a number, or other quantity, arising by multiplying it by itself, any number of times. Any number is called the first or single power of itself. If it be multiplied once by itself, the product is the second power, or square; if this be multiplied by the first power again, the product is the third power, or cube; if this be multiplied by the first power again, the product is the fourth power, or biquadratic; and so on; the power being always denominated from the number which exceeds the multiplications by one or unity, which number is called the index or exponent of the power, and is usually set at the upper corner towards the right of the given quantity or root, to denote or express the power. Thus,

$$\begin{array}{lcl} 3 \quad \text{or } 3^1 = 3 & \text{is the 1st power of } 3, \\ 3 \times 3 \text{ or } 3^2 = 9 & \text{is the 2d power of } 3, \\ 3^2 \times 3 \text{ or } 3^3 = 27 & \text{is the 3d power of } 3, \\ 3^3 \times 3 \text{ or } 3^4 = 81 & \text{is the 4th power of } 3, \\ \&c. & \&c. \end{array}$$

Hence, to raise a quantity to a given power, is the same as to find the product arising from its being multiplied by itself a certain number of times; for example to raise 2 to the 3d power, is the same thing as to find the factum, or product $8 = 2 \times 2 \times 2$. The operation of raising powers, is called Involution.

Powers, of the same degree, are to one another in the ratio of the roots as manifold as their common exponent contains units: thus, squares are in a duplicate ratio of the roots; cubes in a triplicate ratio; 4th powers in a quadruplicate ratio.—And the powers of proportional quantities are also proportional to one another: so, if $a : b :: c : d$, then, in any powers also, $a^n : b^n :: c^n : d^n$.

The particular names of the several powers, as introduced by the Arabians, were, square, cube, quadratoquadratum or biquadrate, sursolid, cube squared, second sursolid, quadrato-quadrato-quadratum, cube of the cube, square of the sursolid, third sursolid, and so on, according to the *products* of the indices.

And the names given by Diophantus, who is followed by Vieta and Oughtred, are, the side or root, square, cube, quadrato-quadratum, quadrato-cubus, cubo-cubus, quadrato-quadrato-cubus, quadrato-cubo-cubus, cubo-cubo-cubus, &c, according to the *sums* of the indices.

But the moderns, after Harriot and Descartes, are contented to distinguish most of the powers by the exponents; as 1st, 2d, 3d, 4th, &c.

The characters by which the several powers are denoted, both in the Arabic and Cartesian notation, are thus:

Arab.	1	*R*	*q*	*c*	*bq*	*s*	*qc*	*Bs*	*tq*	*bc*
Cart.	a^0	a^1	a^2	a^3	a^4	a^5	a^6	a^7	a^8	a^9
	1	2	4	8	16	32	64	128	256	512

Hence, 1st. The powers of any quantity, form a series of geometrical proportionals, and their exponents a series of arithmetical proportionals, in such sort that the addition of the latter answers to the multiplication of the former, and the subtraction of the latter answers to the division of the former, &c; or in short, that the latter, or exponents, are as the logarithms of the former, or powers.

$$\begin{array}{l} \text{Thus, } a^2 \times a^3 = a^5, \text{ and } 2 + 3 = 5; \\ \qquad 4 \times 8 = 32; \\ \text{also } a^5 \div a^3 = a^2, \text{ and } 5 - 3 = 2; \\ \qquad 32 \div 8 = 4. \end{array}$$

2d. The 0 Power of any quantity, as a^0, is $= 1$.

3d. Powers of the same quantity are multiplied, by adding their exponents: Thus,

$$\begin{array}{llllll} \text{Mult.} & a^3 & x^2 & y^m & x^m & a^3 \\ \text{by} & a^4 & x^4 & y^m & x^n & a^n \\ \hline \text{Prod.} & a^7 & x^6 & y^{2m} & x^{m+n} & a^{3+n} \end{array}$$

4th. Powers are divided by subtracting their exponents.

$$\begin{array}{llllll} \text{Div.} & a^7 & x^6 & y^{2m} & x^{m+n} & a^{3+n} \\ \text{by} & a^3 & x^2 & y^m & x^m & a^3 \\ \hline \text{Quot.} & a^4 & x^4 & y^m & x^n & a^n \end{array}$$

5th. Powers are also considered as negative ones, or having negative exponents, when they denote a divisor, or the denominator of a fraction. So $\frac{1}{a^3} = a^{-3}$, and $\frac{2}{a^2} = 2a^{-2}$, and $\frac{a^2}{x^4} = a^2x^{-4}$, &c. And hence any quantity may be changed from the denominator to the numerator, or from a divisor to a multiplier, or vice versa, by changing the sign of its exponent; and the whole series of powers proceeds indefinitely both ways from 1 or the 0 power, positive on the one hand, and negative on the other. Thus,

$$\&c\ a^{-4}\ a^{-3}\ a^{-2}\ a^{-1}\ a^0\ a^1\ a^2\ a^3\ a^4\ \&c,$$

$$\text{or, } \&c\ \frac{1}{a^4}\quad \frac{1}{a^3}\quad \frac{1}{a^2}\quad \frac{1}{a}\quad 1\quad a\quad a^2\quad a^3\quad a^4\ \&c.$$

Powers are also denoted with fractional exponents, or even with surd or irrational ones; and then the numerator denotes the power raised to, and the denominator the exponent of some root to be extracted: Thus,

$\sqrt{a} = a^{\frac{1}{2}}$, and $\sqrt{a^3} = a^{\frac{3}{2}}$, and $\sqrt[3]{a^2} = a^{\frac{2}{3}}$, &c. These are sometimes called imperfect powers, or surds.

When the quantity to be raised to any power is positive, all its powers must be positive. And when the radical quantity is negative, yet all its even powers must be positive: because $-\times-$ gives $+$: the odd powers only being negative, or when their exponents are odd numbers: Thus, the powers of $-a$,

are $+ 1, - a, + a^2, - a^3, + a^4, - a^5, + a^6$, &c.
where the even powers a^2, a^4, a^6 are positive,
and the odd powers a, a^3, a^5 are negative.

Hence, if a power have a negative sign, no even root of it can be assigned; since no quantity multiplied by itself an even number of times, can give a negative product. Thus $\sqrt{-a^2}$, or the square or 2d root of $-a^2$, cannot be assigned; and is called an impossible root, or an imaginary quantity. Every power has as many roots, real and imaginary, as there are units in the exponent.

M. Lahire gives a very odd property common to all powers. M. Carre had observed with regard to the number 6, that all the natural cubic numbers, 8, 27, 64, 125, having their roots less than 6, being divided by 6, the remainder of the division is the root itself; and if we go farther, 216, the cube of 6, being divided by 6, leaves no remainder; but the divisor 6 is itself the root. Again, 343, the cube of 7, being divided by 6, leaves 1; which added to the divisor 6, makes the root 7, &c. M. Lahire, on considering this, has found that all numbers, raised to any power whatever, have divisors, which have the same effect with regard to them, that 6 has with regard to cubic numbers. For finding these divisors, he discovered the following general rule, viz, If the exponent of the power of a number be even, i. e. if the number be raised to the 2d, 4th, 6th, &c, power, it must be divided by 2; the remainder of the division, when there is any, added to 2, or to a multiple of 2, gives the root of this number, corresponding to its power, i. e. the 2d, 4th, 6th, &c root.

But if the exponent of the power be an uneven number, i. e. if the number be raised to the 3d, 5th, 7th, &c power; the double of this exponent will be the divisor, which has the property abovementioned. Thus is it found in 6, the double of 3, the exponent of the power of the cubes: so also 10, the double of 5, is the divisor of all 5th powers; &c.

If r be a prime number, and n any number not divisible by r, then n^r, being divided by r, will leave the same remainder, as n when divided by the same number: and hence it follows that $\frac{n^r - n}{r}$ is always an integer; and since n is prime to r, therefore $\frac{n^{r-1} - 1}{r}$ is always an integer when r is a prime number and n prime to r. This is a very important theorem in the theory of numbers, the invention of which is due to Fermat, though the demonstration of it was first given by Euler in the Petersburg Memoirs.

By means of this theorem we readily deduce the following table of the forms of powers, with regard to certain numbers taken as moduli. Thus all

2nd powers are of the form	$5n$	or	$5n \pm 1$
3d powers - - - - -	$7n$	or	$7n \pm 1$
4th powers - - - - -	$5n$	or	$5n + 1$
5th powers - - - - -	$11n$	or	$11n + 1$
6th powers - - - - -	$13n$	or	$13n \pm 1$
8th powers - - - - -	$17n$	or	$17n \pm 1$
9th powers - - - - -	$19n$	or	$19n \pm 1$
10th powers - - - - -	$11n$	or	$11n \pm 1$
11th powers - - - - -	$23n$	or	$23n \pm 1$
12th powers - - - - -	$13n$	or	$13n + 1$

And generally if $m + 1$ is a prime number, then x^m is of one of the forms $(m + 1)n$ or $(m + 1)n + 1$. And if $2m + 1$ be a prime, then x^m is of one of the three forms $(m + 1)n$ or $(m + 1)n \pm 1$. And since neither $7 + 1$ nor $2.7 + 1$ is a prime, therefore 7th powers cannot be reduced to the laws observed in the foregoing table, for which reason this power is there omitted.

Any power of the natural numbers 1, 2, 3, 4, 5, 6, &c, as the nth power, has as many orders of differences as there are units in the common exponent of all the numbers; and the last of those differences is a constant quantity, and equal to the continual product $1 \times 2 \times 3 \times 4 \times \text{------} \times n$, continued till the last factor, or the number of factors, be n, the exponent of the powers. Thus,

the 1st powers, 1 , 2 , 3 , 4 , 5 , &c, have but one order
of differences 1 1 1 1 &c, and that difference is 1.
The 2d pwrs. 1 , 4 , 9 , 16 , 25, &c, have two orders of
differences 3 5 7 9
2 2 2
and the last of these is $2 = 1 \times 2$.
The 3d pwrs. 1 , 8 , 27 , 64 , 125 , &c, have three orders
of differences 7 19 37 61
12 18 24
6 6
and the last of these is $6 = 1 \times 2 \times 3$.

In like manner, the 4th or last differences of the 4th powers, are each $24 = 1 \times 2 \times 3 \times 4$; and the 5th or last differences of the 5th powers, are each $120 = 1 \times 2 \times 3 \times 4 \times 5$. And so on. Which property was first noticed by Peletarius.

And the same is true of the powers of any other arithmetical progression $1, 1 + d, 1 + 2d, 1 + 3d$, &c,
viz, $1, (1 + a)^n, (1 + 2d)^n, (1 + 3d)^n$, &c,
the number of the orders of differences being still the same exponent n, and the last of those orders each equal to $1 \times 2 \times 3 \text{------} \times nd^n$, the same product of factors as before, multiplied by the same power of the common difference d of the series of roots: as was shown by Briggs.

And hence arises a very easy and general way of raising the powers of all the natural numbers, viz, by common addition only, beginning at the last differences, and adding them all continually, one after another, up to the powers themselves. Thus, to generate the series of cubes, or 3d powers, adding always 6, the common 3d difference gives the 2d differences 12, 18, 24, &c; and these added to the 1st of the 1st differences 7, gives the rest of the said 1st differences; and these again added to the 1st cube 1, gives the rest of the series of cubes, 8, 27, 64, &c, as below.

3dD.	2dD.	1stD.	Cubes.
			1
		7	
	12		8
6		19	
	18		27
6		37	
	24		64
6		61	
	30		125
		91	
			216
			&c.

Commensurable in POWER, is said of quantities which, though not commensurable themselves, have their squares, or some other power of them, commensurable. Euclid confines it to squares. Thus, the diagonal and side of a square are commensurable in power, their squares being as 2 to 1, or commensurable; though they are not commensurable themselves, being as $\sqrt{2}$ to 1.

POWER *of an Hyperbola*, is the square of the 4th part of the conjugate axis.

PRACTICAL Arithmetic, Geometry, Mathematics,

&c, is the part that regards the practice, or application, as distinguished from the theoretical part.

PRACTICE, in Arithmetic, is a rule which expeditiously and compendiously answers questions in the golden rule, or rule-of-three, especially when the first term is 1. See rules for this purpose in all the books of practical arithmetic.

PRECESSION *of the Equinoxes*, is a very slow motion of them, by which they change their place, going from east to west, or backward, *in antecedentia*, as astronomers call it, or contrary to the order of the signs. From the late improvements in astronomy it appears, that the pole, the solstices, the equinoxes, and all the other points of the ecliptic, have a retrograde motion, and are constantly moving from east to west, or from Aries towards Pisces, &c; by means of which, the equinoctial points are carried farther and farther back, among the preceding signs or stars, at the rate of about 50″⅕ each year; which retrograde motion is called the Precession, Recession, or Retrocession of the Equinoxes.

Hence, as the stars remain nearly immoveable, and the equinoxes go backward, the stars will seem to move more and more eastward with respect to them; for which reason the longitudes of all the stars, being reckoned from the first point of Aries, or the vernal equinox, are continually increasing. From this cause it is, that the constellations seem all to have changed the places assigned to them by the ancient astronomers. In the time of Hipparchus, and the oldest astronomers, the equinoctial points were fixed to the first stars of Aries and Libra: but the signs do not now answer to the same points; and the stars which were then in conjunction with the sun when he was in the equinox, are now a whole sign, or 30 degrees, to the eastward of it: so, the first star of Aries is now in the portion of the ecliptic, called Taurus; and the stars of Taurus are now in Gemini; and those of Gemini in Cancer; and so on.

This seeming change of place in the stars was first observed by Hipparchus of Rhodes, who, 128 years before Christ, found that the longitudes of the stars in his time were greater than they had been before observed by Timochares, and than they were in the sphere of Eudoxus, who wrote 380 years before Christ. Ptolemy also perceived the gradual change in the longitudes of the stars; but he stated the quantity at too little, making it but 1° in 100 years, which is at the rate of only 36″ per year. Y-hang, a Chinese, in the year 721, stated the quantity of this change at 1° in 83 years, which is at the rate of 43″½ per year. Other more modern astronomers have made this precession still more, but with some small differences from each other; and it is now usually taken at 50″⅕ per year. All these rates are deduced from a comparison of the longitude of certain stars as observed by more ancient astronomers, with the later observations of the same stars; viz, by subtracting the former from the latter, and dividing the remainder by the number of years in the interval between the dates of the observations. Thus, by a medium of a great number of comparisons, the quantity of the annual change has been fixed at 50″⅕.

Thus, by taking the longitudes of the principal stars established by Tycho Brahé, in his book Astronomiæ Instauratæ Progymnasmata, pa. 208 and 232, for the beginning of 1586, and comparing them with the same as determined for the year 1750, by M. Lacaille, for that interval of 164 years, there will be obtained the following differences of longitude of several stars; viz,

γ Arietis	2°	17′	37″
Aldebaran	2	17	45
μ Geminorum	2	17	1
β Geminorum	2	15	26
Regulus	2	16	32
α Virginis	2	18	18
α Aquilæ	2	19	1
α Pegasi	2	16	12
β Libræ	2	17	52
Antares	2	16	28
ε Tauri	2	17	58
γ Geminorum	2	18	38
γ Cancri	2	19	12
γ Leonis	2	19	38
γ Capricorni	2	16	10
Medium of these 15 stars	2	17	35

which divided by 164, the interval of years, gives 50″·336, or nearly 50″⅓, or after the rate of 1° 23′ 53″⅓ in 100 years. And nearly the same conclusion results from the longitudes of the stars in the Britannic catalogue, compared with those of the still later catalogue. See Lalande's Astronomy, in several places.

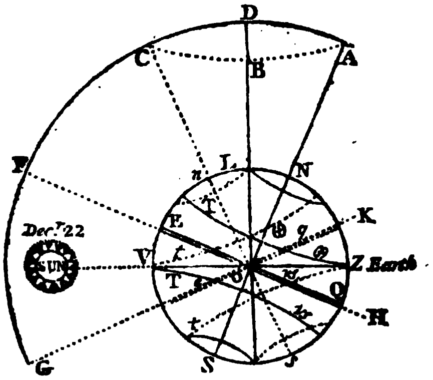
Mr. Mayer, in the construction of his tables, assumed the precession of the equinoxes, or the annual motions of the fixed stars in longitude, to be exactly 50″·3, without paying any regard to the alteration of the place of the equinox arising from the translation of the plane of the ecliptic by the action of the planets. Dr. Bradley, by comparing his own observations of declinations of stars, lying on both sides of the equinoctial colure, with the like observations of Tycho Brahé, found the precession of the equinoxes in longitude, to be exactly 1° in 71½ years, or at the rate of 50″·35 in a year, which is evidently what arises from the motion of the plane of the equator alone, being that which is occasioned by the actions of the sun and moon on the spheroidal figure of the earth. But the equinoctial point is also altered, though in a far less degree, by the continual motion of the plane of the ecliptic, owing to the action of the planets, and goes forward 0″·15 in a year, from that cause, along the ecliptic, which will diminish the precession of the equinoxes, on the apparent annual motions of the fixed stars, lying near the plane of the ecliptic, in longitude as much, and so reduce them from 50″·35 to 50″·20 or 50″⅕. See Naut. Ephemer. for 1797, the preface.

Taking therefore 50″⅕ for the true mean annual precession of the equinoxes, at this rate it will require 25,816¾ years for the equinoxes to make their revolution westward quite around the circle, and return to the same point again.

The ancients, and even some of the moderns, have taken the equinoxes to be immoveable; and ascribed that change in the distance of the stars from it, to a real motion of the orb of the fixed stars, which they supposed had a slow revolution about the poles of the ecliptic; so as that all the stars perform their circuits in the ecliptic, or its parallels, in the space of 25,791 years; after which they should all return again to their former places.

This period the ancients called the Platonic, or great year; and imagined that at its completion every thing would begin as at first, and all things come round in the same order as they have done before.

The phenomena of this retrograde motion of the equinoxes, or intersections of the equinoctial with the ecliptic, and consequently of the conical motion of the earth's axis, by which the pole of the equator describes a small circle in the same period of time, may be understood and illustrated by a scheme, as follows: Let NZSVL be the earth, SONA its axis produced to the starry heavens, and terminating in A, the present north pole of the heavens, which is vertical to N, the north pole of the earth. Let EOQ be the equator, T♋Z the tropic of cancer, and VT♑ the tropic of Capricorn; VOZ the ecliptic, and BO its axis, both of which are immoveable among the stars. But as the equinoctial points recede in the ecliptic, the earth's axis SON is in motion upon the earth's centre O, in such a manner as to describe the double cone NO*n* and SO*s*, round the axis of the ecliptic BO, in the time that the equinoctial points move round the ecliptic, which is 25,791 years; and in that length of time, the north pole of the earth's axis, produced, describes the circle ABCDA in the starry heavens, round the pole of the ecliptic, which keeps immoveable in the centre of that circle. The earth's axis being now 23° 28′ inclined to the axis of the ecliptic, the circle ABCDA, described by the north pole of the earth's axis produced to A, is 46° 56′ in diameter, or double the inclination of the earth's axis. In consequence of this, the point A, which is at present the north pole of the heavens, and near to a star of the 2d magnitude in the end of the Little Bear's tail, must be deserted by the earth's axis; which moving backwards 1 degree every 71⅔ years nearly, will be directed towards the star or point B in 6447¾ years hence; and in double of that time, or 12,895½ years, it will be directed towards the star or point C; which will then be the north pole of the heavens, though it is at present 8½ degrees south of the zenith of London L. The present position of the equator EOQ will then be changed into *eoq*, the tropic of cancer T♋Z into V*t*♋, and the tropic of capricorn VT♑ into *t*♑Z; as is evident by the figure. And the sun, in the same part of the heavens where he is now over the earthly tropic of capricorn, and makes the shortest days and longest nights in the northern hemisphere, will then be over the earthly tropic of cancer, and make the days longest and nights shortest. So that it will require 12,895½ years yet more, or from that time, to bring the north pole N quite round, so as to be directed toward that point of the heavens which is vertical to it at present. And then, and not till then, the same stars which at present describe the equator, tropics, and polar circles, &c, by the earth's diurnal motion, will describe them over again.



From this movement of the equinoctial points, and with them all the signs of the ecliptic, it follows, that those stars which in the infancy of astronomy were in Aries, are now found in Taurus; those of Taurus in Gemini, &c. Hence likewise it is, that the stars which rose or set at any particular season of the year, in the times of Hesiod, Eudoxus, Virgil, Pliny, &c, by no means answer at this time to their descriptions.

As to the physical cause of the precession of the equinoxes, sir Isaac Newton demonstrates, that it arises from the broad or flat spheroidal figure of the earth; which itself arises from the earth's rotation about its axis: for as more matter has thus been accumulated all round the equatorial parts, than any where else on the earth, the sun and moon, when on either side of the equator, by attracting this redundant matter, bring the equator sooner under them, in every return towards it, than if there were no such accumulation.

Sir Isaac Newton, in determining the quantity of the annual precession from the theory of gravity, on supposition that the equatorial diameter of the earth is to the polar diameter, as 230 to 229, finds the sun's action sufficient to produce a precession of 9″⅛ only; and collecting from the tides the proportion between the sun's force and the moon's to be as 1 to 4½, he settles the mean precession resulting from their joint actions, at 50″; which, it must be owned, is nearly the same as it has since been found by the best observations; and yet several other mathematicians have since objected to the truth of Newton's computation.

Indeed, to determine the quantity of the precession arising from the action of the sun, is a problem that has been much agitated among modern mathematicians; and though they seem to agree as to Newton's mistake in the solution of it, they have yet generally disagreed from one another. Dalembert, in 1749, printed a treatise on this subject, and claims the honour of having been the first who rightly determined the method of resolving problems of this kind. The subject has been also considered by Euler, Frisi, Silvabelle, Walmesley, Simpson, Emerson, Laplace, Lagrange, Landen, Milner, and Vince.

M. Silvabelle, stating the ratio of the earth's axis to be that of 178 to 177, makes

the annual precession caused by the sun 13″ 52‴,
and that of the moon - - 34 17;

making the ratio of the lunar force to the solar, to be that of 5 to 2; also the nutation of the earth's axis caused by the moon, during the time of a semirevolution of the pole of the moon's orbit, i. e. in 9⅓ years, he makes 17″ 51‴. —Walmesley, on the supposition that the ratio of the earth's diameters is that of 230 to 229, and the obliquity of the ecliptic to the equator 23° 28′ 30″, makes the annual precession, owing to the sun's force, equal to 10″·583; but supposing the ratio of the diameters to be that of 178 to 177, that precession will be 13″·675.—Mr. Simpson, by a different method of calculation, determines the whole annual precession of the equinoxes caused by the sun, at 21″ 6‴; and he has pointed out the errors of the computations proposed by Silvabelle and Walmesley.—Mr. Milner's deduction agrees with that of Mr. Simpson, as well as Mr. Vince's; and their papers contain besides several curious particulars relative to this subject. But for the various principles and reasonings of these mathematicians, see Philos. Trans. vol. 48, pa. 385; vol. 49, pa. 704; vol. 69, pa. 505; and vol. 77, pa. 363; as also the writings of Simpson, Emerson, Landen, &c; also Lalande's Astronomie, and the Memoirs of the Acad. Sci. in several places.

As to the effect of the planets on the equinoctial points, Laplace, in his new researches on this article, finds that their action causes those points to advance by $0''\cdot2016$ in a year, along the equator, or $0''\cdot1849$ along the ecliptic; whence it follows that the quantity of the luni-solar precession must be $50''\cdot4349$, since the total observed precession is $50''\frac{1}{4}$, or $50''\cdot25$.

To find the Precession in right ascension and declination.

Put $d =$ the declination of a star,
and $a =$ its right ascension;

then their annual variations of precessions will be nearly as follow:

viz, $20''\cdot084 \times \cos. a =$ the annual preces. in declinat.
and $46''\cdot0619 + 20''\cdot084 \times \sin. a \times \text{tang}. d =$ that of right ascension. See the Connoissance des Temps for 1792, pa. 206, &c.

PRESS, in Mechanics, is a machine made of iron or wood, serving to compress or squeeze any body very close, by means of screws. The common presses consist of six members, or pieces; viz, two flat and smooth planks; between which the things to be pressed are laid; two screws, or worms, fastened to the lower plank, and passing through two holes in the upper; and two nuts, serving to drive the upper plank, which is moveable, against the lower, the latter being stable, and without motion.

PRESSION. See Pressure.

PRESSURE, is properly the action of a body which makes a continual effort or endeavour to move another; such as the action of a heavy body supported by a horizontal table; in contradistinction from percussion, or a momentary force or action. Pressure equally respects both bodies, that which presses, and that which is pressed; from the mutual equality of action and reaction.

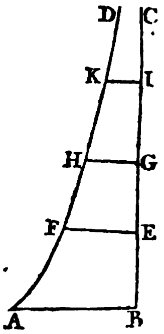
Pressure, in the Cartesian philosophy, is an impulsive kind of motion, or rather an endeavour to move, impressed on a fluid medium, and propagated through it. In such a pressure the Cartesians suppose the action of light to consist. And in the various modifications of this pressure, by the surfaces of bodies, on which that medium presses, they suppose the various colours to consist, &c. But Newton shows, that if light consisted only in a pressure, propagated without actual motion, it could not agitate and warm such bodies as reflect and refract it, as we actually find it does; and if it consisted in an instantaneous motion, or one propagated to all distances in an instant, as such pressure supposes, there would be required an infinite force to produce that motion every moment, in every lucid particle. Further, if light consisted either in pressure, or in motion propagated in a fluid medium, whether instantaneously, or in time, it must follow, that it would inflect itself ad umbram; for pressure, or motion, in a fluid medium, cannot be propagated in right lines, beyond any obstacle which shall hinder any part of the motion; but will inflect and diffuse itself, every way, into those parts of the quiescent medium which lie beyond the said obstacle. Thus the force of gravity tends downward; but the pressure which arises from that force of gravity, tends every way with an equable force; and, with equal ease and force, is propagated in crooked lines, as in straight ones. Waves on the surface of water, while they slide by the sides of any large obstacle, do inflect, dilate, and diffuse themselves gradually into the quiescent water lying beyond the obstacle. The waves, pulses, or vibrations of the air, in which sounds consist, do manifestly inflect themselves, though not so much as the waves of water; for the sound of a bell, or of a cannon, can be heard over a hill, which intercepts the sonorous object from our sight; and sounds are propagated as easily through crooked tubes, as through straight ones. But light is never observed to go in curved lines, nor to inflect itself ad umbram; for the fixed stars do immediately disappear on the interposition of any of the planets; as well as some parts of the sun's body, by the interposition of the moon, or Venus, or Mercury.

Pressure *of Air, Water, &c.* See Air, Water, &c.

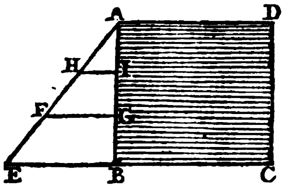
The effects anciently ascribed to the fuga vacui, are now accounted for from the weight and pressure of the air.

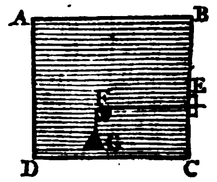
The pressure of the air on the surface of the earth, is balanced by a column of water of the same base, and about 34 feet high; or of one of mercury of near 30 inches high; and upon every square inch at the earth's surface, that pressure amounts to about $14\frac{3}{4}$ pounds avoirdupois. The elasticity of the air is equal to that pressure, and by means of that pressure, or elasticity, the air would rush into a vacuum with a velocity of about 1370 feet per second. At different heights above the earth's surface the pressure of the air is as its density and elasticity, and each decreases in such sort, that as the heights above the surface increase in arithmetical progression, the pressure &c decreases in geometrical progression: and hence if the axis BC of a logarithmic curve AD be erected perpendicular to the horizon, and if the ordinate AB denote the pressure, or elasticity, or density of the air, at the earth's surface, then will any other absciss

$$\left.\begin{array}{l}EF\\GH\\IK\end{array}\right\}\text{denote the pressure \&c at the altitude}\left\{\begin{array}{l}BE,\\BG,\\BI,\end{array}\right.$$



The pressure of water, as this fluid is every-where of the same density, is as its depth at any place, and in all directions the same; and upon a square foot of surface, every foot in height presses with the force of a weight of 1000 ounces or $62\frac{1}{2}$lbs. avoirdupois. And hence, if AB be the depth of water in any vessel, and BE denote its pressure at the depth B; by joining AE and drawing any other ordinates FG, HI; then will these ordinates FG, HI, &c, denote the pressure at the corresponding depths AG, AI, &c; also the area of the triangle ABE will denote the whole pressure against the whole upright side AB and which therefore is but half the pressure on the bottom of the same area as the side. Moreover, if a hole were opened in the bottom or side of the vessel at B, the water, from the pressure of the superincumbent fluid, would issue out with the velocity of $8\sqrt{AB}$ feet per second nearly; AB being estimated in feet.





Pressure of Earth against Walls, &c. This is a circumstance of considerable importance, on many occasions, as in embankments, in fortifications, in docks, in piers, &c. The practice is to have the counterparts equal to, or rather to exceed the pressure, in order to secure stability. For determining this equality, several different principles have been employed, approaching more or less to perfect accuracy, as may be seen in my Course of Mathematics, vol. 2, pa. 196, and vol. 3, pa. 256; where a popular and mechanical theory is delivered, for pretty compact or firm earth, different from former ones, and accompanied with several practical examples, which may be usefully consulted on any real occasion. Below is also inserted another new theory, for the pressure or push of semifluid and cohesive earth, communicated by a learned friend, Dr. Young, Foreign Secr. to the Royal Society.

An Essay on the Pressure of semifluid and cohesive Substances.

The resistance opposed by friction, or adhesion, to the relative motion of any two given solid or semifluid substances, is nearly proportional to the force urging the surfaces into contact. Since, however, this force must necessarily be augmented by the force of direct cohesion, which is proportional to the extent of the surfaces in contact, it follows, that a portion of the resistance to lateral motion, must also, in cohesive substances, be proportional to the magnitude of the surfaces concerned, and independent of the direct pressure. The proportion of the variable resistance, to the force on which it depends, is that of the height to the horizontal extent of an inclined plane, on which the surfaces would begin to slide on each other, if this resistance only were concerned, or if the force or weight were very great, and the extent of the surface very small: and the angle formed by such a plane, with the horizon, is called the angle of repose of the substance. The mutual cohesion of two substances, may be estimated from the thickness of a coat of one of the substances, which would be supported by it in contact with a vertical surface of the other; and both these properties may be practically determined, with respect to any internal surfaces or sections of a given substance, by raising a portion of it, terminated by a horizontal and a vertical surface, until the angle breaks off, observing both the depth and the breadth of the portion thus separating.

A. It is first required to determine the angle of fracture for a semifluid and cohesive substance, terminated by a horizontal and a vertical surface, and supported only by a horizontal force.

We have here a wedge of the given substance, tending to slide down an inclined plane, and to overcome at once the horizontal pressure, and the resistances in the direction of the plane derived from the cohesion, and from the friction produced by the sum of the other forces; and we are to determine the breadth x of that wedge, in which this tendency will be the greatest, its depth being a.

Now the weight of the wedge being expressed by $\frac{1}{2}ax$, its immediate tendency to descend along the inclined plane will be $\frac{1}{2}ax . \frac{a}{\sqrt{(aa + xx)}}$, which will be opposed by the horizontal force f, acting in a contrary direction, and reduced to $f \frac{x}{\sqrt{(aa + xx)}}$, and by the resistance derived from three sources: the first from the cohesion, which is expressed by $b\sqrt{(aa + xx)}$, b being the thickness supported by the lateral adhesion of a vertical surface; the second and third from the two pressures, represented by $\frac{1}{2}tax . \frac{x}{\sqrt{(aa + xx)}}$ and $tf . \frac{a}{\sqrt{(aa + xx)}}$, where t is the tangent of the angle of repose, the resistance being to the direct or perpendicular pressure as t to 1. Hence, for the state of equilibrium, we have the equation $\frac{1}{2}ax . \frac{a}{\sqrt{(aa + xx)}} = f . \frac{x}{\sqrt{(aa + xx)}} + b\sqrt{(aa + xx)} + \frac{1}{2}tax . \frac{x}{\sqrt{(aa + xx)}} + tf . \frac{a}{\sqrt{(aa + xx)}}$, and $\frac{1}{2}a^2x = fx + b(a^2 + x^2) + \frac{1}{2}atx^2 + atf$; whence $f = \frac{\frac{1}{2}aax - aab - bxx - \frac{1}{2}atxx}{x + at}$. This force must be a maximum in the section affording the greatest pressure, and its fluxion must vanish; whence we have $(\frac{1}{2}a^2 - 2bx - atx).(x + at) = \frac{1}{2}a^2x - a^2b - bx^2 - \frac{1}{2}atx^2$; $(b + \frac{1}{2}at)x^2 + (2abt + a^2t^2)x = a^2b + \frac{1}{2}a^3t$; $x^2 + 2atx = a^2$, $x = \sqrt{(a^2 + a^2t^2)} - at$; and if $b = 0$, $f = a^2[\frac{1}{2} + t^2 - t\sqrt{(1 + t^2)}]$. Hence it appears that, as Mr. Prony has already observed, the angle formed by the surface thus determined, with the vertical surface, is half the complement of the angle of repose, since $\sqrt{(1 + t^2)} - t$ is the tangent of half the angle of which the cotangent is t, as is easily shown by a trigonometrical calculation; and that this angle is independent of the magnitude of the cohesive resistance, and determined only by the friction; at the same time, if the friction vanishes, and the cohesion alone remains, we have $x = a$, the angle being 45°.

B. The portion of a semifluid and cohesive substance, of which the surfaces are horizontal and vertical, affording the greatest lateral pressure, is terminated by a plane.

For if we conceive the substance to be divided by a second vertical surface, parallel to the first, the angular situation of the upper part of the oblique termination, cut off by this surface, will obviously be correctly determined, if considered as a plane, according to the principles already laid down; and if any curved surface would afford a greater lateral pressure than a plane, the direction of the lower part of the oblique termination, considered also as a plane, would require to be different from that of the upper, and this difference might be exhibited by supposing its horizontal extent to be varied, that of the upper portion remaining the same. But in fact, the determination of the direction for this part, thus considered, will be precisely the same as for the upper part; since the proportion of the resistance to the pressure remains the same, and the horizontal force acts on the lower part of the oblique surface with the same increased intensity as the weight, the one depending on the other; so that the relations of all the forces concerned in the determination remain unaltered.

C. To determine what portion of a soft and adhesive substance, having a horizontal and a vertical surface, will stand alone.

Put $f = 0$, then $\frac{1}{2}a^2x - a^2b - bxx - \frac{1}{2}atxx = 0$; and if t is given, let $\sqrt{(1 + t^2)} - t$ be r, and $x = ra$, then $\frac{1}{2}ra^2 - a^2b - r^2a^2b - \frac{1}{2}r^2ta^3 = 0$, and $\frac{1}{2}ra - b - r^2b - \frac{1}{2}r^2at = 0$, and $a = \frac{2b + 2rrb}{r - rrt} = \frac{2b}{r} . \frac{1 + rr}{1 - rt} = \frac{4b}{r}$, and $b = \frac{1}{4}ar$; but if we observe a and x, we find $t = \frac{aa - xx}{2ax}$, and $b = \frac{aax - atxx}{2aa + 2xx}$. When t vanishes, x becomes equal to a, and $b = \frac{1}{4}a$: if $t = 1$, $b = {\cdot}1036a$, if $t = \frac{1}{2}$, $b = {\cdot}155a$.

D. When the surface of a soft, or semifluid and cohesive substance, is inclined to the horizon, the portion affording the greatest horizontal pressure is generally terminated by a curve.

We may suppose the substance to be divided into vertical strata; and the mean depth of any stratum being called y, and the difference of the depths of its two surfaces c, we must inquire what must be its thickness x, in order to afford the greatest horizontal thrust. The weight of the stratum will then be represented by yx; and if the tangent of the elevation of the exposed surface, ascending from its angular end, be u, the length of the oblique termination of the stratum will be $\sqrt{(x^2+(c+ux)^2)}=z$: we have then, for the state of equilibrium, the equation $yx\,.\,\frac{c+ux}{z}=f.\frac{x}{z}+bz+tyx\,.\,\frac{x}{z}+tf\,.\,\frac{c+ux}{z}$, and $f=\frac{cyx+uyxx-bzz-tyxx}{x+ct+tux}=\frac{cyx+uyxx-bxx-bcc-2bcux-buuxx-tyxx}{x+ct+tux}$; then putting the fluxion of $f=0$, x only being variable, we obtain $(cy+2uyx-2bx-2bcu-2bu^2x-2tyx).(x+ct+tux)=(1+tu).(cyx+uyx^2-bx^2-bc^2-2bcux-bu^2x^2-tyx^2)$; $(2uy-2b-2bu^3-2ty)x.(1+tu)x+(2uy-2b-2bu^2-2ty)x.ct+(cy-2bcu).(1+tu)x+(cy-2bcu).ct=(1+tu).(uy-b-bu^2-ty)x^2+(1+tu).(cy-2bcu)x-(1+tu).bc^2$; and $x^2+\frac{2ct}{1+tu}.x=\frac{(2bcu-cy).ct-(1+tu).bcc}{(uy-b-buu-ty).(1+tu)}$, whence $\frac{x}{c}$ is found $=\sqrt{(\frac{tt}{(1+tu)^2}+\frac{btu-ty-b}{(uy-b-buu-ty).(1+tu)})}-\frac{t}{1+tu}$. Having thus obtained the angular direction of the termination of the vertical stratum, which affords the greatest lateral thrust when the height is y, we may proceed to find what must be the magnitude of y for different strata, in order that they may all possess this property, and that the whole horizontal force may consequently be the greatest possible. For this purpose we must substitute $\frac{\dot{x}}{-\dot{y}}$ for $\frac{x}{c}$, x being now considered as the whole horizontal thickness, and y the whole vertical ordinate or depth, as before. Hence $-\dot{x}=\frac{\dot{y}}{1+tu}(\sqrt{(t^2+\frac{(1+tu).(btu-ty-b)}{uy-b-buu-ty})}-t)=\frac{\dot{y}}{1+tu}(\sqrt{\frac{-ty-b-btt-t^2y}{uy-b-buu-ty}}-t)=\frac{\dot{y}}{1+tu}\times(\sqrt{\frac{b+btt+(t+t^2)y}{b+buu+(t-u)y}}-t)$. Call $\sqrt{(b+buu+(t-u)y)}$, v, then $y=\frac{v^2-b-buu}{t-u}$, $\dot{y}=\frac{2v\dot{v}}{t-u}$, and $-\dot{x}=\frac{2v\dot{v}}{(1+tu).(t-u)}\sqrt{(b+btt+\frac{(t+t^2)v^2-b-buu}{t-u})}:v-\frac{t\dot{y}}{1+tu}=\frac{2\dot{v}}{(1+tu).(t-u)}\sqrt{(b+btt-(b+buu):(t-u)+(t+t^2):(t-u).v^2)}-\frac{t\dot{y}}{1+tu}$: and if we call $\frac{(t-u)b+btt-(b+buu)}{t+t^2}$, d^2, we have $-\dot{x}=\frac{2\dot{v}}{(1+tu).(t-u)}\sqrt{\frac{t+t^2}{t-u}}\sqrt{(d^2+v^2)}-\frac{t\dot{y}}{1+tu}$. But it is well known that the fluent of $\sqrt{(a^2+x^2)}\,\dot{x}$ is $\frac{1}{2}x\sqrt{(a^2+x^2)}+\frac{1}{2}a^2\,.\,\text{HL}\,(x+\sqrt{(a^2+x^2)})$, and by comparison with this fluent, we obtain the equation $e-x=\frac{1}{(1+tu).(t-u)}\sqrt{\frac{t+t^2}{t-u}}(v\sqrt{(d^2+v^2)}+d^2\text{HL}(v+\sqrt{(d^2+v^2)}))-\frac{ty}{1+tu}+e$. When, however, $t-u$ is negative, that is, when the elevation of the inclined surface is greater than could exist without the cohesion, the fluent assumes a different form, and we must make $d^2=\frac{(u-t).(b+tt)+b+buu}{t+t^2}$; then $-\dot{x}=\frac{-2\dot{v}}{(1+tu).(u-t)}\sqrt{\frac{t+t^2}{u-t}}\sqrt{(d^2-v^2)}-\frac{t\dot{y}}{1+tu}$. But it is known that the fluent of $\sqrt{(a^2-x^2)}\dot{x}$ is $\frac{1}{2}x\sqrt{(a^2-x^2)}+\frac{1}{2}a^2$ arc sine $\frac{x}{a}$; hence $e-x$ becomes $=e-\frac{1}{(1+tu).(u-t)}\sqrt{\frac{t+t^2}{u-t}}(d\sqrt{(d^2-v^2)}+d^2\text{ arc sine }\frac{v}{d})-\frac{ty}{1+tu}$.

E. When the variable resistance vanishes, the curve becomes a parabola.

For if $t=0$, $\frac{x}{c}$ or $\frac{\dot{x}}{-\dot{y}}$, becomes $=\sqrt{\frac{b}{b+buu-uy}}$, whence $x+e=\frac{2}{u}\sqrt{(b^2+b^2u^2-buy)}$; but when $x=0$, $y=a$, and $e=\frac{2}{u}\sqrt{(b^2+b^2uu-bau)}$, $(x+\frac{2}{u}\sqrt{(b^2+b^2uu-bau)})^2=\frac{4bb}{uu}+4b^2-\frac{4b}{u}y=x^2+\frac{4bb}{uu}+4b^2-\frac{4ba}{u}+\frac{4x}{u}\sqrt{(b^2+b^2uu-bau)}$, and $y=a-\sqrt{(1+uu-\frac{a}{b}u)}x-\frac{u}{4b}x^2$. In order to determine the whole horizontal force, we must find its fluxion by substituting $\dot{x}$ for x, and $-\dot{y}$ for c, in the equation for f, which becomes $-y\dot{y}+uy\dot{x}-b\dot{x}-b\frac{\dot{y}\dot{y}}{\dot{x}}+2bu\dot{y}-bu^2\dot{x}$; and since $-\dot{y}=\sqrt{(1+uu-\frac{a}{b}u)}\dot{x}+\frac{u}{2b}x\dot{x}$, we obtain the fluent $=g-\frac{1}{2}y^2+aux-\frac{u}{2}\sqrt{(1+uu-\frac{a}{b}u)}x^2-\frac{uu}{12b}x^3-bx-bx-bu^2x+aux-\frac{u^2}{12b}x^3-\frac{u}{2}\sqrt{(1+uu-\frac{a}{b}u)}x^2+2buy-bu^2x=g+(2au-2b-2bu^2)x-u\sqrt{(1+u^2-\frac{a}{b}u)}x^2-\frac{uu}{6b}x^3+2buy-\frac{1}{2}y^2$, which must vanish when $x=0$, and $y=a$, or $g+2bau-\frac{1}{2}a^2=0$, and $g=\frac{1}{2}a^2-2bau$. When $y=0$, $x+e=\frac{2b}{u}\sqrt{(1+u^2)}$, and $x=\frac{2b}{u}\sqrt{(1+u^2)}-\frac{2b}{u}\sqrt{(1+u^2-\frac{a}{b}u)}$, and the whole force is $\frac{1}{2}a^2-2bau+(2au-2b-2bu^2)x-u\sqrt{(1+u^2-\frac{a}{b}u)}x^2-\frac{uu}{6b}x^3$.

Here it must be observed, that when $\frac{a}{b}u$ is equal to or greater than $1+u^2$, the problem becomes impossible, the value of $\frac{x}{c}$ becoming first infinite, and then imaginary. We may take for an example the case $u=\frac{1}{10}$ and $a=10b$, then $x=2a(\sqrt{1\cdot01}-\cdot1)=1\cdot81a$, and the whole force is $\frac{1}{2}a^2-\cdot02a^2+(\cdot2a-\cdot2a-\cdot002a)x-\cdot01x^2-\frac{x^3}{60a}=\cdot345a^2$. If $u=1$, and $a=2b$, $x=\sqrt{2}a$, and the force $\frac{1}{2}a^2-a^2-\sqrt{2}a^2-\frac{2}{3}\sqrt{2}a^2$, which, being negative, implies that there can be no separation. In order to show how little the force thus determined differs from that which is afforded by a section terminated by a plane surface, even where the variable resistance is supposed to be absent, we may calculate, for the depth of a, the horizontal extent x of a prismatic section affording the greatest pressure, the equation of the forces will then be $\frac{1}{2}ax\,\frac{a+ux}{z} - bz = f\,.\,\frac{x}{z}$, and $f = \frac{1}{2}a^2 + \frac{1}{2}aux - \frac{bz^2}{x} = \frac{1}{2}a^2 + \frac{1}{2}aux - bx - \frac{ba^2}{x} - 2bau - bu^2x$, and when its fluxion vanishes, $\frac{1}{2}au - b + \frac{baa}{xx} - bu^2 = 0$, consequently $\frac{aa}{xx} = 1 + u^2 - \frac{au}{2b}$, which, when $u = \frac{1}{10}$ and $a = 10b$, becomes ·51, and $x = 1{\cdot}4a$, whence f is found $\cdot 337a^2$, which is not one-fortieth part less than the more correct result of the former calculation. When the cohesion vanishes, and the variable resistance alone remains, the maximum of force seems in all cases to be afforded by a plane surface, whether the resistance is horizontal or not.

F. It remains to be determined, what is the proportion of the forces, when the pressure, instead of being horizontal, is supposed to be oblique, as will be the case when the surface of a wall is opposed to the thrust of earth, and exhibits a lateral adhesion or friction, as well as a direct resistance.

We have here two new forces to be considered, the one constant, representing the adhesion of the wall, the other depending on f the horizontal pressure, both tending directly to lessen the weight, if we consider the surface of the wall as vertical. We may still call the horizontal extent of the prismatic portion x, disregarding the slight inaccuracy of supposing the oblique surface a plane; and u being, as above, the tangent of the elevation of the exposed surface, the friction of the wall being, for the sake of simplicity, considered as equal to the internal friction of the materials, which it can never exceed, and of which it will seldom fall short, we have the equation $(\frac{1}{2}ax - ab - tf)\,\frac{a+ux}{z} - bz - t(\frac{1}{2}ax - ab - tf)\,\frac{x}{z} - tf\,\frac{a+ux}{z} = f\frac{x}{z}$, and $f = \frac{(\frac{1}{2}ax - ab).(a+ux) - bz^2 - tx(\frac{1}{2}ax - ab)}{t(a+ux) - ttx + t(a+ux) + x}$; and when its fluxion vanishes, $(\frac{1}{2}a^2 + aux - abu - 2bx - 2abu - 2bu^2x - atx + tab).(2t(a+ux) - t^2x + x) = (2tu - t^2 + 1).[(\frac{1}{2}ax - ab).(a+ux) - bx^2 - b(a+ux)^2) - \frac{1}{2}tax^2 + tabx]$, or $[(au - 2b - 2bu^2 - at)x + \frac{1}{2}a^2 - 3abu + atb].[(2tu - t^2 + 1)x + 2at] = (2tu - t^2 + 1).[(\frac{1}{2}au - b - bu^2 - \frac{1}{2}at)x^2 + (\frac{1}{2}au - abu - 2abu + atb)x - 2a^2b]$; whence $x^2 + \frac{2atx}{2tu - tt + 1} + \frac{2at(\frac{1}{2}a^2 - 3abu + atb)}{(au - 2b - 2buu - at)\,.\,(2tu - tt + 1)} = \frac{1}{2}x^2 - \frac{2a^2b}{au - 2b - 2buu - at}$, or $x^2 + \frac{4atx}{2tu - tt + 1} = 2a^2.\frac{2btu - at - 2b}{(2tu - tt + 1)\,.\,(au - 2b - 2buu - at)}$, and x may be found by completing the square.

But for practical use on a large scale, we may neglect the cohesive resistance without impropriety, its value being generally variable, from the effects of moisture and agitation, so that it would be unsafe to place any dependence on it, even if it were much larger than commonly happens: we may therefore make $b = 0$, and $x^2 + \frac{4atx}{2tu - tt + 1} = 2a^2\frac{t}{(2tu - tt + 1)\,.\,(t - u)}$, and $x + \frac{2at}{2tu - tt + 1} = \pm a\sqrt{(\frac{4t^2}{(2tu - tt + 1)^2} + \frac{2t}{(2tu - tt + 1).(t - u)})} = \pm\frac{a}{2tu - tt + 1}\sqrt{\frac{2t + 2t^3}{t - u}}$; whence f may be readily determined, being equal to $\frac{1}{2}ax.\,\frac{a - (t - u)x}{2at + 2tux - ttx + x}$.

If the wall, instead of being vertical, be inclined towards the bank, which is a condition highly favourable to its stability, the oblique direction of the thrust must also be taken into consideration, in computing its magnitude. Let u be now the tangent of the deviation of the wall from the vertical direction, the surface of the earth being horizontal, and let x be, as above, the whole horizontal extent of the portion affording the greatest thrust, the force f being perpendicular to the wall. We shall then have for the weight, $\frac{1}{2}a(x - au)$, acting in the direction of the oblique surface z with the force $\frac{1}{2}a(x - au)\frac{a}{z}$, and causing a resistance $\frac{1}{2}at(x - au)\frac{x}{z}$. In order to reduce the force f to the same direction, we must find the sine and cosine of the angle contained by the oblique surface z and the wall, which are $\frac{x - au}{\sqrt{(1 + uu)}}$, and $\sqrt{(1 - \frac{(x - au)^2}{z^2 + u^2z^2})} = \frac{1}{z}\sqrt{(a^2 + x^2\frac{(x - au)^2}{1 + u^2})} = \frac{a + ux}{z\sqrt{(1 + uu)}}$; whence we have $f\frac{x - au}{z\sqrt{(1 + uu)}}$, and $tf\frac{a + ux}{z\sqrt{(1 + uu)}}$; and the friction of the wall, ft, being reduced in a similar manner, gives $ft\frac{a + ux}{z\sqrt{(1 + uu)}}$, and $-ft^2\frac{x - au}{z\sqrt{(1 + uu)}}$, whence we have the equation $\frac{1}{2}a(x - au)\,.\frac{a}{z} = \frac{1}{2}at\,.(x - au)\frac{x}{z} + f\frac{x - au}{z\sqrt{(1 + uu)}} + ft\frac{a + ux}{z\sqrt{(1 + uu)}} + ft\frac{a + ux}{z\sqrt{(1 + uu)}} - ft^2\frac{x - au}{z\sqrt{(1 + uu)}}$, or $a^2(x - au) = atx\,(x - au) + 2f\frac{x - au}{\sqrt{(1 + uu)}} + 4ft\frac{(a + ux)}{\sqrt{(1 + uu)}} - 2ft^2\frac{x - au}{\sqrt{(1 + uu)}}$; consequently $\frac{f}{a\sqrt{(1 + uu)}} = \frac{(a - tx)\,.\,(x - au)}{(2 - 2tt)\,.\,(x - au) + 4t(a + ux)}$: this we may call $\frac{b + cx + dxx}{ex + g}$; and when its fluxion vanishes, $(c + 2dx)\,.\,(ex + g) = eb + cex + dex^2 = cex + 2dex^2 + cg + 2dgx$, $x^2 + \frac{2g}{e}x = \frac{b}{d} - \frac{cg}{de}$, and $x = \sqrt{(\frac{b}{d} - \frac{cg}{de} + \frac{gg}{ee})} - \frac{g}{e}$. Here $b = -a^2u$, $c = a + atu$, $d = -t$, $\frac{b}{d} = \frac{au}{t}$, $\frac{c}{d} = -\frac{a}{t} - au$, $e = 2 - 2t^2 + 4tu$, and $g = 4at - (2 - 2tt)\,au$.

G. It will now be easy to find the dimensions of a wall, capable of withstanding the thrust of a given bank of earth, without being overturned or carried away horizontally, provided that we know the elevation at which the surface of the earth is capable of supporting itself.

It is obvious that the whole pressure, like that of fluids, must be proportional to the square of the depth a, neglecting the effect of adhesion; and consequently that the centre of pressure must be at one-third of the height. We may consider the specific gravity of the wall as equal to that of the earth, which will in general allow us some excess of stability for the security of the work: then if the wall be vertical, and its thickness be y, the force being referred to the outside of the base of the wall as the fulcrum of a lever, we must have, in order that it may not be overturned, $\frac{1}{3}af = tfy + \frac{1}{2}ayy$, and $y^2 + \frac{2ft}{a}y = \frac{2}{3}f$, $y = \sqrt{(\frac{2}{3}f + \frac{f^2t^2}{a^2})} - \frac{ft}{a}$. And in the same manner, if we suppose the section of the wall to be triangular, its outer surface being sloped off, we have $\frac{1}{3}af = tfz + \frac{1}{4}azz$, and $z = \sqrt{(\frac{4}{3}f + (\frac{4f^2t^2}{a^2})} - \frac{2ft}{a}$, z being the thickness at the bottom. When the wall is inclined towards the bank, in an angle of which the tangent is u, f being the force perpendicular to it, and y the horizontal thickness of the wall, the force f will act on a lever of which the length is $\frac{1}{3}a\sqrt{(1 + uu)} + \frac{uy}{\sqrt{(1 + uu)}}$, and the friction tf will act at the distance $\frac{y}{\sqrt{(1 + uu)}}$, and the weight at $\frac{1}{2}y + \frac{1}{2}au$, whence $\frac{1}{3}af\sqrt{(1 + uu)} + \frac{fuy}{\sqrt{(1+uu)}} = \frac{fty}{\sqrt{(1 + uu)}} + \frac{1}{2}ay^2 + \frac{1}{2}a^2uy$, and $y^2 + (\frac{2ft - 2fu}{a\sqrt{(1 + uu)}} + au)y = \frac{2}{3}f\sqrt{(1 + uu)}$; consequently $y = \sqrt{[\frac{2}{3}f\sqrt{(1 + uu)} + (\frac{ft - fu}{a\sqrt{(1 + uu)}} + \frac{1}{2}au)^2]} - \frac{ft - fu}{a\sqrt{(1 + uu)}} - \frac{1}{2}au$.

If the wall be not securely fixed at its foundations, for example when the earth is dug away beyond it, it may be liable to slide away laterally more easily than to be overturned. Supposing it simply to rest on materials similar to those which constitute the bank, we may calculate the thickness sufficient to produce a resistance equivalent to the thrust; thus if the wall is vertical, we must have $f = t\,(ay + ft)$, and $ay = \frac{f}{t} - ft$; but when the wall is inclined, the force f takes from the weight the portion $f\frac{u}{\sqrt{(1 + uu)}}$, and the friction adds to it only $ft\frac{1}{\sqrt{(1 + uu)}}$, the horizontal thrust being $f\frac{1}{\sqrt{(1 + uu)}}$; whence $f\frac{1}{\sqrt{(1 + uu)}} = t(ay - f\frac{u}{\sqrt{(1 + uu)}} + f\frac{t}{\sqrt{(1 + uu)}}$, and $ay = \frac{f}{\sqrt{(1 + uu)}}\,(\frac{1}{t} + u - t)$.

H. In the case of driving a pile, the pressure of the soft materials is modified by the inversion of the direction of the friction of the vertical surface, which now acts in conjunction with the weight of the materials, so that $\frac{1}{2}ax - ab - tf$ becomes $\frac{1}{2}ax + ab + tf$, or, if $b = 0$, simply $\frac{1}{2}ax + tf$; and $f = \frac{1}{2} \cdot \frac{aa - atx}{u + 1}$, which is greatest when x is least, and becomes ultimately $\frac{\frac{1}{2}aa}{u + 1}$, and the resistance tf will be $\frac{1}{2}a^2\,\frac{t}{u + 1}$, which is a maximum when $u + 1 = 2t$, or $t = 1$, being then $\frac{1}{4}a^2$; and in this case the resistance derived from the friction, on the whole of the lateral surfaces of a square pile, would be equal to the weight of the earth which would press on one of the surfaces, if it were buried at the depth to which its lower end has penetrated. There would however be other resistances from the tenacity preventing the ready separation of the earth before the pile, which would perhaps considerably exceed the friction thus determined.

I. Such of the results of these calculations, as are most likely to be of practical utility, may be conveniently exhibited in the form of a table: but it must be remembered, in its application, that some additional strength ought always to be given to the works concerned, in order to insure their stability, and that occasional agitation will very much diminish the resistance of almost all kinds of materials; to say nothing of the precaution necessary to obviate the effects of the penetration of water; which will not only act by its own hydrostatic pressure, but also weaken the adhesion of the earth employed, unless a sufficient number of apertures can be provided for allowing it to escape.

Table *of the Thrust of Earth against an upright Wall.*

Surface Horizontal.

Tangent of the angle of repose, expressing the proportion of the resistance to the pressure.	Angle of repose, at which the substance will support itself		Breadth of the surface of the portion affording the greatest pressure, the height being unity.	Horizontal thrust, that of a fluid being unity.	Requisite thickness of a vertical wall of equal specific gravity, its height being unity.	Requisite thickness of a triangular wall, the external surface being oblique.	Thickness required to secure a wall from sliding.
1 : ∞	00°	00′	(1·414)	1 000	·577	·707	∞
1 : 10	5	43	1·234	·761	·491	·591	3·767
1 : 8	7	7	1·194	713	·470	·575	2·812
1 : 6	9	28	1·132	·640	·444	·539	1·867
1 : 5	11	18	1.086	·589	·424	·514	1·414
1 : 4	14	2	1·022	·522	.396	·479	·979
1 . 3	18	26	·927	·430	·355	·430	·573
1 : 2	26	34	·774	·300	·292	·352	.225
2 : 3	33	41	·660	·217	·246	·295	·090
3 : 4	36	52	·611	·186	·226	·270	·059
1 : 1	45	0	·500	·125	·184	·221	·000
Descent of the surface towards the wall							10°
1 : 5	11	18	3·750	·805	·491	·591	
1 : 3	18	26	1·518	·528	·399	·481	
1 : 2	26	34	1·058	·373	·322	·386	
3 : 4	36	52	·753	·211	·240	·287	
1 : 1	45	0	·585	·138	·192	·230	
Descent of the surface towards the wall							20°
1 : 2	26	34	2·022	·452	·350	·418	
3 : 4	36	52	1·040	·247	·255	·304	
1 : 1	45	0	·743	·155	·200	·239	
							30°
3 : 4	36	52	1·581	·302	·282	·335	
1 : 1	45	0	1·076	·186	·220	·262	
							40°
1 : 1	45	0	1·713	·246	·248	·294	
Ascent of the surface towards the wall							10°
1 : 5	11	18	·746	·507	·391	·478	
1 : 3	18	26	·712	·374	·332	·401	
1 : 2	26	34	·639	·267	·276	·333	
3 : 4	36	52	·534	·170	·217	·261	
1 : 1	45	0	·452	·119	·180	·216	
							20°
1 : 2	26	34	·559	·239	·263	·317	
3 : 4	36	52	·486	·154	·208	·250	
1 : 1	45	0	·377	·117	·179	·214	
							30°
3 : 4	36	52	·460	·133	·195	·234	
1 : 1	45	0	·408	·098	·165	·197	
							40°
1 : 1	45	0	·408	·090	·159	·203	

TABLE *of the Thrust of Earth against a Wall inclined towards the Bank in an Angle of* 11° 18′, *of which the Tangent is* 2; *the Surface being horizontal.*

Tangent of the angle of repose.	Angle of repose.		Horizontal extent of the portion affording the greatest pressure.	Direct thrust against the wall, that of a fluid being unity.	Horizontal thrust.	Thickness required to secure a wall from being overturned	Thickness required to secure a wall from sliding back.
1 : 00	00°	00′	(·200)	1 000	·500	·540	∞
1 : 10	5	43	1·419	·721	·360	·440	3·640
1 : 8	7	7	1·373	·661	·330	·410	2·670
1 : 6	9	28	1·296	·579	·289	·367	1·747
1 : 5	11	18	1·242	·523	·261	·337	1·308
1 : 4	14	2	1·166	·452	·226	·300	·847
1 : 3	18	26	1·059	·354	·177	·248	·508
1 : 2	26	34	·885	·225	·112	·176	·191
2 : 3	33	41	·733	·149	·074	·130	·077
3 : 4	36	52	·636	·121	·060	·111	·047
1 : 1	45	0	·586	·071	·035	·074	·007

An instance has occurred on a large scale, where the wall of a dock has given way horizontally, when its mean thickness was about ·230, the ground having been dug away beyond its foundation: it was of brick, and somewhat curved, being vertical at the top, while the inclination of the chord, or the mean inclination, was 11° 18′, as is supposed in the second table. Hence it appears that the friction must have been somewhat less than $\frac{1}{2}$ of the weight, and that the materials would have stood at an angle of about 25°: to have overturned this wall, the materials must have exhibited a friction of about one-third of the weight, and have been incapable of standing at a greater inclination than about 20°.

In general, it will be unquestionably proper to calculate on a friction not exceeding $\frac{1}{2}$ of the weight, and to make the thickness of a wall, if vertical, at least $\frac{3}{10}$ or perhaps $\frac{1}{3}$ of its height, and if inclined in an angle of 10° or 12°, about $\frac{1}{5}$, taking care to secure the foundation from sliding, to which an inclined wall will otherwise be liable if its thickness be less than $\frac{1}{4}$, though a vertical wall would be safe in this respect if its thickness were sufficient to secure it from being overturned. The disposal of a part of the materials of the wall in the form of counterforts, or buttresses, will add to the strength in either case, especially with respect to the danger of overturning: the curvature, which is a considerable convenience in the case of a dock, tends in a slight degree to lessen the stability with respect to sliding, and makes it still more necessary to attend to the security of the foundation. On the other hand, when we have an opportunity of ascertaining, by a simple experiment, the utmost fluidity that can be communicated by accidental moisture to a chalky or gravelly soil, these calculations may often justify us in saving a very great expense, by proportioning the strength of the works to the object required to be attained by them.

Centre of PRESSURE, in Hydrostatics, is that point of any plane, to which, if the total pressure were applied, its effect upon the plane would be the same as when it was distributed unequally over the whole; or it is that point in which the whole pressure may be conceived to be united; or it is that point to which, if a force were applied equal to the total pressure, but with an opposite direction, it would exactly balance, or restrain the effect of the pressure, so that the body pressed on would not incline to either side. Thus, if ABCD (2d fig. above) be a vessel of water, and the side BC be pressed upon with a force equivalent to 20 pounds of water, this force is unequally distributed over BC, for the parts near B are less pressed than those near C, which are at a greater depth; and therefore the efforts of all the particular pressures are united in some point E, which is nearer to C than to B; and that point E is called the centre of pressure: and if to that point a force equivalent to 20 pounds weight be applied, it will affect the plane BC in the same manner as by the pressure of the water distributed unequally over the whole; and if to the same point the same force be applied in a contrary direction to that of the pressure of the water, the force and the pressure will balance each other, and by opposite endeavours destroy each other's effects. Supposing a cord EFG fixed at E, and passing over the pulley F, has a weight of 20 pounds annexed to it, and that the part of the cord FE is perpendicular to BC; then the effort of the weight G is equal, and its direction contrary, to that of the pressure of the water. Now if E be the centre of pressure, these two powers will be in equilibrio, and mutually destroy each other's effects.

This point E, or the centre of pressure, is the same with the centre of percussion of the plane BC, the point of suspension being B, the surface of the water. And if the plane be oblique, the case is still the same, taking for the axis of suspension, the intersection of that plane and the surface of the fluid, both produced if necessary. See Cotes's Lectures, pa. 40, &c.—The centre of pressure upon a plane parallel to the horizon, or upon any plane where the pressure is uniform, is the same as the centre of gravity of that plane. For the pressure acts upon every part in the same manner as gravity does.

PRESTET (JOHN), a priest of the Oratory, was born at Chalons-sur-Saone, in 1658. He went to Paris early in life, where, having finished his studies, he was entertained by father Malbranche, who taught him mathematics, in which his young pupil made so rapid a progress, that at 17 years of age he published the first edition of his Elemens des Mathematiques. In the same year he entered the congregation of the Oratory, and taught mathematics with much reputation, particularly at Angers and at Nantes. But he died in 1690, at 32 years of age.—His Elemens, above noticed, contain many curious problems: the best edition is that of 1689, in 2 vols. 4to.

PRICE (RICHARD), D. D. and F. R. S. was born in Glamorganshire in 1723, and died in 1791, about 68 years of age. He received his education in a private academy, after which he became minister to a congregation at Newington, in Middlesex; whence he removed to that of Hackney. He was also lecturer of the meeting-house in the Old Jewry, in London. In 1764 he became F. R. S. and D. D. by a diploma from a Scotch university. At the time of the American war he made himself conspicuous by his zeal in the cause of liberty, which he also displayed on several other occasions: and for the publication of his Observations on Liberty and Civil Government, he had the thanks of the city of London. Among many other learned accomplishments, Dr. Price was no mean mathematician, which enabled him to treat with peculiar precision, the calculations relating to political arithmetic, population, annuities, &c. It is even said that he had the honour of suggesting to the late prime minister, Mr. Pitt, the measure of the present sinking fund, to extinguish the national debt, by the allotment of an annual million to accumulate at compound interest. Dr. Price's principal works are: 1. Four Dissertations on Providence and Prayer; on the Importance of Christianity, &c. 2. A Review of the principal Questions and Difficulties in Morals. 3. Observations on Reversionary Payments, Annuities, &c, 2 vols. 8vo. 4. Discussion of the Doctrines of Materialism and Necessity, in a correspondence with Dr. Priestley. 5. Essay on the Population of England and Wales. 6. A volume of Sermons.

PRIESTLEY (Joseph), LL.D. and F.R.S. was born on March 13, 1733, at Field-head, in the parish of Birstall, in the west-riding of Yorkshire. His father was concerned in the cloth manufacture, and intended his son Joseph also for trade, but was induced to change his mind by the youth's early attachment to reading and literary pursuits. After a pretty extensive course of classical studies, at 19 years of age he entered, as a divinity student, the academy of Daventry, under Dr. Ashworth, as successor of that kept by Dr. Doddridge at Northampton. He then officiated for some years as a minister at different places: and in 1761 joined the academy of Warrington, as a lecturer in belles lettres; where his Biographical and Historical Charts appeared, as also his writings on subjects of history, general politics, &c: and here, in 1767, was published his History of Electricity. In 1770, Dr. Priestley accepted the situation of domestic librarian to the earl of Shelburne, or rather his literary and philosophical companion, in the hours that could be devoted to such pursuits. His "History and Present State of Discoveries relating to Vision, Light, and Colours," in 2 vols. 4to, appeared in 1772; which may be considered as a 2d part of a general history of the philosophical sciences; and which indeed proved the last, as the encouragement of this work fell far short of that of the History of Electricity. In 1775 came out his "Examination of Dr. Reid on the Human Mind; Dr. Beattie on the Nature and Immutability of Truth; and Dr. Oswald's Appeal to Common Sense." In 1777, "Disquisitions relating to Matter and Spirit." And soon after, his correspondence with Dr. Price, relative to the same points. In several volumes of the Philos. Trans., as well as in separate publications of his own, are seen his numerous papers on discoveries relating to aëriform fluids, and other chemical subjects; besides many others on theology.

Dr. Priestley's engagement with lord Shelburne having ceased in 1780, he accepted the office of pastor to a congregation at Birmingham; whence soon after issued some of the most important of his theological works; from which arose several controversies on such topics, with Dr. Horsley and other learned men. Dr. Priestley remained at Birmingham till 1791, when his house and library were burnt, with many others, in a popular commotion in that place. After some little time an invitation to succeed Dr. Price, in a congregation at Hackney, gave him a temporary residence; till, in 1794, he sailed for North America, where he settled at the town of Northumberland, in the state of Pennsylvania, for the remainder of his life; and where he died the 9th of February 1804, at nearly 71 years of age.

The following has been given as a true character of Dr. Priestley.—"I beg you will insert the following faithful portrait of a man whose character has been grossly misrepresented by interested enemies, and misconceived by a deluded public.—He was a patient, indefatigable, acute, and judicious experimental philosopher; a candid, bold, and unguarded disputant in theology; a sincere and zealous Christian, a serious and rational preacher of the practical morality of religion—but without the least pretension to, or affectation of, oratorical ornaments. His mind embraced the whole extent of the knowledge and literature in his closet: but in the affairs of the world, he was a plain, uninformed, unaccomplished, honest man. What he believed to be true he thought it his duty to propagate, without any regard to his own interest or the prejudices of mankind; but being overpowered by calumny and oppression, he was compelled to seek a residence among strangers, and leave his principles and character to the impartial judgment of posterity."

PRIMARY *Planets*, are those which revolve round the sun as a centre. Such are the planets Mercury, Venus, Terra, (the Earth,) Mars, Vesta, Juno, Pallas, Ceres, Jupiter, Saturn, and Herschel, &c. They are thus called, in contradistinction from the secondary planets, or satellites, which revolve about their respective primaries. See Planet.

Prime and Ultimate Ratios, a method invented by sir Isaac Newton, at once to avoid the tediousness of the ancients and the inaccuracy of the moderns. The foundation of this method is contained in the first lemma of the first book of the Principia.—This lemma may be thus explained. Let there be two quantities, one fixed and the other varying, so related to each other, that, 1st, the varying quantity continually approaches to the fixed quantity; 2d, that the varying quantity never reaches or can pass beyond the fixed one; 3dly, that the varying quantity approaches nearer to the fixed one than by any assigned difference; then is such a fixed quantity called the limit of the varying one; or, in a looser way of speaking, these quantities may be said to be ultimately equal or in a ratio of equality.

On this subject, see Newton's Principia, lib. 1; Smith's Fluxions; Ludlam on Ultimate Ratios; &c.

PRIMES, denote the first divisions into which some whole or integer is divided. As, a minute, or prime minute, the 60th part of a degree; or the first place of decimals, being the 10th parts of units; or the first division of inches in duodecimals, being the 12th parts of inches; &c.

Prime *Numbers*, are those which can only be measured by unity, or exactly divided without a remainder, 1 being the only aliquot part: as 2, 3, 5, 7, 11, 13, 17, &c. And they are otherwise called Simple or Incomposite numbers.

The peculiar property of prime numbers, as to their forms, the method of finding them, and the many collateral truths that have been derived from the investigations of those properties, have rendered them deserving of the particular attention of mathematicians; and accordingly, we find some of the most celebrated analysts of modern times have bestowed on the theory of those numbers many elaborate and ingenious investigations; among whom, those who have more particularly distinguished themselves, are Bachet, Fermat, Euler, Lagrange, Legendre, and Gauss; the united efforts of these celebrated authors to any particular subject, cannot fail of giving it considerable importance in the opinion of mathematicians, at the same time that we may expect from their combined and concentrated labours, that many interesting truths have been the reward of so much talent and ingenuity.

It would be contrary to the plan of this work to enter at any length into the investigations above alluded to, but the result of them will no doubt be acceptable to the reader; we shall therefore content ourselves with recording some of the most important of those propositions, referring for their investigations and demonstrations to the authors above quoted, viz, Bachet's Diophantus published in 1621, and his work entitled, Problems plaisans et delectables &c; Fermat's edition of Bachet's Diophantus, with notes, published in 1670; Euler's Algebra published in German 1770, and since translated into the Russian, French, and English languages, with the additions by Lagrange, on the same subject; also to the Analysis Infinitorum of the same author, and more particularly to the Petersburg Acts, which contain many of the ingenious labours of this celebrated geometer; Lagrange's additions to Euler's Algebra above quoted, and to the Berlin Memoirs from 1760 for several years. But the most elaborate and connected works on the subject of numbers are those of Legendre, entitled Essai sur la Theorie des Nombres, second edition, published in 1808; and the Disquisitiones Arithmeticæ by M. Gauss, published at Leipsick in 1801, and since (1807) translated into French by Poulet Delisle, under the title of Recherches Arithmetiques.

In these works the reader will find the subject of numbers handled in the most masterly manner, many particular properties of the prime numbers accurately demonstrated, and their applications to various parts of the Indeterminate and general Analysis.

Every prime number, greater than 2, is of one of the forms $4n + 1$, or $4n - 1$.

Every prime number, greater than 3, is of one of the forms $6n + 1$, or $6n - 1$.

And as, in the former case, n may be either even or odd; it therefore follows, that every prime number, except 2, is of one of the forms $8n + 1$, $8n + 3$, $8n + 5$, or $8n + 7$.

In the same manner we may divide prime numbers into classes according to any modulus at pleasure, but the last four forms, in which are included the first two forms $4n + 1$ and $4n - 1$, are those which are found to possess the most distinct properties.

But though every prime number, except 2, is contained in one or other of these four forms, the converse of the proposition is not true, namely, that every number in those forms is a prime number. Indeed no formula has yet been discovered that belongs exclusively to prime numbers, nor has any direct rule been given for finding them, or for ascertaining whether a given number be prime or not; Euler has however considerably simplified the method of trials, in this latter case, by means of the different forms of divisors that belong to certain algebraical formulæ; thus, he has shown, in the Berlin Memoirs for 1772, that $A = 2^{31} - 1$ can have no divisors except numbers of the form $248n + 1$, or $248n + 63$; and having made trials of all the prime numbers in those forms, less than 46339, the root of the number A, and finding that none of them divided A, he thence confidently concludes that $2^{31} - 1 = 2147483647$ is a prime number: and this is the greatest of those that have been verified at present.

It is needless to observe, that without some method similar to the one above given, we should have no means of ascertaining whether the number was prime or not, but by trying every prime number for a divisor from 1 to $\sqrt{A}$, that is, from 1 to 46339, which would be too laborious a task for any one to have attempted.

Fermat had asserted that both $2^{31} - 1$ and $2^{32} - 1$ were prime numbers, but Euler has shown that this last may be decomposed into the factors $3 \cdot 5 \cdot 17 \cdot 257 \cdot 65537$.

Eratosthenes invented a method of finding those numbers, but it is rather mechanical than analytical; this is generally spoken of under the appellation of Eratosthenes' Sieve, or the Sieve of Eratosthenes, a description of which is given under that article. See SIEVE.

Properties of Prime Numbers.

Every prime number, $4n + 1$, is the sum of two squares, or is of the form $x^2 + y^2$. Thus $17 = 4^2 + 1^2$; $29 = 5^2 + 2^2$; $37 = 6^2 + 1^2$, &c.

Every prime number, $8n + 1$, is at the same time of the three forms $x^2 + y^2$, $x^2 + 2y^2$, and $x^2 - 2y^2$. Thus $41 = 5^2 + 4^2 = 3^2 + 2 \cdot 4^2 = 7^2 - 2.2^2$.

Every prime number $8n + 3$, is of the form $x^2 + 2y^2$. Thus $43 = 5^2 + 2.3^2$, and $59 = 3^2 + 2.5^2$, &c.

Every prime number, $8n + 7$, is of the form $x^2 - 2y^2$. Thus $31 = 7^2 - 2.3^2$ and $47 = 7^2 - 2.1^2$, &c.

The demonstrations of these four theorems were first given by Lagrange, in the Berlin Memoirs for 1775; they may also be found in the notes subjoined to the second English edition of Euler's Algebra. It is likewise to Lagrange that we are indebted, if not for the demonstration of the properties contained in the following table, at least for pointing out the method that led to them, as is ingenuously acknowledged by Legendre, at pa. 286, 1st edition, and at pa. 262, 2d edition of his Theory of Numbers, whence this table is extracted.

Table of the Forms of Prime Numbers.

	Prime Numbers.	Forms.
1	$4n + 1$	$y^2 + z^2$
2	$6n + 1$	$y^2 + yz + z^2$
3	$8n + 1, 7$	$y^2 - 2z^2$
4	$8n + 1, 3$	$y^2 + 2z^2$
5	$12n + 1$	$y^2 - 3z^2$
6	$12n + 11$	$3y^2 - z^2$
7	$14n + 1, 9, 11$	$y^2 + 7z^2$
8	$20n + 1, 9, 11, 19$	$y^2 - 5z^2$
9	$20n + 1, 9$	$y^2 + 5z^2$
10	$20n + 3, 7$	$2y^2 + 2yz + 3z^2$
11	$24n + 1, 19$	$y^2 - 6z^2$
12	$24n + 5, 25$	$6y^2 - z^2$
13	$24n + 5, 11$	$2y^2 + 3z^2$
14	$24n + 1, 7$	$y^2 + 6z^2$
15	$28n + 1, 9, 25$	$y^2 - 7z^2$
16	$28n + 3, 19, 27$	$7y^2 - z^2$
17	$30n + 1, 19$	$y^2 + 15z^2$
18	$30n + 17, 23$	$3y^2 + 5z^2$
19	$40n + 1, 9, 31, 39$	$y^2 - 10z^2$
20	$40n + 3, 13, 27, 37$	$2y^2 - 5z^2$
21	$40n + 1, 9, 11, 19$	$y^2 + 10z^2$
22	$40n + 7, 13, 23, 37$	$2y^2 + 5z^2$
23	$120n + 11, 29, 59, 101$	$5y^2 + 6z^2$
24	$120n + 13, 37, 43, 67$	$10y^2 + 3z^2$
25	$120n + 1, 31, 49, 79$	$y^2 + 30z^2$
26	$120n + 17, 23, 47, 113$	$2y^2 + 15z^2$

Beside the properties of prime numbers above enumerated, and those which are given under the article NUMBER, we have another to notice, the discovery of which has been the admiration of every mathematician in Europe, namely, the solution of the equation $x^n - 1 = 0$, when n is a prime number.

The first investigation and demonstration of this problem was published by M. Gauss, in his truly ingenious work entitled Disquisitiones Arithmeticæ; though it seems that Vandermonde had asserted, in the Memoirs of the Academy of Sciences, at Paris in 1771, pa. 416, but without explaining the method, that such equations were always resolvable by means of equations of inferior degrees. It is however to M. Gauss that we are indebted for the complete development of this interesting theorem, he having shown, in the most satisfactory manner, that when n is a prime number, and $(n - 1)$ is resolved into its prime factors $a^\alpha\, b^\beta\, c^\gamma$ &c, that the solution of the equation $x^n - 1 = 0$ may be obtained by means of α equations of the degree a,

β of the degree b, γ of the degree c, &c; thus the equation $x^{73}-1=0$, ($73-1$ being equal to $3^2\cdot2^3$) is resolved by means of two equations of the third degree, and three of the second; and $x^{19}-1=0$, (as $18=3^2\cdot2^1$) is resolved by means of two cubic, and one quadratic equation. And hence it follows, that when $n=2^m+1$ that the equation $x^n-1=0$ may be resolved by means of m quadratic equations, in which case the roots may be found by construction, and consequently the circle may, with such values of n, be divided into n equal parts by means of the scale and compasses only, which was always thought to be impossible till the appearance of the work above mentioned.

Since 17 is a prime number of this form, that is, $17=2^4+1$, therefore a circle may be divided geometrically into 17 equal parts; and since $15=3\times5$, and $16=2^4$, therefore the circle may be divided into equal parts represented by the three consecutive numbers 15, 16 and 17; the same is also true of the three numbers 255, 256, and 257, since $255=3\times5\times17$; $256=2$, and $257=2^8+1$: also of the three 65535, 65536, and 65537, because $65535=255\times257$, $65536=2^{16}$, and $65537=2^{16}+1$. But as $2^{32}+1$ is not a prime, we cannot pursue this reasoning any farther. See Gauss's Disquisitiones Arithmeticæ, Legendre's Essai sur la Theorie des Nombres, 2d Edition, and the Complement to Lacloix's Algebra.

The following Table contains all the prime numbers, and all the odd composite numbers, under 10,000, with the least prime divisors of these; the description, nature, and use of which, see immediately following the table.

A Table of Prime and Composite Odd Numbers, under 10,000.

		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
01			3	7		3			3	17	7	3			3	19	
03			7	3	13		3	19	11	3	17		3		23	3	7
07			3		11	3		7	3		19	3	17		3	11	
09	3		11	3			3			3			3	7		3	
11		3			3	7	13	3			3	11	7	3	17		3
13			3		7	3		23	3	11		3		13	3	17	
17		3	7		3	11		3	19	7	3			3	13	37	3
19		7	3	11		3			3			3	23		3	7	
21	3	11	13	3			3	7		3		19	3		7	3	
23		3		17	3		7	3		13	3			3			3
27	3			3	7	17	3			3	13	7	3			3	
29		3		7	3	23	17	3			3			3		11	3
31			3			3		17	3	7		3		11	3		7
33	3	7		3		13	3		7	3		11	3	31		3	23
37			3		19	3	7	11	3		17	3		7	3	29	
39	3			3		7	3			3		17	3	13		3	11
41		3		11	3			3	29		3	7	17	3	11	23	3
43		11	3	7		3			3	23	7	3	11	17	3		31
47		3	13		3			3	7		3	31	29	3		7	3
49	7		3			3	11	7	3	13		3		19	3		17
51	3			3	11	19	3		23	3			3	7		3	13
53		3	11		3	7		3			3		7	3			3
57	3			3			3			3	7	13	3	23	31	3	
59		3	7		3	13		3		7	3	19		3			3
61		7	3	19		3			3	31		3	13		3	7	11
63	3			3			3	7		3			3	29	7	3	
67			3			3	23	13	3		11	3	7		3		
69	3	13		3	7		3		11	3		7	3	37	13	3	
71		3		7	3		11	3	13		3		31	3			3
73			3		11	3			3	7	29	3	19		3	11	7
77	7	3		13	3			3			3	11		3	7	19	3
79			3			3	7	19	3	11	13	3		7	3		23
81	3			3	13	7	3	11		3	23		3			3	41
83		3			3	11		3			3	7		3			3
87	3	11	7	3			3			3			3	19		3	7
89		3	17		3	19	13	3	7	23	3	29		3		7	3
91	7		3	17		3		7	3			3		13	3	37	19
93	3			3	17		3	13	19	3			3	7		3	
97			3		7	3	17		3			3		11	3		
99	3		13	3			3	17	29	3	7	11	3			3	

	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33
01	3			3	11	31	3	7	41	3	37		3		7	3	
03	13	3	11		3		7	3		19	3			3	29		3
07	3	13		3	7		3	29	23	3		7	3	31	13	3	
09		3	23	7	3	47		3	13		3	53		3			3
11	29		3			3			3	7		3	41		3	13	7
13	3	7		3			3	19	7	3		29	3	23	11	3	
17	17	23	3		29	3	7		3		11	3		7	3		31
19	3	17	19	3	13	7	3	41	11	3			3			3	
21		3	17	43	3		11	3			3	7	23	3			3
23			3	7	11	3	23		3	43	7	3	37		3	11	
27	11	3	41		3	17	13	3	7	37	3	11		3	53	7	3
29	7	31	3			3	17	7	3	11		3	29	13	3		
31	3			3		23	3	11		3		19	3	7	31	3	
33		3		19	3	7		3	17		3		7	3	13	53	3
37	3	11	13	3			3		43	3	7		3			3	47
39	37	3	7		3			3		7	3	17		3	43	41	3
41		7	3	13		3			3	19		3	17		3	7	13
43	3	19	29	3			3	7		3	13		3	17	7	3	
47			3	23	19	3			3		41	3	7	11	3	17	
49	3	43		3	7	13	3	31		3		7	3		47	3	17
51	17	3		7	3			3		11	3		13	3	23		3
53		17	3			3	13	11	3	7		3		43	3		7
57	7	3	19	11	3	37		3			3			3	7		3
59		11	3	29	17	3	7		3		31	3	11	7	3		
61	3		37	3		7	3	23	13	3	11		3		29	3	
63	41	3	13		3	31	17	3	11		3	7		3		13	3
67	3		7	3	11		3		17	3		47	3			3	7
69	29	3	11		3		23	3	7	17	3	19		3		7	3
71	7		3	19	13	3		7	3		17	3		37	3		
73	3			3	41		3		31	3	47	13	3	7	19	3	
77			3	31	7	3			3			3	13	17	3	29	11
79	3			3		43	3	37		3	7		3		11	3	31
81	13	3	7		3			3	29	7	3	43	11	3		17	3
83		7	3		37	3		13	3		11	3	19		3	7	17
87		3			3		7	3	13		3		29	3		19	3
89			3		11	3		19	3			3	7		3	11	
91	3	31	11	3	7	29	3	47		3		7	3	11		3	3
93	11	3		7	3			3			3	11	41	3	31	37	43
97	3	7		3	13		3	11	7	3			3	19	23	3	3
99	7	3			3	11		3	23		3	13		3	7		

A Table of Prime and Composite Odd Numbers, under 10,000.

	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
01	19	3	13		3	47		3		11	3	7	43	3		13	3
03	41	31	3	7		3		11	3	13	7	3			3		
07		3		11	3			3	7	59	3		17	3	11	7	3
09	7	11	3		13	3	19	7	3	31		3	11	17	3		
11	3		23	3	37		3			3	11	13	3	7	17	3	
13		3		47	3	7		3	11	19	3		7	3		17	3
17	3			3	11		3	23		3	7		3	53		3	29
19	13	3	7		3			3		7	3		31	3	61		3
21	11	7	3	61		3		13	3	29		3			3	7	
23	3	13		3			3	7	41	3			3		7	3	
27	23		3		43	3			3		19	3	7	29	3	13	11
29	3		19	3	7		3			3	43	7	3		11	3	47
31	47	3		7	3		29	3		61	3	23	11	3			3
33			3			3	37		3	7	11	3	41		3		7
37	7	3		37	3	31	11	3	19		3	13		3	7		3
39	19		3		11	3	7		3		23	3		7	3	11	
41	3		11	3	23	7	3	41		3		19	3	11	47	3	71
43	11	3		19	3		13	3		43	3	7		3	29		3
47	3		7	3			3	11	31	3			3	47	37	3	7
49		3	41	23	3	11		3	7		3			3	13	7	3
51	7	53	3	11		3		7	3	19		3			3		
53	3	11	13	3		59	3			3	61	29	3	7	23	3	31
57			3	13	7	3			3			3		67	3		13
59	3			3	17	37	3			3	7	47	3		43	3	
61		3	7		3	17	31	3		7	3		59	3		11	3
63		7	3	53		3	17	23	3			3		11	3	7	61
67		3	19		3		7	3	17	11	3		13	3	31		3
69		43	3		53	3	13	11	3	17	41	3	7	19	3		37
71	3			3	7	11	3	43		3	17	7	3	13		3	11
73	23	3		7	3	29		3			3	17		3	11		3
77	3	7		3		41	3		7	3	11	23	3	17		3	
79	7	3	13		3	23		3	11	29	3	19		3	7	13	3
81	59		3	19		3	7	37	3	13		3	31	7	3	17	
83	3		29	3	11	7	3	47		3			3		19	3	13
87	11	17	3	7	13	3	61	53	3	41	7	3	43		3		
89	3	37	7	3			3	59		3	67	13	3			3	7
91		3		17	3	13		3	7		3			3	67	7	3
93	7		3		17	3		7	3	23		3	13		3		11
97	13	3			3	7	17	3			3		7	3	59	19	3
99		59	3	29	7	3		13	3	53	11	3	37		3		

	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67
01		7	3	11		3			3	17		3		37	3	7	
03	3	11		3		13	3	7		3	17		3	19	7	3	
07		41	3			3	13		3		31	3	7	43	3		19
09	3			3	7	71	3	37	19	3	41	7	3	13	23	3	
11	19	3	47	7	3	31		3	23		3			3	17	11	3
13		13	3		37	3	29		3	7		3	59	11	3	17	7
17	7	3	13		3	41		3	61	11	3			3	7	13	3
19		7	3			3	7	11	3	13	29	3	71	7	3		
21	3	23	17	3		7	3		31	3			3			3	11
23	47	3		11	3		59	3		19	3	7		3	11	37	3
27	3		7	3		17	3			3	11	13	3		61	3	7
29	23	3	73	61	3	13	17	3	7		3			3		7	3
31	7		3			3	11	7	3	37		3	13	59	3	19	53
33	3			3	11	43	3	19	17	3		23	3	7	47	3	
37	11		3		7	3		13	3		17	3		41	3		
39	3	13	19	3	29		3			3	7	17	3	47	13	3	23
41	53	3	7		3			3	13	7	3	79	17	3	31	29	3
43	37	7	3		23	3			3			3		17	3	7	11
47		3		13	3		7	3	19		3		11	3		17	3
49	19	29	3		31	3			3	23	11	3	7		3	61	17
51	3	59		3	7		3		11	3		7	3			3	43
53		3	53	7	3		11	3			3	13		3			3
57	3	7	11	3			3		7	3	47		3	11	79	3	29
59	7	3	23	53	3		13	3	59	73	3	11		3	7		3
61	13		3	43	67	3	7		3	11	61	3		7	3		
63	3	19	31	3		7	3	11	67	3	7		3	23		3	
67		23	3	7	19	3	73		3			3		29	3	59	67
69	3	11	7	3			3		47	3	31		3			3	7
71		3	41		3	53	29	3	7	13	3		23	3		7	3
73	7		3	13		3	23	7	3			3			3		13
77	31	3	19		3	7	53	3	43	59	3		7	3		11	3
79			3			3			3		37	3		11	3		
81	3			3		13	3			3	7	11	3			3	
83	71	3	7		3			3	31	7	3	61	13	3	29	41	3
87	3	17		3	37	11	3	7		3	23		3	13	7	3	11
89		3	17	11	3		7	3	53		3	19		3	11		3
91	29	11	3	17		3		43	3		41	3	7		3		
93	3	67		3	7		3	71	13	3	11	7	3	43	19	3	
97			3	23	29	3	11		3	7		3		73	3	37	7
99	3	7		3	11	41	3	17		3			3	67		3	13

A Table of Prime and Composite Numbers, under 10,000.

	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83
01	3	67		3	19	7	3	13	11	3	29		3		59	3
03		3	47		3	67	11	3			3	7	53	3	13	19
07	3		7	3			3			3	37		3	11	29	3
09	11	3	43		3		31	3	7	13	3	11		3		7
11	7		3	13		3		7	3	11	73	3			3	
13	3	31		3		71	3	11	23	3	13	41	3	7	43	3
17	17		3	11	7	3			3			3			3	
19	3	11		3		13	3	73	19	3	7		3	23		3
21	19	3	7		3		41	3		7	3	89	3	3		53
23		7	3	17	31	3	13		3			3	71		3	7
27		3			3	17	7	3	29		3		23	3	19	11
29		13	3			3	17		3	59		3	7	11	3	
31	3	29	79	3	7		3	17	13	3	41	7	3	47		3
33		3	13	7	3			3	17	11	3		29	3		13
37	3	7	31	3		11	3		7	3	17		3	79		3
39	7	3		11	3	41	43	3		71	3	17		3	7	31
41		11	3	37	13	3	7		3			3	11	7	3	19
43	3	53		3		7	3	19		3	11	13	3	17		3
47	41		3	7		3	11		3	61	7	3	13		3	17
49	3		7	3	11		3			3	47		3	29	73	3
51	13	3	11		3			3	7	23	3		83	3	37	7
53	7	17	3	23		3	29	7	3			3		31	3	
57		3		17	3	7		3	13		3	73	7	3	23	61
59	19		3		7	3			3		29	3		41	3	13
61	3		23	3	53	17	3		47	3	7	19	3		11	3
63		3	7	13	3	37	17	3	79	7	3		11	3		
67	3		37	3	13	53	3	7	11	3		31	3		7	3
69		3		67	3		7	3		17	3	13		3		
71			3	71	11	3	31	67	3	19	17	3	7		3	11
73	3	19	11	3	7	73	3			3		7	3	11		3
77	13		3		19	3			3	7		3	41	13	3	
79	3	7		3	29	47	3	11	7	3		79	3		17	3
81	7	3	73	43	3	11		3		31	3	23		3	7	17
83			3	11		3	7		3	43		3	59	7	3	83
87	71	3	19		3	83		3		13	3	7		3		
89	83	29	3	7	37	3			3		7	3		19	3	
91	3		7	3	23	19	3			3	13	61	3			3
93	61	3	41		3		59	3	7		3			3		7
97	3		47	3		13	3	71	43	3	53	11	3	7		3
99		3	31	23	3	7		3		11	3	19	7	3	43	37

	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99
01	31		3	7	13	3		19	3	71	7	3		89	3	
03	3	11	7	3		29	3			3		13	3	31		3
07	7	47	3			3		7	3	41	23	3	13	17	3	
09	3	67		3	23	59	3			3	97	37	3	7	17	3
11	13	3	79	31	3	7		3	19		3		7	3		11
13	47		3		7	3		13	3	67		3		11	3	23
17	19	3	7	23	3	37	71	3	13	7	3	31	59	3		47
19		7	3			3	29	11	3			3			3	7
21	3		37	3		11	3	7		3			3		7	3
23		3		11	3		7	3	23		3	89		3	11	
27	3			3	7	79	3			3	11	7	3	71	31	3
29		3		7	3			3	11	19	3	13		3		
31		19	3			3	11	23	3	7		3		37	3	
33	3	7	89	3	11		3		7	3			3			3
37	11		3			3	7		3			3	23	7	3	19
39	3		53	3		7	3	13		3			3			3
41	23	3			3			3			3	7	31	3	13	
43			3	7	37	3		41	3		7	3			3	61
47		3			3	23	83	3	7	13	3		11	3	43	7
49	7	83	3	13		3		7	3		11	3			3	
51	3	17	41	3	53		3		11	3	13		3	7		3
53	79	3	17		3	7	11	3	19	47	3	41	7	3	59	37
57	3	43	11	3	17	13	3			3	7	19	3	11		3
59	11	3	7	19	3	17		3	47	7	3	11	13	3		23
61		7	3			3	13		3	11		3		43	3	7
63	3			3			3	7	59	3		73	3	13	7	3
67		13	3	11		3		89	3	17		3	7		3	
69	3	11		3	7		3	53	13	3	17	7	3		71	3
71	43	3	13	7	3		47	3	73		3	17	19	3		13
73	37		3	31	19	3	43		3	7		3	17	29	3	
77	7	3		67	3	47	29	3			3	61		3	7	11
79	61	23	3		13	3	7	67	3	83		3		7	3	17
81	3			3	83	7	3			3	19	11	3		41	3
83	17	3	19		3	13	31	3		11	3	7	23	3		67
87	3	31	7	3		11	3		19	3	53		3			3
89	13	3		11	3	89	61	3	7	41	3	43		3	11	7
91	7	11	3	59	17	3		7	3			3	11		3	97
93	3	13		3		17	3	29		3	11	53	3	7	13	3
97	29		3	19	7	3	11	17	3			3		97	3	13
99	3			3	11		3		17	3	7	29	3	41	19	3

In the foregoing table, all the odd numbers that end with 5 are omitted, because it is known that 5 is a divisor, or aliquot part of every such number.—The disposition of the prime and composite odd numbers in this table, is along the top line, and down the first or left-hand column; while their least prime divisors are placed in the angles of meeting in the body of the page. Thus, the figures along the top line, viz, 0, 1, 2, 3, 4, &c, to 99, are so many hundreds; and those down the first column, from 1 to 99 also, are units or ones; and the former of these set before the latter, make up the whole number, whether it be prime or composite; just like the disposition of the natural numbers in a table of logarithms. Thus the 16 in the top line, joined with the 19 in the first column, makes the number 1619: the angle of their meeting, viz, of the column under 16, and of the line of 19, being blank, shows that the number 1619 has no aliquot part or divisor, or that it is a prime number. In like manner, all the other numbers are primes that have no figure in their angle of meeting, as the numbers 41, 401, 919, &c. But when the two parts of any number have some figure in their angle of meeting, that figure is the least divisor of the number, which is therefore not a prime, but a composite number: so 301 has 7 for its least divisor, and 803 has 11 for its least divisor, and 1633 has 23 for its least divisor.

Hence, by the foregoing table, are immediately known at sight all the prime numbers up to 10,000; and hence also are readily found all the divisors or aliquot parts of the composite numbers, namely in this manner: Find the least divisor of the given number in the table, as above; divide the given number by this divisor, and consider the quotient as another or new number, of which find the least divisor also in the table, dividing the said quotient by this last divisor; and so on, dividing always the last quotient by its least divisor found in the table, till a quotient be found that is a prime number: then are the said divisors and the last or prime quotient, all the simple or prime divisors of the first given number; and if these simple divisors be multiplied together thus, viz, every two, and every three, and every four, &c, of them together, the several products will make up the compound divisors or aliquot parts of the first given number; noting, that if the given number be an even one, divide it by 2 till an odd number come out.

For example, to find all the divisors or component factors of the number 210. This being an even number, dividing it by 2, one of its divisors, gives 105; and this ending with 5, dividing it by 5, another of its factors, gives 21; and the least divisor of 21, by the table is 3, the quotient from which is 7; therefore all the prime or simple factors of the given number, are 2, 3, 5, 7. Set these therefore down in the first line as in the margin; then multiply the 2 by the 3, and set the product 6 below the 3; next multiply the 5 by all that precede it, viz, 2, 3, 6, and set the products below the 5; lastly multiply the 7 by all the seven factors preceding it, and set the products below the 7; so shall we have all the factors or divisors of the given number 210, which are these, viz,

2	3	5	7
—	6	10	14
	—	15	21
		30	42
		—	35
			70
			105
			210

2, 3, 5, 6, 7, 10, 14, 15, 21, 30, 35, 42, 70, 105.

A table containing every divisor of every number from 1 to 10,000, was given by Anjema, and was reprinted in London in 1747. And a much more extensive table of this kind is given in the second edition of Vega's tables, where also is a table of all prime numbers to 400,000.

Prime *Vertical*, is that vertical circle, or azimuth, which is perpendicular to the meridian, and passes through the east and west points of the horizon.

Prime *Verticals*, in Dialling, or Prime-*Vertical* Dials, are those that are projected on the plane of the prime vertical circle, or on a plane parallel to it. These are otherwise called direct, erect, north, or south dials.

Prime *of the Moon*, is the new moon at her first appearance, for about 3 days after her change. It means also the Golden *Number;* which see.

PRIMUM *Mobile*, in the Ptolemaic Astronomy, is supposed to be a vast sphere, whose centre is that of the world, and in comparison of which the earth is but a point. This they describe as including all other spheres within it, and giving motion to them, turning itself and all the rest quite round in 24 hours.

PRINCIPAL, in Arithmetic, or in Commerce, is the sum lent upon interest, either simple or compound.

Principal *Point*, in Perspective, is a point in the perspective plane, upon which falls the principal ray, or line from the eye perpendicular to the plane. This point is in the intersection of the horizontal and vertical planes; and is also called the point of sight, and point of the eye, or centre of the picture, or again the point of concurrence.

Principal *Ray*, in Perspective, is that which passes from the spectator's eye perpendicular to the picture or perspective plane, and so meeting it in the principal point.

PRINGLE (*Sir* John), Baronet, the late worthy president of the Royal Society, was born at Stichel-house, in the county of Roxburgh, North Britain, April 10, 1707. His father was Sir John Pringle, of Stichel, Bart. and his mother Magdalen Elliott, was sister to Sir Gilbert Elliot, of Stobs, Baronet. He was the youngest of several sons, three of whom, besides himself, arrived to years of maturity. After receiving his grammatical education at home, he was sent to the university of St. Andrews, where having staid some years, he removed to Edinburgh in 1727, to study physic, that being the profession which he now determined to follow. He staid however only one year at Edinburgh, being desirous of going to Leyden, which was then the most celebrated school for medicine in Europe. Dr. Boerhaave, who had brought that university into great reputation, was considerably advanced in years, and Mr. Pringle was desirous of benefiting by that great man's lectures. After having gone through his proper course of studies at Leyden, he was admitted, in 1730, to his doctor of physic's degree; upon which occasion his inaugural dissertation, De Marcore Senili, was printed. On quitting Leyden, Dr. Pringle returned and settled at Edinburgh as a physician, where, in 1734, he was appointed, by the magistrates and council of the city, to be joint professor of pneumatics and moral philosophy with Mr. Scott, during this gentleman's life, and sole professor after his decease; being also admitted at the same time a member of the university. In dischasging the duties of this new employment, his text-book was Puffendorff De Officio Hominis et Civis; agreeably to the method he pursued through life, of making fact and experiment the basis of science.

Dr. Pringle continued in the practice of Physic at Edinburgh, and in duly performing the office of professor, till 1742, when he was appointed physician to the earl of Stair, who then commanded the British army. By the interest of this nobleman, Dr. Pringle was constituted, the same year, physician to the military hospital in Flanders, with a salary of 20 shillings a-day, and the right to half-pay for life. On this occasion he was permitted to retain his professorship of moral philosophy; two gentlemen, Messrs. Muirhead and Cleghorn teaching in his absence, as long as he requested it. The great attention which Dr. Pringle paid to his duty as an army physician, is evident from every page of his Treatise on the Diseases of the Army, in the execution of which office he was sometimes exposed to very imminent dangers. He soon after also met with no small affliction in the retirement of his great friend the earl of Stair, from the army. He offered to resign with his noble patron, but was not permitted: he was therefore obliged to content himself with testifying his respect and gratitude to him, by accompanying the earl 40 miles on his return to England; after which he took leave of him with the utmost regret.

But though Dr. Pringle was thus deprived of the immediate protection of a nobleman who knew and esteemed his worth, his conduct in the duties of his station procured him effectual support. He attended the army in Flanders through the campaign of 1744, and so powerfully recommended himself to the duke of Cumberland, that in the spring following he had a commission, appointing him physician-general to the king's forces in the Low-Countries, and parts beyond the seas; and on the next day he received a second commission from the duke, constituting him physician to the royal hospitals in those countries. In consequence of these promotions, he the same year resigned his professorship in the university of Edinburgh.

In 1745 he was also with the army in Flanders; but was recalled from that country in the latter end of the year, to attend the forces which were to be sent against the rebels in Scotland. At this time he had the honour of being chosen F. R. S. and the Society had good reason to be pleased with the addition of such a member. In the beginning of 1746, Dr. Pringle accompanied, in his official capacity, the duke of Cumberland in his expedition against the rebels; and remained with the forces, after the battle of Culloden, till their return to England the following summer. In 1747 and 1748, he again attended the army abroad; but in the autumn of 1748, he embarked with the forces for England, on the signing of the treaty of Aix-la-Chapelle.

From that time he mostly resided in London, where, from his known skill and experience, and the reputation he had acquired, he might reasonably expect to succeed as a physician. In 1749 he was appointed physician in ordinary to the duke of Cumberland. And in 1750 he published, in a letter to Dr. Mead, Observations on the Gaol or Hospital Fever: this piece, with some alterations, was afterwards included in his grand work on the Diseases of the Army.

In this and the two following years Dr. Pringle communicated to the Royal Society his celebrated Experiments upon Septic and Antiseptic Substances, with Remarks relating to their Use in the Theory of Medicine; some of which were printed in the Philosophical Transactions, and the whole were subjoined, as an appendix, to his Observations on the Diseases of the Army. Those experiments procured for the ingenious author the honour of Sir Godfrey Copley's gold medal; besides gaining him a high and just reputation as an experimental philosopher. He gave also many other curious papers to the Royal Society: thus, in 1753, he presented, An Account of several Persons seized with the Gaol Fever by working in Newgate; and of the Manner by which the Infection was communicated to one entire Family; in the Philos. Trans. vol. 48. His next communication was, A remarkable case of Fragility, Flexibility, and Dissolution of the Bones; in the same vol.—In the 49th volume, are accounts which he gave of an Earthquake felt at Brussels; of another at Glasgow and Dunbarton; and of the Agitation of the Waters, Nov. 1, 1756, in Scotland and at Hamburgh.—The 50th volume contains his Observations on the Case of lord Walpole, of Woollerton; and a Relation of the Virtues of Soap, in Dissolving the Stone.—The next volume is enriched with two of the doctor's articles, of considerable length, as well as value. In the first, he hath collected, digested, and related, the different accounts that had been given of a very extraordinary Fiery Meteor, which appeared the 26th of November 1758; and in the second he has made a variety of remarks upon the whole, displaying a great degree of philosophical sagacity.—Besides his communications in the Philosophical Transactions, he gave, in the 5th volume of the Edinburgh Medical Essays, an account of the Success of the Vitrum ceratum Antimonii.

In 1752, Dr. Pringle married Charlotte, the second daughter of Dr. Oliver, an eminent physician at Bath: a connexion which however did not last long, the lady dying in the space of a few years. And nearly about the time of his marriage, he gave to the public the first edition of his Observations on the Diseases of the Army; which afterwards went through many editions with improvements, was translated into the French, the German, and the Italian languages, and deservedly gained the author the highest credit and encomiums. The utility of this work however was of still greater importance than its reputation. From the time that the doctor was appointed a physician to the army, it seems to have been his grand object to lessen, as far as lay in his power, the calamities of war; nor was he without considerable success in his noble and benevolent design. The benefits which may be derived from our author's great work, are not solely confined to gentlemen of the medical profession. General Melville, a gentleman who united with his military abilities the spirit of philosophy, and the feelings of humanity, was enabled, when governor of the Neutral Islands, to be singularly useful, in consequence of the instructions he had received from Dr. Pringle's book, and from personal conversation with him. By taking care to have his men always lodged in large, open, and airy apartments, and by never letting his forces remain long enough in swampy places to be injured by the noxious air which they are subject to, the general was the happy instrument of saving the lives of 700 soldiers.

Though Dr. Pringle had not for some years been called abroad, he still held his place of physician to the army; and in the war that began in 1755, he attended the camps in England during three seasons. In 1758, however, he entirely quitted the service of the army; and being now determined to fix wholly in London, he was the same year admitted a licenciate of the college of physicians.—After the accession of king George the 3d to the throne of Great Britain, Dr. Pringle was appointed, in 1761, physician to the queen's household; and this honour was succeeded, by his being constituted, in 1763, physician extraordinary to the queen. The same year he was chosen a member of the Academy of Sciences at Haarlem, and elected a fellow of the Royal College of Physicians in London.—In 1764, on the decease of Dr. Wollaston, he was made physician-in-ordinary to the queen. In 1766 he was elected a foreign member, in the physical line, of the Royal Society of Sciences at Gottingen, and the same year he was raised to the dignity of a baronet of Great-Britain. In 1768 he was appointed physician in ordinary to the late princess-dowager of Wales.

After having had the honour to be several times elected into the council of the Royal Society, sir John Pringle was at length, viz, Nov. 30, 1772, in consequence of the death of James West, esq. elected president of that learned body. His election to this high station, though he had so respectable a character as the late sir James Porter for his opponent, was carried by a very considerable majority. Sir John Pringle's conduct in this honourable station fully justified the choice the Society made of him as their president. By his equal, impartial, and encouraging behaviour, he secured the good will and best exertions of all for the general benefit of science, and true interests of the Society, which in his time was raised to the pinnacle of honour and credit. Instead of splitting the members into opposite parties, by cruel, unjust, and tyrannical conduct, as has sometimes been the case, to the ruin of the best interests of the Society, sir John Pringle cherished and happily united the endeavours of all, collecting and directing the energy of every one to the common good of the whole. He happily also struck out a new way to distinction and usefulness, by the discourses which were delivered by him, on the annual assignment of sir Godfrey Copley's medal. This gentleman had originally bequeathed five guineas, to be given at each anniversary meeting of the Royal Society, by the determination of the president and council, to the person who should be the author of the best paper of experimental observations for the year. In process of time, this pecuniary reward, which could never be an important consideration to a man of an enlarged and philosophical mind, however narrow his circumstances might be, was changed into the more liberal form of a gold medal; in which form it is become a truly honourable mark of distinction, and a just and laudable object of ambition. No doubt it was always usual for the president, on the delivery of the medal, to pay some compliment to the gentleman on whom it was bestowed; but the custom of making a set speech on the occasion, and of entering into the history of that part of philosophy to which the experiments, or the subject of the paper related, was first introduced by Martin Folkes, esq. The discourses however which he and his successors delivered, were very short, and were only inserted in the minute-books of the Society. None of them had ever been printed before sir John Pringle was raised to the chair. The first speech that was made by him being much more elaborate and extended than usual, the publication of it was desired; and with this request, it is said, he was the more ready to comply, as an absurd account of what he had delivered had appeared in a newspaper. Sir John was very happy in the subject of his first discourse. The discoveries in magnetism and electricity had been succeeded by the inquiries into the various species of air. In these inquiries, Dr. Priestley, who had already greatly distinguished himself by his electrical experiments, and his other philosophical pursuits and labours, took the principal lead. A paper of his, entitled, Observations on different Kinds of Air, having been read before the Society in March 1772, was adjudged to be deserving of the gold medal; and sir John Pringle embraced with pleasure the occasion of celebrating the important communications of his friend, and of relating with accuracy and fidelity what had previously been discovered upon the subject.

It was not intended, we believe, when sir John's first speech was printed, that the example should be followed: but the second discourse was so well received by the Society, that the publication of it was unanimously requested. Both the discourse itself, and the subject on which it was delivered, merited such a distinction. The composition of the second speech is evidently superior to that of the former one; sir John having probably been animated by the favourable reception of his first effort. His account of the Torpedo, and of Mr. Walsh's ingenious and admirable experiments relative to the electrical properties of that extraordinary fish, is singularly curious. The whole discourse abounds with ancient and modern learning, and exhibits the worthy president's knowledge in natural history, as well as in medicine, to great advantage.

The third time that he was called upon to display his abilities at the delivery of the annual medal, was on a very beautiful and important occasion. This was no less than Mr. Maskelyne's successful attempt completely to establish Newton's system of the universe, by his observations made on the Mountain Schihallien, for finding its attraction. Sir John laid hold of this opportunity to give a perspicuous and accurate relation of the several hypotheses of the ancients, with regard to the revolutions of the heavenly bodies, and of the noble discoveries with which Copernicus enriched the astronomical world. He then traces the progress of the grand principle of gravitation down to sir Isaac's illustrious confirmation of it; to which he adds a concise account of Messrs. Bouguer's and Condamine's experiment at Chimboraço, and of Mr. Maskelyne's at Schihallien. If any doubts still remained with respect to the truth of the Newtonian system, they were now completely removed.

Sir John Pringle had reason to be peculiarly satisfied with the subject of his fourth discourse; that subject being perfectly congenial to his disposition and studies. His own life had been much employed in pointing out the means which tended not only to cure, but to prevent the diseases of mankind; and it is probable, from his intimate friendship with captain Cook, that he might suggest to that sagacious commander some of the rules which he followed, in order to preserve the health of the crew of his ship, during his voyage round the world. Whether this was the case, or whether the method pursued by the captain to attain so salutary an end, was the result alone of his own reflections, the success of it was astonishing; and this celebrated voyager seemed well entitled to every honour which could be bestowed. To him the Society assigned their gold medal, but he was not present to receive the honour. He was gone out upon the voyage, from which he never returned: but in this last voyage he continued equally successful in maintaining the health of his men.

The learned president, in his fifth annual dissertation, had an opportunity of displaying his knowledge in a way in which it had not hitherto appeared. The discourse took its rise from the adjudication of the prize medal to Mr. Mudge, then an eminent surgeon at Plymouth, on account of his valuable paper, containing Directions for making the best Composition for the Metals of Reflecting Telescopes, together with a Description of the Process for Grinding, Polishing, and giving the Great Speculum the true Parabolic Form. Sir John has accurately related a variety of particulars, concerning the invention of reflecting telescopes, the subsequent improvements of these instruments, and the state in which Mr. Mudge found them, when he first set about working them to a greater perfection, till he had truly realized the expectation of Newton, who, above an hundred years ago, presaged that the public would one day possess a parabolic speculum, not accomplished by mathematical rules, but by mechanical devices.

Sir John Pringle's sixth and last discourse, to which he was led by the assignment of the gold medal to myself, on account of my paper entitled, The Force of fired Gunpowder, and the Initial Velocity of Cannon Balls, determined by Experiments, was on the theory of gunnery. Though sir John had so long attended the army, this was probably a subject to which he had heretofore paid very little attention. We cannot however help admiring with what perspicuity and judgment he stated the progress that was made, from time to time, in the knowledge of projectiles, and the scientific perfection to which it has been said to be carried in my paper. As sir John Pringle was not one of those who delighted in war, and in the shedding of human blood, he was happy in being able to show that even the study of artillery might be useful to mankind; and therefore this is a topic which he has not forgotten to mention. Here ended our author's discourses on the delivery of sir Godfrey Copley's medal, and his presidency over the Royal Society at the same time, the delivering that medal into my hand being the last office he ever performed in that capacity; a ceremony which was attended by a greater number of the members, than had ever met together before upon any other occasion. Had he been permitted to preside longer in that chair, he would doubtless have found other occasions of displaying his acquaintance with the history of philosophy. But the opportunities which he had of signalizing himself in this respect were important in themselves, happily varied, and sufficient to gain him a solid and lasting reputation.

Several marks of literary distinction, as we have already seen, had been conferred on sir John Pringle, before he was raised to the president's chair. But after that event they were bestowed upon him in great abundance, having been elected a member of almost all the literary societies and institutions in Europe. He was also, in 1774, appointed physician-extraordinary to the king.

It was at rather a late period of life when sir John Pringle was chosen to be president of the Royal Society, being then 65 years of age. Considering therefore the great attention that was paid by him to the various and important duties of his office, and the great pains he took in the preparation of his discourses, it was natural to expect that the burthen of his honourable station should grow heavy upon him in a course of time. This burthen, though not increased by any great addition to his life, for he was only 6 years president, was somewhat augmented by the accident of a fall in the area in the back part of his house, from which he received some hurt. From these circumstances some persons have affected to account for his resigning the chair at the time when he did. But sir John Pringle was naturally of a strong and robust frame and constitution, and had a fair prospect of being well able to discharge the duties of his situation for many years to come, had his spirits not been broken by the most cruel harassings and baitings in his office. His resolution to quit the chair originated from the disputes introduced into the Society, concerning the question, whether pointed or blunt electrical conductors are the most efficacious in preserving buildings from the pernicious effects of lightning, and from the cruel circumstances attending those disputes. These drove him from the chair. Such of those circumstances as were open and manifest to every one, were even of themselves perhaps quite sufficient to drive him to that resolution. But there were yet others of a more private nature, which operated still more powerfully and directly to produce that event; which may probably hereafter be laid before the public.

His intention of resigning however, was disagreeable to his friends, and the most distinguished members of the Society, who were many of them perhaps ignorant of the true motive for it. Accordingly, they earnestly solicited him to continue in the chair; but, his resolution being fixed, he resigned it at the anniversary meeting in 1778, immediately on delivering the medal, at the conclusion of his speech, as mentioned above.

Though sir John Pringle thus quitted his particular relation to the Royal Society; and did not attend its meetings so constantly as he had formerly done, he still retained his literary connexions in general. His house continued to be the resort of ingenious and philosophical men, whether of his own country, or from abroad; and he was frequent in his visits to his friends. He was held in particular esteem by eminent and learned foreigners, none of whom came to England without waiting upon him, and paying him the greatest respect. He treated them, in return, with distinguished civility and regard. When a number of gentlemen met at his table, foreigners were usually a part of the company.

In 1780 sir John spent the summer on a visit to Edinburgh; as he did also that of 1781; where he was treated with the greatest respect. In this last visit he presented to the Royal College of Physicians in that city, the result of many years labour, being ten folio volumes of Medical and Physical Observations, in manuscript, on condition that they should neither be published, nor lent out of the library of the college on any account whatever. He was at the same time preparing two other volumes, to be given to the university, containing the formulas referred to in his annotations. He returned again to London, and continued for some time his usual course of life, receiving and paying visits to the most eminent literary men, but languishing and declining in his health and spirits, till the 18th of January 1782, when he died, in the 75th year of his age; the account of his death being every where received in a manner which showed the high sense that was entertained of his merit.

Sir John Pringle's eminent character as a practical physician, as well as a medical author, is so well known, and so universally acknowledged, that an enlargement upon it cannot be necessary. In the exercise of his profession he was not rapacious; being ready, on various occasions, to give his advice without pecuniary views. The turn of his mind led him chiefly to the love of science, which he built on the firm basis of fact. With regard to philosophy in general, he was as averse to theory, unsupported by experiments, as he was with respect to medicine in particular. Lord Bacon was his favourite author; and to the method of investigation recommended by that great man, he steadily adhered. Such being his intellectual character, it will not be thought surprising that he had a dislike to Plato; and that to metaphysical disquisitions he lost all regard in the latter part of his life.

Sir John had no great fondness for poetry: he had not even any distinguished relish for the immortal Shakespear: at least he seemed too highly sensible of the defects of that illustrious bard, to give him the proper degree of estimation. Sir John had not in his youth been neglectful of philological inquiries, nor did he desert them in the last stages of his life, but cultivated even to the last a knowledge of the Greek language. He paid a great attention to the French language; and it is said that he was fond of Voltaire's critical writings. Among all his other pursuits, he never forgot the study of the English language. This he regarded as a matter of so much consequence, that he took uncommon pains with regard to the style of his compositions; and it cannot be denied, that he excelled in perspicuity, correctness, and propriety of expression. His six discourses in particular, delivered at the annual meetings of the Royal Society, on occasion of the prize medals, have been universally admired as elegant compositions, as well as critical and learned dissertations. And this characteristic of them, seemed to increase and heighten, from year to year: a circumstance which argues rather an improvement of his faculties, than any decline of them, and that even after the accident which it was pretended occasioned his descent from the president's chair. So excellent indeed were these compositions esteemed, that envy used to asperse his character with the imputation of borrowing the hand of another in those learned discourses. But how false such aspersion was, I, and I believe most of the other gentlemen who had the honour of receiving the annual medal from his hands, can fully testify. For myself in particular, I can witness for the last, and perhaps the best, that on the theory and improvements in gunnery, having been present or privy to his composition of every part of it.—Though our author was not fond of poetry, he had a great affection for the sister art music: of this he was not merely an admirer, but became so far a practitioner in it, as to be a performer on the violoncello, at a weekly concert given by a society of gentlemen at Edinburgh. Besides a close application to medical and philosophical science, during the latter part of his life, he devoted much time to the study of divinity: this being with him a very favourite and interesting object.

If, from the intellectual, we pass on to the moral character of sir John Pringle, we shall find that the ruling feature of it was integrity: and by this principle he was uniformly actuated in the whole of his conduct and behaviour. He was equally distinguished for his sobriety, having been heard to declare, that he had never once in his life been intoxicated with liquor. In his friendships, he was ardent and steady. The intimacies which were formed by him, in the early part of his life, continued unbroken to the decease of the gentlemen with whom they were made; and were kept up by a regular correspondence, and by all the good offices that lay in his power.

With regard to sir John's external manner of deportment, he paid a very respectful attention to those who were honoured with his friendship and esteem, and to such strangers as came to him well recommended. Foreigners in particular had good reason to be satisfied with the uncommon pains which he took to show them every mark of civility and regard. He had however at times somewhat of a dryness and reserve in his behaviour, which had the appearance of coldness; and this was the case when he was not perfectly pleased with the persons who were introduced to him, or who happened to be in his company. His sense of integrity and dignity would not permit him to adopt that false and superficial politeness, which treats all men alike, though ever so different in point of real estimation and merit, with the same show of cordiality and kindness. He was above assuming the profession, without the reality of respect.

PRISM, in Geometry, is a body, or solid, whose two ends are any plane figures which are parallel, equal, and similar; and its sides, connecting those ends, are parallelograms.—Hence, every section parallel to the ends, is the same kind of equal and similar figure as the ends themselves are; and the prism may be considered as generated by the parallel motion of this plane figure.

Prisms take their several particular names from the figure of their ends. Thus, when the end is a triangle, it is a triangular prism; when a square, a square prism; when a pentagon, a pentagonal prism; when a hexagon, a hexagonal prism; and so on. And hence the denomination prism comprises also the cube and parallelopipedon, the former being a square prism, and the latter a rectangular one. And even a cylinder may be considered as a round prism, or one that has an infinite number of sides. Also a prism is said to be regular or irregular, according as the figure of its end is a regular or an irregular polygon.

The axis of a prism, is the line conceived to be drawn lengthways through the middle of it, connecting the centre of one end with that of the other end.

Prisms, again, are either right or oblique.

A *Right* Prism is that whose sides, and its axis, are perpendicular to its ends; like an upright tower.

An *Oblique* Prism, is when the axis and sides are oblique to the ends; so that, when set upon one end, it inclines on one side, like an inclined tower.

The principal properties of prisms, are,

1. That all prisms are to one another in the ratio compounded of their bases and heights.

2. Similar prisms are to one another in the triplicate ratio of their like sides.

3. A prism is triple of a pyramid of equal base and height; and the solid content of a prism is found by multiplying the base by the perpendicular height.

4. The upright surface of a right prism, is equal to a rectangle of the same height, and its breadth equal to the perimeter of the base or end. And therefore such upright surface of a right prism, is found by multiplying the perimeter of the base by the perpendicular height. Also the upright surface of an oblique prism is found by multiplying the perimeter of the base by the slant height. And if to the upright surface be added the areas of the two ends, the sum will be the whole surface of the prism.

Prism, in Dioptrics, is a piece of glass in form of a triangular prism: which is much used in experiments concerning the nature of light and colours.—The use and phenomena of the prism arise from its sides not being parallel to each other; whence it separates the rays of light in their passage through it, by coming through two sides of one and the same angle.

The more general of these phenomena are enumerated and illustrated under the article Colour; which are sufficient to prove, that colours do not either consist in the contorsion of the globules of light, as Descartes imagined; nor in the obliquity of the pulses of the ethereal matter, as Hooke fancied; nor in the constipation of light, and its greater or less concitation, as Dr. Barrow conjectured; but that they are original and unchangeable properties of light itself.

PRISMOID, is a solid, or body, somewhat resembling a prism, but that its ends are any dissimilar parallel plane figures of the same number of sides; the upright sides being trapezoids.—If the ends of the prismoid be bounded by dissimilar curves, it is sometimes called a cylindroid.

PROBABILITY *of an Event*, in the Doctrine of Chances, is the ratio of the number of chances by which the event may happen, to the number by which it may both happen and fail. So that, if there be constituted a fraction, of which the numerator is the number of chances for the event's happening, and the denominator the number for both happening and failing, that fraction will properly express the value of the probability of the event's happening. Thus, if an event have 3 chances for happening, and 2 for failing, the sum of which being 5, the fraction $\frac{3}{5}$ will properly represent the probability of its happening, and may be taken to be the measure of it. The same thing may be said of the probability of failing, which will likewise be measured by a fraction, whose numerator is the number of chances by which it may fail, and its denominator the whole number of chances both for its happening and failing: so the probability of the failing of the above event, which has 2 chances to fail, and 3 to happen, will be expressed or measured by the fraction $\frac{2}{5}$.

Hence, if there be added together the fractions which express the probability for both happening and failing, their sum will always be equal to unity or 1; since the sum of their numerators will be equal to their common denominator. And since it is a certainty that an event will either happen or fail, it follows that a certainty, which may be considered as an infinitely great degree of probability, is fitly represented by unity. If it be required, what the probability is of an event happening in two trials, then we must estimate the probability of its failing twice, which taken from unity will be the probability of its happening. Thus if it was asked what is the probability of a person's casting an ace in two throws with a die of 6 faces. Here the probability of its failing the first time is $\frac{5}{6}$, there being 5 sides that may come up without the ace; also the probability of its failing the second throw is the same, therefore $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$ the probability of its failing both times, and consequently $\frac{36}{36} - \frac{25}{36} = \frac{11}{36}$ is the probability of its coming up one time at least in two throws. This circumstance is not readily comprehended by persons unskilled in the doctrine of chances; for, say they, the probability of its coming up the first time being $\frac{1}{6}$, and the probability of its coming up the second time being also $\frac{1}{6}$, therefore the two chances together must be $\frac{1}{6} + \frac{1}{6} = \frac{1}{3}$. But in this they deceive themselves, since it is not certain that they will have to throw a second time. See Simpson's or Demoivre's Doctrine of Chances; also Bernoulli's Ars Conjectandi; Monmort's Analyse des Jeux de Hasard; or M. De Parcieu's Essais sur les Probabilites de la Vie humaine. See also CHANCES, EXPECTATION, and GAMING.

PROBABILITY *of Life*. See EXPECTATION *of Life*, and LIFE-*Annuities*.

PROBLEM, in Geometry, is a proposition in which some operation or construction is required. As, to bisect a line, to make a triangle, to raise a perpendicular, to draw a circle through three points, &c. A problem, according to Wolfius, consists of three parts: The proposition, which expresses what is to be done; the resolution or solution, in which are orderly rehearsed the several steps of the process or operation; and the demonstration, in which it is shown, that by doing the several things prescribed in the resolution, the thing required is obtained.

PROBLEM, in Algebra, is a proposition which requires some unknown truth to be investigated or discovered; and the truth of the discovery demonstrated.

PROBLEM, *Kepler's*. See KEPLER'S *Problem*.

PROBLEM, *Determinate*, *Diophantine*, *Indeterminate*, *Limited*, *Linear*, *Local*, *Plane*, *Solid*, *Sursolid*, and *Unlimited*. See the adjectives.

Deliacal PROBLEM, in Geometry, is the doubling of a cube. This amounts to the same thing as the finding of two mean proportionals between two given lines: whence this also is called the Deliacal Problem. See DUPLICATION.

PROCLUS, an eminent philosopher and mathematician among the later Platonists, was born at Constantinople in the year 410, of parents who were both able and willing to provide for his instruction in all the various branches of learning and knowledge. He was first sent to Xanthus, a city of Lycia, to learn grammar: from thence to Alexandria, where he was under the best masters in rhetoric, philosophy, and mathematics: and from Alexandria he removed to Athens, where he attended the younger Plutarch, and Syrian, both of them celebrated philosophers. He succeeded the latter in the government of the Platonic school at Athens; where he died in 485, at 75 years of age.

Marinus of Naples, who was his successor in the school, wrote his life; the first perfect copy of which was published, with a Latin version and notes, by Fabricius at Hamburgh, 1700, in 4to; and afterwards subjoined to his Bibliotheca Latina, 1703, in 8vo. Marinus was also author of a learned commentary on Euclid's Data.

Proclus wrote a great number of pieces, and on many different subjects; as, commentaries on philosophy, mathematics, and grammar; on the whole works of Homer, Hesiod, and Plato's books of the republic: he wrote also on the construction of the Astrolabe: but many of his pieces are lost; some have been published; and a few remain still in manuscript only. Of the published, there are four very elegant hymns; one to the Sun, two to Venus, and one to the Muses. There are commentaries on several pieces of Plato; on the four books of Ptolemy's work De Judiciis Astrorum; on the first book of Euclid's Elements; and on Hesiod's Opera et Dies. There are also works of Proclus on philosophical and astronomical subjects; particularly the piece De Sphera, which was published, 1620, in 4to, by Bainbridge, the Savilian professor of astronomy at Oxford. He wrote also 18 arguments against the Christians, which are still extant, and in which he attacks them on the question, whether the world be eternal? the affirmative of which he maintains.

The character of Proclus is the same as that of all the later Platonists, who it seems were not less enthusiasts and madmen, than the Christians their contemporaries, whom they resembled in this respect. Proclus was not reckoned quite orthodox by his own order: he did not adhere so rigorously, as Julian and Porphyry, to the doctrines and principles of his master; "He had," says Cudworth, "some peculiar fancies and whimsies of his own, and was indeed a confounder of the Platonic theology, and a mingler of much unintelligible stuff with it."

PROCYON, in Astronomy, a fixed star, of the second magnitude, in Canis Minor, or the Little Dog.

PRODUCING, in Geometry, denotes the continuing a line, or drawing it farther out, till it have an assigned length.

PRODUCT, in Arithmetic, or Algebra, is the quantity arising from, or produced by, the multiplication of, two or more numbers &c together. Thus, 48 is the product of 6 multiplied by 8.—In multiplication, unity is in proportion to one factor, as the other factor is to the product. So 1 : 6 : : 8 : 48.

In algebra, the product of simple quantities is expressed by joining the letters together like a word, and prefixing the product of the numeral coefficients: So the product of a and b is ab, of $3a$ and $4bc$ is $12abc$. But the product of compound factors or quantities is expressed by setting the sign of multiplication between them, and binding each compound factor in a vinculum: so the product of $2a + 3b$ and $a - 4c$ is $(2a + 3b) \times (a - 4c)$.

In geometry, a rectangle answers to a product, its length and breadth being the two factors; because the numbers expressing the length and breadth being multiplied together, produce the content or area of the rectangle.

The term product, or continual product, is also sometimes used when the factors are more than two.

In algebra there are several curious properties relating to the particular forms of the product of certain formulæ, which are of great importance in the theory of numbers, and the indeterminate analysis; the most remarkable of which are as follows:

1. The product of a sum of two squares by double a square, is also the sum of two squares.

For $(x^2 + y^2) . 2z^2 = (x + y)^2 . z^2 + (x - y)^2 . z^2$.

2. The product of the sum of two squares, by the sum of two squares, is itself the sum of two squares.

$$\text{For } (x^2 + y^2) . (x'^2 + y'^2) = \begin{cases} (xx' + yy')^2 + (xy' - x'y)^2 \\ \text{or } (xx' - yy')^2 + (xy' + x'y)^2 \end{cases}$$

The product may therefore be divided into two squares two different ways. And if this product be again multiplied by the sum of two squares, the product may be divided into two squares four different ways; and so on.

3. The product of the sum of three squares by the sum of two squares, is the sum of four squares.

For $(x^2 + y^2 + z^2) \times (x'^2 + y'^2) =$
$(xx' + yy')^2 + (xy' - x'y)^2 + z^2x'^2 + z^2y'^2$.

4. The product of the sum of four squares by double a square, is also the sum of four squares.

For $(x^2 + y^2 + z^2 + w^2) .. 2z^2 =$
$z^2((x + y)^2 + (x - y)^2 + (z + w)^2 + (z - w)^2) =$
$z^2(x + y)^2 + z^2 (x - y)^2 + z^2 (z + w)^2 + z^2 (z - w)^2$.

5. The product of the sum of four squares, by the sum of four squares, is itself the sum of four squares.

For $(w^2 + x^2 + y^2 + z^2) . (w'^2 + x'^2 + y'^2 + z'^2)$
$= (ww' + xx' + yy' + zz')^2 + (wx' - xw' + yz' - zy')^2$
$+ (wy' - xz' - yw' + zx')^2 + (wz' + xy' - yx' - zw')^2$.

6. The two formulæ $x^2 + y^2 + z^2$ and $x'^2 + y'^2 + 2z'^2$ are so related to each other, that double the one produces the other.

For $2 . (x^2 + y^2 + z^2) = 2x^2 + 2y^2 + 2z^2 =$
$(x + y)^2 + (x - y)^2 + 2z^2$; and $2 (x'^2 + y'^2 + 2z'^2) =$
$2x'^2 + 2y'^2 + 4z'^2 = (x' + y')^2 + (x' - y')^2 + 4z'^2$.

The truth of the above theorems will be seen immediately by the developement of each respective formula.

PROFILE, in Architecture, the figure or draught of a building, fortification, or the like; in which are expressed the several heights, widths, and thicknesses, such as they would appear, were the building cut down perpendicularly from the roof to the foundation. Whence the profile is also called the section, and sometimes the orthographical section; and by Vitruvius the sciography. In this sense, profile amounts to the same thing with elevation; and so stands opposed to a plan or ichnography.

PROFILE is also used for the contour, or outline of a figure, building, member of architecture, or the like; as a base, a cornice, &c.

PROGRESSION, an orderly advancing or proceeding in the same manner, course, tenor, proportion, &c.

Progression is either arithmetical, geometrical, or harmonical.

Arithmetical **PROGRESSION**, is a series of quantities proceeding by continued equal differences, either increasing or decreasing. Thus,

increasing 1, 3, 5, 7, 9, &c, or
decreasing 21, 18, 15, 12, 9, &c;

where the former progression increases continually by the common difference 2, and the latter decreases continually by the common difference 3.

1. And hence, to construct an arithmetical progression, from any given first term, with a given common difference; add the common difference to the first term, to give the 2d; to the 2d, to give the 3d; to the 3d, to give the 4th; and so on; when the series is ascending or increasing: but subtract the common difference continually, when the series is a descending one.

2. The chief property of an arithmetical progression, and which arises immediately from the nature of its construction, is this; that the sum of its extremes, or first and last terms, is equal to the sum of every pair of intermediate terms that are equidistant from the extremes, or to the double of the middle term when there is an uneven number of the terms.

$$\begin{array}{llllllll} \text{Thus,} & 1, & 3, & 5, & 7, & 9, & 11, & 13, \\ & \underline{13,} & \underline{11,} & \underline{9,} & \underline{7,} & \underline{5,} & \underline{3,} & \underline{1,} \\ \text{Sums} & 14 & 14 & 14 & 14 & 14 & 14 & 14, \end{array}$$

where the sum of every pair of terms is 14.

$$\begin{array}{llllll} \text{Also,} & a, & a + d, & a + 2d, & a + 3d, & a + 4d, \\ & \underline{a + 4d,} & \underline{a + 3d,} & \underline{a + 2d,} & \underline{a + d,} & \underline{a} \\ \text{sums} & 2a + 4d, & 2a + 4d, & 2a + 4d, & 2a + 4d, & 2a + 4d. \end{array}$$

3. And hence it follows, that double the sum of all the terms in the series, is equal to the sum of the two extremes multiplied by the number of the terms; and consequently, that the single sum of all the terms of the series, is equal to half the said product. So the sum of the 7 terms, 1, 3, 5, 7, 9, 11, 13, is $(1 + 13) \times \frac{7}{2} = \frac{14}{2} \times 7 = 49$. And the sum of the five terms
$a, a + d, a + 2d, a + 3d, a + 4d$, is $(2a + 4d) \times \frac{5}{2}$.

4. Hence also, if the first term of the progression be 0, the sum of the series will be equal to half the product of the last term multiplied by the number of terms: i. e. the sum of
$0 + d + 2d + 3d + 4d \text{------} (n - 1)d = \frac{1}{2}n . (n - 1)d$,
where n is the number of terms, supposing 0 to be one of them. That is, in other words, the sum of an arithmetical progression, whether finite or infinite, whose first term is 0, is to the sum of as many times the greatest term, in the ratio of 1 to 2.

5. In like manner, the sum of the squares of the terms of such a series, beginning at 0, is to the sum of as many terms each equal to the greatest, in the ratio of 1 to 3. And

6. The sum of the cubes of the terms of such a series, is to the sum of as many times the greatest term, in the ratio of 1 to 4.

7. And universally, if every term of such a progression be raised to the m power, then the sum of all those powers will be to the sum of as many terms equal to the greatest, in the ratio of 1 to $m + 1$. That is,

the sum $0 + d + 2d + 3d \text{-------} l$,
is to $l^m + l^m + l^m + l^m \text{-------} l^m$,
in the ratio of 1 to $m + 1$.

8. A synopsis of all the theorems, or relations, in an arithmetical progression, between the extremes or first and last term, the sum of the series, the number of terms, and the common difference, is as follows:

viz, if a denote the least term,
z the greatest term,
d the common difference,
n the number of terms,
s the sum of the series;
then will each of these five quantities be expressed in terms of the others, as below:

$$a = z - (n - 1)\,d = \frac{2s}{n} - z = \frac{s}{n} - \frac{n-1}{2}d = \sqrt{((\tfrac{1}{2}d + z)^2 - 2ds)} + \tfrac{1}{2}d.$$

$$z = a + (n - 1)\,d = \frac{2s}{n} - a = \frac{s}{n} + \frac{n-1}{2}d = \sqrt{((\tfrac{1}{2}d - a)^2 + 2ds)} - \tfrac{1}{2}d.$$

$$d = \frac{z-a}{n-1} = \frac{s-na}{n-1}\cdot\frac{2}{n} = \frac{nz-s}{n-1}\cdot\frac{2}{n} = \frac{z+a\,.\,z-a}{2s-a-z}.$$

$$n = \frac{z-a}{d} + 1 = \frac{2s}{a+z} = \frac{\frac{1}{2}d - a + \sqrt{((\frac{1}{2}d - a)^2 + 2ds)}}{d} = \frac{\frac{1}{2}d + z - \sqrt{((\frac{1}{2}d + z)^2 - 2ds)}}{d}.$$

$$s = \frac{a+z}{2}n = \frac{a+z}{2}\cdot\frac{z-a+d}{d} = \frac{2a + (n-1)d}{2}n = \frac{2z - (n-1)d}{2}n.$$

And most of these expressions will become much simpler if the first term be 0 instead of a.

Geometrical PROGRESSION, is a series of quantities proceeding in the same continual ratio or proportion, either increasing or decreasing; or it is a series of quantities that are continually proportional; or which increase by one common multiplier, or decrease by one common divisor; which common multiplier or divisor is called the common ratio. As,

increasing, 1, 2, 4, 8, 16, &c,
decreasing, 81, 27, 9, 3, 1, &c;

where the former progression increases continually by the common multiplier 2, and the latter decreases by the common divisor 3.

Or ascending, $a,\ ra,\ r^2a,\ r^3a$, &c,
or descending, $a,\ \frac{a}{r},\ \frac{a}{r^2},\ \frac{a}{r^3}$, &c;

where the first term is a, and common ratio r.

1. Hence, the same principal properties obtain in a geometrical progression, as have been remarked of the arithmetical one, using only multiplication in the geometricals, for addition in the arithmeticals, and division in the former for subtraction in the latter. So that, to construct a geometrical progression, from any given first term, with a given common ratio; multiply the 1st term continually by the common ratio, for the rest of the terms when the series is an ascending one; or divide continually by the common ratio, when it is a descending progression.

2. In every geometrical progression, the product of the extreme terms, is equal to the product of every pair of the intermediate terms that are equidistant from the extremes, and also equal to the square of the middle term when there is a middle one, or an uneven number of the terms.

$$\begin{array}{llllll}
\text{Thus,} & 1, & 2, & 4, & 8, & 16, \\
 & \underline{16} & \underline{8} & \underline{4} & \underline{2} & \underline{1} \\
\text{prod.} & 16 & 16 & 16 & 16 & 16 \\
\text{Also} & a, & ra, & r^2a, & r^3a, & r^4a, \\
 & \underline{r^4a} & \underline{r^3a} & \underline{r^2a} & \underline{ra} & \underline{a} \\
\text{prod.} & r^4a^2 & r^4a^4 & r^4a^2 & r^4a^2 & r^4a^2
\end{array}$$

3. The last term of a geometrical progression, is equal to the first term multiplied, or divided, by the ratio raised to the power whose exponent is less by 1 than the number of terms in the series; so $z = ar^{n-1}$ when the series is an ascending one, or $z = \frac{a}{r^{n-1}}$, when it is a descending progression.

4. As the sum of all the antecedents, or all the terms except the least, is to the sum of all the consequents, or all the terms except the greatest, so is 1 to r the ratio. For,

if $a + ra + r^2a + r^3a$ be all except the last,
then $ra + r^2a + r^3a + r^4a$ are all except the first;

where it is evident that the former is to the latter as 1 to r, or the former multiplied by r gives the latter. So that, z denoting the last term, a the first term, and r the ratio, also s the sum of all the terms; then $s - z : s - a :: 1 : r$, or $s - a = (s - z)r$. And from this equation all the relations among the four quantities a, z, r, s, are easily derived; such as $s = \frac{rz - a}{r - 1}$; viz, multiply the greatest term by the ratio, subtract the least term from the product, then the remainder divided by 1 less than the ratio, will give the sum of the series. And if the least term a be 0, which happens when the descending progression is infinitely continued, then the sum is barely $\frac{rz}{r-1}$. As in the infinite progression $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ &c, where $z = 1$, and $r = 2$, it is s or $\frac{rz}{r-1} = \frac{2}{2-1} = \frac{2}{1} = 2$.

5. The first or least term of a geometrical progression, is to the sum of all the terms, as the ratio minus 1, to the n power of the ratio minus 1; that is $a : s :: r - 1 : r^n - 1$.

Other relations among the five quantities a, z, r, n, s, where

a denotes the least term,
z the greatest term,
r the common ratio,
n the number of terms,
s the sum of the progression,
are as below; viz,

$$a = \frac{z}{r^{n-1}} = zr - (r - 1)s = \frac{r-1}{r^n - 1}s.$$

$$z = ar^{n-1} = \frac{a + (r-1)s}{r} = \frac{r-1}{r^n - 1} sr^{n-1}.$$

$$r = \frac{s-a}{s-z} = \sqrt[n-1]{\frac{z}{a}}.$$

$$n = \frac{\log.\frac{rz}{a}}{\log.r} = \frac{\log.\frac{a+(r-1)s}{a}}{\log.r} = \frac{\log.\frac{rz}{rz-(r-1)s}}{\log.r} = \frac{\log.\frac{s-a}{s-z}.\frac{z}{a}}{\log.\frac{s-a}{s-z}}.$$

$$s = \frac{rz-a}{r-1} = \frac{r^n - 1}{r-1}a = \frac{r^n-1}{r-1}.\frac{z}{r^{n-1}} = \frac{\sqrt[n-1]{z^n} - \sqrt[n-1]{a^n}}{\sqrt[n-1]{z} - \sqrt[n-1]{a}}.$$

And the other values of a, z, and r are to be found from these equations, viz,

$$(s-z)^{n-1} z = (s-a)^{n-1} a,$$

$$r^n - \frac{s}{a} r = 1 - \frac{s}{a},$$

$$r^n - \frac{s}{s-z} r^{n-1} = \frac{z}{s-z}.$$

Harmonical PROGRESSION, is a continued series of terms in harmonical proportion. The reciprocals of an arithmetical progression form an harmonical progression. Thus, the reciprocals of the arithmetical series 1, 2, 3, 4, 5, 6, &c give $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, &c, fo ran harmonical series.

For other kinds of Progression, see PROPORTION, and SERIES.

PROJECTILE, in Mechanics, is any body which, being put into a violent motion by an external force impressed upon it, is dismissed from the agent, and left to pursue its course. Such as a stone thrown out of the hand or a sling, an arrow from a bow, a ball from a gun, &c.

PROJECTILES, the science of the motion, velocity, flight, range, &c, of a projectile put into violent motion by some external cause, as the force of gunpowder, &c. This is the foundation of gunnery, under which article may be found all that relates peculiarly to that branch.

All bodies, being indifferent as to motion or rest, will necessarily continue in the state they are put into, except so far as they are retarded, and forced to change it by some new cause. Hence, a projectile, put in motion, must continue eternally to move on in the same right line, and with the same uniform or constant velocity, were it to meet with no resistance from the medium, nor had any force of gravity to encounter.

In the first case, the theory of projectiles would be very simple indeed; for there would be nothing more to do, than to compute the space passed over in a given time by a given constant velocity; or either of these, from the other two being given.

But by the constant action of gravity, the projectile is continually deflected more and more from its right-lined course, and that with an accelerated velocity; which, being combined with its projectile impulse, causes the body to move in a curvilineal path, with a variable motion, which path is the curve of a parabola, as will be proved below; and the determination of the range, time of flight, angle of projection, and variable velocity, constitutes what is usually meant by the doctrine of projectiles, in the common acceptation of the term.

What is said above however, is to be understood of projectiles moving in a non-resisting medium; for when the resistance of the air is also considered, which is enormously great, and which very much impedes the first projectile velocity, the path deviates greatly from the parabola, and the determination of the circumstances of its motion becomes one of the most complex and difficult problems in nature.

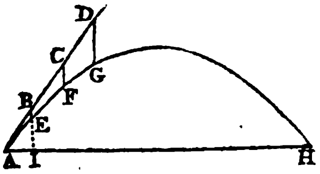
In the first place therefore it will be proper to consider the common doctrine of projectiles, or that on the parabolic theory, or as depending only on the nature of gravity and the projectile motion, as abstracted from the resistance of the medium.

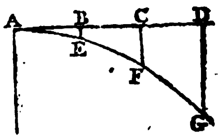
About 300 years ago, philosophers took the line described by a body projected horizontally, such as a bullet out of a cannon, while the force of the powder greatly exceeded the weight of the bullet, to be a right line, after which they allowed it became a curve. Nicholas Tartaglia was the first who perceived the mistake, maintaining that the path of the bullet was a curved line through the whole of its extent. But it was Galileo who first determined what particular curve it is that a projectile describes; showing that the path of a bullet projected horizontally from an eminence, was a parabola; the vertex of which is the point where the bullet quits the cannon. And the same is proved generally, in the 2d section following, when the projection is made in any direction whatever, viz, that the curve is always a parabola, supposing the body moves in a non-resisting medium, and that gravity acts upon it in lines parallel to each other.—It is true, that this is not accurately the case, because this force always tends to the centre of gravity of the earth; but the inclination of these lines is too trifling to affect the parabolic theory of projectiles.

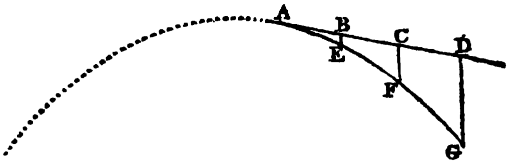
The Laws of the Motion of PROJECTILES.

I. If a heavy body be projected perpendicularly, it will continue to ascend or descend perpendicularly; because both the projecting and the gravitating force are found in the same line of direction.

II. If a body be projected in free space, either parallel to the horizon, or in any oblique direction; it will, by this motion, in conjunction with the action of gravity, describe the curve line of a parabola.







For let the body be projected from A, in the direction AD, with any uniform velocity; then in any equal portions of time it would, by that impulse alone, describe the equal spaces AB, BC, CD, &c in the line AD, if it were not drawn continually down below that line by the action of gravity. Draw BE, CF, DG, &c, in the direction of gravity, or perpendicular to the horizon; and take BE, CF, DG, &c, equal to the spaces through which the body would descend by its gravity in the same times in which it would uniformly pass over the spaces AB, AC, AD, &c, by the projectile motion. Then, since by these motions, the body is carried over the space AB in the same time as the space BE, and the space AC in the same time as the space CF, and the space AD in the same time as the space DG, &c; therefore, by the composition of motions, at the end of those times the body will be found respectively in the points E, F, G, &c, and consequently the real path of the projectile will be the curve line AEFG&c. But the spaces AB, AC, AD, &c, being described by uniform motion, are as the times of description; and the spaces BE, CF, DG, &c, described in the same times by the accelerating force of gravity, are as the squares of the times; consequently the perpendicular descents are as the squares of the spaces in AD,

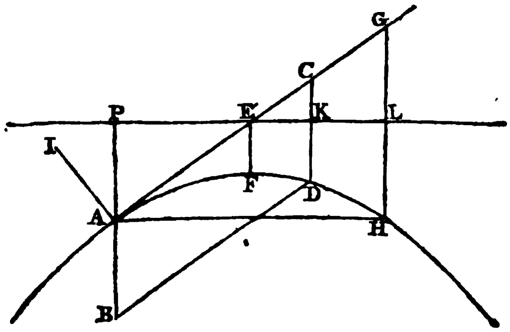
that is - - - BE, CF, DG, &c,
are respectively proportional to AB^2, AC^2, AD^2, &c,
which is the same as the property of the parabola. Therefore the path of the projectile is the parabolic line AEFG &c, to which AD is a tangent at the point A.

Hence, 1. The horizontal velocity of a projectile is always the same constant quantity, in every point of the curve; because the horizontal motion is in a constant ratio to the motion in AD, which is the uniform projectile motion; viz, the constant horizontal velocity being to the projectile velocity, as radius to the cosine of the angle DAH, or angle of elevation or depression of the piece above or below the horizontal line AH.

2. The velocity of the projectile in the direction of the curve, or of its tangent, at any point A, is as the secant of its angle BAI of direction above the horizon. For the motion in the horizontal direction AI being constant, and AI being to AB as radius to the secant of the angle A; therefore the motion at A, in AB, is as the secant of the angle A.

3. The velocity in the direction DG of gravity, or perpendicular to the horizon, at any point G of the curve, is to the first uniform projectile velocity at A, as 2GD to AD. For the times of describing AD and DG being equal, and the velocity acquired by freely descending through DG being such as would carry the body uniformly over twice DG in an equal time, and the spaces described with uniform motions being as the velocities, it follows that the space AD is to the space 2DG, as the projectile velocity at A is to the perpendicular velocity at G.

III. The velocity in the direction of the curve, at any point of it, as A, is equal to that which is generated by gravity in a body freely descending through a space which is equal to one-fourth of the parameter of the diameter to the parabola at that point.



Let PA or AB be the height due to the velocity of the projectile at any point A, in the direction of the curve or tangent AC, or the velocity acquired by falling through that height; and complete the parallelogram ACDB. Then is CD = AB or AP the height due to the velocity in the curve at A; and CD is also the height due to the perpendicular velocity at D, which will therefore be equal to the former: but, by the last corollary, the velocity at A is to the perpendicular velocity at D, as AC to 2CD; and as these velocities are equal, therefore AC or BD is equal to 2CD or 2AB; and hence AB or AP is equal to ¼BD or ¼ of the parameter of the diameter AB by the nature of the parabola.

Hence, 1. If through the point P, the line PL be drawn perpendicular to AP; then the velocity in the curve at every point, will be equal to the velocity acquired by falling through the perpendicular distance of the point from the said line PL; that is, a body falling freely through

PA, acquires the velocity in the curve at A,
EF, - - - at F,
KD, - - - at D,
LH, - - - at H.

The reason of which is, that the line PL is what is called the directrix of the parabola, the property of which is, that the perpendicular to it, from every point of the curve, is equal to one-fourth of the parameter of the diameter at that point, viz,

PA = ¼ the parameter of the diameter at A,
EF = - - - at F,
KD = - - - at D,
LH = - - - at H.

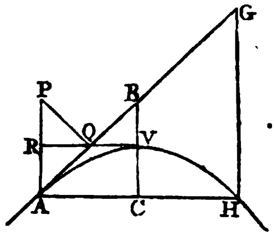
2. If a body, after falling through the height PA, which is equal to AB, and when it arrives at A if its course be changed, by reflection from a firm plane AI, or otherwise, into any direction AC, without altering the velocity; and if AC be taken equal to 2AP or 2AB, and the parallelogram be completed; the body will describe the parabola passing through the point D.

3. Because AC = 2AB or 2CD or 2AP, therefore $AC^2 = 2AP \,.\, 2CD$ or $AP \,.\, 4CD$; and because all the perpendiculars EF, CD, GH are as AE^2, AC^2, AG^2; therefore also $AP \,.\, 4FF = AE^2$, and $AP \,.\, 4GH = AC^2$, &c; and because the rectangle of the extremes is equal to the rectangle of the means, of four proportionals, therefore it is always,

and AP : AE : : AE : 4EF,
and AP : AC : : AC : 4CD,
and AP : AG : : AG : 4GH ;
and so on.

IV. Having given the direction of a projectile, and the impetus or altitude due to the first velocity; to determine the greatest height to which it will rise, and the random or horizontal range.

Let AP be the height due to the projectile velocity at A, or the height which a body must fall to acquire the same velocity as the projectile has in the curve at A; also AG the direction, and AH the horizon. Upon AG let fall the perpendicular PQ, and on AP the perpendicular QR; so shall AR be equal to the greatest altitude CV, and 4RQ equal to the horizontal range AH.



Or, having drawn PQ perpendicular to AG, take AG = 4AQ, and draw GH perpendicular to AH; then AH is the range.

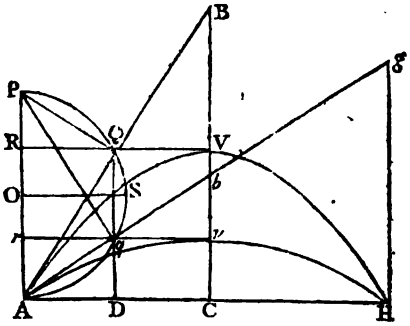
For by the last cor. - - - AP : AG :: AG : 4GH,
and by sim. triangles, - - AP : AG :: AQ: GH,
or AP : AG : : 4AQ: 4GH;

therefore AG = 4AQ; and, by similar triangles, AH = 4RQ.

Also, if V be the vertex of the parabola, then AB or

½AG = 2AQ, or AQ = QB; consequently AR = BV which is = CV by the nature of the parabola.

Hence, 1. Because the angle Q is a right angle, which is the angle in a semicircle, therefore if upon AP as a diameter a semicircle be described, it will pass through the point Q.



2. If the horizontal range and the projectile velocity be given, the direction of the piece so as to strike the object H will be easily found thus: Take AD = ¼AH, and draw DQ perpendicular to AH, meeting the semicircle described on the diameter AP in Q and *q*; then either AQ or A*q* will be the direction of the piece. And hence it appears, that there are two directions AB and A*b* which, with the same projectile velocity, give the very same horizontal range AH; and these two directions make equal angles *q*AD and QAP with AH and AP, because the arc PQ is equal to the arc A*q*.

3. Or if the range AH and direction AB be given; to find the altitude and velocity or impetus: Take AD = ¼AH, and erect the perpendicular DQ meeting AB in Q; so shall DQ be equal to the greatest altitude CV. Also erect AP perpendicular to AH, and QP to AQ; so shall AP be the height due to the velocity.

4. When the body is projected with the same velocity, but in different directions; the horizontal ranges AH will be as the sines of double the angles of elevation. Or, which is the same thing, as the rectangle of the sine and cosine of elevation. For AD or RQ, which is ¼AH, is the sine of the arc AQ, which measures double the angle QAD of elevation.

And when the direction is the same, but the velocities different, the horizontal ranges are as the square of the velocities, or as the height AP which is as the square of the velocity; for the sine AD or RQ, or ¼AH, is as the radius, or as the diameter AP.

Therefore, when both are different, the ranges are in the compound ratio of the squares of the velocities, and the sines of double the angles of elevation.

5. The greatest range is when the angle of elevation is half a right angle, or 45°. For the double of 45 is 90°, which has the greatest sine. Or the radius OS, which is ¼ of the range, is the greatest sine.

And hence the greatest range, or that at an elevation of 45°, is just double the altitude AP which is due to the velocity. Or equal to 4VC. And consequently, in that case, C is the focus of the parabola, and AH its parameter. And the ranges are equal at angles equally above and below 45°.

6. When the elevation is 15°, the double of which, or 30°, having its sine equal to half the radius, consequently its range will be equal to AP, or half the greatest range at the elevation of 45°; that is, the range at 15° is equal to the impetus or height due to the projectile velocity.

7. The greatest altitude CV, being equal to AR, is as the versed sine of double the angle of elevation, and also as AP or the square of the velocity. Or as the square of the sine of elevation, and the square of the velocity; for the square of the sine is as the versed sine of the double angle.

8. The time of flight of the projectile, which is equal to the time of a body falling freely through GH or 4CV, 4 times the altitude, is therefore as the square root of the altitude, or as the projectile velocity and sine of the elevation.

9. And hence may be deduced the following set of theorems, for finding all the circumstances relating to projectiles on horizontal planes, having any two of them given. Thus, let

s, c, t = sine, cosine, and tang. of elevation,

S, v = sine and vers. of double the elevation,

R the horizontal range, T the time of flight, V the projectile velocity, H the greatest height of the projectile, $g = 16\frac{1}{12}$ feet, and a = the impetus or the altitude due to the velocity V. Then,

$$R = 2aS = 4asc = \frac{SV^2}{2g} = \frac{scV^2}{g} = \frac{gcT^2}{s} = \frac{gT^2}{t} = \frac{4H}{t}.$$

$$V = \sqrt{4ag} = \sqrt{\frac{2gR}{S}} = \sqrt{\frac{gR}{sc}} = \frac{gT}{s} = \frac{2\sqrt{gH}}{s}.$$

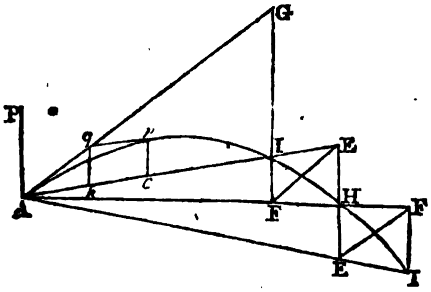
$$T = \frac{sV}{g} = 2s\sqrt{\frac{a}{g}} = \sqrt{\frac{tR}{g}} = \sqrt{\frac{sR}{gc}} = 2\sqrt{\frac{H}{g}}.$$

$$H = as^2 = \tfrac{1}{2}av = \tfrac{1}{4}tR = \frac{sR}{4c} = \frac{sV^2}{4g} = \frac{vV^2}{8g} = \frac{gT^2}{4}.$$

And from any of these, the angle of direction may be found.

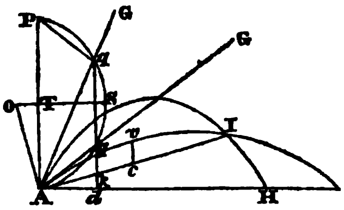
V. To determine the range on an oblique plane; having given the impetus or the velocity, and the angle of direction.

Let AE be the oblique plane, at a given angle above or below the horizontal plane AH; AG the direction of the piece; and AP the altitude due to the projectile velocity at A.



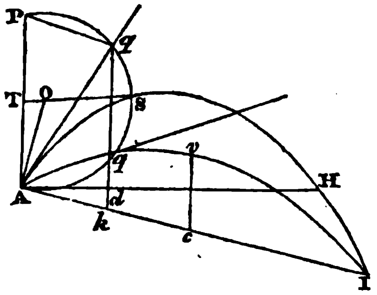
By the last prop. find the horizontal range AH to the given velocity and direction; draw HE perpendicular to AH meeting the oblique plane in E; draw EF parallel to the direction AG, and FI parallel to HE; so shall the projectile pass through I, and the range on the oblique plane will be AI. This is evident from prob. 17 of the parabola in my Treatise on Conic Sections, where it is proved, that if AH, AI be any two lines terminated at the curve, and IF, HE be parallel to the axis; then is EF parallel to the tangent AG.

Hence, 1. If AO be drawn perpendicular to the plane AI, and AP be bisected by the perpendicular STO; then



with the centre O describing a circle through A and P, the same will also pass through q, because the angle GAI, formed by the tangent AG and AI, is equal to the angle APq, which will therefore stand upon the same arc Aq.

2. If there be given the range and velocity, or the impetus, the direction will then be easily found thus: Take A$k = \frac{1}{4}$AI, draw kq perpendicular to AH, meeting the circle described with the radius AO in two points q and q; then Aq or Aq will be the direction of the piece. And hence it appears that there are two directions, which, with the same impetus, give the very same range AI, on the oblique plane. And these two directions make equal angles with AI and AP, the plane and the perpendicular, because the arc Pq = the arc Aq. They also make equal angles with a line drawn from A through S, because the arc Sq = the arc Sq.



3. Or, if there be given the range AI, and the direction Aq; to find the velocity or impetus. Take A$k = \frac{1}{4}$AI, and erect kq perpendicular to AH meeting the line of direction in q; then draw qP making the angle AqP = the angle Akq; so shall AP be the impetus, or altitude due to the projectile velocity.

4. The range on an oblique plane, with a given elevation, is directly as the rectangle of the cosine of the direction of the piece above the horizon and the sine of the direction above the oblique plane, and reciprocally as the square of the cosine of the angle of the plane above or below the horizon.

For put s = sin. ∠ qAI or APq,
c = cos. ∠ qAH or sin. PAq,
C = cos. ∠ AIH or sin. Akd or Akq or AqP.
Then, in the tri. APq, - - - C : s : : AP : Aq,
and in the tri. Akq, - - - C : c : : Aq : Ak,
therefore by compos. - - - C^2 : cs : : AP : Ak = $\frac{1}{4}$AI.

So that the oblique range AI $= \frac{cs}{C^2} \times 4$AP.

Hence the range is the greatest when Ak is the greatest, that is when kq touches the circle in the middle point S, and then the line of direction passes through S, and bisects the angle formed by the oblique plane and the vertex. Also the ranges are equal at equal angles above and below this direction for the maximum.

5. The greatest height cv or kq of the projectile, above the plane, is equal to $\frac{s^2}{C^2} \times$ AP. And therefore it is as the impetus and square of the sine of direction above the plane directly, and square of the cosine of the plane's inclination reciprocally.

For C (sin. AqP) : s (sin. APq) : : AP : Aq,
and C (sin. Akq) : s (sin. kAq) : : Aq : kq,
therefore by comp. C^2 : s^2 : : AP : kq.

6. The time of flight in the curve AvI is $= \frac{2s}{C}\sqrt{\frac{\text{AP}}{g}}$, where $g = 16\frac{1}{12}$ feet. And therefore it is as the velocity and sine of direction above the plane directly, and cosine of the plane's inclination reciprocally. For the time of describing the curve, is equal to the time of falling freely through GI or $4kq$ or $\frac{4s^2}{C^2} \times$ AP. Therefore, the time being as the square root of the distance, $\sqrt{g} : \frac{2s}{C}\sqrt{\text{AP}} : : 1'' : \frac{2s}{C}\sqrt{\frac{\text{AP}}{g}}$ the time of flight.

7. From the foregoing corollaries may be collected the following set of theorems, relating to projectiles made on any given inclined planes, either above or below the horizontal plane. In which the letters denote as before, namely,

c = cos. of direction above the horizon,
C = cos. of inclination of the plane,
s = sin. of direction above the plane,
R the range on the oblique plane,
T the time of flight,
V the projectile velocity,
H the greatest height above the plane,
a the impetus, or alt. due to the velocity V,
$g = 16\frac{1}{12}$ feet. Then

$$R = \frac{cs}{C^2} \times 4a = \frac{cs}{C^2 g}V^2 = \frac{gc}{s}T^2 = \frac{4c}{s}H.$$

$$H = \frac{s^2}{C^2}a = \frac{s^2V^2}{4gC^2} = \frac{sR}{4c} = \frac{g}{4}T^2.$$

$$V = \sqrt{4ag} = C\sqrt{\frac{gR}{cs}} = \frac{gC}{s}T = \frac{2C}{s}\sqrt{gH}.$$

$$T = \frac{2s}{C}\sqrt{\frac{a}{g}} = \frac{sV}{gC} = \sqrt{\frac{sR}{gc}} = 2\sqrt{\frac{H}{g}}.$$

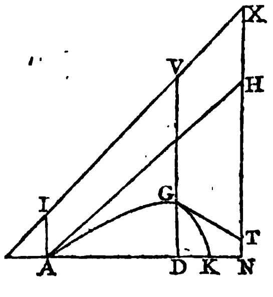
And from any of these, the angle of direction may be found.

Of the Path of PROJECTILES *as depending on the Resistance of the Air.*

For a long time after Galileo, philosophers seemed to be satisfied with the parabolic theory of projectiles, deeming the effect of the air's resisitance on the path as of no consequence. In process of time however, as the true philosophy began to dawn, they began to suspect that the resistance of the medium might have some effect on the projectile curve, and they set themselves to consider this subject with some attention.

Huygens, supposing that the resistance of the air was proportional to the velocity of the moving body, concluded that the line described by it would be a kind of logarithmic curve.

But Newton, having clearly proved, that the resistance to the body is not proportional to the velocity itself, but to the square of it, shows, in his Principia, that the line a projectile describes, approaches nearer to an hyperbola than a parabola. Schol. prop. 10, lib. 2. Thus if AGK be a curve of the hyperbolic kind, one of whose asymptotes is NX, perpendicular to the horizon AK, and the other IX inclined to the same, where VG is reciprocally as DN^n, whose index is n: this curve will nearer represent the path of a projectile thrown in the direction AH in the air, than a para-



bola. Newton indeed says, that these hyperbolas are not accurately the curves that a projectile makes in the air; for the true ones are curves which about the vertex are more distant from the asymptotes, and in the parts remote from the axis approach nearer to the asymptotes than these hyperbolas; but that in practice these hyperbolas may be used instead of those more compounded ones. And if a body be projected from A, in the right line AH, and AI be drawn parallel to the asymptote NX, and GT a tangent to the curve at the vertex: Then the density of the medium in A will be reciprocally as the tangent AH, and the body's velocity will be as $\sqrt{\frac{AH^2}{AI}}$, and the resistance of the medium will be to gravity, as

$$AH \text{ to } \frac{2n^2 + 2n}{n + 2} \times AI.$$

John Bernoulli constructed this curve by means of the quadrature of some transcendental curves, at the request of Dr. Keil, who proposed this problem to him in 1718. It was also resolved by Dr. Taylor; and another solution of it may be found in Hermann's Phoronomia.

The commentators Le Sieur and Jacquier say, that the description of the curve in which a projectile moves, is so very perplexed, that it can scarcely be expected any deduction should be made from it, either to philosophical or mechanical purposes: vol. 2, pa. 118.

Dan. Bernoulli too proved, that the resistance of the air has a very great effect on the swift motions, such as those of cannon shot. He concludes from experiment, that a ball which ascended only 7819 feet in the air, would have ascended 58750 feet in vacuo, being near 8 times as high. Comment. Acad. Petr. tom. 2.

Euler has still farther investigated the nature of this curve, and directed the calculation and use of a number of tables for the solution of all cases that occur in gunnery, which may be accomplished with nearly as much expedition as by the common parabolic principles. Memoirs of the Academy of Berlin, for the year 1753.

But how rash and erroneous the old opinion of the inconsiderable resistance of the air is, will easily appear from the experiments of Mr. Robins, who has shown that, in some cases, this resistance to a cannon ball, amounts to more than 20 times the weight of the ball; and I myself, having prosecuted this subject far beyond any former example, have sometimes found this resistance amount to near 100 times the weight of the ball, viz, when it moved with a velocity of 2000 feet per second, which is a rate of almost 23 miles in a minute. What errors then may not be expected from an hypothesis which neglects this force, as inconsiderable? Indeed it is easy to show, that the path of such projectiles is neither a parabola nor nearly a parabola. For, by that theory, if the ball, in the instance last mentioned, moved in the curve of a parabola, its horizontal range, at 45° elevation, will be found to be almost 24 miles; whereas it often happens that the ball, with such a velocity, ranges far short of even one mile.

Indeed the fallacy of this hypothesis almost appears at sight, even in projectiles slow enough to have their motion traced by the eye; for they are seen to descend through a curve manifestly shorter and more inclined to the horizon than that in which they ascended, and the highest point of their flight, or the vertex of the curve, is much nearer to the place where they fall on the ground, than to that from which they were at first discharged. These things cannot for a moment be doubted of by any one, who in a proper situation views the flight of stones arrows, or shells, thrown to any considerable distance.

Mr. Robins has not only detected the errors of the parabolic theory of gunnery, which takes no account of the resistance of the air, but attempts to show how to compute the real range of resisted bodies. But for the method which he proposes, and the tables he has computed for this purpose, see his Tracts of Gunnery, pa. 183, &c, vol. 1; and also Euler's Commentary on the same, translated by Mr. Hugh Brown, in 1777.

There is an odd circumstance which often takes place in the motion of bodies projected with considerable force, which shows the great complication and difficulty of this subject; namely, that bullets in their flight are not only depressed beneath their original direction by the action of gravity, but are also frequently driven to the right or left of that direction by the action of some other force. Now if it were true that bullets varied their direction by the action of gravity only, then it ought to happen that the errors in their flight to the right or left of the mark they were aimed at, should increase in the proportion of the distance of the mark from the piece only. But this is contrary to all experience; the same piece which will carry its bullet within an inch of the intended mark, at 10 yards distance, cannot be relied on to 10 inches in 100 yards, much less to 30 in 300 yards.

And this inequality can only arise from the track of the bullet being incurvated sideways as well as downwards; for by this means the distance between the incurvated line and the line of direction, will increase in a much greater ratio than that of the distance; these lines coinciding at the mouth of the piece, and afterwards separating in the manner of a curve from its tangent, if the mouth of the piece be considered as the point of contact.

This is put beyond a doubt from the experiments made by Mr. Robins; who found also that the direction of the shot in the perpendicular line was not less uncertain, falling sometimes 200 yards short of what it did at other times, though there was no visible cause of difference in making the experiment. And I myself have often experienced a difference of one-fifth or one-sixth of the whole range, both in the deflection to the right or left, and also in the extent of the range, of cannon shot.

If it be asked, what can be the cause of a motion so different from what has been hitherto supposed? It may be answered, that the deflection in question must be owing to some power acting obliquely to the progressive motion of the body, which power can be no other than the resistance of the air. And this resistance may perhaps act obliquely to the progressive motion of the body, from inequalities in the resisted surface; but its general cause is doubtless a whirling motion acquired by the bullet about an axis, by its friction against the sides of the piece; for by this motion of rotation, combined with the progressive motion, each part of the ball's surface will strike the air in a direction very different from what it would do if there was no such whirl; and the obliquity of the action of the air, arising from this cause, will be greater, according as the rotatory motion of the bullet is greater in proportion to its progressive motion. Tracts, vol. 3.

M. Euler, on the contrary, attributes this deflection of the ball to its figure, and very little to its rotation: for if the ball was perfectly round, though its centre of gravity did not coincide with the centre of spontaneous rotation, the deflection from the axis of the cylinder, or line of direction sideways, would be very inconsiderable. but when it is not round, it will generally go to the right or left of its direction, and so much the more, as its range is greater. From his reasoning on this subject he infers, that cannon shot, which are made of iron, and rounder and less susceptible of a change of figure in passing along the cylinder than those of lead, are more certain than musket shot. True Principles of Gunnery investigated, 1777, pa. 304, &c. And for the experiments on the air's resistance to all balls and velocities, with the application to gunnery, see my Tracts, vols. 2 and 3.

PROJECTION, in Mechanics, the act of giving a projectile its motion.—If the direction of the force, by which the projectile is put in motion, be perpendicular to the horizon, the projection is said to be perpendicular; if parallel to the apparent horizon, it is said to be an horizontal projection; and if it make an oblique angle with the horizon, the projection is oblique. In all cases the angle which the line of direction makes with the horizontal line, is called the angle of elevation of the projectile, or of depression when the line of direction points below the horizontal line.

PROJECTION, in Perspective, denotes the appearance or representation of an object on the perspective plane. So, the projection of a point, is a point, where the optic ray passes from the objective point through the plane to the eye; or it is the point where the plane cuts the optic ray. And hence it is easy to conceive what is meant by the projection of a line, a plane, or a solid.

PROJECTION *of the Sphere in Plano*, is a representation of the several points or places of the surface of the sphere, and of the circles described upon it, on a supposed transparent plane placed between the eye and the sphere, or such as they appear to the eye placed at a given distance. For the laws of this projection, see PERSPECTIVE: the projection of the sphere being only a particular case of perspective.—The chief use of the projection of the sphere, is in the construction of planispheres, maps, and charts; which are said to be of this or that projection, according to the several situations of the eye, and the perspective plane, with regard to the meridians, parallels, and other points or places to be represented.—The most usual projection of maps of the world, is that on the plane of the meridian, which exhibits a right sphere; the first meridian being the horizon. The next is that on the plane of the equator, which has the pole in the centre, and the meridians the radii of a circle, &c; which represents a parallel sphere. See MAP.

The projection of the sphere is usually divided into orthographic and stereographic; to which may be added gnomonic.

Orthographic PROJECTION, is that in which the surface of the sphere is drawn upon a plane, cutting it in the middle; the eye being placed at an infinite distance vertically to one of the hemispheres. And

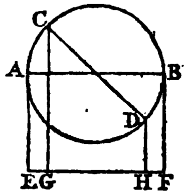
Stereographic PROJECTION of the sphere, is that in which the surface and circles of the sphere are drawn upon the plane of a great circle, the eye being in the pole of that circle.

Gnomonical PROJECTION *of the Sphere*, is that in which the surface of the sphere is drawn upon a plane without side of it, commonly touching it, the eye being at the centre of the sphere. See GNOMONICAL *Projection*.

Laws of the Orthographic Projection.

1. The rays coming from the eye, being at an infinite distance, and making the projection, are parallel to each other, and perpendicular to the plane of projection.

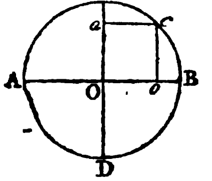
2. A right line perpendicular to the plane of projection, is projected into a point, where that line meets the said plane.



3. A right line, as AB, or CD, not perpendicular, but either parallel or oblique to the plane of the projection, is projected into a right line, as EF or GH, and is always comprehended between the extreme perpendiculars AE and BF, or CG and DH.

4. The projection of the line AB is the greatest, when AB is parallel to the plane of the projection.

5. Hence it is evident, that a line parallel to the plane of projection, is projected into a right line equal to itself; but a line that is oblique to the plane of projection, is projected into one that is less than itself.



6. A plane surface, as ACBD, perpendicular to the plane of projection, is projected into the right line, as AB, in which it cuts that plane.—Hence it is evident, that the circle ACBD perpendicular to the plane of projection, passing through its centre, is projected into that diameter AB in which it cuts the plane of the projection. Also any arch as *cc* is projected into *oo*, equal to *ca*, the right sine of that arch; and the complemental arc *c*B is projected into *o*B, the versed sine of the same arc *c*B.

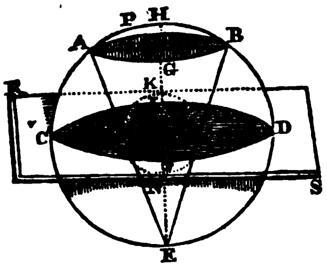
7 A circle parallel to the plane of projection, is projected into a circle equal to itself, having its centre the same with the centre of the projection, and its radius equal to the cosine of its distance from the plane. And a circle oblique to the plane of projection, is projected into an ellipsis, whose greater axis is equal to the diameter of the circle, and its less axis equal to double the cosine of the obliquity of the circle, to a radius equal to half the greater axis.

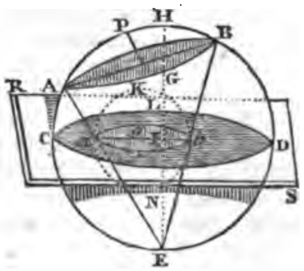
Properties of the Stereographic Projection.

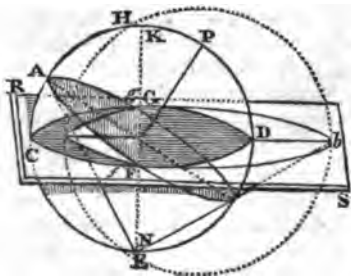
1. In this projection a right circle, or one perpendicular to the plane of projection, and passing through the eye, is projected into a line of half tangents.

2. The projection of all other circles, not passing through the projecting point, whether parallel or oblique, are projected into circles.

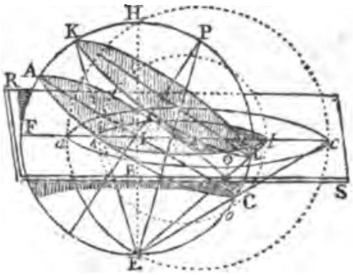
Thus, let ACEDB represent a sphere, cut by a plane RS, passing through the centre I, perpendicular to the diameter EH, drawn from E the place of the eye; and let the section of the sphere by the plane RS be the circle CFDL, whole poles are H and E. Suppose now AGB a circle on the sphere to be projected, whose pole most remote from the eye is P; and the visual rays from the circle AGB meeting in E, to form the cone AGBE, of which the triangle AEB is a section through the vertex E, and diameter of the base AB: then will the figure *agbf*, which is the projection of the circle AGB, be itself a circle. Hence, the middle of the projected diameter is the centre of the projected circle, whether it be a great circle or a small one: Also the poles and centres of all circles, parallel to the plane of projection, fall in the centre of the projection: And all oblique great circles cut the primitive circle in two points diametrically opposite.



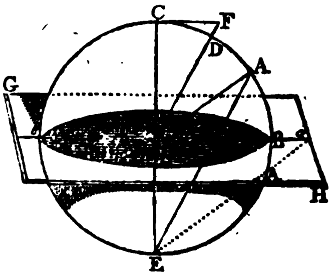




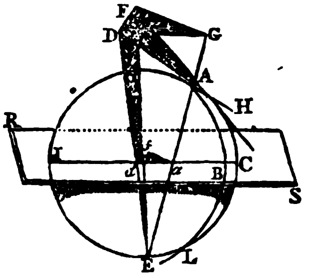
2. The projected diameter of any circle subtends an angle at the eye equal to the distance of that circle from its nearest pole, taken on the sphere; and that angle is bisected by a right line joining the eye and that pole. Thus, let the plane RS cut the sphere HFEG through its centre I; and let ABC be any oblique great circle, whose diameter AC is projected into *ac*; and KOL any small circle parallel to ABC, whose diameter KL is projected in *kl*. Then the distances of those circles from their pole P, being the arcs AHP, KHP; and the angles *a*E*c*, *k*E*l*, being the angles at the eye, subtended by their projected diameters, *ac* and *kl*. It follows that the angle *a*E*c* is measured by the arc AHP, and that the angle *k*E*l* is measured by the arc KHP; and these angles are bisected by EP.



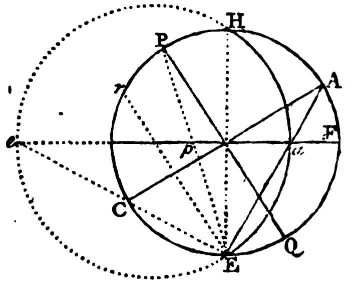
3. Any point of a sphere is projected at such a distance from the centre of projection, as is equal to the tangent of half the arc intercepted between that point and the pole opposite to the eye, the semidiameter of the sphere being radius. Thus, let *cb*Eβ be a great circle of the sphere, whose centre is *c*, GH the plane of projection, cutting the diameter of the sphere in *b* and B; also E and C the poles of the section by that plane; and *a* the projection of A. Then *ca* is equal the tangent of half the arc AC, as is evident by drawing CF equal to the tangent of half that arc, and joining CF.



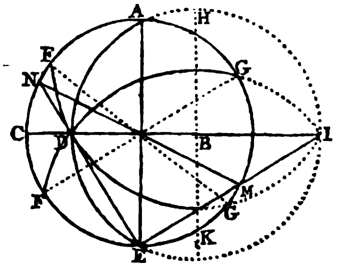
4. The angle made by two projected circles, is equal to the angle which these circles make on the sphere. For let IACE and ABL be two circles on a sphere intersecting in A; E the projecting point; and RS the plane of projection, in which the point A is projected in *a*, in the line IC, the diameter of the circle ACE. Also let DH and FA be tangents to the circles ACE and ABL. Then will the projected angle *daf* be equal to the spherical angle BAC.



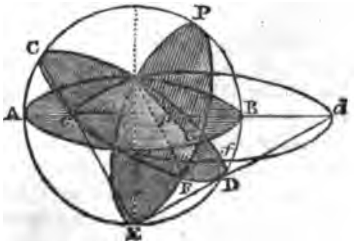
5. The distance between the poles of the primitive circle and an oblique circle, is equal to the tangent of half the inclination of those circles; and the distance of their centres, is equal to the tangent of their inclination; the semidiameter of the primitive circle being radius. For let AC be the diameter of a circle, whose poles are P and Q, and inclined to the plane of projection in the angle AIF; and let *a*, *c*, *p* be the projections of the points A, C, P; also let H*a*E be the projected oblique circle, whose centre is *q*. Now when the plane of projection becomes the primitive circle, whose pole is I; then is I*p* equal to the tangent of half the angle AIF, or of half the arch AF; and I*q* is equal to the tangent of AF, or of the angle FH*a* = AIF.



6. If through any given point in the primitive circle, an oblique circle be described; then the centres of all other oblique circles passing through that point, will be in a right line drawn through the centre of the first oblique circle, and perpendicular to a line passing through that centre, the given point, and the centre of the primitive circle. Thus, let GACE be the primitive circle, ADEI a great circle described through D, its centre being B. HK is a right line drawn through B perpendicular to a right line CI passing through D and B and the centre of the primitive circle. Then the centres of all other great circles, as FDG, passing through D, will fall in the line HK.



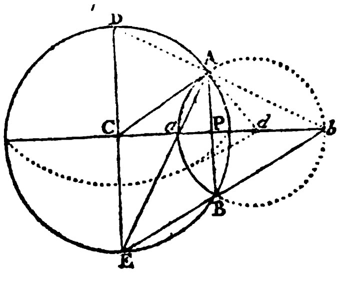
7. Equal arcs of any two great circles of the sphere will be intercepted between two other circles drawn on the sphere through the remotest poles of those great circles. For let PBEA be a sphere, on which AGB and CFD are two great circles, whose remotest poles are E and P; and through these poles let the great circle PBEC and the small circle PGE be drawn, cutting the great cir-



cles AGB and CFD in the points B, G, D, F. Then are the intercepted arcs BG and DF equal to each other.

8. If lines be drawn from the projected pole of any great circle, cutting the peripheries of the projected circle and plane of projection; the intercepted arcs of those peripheries are equal; that is, the arc BG = *df*.

9. The radius of any lesser circle, whose plane is perpendicular to that of the primitive circle, is equal to the tangent of that lesser circle's distance from its pole; and the secant of that distance is equal to the distance of the centres of the primitive and lesser circle. For let P be the pole, and AB the diameter of a lesser circle, its plane being perpendicular to that of the primitive circle, whose centre is C: then *d* being the centre of the projected lesser circle, *da* is equal to the tangent of the arc PA, and *dc* = the secant of PA. See STEREOGRAPHIC *Projection*.



Mercator's PROJECTION. See MERCATOR, and CHART.

PROJECTION of Globes, &c. SEE GLOBE, &c.

Polar PROJECTION. See POLAR.

PROJECTION *of Shadows*. See SHADOW.

PROJECTION, or PROJECTURE, in Building, the outjetting or prominency which the mouldings and members have, beyond the plane or naked of the wall, column, &c.

Monstrous PROJECTION. See ANAMORPHOSIS.

PROJECTIVE *Dialling*, a manner of drawing the hour lines, the furniture &c of dials, by a method of projection on any kind of surface whatever, without regard to the situation of those surfaces, either as to declination, reclination, or inclination. See DIALLING.

PROLATE, or OBLONG *Spheroid*, is a spheroid produced by the revolution of a semiellipsis about its longer diameter; being longest in the direction of that axis, and resembling an egg, or a lemon. It is so called in opposition to the oblate or short spheroid, which is formed by the rotation of a semiellipsis about its shorter axis; being therefore shortest in the direction of its axis, or flatted at the poles, and so resembling an orange, or perhaps a turnip, according to the degree of flatness; and which is also the figure of the earth. See SPHEROID.

PROMONTORY, in Geography, is a rock or high point of land projecting out into the sea. The extremity of which towards the sea is usually called a Cape, or Headland.

PROPORTION, in Arithmetic, &c, the equality or similitude of ratios. As the four numbers 4, 8, 15, 30 are proportionals, or in proportion, because the ratio of 4 to 8 is equal or similar to the ratio of 15 to 30, both of them being the same as the ratio of 1 to 2.

Euclid. in the 5th definition of the 5th book, gives a general definition of four proportionals, or when, of four terms, the first has the same ratio to the 2d, as the 3d has to the 4th, viz. when any equimultiples whatever of the first and third being taken, and any equimultiples whatever of the 2d and 4th; if the multiple of the first be less than that of the 2d, the multiple of the 3d is also less than that of the 4th; or if the multiple of the first be equal to that of the 2d, the multiple of the 3d is also equal to that of the 4th; or if the multiple of the first be greater than that of the 2d, the multiple of the 3d is also greater than that of the 4th. And this definition is general for all kinds of magnitudes or quantities whatever, though a very obscure one.

Also, in the 7th book, Euclid gives another definition of proportionals, viz, when the first is the same equimultiple of the 2d, as the 3d is of the 4th, or the same part or parts of it. But this definition appertains only to numbers and commensurable quantities.

Proportion is often confounded with ratio; but they are quite different things. For, ratio is properly the relation of two magnitudes or quantities of one and the same kind; as the ratio of 4 to 8, or of 15 to 30, or of 1 to 2; and so implies or respects only two terms or things. But proportion respects four terms or things, or two ratios which have each two terms. Though the middle term may be common to both ratios, and then the proportion is expressed by three terms only, as 4, 8, 64, where 4 is to 8 as 8 to 64.

Proportion is also sometimes confounded with progression. In fact, the two often coincide; the difference between them only consisting in this, that progression is a particular species of proportion, being indeed a continued proportion, or such as has all the terms in the same ratio, viz, the 1st to the 2d, the 2d to the 3d, the 3d to the 4th, &c; as the terms 2, 4, 8, 16, &c; so that progression is a series or continuation of proportions.

Proportion is either continual, or discrete or interrupted.

The proportion is continual when every two adjacent terms have the same ratio, or when the consequent of each ratio is the antecedent of the next following ratio, and so all the terms form a progression; as 2, 4, 8, 16, &c; where 2 is to 4 as 4 to 8, and as 8 to 16, &c.

Discrete or interrupted proportion, is when the consequent of the first ratio is different from the antecedent of the 2d, &c; as 2, 4, and 3, 6.

Proportion is also either direct or inverse.

Direct PROPORTION is when more requires more, or less requires less. As it will require more men to perform more work, or fewer men for less work, in the same time.

Inverse or *Reciprocal* PROPORTION, is when more requires less, or less requires more. As it will require more men to perform the same work in less time, or fewer men in more time. Ex. If 6 men can perform a piece of work in 15 days, how many men can do the same in 10 days.

$$\left.\begin{array}{lll}\text{Then, reciprocally} & - & \text{as } \frac{1}{15} \text{ to } \frac{1}{10} \text{ so is } 6 : 9 \\ \text{or inversely} & - & \text{as } 10 \text{ to } 15 \text{ so is } 6 : 9\end{array}\right\} \text{the answer.}$$

Proportion, again, is distinguished into arithmetical, geometrical, and harmonical.

Arithmetical PROPORTION is the equality of two arithmetical ratios, or differences. As in the numbers 12, 9, 6; where the difference between 12 and 9, is the same as the difference between 9 and 6, viz 3. And here the sum of the extreme terms is equal to the sum of the means, or to double the single mean when there is but one. As $12 + 6 = 9 + 9 = 18$.

Geometrical PROPORTION is the equality between two geometrical ratios, or between the quotients of the terms. As in the three 9, 6, 4, where 9 is to 6 as 6 is to 4, thus denoted, $9 : 6 :: 6 : 4$; for $\frac{9}{6} = \frac{6}{4}$, being each equal $\frac{3}{2}$ or $1\frac{1}{2}$. And in this proportion, the rectangle or product of the extreme terms, is equal to that of the two means, or the square of the single mean when there is but one. For $9 \times 4 = 6 \times 6 = 36$.

Harmonical PROPORTION, is when the first term is to the third, as the difference between the 1st and 2d is to the difference between the 2d and 3d; or, in four terms, when the 1st is to the 4th, as the difference between the 1st and 2d is to the difference between the 3d and 4th; or the reciprocals of an arithmetical proportion are in harmonical proportion. As 6, 4, 3; because $6 : 3 :: 6 - 4 = 2 : 4 - 3 = 1$; or because $\frac{1}{6}$, $\frac{1}{4}$, $\frac{1}{3}$, are in arithmetical proportion, making $\frac{1}{6} + \frac{1}{3} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$. Also the four 24, 16, 12, 9 are in harmonical proportion, because $24 : 9 :: 8 : 3$.

See PROPORTIONALS.

Compass of PROPORTION, a name by which the French, and some English authors, call the Sector.

Rule of PROPORTION, in Arithmetic, a rule by which a 4th term is found in proportion to three given terms. It is popularly called the Golden Rule, or Rule of Three.

PROPORTIONAL, relating to proportion. As, Proportional Compasses, Parts, Scales, Spirals, &c. See the several terms.

PROPORTIONAL *Compasses*, are compasses with two pair of opposite legs, like a St. Andrew's cross, by which any space is enlarged or diminished in any proportion.

PROPORTIONAL *Part*, is a part of some number that is analogous to some other part or number; such as the proportional parts in the logarithms, and other tables.

PROPORTIONAL *Scales*, called also logarithmic scales, are the logarithms, or artificial numbers, placed on lines, for the ease and advantage of multiplying and dividing &c, by means of compasses, or of sliding rulers. These are in effect so many lines of numbers, as they are called by Gunter, but made single, double, triple, or quadruple; beyond which they seldom go. See GUNTER'S *Scale*, SCALE, &c.

PROPORTIONAL *Spiral*. See SPIRAL.

PROPORTIONALITY, the quality of proportionals. This term is used by Gregory St. Vincent, for the proportion between the exponents of four ratios.

PROPORTIONALS, are the terms of a proportion; consisting of two extremes, which are the first and last terms of the set, and the means, which are the other terms. These proportionals may be either arithmeticals, geometricals, or harmonicals, and in any number above two, and also either continued or discontinued.

Pappus gives this beautiful and simple comparison of the three kinds of proportionals, arithmetical, geometrical, and harmonical, viz. a, b, c, being the first, second, and third terms in any such proportion, then

$$\left.\begin{array}{l}\text{In the arithmeticals, } a : a \\ \text{in the geometricals, } a : b \\ \text{in the harmonicals, } a : c\end{array}\right\} :: a - b : b - c.$$

See MEAN *Proportional*.

Continued proportionals form what is called a progression; for the properties of which see PROGRESSION.

I. *Properties of Arithmetical* PROPORTIONALS.

(For what respects progressions and mean proportionals of all sorts, see MEAN, and PROGRESSION).

1. Four arithmetical proportionals, as 2, 3, 4, 5,
are still proportionals taken inversely, as 5, 4, 3, 2;
or alternately, thus, 2, 4, 3, 5;
or inversely and alternately, thus 5, 3, 4, 2.

2. If two arithmeticals be added to the like terms of other two arithmeticals, of the same difference, or arithmetical ratio, the sums will have double the same difference or arithmetical ratio.

$$\begin{array}{lrlr}
\text{So,} & 3 \text{ and } 5, & \text{whose difference is} & 2, \\
\text{add} & 7 \text{ and } 9, & \text{whose difference is also} & 2, \\
\text{the sums} & 10 \text{ and } 14 & \text{have a double diff. viz,} & 4.
\end{array}$$

And if to these sums be added two other numbers also in the same difference, the next sums will have a triple ratio or difference; and so on. Also, whatever be the ratios of the terms that are added, whether the same or different, the sums of the terms will have such arithmetical ratio as is composed of the sums of the others that are added.

$$\begin{array}{lrrlr}
\text{So} & 3, & 5, & \text{whose dif. is} & 2 \\
\text{and} & 7, & 10, & \text{whose dif. is} & 3 \\
\text{and} & 12, & 16, & \text{whose dif. is} & 4 \\
\hline
\text{make} & 22, & 31, & \text{whose dif. is} & 9.
\end{array}$$

On the contrary, if from two arithmeticals there be subtracted others, the difference will have such arithmetical ratio as is equal to the differences of those.

$$\begin{array}{lrlr}
\text{So from } 12 \text{ and } 16, & & \text{whose dif. is} & 4 \\
\text{take} & 7 \text{ and } 10, & \text{whose dif. is} & 3 \\
\hline
\text{leaves} & 5 \text{ and } 6, & \text{whose dif. is} & 1 \\
\text{Also from } 7 \text{ and } 9, & & \text{whose dif. is} & 2 \\
\text{take} & 3 \text{ and } 5, & \text{whose dif. is} & 2 \\
\hline
\text{leaves} & 4 \text{ and } 4, & \text{whose dif. is} & 0
\end{array}$$

3. Hence, if arithmetical proportionals be multiplied or divided by the same number, their difference, or arithmetical ratio, is also multiplied or divided by the same number.

II. *Properties of Geometrical Proportionals.*

The properties relating to mean proportionals are given under the term MEAN *Proportional*; some are also given under the article PROPORTION; and some additional ones are as below:

1. To find a 3d proportional to two given numbers, or a 4th proportional to three: In the former case, multiply the 2d term by itself, and divide the product by the 1st: and in the latter case, multiply the 2d term by the 3d, and divide the product by the 1st.

So $2 : 6 :: 6 : 18$, the 3d prop. to 2 and 6:
and $2 : 6 :: 5 : 15$, the 4th prop. to 2, 6, and 5.

2. If the terms of any geometrical ratio be augmented or diminished by any others in the same ratio, or proportion, the sums or differences will still be in the same ratio or proportion.

So if $a : b :: c : d$,
then is $a : b :: a \pm c : b \pm d :: c : d$.

And if the terms of a ratio, or proportion, be multiplied or divided by the same number, the products and quotients will still be in the same ratio, or proportion.

Thus, $a : b :: na : nb :: \frac{a}{n} : \frac{b}{n}$.

3. If a set of continued proportionals be either augmented or diminished by the same part or parts of themselves, the sums or differences will also be proportionals.

Thus if a, b, c, d, &c be propors.
then are $a \pm \frac{a}{n}$, $b \pm \frac{b}{n}$, $c \pm \frac{c}{n}$, &c also propors.
where the common ratio is $1 \pm \frac{1}{n}$.

And if any single quantity be either augmented or diminished by some part of itself, and the result be also increased or diminished by the same part of itself, and this third quantity treated in the same manner, and so on; then shall all these quantities be continued proportionals. So, beginning with the quantity a, and taking always the nth part, then shall

a, $a \pm \frac{a}{n}$, $a \pm \frac{2a}{n} + \frac{a^2}{n^2}$, &c be proportionals,

or a, $a \pm \frac{a}{n}$, $(a \pm \frac{a}{n})^2$, $(a \pm \frac{a}{n})^3$, &c propors.

the common ratio being $1 \pm \frac{a}{n}$.

4. If one set of proportionals be multiplied or divided by any other set of proportionals, each term by each, the products or quotients will also be proportionals.

$$\begin{array}{llcl} \text{Thus, if} & a : na & :: & b : nb, \\ \text{and} & c : mc & :: & d : md; \\ \text{then is} & ac : mnac & :: & bd : mnbd, \\ \text{and} & \frac{a}{c} : \frac{na}{mc} & :: & \frac{b}{d} : \frac{nb}{mb}. \end{array}$$

5. If there be several continued proportionals, then whatever ratio the 1st has to the 2d, the first to the 3d shall have the duplicate of the ratio, the 1st to the 4th the triplicate of it, and so on.

So in a, na, n^2a, n^3a, &c, the ratio being n;
then $a : n^2a$, or 1 to n^2, the duplicate ratio,
and $a : n^3a$, or 1 to n^3, the triplicate ratio, &c.

6. In three continued proportionals, the difference between the 1st and 2d term, is a mean proportional between the 1st term and the 2d difference of all the terms.

Thus, in the three propor. a, na, n^2a;

$$\begin{array}{r|l|l} \text{Terms} & \text{1st difs.} & \text{2d dif.} \\ n^2a & & \\ na & n^2a - na & n^2a - 2na + a, \\ & na - a & \\ a & & \end{array}$$

then $a : na - a :: na - a : n^2a - 2na + a$.

Or in the numbers 2, 6, 18;

$$\begin{array}{r|r|l} 18 & & \\ 6 & 12 & \text{8 the 2d difference;} \\ & 4 & \\ 2 & & \end{array}$$

then 2, 4, 8 are proportionals.

7. When four quantities are in proportion, they are also in proportion by inversion, composition, division, &c; thus a, na, b, nb being in proportion, viz.

$$\begin{array}{llcl} 1. & & a : na :: b : nb; & \text{then by} \\ 2. & \text{Inversion} & na : a :: nb : b; & \\ 3. & \text{Alternation} & a : b :: na : nb; & \\ 4. & \text{Composition} & a + na : na :: b + nb : nb; & \\ 5. & \text{Conversion} & a + na : a :: b + nb : b; & \\ 6. & \text{Division} & \left\{\begin{array}{l} a - na : a :: b - nb : b; \\ a - na : na :: b - nb : nb. \end{array}\right. & \end{array}$$

III. *Properties of Harmonical Proportionals.*

1. If three or four numbers in harmonical proportion, be either multiplied or divided by any number, the products or quotients will also be harmonical proportionals.

Thus, 6, 3, 2 being harmon. propor.
then 12, 6, 4 are also harmon. propor.
and $\frac{6}{2}$, $\frac{3}{2}$, $\frac{2}{2}$ are also harmon. propor.

2. In the three harmonical proportionals a, b, c, when any two of these are given, the 3d can be found from the definition of them, viz, that $a : c :: a - b : b - c$; for hence

$b = \frac{2ac}{a + c}$ the harmonical mean, and

$c = \frac{ab}{2a - b}$ the 3d harmon. to a and b.

3. And of the four harmonicals, a, b, c, d, any three being given, the fourth can be found from the definition of them, viz, that $a : d :: a - b : c - d$; for thence the three b, c, d, will be thus found, viz,

$$b = \frac{2ad - ac}{d}; \quad c = \frac{2ad - bd}{a}; \quad d = \frac{ac}{2a - b}.$$

4. If there be four numbers disposed in order, as 2, 3, 4, 6, of which one extreme and the two middle terms are in arithmetical proportion, and the other extreme and the same middle terms are in harmonical proportion; then are the four terms in geometrical proportion: thus,

the three 2, 3, 4 are arithmeticals,
and the three 3, 4, 6 are harmonicals,
then the four 2, 3, 4, 6 are geometricals.

5. If between any two numbers, as 2 and 6, there be interposed an arithmetical mean 4, and also a harmonical mean 3, the four will then be geometricals, viz, $2 : 3 :: 4 : 6$.

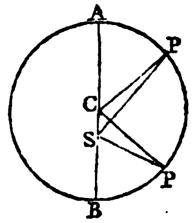
6. Between the three kinds of proportion, there is this remarkable difference; viz, that from any given number there can be raised a continued arithmetical series increasing ad infinitum, but not decreasing; while the harmonical can be decreased ad infinitum, but not increased; and the geometrical admits of both.

PROPOSITION, is either some truth advanced, and shown to be such by demonstration; or some operation proposed, and its solution shown. In short, it is something proposed either to be demonstrated, or to be done or performed. The former is a theorem, and the latter is a problem.

PROSTHAPHERESIS, in Astronomy, the difference between the true and mean motion, or between the true and mean place, of a planet, or between the true and equated anomaly; called also equation of the orbit, or equation of the centre, or simply the equation; and it is equal to the angle formed at the planet, and subtended by the excentricity of its orbit. Thus, if S be the sun, and P the place of a planet in its orbit APB, whose centre is C. Then the

Mean anomaly is the ∠ ACP,
true anomaly is the ∠ ASP,
dif. of which is the ∠ CPS,

which is the prosthapheresis; which is so called, because it is sometimes to be added to, and sometimes to be subtracted from the mean motion, to give the true one; as is evident from the figure.



PROTRACTING, or **Protraction**, in Surveying, the act of plotting or laying down the dimensions taken in the field, by means of a Protractor, &c: Protracting makes one part of surveying.

Protracting-*Pin*, a fine pointed pin, or needle, fitted into a handle, used to prick off degrees and minutes from the limb of the protractor.

PROTRACTOR, a mathematical instrument, used in surveying, for laying down angles on paper, &c.

The simplest, and most natural protractor consists of a semicircular limb ADB (fig. 7, plate 24) commonly of metal, divided into 180°, and subtended by a diameter AB; in the middle of which is a small notch C, called the centre of the protractor. And for the convenience of reckoning both ways, the degrees are numbered from the left hand towards the right, and from the right hand towards the left.

But this instrument is made much more commodious by transferring the divisions from the circumference to the edge of a ruler, whose side EF is parallel to AB, which is easily done by laying a ruler on the centre C, and over the several divisions on the semicircumference ADB, and marking the intersections of that ruler on the line EF: so that a ruler with these divisions marked on one of its sides as above, and returned down the two ends, and numbered both ways as in the circular protractor, the fourth or blank side representing the diameter of the circle, is both a more useful form than the circular protractor, and better adapted for putting into a case.

To make any Angle with the Protractor.—Lay the diameter of the protractor along the given line, which is to be one side of the angle, and its centre at the given angular point; then make a mark opposite the given degree of the angle found on the limb of the instrument, and, removing the protractor, by a plane ruler laid over that point and the centre, draw a line, which will form the angle sought. In the same way is any given angle measured, to find the number of degrees it contains.—This protractor is also very useful in drawing one line perpendicular to another, which is readily done by laying the protractor across the given line, so that both its centre and the 90th degree on the opposite edge fall upon the line, also one of the edges passing over the given point, by which then let the perpendicular be drawn.

The Improved PROTRACTOR is an instrument much like the former, only furnished with a little more apparatus, by which an angle may be set off to a minute.

The chief addition is an index attached to the centre, about which it is moveable, so as to play freely and steadily over the limb: beyond this limb the index is divided, on both edges, into 60 equal parts of the portions of circles, intercepted by two other right lines drawn from the centre, so that each makes an angle of 1° with lines drawn to the assumed points from the centre.

To set off an angle of any number of degrees and minutes with this protractor, move the index, so that one of the lines drawn on the limb, from one of the fore-mentioned points, may fall upon the number of degrees given; and prick off as many of the equal parts on the proper edge of the index as there are minutes given; then drawing a line from the centre to that point so pricked off, the required angle is thus formed with the given line or diameter of the protractor.

The best protractors are now made with a vernier, and fine pins, to prick off angles to minutes.

PROVING *of Gunpowder*. See EPROUVETTE, and GUNPOWDER.

PSEUDO-STELLA, any kind of meteor or phenomenon, appearing in the heavens, and resembling a star.

PTOLEMAIC, or PTOLOMAIC, something relating to Ptolemy; as the Ptolemaic System, the Ptolemaic Sphere, &c. See SYSTEM, SPHERE, &c.

PTOLEMY, or PTOLOMY, (CLAUDIUS), a celebrated geographer, astronomer, and mathematician, was born at Pelusium in Egypt, about the 70th year of the Christian era; and died, it has been said, in the 78th year of his age, and in the year of Christ 147. He taught astronomy at Alexandria in Egypt, where he made many astronomical observations, and composed his other works. It is certain however that he flourished in the reigns of Marcus Antoninus and Adrian: for it is noted in his canon, that Antoninus Pius reigned 23 years, which shows that he himself survived him; he also tells us in one place, that he made a great many observations on the fixed stars at Alexandria, in the second year of Antoninus Pius; and in another, that he observed an eclipse of the moon, in the 9th year of Adrian; from which it is reasonable to conclude that this astronomer's observations on the heavens were many of them made between the years 125 and 140.

Ptolemy has always been reckoned the prince of astronomers among the ancients, and in his works has left us an entire body of that science. He has preserved and transmitted to us the observations and principal discoveries of the ancients, and at the same time augmented and enriched them with his own. He corrected Hipparchus's catalogue of the fixed stars; and formed tables, by which the motions of the sun, moon, and planets, might be calculated and regulated. He was indeed the first who collected the scattered and detached observations of the ancients, and digested them into a system; which he set forth in his Μεγαλη Συνταξις, sive Magna Constructio, divided into 13 books. He there adopts and exhibits the ancient system of the world, which placed the earth in the centre of the universe; and this has been called after him the Ptolemaic System, to distinguish it from those of Copernicus and Tycho Brahé.

About the year 827 this work was translated by the Arabians into their language, in which it was called Almagestum, by order of one of their kings; and from Arabic into Latin, about 1230, by the encouragement of the emperor Frederic the 2d. There were also other versions from the Arabic into Latin; and a manuscript of one, done by Girardus Cremonensis, who flourished about the middle of the 14th century, which, Fabricius says, is still extant in the library of All Souls College in Oxford. The Greek text of this work began to be read in Europe in the 15th century; and was first published by Simon Grynæus at Basil, 1538, in folio, with the eleven books of commentaries by Theon, who flourished at Alexandria in the reign of the elder Theodosius. In 1541 it was reprinted at Basil, with a Latin version by George Trapezond; and again at the same place in 1551, with the addition of other works of Ptolemy, and Latin versions by Camerarius: which last edition, we learn from Kepler, was used by Tycho.

Of this principal work of the ancient astronomers, it may not be improper to give here a more particular account. In general, it may be observed, that it is founded on the hypothesis of the earth's being at rest in the centre of the universe, and that the heavenly bodies, the stars and planets, all move around it in solid orbs, whose motions are all directed by one, which Ptolemy called the primum mobile, or first mover, of which he discourses at large. The whole of this great work is divided into 13 books.

In the first book, Ptolemy shows, that the earth is in the centre of those orbs, and of the universe itself, as he understood it: he represents the earth as of a spherical figure, and but as a point in comparison of the rest of the heavenly bodies: he then treats of the several circles of the earth, and their distances from the equator; as also of the right and oblique ascension of the heavenly bodies in a right sphere.

In the 2d book, he treats of the habitable parts of the earth; of the elevation of the pole in an oblique sphere, and the various angles which the several circles make with the horizon, according to the different latitude of places; also of the phenomena of the heavenly bodies depending on the same.

In the 3d book, he treats of the quantity of the year, and of the unequal motion of the sun through the zodiac:

and he here also gives the method of computing the mean motion of the sun, with tables of the same; and also treats of the inequality of days and nights.

In the 4th book, he treats of the lunar motions, and their various phenomena: and gives tables for finding the moon's mean motions, with her latitude and longitude: he discourses largely concerning lunar epicycles; and by comparing the times of a great number of eclipses, mentioned by Hipparchus, Calippus, and others, he has computed the places of the sun and moon, according to their mean motions, from the first year of Nabonazar, king of Egypt, to his own time.

In the 5th book, he treats of the instrument called the astrolabe, and also of the excentricity of the lunar orbit, and the inequality of the moon's motion, according to her distance from the sun: he also gives tables, and a universal canon for the inequality of the lunar motions: he then treats of the different aspects or phases of the moon, and gives a computation of the diameter of the sun and moon, with the magnitude of the sun, moon and earth compared together; he states also the different measures of the distance of the sun and moon, according as they are determined by ancient mathematicians and philosophers.

In the 6th book, he treats of the conjunctions and oppositions of the sun and moon, with tables for computing the mean time when they happen; of the boundaries of solar and lunar eclipses; of the tables and methods of computing the eclipses of the sun and moon, with many other particulars.

In the 7th book, he speaks of the fixed stars; and shows the methods of describing them, in their various constellations, on the surface of an artificial sphere or globe: he rectifies the places of the stars to his own time, and shows how different those places were then, from what they had been in the times of Timocharis, Hipparchus, Aristillus, Calippus, and others: he then lays down a catalogue of the stars in each of the northern constellations, with their latitude, longitude, and magnitudes.

In the 8th book, he gives a like catalogue of the stars in the constellations of the southern hemisphere, and in the 12 signs or constellations of the zodiac. This is the oldest catalogue of the stars now extant, and forms the most valuable part of Ptolemy's works. He then treats of the galaxy, or milky-way; also of the planetary aspects, with the rising and setting of the sun, moon, and stars.

In the 9th book, he treats of the order of the sun, moon, and planets, with the periodical revolutions of the five planets, Mercury, Venus, Mars, Jupiter, and Saturn; he then gives tables of the mean motions, beginning with the theory of Mercury, and showing its various phenomena with respect to the earth.

The 10th book begins with the theory of the planet Venus, as to its greatest distance from the sun; of its epicycle, excentricity, and periodical motions; and then treats of the same particulars in the planet Mars.

In the 11th book he considers the same circumstances in the theory of the planets Jupiter and Saturn. He also corrects all the planetary motions from observations made from the time of Nabonazar to his own.

The 12th book treats of the retrogressive motion of the several planets; giving also tables of their stations, and of the greatest distances of Venus and Mercury from the sun.

The 13th book relates to the several hypotheses of the latitude of the five planets; of the greatest latitude, or inclination of the orbits of the five planets, which are computed and disposed in tables; of the rising and setting of the planets, with tables of them. Then follows a conclusion or summing up of the whole work.

This great work of Ptolemy will always be valuable on account of the observations he gives of the places of the stars and planets in former times, and according to ancient philosophers and astronomers that were then extant; but principally on account of the large and curious catalogue of the stars, which being compared with their places at present, we thence deduce the true quantity of their slow progressive motion according to the order of the signs, or of the precession of the equinoxes.

Another great and important work of Ptolemy was, his Geography, in 7 books; in which, with his usual sagacity, he searches out and marks the situation of places according to their latitudes and longitudes; being the first that did so. Though this work must needs fall far short of perfection, for the want of necessary observations, yet it is of considerable merit, and has been very useful to modern geographers. Cellarius indeed suspects, and he was a very competent judge, that Ptolemy did not use all the care and application which the nature of his work required; and his reason is, that the author delivers himself with the same fluency and appearance of certainty, concerning things and places at the remotest distance, which it was impossible he could know any thing of, that he does concerning those which lay the nearest to him, and fall the most under his cognizance. Salmasius had before made some remarks to the same purpose on this work of Ptolemy. The Greek text of this work was first published by itself at Basil in 1533, in 4to: afterward with a Latin version and notes by Gerard Mercator at Amsterdam, 1605; which last edition was reprinted at the same place, 1618, in folio, with neat geographical tables, by Bertius.

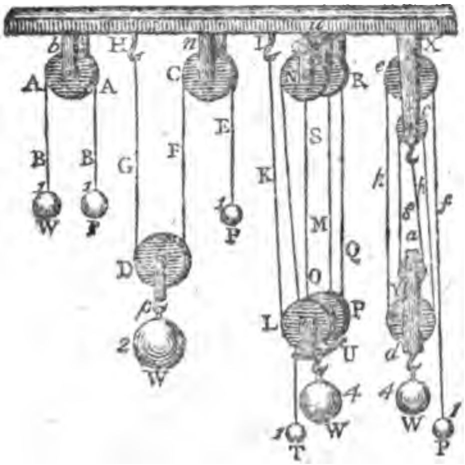
Other works of Ptolemy, though less considerable than these two, are still extant. As, Libri quatuor de Judiciis Astrorum, on the first two books of which Cardan wrote a commentary.—Fructus Librorum suorum; a kind of supplement to the former work.—Recensio Chronologica Regum: this, with another work of Ptolemy, De Hypothesibus Planetarum, was published in 1620, 4to, by John Bainbridge, the Savilian professor of astronomy at Oxford: and Scaliger, Petavius, Dodwell, and the other chronological writers, have made great use of it.—Apparentiæ Stellarum Inerrantium: this was published at Paris by Petavius, with a Latin version, 1630, in folio; but from a mutilated copy, the defects of which have since been supplied from a perfect one, which Sir Henry Saville had communicated to archbishop Usher, by Fabricius, in the 3d volume of his Bibliotheca Græca.—Elementorum Harmonicorum libri tres; published in Greek and Latin, with a commentary by Porphyry the philosopher, by Dr. Wallis at Oxford, 1682, in 4to; and afterwards reprinted there, and inserted in the 3d volume of Wallis's works, 1699, in folio.

Mabillon exhibits, in his German Travels, an effigy of Ptolemy looking at the stars through an optical tube; which effigy, he says, he found in a manuscript of the 13th century, made by Conradus a monk. Hence some have fancied, that the use of the telescope was known to Conradus. But this is only matter of mere conjecture, there being no facts or testimonies, nor even probabilities, to support such an opinion. It is rather likely that the tube was nothing more than a plain open one, employed to strengthen and defend the eye-sight, when looking at particular stars, by excluding adventitious rays from other stars and objects; a contrivance which no observer of the heavens can ever be supposed to have been without.

PULLEY, one of the five mechanical powers; consisting of a little wheel, being a circular piece of wood or metal, turning on an axis, and having a channel around its edge or circumference, in which a cord slides and so raises up weights.

The Latins call it trochlea; and the seamen, when fitted with a rope, a tackle. An assemblage of several pulleys is called a system of pulleys, or polyspaston: some of which are in a block or case, which is fixed; and others in a block which is moveable, and rises with the weight. The wheel or rundle is called the sheave or shiver; the axis on which it turns, the gudgeon; and the fixed piece of wood or iron, into which it is placed, is called the block.

Doctrine of the PULLEY.—1. If the equal weights P and W hang by the cord BB upon the pulley A, whose block *b* is fixed to the beam HI, they will counterpoise each other, just in the same manner as if the cord were cut in the middle, and its two ends hung upon the hooks fixed in the pulley at A and A, equally distant from the centre.



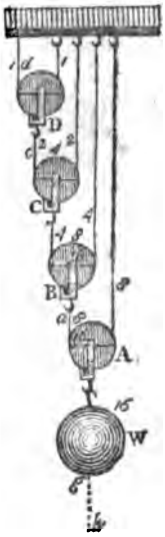
Hence, a single pulley, if the lines of direction of the power and the weight be tangents to the periphery, neither assists nor impedes the power, but only changes its direction. The use of the pulley therefore, is when the vertical direction of a power is to be changed into an horizontal one; or an ascending direction into a descending one; &c. This is found a good provision for the safety of the workmen employed in drawing with the pulley. And this change of direction by means of a pulley has this further advantage; that if any power can exert more force in one direction than another, we are hence enabled to employ it with its greatest effect; as for the convenience of a horse to draw in a horizontal direction, or such like.

But the great use of the pulley is in combining several of them together; thus forming what Vitruvius and others call polyspasta; the advantages of which are, that the machine takes up but little room, is easily removed, and raises a very great weight with a moderate power.

2. When a weight W hangs at the lower end of the moveable block *p* of the pulley D, and the cord GF goes under the pulley, it is evident that the part G of the cord bears one half of the weight W, and the part F the other half of it; for they bear the whole between them; therefore whatever holds the upper end of either rope, sustains one half of the weight; and thus the power P, which draws the cord F by means of the cord E, passing over the fixed pulley C, will sustain the weight W when its intensity is only equal to the half of W; that is, in the case of one moveable pulley, the power gained is as 2 to 1, or as the number of ropes G and F to the one rope E.

In like manner, in the case of two moveable pulleys P and L, each of these also doubles the power, and produces a gain of 4 to 1, or as the number of the ropes Q, M, S, K, sustaining the weight W, to the 1 rope O sustaining the power T; that is, W is to T as 4 to 1. And so on, for any number of moveable pulleys, viz, 3 such pulleys producing an increase of power as 6 to 1; 4 pulleys, as 8 to 1; &c; each pulley adding 2 to the number. Also the effect is the same, when the pulleys are disposed as in the fixed block X, and the other two as in the moveable block Y; these in the lower block giving the same advantage to the power, when they rise all together in one block with the weight.

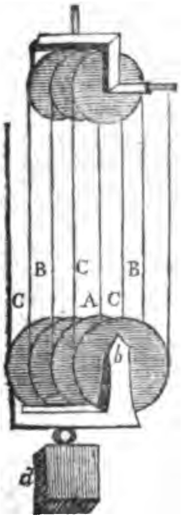
But if the lower pulleys do not rise all together in one block with the weight, but act upon one another, having the weight only fastened to the lowest of them, the force of the power is still more increased, each power doubling the former numbers, the gain of power in this case proceeding in the geometrical progression, 1, 2, 4, 8, 16, &c, according to the powers of 2; whereas in the former case, the gain was only in arithmetical progression, increasing by the addition of 2. Thus, a power whose intensity is equal to 8lb applied at *a* will, by means of the lower pulley A, sustain 16lb; and a power equal to 4lb at *b*, by means of the pulley, will sustain the power of 8lb acting at *a*, and consequently the weight of 16lb at W; also a third power equal to 2lb at *c*, by means of the pulley C, will sustain the power of 4lb at *b*; and a fourth power of 1lb at *d*, by means of the pulley D, will sustain the power 2 at *c*, and consequently the power 4 at B, and the power 8 at A, and the weight 16 at W.

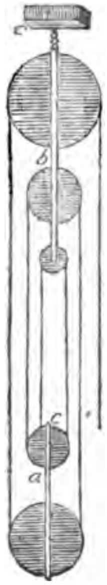


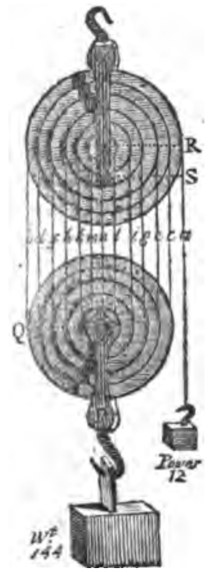
3. It is to be noted however, that, in whatever proportion the power is gained, in that very same proportion is the length of time increased to produce the same effect. For when a power moves a weight by means of several pulleys, the space passed over by the power is to the space passed over by the weight, as the weight is to the power. Hence, the smaller a force is, that sustains a weight by means of pulleys, the slower is the weight raised; so that what is saved or gained in force, is always spent or lost in time: which is the general property of all the mechanical powers.

The usual methods of arranging pulleys in their blocks, may be reduced to two. The first consists in placing them one by the side of another, on the same pin; the other, in placing them directly under each other, on separate pins. Each of these methods however is liable to inconvenience; and Mr. Smeaton, to avoid the impediments to which these combinations are subject, proposes to combine these two methods in one. See the Philos. Trans. vol. 47, pa. 494. Some instances of such combinations of pulleys are exhibited in the following figures; besides which, there are also other varieties of forms.

A very considerable improvement in the construction of pulleys has been made by Mr. James White, who has obtained a patent for his invention, and of which he gives the following description. The last of the three following figures shows the machine, consisting of two pulleys Q and R, one fixed and the other moveable. Each of these has six concentric grooves, capable of having a line put round them, and thus acting like as many different pulleys, having diameters equal to those of the grooves. Supposing then each of the grooves to be a distinct pulley, and that all their diameters were equal, it is evident that if the weight 144 were to be raised by pulling at S till the pulleys touch each other, the first pulley must receive the length of line as many times as there are parts of the line hanging between it and the lower pulley. In the present case, there are 12 lines, *b*, *d*, *f*, &c, hanging between the two pulleys, formed by its revolution about the six upper and lower grooves. Hence as much line must pass over the uppermost pulley as is equal to 12 times the distance of the two. But, from an inspection of the figure, it is evident, that the second pulley cannot receive the full quantity of line by as much as is equal to the distance between it and the first. In like manner, the third pulley receives less than the first by as much as is the distance between the first and third; and so on to the last, which receives only $\frac{1}{12}$th of the whole. For this receives its share of line *n* from a fixed point in the upper frame, which gives it nothing; while all the others in the same frame receive the line partly by turning to meet it, and partly by the line coming to meet them.







Supposing now these pulleys to be equal in size, and to move freely as the line determines them; it appears evident, from the nature of the system, that the number of their revolutions, and consequently their velocities, must be in proportion to the number of suspending parts that are between the fixed point above mentioned, and each pulley, respectively. Thus the outermost pulley would go 12 times round in the time that the pulley under which the part *n* of the line, if equal to it, would revolve only once; and the intermediate times and velocities would be a series of arithmetical proportionals, of which, if the first number were 1, the last would always be equal to the whole number of terms. Since then the revolutions of equal and distinct pulleys are measured by their velocities, and that it is possible to find any proportion of velocity, on a single body running on a centre, viz, by finding proportionate distances from that centre; it follows, that if the diameters of certain grooves in the same substance be exactly adapted to the above series (the line itself being supposed inelastic, and of no magnitude) the necessity of using several pulleys in each frame will be obviated, and with that some of the inconveniencies to which the use of the pulley is liable.

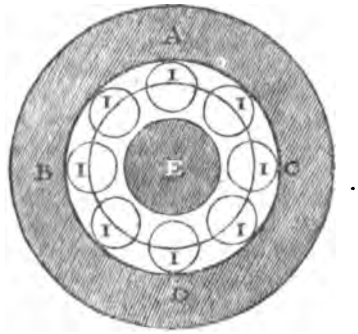
In the figure referred to, the coils of rope by which the weight is supported, are represented by the lines *a*, *b*, *c* &c; *a* is the line of traction, commonly called the fall, which passes over and under the proper grooves, until it is fastened to the upper frame just above *n*. In practice, however, the grooves are not arithmetical proportions, nor can they be so; for the diameter of the rope employed must in all cases be deducted from each term; without which the smaller grooves, to which the said diameter bears a larger proportion than to the larger ones, will tend to rise and fall faster than they, and thus introduce worse defects than those which they were intended to obviate.

The principal advantage of this kind of pulley is, that it destroys lateral friction, and that kind of shaking motion which is so inconvenient in the common pulley. And lest (says Mr. White) this circumstance should give the idea of weakness, I would observe, that to have pins for the pulleys to run on, is not the only nor perhaps the best method; but that I sometimes use centres fixed to the pulleys, and revolving on a very short bearing in the side of the frame, by which strength is increased, and friction very much diminished; for to the last moment the motion of the pulley is perfectly circular: and this very circumstance is the cause of its not wearing out in the centre as soon as it would, assisted by the ever increasing irregularities of a gullied bearing. These pulleys, when well executed, apply to jacks and other machines of that nature with peculiar advantage, both as to the time of going and their own durability; and it is possible to produce a system of pulleys of this kind of six or eight parts only, and adapted to the pockets, which, by means of a skain of sewing silk, or a clue of common thread, will raise upwards of an hundred weight.

As a system of pulleys has no great weight, and lies in a small compass, it is easily carried about, and can be applied for raising weights in a great many cases, where other engines cannot be used. But they are subject to a great deal of friction, on the following accounts; viz, 1st, because the diameters of their axes bear a very considerable proportion to their own diameters; 2d, because in working they are apt to rub against each other, or against the sides of the block; 3dly, because of the stiffness of the rope that goes over and under them. See Ferguson's Mech. pa. 37, 4to.

But the friction of the pulley is now reduced to nothing as it were, by the ingenious Mr. Garnett's patent friction rollers, which produce a great saving of labour and expense, as well as in the wear of the machine, both when applied to pulleys and to the axles of wheel-carriages. His general principle is this: between the axle and nave, or centre pin and box, a hollow space is left, to be filled up by solid equal rollers nearly touching each other. These are furnished with axles inserted into a circular ring at each end, by which their relative distances are preserved; and they are kept parallel by means of wires fastened to the rings between the rollers, and which are rivetted to them.

The above contrivance is exhibited in the annexed figure; where ABCD represents a piece of metal to be inserted into the box or nave, of which E is the centre-pin or axle, and 1, 1, 1, &c, rollers of metal having axes inserted in the brazen circle which passes through their centres; and both circles being rivetted together by means of bolts passing between the rollers from one side of the nave to the other; and thus they are always kept separate and parallel.



PUMP, in Hydraulics, a machine for raising water, and other fluids.—Pumps are probably of very ancient use. Vitruvius ascribes the invention to Ctesebes of Athens, some say of Alexandria, about 120 years before Christ. They are now of various kinds. As the Sucking Pump, the Lifting Pump, the Forcing Pump, Ship Pumps, Chain Pumps, &c. By means of the lifting and forcing pumps, water may be raised to any height, with a sufficient power, and an adequate apparatus: but by the sucking pump, the water being only raised by the general pressure of the atmosphere on the surface of the well, is limited in its ascent to about 33 or 34 feet; though in practice it is seldom applied to the raising it much above 28; because, from the variations observed in the barometer, it appears that the air may sometimes be lighter than 33 feet of water; and whenever that happens, for want of the due counterpoise, this pump will fail in its performance.

The Common Sucking PUMP.—This consists of a pipe, of wood or metal, open at both ends, having a fixed valve in the lower part of it opening upwards, and a moveable valve or bucket by which the water is drawn or lifted up. This bucket is just the size of the bore of the pump-pipe, in that part where it works, and leathered round so as to fit it very close, that no air may pass by the sides of it; the valve hole being in the middle of the bucket. The bucket is commonly worked in the upper part of the barrel by a short rod, and another fixed valve placed just below the descent of the bucket. Thus, (fig. 1, pl. 28), AB is the pump-pipe, C the lower fixed valve, opening upwards, and D is the bucket, or moving valve, also opening upwards.

In working the pump; draw up the bucket D, by means of the pump rod, having any kind of a handle fixed to it: this draws up the water that is above it, or if not, the air; in either case the water pushes up the valve C, and enters to supply the void left between C and D, being forced up by the pressure of the atmosphere on the surface of the water in the well below. Next, the bucket D is pushed down, which shuts the valve C, and prevents the return of the water downwards, which opens the valve D, by which the water ascends above it. And thus, by repeating the strokes of the pump-rod handle, the valves alternately open and shut, and the water is drawn up at every stroke, and runs out at the nozle or spout near the top.

The Lifting PUMP differs from the sucking pump only in the disposition of its valves and the form of its piston frame. This kind of pump is represented in fig. 2, pl. 28; where the lower valve D is moveable, being worked up and down with the pump rod, which lifts the water up, and so opens the upper valve C, which is fixed, and permits the water to issue through it, and run out at top. Then as the piston D descends, the weight of the water above C shuts that valve C, and so prevents its return, till that valve be opened again by another lift of the piston D. And so on alternately.

The Forcing PUMP raises the water through the sucker, or lower valve C (fig. 3, pl. 28), in the same manner as the sucking pump; but as the piston or plunger D has no valve in it, the water cannot get above it when this is pushed down again; instead of which, a side pipe is inserted between C and D, having a fixed valve at E opening upwards, through which the water is forced out of the pump by pushing down the plunger D.

To the forcing pump is sometimes adapted an air vessel, in which the air being compressed by the water, by its elasticity acts upon the water again, and forces it out to a great distance, and in a continued stream, instead of by jets or jerks. So, Newsham's engine, for extinguishing fires, consists of two forcing pumps, which alternately drive water into a close vessel of air, by which means the air in it is condensed, and compresses the water so strongly, that it rushes out with great impetuosity and force through a pipe that comes down into it, making a continued uniform stream.—By means of forcing pumps, water may be raised to any height whatever above the level of a river or spring; and machines may be contrived to work these pumps, either by a running stream, a fall of water, or by horses.

Observations on Pumps.—The force required to work a pump, is equal to the weight of water raised at each stroke, or equal to the weight of water filling the cavity of the pipe, and its height equal to the length of the stroke made by the piston. Hence if d denote the diameter of the pipe, and l the length of the stroke, both in inches; then is $\cdot 7854d^2l$ the content of the water raised at a stroke, in inches, or $\cdot 0028d^2l$ in ale gallons; and the weight of it is $\frac{d^2l}{220}$ ounces or $\frac{d^2l}{3520}$ lb. But if the handle of the pump be a lever which gains in the power of p to 1, the force of the hand to work the pump will be only $\frac{d^2l}{3520p}$ lb, or, when p is 5 for instance, it will be $\frac{d^2l}{17600}$ lb. And all these over and above the friction of the moving parts of the pump.

Ctesebes's PUMP, acts both by suction and by pression. Thus, a brass cylinder ABCD (fig. 5, pl. 28), furnished with a valve at L, is placed in the water. In this is fitted the piston KM, made of green wood, which will not swell in the water, which is adjusted to the aperture of the cylinder with a covering of leather, but without any valve. Another tube NH is fitted on at H, with a valve I opening upwards.—Now the piston being raised, the water opens the valve L, and rises into the cavity of the cylinder.—When the piston is depressed again, the valve I is opened, and the water is driven up through the tube HN. This was the pump used among the ancients, and that from which the others have been deduced. Sir Samuel Morland has endeavoured to increase its force by lessening the friction; which he has done in a great degree. There are various kinds of pumps used in ships, for throwing the water out of the hold, and on other occasions, as the chain pump, &c.

A TABLE by which the Quantity and Weight of Water in a Cylindrical Bore of any given Diameter and Perpendicular Height, may be found; and consequently, the Degree of Power that will be requisite to work any Hydraulic Engine. By JAMES FERGUSON, F. R. S.

Feet High.	Diameter of the Cylindrical Bore one Inch.		
	Quantity of Water in Cubic Inches.	Weight of Water in Troy Ounces.	In Avoirdupois Ounces.
1	9·4247781	4·9712340	5·4541539
2	18·8495562	9·9424680	10·9083078
3	28·2743343	14·9137020	16·3624617
4	37·6991124	19·8849360	21·8166156
5	47·1238905	24·8561700	27·2707695
6	56·5486686	29·8274040	32·7249234
7	65·9734467	34·7986380	38·1790773
8	75·3982248	39·7698720	43·6332312
9	84·8230029	44·7411060	49·0873851

The numbers to the right hand from the points in each column are decimals.

For tens of feet high, remove the decimal points one place forward; for hundreds of feet, two places; for thousands of feet, three places; and so on.

Then, multiply the sums by the square of the diameter of the given bore; and the products will be the quantity of water in the pipe, in cubic inches, and in troy and avoirdupois ounces.

EXAMPLE.—Qu. *The Quantity and Weight of Water in an upright Pipe whose Bore is* 10 *Inches in Diameter, and its Height* 208 *Feet?*—The Square of 10 is 100.

Feet high.	Cubic Inches.	Troy Ounces.	Avoird. Ounces.
200	1884·95562	994·24680	1090·83078
8	75·39822	39·76897	43·63323
208	1960·35384	1034·01577	1134·46401
Multiply by	100	100	100
Ans.	196035·384	103401·577	113446·401

Which number of cubic inches being divided by 231 (the number of cubic inches in a wine gallon) gives $848\frac{6}{10}$ for the number of gallons of water in the pipe; and the respective weights, 103401·577 and 113446·401, being divided by 12 and by 16, give $8616\frac{8}{10}$ for the number of troy pounds, and $7090\frac{4}{10}$ for the number of avoirdupois pounds of water. The power of an engine equal to the weight will just balance the water; but the engine must have as much more power as will be sufficient to overcome the friction of its working parts.—In pumps, it matters not what the diameter of any part of the bore be, besides that part in which the piston or bucket works; for, the power requisite to work them will be the same as if the whole bore was of that diameter throughout.

Air-PUMP, in Pneumatics, is a machine, by means of which the air is emptied out of vessels, and a kind of vacuum produced in them. For the particulars of which, see AIR-*Pump*.

PUNCHEON, a measure for liquids, containing $\frac{1}{3}$ of a tun, or a hogshead and $\frac{1}{3}$, or 84 gallons.

PUNCHINS, or PUNCHIONS, in Building, short pieces of timber placed to support some considerable weight.

PUNCTATED *Hyperbola*, in the higher geometry, an hyperbola, whose conjugate oval is infinitely small, that is, a point.

PUNCTUM *ex Comparatione*, is either focus, in the ellipse or hyperbola; so called by Apollonius, because the rectangle under two abscisses made at the focus, is equal to one-fourth part of what he calls the figure, which is the square of the conjugate axis, or the rectangle under the transverse and the parameter.

PUNCTUM *Duplex*, double point, in the higher geometry, a point where two branches of a curve intersect. See CURVE, LEMNISCATE, &c.

PURBACH (GEORGE), a very eminent mathematician and astronomer, was born at Purbach, a town upon the confines of Bavaria and Austria, in 1423, and educated at Vienna. He afterwards visited the most celebrated universities in Germany, France, and Italy; and found a particular friend and patron in Cardinal Cusa at Rome. Returning to Vienna, he was appointed mathematical professor, in which office he continued till his death, which happened in 1461, in the 39th year of his age only, to the great loss of the learned world.

Purbach composed a great number of pieces, on mathematical and astronomical subjects; and his fame brought many students to Vienna, and among them, the celebrated Regiomontanus, between whom and Purbach there subsisted the strictest friendship and union of studies till the death of the latter. These two celebrated mathematicians laboured together to improve every branch of learning, by all the means in their power, though astronomy seems to have been the favourite of both; and had not the immature death of Purbach prevented his further pursuits, there is no doubt but that, by their joint industry, astronomy would have been cultivated to a very great degree. That this is not merely surmise, may be learnt from those improvements which Purbach actually did make, to render the study of it more easy and practicable. His first essay was, to amend the Latin translation of Ptolemy's Almagest, which had been made from the Arabic version; which he did, not by the help of the Greek text, for he was unacquainted with that language, but by drawing the most probable conjectures from a strict attention to the sense of the author.

He then proceeded to other works, and among them, he wrote a tract, which he entitled, An Introduction to Arithmetic; then a treatise on Gnomonics, or Dialling, with tables suited to the difference of climates or latitudes; also a small tract concerning the Altitudes of the Sun, with a table; also, Astrolabic Canons, with a table of the parallels, proportioned to every degree of the equinoctial.

After this, he constructed Solid Spheres, or Celestial Globes, and composed a new table of fixed stars, adding the longitude by which every star, since the time of Ptolemy, had increased. He also invented various other instruments, among which was the Gnomon, or Geometrical Square, with canons and a table for the use of it.

He not only collected the various tables of the Primum Mobile, but added new ones. He made very great improvements in Trigonometry, and by introducing the table of Sines, by a decimal division of the radius, he quite changed the appearance of that science: he supposed the radius to be divided into 600,000 equal parts, and computed the sines of the arcs, for every ten minutes, in such equal parts of the radius, by the decimal notation, instead of the duodecimal one delivered by the Greeks, and preserved even by the Arabians till our author's time; a project which was completed by his friend Regiomontanus, who computed the sines to every minute of the quadrant, in 1,000,000th parts of the radius.

Having prepared the tables of the fixed stars, he next undertook to reform those of the planets, and constructed some entirely new ones. Having finished his tables, he wrote a kind of Perpetual Almanac, but chiefly for the moon, answering to the periods of Meton and Calippus; also an Almanac for the Planets, or, as Regiomontanus afterwards called it, an Ephemeris, for many years. But observing that there were some planets in the heavens at a great distance from the places where they were described to be in the tables, particularly the sun and moon (the eclipses of which were observed frequently to happen very different from the times predicted), he applied himself to construct new tables, particularly adapted to eclipses; which were long after famous for their exactness. To the same time may be referred his finishing that celebrated work, entitled, A New Theory of the Planets, which Regiomontanus afterwards published the first of all the works executed at his new printing-house.

PURE *Hyperbola*, is an Hyperbola without any oval, node, cusp, or conjugate point; which happens through the impossibility of two of its roots.

Pure *Mathematics*, *Proposition*, *Quadratics*, &c. See the several articles.

PURLINES, in Architecture, those pieces of timber that lie across the rafters on the inside, to keep them from sinking in the middle of their length.

PYRAMID, a solid having any plane figure for its base, and its sides triangles whose vertices all meet in a point at the top, called the vertex of the pyramid; the base of each triangle being the sides of the plane base of the pyramid.—The number of triangles is equal to the number of the sides of the base; and a cone is a round pyramid, or one having an infinite number of sides.—The pyramid is also denominated according to the figure of its base, being triangular when the base is a triangle, quadrangular when a quadrangle, &c.

The *axis* of the pyramid, is the line drawn from the vertex to the centre of the base; and when this axis is perpendicular to the base, the pyramid is said to be a right one; otherwise it is oblique.

1. A pyramid may be conceived to be generated by a line moved about the vertex, and so carried round the perimeter of the base.

2. All pyramids having equal bases and altitudes, are equal to one another: whatever may be the figures of their bases.

3. Every pyramid is equal to one-third of the circumscribed prism, or a prism of the same base and altitude; and therefore the solid content of the pyramid is found by multiplying the base by the perpendicular altitude, and taking $\frac{1}{3}$ of the product.

4. The upright surface of a pyramid, is found by adding together the areas of all the triangles which form that surface.

5. If a pyramid be cut by a plane parallel to the base, the section will be a plane figure similar to the base; and these two figures will be in proportion to each other as the squares of their distances from the vertex of the pyramid.

6. The centre of gravity of a pyramid is distant from the vertex $\frac{3}{4}$ of the axis.

Frustum of a Pyramid, is the part left at the bottom when the top is cut off by a plane parallel to the base.

The solid content of the frustum of a pyramid is found, by first adding into one sum the areas of the two ends and the mean proportional between them, the 3d part of which sum is a medium section, or it is the base of an equal prism of the same altitude; and therefore this medium area or section multiplied by the altitude gives the solid content. So, if A denote the area of one end, a the area of the other end, and h the height; then $\frac{1}{3}(A + a + \sqrt{Aa})$ is the medium area or section: and $\frac{1}{3}(A + a + \sqrt{Aa}) \times h$ is the solid content.

Pyramids *of Egypt*, are very numerous; but the most remarkable are the three pyramids of Memphis, or, as they are now called, of Gheisa or Gize. These are square pyramids, and the greatest of them measures 700 feet on each side of the base, and the oblique height or slant side measures the same; and its base covers, or stands upon, nearly 11 acres of ground. It is thought by some that these pyramids were designed and used as gnomons, for astronomical purposes; and it is remarkable that their four sides are accurately in the direction of the four cardinal points of the compass, east, west, north, and south.

PYRAMIDAL *Numbers*, are the sums of polygonal numbers, collected after the same manner as the polygonal numbers themselves are found from arithmetical progressions. These are particularly called First pyramidals. The sums of first pyramidals are called second pyramidals; and the sums of the 2d are 3d pyramidals; and so on. Particularly, those arising from triangular numbers, are called Prime Triangular Pyramidals; those arising from pentagonal numbers, are called Prime Pentagonal Pyramidals; and so on.

The numbers 1, 4, 10, 20, 35, &c,
formed by adding the triangulars } 1, 3, 6, 10, 15, &c,
are usually called simply by the name of pyramidals; and the general formula for finding them is $n \times \frac{n-1}{2} \times \frac{n-2}{3}$; so the 4th pyramidal is found by substituting 4 for n; the 5th by substituting 5 for n; &c. See Figurate *Numbers*, and Polygonal *Numbers*.

PYRAMIDOID, is sometimes used for the parabolic spindle, or the solid formed by the rotation of a semiparabola about its base or greatest ordinate. See Parabolic *Spindle*.

PYROMETER, or fire-measurer, a machine for measuring the expansion of solid bodies by heat. Musschenbroek was the first inventor of this instrument; though it has since received several improvements by other philosophers. He has given a table of the expansions of the different metals, with various degrees of heat. Having prepared cylindric rods of iron, steel, copper, brass, tin, and lead, he exposed them first to a pyrometer with one flame in the middle; then with two flames; then successively with three, four, and five flames. The effects were as in the following Table, where the degrees of expansion are marked in parts equal to the 12500th part of an inch.

Expansion of	Iron	Steel	Copp.	Brass	Tin	Lead
By 1 flame	80	85	89	110	153	155
By 2 flames placed close together	117	123	115	220		274
By 2 flames at 2½ inches distant	109	94	92	141	219	263
By 3 flames close together	142	168	193	275		
By 4 flames close together	211	270	270	361		
By 5 flames	230	310	310	377		

Tin easily melts when heated by two flames placed close together; and lead with three flames close together, when they burn long.

It hence appears that the expansion of any metal is in a less degree than the number of flames: so two flames give less than a double expansion, three flames less than a triple expansion, and so on, always more and more below the ratio of the number of flames. And the flames placed together cause a greater expansion, than with an interval between them.

For the construction of Musschenbroek's pyrometer, with alterations and improvements upon it by Desaguliers, see Desag. Exper. Philos. vol. 1, pa. 421; see also Musschenbroek's translation of the Experiments of the Academy del Cimento, printed at Leyden in 1731; and for a pyrometer of a new construction, by which the expansions of metals in boiling fluids may be examined and compared with Fahrenheit's thermometer, see Mussch. Introd. Philos. Nat. 4to, 1762, vol. 2, pa. 610.

But as it has been observed, that Musschenbroek's pyrometer was liable to some objections, these have been removed in a great measure by Ellicott, who has given a description of his improved pyrometer in the Philos. Trans. numb. 443. This instrument measures the expansions to the 7200th part of an inch; and by means of it, Mr. Ellicott found, on a medium, that the expansions of bars of different metals, as nearly of the same dimensions as possible, by the same degree of heat, were as below:

Gold	Silver	Brass	Copper	Iron	Steel	Lead
73	103	95	89	60	56	149

The great difference between the expansions of iron and brass, has been applied with good success to remove the irregularities in pendulums arising from heat. Philos. Trans. vol. 47, pa. 485.

Mr. Graham used to measure the minute expansions of metal bars, by advancing the point of a micrometer screw, till it sensibly stopped against the end of the bar to be measured. This screw, being small and very lightly hung, was capable of agreement within the 3000 or 4000th part of an inch. And on this general principle Mr. Smeaton contrived his pyrometer, in which the measures are determined by the contact of a piece of metal with the point of a micrometer-screw. This instrument makes the expansions sensible to the 2345th part of an inch. And when it is used, both the instrument and the bar to be measured are immerged in a cistern of water, heated to any degree, up to boiling, by means of lamps placed under the cistern; and the water communicates the same degree of heat to the instrument and bar, and to a mercurial thermometer immerged in it, for ascertaining that degree.

With this pyrometer Mr. Smeaton made several experiments, which are arranged in a table; and he remarks, that their result agrees very well with the proportions of expansions of several metals given by Mr. Ellicott. The following table shows how much a foot in length of each metal expands by an increase of heat corresponding to 180° of Fahrenheit's thermometer, or to the difference between the temperatures of freezing and boiling water, expressed in the 10000th part of an inch.

1.	White glass barometer tube	100
2.	Martial regulus of antimony	130
3.	Blistered steel	138
4.	Hard steel	147
5.	Iron	151
6.	Bismuth	167
7.	Copper, hammered	204
8.	Copper 8 parts, mixed with 1 part tin	218
9.	Cast brass	225
10.	Brass 16 parts, with 1 of tin	229
11.	Brass wire	232
12.	Speculum metal	232
13.	Spelter solder, viz 2 parts brass and 1 zinc	247
14.	Fine pewter	274
15.	Grain tin	298
16.	Soft solder, viz lead 2 and tin 1	301
17.	Zinc 8 parts, with tin 1, a little hammered	323
18.	Lead	344
19.	Zinc or spelter	353
20.	Zinc hammered half an inch per foot	373

For a further account of this instrument, with its use, see Philos. Trans. vol. 48, pa. 598.

Mr. Ferguson has constructed, and described a pyrometer (Lect. on Mechanics, Suppl. pa. 7, 4to), which makes the expansion of metals by heat visible to the 45000th part of an inch. And another plan of a pyrometer has lately been invented by M. Deluc, in consequence of a hint suggested to him by Mr. Ramsden: for an account of which, with the principle of its construction and use, both in the comparative measure of the expansions of bodies by heat, and the measure of their absolute expansion, as well as the experiments made with it, see M. Deluc's elaborate essay on pyrometry &c, in the Philos. Trans. vol. 68, pa. 419 &c.

Other accurate and ingenious contrivances, for the measuring of expansions by heat, have been made by Mr. Ramsden; which he has successfully applied in the case of the measuring rods and chains lately employed, by General Roy and Col. Williams, in measuring the base on Hounslow Heath, &c; which determine the expansions, to great minuteness, for each degree of the thermometer. See Philos. Trans. 1785, &c.

PYROPHORUS, the name usually given to that substance by some called black phosphorus; being a chemical preparation possessing the singular property of kindling spontaneously when exposed to the air; which was accidentally discovered by M. Homberg, who prepared it of alum and human fæces. Though it has since been found, by the son of M. Lemeri, that the fæces are not necessary to it, but that honey, sugar, flour, and any animal or vegetable matter, may be used instead of the fæces; and M. De Suvigny has shown that most vitriolic salts may be substituted for the alum. See Priestley's Obser. on Air, vol. 3, Append. pa. 386, and vol. 4, Append. pa. 479.

PYROTECHNY, the art of fire, or the science which teaches the application and management of fire in several operations. Pyrotechny is of two kinds, military and chemical.

Military PYROTECHNY, is the science of artificial fire-works, and fire-arms, teaching the structure and use both of those employed in war, as gunpowder, cannon, shells, carcasses, mines, fusees, &c; and of those made for amusement, as rockets, stars, serpents, &c.—Some call pyrotechny by the name artillery; though that word is usually confined to the instruments employed in war. Others choose to call it pyrobology, or rather pyroballogy, or the art of missile fires.—Wolfius has reduced pyrotechny into a kind of mixed mathematical art. Indeed it will not allow of geometrical demonstrations; but he brings it to tolerable rules and reasons; whereas it had formerly been treated by authors at random, and without regard to any reasons at all. See the several articles CANNON, GUNPOWDER, ROCKET, SHELL, &c.

Chemical PYROTECHNY, is the art of managing and applying fire in distillations, calcinations, and other operations of chemistry. Some reckon a third kind of pyrotechny, viz, the art of fusing, refining, and preparing metals.

PYTHAGORAS, one of the most celebrated philosophers of antiquity, was born about the 47th Olympiad, or 590 years before Christ. His father's principal residence was at Samos, but being a travelling merchant, his son Pythagoras was born at Sidon in Syria; but soon returning home again, our philosopher was brought up at Samos, where he was educated in a manner that was answerable to the great hopes that were conceived of him. He was called "the youth with a fine head of hair;" and from the great qualities that soon appeared in him, he was regarded as a good genius sent into the world for the benefit of mankind.

Samos however afforded no philosophers capable of satisfying his thirst for knowledge; and therefore, at 18 years of age, he resolved to travel in quest of them elsewhere. The fame of Pherecydes drew him first to the island of Syros: from hence he went to Miletus, where he conversed with Thales. He then travelled to Phœnicia, and stayed some time at Sidon, the place of his birth; and from hence he passed into Egypt, where Thales and Solon had been before him.

Having spent 25 years in Egypt, to acquire all the learning and knowledge he could procure in that country, he travelled with the same view through Chaldea, and visited Babylon and India. Returning after some time, he went to Crete; and from hence to Sparta, to be instructed in the laws of Minos and Lycurgus. He then returned again to Samos; but finding it under the tyranny of Polycrates, he quitted it again, and visited the several countries of Greece: passing through Peloponnesus, he stopped at Phlius, where Leo then reigned, who was much surprised with his eloquence and wisdom.

From Peloponnesus he went into Italy, and passed some time at Heraclea, and at Tarentum, but made his chief residence at Croton; where, after reforming the manners of the citizens by preaching, and establishing the city by wise and prudent counsels, he opened a school to display the treasures of wisdom and learning he possessed. It is not to be wondered, that he was soon attended by a crowd of disciples, who repaired to him from different parts of Greece and Italy.

He gave his scholars the rules of the Egyptian priests, and made them pass through the austerities which he himself had endured. He at first enjoined them a five years silence in the school, during which they were only to hear; after which, leave was given them to start questions, and to propose doubts, under the caution however, to say, "not a little in many words, but much in a few." Having gone through their probation, they were obliged, before they were admitted, to bring all their fortune into the common stock, which was managed by persons chosen on purpose, and called œconomists, and the whole community had all things in common.

The desire of concealing their mysteries induced the Egyptians to make use of three kinds of styles, or ways of expressing their thoughts; the simple, the hieroglyphical, and the symbolical. In the simple, they spoke plainly and intelligibly, as in common conversation; in the hieroglyphical, they concealed their thoughts under certain images and characters; and in the symbolical, they explained them by short expressions, which, under a sense plain and simple, included another wholly figurative. Pythagoras borrowed these three different ways from the Egyptians, in all the instructions he gave; but chiefly imitated the symbolical style, which he thought very proper to inculcate the greatest and most important truths: for a symbol, by its double sense, the proper and the figurative, teaches two things at once; and nothing pleases the mind more, than the double image it represents to our view.

In this manner Pythagoras delivered many excellent things concerning God and the human soul, and a great variety of precepts, relating to the conduct of life, both political and civil; he made also some considerable discoveries and advances in the arts and sciences. Thus, among the works ascribed to him, there are not only books of physic, and books of morality, like that contained in what are called his Golden Verses, but treatises on politics and theology. All these works are lost: but the vastness of his mind appears from the wonderful things he performed. He delivered, as antiquity relates, several cities of Italy and Sicily from the yoke of slavery; he appeased seditions in others; and he softened the manners, and brought to temper the most savage and unruly spirits, of several people and tyrants. Phalaris, the tyrant of Sicily, it is said, was the only one who could withstand the remonstrances of Pythagoras; and he it seems was so enraged at his discourses, that he ordered him to be put to death. But though the lectures of the philosopher could make no impression on the tyrant, yet they were sufficient to reanimate the Sicilians, and to put them upon a bold action. In short, Phalaris was killed the same day that he had fixed for the death of the philosopher.

Pythagoras had a great veneration for marriage; and therefore himself married at Croton a daughter of one of the chief men of that city, by whom he had two sons and a daughter: one of the sons succeeded his father in the school, and became the master of Empedocles: the daughter, named Damo, was distinguished both by her learning and her virtues, and wrote an excellent commentary upon Homer. It is also related, that Pythagoras had given her some of his writings, with express commands not to impart them to any but those of his own family; to which Damo was so scrupulously obedient, that even when she was reduced to extreme poverty, she refused a great sum of money for them.

From the country in which Pythagoras thus settled and gave his instructions, his society of disciples was called the Italic sect of philosophers, and their reputation continued for some ages afterwards, when the Academy and the Lycæum united to obscure and swallow up the Italic sect. Pythagoras's disciples regarded the words of their master as the oracles of a god; his authority alone, though unsupported by reason, passed with them for reason itself: they looked on him as the most perfect image of God among men. His house was called the temple of Ceres, and his court-yard the temple of the Muses: and when he went into towns, it was said he went thither, "not to teach men, but to heal them."

Pythagoras however was persecuted by bad men in the latter years of his life; and some say he was killed in a tumult raised by them against him; but according to others, he died a natural death, at 90 years of age, about 497 years before Christ.

Besides the high respect and veneration the world has always had for Pythagoras, on account of the excellence of his wisdom, his morality, his theology, and politics, he was renowned as learned in all the sciences, and a considerable inventor of many things in them; as arithmetic, geometry, astronomy, music, &c. In arithmetic, the common multiplication table is, to this day, still called Pythagoras's table. In geometry, it is said he invented many theorems, particularly these three; 1st, Only three polygons, or regular plane figures, can fill up the space about a point, viz, the equilateral triangle, the square, and the hexagon: 2d, The sum of the three angles of every triangle, is equal to two right angles: 3d, In any right-angled triangle, the square on the longest side is equal to both the squares on the two shorter sides: for the discovery of this last theorem, some authors say he offered to the gods a hecatomb, or a sacrifice of a hundred oxen; Plutarch however says it was only one ox, and even that is questioned by Cicero, as inconsistent with his doctrine, which forbade bloody sacrifices: the more accurate therefore say, he sacrificed an ox made of flour, or of clay; and Plutarch even doubts whether such sacrifice, whatever it was, was made for the said theorem, or for the area of the parabola, which it was said Pythagoras also found out.

In astronomy his inventions were many, and great. It is said he discovered, or maintained the true system of the world, which places the sun in the centre, and makes all the planes revolve about him; and from him it is to this day called the old or Pythagorean system; and is the same as that revived by Copernicus. He first discovered, that Lucifer and Hesperus were but one and the same, being the planet Venus, though formerly thought to be two different stars. The invention of the obliquity of the zodiac is likewise ascribed to him. He first gave to the world the name Κοσμος, *Kosmos*, from the order and beauty of all things comprehended in it; asserting that it was made according to musical proportion: for, as he held that the sun, by him and his followers termed the fiery globe of unity, was seated in the midst of the universe, and the earth and planets moving around him, so he held that the seven planets had an harmonious motion, and their distances from the sun corresponded to the musical intervals or divisions of the monochord.

Pythagoras and his followers held the transmigration of souls, making them successively occupy one body after another: on which account they abstained from flesh, and lived chiefly on vegetables. This he probably learnt in India.

PYTHAGORAS'S *Table*, the same as the multiplication-table; which see.

PYTHAGOREAN, or PYTHAGORIC, *System*, among the ancients, was the same as the Copernican system among the moderns. In this system, the sun is supposed at rest in the centre, with the earth and all the planets revolving about him, each in their respective orbits. See SYSTEM.

It was so called, as having been maintained and cultivated by Pythagoras, and his followers; not that it was invented by him, for it was much older.

PYTHAGOREAN *Theorem*, is that in the 47th proposition of the first book of Euclid's Elements; viz, that in a right-angled triangle, the square of the longest side is equal to the sum of both the squares of the two shorter sides. It has been said that Pythagoras offered a hecatomb, or sacrifice of 100 oxen, to the gods, for inspiring him with the discovery of so remarkable a property.

PYTHAGOREANS, a sect of ancient philosophers, who followed the doctrines of Pythagoras. They were called the Italic sect, from the circumstance of his having settled in Italy. Out of his school proceeded the greatest philosophers and legislators, Zaleucus, Charondas, Archytas, &c. See the article PYTHAGORAS.

PYXIS *Nautica*, the seaman's compass.

Q.

QUADRAGESIMA, a denomination given to the time of lent, from its consisting of about 40 days; commencing on ash-wednesday.

QUADRAGESIMA *Sunday*, is the first sunday in lent, or the first sunday after ash-wednesday.

QUADRANGLE, or QUADRANGULAR *figure*, in Geometry, is a plane figure having four angles; and consequently four sides also.—To the class of quadrangles belong the square, parallelogram, trapezium, rhombus, and rhomboides.—A square is a regular quadrangle; a trapezium an irregular one.

QUADRANT, in Geometry, is either the quarter or 4th part of a circle, or the 4th part of its circumference; the arch of which therefore contains 90 degrees.

QUADRANT also denotes a mathematical instrument of great use in astronomy and navigation, for taking the altitudes of the sun and stars, as also taking angles in surveying, heights-and-distances, &c.—This instrument is variously contrived, and furnished with different apparatus, according to the various uses it is intended for; but they have all this in common, that they consist of the quarter of a circle, whose limb or arch is divided into 90° &c. Some have a plummet suspended from the centre, and are furnished either with plain sights, or a telescope, to look through.

The principal and most useful quadrants, are the common Surveying quadrant, the Astronomical quadrant, Adams's quadrant, Cole's quadrant, Collins's or Sutton's quadrant, Davis's quadrant, Gunter's quadrant, Hadley's quadrant, the Horodictical quadrant, and the Sinical quadrant, &c. Of these, the two most deserving of notice, are Hadley's quadrant, and the mural or astronomical quadrant.

1. *The Common, or Surveying* QUADRANT, is the instrument the use of which may be seen in my Mensuration, in the section on heights-and-distances.

2. *The Astronomical* QUADRANT, is a large one, usually made of brass or iron bars; having its limb EF (fig.3 pl.29) accurately divided, either diagonally or otherwise, into degrees, minutes, and seconds, if room will permit, and furnished either with two pair of plain sights or two telescopes, one on the side of the quadrant at AB, and the other, CD, moveable about the centre by means of the screw G. The dented wheels I and H serve to direct the instrument to any object or phenomenon.—The application of this useful instrument, in taking observations of the sun, planets, and fixed stars, is obvious; for being turned horizontally on its axis, by means of the telescope AB, till the object is seen through the moveable telescope, then the degrees &c cut by the index, give the altitude &c required.

3. *Cole's* QUADRANT, is a very useful instrument, invented by Mr. Benjamin Cole. It consists of six parts, viz, the staff AB (fig.11, pl.29); the quadrantal arch DE; three vanes, A, B, C; and the vernier FG. The staff is a bar of wood about 2 feet long, an inch and a quarter broad, and of a sufficient thickness to prevent it from bending or warping. The quadrantal arch is also of wood; and is divided into degrees and 3d parts of degrees, to a radius of about 9 inches; and to its extremities are fitted two radii, which meet in the centre of the quadrant by a pin, about which it easily moves. The sight-vane A is a thin piece of brass, near 2 inches in height, and one broad, set perpendicularly on the end of the staff A, by means of two screws passing through its foot. In the middle of this vane is drilled a small hole, through which the coincidence or meeting of the horizon and solar spot is to be viewed. The horizon-vane B is about an inch broad, and two inches and a half high, having a slit cut through it of near an inch long, and a quarter of an inch broad; this vane is fixed in the centre pin of the instrument, in a perpendicular position, by means of two screws passing through its foot, by which its position with respect to the sight-vane is always the same, their angle of inclination being equal to 45 degrees. The shade-vane C is composed of two brass plates; one of which serves as an arm, and is about 4½ inches long, and ¾ of an inch broad, being pinned at one end to the upper limb of the quadrant by a screw, about which it has a small motion; the other end lies in the arch, and the lower edge of the arm is directed to the middle of the centre-pin: the other plate, which is properly the vane, is about 2 inches long, being fixed perpendicularly to the other plate, at about half an inch distance from that end next the arch; this vane may be used either by its shade, or by the solar spot cast by a convex lens placed in it. And because the wood-work is often subject to warp or twist, therefore this vane may be rectified by means of a screw, so that the warping of the instrument may occasion no error in the observation, which is performed in the following manner: Set the line G on the vernier against a degree on the upper limb of the quadrant, and turn the screw on the backside of the limb forward or backward, till the hole in the sight-vane, the centre of the glass, and the sunk spot in the horizon-vane, lie in a right line.

To find the Sun's Altitude by this instrument. Turn your back to the sun, holding the staff of the instrument with the right hand, so that it be in a vertical plane passing through the sun; apply one eye to the sight-vane, looking through that and the horizon-vane till the horizon be seen; with the left hand slide the quadrantal arch upwards, till the solar spot, or shade, cast by the shade-vane, fall directly upon the spot or slit in the horizon-vane; then will that part of the quadrantal arch, which is raised above G or S (according as the observation respects either the solar spot or shade) show the altitude of the sun at that time. But for the meridian altitude, the observation must be continued, and as the sun approaches the meridian, the sea will appear through the horizon-vane, which completes the observation; and the degrees and minutes, counted as before, will give the sun's meridian altitude: or the degrees counted from the lower limb upwards will give the zenith distance.

4. *Adams's* QUADRANT, differs only from Cole's, just described, in having an horizontal vane, with the upper part of the limb lengthened; so that the glass, which casts the solar spot on the horizon-vane, is at the same distance from the horizon-vane as the sight-vane at the end of the index.

5. *Collins's* or *Sutton's* QUADRANT, is a stereographic projection of one quarter of the sphere between the tropics, on the plane of the ecliptic, the eye being in its north pole; and fitted to the latitude of London. The lines running from right to left, are parallels of altitude; and those crossing them are azimuths. The smaller of the two circles, bounding the projection, is one quarter of the tropic of Capricorn; and the greater is a quarter of the tropic of Cancer. The two ecliptics are drawn from a point on the left edge of the quadrant, with the characters of the signs upon them; and the two horizons are drawn from the same point. The limb is divided both into degrees and time; and by having the sun's altitude, the hour of the day may here be found to a minute. The quadrantal arches next the centre contain the calendar of months; and under them, in another arch, is the sun's declination. On the projection are placed several of the most remarkable fixed stars between the tropics; and the next below the projection is the quadrant and line of shadows.

6. *Davis's* QUADRANT, the same as the BACKSTAFF; which see.

7. *Gunner's* QUADRANT, (fig. 6, pl. 29), sometimes called the *Gunner's Square*, is used for elevating and pointing cannon, mortars, &c, and consists of two branches either of wood or brass, between which is a quadrantal arch divided into 90°, and furnished with a thread and plummet.—The use of this instrument is very easy; for if the longer branch, or bar, be placed in the mouth of the piece and it be elevated till the plummet cut the degree necessary to hit a proposed object, the thing is done.—Sometimes on the sides of the longer bar,

are noted the division of diameters and weights of iron balls, as also the bores of pieces.

8. *Gunter's* QUADRANT, so called from its inventor Edmund Gunter (fig. 4, pl. 29) besides the apparatus of other quadrants, has a stereographic projection of the sphere on the plane of the equinoctial; and also a calendar of the months, next to the divisions of the limb; by which means, besides the common purposes of other quadrants, several useful questions in astronomy, &c, are easily resolved.

Use of Gunter's Quadrant.—(1) To find the sun's meridian altitude for any given day, or conversely the day of the year answering to any given meridian altitude. Lay the thread to the day of the month in the scale next the limb; then the degree it cuts in the limb is the sun's meridian altitude. And, contrariwise, the thread being set to the meridian altitude, it shows the day of the month.

(2) To find the hour of the day. Having set the bead, which slides on the thread, to the sun's place in the ecliptic, observe the sun's altitude by the quadrant; then if the bead be laid over the same in a limb, it will fall upon the hour required. On the contrary, laying the bead on a given hour, having first rectified or set it to the sun's place, the degree cut by the thread on the limb gives the altitude.—The bead may be rectified otherwise, by bringing the thread to the day of the month, and the bead to the hour-line of 12.

(3) To find the sun's declination from his place given; and the contrary. Bring the bead to the sun's place in the ecliptic, and move the thread to the line of declination ET, so shall the bead cut the degree of declination required. On the contrary, the bead being adjusted to a given declination, and the thread moved to the ecliptic, it will cut the sun's place.

(4) The sun's place being given, to find the right ascension; or the contrary. Lay the thread on the sun's place in the ecliptic, and the degree it cuts on the limb is the right ascension sought. And the converse.

(5) The sun's altitude being given, to find his azimuth; and the contrary. Rectify the bead for the time, as in the second article, and observe the sun's altitude; bring the thread to the complement of that altitude; then the bead will give the azimuth sought, among the azimuth-lines.

9. *Hadley's* QUADRANT, (fig. 7, pl. 29) so called from its inventor John Hadley, esq. is now universally used as the best of any for nautical and other observations. It seems the first idea of this excellent instrument was suggested by Dr. Hooke; for Dr. Sprat, in his History of the Royal Society, pa. 246, mentions the invention of a new instrument for taking angles by reflection, by which means the eye at once sees the two objects both as touching the same point, though distant almost to a semicircle; which is of great use for making exact observations at sea. This instrument is described and illustrated by a figure in Hooke's Posthumous Works, pa. 503. But as it admitted of only one reflection, it would not answer the purpose. The matter however was at last effected by Sir Isaac Newton, who communicated to Dr. Halley a paper of his own writing, containing the description of an instrument with two reflections, which soon after the doctor's death was found among his papers by Mr. Jones, by whom it was communicated to the Royal Society, and it was published in the Philos. Trans. for the year 1742. How it happened that Dr. Halley never mentioned this in his lifetime, is difficult to account for; more especially as Mr. Hadley had described, in the Transac. for 1731, his instrument, which is constructed on the same principles. Mr. Hadley, who was well acquainted with Sir Isaac Newton, might have heard him say, that Dr. Hooke's proposal could be effected by means of a double reflection; and perhaps in consequence of this hint, he might apply himself, without any previous knowledge of what Newton had actually done, to the construction of his instrument. Mr. Godfrey too, of Pennsylvania, had recourse to a similar expedient; for which reason some gentlemen of that colony have ascribed the invention of this excellent instrument to him. The truth may probably be, that each of these gentlemen discovered the method independent of one another. See Trans. of the American Society, vol. 1, pa. 21 Appendix.

This instrument consists of the following particulars: 1. An octant, or the 8th part of a circle, ABC. 2. An index D. 3. The speculum E. 4. Two horizontal glasses, F, G. 5. Two screens, K and K. 6. Two sight-vanes, H and I.—The octant consists of two radii, AB, AC, strengthened by the braces L, M, and the arch BC; which, though containing only 45°, is nevertheless divided into 90 primary divisions, each of which stands for degrees, and are numbered 0, 10, 20, 30, &c, to 90; beginning at each end of the arch for the convenience of numbering both ways, either for altitudes or zenith distances: also each degree is subdivided into minutes, by means of a vernier. But the number of these divisions varies with the size of the instrument.

The index D, is a flat bar, moveable about the centre of the instrument; and that part of it which slides over the graduated arch, BC, is open in the middle, with Vernier's scale on the lower part of it; and underneath is a screw, serving to fasten the index against any division.

The speculum E is a piece of flat glass, quicksilvered on one side, set in a brass box, and placed perpendicular to the plane of the instrument, the middle part of the former coinciding with the centre of the latter: and because the speculum is fixed to the index, the position of it will be altered by the moving of the index along the arch. The rays of an observed object are received on the speculum, and from thence reflected on one of the horizon glasses, F or G; which are two small pieces of looking-glass placed on one of the limbs, their faces being turned obliquely to the speculum, from which they receive the reflected rays of objects. The glass F has only its lower part silvered, and set in brass-work; the upper part being left transparent to view the horizon. The glass G has in its middle a transparent slit, through which the horizon is to be seen. And because the warping of the materials, and other accidents, may distend them from their true situation, there are three screws passing through their feet, by which they may be easily replaced.

The screens are two pieces of coloured glass, set in two square brass frames K, K, which serve as screens to take off the glare of the sun's rays, which would otherwise be too strong for the eye; the one is tinged much deeper than the other; as they both move on the same centre, they may be both or either of them used: in the situation they have in the figure, they serve for the horizon-glass F; but when they are wanted for the horizon-glass G, they must be taken from their present situation, and placed on the quadrant above G.

The sight-vanes are two pins, H and I, standing perpendicularly to the plane of the instrument: that at H having a hole in it, opposite to the transparent slit in the horizon-glass G; the other, at I, has two holes in it, the one opposite to the middle of the transparent part of the horizon-glass F, and the other rather lower than the quick-silvered part: this vane has a piece of brass on the back of it, which moves round a centre, and serves to cover either of the holes.

Of the Observations.—There are two kinds of observations to be made with this instrument: the one is when the back of the observer is turned towards the object, and therefore called the back observation; the other when his face is turned towards the object, which is called the fore-observation.

To Rectify the Instrument for the Fore-observation.—Slacken the screw in the middle of the handle behind the glass F; and bring the index close to the button *h*; hold the instrument in a vertical position, with the arch downwards; look through the right-hand hole in the vane I, and through the transparent part of the glass F, for the horizon; and if it lie in the same right line with the image of the horizon seen on the silvered part, the glass F is rightly adjusted; but if the two horizontal lines disagree, turn the screw which is at the end of the handle backward or forward, till those lines coincide; then fasten the middle screw of the handle, and the glass is rightly adjusted.

To take the Sun's Altitude by the Fore-observation.—Having fixed the screens above the horizon-glass F, and suited them proportionally to the strength of the sun's rays, turn your face towards the sun, holding the instrument with your right hand, by the braces L and M, in a vertical position, with the arch downward; place your eye close to the right-hand hole in the vane I, and view the horizon through the transparent part of the horizon-glass F, at the same time moving the index D with the left hand, till the reflex solar spot coincides with the line of the horizon; then the degrees counted from C, or that end next your body, will give the sun's altitude at that time, observing to add or subtract 16 minutes according as the upper or lower edge of the sun's reflex image is made use of.

But to get the sun's meridian altitude, being what is wanted for finding the latitude; the observations must be continued; and as the sun approaches the meridian, the index D must be continually moved towards B, to maintain the coincidence between the reflex solar spot and the horizon; and consequently as long as this motion can maintain the same coincidence, the observation must be continued, till the sun has reached the meridian, and begins to descend, when the coincidence will require a retrograde motion of the index, or towards C; then the observation is finished, and the degrees counted as before will give the sun's meridian altitude, or those from B will give the zenith distance; observing to add the semi-diameter, or 16′, when his lower edge is brought to the horizon; or to subtract 16′, when the horizon and upper edge coincide.

To take the Altitude of a Star by the Fore-observation.—Through the vane H, and the transparent slit in the glass G, look directly to the star; and at the same time move the index, till the image of the horizon behind you, being reflected by the great speculum, be seen in the silvered part of G, and meet the star; then will the index show the degrees of the star's altitude.

To Rectify the Instrument for the Back-observation.—Slacken the screw in the middle of the handle, behind the glass G; turn the button *h* on one side, and bring the index as many degrees before 0 as is equal to double the dip of the horizon at your height above the water; hold the instrument vertical, with the arch downward; look through the hole of the vane H; and if the horizon, seen through the transparent slit in the glass G, coincide with the image of the horizon seen in the silvered part of the same glass, then the glass G is in its proper position; but if not, set it by the handle, and fasten the screw as before.

To take the Sun's Altitude by the Back-observation.—Put the stem of the screens, K, K, into the hole *r*, and in proportion to the strength or faintness of the sun's rays, let either one or both or neither of the frames of those glasses be turned close to the face of the limb; hold the instrument in a vertical position, with the arch downward, by the braces L and M, with the left hand; then turn your back to the sun, and put one eye close to the hole in the vane H, observing the horizon through the transparent slit in the horizon glass G; with the right hand move the index D, till the reflected image of the sun be seen in the silvered part of the glass G, and in a right line with the horizon; swing your body to and fro, and if the observation be well made, the sun's image will be observed to brush the horizon, and the degrees reckoned from C, or that part of the arch furthest from your body, will give the sun's altitude at the time of observation; observing to add 16′, for the sun's semidiameter, if the sun's upper edge be used, or subtract the same for the lower edge.

The directions just given, for taking altitudes at sea, would be sufficient, but for two corrections that are necessary to be made before the altitude can be accurately determined, viz, one on account of the observer's eye being raised above the level of the sea, and the other on account of the refraction of the atmosphere, especially in small altitudes. The following tables show the corrections to be made on both these accounts.

TABLE I.

Dip of the Horizon of the Sea.

Height of the Eye.	Dip of the Horizon.
Feet.	
1	0′ 57″
2	1 21
3	1 39
5	2 8
10	3 1
15	3 42
20	4 16
25	4 46
30	5 14
35	5 39
40	6 2
45	6 24
50	6 44

TABLE II.

Refractions of the Stars &c in Altitude.

Appar. Altitude in Deg.	Refraction.	Appar. Altitude. in Deg.	Refraction.
0°	33′ 0″	11°	4′ 47″
¼	30 35	12	4 23
½	28 22	15	3 30
1	24 29	20	2 35
2	18 35	25	2 2
3	14 36	30	1 38
4	11 51	35	1 21
5	9 54	40	1 8
6	8 29	45	0 57
7	7 20	50	0 48
8	6 29	60	0 33
9	5 48	70	0 21
10	5 15	80	0 10

General Rules for these Corrections.

1. In the fore-observations, add the sum of both corrections to the observed zenith distance, for the true zenith distance: or subtract the said sum from the observed altitude, for the true one. 2. In the back-observation, add the dip and subtract the refraction for altitudes; and for zenith distances, do the contrary, viz, subtract the dip, and add the refraction.

Example. By a back-observation, the altitude of the sun's lower edge was found by Hadley's quadrant to be 25° 12′; the eye being 30 feet above the horizon. By the tables the dip on 30 feet is 5′ 14″, and the refraction on 25° 12′ is 2′ 1″. Hence

Appar. alt. lower limb	25°	12′	0″
Sun's semidiameter, sub.	0	16	0
Appar. alt. of centre	24	56	0
Dip. of horizon, add	0	5	14
	25	1	14
Refraction, subtract	0	2	1
True alt. of centre	24	59	13

In the case of the moon, besides the true corrections above, another is to be made for her parallaxes. But for all these particulars, see the Requisite Tables for the Nautical Almanac, also Robertson's Navigation, vol. 2, pa. 340 &c, edit. 1780.

10. *Horodictical* QUADRANT, a pretty commodious instrument, which is so called from its use in telling the hour of the day. Its construction is as follows. From the centre of the quadrant C, (fig. 5, pl. 29), whose limb AB is divided into 90°, describe seven concentric circles at any intervals; and to these add the signs of the zodiac, in the order represented in the figure. Then, applying a ruler to the centre C and the limb AB, mark upon the several parallels the degrees corresponding to the altitude of the sun, when in them, for the given hours; connect the points belonging to the same hour with a curve line, to which add the number of the hour. To the radius CA fit a couple of sights, and to the centre of the quadrant C tie a thread with a plummet, and on the thread a bead to slide.

11. *Sinical* QUADRANT, is one of some use in Navigation. It consists of several concentric quadrantal arches, divided into 8 equal parts by means of radii, with parallel right lines crossing each other at right angles. Now any one of the arches may represent a quadrant of any great circle of the sphere, but is chiefly used for the horizon or meridian. The chief use of the sinical quadrant, is to form upon it triangles similar to those made by a ship's way with the meridians and parallels; the sides of which triangles are measured by the equal intervals between the concentric quadrants and the lines N and S, E and W: every 5th line and arch being made deeper than the rest. Now suppose a ship has sailed 150 leagues north-east-by-north, or making an angle of 33° 45′ with the north part of the meridian: here are given the course and distance sailed, by which a triangle may be formed on the instrument similar to that made by the ship's course; and hence the unknown parts of the triangle may be found.

Sutton's QUADRANT. See *Collins's* QUADRANT.

12. QUADRANT *of Altitude,* (fig. 9, pl. 29) is an appendix to the artificial globe, consisting of a thin slip of brass, the length of a quarter part of one of the great circles of the globe, and graduated. At the end, where the division terminates, is a nut riveted on, and furnished with a screw, by means of which the instrument is fitted on the meridian, and moveable round upon the rivet to all points of the horizon, as represented in the figure referred to. Its use is to serve as a scale in measuring altitudes, amplitudes, azimuths, &c.

QUADRANTAL *Triangle,* is a spherical triangle, which has one side equal to a quadrant or quarter part of a circle.

QUADRAT, called also *Geometrical Square,* and *Line of Shadows:* it is often an additional member on the face of Gunter's and Sutton's quadrants; and is chiefly useful in taking heights or depths. See my Mensuration, the chap. on altimetry and longimetry, or heights-and-distances.

QUADRATIC *Equations,* in Algebra, are those in which the unknown quantity is of two dimensions, or raised to the 2d power. Quadratic equations are either simple, or affected, that is compound.

A Simple QUADRATIC *equation,* is that which contains the 2d power only of the unknown quantity, without any other power of it; as $x^2 = 25$, or $y^2 = ab$. And in this case, the value of the unknown quantity is found by barely extracting the square root on both sides of the equation: thus, in the equations above, it will be $x = \pm 5$, and $y = \pm \sqrt{ab}$; where the sine of the root of the known quantity is to be taken either plus or minus, for either of these may be considered as the sign of the value of the root x, since either of them, when squared, make the same square, $(+5)^2 = 25$, and $(-5)^2 = 25$ also; and hence the root of every quadratic or square, has two values.

Compound or *affected* QUADRATICS, are those which contain both the 1st and 2d powers of the unknown quantity; as $x^2 + ax = b$, or $x^{2n} - ax^n = \pm b$, where n may be of any value, and then x^n is to be considered as the root or unknown quantity.—Affected quadratics are usually distinguished into three forms, according to the signs of the terms of the equation:

Thus, 1st form, $x^2 + ax = b$,
2d form, $x^2 - ax = b$,
3d form, $x^2 - ax = -b$.

But the method of extracting the root, or finding the value of the unknown quantity x, is the same in all of them. And that method is usually performed by what is called completing the square, which is done by taking half the coefficient of the 2d term or single power of the unknown quantity, then squaring it, and adding that square to both sides of the equation, which makes the unknown side a complete square. Thus, in the equation $x^2 + ax = b$, the coefficient of the 2d term being a, its half is $\frac{1}{2}a$, the square of which is $\frac{1}{4}a^2$, and this added to both sides of equation, it becomes $x^2 + ax + \frac{1}{4}a^2 + = \frac{1}{4}a^2 b$, the former side of which is now a complete square, and the scond a known quantity.

The square being thus completed, its root is next to be extracted; in order to which, it is to be observed that the root on the unknown side consists of two terms, the one of which is always x the square root of the first term of the equation, and the other part is $\frac{1}{2}a$ or half the coefficient of the second term: thus then the root of $x^2 + ax + \frac{1}{4}a^2$ the first side of the completed equation being $x + \frac{1}{2}a$, and the root of the other side $\frac{1}{4}a^2 + b$ being $\pm\sqrt{(\frac{1}{4}a^2 + b)}$, it follows that $x + \frac{1}{2}a = \pm\sqrt{(\frac{1}{4}a^2 + b)}$, and hence, by transposing $\frac{1}{2}a$, it is $x = -\frac{1}{2}a \pm\sqrt{(\frac{1}{4}a^2 + b)}$, the two values of x, or the roots of the given equation $x^2 + ax = b$. And thus is found the root, or value of x, in the three forms of equations above mentioned: viz,

1st form $x = -\frac{1}{2}a \pm \sqrt{(\frac{1}{4}a^2 + b)}$,
2d form $x = +\frac{1}{2}a \pm \sqrt{(\frac{1}{4}a^2 + b)}$,
3d form $x = +\frac{1}{2}a \pm \sqrt{(\frac{1}{4}a^2 - b)}$.

Where it is observable that, because of the double sign ±, every form has two roots: in the 1st and 2d forms those roots are, the one positive and the other negative, the positive root being the less of the two in the 1st form, but the greater in the 2d form; and in the 3d form the roots are both positive. Again, the two roots of the 1st and 2d forms, are always both of them real; but in the 3d form, the two roots are either both real or both imaginary, viz, both real when $\frac{1}{4}a^2$ is greater than b, or both imaginary when $\frac{1}{4}a^2$ is less than b, because in this case $\frac{1}{4}a^2 - b$ will be a negative quantity, the root of which is impossible, or an imaginary quantity.

Example of the 1st form, let $x^2 + 6x = 7$. Here then $a = 6$, and $b = 7$; then $x = -\frac{1}{2}a \pm \sqrt{(\frac{1}{4}a^2 + b)} = -3 \pm \sqrt{16} = -3 \pm 4 = +1$ or -7.

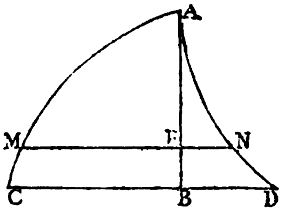
Example of the 2d form, let $x^2 - 6x = 7$. Here also $a = 6$, and $b = 7$; then $x = +\frac{1}{2}a \pm \sqrt{(\frac{1}{4}a^2 + b)} = +3 \pm \sqrt{16} = +3 \pm 4 = +7$ or -1; the same two roots as before, with the signs changed.

Example of the 3d form, let $x^2 - 6x = -7$. Here again $a = 6$, and $b = 7$; then $x = +\frac{1}{2}a \pm \sqrt{(\frac{1}{4}a^2 - b)} = +3 \pm \sqrt{2}$, the two roots both real.

But if $x^2 - 6x = -11$; then $a = 6$, and $b = 11$, which gives x or $+\frac{1}{2}a \pm \sqrt{(\frac{1}{4}a^2 - b)} = +3 \pm \sqrt{-2}$, the two roots both imaginary.

All equations whatever that have only two different powers of the unknown quantity, of which the index of the one is just double that of the other, are resolved like quadratics, by completing the square. Thus, the equation $x^4 + ax^2 = b$, by completing the square becomes $x^4 + ax^2 + \frac{1}{4}a^2 = \frac{1}{4}a^2 + b$; whence, extracting the root on both sides, $x^2 + \frac{1}{2}a = \pm\sqrt{(\frac{1}{4}a^2 + b)}$; therefore $x^2 = -\frac{1}{2}a \pm \sqrt{(\frac{1}{4}a^2 + b)}$, and consequently $x = \pm\sqrt{(-\frac{1}{2}a \pm \sqrt{(\frac{1}{4}a^2 + b)})}$, where the root x has four values, because the given equation $x^4 + ax^2 = b$ rises to the 4th power. See EQUATION.

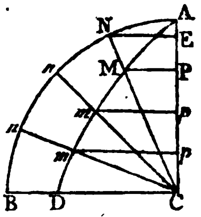
QUADRATRIX, or QUADRATIX, in Geometry, is a mechanical line, by means of which, right lines are found equal to the circumference of circles, or other curves, and of the parts of the same. Or, more accurately, the quadratrix of a curve, is a transcendental curve described on the same axis, the ordinates of which being given, the quadrature of the correspondent parts in the other curve, is also given. See CURVE. Thus, for ex. the curve AND may be called the quadratrix of the parabola AMC, when the area APMA bears some such relation as the following to the absciss AP or ordinate PN, viz,



when $APM = PN^2$,
or $APM = AP \times PN$,
or $APM = a \times PN$,

where a is some given constant quantity.

The most distinguished of these quadratices are, those of Dinostrates and of Tschirnhausen for the circle, and that of Mr. Perks for the hyperbola.



QUADRATRIX *of Dinostrates*, is a curve AMD, by which the quadrature of the circle is effected, though not geometrically, but only mechanically. It is so called from its inventor Dinostrates; and the genesis or description of which is as follows: Divide the quadrantal arc ANB into any number of equal parts, in the points N, *n*, *n*, &c; and also the radius AC into the same number of parts at the points P, *p*, *p*, &c. To the points N, *n*, *n*, &c, draw the radii CN, C*n*, &c; and from the points P, *p*, &c, the parallels to CB, as PM, P*m*, &c: then through all the points of intersection draw the curve AM*m*D, and it will be the quadratrix of Dinostrates.

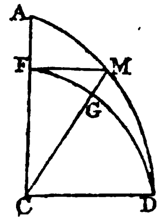
Or the same curve may be conceived to be described by a continued motion, by conceiving a radius CN to revolve with a uniform motion about the centre C, from the position AC to the position BC; at the same time a ruler PM moves uniformly parallel towards CB; then the two uniform motions being so regulated that the radius and the ruler shall arrive at the position BC at the same time; for thus the continual intersection M, *m*, &c. of the revolving radius, and moving ruler, will describe the quadratrix of Dinostrates. Hence,

1. *For the Equation of the Quadratrix*: Since, from the relation of the uniform motions, it is always, AB : AN : : AC : AP; therefore if $AB = a$, $AC = r$, $AP = x$, and $AN = z$, it will be $a : z :: r : x$, or $ax = rz$, which is the equation of the curve.

Or, if s denote the sine NE of the arc AN, and $y = PM$ the ordinate of the curve AM, its absciss AP being x; then, by similar triangles, CE : CP : : EN : PM, that is, $(r^2\sqrt{} - s^2) : r - x :: s : y$, and hence $y\sqrt{(r^2 - s^2)} = (r - x)s$, the equation of the curve. And when the relation between AB and AN is given, in terms of that between AC and AP, hence will be expressed the relation between the sine EN and the radius CB, or s will be expressed in terms of r and x; and consequently, the equation of the curve will be expressed in terms of r, x, and y only.

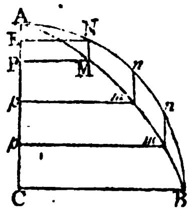
2. The base of the quadratrix CD is a third proportional to the quadrant AB and the radius AC or CB; i. e. CD : CB : : CB : AB. Hence the rectification and quadrature of the circle.

3. A quadrantal arc DF described with the centre C and radius CD, will be equal in length to the radius CA or CB.



4. CDF being a quadrant inscribed in the quadratrix AMD, if the base CD be $= 1$, and the circular arc $DG = x$; then is the area $CFMD = x - \frac{1}{9}x^3 - \frac{1}{225}x^5 - \frac{2}{6615}x^7$ &c. See QUADRATURE. Also Emerson's Curve Lines, pa. 16.

QUADRATRIX *of Tschirnhausen*, is a transcendental curve AM*m*B by which the quadrature of the circle is also effected. This was invented by M. Tschirnhausen, and its genesis, in imitation of that of Dinostrates, is as follows: Divide the quadrant ANB, and the radius AC,



each into equal parts, as before; and from the points P, p, &c, draw the lines PM, pm, &c, parallel to CB; also from the points N, n, &c, the lines NM, nm, &c, parallel to the other radius AC; so shall all the intersections M, m, &c, be in the curve of the quadratrix AMmB.

Now for the Equation of this Quadratrix; it is, as before, AB : AN : : AC : AP, or $a : z : : r : x$, or $ax = rz$.

Or, because here $y =$ PM $=$ EN $= s$; therefore s, as before, expressed in terms of r and x, gives the equation of this quadratrix in terms of r, x, and y, and that in a simpler form than the other. Thus, from the nature of the circle and the construction of the quadratrix, it is

$$y \text{ or } s = x + \frac{r^2 - x^2}{2 \cdot 3r^2}\text{A} + \frac{3^2r^2 - x^2}{4 \cdot 5r^2}\text{B} + \frac{5^2r^2 - x^2}{6 \cdot 7r^2}\text{C} \ \&\text{c},$$

where A, B, C, &c, are the preceding terms; which is the equation of the quadratrix of Tschirnhausen.

By either quadratrix, it is evident that an arc or angle is easily divided into three, or any other number of equal parts; viz, by dividing the corresponding radius, or part of it, into the same number of equal parts: for AN is always the same part of AB, that AP is of AC.

QUADRATURE, in Astronomy, that aspect or position of the moon when she is 90° distant from the sun. Or, the quadratures or quarters are the two middle points of the moon's orbit between the points of conjunction and opposition, viz, the points of the 1st and 3d quarters; at which times the moon's face shows half full, being dichotomized or bisected.

The moon's orbit is more convex in the quadratures than in the syzygies, and the greater axis of her orbit passes through the quadratures, at which points also she is most distant from the earth.—In the quadratures, and within 35° of them, the apses of the moon go backwards, or move in antecedentia; but in the syzygies the contrary.—When the nodes are in the quadratures, the inclination of the moon's orbit is greatest, but least when they are in the syzygies.

QUADRATURE *Lines*, or *Lines of* **QUADRATURE**, are two lines often placed on Gunter's sector. They are marked with the letter Q, and the figures 5, 6, 7, 8, 9, 10; of which Q denotes the side of a square, and the figures denote the sides of polygons of 5, 6, 7, &c sides. Also S denotes the semidiameter of a circle, and 90 a line equal to the quadrant or 90° in circumference.

QUADRATURE, in Geometry, is the squaring of a figure, or reducing it to an equal square, or finding a square equal to the area of it.—The quadrature of rectilineal figures falls under common geometry, or mensuration; as amounting to no more than the finding their areas, or superficies; which are in effect their squares: which was fully effected by Euclid.

The **QUADRATURE** *of Curves*, that is, the measuring of their areas, or the finding a rectilineal space equal to a proposed curvilineal one, is a matter of much deeper speculation; and makes a part of the sublime or higher geometry. The lunes of Hypocrates are the first curves that were squared, as far as we know. The circle was attempted by Euclid and others before him: he showed indeed the proportion of one circle to another, and gave a good method of approximating to the area of the circle, by describing a polygon between any two concentric circles, however near their circumferences might be to each other. At that time the conic sections were admitted into geometry, and Archimedes, perfectly, for the first time, squared the parabola, and he determined the relations of spheres, spheroids, and conoids, to cylinders and cones; and by pursuing the method of exhaustions, or by means of inscribed and circumscribed polygons, he approximated to the periphery and area of the circle; showing that the diameter is to the circumference nearly as 7 to 22, and the area of the circle to the square of the diameter as 11 to 14 nearly. Archimedes also determined the relation between the circle and ellipse, as well as that of their similar parts: and it is probable also that he attempted the hyperbola; but it is not likely that he met with any success, since approximations to its area are all that can be given by the various methods that have since been invented. Besides these figures, he left a treatise on a spiral curve; in which he determined the relation of its area to that of the circumscribed circle; as also the relation of their sectors.

Several other eminent men among the ancients wrote upon this subject, both before and after Euclid and Archimedes; but their attempts were usually confined to particular parts of it, and made according to methods not essentially different from theirs. Among these are to be reckoned Thales, Anaxagoras, Pythagoras, Bryson, Antiphon, Hippocrates of Chios, Plato, Apollonius, Philo, and Ptolemy; most of whom wrote upon the quadrature of the circle; and those after Archimedes, by his method, usually extended the approximation to a further degree of accuracy.

Many of the moderns have also prosecuted the same problem of the quadrature of the circle, after the same methods, to still greater lengths; such are Vieta, and Metius; whose ratio between the diameter and the circumference, is that of 113 to 355, which is within about $\frac{3}{10000000}$ of the true ratio; but above all, Ludolph van Ceulen, or Cologne, who, with an amazing degree of industry and patience, by the same methods, extended the ratio to 36 places of figures, making the ratio to be that of 1 to 3·14159, 26535, 89793, 23846, 26433, 83279, 50288 + or 9 −.

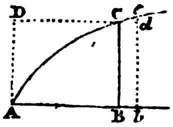
Of this labour, which was rather the exercise of patience than his ingenuity, he was so proud, that, after the example of the profound geometrician of Syracuse, with respect to the sphere and cylinder, he requested it might be inscribed on his tombstone, and it is said that this monument of his patient industry is still to be seen in one of the towns of Flanders. Willebrode Snell, the editor of Van Ceulen, also made several additions to what had been previously done on this subject. He discovered and published in his work, entitled Cyclometriæ, the method of expressing, by an approximate proportion, and a very simple calculation, the magnitude of any arc; and he made use of this method in examining the calculation of Van Ceulen, which he found to be correct. By this method he also calculated a series of both inscribed and circumscribed polygons, beginning with the decagon, and always doubling the number of sides, until the number was 5242880; and ranged the results of his computations in a table, for the purpose of detecting the falsity of any pretended quadrature of the circle. The celebrated Huygens, when very young, enriched this measure of the circle with several new theorems; and successfully combated the pretended quadrature of Gregory St. Vincent, a Jesuit of the Netherlands, who announced his discovery as only wanting a few calculations to render it complete, but which he dexterously forgot to perform. James Gregory and Leibnitz, about the same time, discovered, independent of each other, a very simple series for expressing the length of an arc of a circle, and which was first given in a letter of the 15th of February 1671, from Gregory to Mr. Collins. If a be an arc, t its tangent, and r the radius, then $a = t \times (1 - \frac{t^2}{3r^2} + \frac{t^4}{5r^4} - \frac{t^6}{7r^6} + \frac{t^8}{9r^8} - \&c)$. But the arc must not be assumed greater than half a quadrant, otherwise the series will not converge. Dr. Halley also discovered a simple series for expressing the arc of 30°; which is, $a = \sqrt{\tfrac{1}{3}} \times (1 - \frac{1}{3.3} + \frac{1}{5.3^2} - \frac{1}{7.3^3} + \frac{1}{9.3^4} - \&c)$; and which converges very quickly, and being multiplied by 12, gives the whole circumference. Mr. Sharp, an English mathematician, in 1699, undertook the quadrature of the circle for his own private amusement, and deduced it from two different series, by which the truth of it was proved to 72 places of figures. But Mr. John Machin, professor of astronomy in Gresham College, discovered another very expeditious series for expressing the length of the circumference of a circle, depending on the differences of arcs, the tangents of which have certain relations to each other, and thus extended Mr. Sharp's number to 100 places of figures. And M. De Lagny, a French mathematician, continued this computation to 128 places of figures; on which Montucla has observed, that, "if we suppose a circle, the diameter of which is a thousand millions of times greater than the distance between the sun and the earth, the error in the circumference would be a thousand millions of times less than the thickness of a hair." Nay, it is even possible to surpass this, and Euler has pointed out the method of accomplishing it, in the Transactions of the Imperial Academy of Sciences at Petersburg; but any thing farther than what has been done, could only be considered as superfluous labour. I have also given several series for the same purpose, which converge much easier and quicker than any others; some of which may be seen in my Mensuration, and more especially in my Tracts, vol. 1, pa. 268.

While these, and other mathematicians, were extending the approximative methods for finding the circumference and area of the circle, some were endeavouring to obtain, and even asserting that they had obtained, their true measures; at the same time others were denying the possibility of exhibiting the true ratio. Mr. James Gregory undertook, in 1668, to demonstrate the absolute impossibility of the quadrature of the circle. This he did by a very ingenious method of reasoning, which might deserve to be better examined. However it did not meet with the approbation of Mr. Huygens; which produced a very warm dispute between these two geometricians. Mr. Gregory gave also some ingenious methods for approaching near to the measure of the circle, and even to that of the hyperbola. Dr. Barrow and several other persons have also attempted the demonstration of the same impossibility, with various degrees of success. Of this latter opinion was the celebrated Dr. Barrow, who in his mathematical lecture, observes, that the radius and circumference of a circle are lines of such a nature as to be not only incommensurable in length and square, but also in cube, biquadrate, and all higher powers to infinity. But, notwithstanding the attempts of Dr. Barrow, and many other celebrated mathematicians, to prove the absolute impossibility of resolving this interesting problem, they have been as unsuccessful on this head as those who have endeavoured to find its true quadrature. Legendre, however, in the fourth note prefixed to his Geometry, has proved that the ratio of the circumference to the diameter, and its square, are irrational numbers. Besides the efforts above enumerated, which were made principally by men of great talents, many other vain attempts at squaring the circle have been made by men of less acquirements than vanity, who have endeavoured to persuade us that they had discovered the true quadrature of the circle, which so many able mathematicians had so long sought in vain; and whose pretensions, like falling stars, attracted notice for a moment, and then, like them, sank into eternal oblivion. The first among the moderns who pretended to have solved this problem, was Cardinal de Cusa. He rolled a cylinder over a plane, till the point which first was in contact with it touched it again; and then, by a train of reasoning, wholly destitute of geometrical precision, he endeavoured to determine the length of a line thus described; but he was easily refuted by Regiomontanus about 1465. Near a century after, Orontius Finæus attracted notice by his parallogisms on this subject; but the fallacy of his reasoning was clearly shown by Peter Nunez, and J. Borelli. The celebrated Joseph Scaliger also ranks under this class; who having no great esteem for geometricians, he endeavoured to show them his superiority, in undertaking, by way of amusement, the quadrature of the circle, and seriously imagined that he had obtained it; but Vieta, Clavius, and others, found no difficulty in refuting him. Longomontanus, the celebrated Danish astronomer, was also of the number who asserted he had obtained a finite ratio between the diameter and circumference, which was exactly 1 to 3·14185. And our countryman, Mr. Hobbes, also rendered himself remarkable as a member of this class, but his pretensions were refuted by Dr. Wallis. Oliver de Serres weighed a circle, and a triangle equal to the equilateral inscribed triangle, and believed that the one was exactly double of the others; but, a very little knowledge of the subject would have been sufficient to have shown him, that the double of this triangle is the hexagon inscribed in the same circle. It would be tedious and uninteresting to go through the history of all these pretended quadratures, the authors of which would at this day have been totally unknown, had they not erected a monument to their own ignorance and vanity, by attempting that which they were totally unacquainted with. We shall, however, for the amusement of the reader, furnish him with a few more anecdotes on this head, in order to show to what a degree of enthusiasm some have suffered themselves to be carried in their erroneous speculations. One Mathulon, who from being a manufacturer of stuffs at Lyons, commenced geometer, claimed the merit of having solved this problem, and deposited 1000 crowns as a reward for the person who should prove that his solution was not correct; which was done by M. Nicole, a member of the French Academy of Sciences, who gave the reward to the General Hospital at Lyons; and a similar circumstance happened some time afterwards: a Frenchman announced the quadrature of the circle, and challenged the whole world to refute him; and deposited 10,000 livres for any person who should do it. This grand problem he reduced to the mechanical process of dividing a circle into quadrants, and then turning these with their angles outwards, so as to form a square, which he asserted to be equal to the circle. Three persons claimed the reward, and the cause was tried at the Chatelet of Paris; but the judges thought a person's fortune ought not to be diminished on account of the error of his judgment, unless they were prejudicial to society: whereon the king decreed that the proposal should be void: and the Academy of Sciences recommended him to study the elements of geometry; but still he fancied that future ages would blush for the injustice that was done him. M. Le Rohberges de Vausenville, in a work, entitled, "Consultation sur la Quadrature du Cercle," inquires of a mathematician if the quadrature of the circle would not be obtained if any means were devised for finding the centre of gravity of a sector of a circle in common parts of the radius and the circumference of the same circle; the meaning of this last clause is not clear, but it may be observed that when this can be done without the arc being one of the terms, the business will be accomplished.

The preceding examples, one might suppose, would be sufficient for deterring men from farther pursuing this hopeless speculation; yet such is the weakness and vanity of some pretenders to science, that hopes are still entertained by them of obtaining the solution within very narrow limits. We have an instance of this infatuation in Signor Rossi, an Italian attorney, who visited London about five years ago, to claim the reward of his ingenuity in squaring the circle; but, unfortunately, it rested upon the supposition that the side of a square is to its diagonal as 5 to 7, or in other words that 49 is equal to 50; he was, notwithstanding, very much dissatisfied at not receiving the reward he fancied himself entitled to, and returned with a perfect conviction that the English had not done him justice. See my Translation of Montucla's Recreations, vol. 1, pa. 299, &c, 2d edition.

But though a definite quadrature of the whole circle was never yet given, nor of any aliquot part of it; yet certain other portions of it have been squared. The first partial quadrature was given by Hippocrates of Chios; who squared a portion called, from its figure, the Lune, or Lunule; but this quadrature has no dependence on that of the circle. And some modern geometricians have found the quadrature of any portion of the lune taken at pleasure, independently of the quadrature of the circle; though still subject to a certain restriction, which prevents the quadrature from being perfect, and what the geometricians call absolute and indefinite. See LUNE. And for the quadrature of the different kinds of curves, see their several particular names.

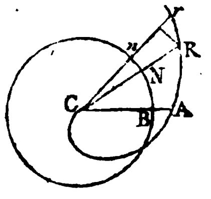
QUADRATURES, *by Fluxions.*—The most general method of quadratures yet discovered, is that of Newton, by means of fluxions, and is as follows. AC being any curve to be squared, AB an absciss, and BC an ordinate perpendicular to it, also bc another ordinate indefinitely near to the former. Putting AB $= x$, and BC $= y$; then is B$b = \dot{x}$ the fluxion of the absciss, and $y\dot{x} =$ Cb the fluxion of the area ABC sought. Now let the value of the ordinate y be found in terms of the absciss x, or in a function of the absciss, and let that function be called X, that is $y = \mathrm{X}$; then substituting X for y in $y\dot{x}$, gives $\mathrm{X}\dot{x}$ the fluxion of the area; and the fluent of this, being taken, gives the area or quadrature of ABC as required, for any curve, whatever its nature may be.



Ex. Suppose, for example, AC to be a common parabola; then its equation is $px = y^2$, where p is the parameter; which gives $y = \sqrt{px}$, the value of y in a function of x, and is what is called X above; hence then $y\dot{x} = \dot{x}\sqrt{px} = p^{\frac{1}{2}}x^{\frac{1}{2}}\dot{x}$ is the fluxion of the area; and the fluent of this is $\frac{2}{3}p^{\frac{1}{2}}x^{\frac{3}{2}} = \frac{2}{3}x\sqrt{px} = \frac{2}{3}xy = \frac{2}{3}$ of the circumscribing rectangle BD; which therefore is the quadrature of the parabola.

Again, if AC be a circle whose diameter is d; then its equation is $y^2 = dx - x^2$, which gives $y = \sqrt{(dx - x^2)}$, and the fluxion of the area $y\dot{x} = \dot{x}\sqrt{(dx - x^2)}$. But as the fluent of this cannot be found in finite terms, the quantity $\sqrt{(dx - x^2)}$ is developed or thrown into a series, and then the fluxion of the area is $y\dot{x} = \dot{x}\sqrt{(dx - x^2)} = \dot{x}\sqrt{dx} \times (1 - \frac{x}{2d} - \frac{x^2}{2.4d^2} - \frac{1.3x^3}{2.4.6d^3}$ &c); and the fluent of this gives $x\sqrt{dx} \times (\frac{2}{3} - \frac{1}{5} . \frac{x}{d} - \frac{1}{4.7} . \frac{x^2}{d^2} - \frac{1.3}{4.6.9} . \frac{x^3}{d^3}$ &c) for the general expression of the area ABC. Now when the space becomes a semicircle, x becomes $= d$, and then the series above becomes $d^2(\frac{2}{3} - \frac{1}{5} - \frac{1}{4.7} - \frac{1.3}{4.6.9}$ &c) for the area of the semicircle whose diameter is d.

In spirals CAR, or any curves referred to a centre C; putting $y =$ any radius CR, $x =$ BN the arc of a circle described about the centre C, at any distance CB $= a$, and Cnr another ray indefinitely near CNR: then $\frac{1}{2}$CN . Nn $= \frac{1}{2}a\dot{x} =$ CNn, and by sim. fig. CN^2 : CR^2 or $a^2 : y^2 ::$ CNn : $\frac{y^2\dot{x}}{2a} =$ CRr the fluxion of the area described by the revolving ray CR; then the fluent of this, for any particular case, will be the quadrature of the spiral. So if, for instance, it be Archimedes's spiral, in which $x : y$ in a constant ratio, suppose as $m : n$, or $my = nx$, and $y^2 = \frac{n^2x^2}{m^2}$; hence then CR$r = \frac{y^2\dot{x}}{2a} = \frac{n^2x^2\dot{x}}{2am^2}$ the fluxion of the area; the fluent of which is $\frac{n^2x^3}{6am^2} = \frac{xy^2}{6a}$ the general quadrature of the spiral of Archimedes.



QUADRIBLE. Squarable.

QUADRILATERAL, or QUADRILATERAL *Figure*, is a figure comprehended by four right lines; and having consequently also four angles; for which reason it is otherwise called a quadrangle. The general term quadrilateral comprehends these several particular species or figures, viz, the square, parallelogram, rectangle, rhombus, rhomboides, and trapezium. If the opposite sides be parallel, the quadrilateral is a parallelogram. If the parallelogram have its angles right ones, it is a rectangle; if oblique, it is an oblique one. The rectangle having all its sides equal, becomes a square; and the oblique parallelogram having all its sides equal, is a rhombus, but if only the opposites be equal, it is a rhomboides. All other forms of the quadrilateral, are trapeziums, including all the irregular shapes of it.

The sum of all the four angles of any quadrilateral, is equal to 4 right angles. Also, the two opposite angles of a quadrilateral inscribed in a circle, taken together, are equal to two right angles. And in this case the rectangle of the two diagonals is equal to the sum of the two rectangles of the opposite sides. For the properties of the particular species of quadrilaterals, see their respective names, SQUARE, RECTANGLE, PARALLELOGRAM, RHOMBUS, RHOMBOIDES, TRAPEZIUM, and TRAPEZOID.

QUADRIPARTITION, is the dividing by 4, or into four equal parts.

QUADRUPLE, is four-fold, or something taken four times, or multiplied by 4.

QUALITY, denotes generally the property or affection of some being, by which it affects our senses in a certain way, &c.

Sensible QUALITIES are such as are the more immediate object of the senses: as figure, taste, colour, smell, hardness, &c.

Occult QUALITIES, among the ancients, were such as did not admit of a rational solution in their way.

Dr. Keil demonstrates, that every quality which is propagated in orbem, such as light, heat, cold, odour, &c, has its efficacy or intensity either increased, or decreased, in a duplicate ratio of the distances from the centre of radiation inversely. So at double the distance from the earth's centre, or from a luminous or hot body, the weight or light or heat, is but a 4th part; and at 3 times the distance, they are 9 times less, or a 9th part, &c.

Sir Isaac Newton lays it down as one of the rules of philosophizing, that those qualities of bodies that are incapable of being intended and remitted, and which are found to obtain in all bodies on which experiment could ever be tried, are to be esteemed universal qualities of all bodies.

QUALITY *of Curvature*, in the higher Geometry, is used to signify its form, as it is more or less inequable, or as it is varied more or less in its progress through different parts of the curve. Newton's Method of Fluxions, pa. 75; and Maclaurin's Fluxions, art. 369.

QUANTITY, denotes any thing capable of estimation, or mensuration; or which, being compared with another thing of the same kind, may be said to be either greater or less, equal or unequal to it. Mathematics is the doctrine or science of quantity.

Physical or *Natural* QUANTITY, is of two kinds: 1st, that which nature exhibits in matter, and its extension; and 2dly, in the powers and properties of natural bodies; as gravity, motion, light, heat, cold, density, &c. Quantity is popularly distinguished into continued and discrete.

Continued QUANTITY, is when the parts are connected together, and is commonly called magnitude; which is the object of geometry.

Discrete QUANTITY, is when the parts, of which it consists, exist distinctly, and unconnected; which makes what is called multitude or number, the object of arithmetic.

The notion of continued quantity, and its difference from discrete, appears to some without foundation. Mr. Machin considers all mathematical quantity, or that for which any symbol is put, as nothing else but number, with regard to some measure, which is considered as 1; for that we know nothing precisely how much any thing is, but by means of number. The notion of continued quantity, without regard to some measure, is indistinct and confused; and though some species of such quantity, considered physically, may be described by motion, as lines by the motion of points, and surfaces by the motion of lines; yet the magnitudes, or mathematical quantities, are not made by the motion, but by numbering according to a measure. Philos. Trans. numb. 447, pa. 228.

QUANTITY *of Action*. See ACTION.

QUANTITY *of Curvature* at any point of a curve is determined by the circle of curvature at that point, and is reciprocally proportional to the radius of curvature.

QUANTITY *of Matter* in any body, is its measure arising from the joint consideration of its magnitude and density, being expressed by, or proportional to the product of the two. So,

if M and m denote the magnitude of two bodies,
and D and d their densities;
then DM and dm will be as their quantities of matter.

The quantity of matter of a body is best discovered by its absolute weight, to which it is always proportional, and by which it is measured.

QUANTITY *of Motion*, or the *Momentum*, of any body, is its measure arising from the joint consideration of its quantity, and the velocity with which it moves. So,

if q denote the quantity of matter,
and v the velocity of any body;
then qv will be its quantity of motion.

QUANTITIES, in Algebra, are the expressions of indefinite numbers, that are usually represented by letters. Quantities are properly the subject of algebra; which consists in the computation of such quantities.

Algebraic quantities are either given and known, or else they are unknown and sought. The given or known quantities are usually represented by the first letters of the alphabet, as a, b, c, d, e, &c, and the unknown or required quantities, by the last letters, as z, y, x, w, &c. and also indeterminate, or such as may be assumed at pleasure, by some of the middle letters, as m, n, p, &c.

Again, algebraic quantities are either positive or negative.—A positive or affirmative quantity, is one that is to be added, and has the sign + or plus prefixed, or understood; as ab or $+ab$. And a negative or privative quantity, is one that is to be subtracted, and has the sign − or minus prefixed; as $-ab$.

QUART, a measure of capacity, being the quarter or 4th part of some other measure. The English quart is the 4th part of the gallon, and contains two pints. The Roman quart, or quartarius, was the 4th part of their congius. The French had, besides their quart or pot of two pints, various other quarts, distinguished by the whole of which they are quarters; as quart de muid, and quart de boisseau.

QUARTER, the 4th part of a whole, or one part of an integer, which is divided into four equal portions.

Quarter, in weights, is the 4th part of the quintal, or hundred weight; and so contains 28 pounds.

Quarter is also a dry measure, containing of corn 8 bushels striked; and of coals the 4th part of a chaldron.

Quarter, in Astronomy; the moon's period, or lunation, is divided into 4 stages or quarters, each containing between 7 and 8 days. The first quarter is from the new moon to the quadrature; the second is from thence to the full moon, and so on.

Quarter, in Navigation, is the quarter or 4th part of a point, wind, or rhumb; or of the distance between two points &c. The quarter contains an arch of 2° 48′ 45″, being the 4th part of 11° 15′, which is one point.

Quarter *Round*, in Architecture, is a term used by the workmen for any projecting moulding, whose contour is a quarter of a circle, or nearly so.

QUARTILE, an aspect of the planets when they are at the distance of 3 signs or 90° from each other: and is denoted by the character □.

QUEUE d'Aronde, or *Swallow's Tail*, in Fortification, is a detached or outwork, whose sides spread or open towards the campaign, or draw narrower and closer towards the gorge. Of this kind are either single or double tenailles, and some horn-works, whose sides are not parallel, but are narrow at the gorge, and open at the head, like the figure of a swallow's tail. On the contrary, when the sides are less than the gorge, the work is called contre Queue d'aronde.

Queue *d'Aronde*, in Carpentry, a method of jointing, called also dove-tailing.

QUICKSILVER, the same as Mercury; which see.

QUINCUNX, in Astronomy, is that position, or aspect, of the planets, when distant from each other by $\frac{5}{12}$ths of the whole circle, or 5 signs out of the 12, that is 150 degrees. The quincunx is marked Q, or *Vc*.

QUINDECAGON, is a plane figure of 15 sides, and consequently the same number of angles. When those are all equal, it is a regular quindecagon, otherwise not. Euclid shows how to inscribe this figure in a circle, prop. 16, lib. 4. And the side of a regular quindecagon, so inscribed, is equal in power to the half difference between the side of the equilateral triangle, and the side of the pentagon; and also the difference of the perpendiculars let fall on both sides, taken together.

QUINQUAGESIMA-*Sunday*, is the same as Shrove-Sunday, and is so called as being about the 50th day before Easter, being indeed the 7th Sunday before it. Anciently the term quinquagesima was used for Whitsunday, and for the 50 days between Easter and Whitsunday; but to distinguish this quinquagesima from that before Easter, it was called the paschal quinquagesima.

QUINQUEANGLED, or **Quinqueangular**, consisting of 5 angles.

QUINTAL, the weight of a hundred pounds, in most countries; but in England it is the hundred weight, or 112 pounds. Quintal was also formerly used for a weight of lead, iron, or other common metal, usually equal to a hundred pounds, at 6 score to the hundred.

QUINTILE, in Astronomy, an aspect of the planets when they are distant the 5th part of the zodiac, or 72 degrees; and is marked thus, c, or o.

QUINTUPLE, 5 times as much as another thing.

QUOIN, in Architecture, an angle or corner of stone or brick walls. When these stand out beyond the rest of the wall, their edges being chamferred off, they are called rustic quoins.

Quoin, in Artillery, is a loose wedge of wood, which is put in below the breech of a cannon, to raise or depress it more or less.

QUOTIENT, in Arithmetic, is the result of the operation of division, or the number that arises by dividing the dividend by the divisor, showing how often the latter is contained in the former. Thus the quotient of 12 divided by 3 is 4; which is usually thus disposed, or expressed, 3) 12 (4 the quotient, or thus $12 \div 3 = 4$ the quotient, or thus $\frac{12}{3}$, like a vulgar fraction; all these meaning the same thing.—In division, as the divisor is to the dividend, so is unity or 1 to the quotient; thus 3 : 12 : : 1 : 4 is the quotient.

R.

RADIANT *Point*, or **Radiating** *Point*, is any point from which rays proceed. Every radiant point diffuses innumerable rays in all directions: but those rays are only visible from which right lines can be drawn to the pupil of the eye; because the rays are all in right lines. All the rays proceeding from the same radiant continually diverge; but the crystalline collects or reunites them again.

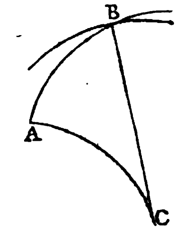
RADIATION, is the casting or shooting forth of rays of light as from a centre.—Every visible body is a radiating body; it being only by means of its rays that it affects the eye.—The surface of a radiating or visible body, may be conceived as consisting of radiant points.

RADICAL *Sign*, in Algebra, the sign or character denoting the root of a quantity; and is this, $\surd$. So $\sqrt{2}$ is the square root of 2, and $\sqrt[3]{2}$ is the cube root of 2, &c.

RADIOMETER, a name which some writers give to the radius astronomicus, or Jacob's staff. See Fore-Staff.

RADIUS, in Geometry, the semidiameter of a circle; or a right line drawn from the centre to the circumference.—It is implied in the definition of a circle, and it is apparent from its construction, that all the radii of the same circle are equal.—The radius is sometimes called, in trigonometry, the sinus totus, or whole sine.

Radius, in the Higher Geometry. **Radius** *of the Evoluta*, **Radius** *Osculi*, called also the *Radius of concavity*, and the *Radius of curvature*, is the right line CB, representing a thread, by whose evolution from off the curve AC, upon which it was wound, the curve AB is formed. Or it is the radius of a circle having the same curvature, in a given point of the curve at B, with that of the curve in that point. See **Curvature** and **Evolute**, where the method of finding this radius may be seen.



Radius *Astronomicus*, an instrument usually called Jacob's staff, the Cross-staff, or Fore-staff.

Radius, in Mechanics, is applied to the spokes of a wheel; because issuing like rays from its centre.

Radius, in Optics. See **Ray**.

Radius *Vector*, is used for a right line drawn from the centre of force in any curve in which a body is supposed to move by a centripetal force, to that point of the curve where the body is supposed to be. In the elliptical orbit of a planet, let a = the greater semiaxis; ae = distance from the centre to the focus, or e = excentricity for the greater semiaxis 1, v = true anomaly, and u = excentric anomaly; then the radius vector r is expressed by either of the following formulæ, $r = a(1 + e \cos. u)$ or $r = \frac{a(1-e^2)}{1-e \cos. v}$.

RADIX, or *Root*, is a certain finite expression or function, which, being evolved or expanded according to the rules proper to its form, produces a series. That finite expression, or radix, is also the value of the infinite series. So $\frac{1}{3}$ is the radix of ·3333 &c, because $\frac{1}{3}$ being evolved or expanded, by dividing 1 by 3, gives the infinite series ·3333 &c. In like manner, the radix

of $1 - r + r^2 - r^3 + r^4$ &c is $\frac{1}{1+r}$,

of $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16}$ &c is $\frac{1}{1+\frac{1}{2}}$,

of $1 - 1 + 1 - 1 + 1$ &c is $\frac{1}{1+1}$,

of $1 - 2 + 4 - 8 + 16$ &c is $\frac{1}{1+2}$,

of $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32}$ &c is $\frac{1}{2+1}$,

of $1 + x + x^2 + x^3 + x^4$ &c is $\frac{1}{1-x}$,

of $1 + 2x + 3x^2 + 4x^3 + 5x^4$ &c is $\frac{1}{(1-x)^2}$,

of $1 + \frac{x^2}{2} + \frac{3x^4}{8} + \frac{5x^6}{16} + \frac{35}{128}x^8$ &c is $\sqrt{\frac{1}{1-x^2}}$.

See my Tracts, vol. 1, tracts 7 and 8.

RAFTERS, in Architecture, are pieces of timber which stand by pairs on the raising-piece, or wall plate, and meet in an angle at the top, forming the roof of a building. These commonly rise at 45°, and meet in a right angle at top; and then the roof is said to be of a true pitch.

RAIN, water that descends from the atmosphere in the form of drops of a considerable size. Rain is apparently a precipitated cloud; as clouds are nothing but vapours raised from moisture, waters, &c. By this circumstance it is distinguished from dew and fog: in the former of which the drops are so small that they are quite invisible; and in the latter, though their size be larger, they seem to have very little more specific gravity than the atmosphere itself, and may therefore be reckoned hollow spherules rather than drops.

It is universally agreed, that rain is produced by the water previously absorbed by the heat of the sun, or otherwise, from the terraqueous globe, into the atmosphere, as vapours, or vesiculæ. These vesiculæ, being specifically lighter than the atmosphere, are buoyed up by it, till they arrive at a region where the air is in a just balance with them; and there they float, till by some new agent they are converted into clouds, and thence either into rain, snow, hail, mist, or the like.

But the agent in this formation of the clouds into rain, and even of the vapours into clouds, has been much controverted. Most philosophers will have it, that the cold, which constantly occupies the superior regions of the air, chills and condenses the vesiculæ, at their arrival from a warmer quarter; congregates them together, and occasions several of them to coalesce into little masses: and thus their quantity of matter increasing in a higher proportion than their surface, they become an overload to the thin air, and so descend in rain.

Dr. Derham accounts for the precipitation, from the vesiculæ being full of air; when they meet with a colder air than that they contain, this is then contracted into a less space: and consequently the watry shell or case becomes thicker, so as to become heavier than the air, &c.

But this separation cannot be ascribed to cold, since rain often takes place in very warm weather. And though we should suppose the condensation owing to the cold of the higher regions, yet there is a remarkable fact which will not allow us to have recourse to this supposition: for it is certain that the drops of rain increase in size considerably as they descend. On the top of a hill for instance, they will be small and inconsiderable, forming only a drizzling shower; but half way down the hill it is much more considerable; and at the bottom the drops will be very large, descending in an impetuous rain. Which shows that the atmosphere condenses the vapours as well where it is warm as where it is cold.

Others allow the cold only a part in the action, attributing to the wind a considerable part of the agency: alleging, that a wind blowing against a cloud will drive its vesiculæ upon one another, by which means several of them coalescing as before, will be enabled to descend; and that the effect will be still more considerable, if two opposite winds blow together towards the same place: they add, that clouds already formed, happening to be aggregated by fresh accessions of vapour continually ascending, may thence be enabled to descend.

Yet the grand cause, according to Rohault, is still behind. That author conceives it to be the heat of the air, which, after continuing for some time near the earth, is at length carried up on high by a wind, and there thawing the snowy villi or flocks of the half-frozen vesiculæ, it reduces them into drops; which, coalescing, descend, and have their dissolution perfected in their progress through the lower and warmer stages of the atmosphere.

Others, as Dr. Clarke, &c, ascribe this descent of the clouds rather to an alteration of the atmosphere than of the vesiculæ; and suppose it to arise from a diminution of the spring or elastic force of the air. This elasticity, which depends chiefly or wholly on the dry terrene exhalations, being weakened, the atmosphere sinks under its burden; and the clouds fall, on the common principle of precipitation.

Now the small vesiculæ, by these or any other causes, being once upon the descent, will continue to descend notwithstanding the increase of resistance they every moment meet with in their progress through still denser and denser parts of the atmosphere. For as they all tend toward the same point, viz, the centre of the earth, the farther they fall, the more coalitions will they make; and the more coalitions, the more matter will there be under the same surface; the surface only increasing as the squares, but the solidity as the cubes of the diameters: and the more matter under the same surface, the less friction or resistance there will be to the same matter.

Thus then, if the causes of rain happen to act early enough to precipitate the ascending vesiculæ, before they are arrived at any considerable height, the coalitions being few in so short a descent, the drops will be proportionably small; thus forming what is called dew. If the vapours prove more copious, and rise a little higher, there is produced a mist or fog. A little higher still, and they produce a small rain, &c. If they neither meet with cold nor wind enough to condense or dissipate them; they form a heavy, thick, dark sky, which lasts sometimes several days, or even weeks.

But later writers on this part of philosophical science have, with greater show of truth, considered rain as an electrical phenomenon. Signior Beccaria counts rain, hail, and snow, among the effects of a moderate electricity in the atmosphere. Clouds that bring rain, he thinks are produced in the same manner as thunder clouds, only by a moderate electricity. He describes them at large; and the resemblance which all their phenomena bear to those of thunder clouds, is very striking. He notes several circumstances attending rain without lightning, which render it probable that it is produced by the same cause as when it is accompanied with lightning. Light has been seen among the clouds by night in rainy weather; and even by day rainy clouds are sometimes seen to have a brightness evidently independent of the sun. The uniformity with which the clouds are spread, and with which the rain falls, he thinks are evidences of a uniform cause like that of electricity. The intensity also of electricity in his apparatus usually corresponded very nearly to the quantity of rain that fell in the same time. Sometimes all the phenomena of thunder, lightning, hail, rain, snow, and wind, have been observed at one time; which shows the connection they all have with some common cause. Signior Beccaria therefore supposes that, previous to rain, a quantity of electric matter escapes out of the earth, in some place where there is a redundancy of it; and in its ascent to the higher regions of the atmosphere, collects and conducts into its path a great quantity of vapours. The same cause that collects, will condense them more and more; till, in the places of the nearest intervals, they come almost into contact, so as to form small drops; which, uniting with others as they fall, come down in the form of rain. The rain will be heavier in proportion as the electricity is more vigorous, and the cloud approaches more nearly to a thunder cloud: &c. See Lettere dell Elettricismo; and Priestley's Hist. &c of Electricity, vol. 1, pa. 427, &c, 8vo. And for further accounts of the phenomena of rain, &c, see BAROMETER, EVAPORATION, OMBROMETER, PLUVIAMETER, VAPOUR, &c. See also the Theory of Rain, by Dr. James Hutton, art 2, vol. 1. of Transactions of the Royal Society of Edinburgh.

Quantity of RAIN. As to the general quantity of rain that falls, with its proportion in several places at the same time, and at the same place in different times, there are many observations, journals, &c, in the Philos. Trans., the Memoirs of the French Academy, &c.

It has been ascertained by observation, that the mean annual quantity of rain is greatest at the equator, where it decreases gradually towards the poles. Thus, at

Granada, Antilles, lat.	12°		it is 126 inches.
St. Domingo - -	19	46′	- 120
Calcutta - -	22	23	- 81
Rome - -	41	54	- 39
England - -	33	00	- 32
Petersburg - -	59	16	- 16

Philos. Mag. vol. 44, pa. 350. Hence it appears that the quantity of rain is influenced generally by the heat of the climate. But it is also much influenced by particular local causes and circumstances, as affected by hills and mountains, and by the vicinity of seas, &c, as further appears by the following tables and observations. Thus on measuring the rain that falls annually, its depth, on a medium, in several places, is found as in the following table:

Mean Annual Depth of Rain for several Places.

At	*Observed by*	*Inch.*
Townley, in Lancas.	Mr. Townley	$42\frac{1}{2}$
Upminster, in Essex	Dr. Derham	$19\frac{1}{4}$
Zurich, Swisserland	Dr. Scheuchzer	$32\frac{1}{4}$
Pisa, in Italy	Dr. Mich. Ang. Tilli	$43\frac{1}{4}$
Paris, in France	M. Lahire	19
Lisle, Flanders	M. De Vauban	24

Quantity of Rain fallen in several Years at Paris and Upminster.

At Paris.	*Years.*	*At Upminster.*
Inches 21·37	1700	19·03 inches.
27·77	1701	18·69
17·45	1702	20·38
18.51	1703	23·99
21·20	1704	15·80
14·82	1705	16·93
20·19	Mediums	19·14

Medium Quantity of Rain at London, for several Years, from the Philos. Trans.

Viz, in 1774	26·328 inches.
1775	24·083
1776	20·354
1777	25·371
1778	20·772
1779	26·785
1780	17·313
Medium of these 7 years	23·001

See also the Meteorological Journal of the Royal Society, published annually in the Philos. Trans. and the article PLUVIAMETER or OMBROMETER.

It is reasonably to be expected, and all experience shows, that the most rain falls in places near the sea coast, and less and less as the places are situated more inland. Some differences also arise from the circumstances of hills, valleys, &c. So when the quantity of rain fallen in one year at London, is 20 inches, that on the western coast of England will often be twice as much, or 40 inches, or more. Those winds also bring most rain, that blow from the quarter in which is the most and nearest sea; as our west and south-west winds.

It is also found, by the pluviameter or rain-gage, that, in any one place, the more rain is collected in the instrument, as it is placed nearer the ground; without any appearance of a difference, between two places, on account of their difference of level above the sea, provided the instrument is but as far from the ground at the one place as at the other. These effects are remarked in the Philos. Trans. for 1769 and 1771, the former by Dr. Heberden, and the latter by Mr. Daines Barrington. Dr. Heberden says, "A comparison having been made between the quantity of rain, which fell in two places in London, about a mile distant from one another, it was found, that the rain in one of them constantly exceeded that in the other, not only every month, but almost every time that it rained. The apparatus used in each of them was very exact, and both made by the same artist; and upon examining every probable cause, this unexpected variation did not appear to be owing to any mistake, but to the constant effect of some circumstance, which not being supposed to be of any moment, had never been attended to. The rain-gage in one of these cases was fixed so high, as to rise above all the neighbouring chimneys; the other was considerably below them; and there appeared reason to believe, that the difference of the quantity of rain in these two places, was owing to this difference in the placing of the vessel in which it was received. A funnel was therefore placed above the highest chimneys, and another on the ground of the garden belonging to the same house, and there was found the same difference between these two, though placed so near one another, which there had been between them, when placed at similar heights in different parts of the town. After this fact was sufficiently ascertained, is was thought proper to try whether the difference would be greater at a much greater height; and a rain-gage was therefore placed upon the square part of the roof of Westminster Abbey. Here the quantity of rain was observed for a twelvemonth, the rain being measured at the end of every month, and care being taken that none should evaporate by passing a very long tube of the funnel into a bottle through a cork, to which it was exactly fitted. The tube went down very near to the bottom of the bottle, and therefore the rain which fell into it would soon rise above the end of the tube, so that the water was no-where open to the air except for the small space of the area of the tube: and by trial it was found that there was no sensible evaporation through the tube thus fitted up.—The following table shows the result of these observations.

From July the 7th 1766, to July the 7th 1767, there fell in a rain-gage, fixed

1766.	Below the top of a house.	Upon the top of a house.	Upon Westminster Abbey.
	Inches.	Inches.	Inches.
From the 7th to the end of July	3·591	3·210	2·311
August	0·558	0·479	0·508
September	0·421	0·344	
October	2·364	2·061	1·416
November	1·079	0·842	0·632
December	1·612	1·258	0·994
1767, January	2·071	1·455	1·035
February	2·864	2·494	1·335
March	1·807	1·303	0·587
April	1·437	1·213	0·994
May	2·432	1·745	1·142
June	1·997	1·426	1·145
July 7	0·395	0·309	
	22·608	18·139	12·099

By this table it appears, that there fell below the top of a house above a fifth part more rain, than what fell in the same space above the top of the same house; and that there fell upon Westminster Abbey not much above one half of what was found to fall in the same space below the tops of the houses. This experiment has been repeated in other places with the same result. What may be the cause of this extraordinary difference, has not yet been discovered; but it may be useful to notice it, in order to prevent that error, which would frequently be committed in comparing the rain of two places without attending to this circumstance."

Such were the observations of Dr. Heberden on first announcing this circumstance, viz, of different quantities of rain falling at different heights above the ground. Two years afterwards, Daines Barrington Esq. made the following experiments and observations, to show that this effect, with respect to different places, respected only the several heights of the instrument above the ground at those places, without regard to any real difference of level in the ground at those places.

Mr. Barrington caused two other rain-gages, exactly like those of Dr. Heberden, to be placed, the one upon mount Rennig, in Wales, and the other on the plane below, at about half a mile's distance, the perpendicular height of the mountain being 450 yards, or 1350 feet; each gage being at the same height above the surface of the ground at the two stations.

The results of the experiment are as below:

1770.	Bottom of the mountain.	Top of the mountain.
	Inches.	Inches.
From July 6 to 16	0·709	0·648
July 16 to 29	2·185	2·124
July 29 to Aug. 10.	0·610	0·656
Sept. 9 both bottles had run over.		
Sept. 9 to 30	3·234	2·464
Oct. 17. both bottles had run over.		
Oct. 17 to 22	0·747	0·885
Oct. 22 to 29	1·281	1·388
Nov. 20 both bottles were broken by the frost.	8·766	8·165

"The inference to be drawn from these experiments," Mr. Barrington observes, "seems to be, that the increase of the quantity of rain depends upon its nearer proximity to the earth, and scarcely at all on the height of places, provided the rain-gages are fixed at about the same distance from the ground.

"Possibly also a much controverted point between the inhabitants of mountains and plains may receive a solution from these experiments; as in an adjacent valley, at least, very nearly the same quantity of rain appears to fall within the same period of time as on the neighbouring mountains."

Dr. Heberden also adds the following note. "It may not be improper to subjoin to the foregoing account, that, in places where it was first observed, a different quantity of rain would be collected, according as the rain-gages were placed above or below the tops of the neighbouring buildings; the rain-gage below the top of the house, into which the greater quantity of rain had for several years been found to fall, was above 15 feet above the level of the other rain-gage, which in another part of London was placed above the top of the house, and into which the lesser quantity always fell. This difference therefore does not, as Mr. Barrington justly remarks, depend on the greater quantity of atmosphere, through which the rain descends: though this has been supposed by some, who have thence concluded that this appearance might readily be solved by the accumulation of more drops, in a descent, through a great depth of atmosphere."

The quantity of rain that falls at Bombay is very extraordinary. The following register of the quantity fallen there in 8 successive years, is extracted from the Monthly Magazine for 1796, pa. 99.

In 1780. From	Inches
July 4 to Aug 4 -	20·4
Aug 5, to Sep. 7 -	17·4
Sep. 8, to Oct. 14 -	15·6
	53·4
1781. From	
June 14, to July 3 -	23·3
July 3, to Aug. 10 -	8·7
Aug. 11, to Sep. 3 -	24·1
Sep. 4, to Oct. 14, -	14·4
	70·5
1782. From	
May 28, to May 31	2·2
June 1, to July 2 -	8·3
July 3, to Aug. 9, -	29·0
Aug. 10, to 31 -	5·6
Sep. 1, to Oct. 5 -	6·7
	51·8
1783. From	
June 1, to July 3 -	25·7
July 4, to Aug. 7 -	30·3
Aug. 8, to 31 -	7·1
Sep. 1, to Oct. 4 -	9·9
	73 0
1784. From	
June 6, to 30 -	8·2
July - - -	9·4
1784—*continued.*	
August - - -	17·7
Sep. to Oct. 6 -	12·2
	47·5
1785. From	
May 29, to 31 -	5·5
June - - -	9·0
July - - -	25·3
August - - -	13·3
September - -	14·5
October 27 - -	2·6
	70·2
1786. From	
June 12, to 30 -	26·9
July - - -	25·6
August - - -	10·1
September - -	10·4
October 12 - -	1·0
	74·0
1787. From	
June 11, to 26 -	12·2
June 27, to July 31	32·9
August - -	15·5
September - -	9·3
October 12 - -	0·5
	70·4
Gen. yearly average	63·85

From this abstract it appears, that the rainy season commences about the beginning of June, and ends in the 2d week of October; and that July is the most rainy month, the general average of July being 22·7 inches, or above one-third of the whole. The heaviest rain that fell during these 8 years, was in 1782, on July 19, 6 inches, 20th, 5·6, 21st 6·4.

RAINBOW, *Iris*, or simply the *Bow*, is a meteor in form of a party-coloured arch, or semicircle, exhibited in a rainy sky, opposite to the sun, by the refraction and reflection of his rays in the drops of falling rain. There is also a secondary, or fainter bow, usually seen investing the former at some distance. Among naturalists, we also read of lunar rainbows, marine rainbows, &c.

The rainbow, Sir Isaac Newton observes, never appears but where it rains in the sunshine; and it may be represented artificially, by contriving water to fall in small drops like rain, through which the sun shining, exhibits a bow to the spectator placed between the sun and the drops, especially if there be disposed beyond the drops some dark body, as a black cloth, or such like.

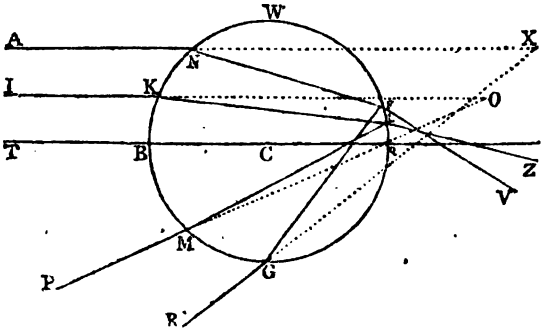
Some of the ancients, as appears by Aristotle's tract on Meteors, knew that the rainbow was caused by the refraction of the sun's light in drops of falling rain. Long afterwards, one Fletcher of Breslaw, in a treatise which he published in 1571, endeavoured more particularly to account for the colours of the rainbow by means of a double refraction, and one reflection. But he imagined that a ray of light, after entering a drop of rain, and suffering a refraction, both at its entrance and exit, was afterwards reflected from another drop, before it reached the eye of the spectator. It seems he overlooked the reflection at the farther side of the drop, or else he imagined that all the bendings of the light within the drop would not make a sufficient curvature, to bring the ray of the sun to the eye of the spectator. But Antonio de Dominis, Bishop of Spalato, about the year 1590, whose treatise De Radiis Visûs et Lucis was published in 1611 by J. Bartolus, first advanced, that the double refraction of Fletcher, with an intervening reflection, was sufficient to produce the colours of the rainbow, and also to bring the rays that formed them to the eye of the spectator, without any subsequent reflection. He distinctly describes the progress of a ray of light entering the upper part of the drop, where it suffers one refraction, and after being by that thrown upon the back part of the inner surface, is from thence reflected to the lower part of the drop; at which place undergoing a second refraction, it is thereby bent so as to come directly to the eye. To verify this hypothesis, he procured a small globe of solid glass, and viewing it when it was exposed to the rays of the sun, in the same manner in which he had supposed the drops of rain were situated with respect to them, he actually observed the same colours which he had seen in the true rainbow, and in the same order. Thus this author showed how the interior bow is formed in round drops of rain, viz, by two refractions of the sun's rays and one reflection between them; and he likewise showed that the exterior bow is formed by two refractions and two sorts of reflections between them in each drop of water.

The theory of A. de Dominis was adopted, and in some degree improved with respect to the exterior bow, by Descartes, in his treatise on Meteors; and indeed he was the first who, by applying mathematics to the investigation of this surprising appearance, ever gave a tolerable theory of the rainbow. Philosophers were however still at a loss when they endeavoured to assign reasons for all the particular colours, and for the order of them. Indeed nothing but the doctrine of the different refrangibility of the rays of light, a discovery which was reserved for the great Newton, could furnish a complete solution of this difficulty.

Dr. Barrow, in his Lectiones Opticæ, at Lect. 12, n. 14, says, that a friend of his (by whom we are to understand Mr. Newton) communicated to him a method of determining the angle of the rainbow, which was hinted to Newton by Slusius, without making a table of the refractions, as Descartes did. The doctor shows the method; as also several other matters, at n. 14, 15, 16, relating to the rainbow, worthy the genius of those two eminent men. But the subject was given more perfectly by Newton afterwards, viz, in his Optics, prop. 9; where he makes the breadth of the interior bow to be nearly 2° 15′, that of the exterior 3° 40′, their distance 8° 25′, the greatest semidiameter of the interior bow 42° 17′, and the least of the exterior 50° 42′, when their colours appear strong and perfect.

The doctrine of the rainbow may be illustrated and confirmed by experiment, in several different ways. Thus, by hanging up a glass globe, full of water, in the sunshine, and viewing it in such a posture that the rays which come from the globe to the eye, may include an angle either of 42° or 50°, with the sun's rays; for ex. if the angle be about 42°, the spectator will see a full red colour in that side of the globe opposite to the sun. And by varying the position so as to make that angle gradually less, the other colours, yellow, green, and blue, will appear successively, in the same side of the globe, and that very bright. But if the angle be made about 50°, suppose by raising the globe, there will appear a red colour in that side of the globe towards the sun, though somewhat faint; and if the angle be made greater, as by raising the globe still higher, this red will change successively to the other colours, yellow, green, and blue. And the same changes are observed by raising or depressing the eye, while the globe is at rest. Newton's Optics, pt. 2, prop. 9, prob. 4.

Again, a similar bow is often observed among the waves of the sea (called the marine rainbow), the upper parts of the waves being blown about by the wind, and so falling in drops. This appearance is also seen by moonlight (called the lunar rainbow), though seldom vivid enough to render the colours distinguishable. Also it is sometimes seen on the ground, when the sun shines on a very thick dew. Cascades and fountains too, whose waters are in their fall divided into drops, exhibit rainbows to a spectator, if properly situated during the time of the sun's shining; and even water blown violently out of the mouth of an observer, standing with his back to the sun, never fails to produce the same phenomenon. The artificial rainbow may even be produced by candlelight on the water which is ejected by a small fountain or jet d'eau. All these are of the same nature, and they depend on the same causes; some account of which is as follows.

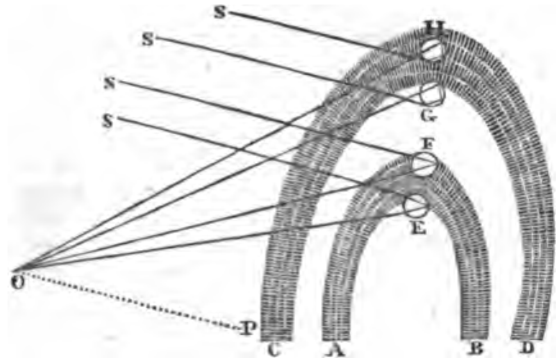


Let the circle WQGB represent a drop of water, or a globe, upon which a beam of parallel light falls, of which let TB represent a ray falling perpendicularly at B, and which consequently either passes through without refraction, or is reflected directly back from Q: suppose another ray IK, incident at K, at a distance from B; then this will be refracted according to a certain ratio of the sines of incidence and refraction to each other, which in rain water is as 529 to 396, to a point L, whence it will be in part transmitted in the direction LZ and in part reflected to M, where it will again in part be reflected, and in part transmitted in the direction MP, being inclined to the line described by the incident ray in the angle IOP. Another ray AN, still farther from B, and consequently incident under a greater angle, will be refracted to a point F, still farther from Q, whence it will be in part reflected to G, from which place it will in part emerge, forming an angle AXR with the incident AN, greater than that which was formed between the ray MP and its incident ray. And thus, while the angle of incidence, or distance of the point of incidence from B, increases, the distance between the point of reflection and Q, and the angle formed between the incident and emergent reflected rays, will also increase; that is, as far as it depends on the distance from B: but as the refraction of the ray tends to carry the point of reflection towards Q, and to diminish the angle formed between the incident and emergent reflected ray, and that the more the greater the distance of the point of incidence from B, there will be a certain point of incidence between B and W, with which the greatest possible distance between the point of reflection and Q, and the greatest possible angle between the incident and emergent reflected ray, will correspond. So that a ray incident nearer to B shall, at its emergence after reflection, form a less angle with the incident, by reason of its more direct reflection from a point nearer to Q; and a ray incident nearer to W, shall at its emergence form a less angle with the incident, by reason of the greater quantity of the angles of refraction at its incidence and emergence. The rays which fall for a considerable space in the vicinity of that point of incidence with which the greatest angle of emergence corresponds, will, after emerging, form an angle with the incident rays differing insensibly from that greatest angle, and consequently will proceed nearly parallel to each other; and those rays which fall at a distance from that point will emerge at various angles, and consequently will diverge. Now, to a spectator, whose back is turned towards the radiant body, and whose eye is at a considerable distance from the globe or drop, the divergent light will be scarcely, if at all, perceptible; but if the globe be so situated, that those rays that emerge parallel to each other, or at the greatest possible angle with the incident, may arrive at the eye of the spectator, he will, by means of those rays, behold it nearly with the same splendour at any distance.

In like manner, those rays which fall parallel on a globe, and are emitted after two reflections, suppose at the points F and G, will emerge at H parallel to each other, when the angle they make with the incident AN is the least possible; and the globe must be seen very resplendent when its position is such, that those parallel rays fall on the eye of the spectator.

The quantities of these angles are determined by calculation, the proportion of the sines of incidence and refraction to each other being known. And this proportion being different in rays which produce different colours, the angles must vary in each. Thus it is found, that the greatest angle in rain water for the least refrangible, or red rays, emitted parallel after one reflection, is 42° 2′, and for the most refrangible or violet rays, emitted parallel after one reflection, 40° 17′; likewise, after two reflections, the least refrangible, or red rays, will be emitted nearly parallel under an angle of 50° 57′, and the most refrangible, or violet, under an angle of 54° 7′; and the intermediate colours will be emitted nearly parallel at intermediate angles.

Suppose now, that O is the spectator's eye, and OP a line drawn parallel to the sun's rays, SE, SF, SG, and SH; and let POE, POF, POG, POH be angles of 40° 17′, 42° 2′, 50° 57′, and 54° 7′ respectively; then these angles turned about their common side OP, will with their other sides OE, OF, OG, OH, describe the verges of the two rainbows, as in the figure.



For, if E, F, G, H be drops placed any where in the conical superficies described by OE, OF, OG, OH, and be illuminated by the sun's rays SE, SF, SG, SH; the angle SEO being equal to the angle POE, or 40° 17′, will be the greatest angle in which the most refrangible rays can, after one reflection, be refracted to the eye, and therefore all the drops in the line OE must send the most refrangible rays most copiously to the eye, and so strike the sense with the deepest violet colour in that region. In like manner, the angle SFO being equal to the angle POF, or 42° 2′, will be the greatest in which the least refrangible rays after one reflection can emerge out of the drops, and therefore those rays must come most copiously to the eye from the drops in the line OF, and strike the sense with the deepest red colour in that region. And, by the same argument, the rays which have the intermediate degrees of refrangibility will come most copiously from drops between E and F, and strike the senses with the intermediate colours in the order which their degrees of refrangibility require; that is, in the progress from E to F, or from the inside of the bow to the outside, in this order, violet, indigo, blue, green, yellow, orange, red. But the violet, by the mixture of the white light of the clouds, will appear faint, and inclined to purple.

Again, the angle SGO being equal to the angle POG, or 50° 57′, will be the least angle in which the least refrangible rays can, after two reflections, emerge out of the drops, and therefore the least refrangible rays must come most copiously to the eye from the drops in the line OG, and strike the sense with the deepest red in that region. And the angle SHO being equal to the angle POH, or 54° 7′, will be the least angle in which the most refrangible rays, after two reflections, can emerge out of the drops, and therefore those rays must come most copiously to the eye from the drops in the line OH, and strike the sense with the deepest violet in that region. And, by the same argument, the drops in the regions between G and H will strike the sense with the intermediate colours in the order which their degrees of refrangibility require; that is, in the progress from G to H, or from the inside of the bow to the outside, in this order, red, orange, yellow, green, blue, indigo, and violet. And since the four lines OE, OF, OG, OH, may be situated any where in the above-mentioned conical superficies, what is said of the drops and colours in these lines, is to be understood of the drops and colours every where in those superficies.

Thus there will be made two bows of colours, an interior and stronger, by one reflection in the drops, and an exterior and fainter by two; for the light becomes fainter by every reflection; and their colours will lie in a contrary order to each other, the red of both bows bordering upon the space GF, which is between the bows. The breadth of the interior bow, EOF, measured across the colours, will be 1° 15′, and the breadth of the exterior GOH, will be 3° 10′, also the distance between them GOF, will be 8° 55′, the greatest semidiameter of the innermost, that is, the angle POF, being 42° 2′, and the least semidiameter of the outermost POG being 50° 57′. These are the measures of the bows as they would be, were the sun but a point; but by the breadth of his disc, the breadth of the bows will be increased by half a degree, and their distance diminished by as much; so that the breadth of the inner bow will be 2° 15′, that of the outer 3° 40′, their distance 8° 25′; the greatest semidiameter of the interior bow 42° 17′, and the least of the exterior 50° 42′. And such are the dimensions of the bows in the heavens found to be, very nearly, when their colours appear strong and perfect.

The light which comes through drops of rain by two refractions without any reflection, ought to appear strongest at the distance of about 26 degrees from the sun, and to decay gradually both ways as the distance from the sun increases and decreases. And the same is to be understood of light transmitted through spherical hailstones. If the hail be a little flatted, as it often is, the light transmitted may grow so strong at a little less distance than that of 26°, as to form a halo about the sun and moon; which halo, when the stones are duly figured, may be coloured, and then it must be red within, by the least refrangible rays, and blue without, by the most refrangible ones.

The light which passes through a drop of rain after two refractions, and three or more reflections, is scarce strong enough to cause a sensible bow.

As to the dimension of the rainbow, Descartes first determined its diameter by a tentative and indirect method; laying it down, that the magnitude of the bow depends on the degree of refraction of the fluid; and assuming the ratio of the sine of incidence to that of refraction, to be in water as 250 to 187. But Dr. Halley, in the Philos. Trans. number 267, gave a simple direct method of determining the diameter of the rainbow from the ratio of the refraction of the fluid being given; or, vice versa, the diameter of the rainbow being given, to determine the refractive power of the fluid. And Dr. Halley's principles and construction were further explained by Dr. Morgan, master of Clare Hall, Cambridge, in his Dissertation on the Rainbow, among the notes upon Rohault's System of Philosophy, part 3, chap. 17.

From the theory of the rainbow, all the particular phenomena of it are easily deducible. Hence we see, 1st, Why the iris is always of the same breadth; because the intermediate degrees of refrangibility of the rays between red and violet, which are its extreme colours, are always the same.

2dly, Why the bow shifts its situation as the eye does; and, according to the popular phrase, flies from those who follow it, and follows those that fly from it; the coloured drops being disposed under a certain angle, about the axis of vision, which is different in different places: whence also it follows, that every different spectator sees a different bow.

3dly, Why the bow is sometimes a larger portion of a circle, sometimes a less: its magnitude depending on the greater or less part of the surface of the cone, above the surface of the earth, at the time of its appearance; and the higher the sun is, the less will be the rainbow.

4thly, Why the bow never appears when the sun is above a certain altitude; the surface of the cone, in which it should be seen, being lost in the ground at a little distance from the eye, when the sun is above 42° high.

5thly, Why the bow never appears greater than a semicircle, on a plane; since, be the sun never so low, and even in the horizon, the centre of the bow is still in the line of aspect; which in this case runs along the earth, and is not at all raised above the surface. Indeed if the spectator be placed on a very considerable eminence, and the sun in the horizon, the line of aspect, in which the centre of the bow is, will be considerably raised above the horizon. And if the eminence be very high, and the rain near, it is possible the bow may be an entire circle.

6thly, How the bow may chance to appear inverted, or the concave side turned upwards; viz, a cloud happening to intercept the rays, and prevent their shining on the upper part of the arch: in which case, only the lower part appearing, the bow will seem as if turned upside down; which has probably been the case in several prodigies of this kind, related by authors.

Lunar Rainbow. The moon sometimes also exhibits the phenomenon of an iris, by the refraction of her rays in the drops of rain in the night-time. Aristotle says, he was the first that ever observed it; and adds, it is never seen but at the time of the full moon; her light at other times being too faint to affect the sight after two refractions and one reflection. The lunar iris has all the colours of the solar, very distinct and pleasant; only fainter, both from the different intensity of the rays, and the different disposition of the medium.

Marine Rainbow. This is a phenomenon sometimes observed in a much agitated sea; when the wind, sweeping part of the tops of the waves, carries them up; so that the sun's rays, falling upon them, are refracted, &c, as in a common shower, and there paint the colours of the bow. These bows are less distinguishable and bright than the common bow: but then they exceed as to numbers, there being sometimes 20 or 30 seen together. They appear at noon day, and in a position opposite to that of the common bow, the concave side being turned upwards, as indeed it ought to be.

RAIN-Gage, an instrument for measuring the quantity of rain that falls. It is the same as Ombrometer, or Pluviameter; which see.

RAKED *Table*, or Raking *Table*, in Architecture, a member hollowed in the square of a pedestal.

RAM, in Astronomy. See Aries,

Ram, *Battering*. See Battering *Ram*.

Rams-Horns, in Fortification, a name given by Belidor to the tenailles.

RAMPART, or Rampier, in Fortification, a massy bank or elevation of earth around a place, to cover it from the direct fire of an enemy, and of sufficient thickness to resist the efforts of their cannon for many days. It is formed into bastions, curtains, &c.

Upon the rampart the soldiers continually keep guard, and the pieces of artillery are planted for defence. Also, to shelter the men from the enemy's shot, the outside of the rampart is built higher than the rest, i. e. a parapet is raised upon it with a platform. It is encompassed with a moat or ditch, out of which is dug the earth that forms the rampart, which is raised sloping, that the earth may not slip down, and having a berme at bottom, or is otherwise fortified, being lined with a facing of brick or stone.

The height of the rampart need not be more than 3 fathoms, this being sufficient to cover the houses from the battery of the cannon; neither need its thickness be more than 10 or 12, unless more earth come out of the ditch than can otherwise be bestowed.—The ramparts of half-moons are the better for being low, that the small fire of the defendants may the better reach the bottom of the ditch; yet they must be so high as not to be commanded by the covert-way.

Rampart is also used, in civil architecture, for the void space left between the wall of a city and the houses. This is what the Romans called Pomœrium, where it was forbidden to build, and where they planted rows of trees for the people to walk and amuse themselves under.

RAMSDEN (Jesse), f.r.s. an excellent optician and mechanist, was born at Halifax in Yorkshire, 1730; and died at Brighthelmstone, Nov. 5, 1800. He served his apprenticeship in his native place, to the trade of a hot-presser; after which, about 1751, he came to London, and applied himself to engraving. In the course of this employment, mathematical instruments were often brought to him to be engraved, which induced him to try his genius in that way; and with such success, that by the year 1763 he made instruments for several of the best artists. Soon after his coming up to London, he married the daughter of Mr. Dollond, the celebrated optician in St. Paul's Church-yard; by which means he was introduced to the knowledge of a profession in which his genius enabled him to excel, and attract the approbation of the public, in the same manner as his private worth endeared him to society. In 1763 he opened a shop in the Haymarket; but in 1775 he removed to Piccadilly, where he carried on business till his death.

Mr. R. greatly improved Hadley's quadrant, or sextant; and he invented a curious machine for dividing mathematical instruments; for which discovery he received a premium from the board of longitude. He also improved the construction of the theodolite, as well as the barometer for measuring the heights of mountains. The pyrometer for measuring the dilatation of bodies by heat, also employed his talents; and he made many important discoveries and improvements in optics. But his astronomical instruments appear to have been the principal of his works. He improved the refracting micrometer, as also the transit instrument and quadrant. He procured a patent for an improved equatorial. His mural quadrants were excellent, and much sought for.—Mr. Ramsden was chosen a fellow of the Royal Society in 1786.—Being always of a slender frame of body, as well as of delicate constitution, in his latter years his health gradually declined; to recruit which, he had retired to Brighthelmstone, where he died as before observed.

RAMUS (Peter), a celebrated French mathematician and philosopher, was born in 1515, in a village of Vermandois in Picardy. He was descended of a respectable family, which had been reduced to extreme poverty by the wars and other misfortunes. His own life too, says Bayle, was the sport of fortune. In his infancy he was twice attacked by the plague. At an early age, a thirst for learning urged him to go to Paris; but he was soon forced by poverty to leave that city. He returned to it again as soon as he could; but, being unable to support himself, he left it a second time; yet his passion for study was so violent, that notwithstanding his bad success in the two former visits, he ventured on a third. He was maintained there some months by one of his uncles; after which he was obliged to become a servant in the college of Navarre. Here he spent the day in waiting on his masters, and the greatest part of the night in study.

After having finished classical learning and rhetoric, he went through a course of philosophy, which took him up three years and a half in the schools; but the thesis, which he made for his master-of-arts degrees, offended every one; for he maintained in it, that all that Aristotle had advanced was false; and he gave very good answers to the objections of the professors. This success encouraged him to examine the doctrine of Aristotle more closely, and to combat it vigorously: but he confined himself chiefly to his logic. The two first books he published, the one entitled, Institutiones Dialecticæ, the other Aristotelicæ Animadversiones, occasioned great disturbances in the university of Paris. The professors there, who were adorers of Aristotle, ought to have refuted Ramus's books, if they could, by writings and lectures: but instead of confining themselves within the just bounds of academical wars, they prosecuted this anti-peripatetic before the civil magistrate, as a man who was going to sap the foundations of religion. They raised such clamours, that the cause was carried before the parliament of Paris: but, perceiving that it would be examined equably, his enemies by their intrigues took it from that tribunal, to bring it before the king's council, in 1543. The king ordered, that Ramus and Anthony Govea, who was his principal adversary, should choose two judges each, to pronounce on the controversy, after they should have ended their disputation: while he himself appointed a deputy. Ramus appeared before the five judges, though three of them were his declared enemies. The dispute lasted two days, and Govea had all the advantages he could desire; Ramus's books being prohibited in all parts of the kingdom, and their author sentenced not to teach philosophy any longer; upon which his enemies triumphed in the most indecent manner.

The year after, the plague made great havoc in Paris, and forced most of the students in the college of Presle to quit it; but Ramus, being prevailed on to teach in it, soon drew together a great number of auditors. The Sorbonne attempted in vain to drive him from that college; for he held the headship of that house by arrêt of parliament: and through the patronage and protection of the cardinal of Lorrain, he obtained from Henry the 2d, in 1547, the liberty of speaking and writing, and the regal professorship of philosophy and eloquence in 1551. The parliament of Paris had, before this, maintained him in the liberty of joining philosophical lectures to those of eloquence; and this arrêt or decree had put an end to several prosecutions, which Ramus and his pupils had suffered. As soon as he was made regius professor, he was fired with a new zeal for improving the sciences, notwithstanding the hatred of his enemies, who were never at rest.

Ramus bore at that time a part in a very singular affair. About the year 1550, the royal professors corrected, among other abuses, that which had crept into the pronunciation of the Latin tongue. Some of the clergy followed this regulation; but the Sorbonnists were much offended at it as an innovation, and defended the old pronunciation with great zeal. Things at length were carried so far, that a minister, who had a good living, was very ill treated by them; and caused to be ejected from his benefice for having pronounced *quisquis*, *quanquam*, according to the new way, instead of *kiskis*, *kankam*, according to the old. The minister applied to the parliament; and the royal professors, with Ramus among them, fearing he would fall a victim to the credit and authority of the faculty of divines, for presuming to pronounce the Latin tongue according to their regulations, thought it incumbent on them to assist him. They accordingly went to the court of justice, and represented in such strong terms the indignity of the prosecution, that the minister was cleared, and every person had the liberty of pronouncing as he pleased.

Ramus was bred up in the Catholic religion, but afterwards deserted it. He first began to discover his new principles by removing the images from the chapel of his college of Presle, in 1552. On this account such a persecution was raised against him by the religionists, as well as Aristotelians, that he was driven out of his professorship, and obliged to conceal himself. For that purpose, with the king's leave he went to Fontainbleau; where, by the help of books in the king's library, he prosecuted geometrical and astronomical studies. As soon as his enemies found out his retreat, they renewed their persecutions; and he was forced to conceal himself in several other places. In the mean time, his curious and excellent collection of books in the college of Presle was plundered: but after a peace was concluded in 1563, between Charles the 9th and the Protestants, he again took possession of his employment, maintained himself in it with vigour, and was particularly zealous in promoting the study of the mathematics.

This continued till the second civil war in 1567, when he was forced to leave Paris, and shelter himself among the Hugonots, in whose army he was at the battle of St. Denys. Peace having been concluded some months after, he was restored to his professorship; but, foreseeing that the war would soon break out again, he did not care to venture himself in a fresh storm, and therefore obtained the king's leave to visit the universities of Germany. He accordingly undertook this journey in 1568, and received great honours wherever he came. He returned to France, after the third war in 1571; and lost his life miserably, in the massacre of St. Bartholomew's day, 1572, at 57 years of age. It is said, that he was concealed in a granary during the tumult; but discovered and dragged out by some peripatetic doctors who hated him; who, after stripping him of all his money under pretence of preserving his life, gave him up to the assassins, who, after cutting his throat and giving him many wounds, threw him out of the window; and his bowels gushing out in the fall, some Aristotelian scholars, encouraged by their masters, spread them about the streets; then dragged his body in a most ignominious manner, and threw it into the river.

Ramus was a great orator, a man of universal learning, and endowed with very fine qualities. He was sober, temperate, and chaste. He ate but little, and that of boiled meat; and drank no wine till the latter part of his life, when it was prescribed by the physicians. He lay upon straw; rose early, and studied hard all day; and led a single life with the utmost purity. He was zealous for the protestant religion, but at the same time a little obstinate, and given to contradiction. The protestant ministers did not love him much, for he made himself a kind of head of a party, to change the discipline of the protestant churches: his design was to introduce a democratical government in the church, but this design was traversed, and defeated in a national synod. His sect flourished however for some time afterwards, spreading pretty much in Scotland and England, and still more in Germany.

He published a great many books; but mathematics was chiefly obliged to him. Of this kind, his writings were principally these following:

1. Scholarum Mathematicarum libri 31.

2. Arithmeticæ libri duo.—Algebræ libri duo.—Geometriæ libri 27.

These were greatly enlarged and explained by Schoner, and published in 2 volumes 4to. There were several editions of them; mine is that of 1627, at Frankfort.—The geometry, which is chiefly practical, was translated into English by William Bedwell, and published in 4to, at London, 1636.

3. He published also a singular book on geometry, being the 15 books of Euclid; containing only the definitions, and general enunciations of the propositions, without diagrams or demonstrations. In a kind of preface, he says he thinks it better for the teacher to suppress these. Paris, 1558, 4to, fol. 44.

RANDOM-Shot, is a shot discharged with the axis of the gun elevated above the horizontal or point-blank direction.

Random, of a shot, also sometimes means the range of it, or the distance to which it goes at the first graze, or where it strikes the ground. See Range.

RANGE, in Gunnery, sometimes means the path a shot flies in. But more usually

Range means the distance to which the shot flies when it strikes the ground or other object, called also the amplitude of the shot. But range is the term in present use.

Were it not for the resistance of the air, the greatest range, on a horizontal plane, would be when the shot is discharged at an angle of 45° above the horizon; and all other ranges would be the less, the more the angle of elevation is above or below 45°; but so as that at equal distances above and below 45°, the two ranges are equal to each other. But, on account of the resistance of the air, the ranges are altered, and that in different proportions, both for the different sizes of the shot, and their different velocities: so that the greatest range, in practice, always lies below the elevation of 45°, and the more below it as the shot is smaller, and as its velocity is greater; thus the smallest balls, discharged with the greatest velocity in practice, range the farthest with an elevation of 30° or under, while the largest shot, with very small velocities, range farthest with nearly 45° elevation; and at all the intermediate degrees in the other cases. See Projectiles.

RARE, in Physics, is the quality of a body that is very porous, whose parts are at a great distance from one another, and which contains but little matter under a great magnitude. In which sense rare stands opposed to dense.

The corpuscular philosophers, viz, the Epicureans, Gassendists, Newtonians, &c, assert that some bodies are rarer than others, in virtue of a greater quantity of pores, or of vacuity lying between their parts or particles. The Cartesians hold, that a greater rarity only consists in a greater quantity of materia subtilis contained in the pores. And lastly, the Peripatetics contend, that rarity is a new quality superinduced on a body, without any dependence on either vacuity or subtile matter.

RAREFACTION, in Physics, the rendering a body rarer, that is bringing it to expand or occupy more room or space, without the accession of new matter: being thus opposed to condensation. The more accurate writers restrict the term rarefaction to that kind of expansion which is effected by means of heat: and the expansion from other causes they term Dilatation; if indeed there be other causes; for though some philosophers have attributed it to the action of a repulsive principle in the matter itself; yet from the many discoveries concerning the nature and properties of the electric fluid and fire, there is great reason to believe that this repulsive principle is no other than elementary fire.

The Cartesians deny any such thing as absolute rarefaction: extension, according to them, constituting the essence of matter, being obliged to hold all extension equally full. Hence they make rarefaction to be no other than an accession of fresh, subtile, and insensible matter, which, entering the parts of bodies, sensibly distends them.

It is by rarefaction that gunpowder has its effect; and to the same principle also we owe eolipiles, thermometers, &c. As to the air, the degree to which it is rarefiable exceeds all imagination, experience having shown it to be far above 10,000 times more than the usual state of the atmosphere; and as it is found to be above 1500 times denser in gunpowder than the atmosphere, it follows that experience has found it differ by about 15 millions of times. Perhaps indeed its degree of expansion is absolutely beyond all limits.

Such immense rarefaction, Newton observes, is inconceivable on any other principle than that of a repelling force inherent in the air, by which its particles mutually fly from one another. This repelling force, he observes, is much more considerable in air than in other bodies, as being generated from the most fixed bodies, and that with much difficulty, and scarcely without fermentation; those particles being always found to fly from each other with the greatest force, which, when in contact, cohere the most firmly together. See Air.

On the rarefaction of the air is founded the useful method of measuring altitudes by the barometer, in all the cases of which, the rarity of the air is found to be inversely as the force that compresses it, or inversely as the weight of all the air above it at any place.

RARITY, thinness, subtlety; the contrary to density.

RATCH, or Rash, in Clock-Work, a kind of wheel having 12 fangs, which serve to lift up the detents every hour, to make the clock strike.

RATCHETS, in a Watch, are the small teeth at the bottom of the fusee, or barrel, that stop it in winding up.

RATIO, according to Euclid, is the habitude or relation of two magnitudes of the same kind in respect of quantity. So the ratio of 2 to 1 is double, that of 3 to 1 triple, &c. Several mathematicians have found fault with Euclid's definition of a ratio, and others have as much defended it, especially Dr. Barrow, in his Mathematical Lectures, with great skill and learning.

Ratio is sometimes confounded with proportion, but very improperly, as being quite different things; for proportion is the similitude or equality or identity of two ratios. So the ratio of 6 to 2 is the same as that of 3 to 1, and the ratio of 15 to 5 is that of 3 to 1 also; and therefore the ratio of 6 to 2 is similar or equal or the same with that of 15 to 5, which constitutes proportion, being thus expressed, 6 is to 2 as 15 to 5, or thus $6 : 2 :: 15 : 5$, which means the same thing. So that ratio exists between two terms, but proportion between two ratios or four terms.

The two quantities that are compared, are called the Terms of the ratio, as 6 and 2; the first of these 6 being called the Antecedent, and the latter 2 the Consequent. Also the Index or Exponent of the ratio, is the quotient of the two terms: so the index of the ratio of 6 to 2 is $\frac{6}{2}$ or 3, and which is therefore called a Triple ratio.

Wolfius divides ratios into Rational and Irrational.

Rational RATIO is that which can be expressed between two rational numbers; as the ratio of 6 to 2, or of $6\sqrt{3}$ to $2\sqrt{3}$, 3 to 1. And

Irrational RATIO is that which cannot be expressed by that of one rational number to another; as the ratio of $\sqrt{6}$ to $\sqrt{2}$, or of $\sqrt{3}$ to root $\sqrt{1}$, that is $\sqrt{3}$ to 1, which cannot be expressed in rational numbers.

When the two terms of a ratio are equal, the ratio is said to be that of Equality; as of 3 to 3, whose index is 1, denoting the single or equal ratio. But when the terms are not equal, as of 6 to 2, it is a Ratio of Inequality.

Further, when the antecedent is the greater term, as in 6 to 2, it is said to be the Ratio of Greater Inequality: but when the antecedent is the less term, as in the ratio of 2 to 6, it is said to be the Ratio of Less Inequality. In the former case, if the less term be an aliquot part of the greater, the ratio of greater inequality is said to be Multiplex or Multiple; and the ratio of the less inequality, Sub-multiple. Particularly, in the first case, if the exponent of the ratio be 2, as in 6 to 3, the ratio is called Duple or Double; if 3, as in 6 to 2, it is Triple; and so on. In the second case, if the ratio be $\frac{1}{2}$, as in 3 to 6, the ratio is called Subduple; if $\frac{1}{3}$, as in 2 to 6, it is Subtriple; and so on.

If the greater term contain the less once, and one aliquot part of the same over; the ratio of the greater inequality is called Superparticular, and the ratio of the less Subsuperparticular. Particularly, in the first case, if the exponent be $\frac{3}{2}$ or $1\frac{1}{2}$, it is called Sesquialterate; if $\frac{4}{3}$ or $1\frac{1}{3}$, Sesquitertial; &c. In the other case, if the exponent be $\frac{2}{3}$, the ratio is called Subsesquialterate; if $\frac{3}{4}$, it is subsesquitertial.

When the greater term contains the less once and several aliquot parts over, the ratio of the greater inequality is called Superpartiens, and that of the less inequality is Subsuperpartiens. Particularly, in the former case, if the exponent be $\frac{5}{3}$ or $1\frac{2}{3}$, the ratio is called Superbipartiens tertias; if the exponent be $\frac{7}{4}$ or $1\frac{3}{4}$, Supertripartiens quartas; if $\frac{11}{7}$ or $1\frac{4}{7}$, Superquadripartiens septimas; &c. In the latter case, if the exponent be the reciprocals of the former, or $\frac{3}{5}$, the ratio is called Subsuperbipartiens tertias; if $\frac{4}{7}$, Subsupertripartiens quartas; if $\frac{7}{11}$, Subsuperquadripartiens septimas; &c.

When the greater term contains the less several times, and some one part over; the ratio of the greater inequality is called Multiplex-superparticular; and the ratio of the less inequality is called Submultiplex subsuperparticular. Particularly, in the former case, if the exponent be $\frac{5}{2}$ or $2\frac{1}{2}$, the ratio is called Dupla sesquialtera; if $\frac{13}{4}$ or $3\frac{1}{4}$, Tripla sesquiquarta, &c. In the latter case, if the exponent be $\frac{2}{5}$, the ratio is called Subdupla subsesquialtera; if $\frac{4}{13}$, Subtripla subsesquiquarta, &c. Lastly, when the greater term contains the less several times, and several aliquot parts over; the ratio of the greater inequality is called Multiplex superpartiens; that of the less inequality, Submultiplex subsuperpartiens. Particularly, in the former case, if the exponent be $\frac{8}{3}$ or $2\frac{2}{3}$, the ratio is called Dupla superbipartiens tertias; if $\frac{25}{7}$ or $3\frac{4}{7}$, Tripla superbiquadripartiens septimas, &c. In the latter case, if the exponent be $\frac{3}{8}$, the ratio is called Subdupla subsuperbipartiens tertias; if $\frac{7}{25}$, Subtripla subsuperquadripartiens septimas; &c.

These are the various denominations of rational ratios, names which are very necessary to the reading of the ancient authors; though they occur but rarely among the modern writers, who use instead of them the smallest numeral terms of the ratios; such as 2 to 1 for duple, and 3 to 2 for sesquialterate, &c.

Compound RATIO, is that which is made up of two or more other ratios, viz, by multiplying the exponents together, and so producing the compound ratio of the product of all the antecedents to the product of all the consequents. Thus the compound ratio of 5 to 3,
and 7 to 4,
is the ratio of - - - - - - 35 to 12.

Particularly, if a ratio be compounded of two equal ratios, it is called the Duplicate ratio; if of three equal ratios, the Triplicate ratio; if of four equal ratios, the Quadruplicate ratio; and so on, according to the powers of the exponents, for all Multiplicate ratios. So the several multiplicate ratios of

the simple ratio of - - 3 to 2, are thus, viz,
the duplicate ratio - - 9 : 4,
the triplicate ratio - - 27 : 8,
the quadruplicate ratio 81 : 16, &c.

Properties of RATIOS. Some of the more remarkable properties of ratios are as follow:

1. The like multiples, or the like parts, of the terms of a ratio, have the same ratio as the terms themselves. So $a : b$, and $na : nb$, and $\frac{a}{n} : \frac{b}{n}$ are all the same ratio.

2. If to, or from, the terms of any ratio, be added or subtracted either their like parts, or their like multiples, the sums or remainders will still have the same ratio. So $a : b$, and $a \pm na : b \pm nb$, and $a \pm \frac{a}{n} : b \pm \frac{b}{n}$ are all the same ratio.

3. When there are several quantities in the same continued ratio, a, b, c, d, e, &c; whatever ratio the first has to the 2d,

the 1st to the 3d has the duplicate of that ratio,
the 1st to the 4th has the triplicate of that ratio,
the 1st to the 5th has the quadruplicate of it,

and so on. Thus, the terms of the continued ratio being $1, r, r^2, r^3, r^4, r^5$, &c, where each term has to the following one the ratio of 1 to r, the ratio of the 1st to the 2d; then $1 : r^2$ is the duplicate, $1 : r^3$ the triplicate, $1 : r^4$ the quadruplicate, and so on, according to the powers of $1 : r$.

For other properties see PROPORTION.

To approximate to a RATIO *in smaller Terms.*—Dr. Wallis, in a small tract at the end of Horrox's works, treats of the nature and solution of this problem, but in a very tedious way; and he has prosecuted the same to a great length in his Algebra, chap. 10 and 11, where he particularly applies it to the ratio of the diameter of a circle to its circumference. Mr. Huygens too has given a solution, with the reasons of it, in a much shorter and more natural way, in his Descrip. Autom. Planet. Opera Reliqua, vol. 1, pa. 174. The same has also Mr. Cotes, at the beginning of his Harmon. Mensurarum. And several other persons have done the same thing, by the same or similar methods.

The problem is very useful, for expressing a ratio in small numbers, that shall be near enough in practice, to any given ratio in large numbers, such as that of the diameter of a circle to its circumference. The principle of all these methods, consists in reducing the terms of the proposed ratio into a series of what are called continued fractions, by dividing the greater term by the less, and the less by the remainder, and so on, always the last divisor by the last remainder, after the manner of finding the greatest common measure of the two terms; then connecting all the quotients &c together in a series of continued fractions; and lastly collecting gradually these fractions together one after another. So if $\frac{b}{a}$ be any fraction, or the exponent of any ratio; then dividing thus,

$$\begin{array}{l} a)\,\underline{b}\,(c \\ \quad d)\,\underline{a}\,(e \\ \qquad f)\,\underline{d}\,(g \\ \qquad\quad h)\,\underline{f}\,(i \\ \qquad\qquad k)\,\underline{h}\,(l \\ \qquad\qquad\quad \&c, \end{array}$$

gives c, e, g, i, &c, for the several quotients, and these, formed in the usual way, give the approximate value of the given ratio in a series of continued fractions; thus,

$$\frac{b}{a} = c + \cfrac{1}{e + \cfrac{1}{g + \cfrac{1}{i + \&c.}}}$$

Then collecting the terms of this series, one after another, so many values of $\frac{b}{a}$ are obtained, always nearer and nearer; the first value being c or $\frac{c}{1}$, the next

$$c + \frac{1}{e} = \frac{ce+1}{e} = \frac{A}{B},$$

the 3d value $c + \cfrac{1}{e + \cfrac{1}{g}} = c + \cfrac{1}{\cfrac{ge+1}{g}} = c + \frac{g}{ge+1} =$

$\frac{cge+c+g}{ge+1} = \frac{(ce+1)g+c}{ge+1} = \frac{Ag+c}{Bg+1} = \frac{C}{D}$; in like manner,

the 4th value is $\frac{Ci+A}{Di+B} = \frac{E}{F}$;

the 5th value is $\frac{El+C}{Fl+D} = \frac{G}{H}$; &c.

Hence comes this general rule: Having found any two of these values, multiply the terms of the latter of them by the next quotient, and to the two products add the corresponding terms of the former value, and the sums will be the terms of the next value, &c.

For example, let it be required to find a series of ratios in lesser numbers, constantly approaching to the ratio of 100000 to 314159, or nearly the ratio of the diameter of a circle to its circumference. Here first dividing, thus,

$$\begin{array}{l} 100000\,)\,\underline{314159}\,(\,3 = c \\ \quad d = 14159\,)\,\underline{100000}\,(\,7 = e \\ \qquad\quad f = 887\,)\,\underline{14159}\,(\,15 = g \\ \qquad\qquad\quad h = 854\,)\,\underline{887}\,(\,1 = i, \&c. \\ \qquad\qquad\qquad\qquad\quad \&c, \end{array}$$

there are obtained the quotients 3, 7, 15, 1, 25, 1, 7, 4. Hence 3 or $\frac{3}{1} = c$, the 1st value;

$\frac{ce+1}{e} = \frac{3.7+1}{1.7} = \frac{22}{7} = \frac{A}{B}$, the 2d value;

$\frac{Ag+c}{Bg+1} = \frac{22.15+3}{7.15+1} = \frac{333}{106} = \frac{C}{D}$, the 3d value;

$\frac{Ci+A}{Di+B} = \frac{333.1+22}{106.1+7} = \frac{355}{113} = \frac{E}{F}$, the 4th value; and so on; where the successive continual approximating values of the proposed ratio are $\frac{3}{1}$, $\frac{22}{7}$, $\frac{333}{106}$, $\frac{355}{113}$, &c; the 2d of these, viz, $\frac{22}{7}$, being the approximation of Archimedes; and the 4th, viz, $\frac{355}{113}$, is that of Metius, which is very near the truth, being equal to - - 3·1415929,
the more accurate ratio being - - 3·1415927.

The Doctrine of Ratios and Proportions, as delivered by Euclid, in the fifth book of his Elements, is considered by most persons as very obscure and objectionable, particularly the definition of proportionality; and several ingenious men have endeavoured to elucidate that subject. Among these, the Rev. Dr. Abram Robertson, of Oxford, profes. of Astron. printed a neat little paper there in 1789, for the use of his classes, being a demonstration of that definition, in 7 propositions, the substance of which is as follows. He first premises this advertisement:

"As demonstrations depending upon proportionality pervade every branch of mathematical science, it is a matter of the highest importance to establish it upon clear and indisputable principles. Most mathematicians, both ancient and modern, have been of opinion that Euclid has fallen short of his usual perspicuity in this particular. Some have questioned the truth of the definition upon which he has founded it, and almost all who have admitted its truth and validity have objected to it, as a definition. The author of the following propositions ranks himself amongst objectors of the last-mentioned description. He thinks that Euclid must have founded the definition in question upon the reasoning contained in the first six demonstrations here given, or upon a similar train of thinking; and in his opinion a definition ought to be as simple, or as free from a multiplicity of conditions, as the subject will admit."

He then lays down these four definitions:

"1. Ratio is the relation which one magnitude has to another, of the same kind, with respect to quantity."

"2. If the first of four magnitudes be exactly as great when compared to the second, as the 3d is when compared to the fourth, the first is said to have to the second the same ratio that the third has to the fourth."

"3. If the first of four magnitudes be greater, when compared to the second, than the third is when compared to the fourth, the first is said to have to the second a greater ratio than the third has to the fourth."

"4. If the first of four magnitudes be less, when compared to the second, than the third is when compared to the fourth, the first is said to have to the second a less ratio than the third has to the fourth."

Dr. Robertson then delivers the propositions, which are the following:

"*Prop.* 1. If the first of four magnitudes have to the second, the same ratio which the third has to the fourth; then, if the first be equal to the second, the third is equal to the fourth; if greater, greater; if less, less."

"*Prop.* 2. If the first of four magnitudes be to the second as the third to the fourth, and if any equimultiples whatever of the first and third be taken, and also any equimultiples of the second and fourth; the multiple of the first will be to the multiple of the second as the multiple of the third to the multiple of the fourth."

"*Prop.* 3. If the first of four magnitudes be to the second as the third to the fourth, and if any like aliquot parts whatever be taken of the first and third, and any like aliquot parts whatever of the second and fourth, the part of the first will be to the part of the second as the part of the third to the part of the fourth."

"*Prop.* 4. If the first of four magnitudes be to the second as the third to the fourth, and if any equimultiples whatever be taken of the first and third, and any whatever of the second and fourth; if the multiple of the first be equal to the multiple of the second, the multiple of the third will be equal to the multiple of the fourth; if greater, greater; if less, less."

"*Prop.* 5. If the first of four magnitudes be to the second as the third is to a magnitude less than the fourth, then it is possible to take certain equimultiples of the first and third, and certain equimultiples of the second and fourth, such, that the multiple of the first shall be greater than the multiple of the second, but the multiple of the third not greater than the multiple of the fourth."

"*Prop.* 6. If the first of four magnitudes be to the second as the third is to a magnitude greater than the fourth, then certain equimultiples can be taken of the first and third, and certain equimultiples of the second and fourth, such, that the multiple of the first shall be less than the multiple of the second, but the multiple of the third not less than the multiple of the fourth."

"*Prop.* 7. If any equimultiples whatever be taken of the first and third of four magnitudes, and any equimultiples whatever of the second and fourth; and if when the multiple of the first is less than that of the second, the multiple of the third is also less than that of the fourth; or if when the multiple of the first is equal to that of the second, the multiple of the third is also equal to that of the fourth; or if when the multiple of the first is greater than that of the second, the multiple of the third is also greater than that of the fourth: then, the first of the four magnitudes shall be to the second as the third to the fourth."

And all these propositions Dr. Robertson demonstrates by strict mathematical reasoning.

RATIO, *Section of.* See SECTION *of a Ratio.*

RATIONAL, in Arithmetic &c, the quality of numbers, fractions, quantities, &c, when they can be expressed by common numbers; in contradistinction to irrational or surd ones, which cannot be expressed in common numbers. Suppose any quantity to be 1; there are infinite other quantities, some of which are commensurable to it, either simply, or in power: these Euclid calls Rational quantities. The rest, that are incommensurable to 1, he calls Irrational quantities, or Surds.

RATIONAL *Horizon*, or *True Horizon*, is that whose plane is conceived to pass through the centre of the earth; and which therefore divides the globe into two equal portions or hemispheres. It is called the rational horizon, because only conceived by the understanding; in opposition to the sensible or apparent horizon, or that which is visible to the eye.

RAVELIN, in Fortification, was anciently a flat bastion, placed in the middle of a curtain. But

RAVELIN, is now a detached work, composed only of two faces, which form a salient angle usually without flanks; being a triangular work resembling the point of a bastion with the flanks cut off. It is raised before the curtain, on the counterscarp of the place; and serving to cover it and the adjacent flanks from the direct fire of an enemy. It is also used to cover a bridge or a gate, and is always placed without the moat.—There are also double ravelins, which serve to defend each other; being so called when they are joined by a curtain.—What the engineers call a ravelin, the men usually call a demilune, or half-moon.

RAY, in Geometry, the same as RADIUS.

RAY, in Optics, a beam or line of light, propagated from a radiant point, through any medium. If the parts of a ray of light lie all in a straight line between the radiant point and the eye, the ray is said to be Direct: the laws and properties of which make the subject of Optics. —If any of them be turned out of that direction, or bent in their passage, the ray is said to be Refracted.—If it strike on the surface of any body, and be thrown off again, it is said to be Reflected.—In each case, the ray, as it falls either directly on the eye, or on the point of reflection, or of refraction, is said to be Incident.

Again, if several rays be propagated from the radiant object equidistantly from one another, they are called Parallel rays. If they come inclining towards each other, they are called Converging rays. And if they go continually receding from each other, they are called Diverging rays.

It is from the different circumstances of rays, that the several kinds of bodies are distinguished in optics. A body, for example, that diffuses its own light, or emits rays of its own, is called a Radiating or Lucid or Luminous body. If it only reflect rays which it receives from another, it is called an Illuminated body. If it only transmit rays, it is called a Transparent or Translucent body. If it intercept the rays, or refuse them passage, it is called an Opaque body.

It is by means of rays reflected from the several points of illuminated objects to the eye, that these become visible, and that vision is performed; whence such Rays are called Visual rays.

The rays of light are not homogeneous, or similar, but differ in all the properties we know of; viz, refrangibility, reflexibility, and colour, and even heat. It is probably from the different refrangibility that the other differences have their rise; at least it appears that those rays which agree or differ in this, do so in all the rest. It is not however to be understood that the property or effect called colour, exists in the rays of light themselves; but from the different sensations the differently disposed rays excite in us, we call them Red rays, Yellow rays, &c. Each beam of light however, as it comes from the sun, seems to be compounded of all the kinds of rays mixed together; and it is only by splitting or separating the parts of it, that these different sorts become observable; and this is done by transmitting the beam through a glass prism, which refracting it in the passage, and the parts that excite the different colours having different degrees of refrangibility, they are thus separated from one another, and exhibited each apart, and appearing of the different colours.

Besides refrangibility, and the other properties of the rays of light already ascertained by observation and experiment, sir I. Newton suspects they may have many more; particularly a power of being inflected or bent by the action of distant bodies; and those rays which differ in refrangibility, he conceives likewise to differ in flexibility.

These rays he suspects may be very small bodies emitted from shining substances. Such bodies may have all the conditions of light: and there is that action and reaction between transparent bodies and light, which very much resembles the attractive force between other bodies. Nothing more is required for the production of all the various colours, and all the degrees of refrangibility, but that the rays of light be bodies of different sizes; the least of which may make violet the weakest and darkest of the colours, and be the most easily diverted by refracting surfaces from its rectilinear course; and the rest, as they are larger and larger, may make the stronger and more lucid colours, blue, green, yellow, and red. See COLOUR, LIGHT, REFRACTION, REFLECTION, INFLECTION, CONVERGING, DIVERGING, &c, &c.

Among other qualities of rays, it has been found by experiment, that there is a great difference in the heating power of solar rays. From Dr. Herschel's experiments it appears, that this heating power increases from the middle of the spectrum to the red ray, and is greatest beyond it, where the rays are invisible. Hence it is inferred that the rays of light and caloric nearly accompany each other, and the latter are in different proportions in the different coloured rays. They are easily separated from each other; as when the sun's rays are transmitted through a transparent body, the rays of light pass on seemingly undiminished, but the rays of caloric are intercepted. When the sun's rays are directed to an opaque body, the rays of light are reflected, but the rays of caloric are absorbed and retained. This is the case with the moon's light, which, however much it be concentrated, is not accompanied with heat. It has also been shown, that the different rays of light produce different chemical effects on the metallic salts and oxyds. These effects increase on the opposite direction of the spectrum, from the heating power of the rays. From the middle of the spectrum, towards the violet end, they become more powerful, and produce the greatest effect beyond the visible rays. From these discoveries it appears that the solar rays are of three kinds: 1. Rays which produce heat; 2. Rays which produce colour; and 3. Rays which deprive metallic substances of their oxygen. The first set of rays is in the greatest abundance, or are most powerful towards the red end of the spectrum, and are least refracted. The 2d set, or those which illuminate objects, are most powerful in the middle of the spectrum. And the 3d set produce the greatest effect towards the violet end, where the rays are most refracted. The solar rays pass through transparent bodies, without heating them. The atmosphere, for instance, receives no increase of heat by transmitting the sun's rays, till these rays are reflected from other bodies, or are communicated to it by bodies which have absorbed them. This is also proved by the sun's rays being transmitted through convex lenses, producing a high degree of temperature when they are concentrated, but giving no increase of heat to the glass itself. By this method the heat which proceeds from the sun can be greatly increased. Indeed, the intensity of heat produced in this way is equal to that of the hottest furnace. This is done, either by reflecting the sun's rays from a concave polished mirror, or by concentrating or collecting them, by the refractive power of convex lenses, and directing the rays, thus concentrated, on the combustible body.

Reflected RAYS, those rays of light which are reflected, or thrown back again, from the surfaces of bodies upon which they strike. It is found that, in all the rays of light, the angle of reflection is equal to the angle of incidence.

Refracted RAYS, are those rays of light, which are bent or broken, in passing out of one medium into another.

Pencil of RAYS, a number of rays emitted from a point of an object, and diverging in the form of a cone.

Principal RAY, in Perspective, is the perpendicular distance between the eye and the vertical plane or table, as it is sometimes named.

RAY *of Curvature.* See *Radius of* CURVATURE.

REAUMUR (RENE-ANTOINE-FERCHAULT, Sieur de), a respectable French philosopher, was born at Rochelle in 1683. After the usual course of school education, he was sent to Poitiers to study philosophy, and, in 1699, to Bourges to study the law, the profession for which he was intended. But philosophy and mathematics having very early been his favourite pursuits, he quitted the law, and repaired to Paris in 1703, to pursue those sciences to the best advantage; and here his character procured him a seat in the academy in the year 1708; which he held till the time of his death, which happened the 18th of November 1757, at 74 years of age.

Reaumur soon justified the choice that was made of him by the academy. He made innumerable observations, and wrote a great number of pieces on the various branches of natural philosophy. His History of Insects, in 6 vols. quarto, at Paris, is his principal work. Another edition was printed in Holland, in 12 vols. 12mo. He made also great and useful discoveries concerning iron; showing how to change common wrought iron into steel, how to soften cast iron, and to make works in cast iron as fine as in wrought iron. His labours and discoveries on this subject were rewarded by the duke of Orleans, regent of the kingdom, by a pension of 12 thousand livres, equal to about 500l. sterling; which however he would not accept but on condition of its being put under the name of the academy, who might enjoy it after his death. It was owing to Reaumur's endeavours that there were established in France manufactures of tin-plates, of porcelain in imitation of china-ware, &c. They owe to him also a new thermometer, which bears his name, and is pretty generally used on the continent, while that of Fahrenheit is used in England, and some few other places. Reaumur's thermometer is a spirit one, having the freezing point at 0, and the boiling point at 80.

Reaumur is esteemed an exact and clear writer; and there is an elegance in his style and manner, which is not commonly found among those who have made only the sciences their study. He is represented also as a man of a most amiable disposition, and with qualities to make him beloved as well as admired. He left a great variety of papers and natural curiosities, which he bequeathed to the Academy of Sciences.—The works published by him, are the following:

1. The Art of changing Forged Iron into Steel; of Softening Cast Iron; and of making works of Cast Iron, as fine as of Wrought Iron. Paris, 1722, 1 vol. in 4to.

2. Natural History of Insects, 6 vols. in 4to.

His memoirs printed in the volumes of the Academy of Sciences, are very numerous, amounting to upwards of a hundred, and on various subjects, from the year 1708 to 1763, several papers in almost every volume.

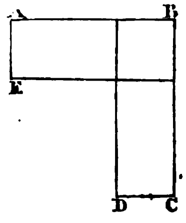
RECEIVER, of an Air-Pump, is part of its apparatus; being a glass vessel placed on the top of the plate, out of which the air is to be exhausted.

RECESSION *of the Equinoxes*. See Precession.

RECIPROCAL, in Arithmetic, &c, is the quotient arising by dividing 1 by any number or quantity. So, the reciprocal of 2 is $\frac{1}{2}$; of 3 is $\frac{1}{3}$, and of a is $\frac{1}{a}$, &c. Hence, the reciprocal of a vulgar fraction is found, by barely making the numerator and the denominator mutually change places: so the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$ or 2; of $\frac{2}{3}$, is $\frac{3}{2}$; of $\frac{a}{b}$, is $\frac{b}{a}$, &c. Hence also, any quantity being multiplied by its reciprocal, the product is always equal to unity or 1: so $\frac{1}{2} \times \frac{2}{1} = \frac{2}{2} = 1$, and $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$, and $\frac{a}{b} \times \frac{b}{a} = \frac{ab}{ab} = 1$.

See a large table of reciprocals of numbers, in my Tracts, vol. 1, at the end, also a method of finding them, pa. 463.

Reciprocal *Figures*, in Geometry, are such as have the antecedents and consequents of the same ratio in both figures. So, in the two rectangles BE and BD, if AB : DC : : BC : AE, then those rectangles are reciprocal figures; and are also equal.



Reciprocal *Proportion*, is when, in four quantities, the two latter terms have the reciprocal ratio of the two former, or are proportional to the reciprocals of them. Thus, 24, 15, 5, 8 form a reciprocal proportion, because $\frac{1}{24} : \frac{1}{15} : : 5 : 8$, or 15 : 24 : : 5 : 8.

Reciprocal *Ratio*, of any quantity, is the ratio of the reciprocal of the quantity.

Reciprocally. One quantity is reciprocally as another, when the one is greater in proportion as the other is less; or when the one is proportional to the reciprocal of the other. So a is reciprocally as b, when a is always proportional to $\frac{1}{b}$. Like as in the mechanic powers, to perform any effect, the less the power is, the greater must be the time of performing it: or, as it is said, what is gained in power, is lost in time. So that, if p denote any power or agent, and t the time of its performing any given service; then p is as $\frac{1}{t}$, and t is as $\frac{1}{p}$; that is, p and t are reciprocally proportionals to each other.

RECIPROCITY of prime numbers, a certain law that obtains with regard to the remainders of the formulæ $\frac{m^{\frac{n-1}{2}}}{n}$ and $\frac{n^{\frac{m-1}{2}}}{m}$, when n and m are both primes, first demonstrated by Legendre, and on which he has founded the demonstration of several curious numerical propositions. These remainders, for the sake of abridgement, may be written $(\frac{m}{n})$ and $(\frac{n}{m})$; that is, $(\frac{m}{n})$ representing the remainders of $\frac{m^{\frac{n-1}{n}}}{n}$, and $(\frac{n}{m})$ representing the remainders of $\frac{n^{\frac{m-1}{2}}}{m}$. Then there always subsists such a relation between these two expressions, that one being given, the other is immediately determined. For whatever may be the form of m and n, provided they are not both of the form $4a-1$, we shall always have $(\frac{n}{m}) = (\frac{m}{n})$; and if they are both of the form $4a-1$, then will $(\frac{n}{m}) = -(\frac{m}{n})$. It is not expected we should here enter upon the investigation of this theorem, but the reader who wishes to see the demonstration, will find ample gratification by consulting the work above alluded to.

RECKONING, in Navigation, is the estimating the quantity of a ship's way; or of the course and distance run. Or, more generally, a ship's reckoning is that account, by which it may at any time be known where the ship is, and consequently on what course or courses she must steer to gain her intended port. The reckoning is usually performed by keeping an account of the courses steered, and the distance run, with any accidental circumstances that occur. The courses steered are observed by the compass; and the distances run are estimated from the rate of running, and the time run upon each course. The rate of running is measured by the log, from time to time; which however is liable to great irregularities. Anciently Vitruvius, for measuring the rate of sailing, advised an axis to be passed through the sides of the ship, with two large heads protending out of of the ship, including wheels touching the water, by the revolution of which the space passed over in a given time is measured. And the same has been since recommended by Snell.

Reckoning, *Dead*. See Dead *Reckoning*.

RECLINATION *of a Plane*, in Dialling, is the angular quantity which a dial plane leans backwards, from an exactly upright or vertical plane, or from the zenith.

RECLINER, or Reclining *Dial*, is a dial whose plane reclines from the perpendicular, that is, leans backwards, or from you, when you stand before it.

Recliner, *Declining*, or *Declining* Reclining *Dial*, is one which neither stands perpendicularly, nor opposite to one of the cardinal points.

RECOIL, or Rebound, the resilition, or flying backward, of a body, especially a fire-arm. This is the motion by which, on explosion, it starts or flies backwards; and the cause of it is the resistance of the ball and the impelling force of the powder, which acts equally on the gun and on the ball. It has been commonly said by authors, that the momentum of the ball is equal to that of the gun with its carriage together; but this is a mistake; for the latter momentum is nearly equal to that of the ball and half the weight of the powder together, moving with the velocity of the ball. So that, if the gun, and the ball with half the powder, were of equal weight, the piece would recoil with the same velocity as the ball is discharged. But the heavier any body is, the less will its velocity be, to have the same momentum, or force; and therefore so many times as the cannon and carriage is heavier than the ball and half the powder, just as many times will the velocity of the ball be greater than that of the gun; and in the same ratio nearly is the length of the barrel before the charge, to the quantity the gun recoils in the time the ball is passing along the bore of the gun. So, if a 24 pounder of 10 feet long be 6400lb weight, and charged with 8lb of powder; then, when the ball quits the piece, the gun will have recoiled $\frac{28}{6400} \times 10 = \frac{7}{160}$ of a foot, or nearly half an inch.

RECORDE (Robert), a learned physician and mathematician, was born of a good family in Wales, and flourished in the reigns of Henry the 8th, Edward the 6th, and Mary. There is no account of the exact time of his birth, though it must have been early in the 16th century, as he was entered of the university of Oxford about the year 1525, where he was elected fellow of All-souls college in 1531. Making physic his profession, he went to Cambridge, where he was honoured with the degree of doctor in that faculty, in 1545, and was highly esteemed by all who knew him, for his great knowledge in several arts and sciences. He afterwards returned to Oxford, where, as he had done before he went to Cambridge, he publicly taught arithmetic, and other branches of the mathematics, with great applause. It seems he afterwards repaired to London, and it has been said he was physician to Edward the 6th and Mary, to which princes he dedicates some of his books; and yet he ended his days in the Fleet, where he was confined for debt, in the year 1558, at a very immature age. See other curious particulars of this author in my Tracts, vol. 2, pa. 243.

Recorde published several mathematical books, which are mostly in dialogue, between the master and scholar. They are as follow:

1. *The Pathway to Knowledge*, containing the first Principles of Geometrie, as they may moste aptly be applied unto practise, bothe for use of Instrumentes Geometricall and Astronomicall, and also for Projection of Plattes much necessary for all sortes of men. Lond. 4to, 1551.—This work, the author says, is the first book on Geometry ever printed in the English language.

2. *The Ground of Arts*, teaching the perfect worke and practice of Arithmeticke, both in whole numbers and fractions, after a more easie and exact forme then in former time hath beene set forth, 8vo. 1552.—It would seem however that there must have been some earlier edition than this, or another like work, since the author, in the dedication of his Geometry, or Pathway, Jan. 1551, says that he has "allready set forth somwhat of Arithmetike."—This work went through many editions, and was correcteed and augmented by several other persons; as first by the famous Dr. John Dee; then by John Mellis, a schoolmaster, 1590; next by Robert Norton; then by Robert Hartwell, practitioner in mathematics, in London; and lastly by R. C. and printed in 8vo, 1623.

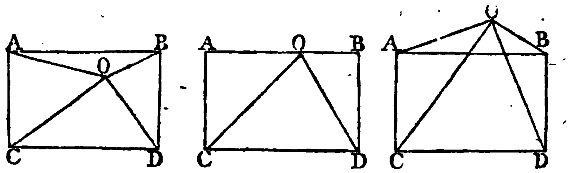
3. *The Castle of Knowledge*, containing the Explication of the Sphere bothe Celestiall and Materiall, and divers other things incident thereto. With sundry pleasaunt proofes and certaine newe demonstrations not written before in any vulgare woorkes. Lond. folio, 1556.

4. *The Whetstone of Witte*, which is the seconde part of Arithmetike: containing the Extraction of Rootes: the Cossike Practise, with the rules of Equation: and the woorkes of Surde Nombers. Lond. 4to, 1557.—For an analysis of this work on Algebra, with an account of what is new in it, see vol. 1, under the article Algebra.

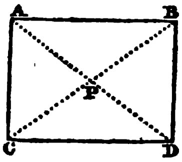
Wood says he wrote also several pieces on physic, anatomy, politics, and divinity; but I know not whether they were ever published. And Sherburne says that he published Cosmographiæ Isagogen; also that he wrote a book, De Arte faciendi Horologium; and another, De Usu Globorum, & de Statu Temporum; which I have never seen.—In the end of the preface to the Geometrical Theorems, in The Pathway to Knowledge, he sets down a list of many other books, partly mathematical and partly other subjects, which he says he had written, but not then published.

RECTANGLE, in Geometry, is a right-angled parallelogram, or a right-angled quadrilateral figure.

If from any point o, lines be drawn to all the four angles of a rectangle; then the sums of the squares of the lines drawn to the opposite corners will be equal, in whatever part of the plane the point o is situated; viz, $OA^2 + OD^2 = OB^2 + OC^2$. For other properties of the rectangle, see Parallelogram; for the rectangle being a species of the parallelogram, whatever properties belong to the latter, must equally hold in the former.



For the Area of a Rectangle. Multiply the length by the breadth or height. *Otherwise;* Multiply the product of the two diagonals by half the sine of their angle at the intersection.



That is, $AB \times AC$, or $AD \times BC \times \frac{1}{2} \sin. \angle P =$ area. A rectangle, as of two lines AB and AC, is thus denoted, $AB \times AC$, or $AB . AC$; or else thus expressed, the rectangle of, or under, AB and AC.

Rectangle, in Arithmetic, is the same with product or factum. So the rectangle of 3 and 4, is 3×4 or 12; and of a and b is $a \times b$ or ab.

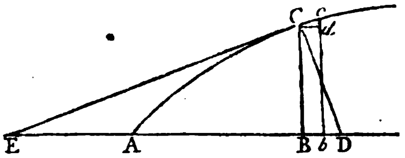
RECTANGLED, Right-angled, or Rectangular, is applied to figures and solids that have at least one right angle, if not more. So a right-angled triangle has one right angle: a right-angled parallelogram is a rectangle, and has four right angles. Such also are squares, cubes, and parallelopipedons. Solids are also said to be rectangular with respect to their situation, viz, when their axis is perpendicular to their base; as right cones, pyramids, cylinders, &c.

The ancients used the phrase Rectangular Section of a Cone, to denote a parabola; that conic section, before Apollonius, being only considered in a cone having its vertex a right angle. And hence it was, that Archimedes entitled his book of the quadrature of the parabola, by the name of Rectanguli Coni Sectio.

RECTIFICATION, in Geometry, is the finding of a right line equal to a curve. The rectification of curves is a branch of the higher geometry, in which the use of the inverse method of fluxions is particularly useful. This is a problem to which all mathematicians, both ancient and modern, have paid the greatest attention, and particularly as to the rectification of the circle, or finding the length of the circumference, or a right line equal to it; but hitherto without the perfect effect: on this also depends the quadrature of the circle, since it is demonstrated that the area of a circle is equal to a right-angled triangle, of which one of the sides about the right angle is the radius, and the other equal to the circumference: but it is much to be feared that neither the one nor the other will ever be accomplished. Innumerable approximations however have been made, from Archimedes, down to the mathematicians of the present day. See CIRCLE, and CIRCUMFERENCE.

The first person who gave the rectification of any curve, was Mr. Neal, son of Sir Paul Neal, as we find at the end of Dr. Wallis's Treatise on the Cissoid; where he says, that Mr. Neal's rectification of the curve of the semicubical parabola, was published in July or August, 1657. Two years after, viz, in 1659, Van Hureat, in Holland, also gave the rectification of the same curve; as may be seen in Schooten's Commentary on Descartes's Geometry.

The most comprehensive method of rectification of curves, is by the inverse method of fluxions, which is thus: Let ACc be any curve line, AB an absciss, and BC a perpendicular ordinate; also bc another ordinate indefinitely near to BC; and Cd drawn parallel to the absciss AB. Put the absciss AB $= x$, the ordinate BC $= y$, and the curve AC $= z$: then is Cd $=$ Bb $= \dot{x}$ the fluxion of the absciss AB, and $cd = \dot{y}$ the fluxion of the ordinate BC, also Cc $= \dot{z}$ the fluxion of the curve AB. Hence, because Ccd may be considered as a plane right-angled triangle, $Cc^2 = Cd^2 + cd^2$, or $\dot{z}^2 = \dot{x}^2 + \dot{y}^2$; and therefore $\dot{z} = \sqrt{(\dot{x}^2 + \dot{y}^2)}$; which is the fluxion of the length of any curve; and consequently, out of this equation expelling either $\dot{x}$ or $\dot{y}$, by means of the particular equation expressing the nature of the curve in question, the fluents of the resulting equation, being then taken, will give the length of the curve, in finite terms when it is rectifiable, otherwise in an infinite series, or in a logarithmic or exponential &c expression, or by means of some other curve, &c.



Ex. 1. *To rectify the common parabola.*—In this case, the equation of the curve is $2ax = y^2$, where a is half the parameter. The fluxion of this equation is $a\dot{x} = y\dot{y}$, and hence $\dot{x}^2 = \frac{y^2\dot{y}^2}{a^2}$; this being substituted in the general equation $\dot{z} = \sqrt{(\dot{x}^2 + \dot{y}^2)}$, it becomes $\dot{z} = \frac{\dot{y}\sqrt{(aa+yy)}}{a}$; the correct fluents of which give $z = \frac{y\sqrt{(aa + yy)}}{2a} + \frac{1}{2}a \times$ hyp. log. of $\frac{y + \sqrt{(aa + yy)}}{a}$, which is the length of the curve AC, when it is a parabola.

And the same might be expressed by an infinite series, by expanding the quantity $\sqrt{(aa + yy)}$. See my Mensuration, pa. 271, 4th edition.

Ex. 2. *To rectify the Circle.*—The equation of the circle may be expressed either in terms of the sine, or versed sine, or tangent, or secant, &c, and the radius. Let therefore the radius of the circle be DA or DC $= r$, the versed sine AB $= x$, the right sine BC $= y$, the tangent CE $= t$, and the secant DE $= s$; then, by the nature of the circle, we have these equations, $y^2 = 2rx - x^2 = \frac{r^2t^2}{r^2 + t^2} = \frac{s^2 - r^2}{s^2}r^2$; and by means of the fluxions of these equations, with the general equation $\dot{z}^2 = \dot{x}^2 + \dot{y}^2$, are obtained the following fluxional forms for the fluxion of the curve, the fluent of any one of which will be the curve itself, viz,

$$\dot{z} = \frac{r\dot{x}}{\sqrt{(2rx - xx)}} = \frac{r\dot{y}}{\sqrt{(r^2 - y^2)}} = \frac{r^2\dot{t}}{r^2 + t^2} = \frac{r^2\dot{s}}{\sqrt{(s^2 - r^2)}}.$$

Hence the value of the curve, from the fluent of each of these, gives the four following forms, in series, viz, putting $d = 2r$ the diameter, the curve is, z

$$= (1 + \frac{x}{2 \cdot 3d} + \frac{3x^2}{2 \cdot 4 \cdot 5d^2} + \frac{3 \cdot 5x^3}{2 \cdot 4 \cdot 6 \cdot 7d^3} \text{ \&c})\sqrt{dx},$$

$$= (1 + \frac{y^2}{2 \cdot 3r^2} + \frac{3y^4}{2 \cdot 4 \cdot 5r^4} + \frac{3 \cdot 5y^6}{2 \cdot 4 \cdot 6 \cdot 7r^6} \text{ \&c})y,$$

$$= (1 - \frac{t^2}{3r^2} + \frac{t^4}{5r^4} - \frac{t^6}{7r^6} + \frac{t^8}{9r^8} - \text{ \&c})t,$$

$$= (\frac{s - r}{s} + \frac{s^3 - r^3}{2 \cdot 3s^3} + \frac{3(s^5 - r^5)}{2 \cdot 4 \cdot 5s^5} + \text{ \&c})r.$$

See my Mensur. 4th edit. pa. 91 &c, also most treatises on Fluxions.

It is evident that the simplest of these series is the third, or that which is expressed in terms of the tangent. It will therefore be the properest form to calculate an example by in numbers. And for this purpose it will be convenient to assume some arc whose tangent, or at least its square, is known to be some small simple number. Now the arc of 45° it is known has its tangent equal to the radius; and therefore, taking the radius $r = 1$, and consequently the tangent of 45° or $t = 1$ also, in this case the arc of 45° to the radius 1, or the quadrant to the diameter 1, will be $= 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}$ &c. But as this series converges very slowly, some smaller arch must be taken, that the series may converge faster; such as the arc of 30°, whose tangent is $= \sqrt{\frac{1}{3}} = \cdot5773502$, or its square $t^2 = \frac{1}{3}$; and hence, after the first term, the succeeding terms will be found by dividing always by 3, and these quotients divided by the absolute numbers 3, 5, 7, 9, &c; and lastly adding every other term together into two sums, the one the sum of the positive terms, and the other the sum of the negative ones, then lastly the one sum taken from the other leaves the length of the arc of 30°, which is the 12th part of the whole circumference when the radius is 1, or the 6th part when the diameter is 1, and consequently 6 times that arc will be the length of the whole circumference to the diameter 1; therefore multiply the 1st term $\sqrt{\frac{1}{3}}$ by 6, and the product is $\sqrt{\frac{36}{3}}$ or $\sqrt{12} = 3\cdot4641016$; hence the operation will be conveniently made as follows:

		+ Terms.	− Terms.
1)	3.4641016	(3·4641016	
3)	1·1547005	(	0·3849002
5)	3849002	(769800	
7)	1283001	(	183286
9)	427667	(47519	
11)	142556	(	12960
13)	47519	(3655	
15)	15840	(	1056
17)	5280	(311	
19)	1760	(	93
21)	587	(28	
23)	196	(	8
25)	65	(3	
27)	22	(	1
		+ 3·5462332	− 0·4046406
		− 0·4046406	
		3·1415926	the circumference.

Various other series for the rectification of the circle may be seen in different parts of my Mensuration. See also my Tracts, vol. 1, Tr. 17 and 18.

RECTIFIER, in Navigation, is an instrument used for determining the variation of the compass, in order to rectify the ship's course. It consists of two circles, either laid upon, or let in to one another, and so fastened together in their centres that they represent two compasses, the one fixed, and the other moveable. Each is divided into 32 points of the compass, and 360°, and numbered both ways, from the north and the south, ending at the east and west in 90°. The fixed compass represents the horizon, in which the north, and all the other points, are liable to variation. In the centre of the moveable compass is fastened a silk thread, long enough to reach the outside of the fixed compass, except when the instrument is made of wood, in which case an index is used instead of the thread.

RECTIFYING *of Curves*. See Rectification.

Rectifying *of the Globe or Sphere*, is a previous adjustment of it, to prepare it for the solution of problems. This usually consists in placing it in the same position as the true sphere of the world has at some certain time proposed: which is done first by elevating the pole above the horizon as much as the latitude of the place is, then bringing the sun's place for the given day, found in the ecliptic, to the graduated side of the brass or general meridian, next move the hour-index to the upper hour of 12, so shall the globe be rectified for noon of that day; and if the globe be turned about till the hour-index point at any proposed hour, then is the globe in the real position of the earth at that time, if the whole globe be set in the north and south position by means of the compass.

RECTILINEAL, Rectilinear, or *Right-lined*, is the quality or nature of figures that are bounded by right lines, or formed by right lines.

RECURRING *Series*, is a series constituted in such a manner, that having taken at pleasure any number of its terms, each following term shall be related to the same number of preceding terms according to a constant law of relation. See *Recurring* Series.

Recurring *Decimals*. See Repetend.

RED, in Physics, or Optics, one of the simple or primary colours of natural bodies, or rather of the rays of light.—The red rays are the least refrangible of all the rays of light. And hence, as Newton supposes the different degrees of refrangibility to arise from the different magnitudes of the luminous particles of which the rays consist; therefore the red rays, or red light, is concluded to be that which consists of the largest particles. See Colour, and Light.—Authors distinguish three general kinds of red: one bordering on the blue, as colombine, or dove-colour, purple, and crimson; another bordering on yellow, as flame-colour and orange; and between these extremes is a medium, which is that which is properly called red.

REDANS, or Redant, or Redent, in Fortification, is a kind of work indented like the teeth of a saw, with salient and re-entering angles; to the end that one part may flank or defend another. It is called also Saw-work, and Indented work.—Redans are often used in fortifying of walls, where it is not necessary to be at the expense of building bastions; as when they stand on the side of a river, or a marsh, or the sea, &c. But the fault of such fortification is, that the besiegers from one battery may ruin both sides of the tenaille or front of a place, and make an assault without fear of being enfiladed, since the defences are ruined.—The parapet of the corridor also is frequently redented, or carried on by the way of redans.

REDOUBT, or Redoute, in Fortification, a small fort, without any defence but in front, used in trenches, lines of circumvallation, contravallation, and approach, as also for the lodging of corps de garde, and to defend passages.

A Detached Redoubt, is a kind of work resembling a ravelin, with flanks, placed beyond the glacis.—It is made to occupy some spot of ground which might be advantageous to the besiegers; and also to oblige the enemy to open his trenches farther off than he would otherwise do.

REDUCING *Scale*, or Surveying *Scale*, is a broad thin slip of box, or ivory, having several lines and scales of equal parts upon it; used by surveyors for turning chains and links into roods and acres, by inspection. They use it also to reduce maps and draughts from one dimension to another.

REDUCTION, in general, is the bringing or changing some thing to a different form, state, or denomination.

Reduction, in *Arithmetic*, is commonly understood of the changing of money, weights, or measures, to other denominations, of the same value; and it is of two kinds, Reduction Descending, which is the changing a number to its equivalent value in a lower denomination; as pounds into shillings or pence: and Reduction Ascending, which is the changing numbers to higher denominations; as pence to shillings or pounds. See the books on arithmetic.

Reduction *of Fractions*. See Fraction, and Decimal.

Reduction *of Equations*. See Equation.

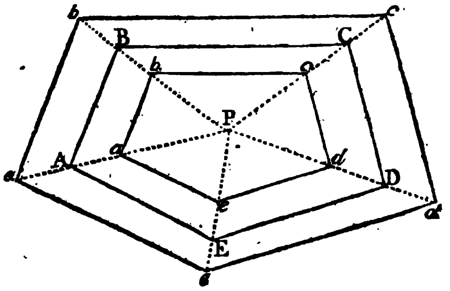
Reduction *of Curves*. See Curve.

Reduction *of a Figure, Design, or Draught*, is the making a copy of it, either larger or smaller than the original, but still preserving the form and proportion.

Figures and plans are reduced, and copied, in various ways; as by the pentagraph, and proportional compasses. See Pentagraph, and *Proportional* Compasses. The best of the other methods of reducing are as below.

To reduce a simple Rectilinear Figure by Lines.

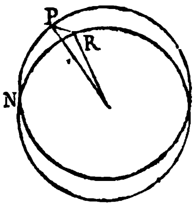
Choose a point P any where about the given figure ABCDE, either within it, or without it, or in one side or angle; but near the middle is best. From that point P draw lines through all the angles; on one of which take Pa to PA in the proposed proportion of the scales, or linear dimensions; then draw *ab* parallel to AB, *bc* to BC, &c; so shall *abcde* be the reduced figure sought, either greater or smaller than the original.



To Reduce a Figure by a Scale.—Measure all the sides, and diagonals, of the figure, as ABCDE, by a scale; and lay down the same measures respectively, from another scale, in the proportion required.

To Reduce a Map, Design, or Figure, by Squares. See Surveying, art. 21.

Reduction *to the Ecliptic*, in Astronomy, is the difference between the argument of latitude, as NP, and an arc of the ecliptic NR, intercepted between the place of a planet, and the node.—To find this reduction, or difference; in the right-angled spherical triangle NPR, are given the angle of inclination, and the argument of latitude NP; to find NR; then the dif. between NP and NR is the reduction sought.



REDUNDANT *Hyperbola*, is a curve of the higher kind, so called because it exceeds the conical hyperbola in the number of legs; being a triple hyperbola, with 6 hyperbolic legs. See Newton's Enum. Lin. tertii Ordinis, nomina formarum, &c.

RE-ENTERING *Angle*, in Fortification, is an angle whose point is turned inwards, or towards the place.

REFLECTED *Ray*, or *Vision*, is that which is made by the reflection of light, or by light first received upon the surface of some body, and thence reflected again. See Ray, Vision, and Reflection.

REFLECTING *Circle*, or *Semicircle*, an ingenious and useful instrument, adapted to the purposes of surveys, especially those of the military kind, in forming sketches in the practice of reconnoitering.

This instrument is the invention of sir Howard Douglas Bart. lieutenant-governor of the senior department of the Royal Military College, at Farnham; and which, with the many useful regulations and good management of the college, are so many verifications of the promising hopes indicated by his talents and regular good conduct in the Royal Mil. Acad. at Woolwich, where sir Howard received his military education.

The instrument combines the effect of a Hadley's quadrant and of a protractor, together; or it combines the measuring principle with a circular protractor, in such a manner, that the index or limb of the instrument shall describe the *whole* of the measured angle. By this contrivance, the angles taken in the field, may be protracted at once, in their real magnitude, on the sketch, without the trouble of reading off the degrees. It is therefore particularly useful in surveying, to determine the true situations of objects, at the same time that the ground is sketched.

To the radius or limb of a semicircular, or circular, protractor, ABC (pl. 31, fig. 2), is fixed the index glass DE; and the horizon glass FG is fixed on a bar, HI, which has a motion on the centre K. This bar slides on a pin O, attached to the limb or radius carrying the index glass; the pin being adjusted so, that there shall be no apparent index-error, and exactly in the same circle with the point K: the principal limb will then describe the whole angle measured.

Thus the new reflecting circle, or semircircle, is divided into 360°, 180°, instead of the double number, as in the repeating circle, and the length of the arc of the latter is equal to that of a sextant, whose radius is the length of the sliding bar, that is, the diameter of the circle. A vernier is applied to read off with accuracy. A 4-inch plotting or diagonal scale of 1 mile, divided into yards, is engraven on the fixed limb of the instrument; by help of which all the cases of trigonometry can be solved by construction.

To those acquainted with the common sextant, the use of the reflecting semicircle will be obvious. The eye is applied to the end of the bar Q: the instrument is held in the right hand, by the end of the fixed limb, and is directed so that the left, or direct object, is seen through the unsilvered part of the horizon glass. Apply the thumb of the left hand to the end of the moveable limb, and turn it till the other object is seen reflected in the lower part of the horizon glass; then PRS is the measured angle, which can be protracted at once, placing the centre R over the station.—The errors or mistakes arising from reading off in a hurry, are thus avoided; the operations of protracting the points, and sketching the features of the ground, are combined; and the sketch much sooner completed.

M is a small screw, to adjust the horizon glass perpendicular to the plane of the instrument; and N is another small screw, behind the index glass, to adjust it parallel to the horizon glass, when the vernier cuts 0 degree on the arc.

For other instruments of reflection, see Circular *Instrument*.

REFLECTING, or Reflexive, *Dial*, is a kind of dial which shows the hour by means of a thin piece of looking-glass plate, duly placed to throw the sun's rays to the top of a cieling, on which the hour-lines are drawn.

Reflecting *Telescope*, is one in which the rays, from the object to be viewed, are first received on a speculum, or polished reflecting surface, of a proper form, thence to another speculum, and so to the eye. See Telescope.

REFLECTION, or Reflexion, in Mechanics, is the return, or regressive motion of a moveable body, occasioned by the resistance of another body, which hinders it from pursuing its former course of direction.

Reflection is conceived, by the latest and best authors, as a motion peculiar to elastic bodies, by which, after striking on others which they cannot remove, they recede, or turn back, or aside, by their elastic power. On this principle it is asserted, that there may be, and is, a period of rest between the incidence and the reflection; since the reflected motion is not a continuation of the other, but a new motion, arising from a new cause or principle, viz, the power of elasticity.

It is one of the great laws of reflection, that the angle of incidence is equal to the angle of reflection; i. e. that the angle which the direction of motion of a striking body makes with the surface of the body struck, is equal to the angle made between the same surface and the direction of motion after the stroke. See Incidence and Percussion.

Reflection *of the Rays of Light*, like that of other bodies, is their motion after being reflected from the surfaces of bodies. The reflection of the rays of light from the surfaces of bodies, is the means by which those bodies become visible. And the disposition of bodies to reflect this or that kind of rays most copiously, is the cause of their being of this or that colour. Also, the reflection of light, from the surfaces of mirrors, makes the subject of catoptrics.

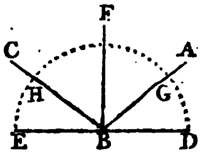
The reflection of light, Newton has shown, is not effected by the rays striking on the very parts of the bodies;

but by some power of the body equally diffused throughout its whole surface, by which it acts upon the ray, attracting or repelling it without any real immediate contact. This power he also shows is the same by which, in other circumstances, the rays are refracted; and by which they are at first emitted from the lucid body.

Dr. Priestley says, it is not more probable, that the rays of light are transmitted from the sun, with an uniform disposition to be reflected or refracted, according to the circumstances of the bodies on which they impinge; and that the transmission of some of the rays, apparently under the same circumstances, with others that are reflected; is owing to the minute vibrations of the small parts of the surfaces of the mediums through which the rays pass; vibrations that are independent of action and reaction between the bodies and the particles of light at the time of their impingeing, though probably excited by the action of preceding rays. Hist. of Light and Colours, pa. 309.

Newton concludes his account of the reflection of light with observing, that if light be reflected not by impingeing on the solid parts of bodies, but by some other principle, it is probable that as many of its rays as impinge on the solid parts of bodies are not reflected, but stifled and lost in the bodies. Otherwise, he says, we must suppose two kinds of reflection; for should all the rays be reflected which impinge on the internal parts of clear water or crystal, those substances would rather have a cloudy colour, than a clear transparency. To make bodies look black, it is necessary that many rays be stopped, retained and lost in them; and it does not seem probable that any rays can be stopped and stifled in them, which do not impinge on their parts: and hence, he says, we may understand, that bodies are much more rare and porous than is commonly believed. However, M. Bouguer disputes the fact of light being stifled or lost by impingeing on the solid parts of bodies.

Reflection, in Catoptrics, is the return of a ray of light from the polished surface of a speculum or mirror, as driven thence by some power residing in it. The ray thus returned is called a reflex or reflected ray, or a ray of reflection; and the point of the speculum where the ray commences, is called the point of reflection. Thus, the ray AB, proceeding from the radiant A, and striking on the point of the speculum B, being returned thence to C, BC represents the reflected ray, and B the point of reflection; in respect of which, AB represents the incident ray, or ray of incidence, and B the point of incidence; also the angle CBE is the angle of reflection, and ABD the angle of incidence; where DE is the reflecting surface, or at least a tangent to it at the point B. Though some count the angle of incidence and of reflection from the perpendicular BF.



General Laws of Reflection.—1. When a ray of light is reflected from a speculum of any form, the angle of incidence is always equal to the angle of reflection. This law obtains in the percussions of all kinds of bodies; and consequently must do so in those of light; the proof of which may be seen at the article Incidence. This law is confirmed also by experiments on all bodies: and on the rays of light in this manner: A ray from the sun falling on a mirror, in a dark room, through a small hole, will be seen to rebound, so as to make the angle of reflection equal to the angle of incidence. And the same may be shown in various other ways: thus ex. gr. placing a semicircle DFE on a mirror DE, its centre on B, and its limb or plane perpendicular to the speculum; then assuming equal arcs DG and EH; place an object in A, and the eye in C: then will the object be seen by a ray reflected from the point B. But by covering B, the object will cease to be seen.

II. Every point of a speculum reflects rays falling on it, from every point of an object.

III. If the eye C and the radiant point A change places, the point will continue to radiate upon the eye, in the same course or path as before.

IV. The plane of reflection is perpendicular to the surface of the speculum; and it passes through the centre in spherical specula.

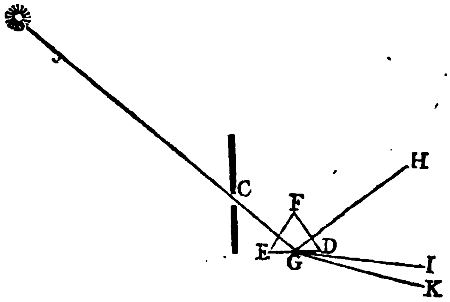
Reflection *of the Moon*, is a term used by some authors for what is otherwise called her variation; being the 3d inequality in her motion, by which her true place out of the quadratures differs from her place twice equated.

Reflection is also used in the Copernican system, for the distance of the pole from the horizon of the disc; which is the same thing as the sun's declination in the Ptolemaic system.

REFLECTOIRE Curve. See *Reflectoire* Curve.

REFLEXIBILITY *of the Rays of Light*, is that property by which they are disposed to be reflected. Or, it is their disposition to be turned back into the same medium, from any other medium on whose surface they fall. Hence those rays are said to be more or less reflexible, which are returned back more or less easily under the same incidence. Thus, if light pass out of glass into air, and by being inclined more and more to the common surface of the glass and air, begins at length to be totally reflected by that surface, those kinds of rays which at like incidences are reflected most copiously, or the rays which by being inclined begin soonest to be totally reflected, are the most reflexible rays.

That rays of light are of different colours, and endued with different degrees of reflexibility, was first discovered by sir I. Newton; and it is shown by the following experiment. Applying a prism DFE to the aperture C of a darkened room, in such manner that the light be reflected from the base in G; the violet rays are seen first reflected into HG; the other rays continuing still refracted to I and K. After the violet, the blue are all reflected; then the green, &c.—Hence it appears, that the differently coloured rays differ in degree of reflexibility. And from other experiments it appears, that those rays which are most reflexible, are also most refrangible.



REFLUX *of the Sea*, is the ebbing of the water, or its return from the shore; being so called, because it is the opposite motion to the flood or flux. See Tide.

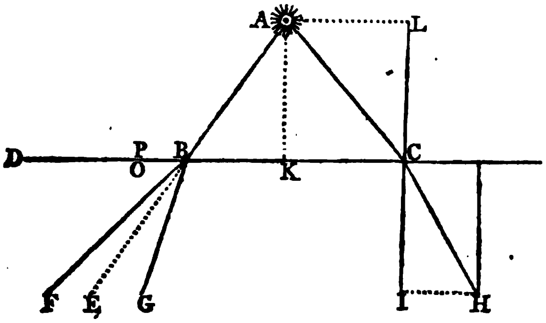
REFRACTED *Angle*, or *Angle of Refraction*, in Optics, is the angle which the refracted ray makes with the refracting surface; or sometimes it denotes the complement of that, or the angle it makes with the perpendicular to the said surface.

Refracted *Dials*, or *Refracting Dials*, are such as show the hour by means of some refracting transparent fluid.

Refracted *Ray*, or *Ray of* Refraction, is a ray after it is broken or bent, at the common surface of two different mediums, where it passes from the one into the other. See Ray, and Refraction.

Refracting *Telescope*, is one by which the rays from an object are transmitted to the eye through certain lenses of a proper form. See Telescope.

REFRACTION, in Mechanics, is the deviation of a moving body from its direct course, by reason of the different density of the medium it moves in; or a flexion and change of determination, occasioned by a body's passing obliquely out of one medium into another of a different density.



Thus, a ball A, moving in the air in the line AB, and falling obliquely on the surface of the water CD, does not proceed straight in the same direction, as to E, but deviates or is deflected to F. Again, if the ball move in water in the line AB, and fall obliquely on a surface of air CD; it will in this case also deviate from the same continued direction BE, but the contrary way, and will go to G, on the other side of it. Now the deflection in either case is called the Refraction, the refraction being towards the denser surface BD in the former case, but from it in the latter.

These refractions are supposed to arise from hence; that the ball arriving at B, in the first case finds more resistance or opposition on the one side O, or from the side of the water, than it did from the side P, or that of the air; and in the latter more resistance from the side P, which is now the side of the water, than the side O, which is that of the air. And so for any other different media: a visible instance of which is often perceived in the falling of shot or shells into the earth, as clay &c, when the perforation is found to rise a little upwards, toward the surface. However, another reason is assigned for the refraction of the rays of light, whose refractions lie the contrary way to those above, as will be seen in what follows, viz, that water by its greater attraction accelerates the motion of the rays of light more than air does.

Refraction *of Light*, in Optics, is an inflection or deviation of the rays from their rectilinear course on passing obliquely out of one medium into another, of a different density. That a body may be refracted, it is necessary that it should fall obliquely on the second medium: in perpendicular incidence there is no refraction. Yet Voscius and Snell imagined they had observed a perpendicular ray of light undergo a refraction; a perpendicular object appearing in the water nearer than it really was: but this was attributing that to a refraction of the perpendicular rays, which was owing to the divergency of the oblique rays after refraction, from a nearer point. Yet there is a manifest refraction even of perpendicular rays found in island crystal.

Rohault adds, that though an oblique incidence be necessary in all other mediums we know of, yet the obliquity must not exceed a certain degree; if it do, the body will not penetrate the medium, but will be reflected, instead of being refracted. Thus, cannon-balls, in sea engagements, falling very obliquely on the surface of the water, are observed to bound or rise from it, and to sweep the men from off the enemy's decks. And the same thing happens to the little stones with which children make their ducks and drakes along the surface of the water.—The ancients confounded refraction with reflection; and it was Newton who first taught the true difference between them. He shows however that there is a good deal of analogy between them, and particularly in the case of light.

The laws of refraction of the rays of light in mediums differently terminated, i. e. whose surfaces are plane, concave, and convex, make the subject of Dioptrics. By refraction it is, that convex glasses, or lenses, collect the rays, magnify objects, burn, &c; and hence the foundation of microscopes, telescopes, &c.—And by refraction it is, that all remote objects are seen out of their real places; particularly, that the heavenly bodies are apparently higher than they are in reality. The refraction of the air has many times so uncertain an influence on the places of celestial objects, near the horizon, that wherever refraction is concerned, the conclusions deduced from observations that are much affected by it, will always remain doubtful, and sometimes too precarious to be relied on. See Dr. Bradley in Philos. Trans. number 485.

As to the cause of refraction, it does not appear that any person before Descartes attempted to explain it: this he undertook to do by the resolution of forces, on the principles of mechanics; in consequence of which, he was obliged to suppose that light passes with more ease through a dense medium than a rare one; thus, the ray AC falling obliquely on a denser medium at C is supposed to be acted on by two forces, one of them impelling it in the direction AL, and the other in AK, which alone can be effected by the change of medium; and since, after the ray has entered the denser medium, it approaches the perpendicular CI, it is plain that this force must have received an increase, while the other continued the same.

The first person who questioned the truth of this explanation of the cause of refraction, was Fermat: he asserted, contrary to Descartes, that light suffers greater resistance in water than in air, and greater in glass than in water; and he maintained that the resistance of different mediums, with respect to light, is in proportion to their densities. Leibnitz also adopted the same general idea; and they reasoned on the subject in the following manner. Nature, say they, accomplishes her ends by the shortest methods; and therefore light ought to pass from one point to another, either by the shortest course, or by that in which the least time is required. But it is plain that the path in which light passes, when it falls obliquely on a denser medium, is not the most direct or the shortest; and therefore it must be that in which the least time is spent. And whereas it is demonstrable, that light falling obliquely upon a denser medium (in order to take up the least time possible, in passing from a point in one medium to a point in the other) must be refracted in such a manner, that the sine of the angles of incidence and refraction must be to one another, as the different facilities with which light is transmitted in those mediums; it follows that, since light approaches the perpendicular when it passes obliquely from air into water, the facility with which water suffers light to pass through it, is less than that of the air; so that the light meets with greater resistance in water than in air.

This method of arguing from final causes could not satisfy philosophers. Dr. Smith observes, that it agrees only to the case of refraction at a plane surface; and that the hypothesis is altogether arbitrary.

Dechales, in explaining the law of refraction, supposes that every ray of light is composed of several smaller rays, which adhere to one another; and that they are refracted towards the perpendicular, in passing into a denser medium, because one part of the ray meets with more resistance than another part; so that the former traverses a smaller space than the latter; in consequence of which the ray must necessarily bend a little towards the perpendicular. This hypothesis was adopted by the celebrated Dr. Barrow, and indeed some say, he was the author of it. Now on this hypothesis it is plain, that mediums of a greater refractive power, must give a greater resistance to the passage of the rays of light, than mediums of a less refractive power; which is contrary to fact.

The Bernoullis, both father and son, have attempted to explain the cause of refraction on mechanical principles; the former on the equilibrium of forces, and the latter on the same principles with the supposition of ethereal vortices: but neither of these hypotheses has gained much credit.

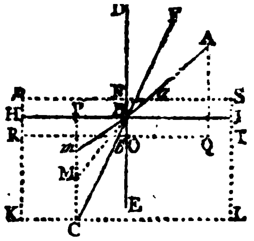
M. Mairan supposes a subtle fluid, filling the pores of all bodies, and extending, like an atmosphere, to a small distance beyond their surfaces; and then he supposes that the refraction of light is nothing more than a necessary and mechanical effect of the incidence of a small body in those circumstances. There is more, he says, of the refracting fluid, in water than in air, more in glass than in water, and in general more in a dense medium than in one that is rarer.

Maupertuis supposes that the course which every ray takes, in passing out of one medium into another, is that which requires the least quantity of action, which depends on the velocity of the body and the space it passes over; so that it is in proportion to the sum of the products arising from the spaces multiplied by the velocities with which bodies pass over them. From this principle he deduces the necessity of the sine of the angle of incidence being in a constant ratio to that of refraction; and also all the other laws relating to the propagation and reflection of light.

Dr. Smith (in his Optics, Remarks, p. 70) observes, that all other theories for explaining the reflexion and refraction of light, except that of Newton, suppose that it strikes upon bodies and is resisted by them; which has never been proved by any deduction from experience. On the contrary, it appears from various considerations, and might be shown by the observations of Mr. Molyneux and Dr. Bradley on the parallax of the fixed stars, that their rays are not at all impeded by the rapid motion of the earth's atmosphere, nor by the object-glass of the telescope, through which they pass. And by Newton's theory of refraction, which is grounded on experience only, it appears that light is so far from being resisted and retarded by refraction into any dense medium, that it is swifter there than in vacuo in the ratio of the sine of incidence in vacuo to the sine of refraction into the dense medium. Priestley's Hist. of Light, &c, p. 102 and 333.

Newton shows that the refraction of light is not performed by the rays falling on the very surface of bodies; but that it is effected, without any contact, by the action of some power belonging to bodies, and extending to a certain distance beyond their surfaces; by which same power, acting in other circumstances, they are also emitted and reflected.

The manner in which refraction is performed by mere attraction, without contact, may be thus accounted for: Now suppose HI the boundary of two mediums, N and O; the first the rarer, ex. gr. air; the second the denser, ex. gr. glass; the attraction of the mediums here will be as their densities. Suppose *ps* to be the distance to which the attracting force of the denser medium exerts itself within the rarer. And let a ray of light A*a* fall obliquely on the surface which separates the mediums, or rather on the surface *ps*, where the action of the second and more resisting medium commences; then as the ray arrives at *a*, it will begin to be turned out of its rectilinear course by a superior force, with which it is attracted by the medium O, more than by the medium N; hence the ray is bent out of its right line in every point of its passage between *ps* and RT, within which distance the attraction acts; and therefore between these lines it describes a curve *a*B*b*; but beyond RT, being out of the sphere of attraction of the medium N, it will proceed uniformly in a right line, according to the direction of the curve in the point *b*.



Again, suppose N the denser and more attracting medium, O the rarer, and HI the boundary as before; and let RT be the distance to which the denser medium exerts its attractive force within the rarer: then even when the ray has passed the point B, it will be within the sphere of the superior attraction of the denser medium; but that attraction acting in lines perpendicular to its surface, the ray will be continually drawn from its straight course BM perpendicularly towards HI: thus, having two forces or directions, it will have a compound motion, by which, instead of BM, it will describe B*m*, which will in strictness be a curve. Lastly, after it has arrived at *m*, being out of the influence of the medium N, it will persist uniformly, in a right line, in the direction in which the extremity of the curve leaves it.—Thus we see how refraction is performed, both towards the perpendicular DE, and from it.

Refraction *in Dioptrics*, is the inflexion or bending of the rays of light, in passing the surfaces of glasses, lenses, and other transparent bodies of different densities. Thus, a ray, as AB, falling obliquely from the radiant A, upon a point B, in a diaphanous surface HI, rarer or denser than the medium along which it was propagated from the radiant, has its direction there altered by the action of the new medium; and instead of proceeding to M, it deviates, as for ex. to C.

This deviation is called the Refraction of the ray; BC the Refracted ray, or Line of Refraction; and B the Point of Refraction.—The line AB is also called the Line of Incidence; and in respect of it, B is also called the Point of Incidence. The plane in which both the incident and refracted ray are found, is called the Plane of Refraction; also a right line BE drawn in the refracting medium perpendicular to the refracting surface at the point of refraction B, is called the Axis of Refraction; and its continuation DB along the medium through which the ray falls, is called the Axis of Incidence.—Further, the angle ABI, made by the incident ray and the refracting surface, is usually called the Angle of Incidence; and the angle ABD, between the incident ray and the axis of incidence, is the Angle of Inclination. Moreover, the angle MBC, between the refracted and incident rays, is called the Angle of Refraction; and the angle CBE, between the refracted ray and the axis of refraction, is the Refracted Angle. But it is also very common to call the angles ABD and CBE, made by the perpendicular with the incident and refracted rays, the Angles of Incidence and Refraction.

General Laws of REFRACTION.—I. A ray of light in its passage out of a rarer medium into a denser, ex. gr. out of air into water or into glass, is refracted towards the perpendicular, i. e. towards the axis of refraction. Hence, the refracted angle is less than the angle of inclination; and the angle of refraction less than that of incidence; as they would be equal were the ray to proceed straight from A to M.

II. The ratio of the sines of the angles ABD, CBE, made by the perpendicular with the incident and refracted rays, is a constant and fixed ratio; whatever be the obliquity of the incident ray, the mediums remaining. Thus, the refraction out of air into water, is nearly as 4 to 3, and into glass it is nearly as 3 to 2. As to air in particular, it is shown by Newton, that a ray of light, in traversing quite through the atmosphere, is refracted the same as it would be, were it to pass with the same obliquity out of a vacuum immediately into air of equal density with that in the lowest part of the atmosphere.

It appears, from Ptolemy's Optics, that he was well acquainted with the phenomena of the refraction of light, in passing from one medium to another; but he knew neither the law nor the exact quantity of it, though he made some experiments on it. Vitello, who collected the knowledge of the ancients on this subject, and their experiments, gave a false law for the comparison of the effect, erroneously stating that the angles of incidence and reflexion are always in a constant ratio.

The true law of refraction was first discovered by Willebrord Snell, professor of mathematics at Leyden; who found by experiment that the cosecants of the angles of incidence and refraction are always in the same ratio. It was commonly attributed however to Descartes; who, having seen it in a MS. of Snell's, first published it in his Dioptrics, without naming Snell, as Huygens asserts; Descartes having only altered the form of the law, from the ratio of the cosecants, to that of the sines, which is the same thing.

It is to be observed however, that as the rays of light are not all of the same degree of refrangibility, this constant ratio must be different in different kinds: so that the ratio mentioned by authors, is to be understood of rays of the mean refrangibility, i. e. of green rays. The difference of refraction between the least and most refrangible rays, that is, between violet and red rays, Newton shows, is about the $\frac{2}{55}$ of the whole refraction of the mean refrangible; which difference, he allows, is so small, that it seldom needs to be regarded.

Different transparent substances have indeed very different degrees of refraction, and those not according to any regular law; as appears by many experiments of Newton, Euler, Hawksbee, &c. See Newton's Optics, 3d edit. pa. 247; Hawksbee's Experim. pa. 292; Act. Berlin. 1762, pa. 302; Priestley's Hist. of Light &c, pa. 479.

Whence the different refractive powers in different fluids arise, has not been determined. Newton shows, that in many bodies, as glass, crystal, selenites, pseudo-topaz, &c, the refractive power is indeed proportionable to their densities; while in sulphureous bodies, as camphor, linseed, and olive oil, amber, spirit of turpentine, &c, the power is 2 or 3 times greater than in other bodies of equal density; and yet even these have the refractive power with respect to each other, nearly as their densities. Water has a refractive power in a medium degree between those two kinds of substances; while salts and vitriols have refractive powers in a middle degree between those of earthy substances and water, and accordingly are composed of those two kinds of matter. Spirit of wine has a refractive power in a middle degree between those of water and oily substances; and accordingly it seems to be composed of both, united by fermentation. It appears therefore, that all bodies seem to have their refractive powers nearly proportional to their densities, excepting so far as they partake more or less of sulphureous oily particles, by which those powers are altered.

Newton suspected that different degrees of heat might have some effect on the refractive power of bodies; but his method of determining the general refraction was not sufficiently accurate to ascertain this circumstance. Euler's method however was well adapted to this purpose: and from his experiments he infers, that the focal distance of a single lens of glass diminishes with the heat communicated to it; which diminution is owing to a change in the refractive power of the glass itself, which is probably increased by heat, and diminished by cold, as well as that of all other translucent substances.

From the law above laid down it follows, that one angle of inclination, and its corresponding refracted angle, being found by observation, the refracted angles corresponding to the several other angles of inclination are thence easily computed. Now, Zahnius and Kircher have found, that if the angle of inclination be 70°, the refracted angle, out of air into glass, will be 38° 50′; on which principle Zahnius has constructed a table of those refractions for the several degrees of the angle of inclination; a specimen of which here follows:

Angle of Inclination.	Refracted Angle.			Angle of Refraction.		
1°	0°	40′	5″	0°	19′	55″
2	1	20	6	0	39	54
3	2	0	4	0	59	56
4	2	40	5	1	19	55
5	3	20	3	1	39	57
10	6	39	16	3	20	44
20	13	11	35	6	48	25
30	19	29	29	10	30	31
45	28	9	19	16	50	41
90	41	51	40	48	8	20

Hence it appears, that if the angle of inclination be less than 20°, the angle of refraction out of air into glass is almost ⅓ of the angle of inclination; and therefore a ray is refracted to the axis of refraction by almost a third part of the quantity of its angle of inclination. And on this principle it is that Kepler, and most other dioptrical writers, demonstrate the refractions in glasses; though in estimating the law of these refractions he followed the example of Alhazen and Vitello, and sought to discover it in the proportion of the angles, and not in that of the sines, or cosecants, as discovered by Snell.

The refractive powers of several substances, as determined by different philosophers, may be seen in the following tables; in which the ray is supposed to pass out of air into each of the substances, and the annexed numbers show the ratio to unity or 1, between the sines of the angles of incidence and refraction.

1. By Sir Isaac Newton's Observations.

Air - - -	0·9997
Rain water - -	1·3358
Spirit of wine - -	1.3698
Oil of vitriol - -	1·4285
Alum - - -	1·4577
Oil of olive - -	1·4666
Borax - - -	1·4667
Gum arabic - -	1·4771
Linseed oil - -	1·4814
Selenites - -	1·4878
Camphor - -	1·5000
Dantzick vitriol - -	1·5000
Nitre - - -	1·5238
Sal gem - - -	1·5455
Glass - - -	1·5500
Amber - - -	1·5556
Rock crystal - -	1·5620
Spirit of turpentine -	1·5625
A yellow pseudo-topaz -	1·6429
Island crystal - -	1·6666
Glass of antimony - -	1·8889
A diamond - -	2·4390

2. By Mr. Hawksbee.

Water - - -	1·3359
Spirit of honey - -	1·3359
Oil of amber - -	1·3377
Human urine - -	1·3419
White of an egg - -	1·3511
French brandy - -	1·3625
Spirit of wine - -	1.3721
Distilled vinegar -	1·3721
Gum ammoniac - -	1·3723
Aqua regia - - -	1.3898
Aqua fortis - -	1·4044
Spirit of nitre - -	1·4076
Crystalline humour of an ox's eye	1.4635
Oil of vitriol - -	1.4262
Oil of turpentine - -	1·4833
Oil of amber - -	1·5010
Oil of cloves - -	1·5136
Oil of cinnamon - -	1·5340

3. By Mr. Euler, junior.

Rain or distilled water -	1·3358
Well water - -	1·3362
Distilled vinegar - -	1·3442
French wine - -	1·3458
A solution of gum arabic -	1·3467
French brandy - -	1·3600
Ditto a stronger kind -	1·3618
Spirit of wine rectified -	1·3683
Ditto more highly rectified	1·3706
White of an egg - -	1·3685
Spirit of nitre - -	1.4025
Oil of Provence - -	1.4651
Oil of turpentine - -	1·4822

III. When a ray passes out of a denser medium into a rarer, it is refracted from the perpendicular, or from the axis of refraction.

This is exactly the reverse of the 2d law, and the quantity of refraction is equal in both cases, or both forwards and backwards; so that a ray would take the same course back, by which another passed forward, viz, if a ray were to pass from A by B to C, another would pass from C by B to A. Hence, in this case, the angle of refraction is greater than the angle of inclination. And also, if the angle of inclination be less than 30°, MBC is nearly equal to ⅓ of MBE; therefore MBC is ½ of CBE; consequently, if the refraction be out of glass into air, and the angle of inclination less than 30°, the ray is refracted from the axis of refraction by almost the half of the angle of inclination. And this is the other dioptrical principle used by most authors after Kepler, to demonstrate the refractions of glasses.

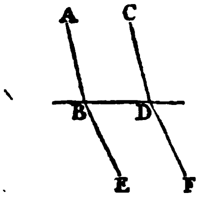
If the refraction be out of air into glass, the ratio of the sines of inclination and refraction is as 3 to 2, or more accurately as 17 to 11; if out of air into water as 4 to 3; therefore if the course be the contrary way, viz, out of glass or water into air, the ratio of the sines will be, in the former case as 2 to 3 or 11 to 17, and in the latter as 3 to 4. So that, if the refraction be from water or glass into air, and the angle of incidence or inclination be greater than about 48½ degrees in water, or greater than about 40° in glass, the ray will not be refracted into air; but will be reflected into a line which makes the angle of reflection equal to the angle of incidence; because the sines of 48½ and 40° are to the radius, as 3 to 4, and as 11 to 17 nearly; and therefore, when the sine has a greater proportion to the radius than as above, the ray will not be refracted.

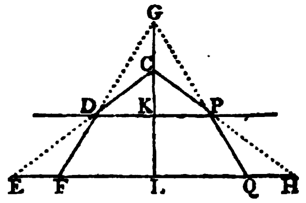
IV. A ray falling on a curve surface, whether concave or convex, is refracted after the same manner as if it fell on a plane which is a tangent to the curve in the point of incidence. Because the curve and its tangent have the point of contact common to both, where the ray is refracted.

Laws of REFRACTION *in Plane Surfaces.*

1. If parallel rays, AB and CD, be refracted out of one transparent medium into another of a different density,

they will continue parallel after refraction, as BE and DF. Hence a glass that is plane on both sides, being turned either directly or obliquely to the sun, &c, the light passing through it will be propagated in the same manner as if the glass were away.



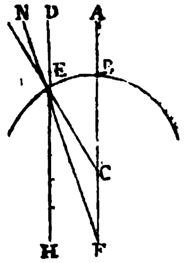


2. If two rays CD and CP, proceeding from the same radiant C, and falling on a plane surface of a different density, so that the points of refraction D and P be equally distant from the perpendicular of incidence GK, the refracted rays DF and PQ have the same virtual focus, or the same point of dispersion G.—Hence, when refracted rays, falling on the eye placed out of the perpendicular of incidence, are either equally distant from the perpendicular, or very near each other, they will flow upon the eye as if they came to it from the point G; consequently the point C will be seen by the refracted rays as in G. And hence also, if the eye be placed in a dense medium, objects in a rarer will appear more remote than they are; and the place of the image, in any case, may be determined from the ratio of refraction: Thus, to fishes swimming under water, objects out of the water must appear farther distant than in reality they are. But on the contrary, if the eye at E be placed in a rarer medium, then an object G placed in a denser, appears, at C, nearer than it is; and the place of the image may be determined in any given case by the ratio of refraction; and thus the bottom of a vessel full of water is raised by refraction a third part of its depth, with respect to an eye placed perpendicularly over the refracting surface; and thus also fishes and other bodies, under water, appear nearer than they really are.

3. If the eye be placed in a rarer medium; then an object seen in a denser, by a ray refracted in a plane surface, will appear larger than it really is. But if the eye be in a denser medium, and the object in a rarer, the object will appear less than it is. And in each case, the apparent magnitude FQ is to the real one EH, as the rectangle CK.GL to GK.CL, or in the compound ratio of the distance CK of the point to which the rays tend before refraction, from the refracting surface DP, to the distance GK of the eye from the same, and of the distance GL of the object EH from the eye, to its distance CL from the point to which the rays tend before refraction.—Hence, if the object be very remote, CL will be physically equal to GL; and then the real magnitude EL is to the apparent magnitude FL, as GK to CK, or as the distance of the eye G from the refracting plane, to the distance of the point of convergence F from the same plane. And hence also, objects under water, to an eye in the air, appear larger than they are; and to fishes under water, objects in the air appear less than they are.

Laws of REFRACTION *in Spherical Surfaces, both concave and convex.*

1. A ray of light DE, parallel to the axis, after a single refraction at E, meets the axis in the point F, beyond the centre C.



2. Also in that case, the semidiameter CB or CE will be to the refracted ray EF, as the sine of the angle of refraction to the sine of the angle of inclination BCE. But the distance of the focus, or point of concurrence from the centre, CF, is to the refracted ray EF, as the sine of the refracted angle to the sine of the angle of inclination.

3. Hence also, in this case, the distance BF of the focus from the refracting surface, must be to CF its distance from the centre, in a ratio greater than that of the sine of the angle of inclination to the sine of the refracted angle. But those ratios will be nearly equal when the rays are very near the axis, and the angle of inclination BCE is only of a few degrees. And when the refraction is out of air into glass, then

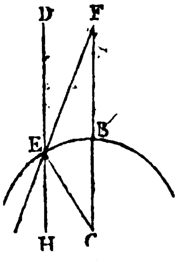
For rays near the axis,	For more distant rays,
$BF : FC :: 3 : 2$,	$BF : FC > 3 : 2$,
$BC : BF :: 1 : 3$.	$BC : BF < 1 : 3$.

But if the refraction be out of air into water, then

For rays near the axis,	For more distant rays,
$BF : FC :: 4 : 3$,	$BF : FC > 4 : 3$,
$BC : BF :: 1 : 4$.	$BC : BF < 1 : 4$.

Hence, as the sun's rays are parallel as to sense, if they fall on the surface of a solid glass sphere, or of a sphere full of water, they will not meet the axis within the sphere: so that Vitello was mistaken when he imagined that the sun's rays, falling on the surface of a crystalline sphere, were refracted to the centre.

4. If a ray HE fall parallel to the axis FA, out of a rarer medium, on the concave spherical surface BE of a denser one; the refracted ray EN will diverge from the point of the axis F, so that FE will be to FC, in the ratio of the sine of the angle of inclination, to the sine of the refracted angle. Consequently FB to FC is in a greater ratio than that; unless when the rays are very near the axis, and the angle BCE is very small, for then FB will be to FC nearly in that ratio. And hence, in the cases of refraction out of air into water or glass, the ratios of BC, BF and CF, will be the same as specified in the last article.



5. If a ray DE, parallel to the axis FC, pass out of a denser into a rarer spherical convex medium, it will diverge from the axis after refraction; and the distance FC of the point of dispersion, or of the virtual focus F, from the centre of the sphere, will be to its semidiameter CE or CB, as the sine of the refracted angle is to the sine of the angle of refraction; but to the portion of the refracted ray drawn back, FE, it will be in the ratio of the sine of the refracted angle to the sine of the angle of inclination. Consequently FC will be to FB, in a greater ratio than this last: unless when the rays DE fall very near the axis FC, for then FC to FB will be very nearly in that ratio.

Hence, when refraction is out of glass into air; then,

For rays near the axis,	For more distant rays,
$FC : FB :: 3 : 2$,	$FC : FB > 3 : 2$,
$BC : BF :: 1 : 2$.	$BC : BF > 1 : 2$.

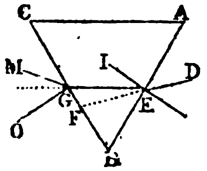
But when the refraction is out of water into air; then,

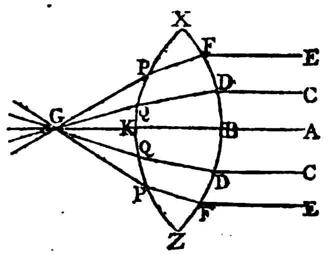
For rays near the axis,	For more distant rays,
FC : FB : : 4 : 3,	FC : FB > 4 : 3,
BC : BF : : 1 : 3.	BC : BF > 1 : 3.

6. If the ray HE fall parallel to the axis CF, from a denser medium, upon the surface of a spherically concave rarer one; the refracted ray will meet with the axis in the point F, so that the distance CF from the centre, will be to the refracted ray FE, as the sine of the refracted angle, to the sine of the angle of inclination. Consequently FC will be to FB, in a greater ratio than that above mentioned: unless when the rays are very near the axis, for then FC is to FB very nearly in that ratio; and the three FB, FC, BC are, in the cases of air, water, and glass, in the numeral ratios as specified at the end of the last article. See Wolfius, Elem. Mathes. tom. 3, pa. 179 &c.

Refraction *in a Glass Prism.*

ABC being the transverse section of a prism; if a ray of light DE fall obliquely upon it out of the air; instead of proceeding straight on to F, being refracted towards the perpendicular IE, it will decline to G. Again, since the ray EG, passing out of glass into air, falls obliquely on BC, it will be refracted to M, so as to recede from the perpendicular GO.' And hence arise the various phenomena of the prism. See Colour.



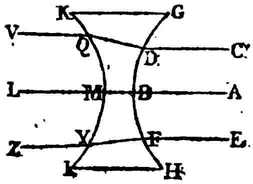


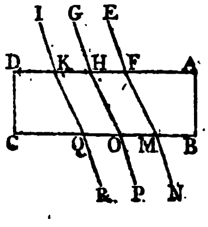
Refraction *in a Convex Lens.*

If parallel rays, AB, CD, EF, fall on the surface of a convex lens XBZ (the last fig. above); the perpendicular ray AB will pass unrefracted to K, where emerging, as before, perpendicularly, into air, it will proceed straight on to G. But the rays CD and EF, falling obliquely out of air into glass, at D and F, will be refracted towards the axis of refraction, or towards the perpendiculars at D and F, and so decline to Q and P: where emerging again obliquely out of the glass into the surface of the air, they will be refracted from the perpendicular, and proceed in the directions QG and PG, meeting in G. And thus also will all the other rays be refracted so as to meet the rest near the place G. See Focus and Lens.—Hence the great property of convex glasses; viz, that they collect parallel rays, or make them converge into a point.

Refraction *in a Concave Lens.*

Parallel rays AB, CD, EF, falling on a concave lens GBHIMK, the ray AB falling perpendicularly on the glass at B, will pass unrefracted to M; where, being still perpendicular, it will pass into the air to L, without refraction. But the ray CD, falling obliquely on the surface of the glass, will be refracted towards the perpendicular at D, and proceed to Q; where again falling obliquely out of the glass upon the surface of air, it will be refracted from the perpendicular at Q, and proceed to V. After the same manner the ray EF is first refracted to Y, and thence to Z.—Hence the great property of concave glasses; viz, that they disperse parallel rays, or make them diverge. See Lens.

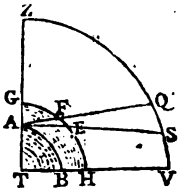




Refraction *in a Plane Glass.*

If parallel rays EF, GH, IK, (the last fig. above) fall obliquely on a plane glass ABCD; the obliquity being the same in all, by reason of their parallelism, they will be all equally refracted towards the perpendicular; and accordingly, being still parallel at M, O, and Q, they will pass out into the air equally refracted again from the perpendicular, and still parallel. Thus will the rays EF, GH, and IK, at their entering the glass, be inflected towards the right; and in their going out as much inflected to the left; so that the first refraction is here undone by the second, thereby causing the rays on their emerging from the glass, to be parallel to their first direction before they entered it; though not so as that the object is seen in its true place; for the ray RQ, being produced back again, will not coincide with the ray IK, but will fall to the right of it; and this the more as the glass is thicker; however, as to the colour, the second refraction does really destroy the first. See Colour.

Refraction *in Astronomy*, or **Refraction** *of the Stars*, is an inflexion of the rays of those luminaries, in passing through our atmosphere; by which the apparent altitudes of the heavenly bodies are increased. This refraction arises from hence, that the atmosphere is unequally dense in different stages or regions; rarest of all at the top, and densest of all at the bottom; which inequality in the same medium, makes it equivalent to several unequal mediums, by which the course of the ray of light is continually bent into a continued curve line. See Atmosphere.—And Sir Isaac Newton has shown, that a ray of light, in passing from the highest and rarest part of the atmosphere, down to the lowest and densest, undergoes the same quantity of refraction that it would do in passing immediately, at the same obliquity, out of a vacuum into air of equal density with that in the lowest part of the atmosphere.



The effect of this refraction may be thus conceived. Suppose ZV a quadrant of a vertical circle described from the centre of the earth T, under which is AB a quadrant of a circle on the surface of the earth, and GH a quadrant of the surface of the atmosphere. Then suppose SE a ray of light emitted by a star at S, and falling on the atmosphere at E: this ray coming out of the ethereal medium, which is much rarer than our air, or perhaps out of a perfect vacuum, and falling on the surface of the atmosphere, will be refracted towards the perpendicular, or inclined down more towards the earth; and since the upper strata of air are rarer than those near the earth, and becomes still denser as they approach the earth's surface, the ray in its progress will be continually refracted, so as to arrive at the eye in the curve line EA. Then supposing the right line AF to be a tangent to the arch at A, the ray will enter the eye at A in the direction of AF; and therefore the star will appear in the heavens at Q, instead of S, higher or nearer the zenith than it really is.—

Hence arise the phenomena of the crepusculum or twilight; and hence also it is that the moon is sometimes seen eclipsed, when she is really below the horizon, and the sun above it.

That there is a real refraction of the stars &c, is deduced not only from physical considerations, and from arguments a priori, and a similitudine, but also from precise astronomical observation: for there are numberless observations by which it appears that the sun, moon, and stars rise much sooner, and appear higher, than they should do according to astronomical calculations. Hence it is argued, that as light is propagated in right lines, no rays could reach the eye from a luminary below the horizon, unless they were deflected out of their course, at their entrance into the atmosphere: and therefore it appears that the rays are refracted in passing through the atmosphere.

Hence the stars appear higher by refraction than they really are; so that to bring the observed or apparent altitudes to the true ones, the quantity of refraction must be subtracted. And hence, some of the ancients, as they were not acquainted with this refraction, reckoned their altitudes too great, so that it is no wonder they sometimes committed considerable errors. Hence also, refraction lengthens the day, and shortens the night, by making the sun appear above the horizon a little before his rising, and a little after his setting. Refraction also makes the moon and stars appear to rise sooner and set later than they really do. The apparent diameter of the sun or moon is about 32′; the horizontal refraction is about 33′; whence the sun and moon appear wholly above the horizon when they are entirely below it. Also, from observations it appears that the refractions are greater nearer the pole than at lesser latitudes, which causes the sun to appear some days above the horizon, when he is really below it; doubtless from the greater density of the atmosphere, and the greater obliquity of the incidence.

Stars in the zenith are not subject to any refraction; and those in the horizon have the greatest of all; the refraction continually decreasing from thence to the zenith. All which follows from hence, that in the first case, the rays are perpendicular to the medium; in the second, their obliquity is the greatest, and they pass through the largest space of the lower and denser part of the air, and through the thickest vapours; and in the third, the obliquity is continually decreasing.

The air is condensed, and consequently refraction is increased, by cold; for which reason it is greater in cold countries than in hot ones. It is also greater in cold weather than in hot, in the same country; and the morning refraction is greater than that of the evening, because the air is rarefied by the heat of the sun in the day, and condensed by the coldness of the night. Refraction is also subject to some small variation at the same time of the day in the finest weather.

At the same altitudes, the sun, moon, and stars all undergo the same refraction: for at equal altitudes the incident rays have the same inclinations; and the sines of the refracted angles are as the sines of the angles of inclination, &c.

Ptolemy, Alhazen, and Vitello, were all acquainted with this refraction, having given many observations on it, though imperfect on the score of accuracy. But Tycho Brahé, who deduced the refractions of the sun, moon, and stars from good observations, and whose table of the refractions of the stars is not much different from those of Flamsteed and Newton, except near the horizon, makes the solar refractions about 4′ greater than those of the fixed stars, and the lunar refractions also sometimes greater than those of the stars, and sometimes less. But the theory of refractions discovered by Snell, was not fully understood in his time.

The horizontal refraction, being the greatest, is the cause that the sun and moon appear of an oval form at their rising and setting; for the lower edge of each being more refracted than the upper edge, the perpendicular diameter is shortened, and the under edge appears more flatted also.—Hence also, if we take with an instrument the distance of two stars when they are in the same vertical and near the horizon, we shall find it considerably less than if we measure it when they are both at such a height as to suffer little or no refraction; because the lower star is more elevated than the higher. There is also another alteration made by refraction in the apparent distance of stars: when two stars are in the same almacantar, or parallel of declination, their apparent distance is less than the true; for since refraction makes each of them higher in the azimuth or vertical in which they appear, it must bring them into parts of the vertical where they come nearer to each other; because all vertical circles converge and meet in the zenith. This contraction of distance, according to Dr. Halley (Philos. Trans. numb. 368) is at the rate of at least one second in a degree; so that, if the distance between two stars in a position parallel to the horizon measure 30°, it is at most to be reckoned only 29° 59′ 30″.

The quantity of the refraction at every altitude, from the horizon, where it is greatest, to the zenith where it is nothing, has been determined by observation, by many astronomers; those of Dr. Bradley and Mr. Mayer are nearly alike, and have been used by most astronomers. Doctor Bradley, from his observations, deduced this very simple and general rule for the refraction r at any altitude a whatever; viz, as rad. $1 : \text{cotang.}\ a + 3r :: 57'' : r''$ the refraction in seconds; that is, refr. $r = 57'' \times \text{cot.}\ (a+3r)$, or, which is the same, refr. $r = 57'' \times \text{tan.}\ (z-3r)$, where z is the zenith distance.

This rule, of Dr. Bradley's, is adapted to these states of the barometer and thermometer, viz,

either 29·6 inc. barom. and 50° thermometer,
or 30 — barom. and 55 thermometer,

for both which states it answers equally the same. But for any other states of the barometer and thermometer, the refraction above-found is to be corrected in this manner, viz, by either of the two following rules, the first of them given by Dr. Maskelyne, and the 2d by Dr. Brinkley.

$$\text{Refrac.} = \frac{b}{23 \cdot 6} \times \text{tan.}\ (z-3r) \times 57'' \times \frac{400}{350+t}.$$

$$\text{Refrac.} = \frac{b}{23 \cdot 6} \times \text{tan.}\ (z-3 \cdot 2r) \times 56'' \cdot 9 \times \frac{500}{450+t}.$$

Where b = altitude of barometer in inches,
t = height of Fahrenheit's thermometer in deg,
$r = 57''$ tan. z the appar. zenith dist.

From Dr. Bradley's rule, $r = 57'' \times \text{cot.}\ (a+3r)$ was computed the table of mean astronomical refractions, given in pa. 1 of Dr. Maskelyne's requisite tables.

M. Laplace gave also a rule for the refractions, in vol. 4 of his Mecanique Celeste. He first assumed it of the same form as Dr. Bradley's, viz, $r = m \times \text{cot.}\ (a+nr) = m \times \text{tan.}\ (z-nr)$, with general coefficients m and n, to be determined by comparing this general formula with two observations; where a denotes the true altitude, or z the true zenith distance, and r the refraction. Besides determining these co-efficients, m, n, more accurately, than in Dr. Bradley's rule, he also reduced the result to a more convenient form for use, which is expressed by this general equation, $·99765175 \sin z = \sin (z - 8r)$. But by computation I find that the first numeral co-efficient ought more correctly to be ·997684, and then the rule will be $·997684 \sin z = \sin (z - 8r)$. That is, to use it, multiply the sine of the true zenith distance by ·997684, or add their logarithms, the result will be the sine of the other arc $(z - 8r)$; therefore subtract this last arc from z, the remainder will be $8r$, which therefore divided by 8, gives r the fraction sought. By this rule then the numbers in the following table have been calculated, though the apparent altitudes are set down in the table, in order to adapt it to the purposes of observation.

TABLE OF REFRACTIONS.—*Barom.* 29·92. *Therm.* 54°.

Altit. appar.	Refractions.	Altit. appar.	Refractions.	Alt. app.	Refractions.	Alt. app.	Refract.
0° 0′	33′ 46·3″	7° 0′	7′ 24·8″	14°	3′ 49·8″	56	39·3″
10	31 54·3	10	7 15·3	15	3 34 3	57	37·8
20	30 9·3	20	7 6·3	16	3 20·6	58	36·4
30	28 32·1	30	6 57·7	17	3 8·5	59	35·0
40	27 2·2	40	6 49·6	18	2 57·6	60	33·6
50	25 38·6	50	6 41·9	19	2 47·7	61	32·3
1 0	24 21·2	8 0	6 34·4	20	2 38·8	62	31·0
10	23 9·6	10	6 27·1	21	2 30·6	63	29·7
20	22 3·4	20	6 20·0	22	2 23·2	64	28·4
30	21 1·9	30	6 13·1	23	2 16·5	65	27·2
40	20 4·8	40	6 0·4	24	2 10·2	66	25·9
50	19 11·5	50	5 59·9	25	2 4·3	67	24·7
2 0	18 22·2	9 0	5 53.6	26	1 58·9	68	23·5
10	17 36·3	10	5 47 4	27	1 53·9	69	22·4
20	16 53·2	20	5 41·5	28	1 49·2	70	21·2
30	16 13·4	30	5 35·8	29	1 44·8	71	20·0
40	15 36·0	40	5 30·3	30	1 40·6	72	18·9
50	15 0·9	50	5 25·0	31	1 36·7	73	17·8
3 0	14 28·1	10 0	5 19·8	32	1 33·1	74	16·7
10	13 57·3	10	5 14·7	33	1 29·6	75	15 6
20	13 28·5	20	5 9·7	34	1 26·2	76	14·5
30	13 1·3	30	5 4·9	35	1 23·1	77	13·5
40	12 35·6	40	5 0·3	36	1 20·1	78	12·4
50	12 11·3	50	4 55 9	37	1 17·2	79	11·3
4 0	11 48·3	11 0	4 51·7	38	1 14·4	80	10·3
10	11 26·6	10	4 47·6	39	1 11·8	81	9·2
20	11 6·1	20	4 43·6	40	1 9·3	82	8·2
30	10 46·7	30	4 39·6	41	1 6 9	83	7·2
40	10 28·3	40	4 35·7	42	1 4·6	84	6·1
50	10 10·9	50	4 31·8	43	1 2·4	85	5·1
5 0	9 54·3	12 0	4 28·0	44	1 0·3	86	4·1
10	9 38·4	10	4 24·3	45	0 58·2	87	3·1
20	9 23·4	20	4 20·7	46	0 56·2	88	2·0
30	9 9·0	30	4 17·2	47	0 54·3	89	1·0
40	8 55·3	40	4 13·8	48	0 52·4	90	0·0
50	8 42·3	50	4 10·6	49	0 50·6		
6 0	8 29·9	13 0	4 7·5	50	0 48.9		
10	8 18·1	10	4 4·4	51	0 47·2		
20	8 6·6	20	4 1·4	52	0 45·5		
30	7 55·6	30	3 58·4	53	0 43·9		
40	7 45·0	40	3 55·5	54	0 42·3		
50	7 34·7	50	3 52·6	55	0 40·8		
7 0	7 24·8	4 0	3 49·8	56	0 39·3		

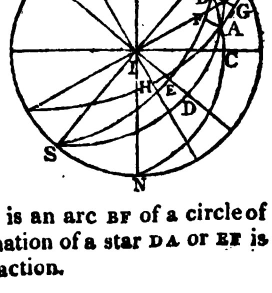
Mr. Mayer says his rule was deduced from theory, and when reduced from French measure and Reaumur's thermometer, to English measure and Fahrenheit's thermometer, it is this,

$r = \frac{74·4b \times \cos. a \times \text{tang.} \frac{1}{2}A}{(1 + ·00248t) \frac{1}{2}}$ the refraction in seconds, corrected for both barometer and thermometer: where the letters denote the same things as before, except A, which denotes the angle whose tangent is $\frac{\sqrt{(1 + ·00248t)}}{17·14 \sin. a}$.

Mr. Simpson too (Dissert. pa. 46 &c) ingeniously determined by theory the astronomical refractions, from which he formed this rule, viz, As 1 to ·9986 or as radius to sine of 86° 58′ 30″, (or rather 4″,) so is the sine of any given zenith distance, to the sine of an arc; then $\frac{3}{11}$ of the difference between this arc and the zenith distance, is the refraction sought for that zenith distance. And by this rule Mr. Simpson computed a table of the mean refractions, which are not much different from those of Dr. Bradley and Mr. Mayer, being uniformly a few seconds less in every case.

Besides the above, the public have been favoured with other rules, deduced from numerous observations, made by Ste. Groombridge, esq. of Blackheath, a gentleman of fortune, who very laudably amuses himself, and benefits science, by cultivating the practice of astronomy. The results of extensive series of observations on astronomical refractions, he has given in two volumes of the Philos. Trans. both in general rules and large tables of results, differing but very little from those above inserted, and that chiefly in the refractions very near the horizon. In the former volume, viz that for the year 1810, Mr. Groombridge's rule for the mean refraction is $58''·1192 \times \text{tang.} (z - 3·3625r)$, where z is the zenith distance, and r an assumed near value of the refraction. But after numerous other observations, especially on stars at very low altitudes, in the vol. for 1814, Mr. G. by further corrections, reduces the rule to this form, viz, the mean refraction $= 58·132967 \times \text{tang.} (z - 3·6342956)$; from which he has calculated an extensive table of refractions, for every 10′ of altitude; accompanied with other tables, showing the corrections on account of the difference of the barometer and thermometer from their mean states.

It is evident that all observed altitudes of the heavenly bodies ought to be diminished by the numbers taken out of the foregoing tables. It is also evident that the refraction diminishes the right and oblique ascensions of a star, and increases the descensions: it increases the northern declination and latitude, but decreases the southern: in the eastern part of the heavens it diminishes the longitude of a star, but in the western part it increases the same.



REFRACTION *of Altitude*, is an arc of a vertical circle, as AB, by which the altitude of a star AC is increased by the refraction.

REFRACTION *of Ascension and Descension*, is an arc DE of the equator, by which the ascension and descension of a star, whether right or oblique, is increased or diminished by the refraction.

REFRACTION *of Declination*, is an arc BF of a circle of declination, by which the declination of a star DA or EF is increased or diminished by refraction.

Refraction *of Latitude* is an arc AG of a circle of latitude, by which the latitude of a star AH is increased or diminished by the refraction.

Refraction *of Longitude*, is an arc IH of the ecliptic, by which the longitude of a star is increased or diminished by means of the refraction.

Terrestrial **Refraction**, is that by which terrestrial objects appear to be raised higher than they really are, in observing their altitudes. The quantity of this refraction is estimated by Dr. Maskelyne at one-tenth of the distance of the object observed, expressed in degrees of a great circle. So, if the distance be 10000 fathoms, its 10th part 1000 fathoms, is the 60th part of a degree of a great circle on the earth, or 1′, which therefore is the refraction in the altitude of the object at that distance. (Requisite Tables, 1766, pa. 134).

But M. Legendre is induced, he says, by several experiments, to allow only $\frac{1}{14}$th part of the distance for the refraction in altitude. So that, on the distance of 10000 fathoms, the 14th part of which is 714 fathoms, he allows only 44″ of terrestrial refraction, so many being contained in the 714 fathoms. See his Memoir concerning the Trigonometrical operations, &c.

Again, M. Delambre, an ingenious French astronomer, makes the quantity of the terrestrial refraction to be the 11th part of the arch of distance. But the English measurers, Col. Edw. Williams, Capt. Mudge, and Mr. Dalby, from a multitude of exact observations made by them, determine the quantity of the medium refraction to be the 12th part of the said distance. The quantity of this refraction, however, is found to vary considerably, with the different states of the weather and atmosphere, from the 15th part of the distance, to the 9th part of the same; the medium of which is the 12th part, as above mentioned.

Some whimsical effects of this refraction are also related, arising from peculiar situations and circumstances. Thus, it is said, that any person standing by the side of the river Thames at Greenwich, when it is high-water there, can see the cattle grazing on the Isle of Dogs, which is the marshy meadow on the other side of the river at that place; but when it is low water, he cannot see any thing of them, as they are hid from his view by the land wall or bank on the other side, which is raised higher than the marsh, to keep out the waters of the river. This curious effect is probably owing to the moist and dense vapours, just above and rising from the surface of the water, being raised higher or lifted up with the surface of the water at the time of high tide, through which the rays pass, and are the more refracted.

In like manner, Calais sometimes is seen from the sea side at Dover.

And other more extraordinary circumstances have been communicated in the following letter from an ingenious friend, Mr. John Andrews.

An account of some remarkable appearances arising from Terrestrial Refraction.

In the year 1792, at Traine, near Modbury, in Devon, for the purpose of obtaining an improved prospect from the garden, a railed platform was erected among the branches of a large spreading laurel-tree, to which was given the name of The Laurel Mount.—It was furnished with chairs, &c, and had moveable staffs, made convenient for resting a telescope at different elevations, so as to form a kind of observatory, for viewing both celestial and terrestrial objects.—Some time after its erection, it was unexpectedly discovered that the pinnacles and flag-staff, on the tower of Maker Church, west of Plymouth Sound, (where signals are made of the ships which pass by that harbour,) might be seen with a telescope, just appearing above the horizon, distance from hence about 12 miles. This object, being frequently looked at, was perceived to appear at certain times higher than usual; and some of the parapet of the tower (below the pinnacles) was evidently seen above the horizon, which was not the case in general; and which, being considered as a curious and remarkable circumstance, occasioned the object to be more frequently and more attentively observed. It was found that the morning was the time best suited for these appearances, which in certain instances were much more remarkable and striking than in others; and that not only the tower, but the scenery of the country, at different distances between it and the place of observation, was in like manner affected by the peculiar state of the atmosphere.—The figures on the plate (plate 30), with the references underneath, will help to explain the following descriptions.

The first instance of this sort occurred on the 9th of January 1794, a little after sunrise; the weather bright, with hard frost, and thick hoary incrustations.—At this time half or more of the shaft of the tower appeared conspicuously above the horizon; its height and appearance frequently varying; sometimes the pinnacles were scarce discernible, and the whole body seemed to be solid; when presently the pinnacles would begin to appear again, as if growing suddenly out of the body of the tower, and shooting up to a greater length than they are of in reality. The horizon itself was also subject to the like mutations, and the trees in Mount Edgcumbe Park, (which is just below the tower,) were sometimes more and sometimes less elevated, and sometimes not visible at all. The intervening scenery assumed the appearance represented in the 3d figure on the plate; objects before concealed by the horizon were elevated considerably above it. Among these was a conical object, supposed then to be a large edifice; and which was, in two journeys for that purpose, searched for in vain; but, being afterwards seen again, from the Mount, altered in size and shape, was found, on a third journey, to be a mow of hay, and was the means of ascertaining the situation of the range of ground at CC, whereon it stood, which might not have been easily done otherwise. It was therefore an object of some importance, and is shown in the 3d drawing at *g*. The enlargement of the prospect was so very singular and striking, as to seem almost as if produced by enchantment.

At this time another phenomenon was observed, which I could scarce believe to be real, till on the 27th of September following it was confirmed by another instance. This was a fluctuating appearance of two horizons, one above the other, with a complete vacancy between them, like what is sometimes observed in looking through an uneven pane of glass. On the day last mentioned, about 6 in the morning, the horizon being perfectly clear, the pinnacles of the tower were observed to have a taller appearance than ordinary; and at about half an hour past 6, a flag being hoisted on the staff, (which is considerably higher than the pinnacles,) the latter appeared to reach farther up towards the flag than they usually did. They also continually varied their appearance, being sometimes longer and sometimes shorter, and sometimes of unequal lengths; and at other times they could not be seen at all, though the flag still continued visible, and was seemingly unaffected. In one of these intervals (of the pinnacles disappearing), the horizon at *c c* being perfectly clear, I began to discern over it a faint stratum of vapour, the upper boundary of which (unevenly terminated) passed just below the flag, which seemed as if in a kind of insulated state. Soon afterwards the stratum extended itself higher, and the flag also became invisible; but in a little while the whole appeared again, when the parapet, pinnacles, and staff, seemed all to have a long and tall appearance, and the flag also to be altered in form from what it had before the extension of the stratum. Not long after this I observed the intervening horizon *c c* begin to be somewhat obscured, the wind probably wafting the stratum of vapour over it, and this obscurity increasing extended itself over all the ground at *c c*, which, (as well as the tower, &c) I then conceived to have a higher elevation than before. The vapour continuing to increase, it became so dense at 40 minutes after 7, as to hide all those objects from my sight, though the hill *b*, and other parts of the horizon, remained exceeding clear.

Another very remarkable instance was observed on the 6th of January 1795, at which time the elevation of the objects was equal to that represented in the 3d drawing, and the phenomenon of the double horizon very distinctly observed both by my brother and myself. The appearances were continually varying and intermitting, but not rapidly, so that sufficient time was afforded for ascertaining their reality beyond a doubt. The vacant line of separation (having the appearance of a whitish stratum of vapour), would often increase its breadth, so as to efface entirely the uppermost of the two horizons; forming then a kind of dent or gap in the remaining horizon; which horizon, at the extremities of the vacancy, seemed to be of the same height as the upper horizon was before its being effaced. This vacancy (continually varying in length as well as breadth), was several times seen to approach and take in the tower, and immediately to admit an apparent view of the whole or the most part of its body (like that in the third drawing), which was not the case before; exactly, to all appearance, as if it had opened a gap for that purpose in the intercepting ground. This phenomenon excited great surprise, and seemed to be inexplicable.

A great many other observations were made in the year 1794 and 1795, and minutes thereof taken, but the above were the most remarkable. The certainty of the phenomena being fully confirmed, less attention was thought necessary, and no further memorandums have been made. After some years the observatory getting into decay, and becoming dangerous, it was taken down, and hath not been since renewed.

In the course of these observations it was remarked that a hoar frost, or that kind of dewy vapour which in a sufficient degree of cold occasions a hoar frost, accompanied by an air rather calm than otherwise, seemed requisite for the elevation of the objects; and that a dry frost, however intense, especially if attended with wind, had no tendency to produce it. Indeed, in several instances of that sort, I have observed the objects very sensibly depressed below their usual pitch. I know at present no other instance of the double horizon having been observed, except by Mr. Isaac Dalby; who (as appears by Phil. Trans. for 1795, pa. 587) noticed an appearance of that sort about nine months before I did.

The telescope made use of was a 3-foot refractor of Dollond's, of the sort with long polygonal tubes of wood. What served me for a micrometer (and from which the scale on the plate was deduced) was a notched bar, made of a piece of fine screw, filed flat, and laid across the focus of the eye-glass.—Its value was ascertained by computing the distance of the stars γ and δ Coronæ, which was found to be the extent of the telescope's field of view. The quantities of elevation are to be understood as judged of by comparing the objects among each other; for having no graduated instrument or level to which the telescope could be attached, and the objects not being sufficiently distinguishable without it, I had no means whereby to determine their absolute elevation in respect of the horizon.

As far as the mere elevation of objects is concerned, the phenomena seem not difficult to be accounted for; but the double horizon, and especially the peculiar circumstances observed on the 6th of January 1795, appear not easy to be explained. They furnish two material questions; first, whether the separation is effected by the refracting matter elevating the upper, or depressing the lower visible horizon? and, secondly, why the apparent vacancy, or gap, described as above, did not cause the tower to disappear, as well as the horizon which intercepted it? My own idea at present is, that the appearance of the lower horizon is effected either by depression, or else by the mass of refracting matter, which, causes the elevation, detaching itself from the ground, so as to admit of the natural (unrefracted) horizon being seen below it, at the same time that an elevated one is visible through its body. I also conceived it possible that the lengthened appearance of the tower (then observed) might have arisen from the connexion of two images thereof, viz. the upper and the lower; having noticed something similar in the instance of a tree, partly intercepted by the ridge of a building, and viewed through an irregular spot (which seems to me to be a bubble) in the glass of a window: and probably an attentive observation of objects seen through such irregularities in glass, may help to illustrate all its different phenomena. The tower being a body of an uniform breadth, a deception of the sort alluded to is not perceivable; but perhaps would have been manifest had the object been of the pyramidical form, as many steeples are. And, as the distance of the tower, beyond the intercepting ground at *c*, is only about 3 miles, it seems rather extraordinary that the difference of their absolute elevations should be sufficient to bring so much of the tower into view. At the times of these extraordinary refractions, it was a sharp white frost, with a calm hazy atmosphere.

Modbury, 3d Jan. 1815. JOHN ANDREWS.

See the representations in plate 30, of the appearances, in three different states of the atmosphere, with the explanations of them.

The following curious instance of refraction was given in the 3d vol. of the Trans. of the American Philos. Trans. by Mr. Andrew Ellicott, at Pittsburg, Nov. 5, 1787, from observations at Lake Erie.—On the evening of Sept. 12, there was a fine aurora borealis. The next day was cloudy, but without rain. About noon, the low peninsula, called Presque-isle, which, at its then distance of 25 miles, is commonly invisible, was descried from the borders of the lake, considerably elevated above the horizon; and, viewed through a telescope, the branches of the trees could be plainly discovered. It is very singular that the peninsula was frequently seen doubled, the images, one above the other, separating and coinciding repeatedly, like those observed in shifting the index of a Godfrey's quadrant. In the evening it began to blow a fresh breeze; which, in the following days, increased into a most violent hurricane. These distinct facts afford some data for the investigation of the curious phenomenon which sailors term Looming. We may offer the following attempt at an explication. It is easy to perceive that, owing to the successive increase of rarity at different heights in the atmosphere, the rays of light, transmitted from a distance, are invariably bent towards the surface of the earth, and therefore bestow on objects an apparent elevation. If this progression of rarity be, from some accidental cause, augmented, the refraction, and its consequent effect, must then become proportionally greater; and this actually takes place in the case under consideration. The lucid complexion of the sky, and the storm which commonly ensues, conspire to indicate that, at no great height, the air is replete with humidity. The double appearance above described may be owing to two fluctuative strata of air, differently charged with moisture, and occasioned probably by opposite currents.

The following is the substance of a letter, on a similar subject, from W. Latham, esq. inserted in the Philos. Trans. of 1798.—On the 26th of July, 1797, about 5 o'clock afternoon, while sitting in his room at Hastings, on the parade, close to the sea shore, nearly fronting the south, Mr. Latham's attention was excited by a number of people running down to the sea side. On inquiring the reason, he was informed that the coast of France was plainly to be distinguished by the naked eye. He immediately went down to the shore, and was surprised to find that, even without the assistance of a telescope, he could plainly see the cliffs on the opposite coast; which, at the very nearest part, are between 40 and 50 miles distant, and are not to be discerned from that low situation by the aid of the best glasses. They appeared to be only a few miles off, and seemed to extend for some leagues along the coast. Mr. L. pursued his walk along the shore, close to the water's edge, conversing with the sailors and fishermen on the subject. At first these could not be persuaded of the reality of the appearance; but soon became so fully convinced, by the cliffs gradually appearing more elevated, and approaching nearer, as it were, that they pointed out, and named to him, the different places they had been accustomed to visit; such as the Bay, the Old Head or Man, the windmill &c, at Boulogne, St. Vallery, and other places on the coast of Picardy; which they afterwards confirmed, when they viewed them through their telescopes. Their remarks were, that the places appeared as near as if they were sailing at a small distance into the harbour.

Having indulged his curiosity on the shore for near an hour, during which time the cliffs appeared to be at some times more bright and near, at others more faint and distant, but never out of sight, Mr. L. went upon the eastern cliff or hill, which is of a considerable height, when a most beautiful scene presented itself to view; for he could at once see Dengeness, Dover cliffs, and the French coast, all along from Calais, Boulogne, &c, to St. Vallery; and as some of the fishermen affirmed, as far to the westward even as Dieppe. By the telescope, the French fishing-boats were plainly to be seen at anchor; and the different colours of the land upon the heights, together with the buildings, were perfectly discernible. This curious phenomenon continued in the highest splendour till past 8 o'clock (though a black cloud totally obscured the face of the sun for some time) when it gradually vanished.

The day was extremely hot, 76° at 5 afternoon, and the three preceding days remarkably fine and clear. Not a breath of wind was stirring the whole of the day; but the small pennons at the mast-heads of the fishing-boats in the harbour were in the morning at all points of the compass.—Mr. L. was, a few days afterwards, at Winchelsea, and at several places along the coast, where he was informed that the above phenomenon had been equally visible.—The cape of land called Dengeness, which extends nearly 2 miles into the sea, and is about 16 miles distant from Hastings, in a straight line, appeared as if quite close to it, as did the fishing-boats, and other vessels which were sailing between the two places.

Similar and still more extraordinary instances of atmospherical refraction have been since described in different volumes of the Philos. Trans. for the years 1795, 1797, 1799, 1800 &c, by Mr. Dalby, Cap. Huddart, Sir Henry Englefield, Mr. Latham, Mr. Vince, and Dr. Wollaston. Mr. Huddart first noticed a distinct image, inverted beneath the object itself; and described several such appearances, accompanied with an optical explanation, remarking that the lowest strata of the air were at the time endued with a weaker refractive power, than others at a small elevation. Mr. Vince has given an instance where erect, as well as inverted images, were visible above, instead of beneath, the objects themselves; and, by tracing the progress of the rays of light, in a manner similar to Mr. Huddart's, concludes that these phenomena arose from unusual variations of increasing density in the lower strata of the atmosphere. In the vol. for 1795, Mr. Dalby mentions having seen the top of a hill appear detached; for the sky was seen under it. In this case, as well as in the preceding, says Dr. W., it is probable that inversion took place, and that the lower half of the portion detached was an inverted image of the upper, as the sky could only be seen beneath it by an inverted course of the rays.

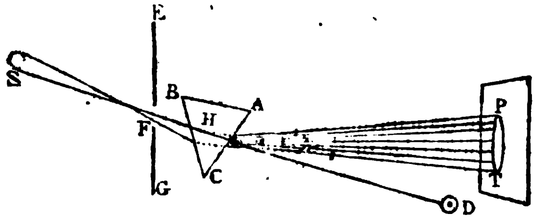
Since the causes of such peculiarities of terrestrial refraction had not received so full an explanation as might be wished, Dr. Wollaston has endeavoured, 1st, To investigate theoretically the successive variations of increasing or decreasing density to which fluids in general are liable, and the laws of the refractions occasioned by them. 2dly. To illustrate and confirm the truth of this theory by experiments with fluids of known density. And lastly, to ascertain, by trial on the air itself, the causes and extent of those variations of its refractive density, on which the inversions of objects, &c, appear to depend. See vol. 90, or my Abridgement, vol. 18, p. 667.

In a late letter from Mr. Dalby he says, he repeatedly observed, when measuring the base on King's Sedgemoor, those extraordinary refractions. The moor is several miles in length, and as level as the sea. When the sun shone out after a shower of rain, he placed a telescope on the top of the front wheel of a carriage, and then the cattle grazing on the moor, at the distance of 4 or 5 miles, appeared through the telescope in their proper shapes and position, without any inversion; but when the telescope was laid on the box near the axle, at about 2 feet below the top of the wheel, or 2 feet from the ground, he saw the inverted images of the cattle complete. Suppose, says he, a looking-glass laid on a table before you; then if you conceive flies, or mice, or any small animals, to be walking on the glass, you will have a perfect idea of the appearance. (See an idea of it represented, plate xxxi, fig. 1). It was curious to see cows and horses with their backs downwards, walking foot to foot against others above. The lower or reflected images were as bright and well defined, as the upper or real objects. In moving the telescope from the top of the wheel, down towards the axle, the first change observed, was the lengthening of the animals' legs; afterwards, before the complete inversion took place, the appearances were so singularly fantastical, that it is impossible to describe them.

The inversion, above mentioned, is evidently the effect of reflection from a stratum of dense vapour; for I never could perceive any thing of the kind but when the sun shone out immediately after a shower of rain, and the evaporation was copious. Such refractions and reflections will account for those strange appearances noticed by some travellers while they were crossing the extensive flats in Arabia and Egypt.

A similar phenomenon Mr. Dalby observed while he was crossing, in a small boat, from Mutton Cove, Plymouth Dock, to the Passage-house below Mount Egdecumbe. He says, "When my eye was brought down to the edge of the boat, about a foot from the surface of the water, the summit of the distant rock called the Mewer-stone, in Plymouth harbour, appeared totally detached, or lifted up, from the lower part. This proves that the vapour rising from the sea must have had a great refractive power near the surface; for no apparent separation took place when the eye was 2 or 3 feet from the water."

REFRANGIBILITY *of Light*, the disposition of the rays to be refracted. And a greater or less refrangibility, is a disposition to be more or less refracted, in passing at equal angles of incidence into the same medium.—That the rays of light are differently refrangible, is the foundation of Newton's whole theory of light and colours; and the truth and circumstances of the principle he evinced from such experiments as the following.



Let EG represent the window-shutter of a dark room, and F a hole in it, through which the light passes, from the luminous object S, to the glass prism ABC within the room, which refracts it towards the opposite side, or a screen, at PT, where it appears of an oblong form; its length being about 5 times the breadth, and exhibiting the various colours of the rainbow; whereas without the interposition of the prism, the ray of light would have proceeded on in its first direction to D. Hence then it follows, 1. That the rays of light are refrangible. This appears by the ray being refracted from its original direction SHD, into another, HP or HT, by passing through a different medium.—2. That the ray SFH is a compound one, which, by means of the prism, is decompounded or separated into its parts, HP, HT, &c, which it hence appears are all endued with different degrees of refrangibility, as they are transmitted to all the intermediate points from T to P, and there painting all the different colours.—From this, and a great variety of other experiments, Newton proved, that the blue rays are more refracted than the red ones, and that there is likewise unequal refraction in the intermediate rays; and upon the whole it appears that the sun's rays have not all the same refrangibility, and consequently are not of the same nature. It is also observed that those rays which are most refrangible, are also most reflexible. See Reflexibility; also Newton's Optics, pa. 22 &c, 3d edit.

The difference between refrangibility and reflexibility was first discovered by Sir Isaac Newton, in 1671-2, and communicated to the Royal Society, in a letter dated Feb. 6 of that year, which was published in the Philos. Trans. numb. 80, pa. 3075; and from that time it was vindicated by him, from the objections of several persons; particularly Pardies, Mariotte, Linus or Lin, and other gentlemen of the English college at Liege; and at length it was more fully laid down, illustrated, and confirmed, by a great variety of experiments, related in his excellent treatise on Optics.

But further, as not only these colours of light produced by refraction in a prism, but also those reflected from opaque bodies, have their different degrees of refrangibility and reflexibility; and as a white light arises from a mixture of the several coloured rays together, the same great author concluded that all homogeneous light has its proper colour, corresponding to its degree of refrangibility, and not capable of being changed by any reflections, or any refractions; that the sun's light is composed of all the primary colours; and that all compound colours arise from the mixture of the primary ones, &c.

The different degrees of refrangibility, he conjectures to arise from the different magnitude of the particles composing the different rays. Thus, the most refrangible rays, that is the red ones, he supposes may consist of the largest particles; the least refrangible, i. e. the violet rays, of the smallest particles; and the intermediate rays, yellow, green, and blue, of particles of intermediate sizes. See Colour.

Dr. Herschel has made many ingenious observations and experiments on the different degrees of refrangibility of the sun's rays; from which it appears, that beside the seven coloured rays of light which formed the basis of Newton's theory, there are other rays that are perfectly colourless; a summary of which experiments is given under the article Sun, in treating of the nature of his rays. See Sun.

For the method of correcting the effect of the different refrangibility of the rays of light in glasses, see Aberration and Telescope.

REGEL, or Rigel, a fixed star of the first magnitude, in the left foot of Orion.

REGIOMONTANUS. See *John* Muller.

REGION, of the Air or Atmosphere. Authors divide the atmosphere into three stages, called the upper, middle, and lower regions.—The lowest region is that in which we breathe, and is bounded by the reflection of the sun's rays, that is, by the height to which they rebound from the earth.—The middle region is that in which the clouds reside, and where meteors are formed, &c; extending from the extremity of the lowest, to the tops of the highest mountains.—The upper region commences from the tops of the mountains, and reaches to the utmost limits of the atmosphere. In this region there probably reigns a perpetual equable calmness, clearness, and serenity.

Elementary Region, according to the Aristotelians, is a sphere terminated by the concavity of the moon's orb, comprehending the earth's atmosphere,

Ethereal Region, is the whole extent of the universe, comprising all the heavens with the orbs of the fixed stars and other celestial bodies.

Region, in Geography, a country or particular division of the earth, or a tract of land inhabited by people of the same nation.

Regions *of the Moon*. Modern astronomers divide the moon into several regions, or provinces, to each of which they give its proper name.

Regions *of the Sea*, are the two parts into which the whole depth of the sea is conceived to be divided. The upper of these extends from the surface of the water, down as low as the rays of the sun can pierce, and extend their influence; and the lower region extends from thence to the bottom of the sea.

Subterranean Regions. These are three, into which the earth is divided, at different depths below the surface, according to different degrees of cold or warmth; and it is imagined that the 2d or middlemost of these regions is the coldest of the three.

REGIS (Peter Sylvain), a French philosopher, and great propagator of Cartesianism, was born in Agenois 1632. He studied the languages and philosophy under the Jesuits at Cahors, and afterwards divinity in the university of that town, being designed for the church. His progress in learning was so uncommon, that at the end of four years he was offered a doctor's degree without the usual charges; but he did not think it became him till he should study also in the Sorbonne at Paris. He accordingly repaired to the capital for that purpose; but he soon became disgusted with theology; and, as the philosophy of Descartes began at that time to become popular through the lectures of Rohault, he conceived a taste for it, and gave himself up entirely to its doctrines. Having, by attending those lectures, and by close study, become an adept in that philosophy, he went to Toulouse in 1665, where he gave lectures in it himself. Having a clear and fluent manner, and a happy way of explaining his subject, he drew many persons to his discourses; the magistrates, the literati, the ecclesiastics, and the very women, who all now affected to renounce the ancient philosophy.

In 1671, he received at Montpellier the same applauses for his lectures as at Toulouse. Finally, in 1680 he returned to Paris; where the concourse about him was such, that the sticklers for Peripateticism began to be alarmed. These applying to the archbishop of Paris, he thought it expedient, in the name of the king, to put a stop to the lectures; which accordingly were discontinued for several months. Afterwards his whole time was spent in propagating the new philosophy, both by lectures, and by publishing books; and in defence of his system, he had disputes with Huet, Du Hamel, Malbranche, and others. His works, though abounding with ingenuity and learning, have been neglected in consequence of the great discoveries and advancement in philosophic knowledge that has been since made.—He was chosen a member of the Academy of Sciences in 1699; and died in 1707, at 75 years of age.

His works, which he published, are,

1. A system of Philosophy; containing Logic, Metaphysics, and Morals: in 1690, 3 vols in 4to. being a compilation of the different ideas of Descartes.—It was reprinted the year after at Amsterdam, with the addition of a Discourse on Ancient and Modern Philosophy.

2. The Use of Reason and of Faith.

3. An Answer to Huet's Censures of the Cartesian Philosophy; and an Answer to Du Hamel's Critical Reflections.

4. Some pieces against Malbranche, to show that the apparent magnitude of an object depends solely on the magnitude of its image, traced on the retina.

5. A small piece on the question, Whether Pleasure makes our present Happiness?

REGRESSION, or Retrogradation *of Curves*, &c. See Retrogradation.

REGULAR *Figure*, in Geometry, is a figure that is both equilateral and equiangular, or having all its sides and angles equal to one another.—For the dimensions, properties, &c, of regular figures, see Polygon.

Regular *Body*, called also, *Platonic Body*, is a body or solid comprehended by like, equal, and regular plane figures, and whose solid angles are all equal. The plane figures by which the solid is contained, are the faces of the solid. And the sides of the plane figures are the edges, or linear sides of the solid.

There are only five regular solids, viz,

The tetraedron, or regular triangular pyramid, having 4 triangular faces;

The hexaedron, or cube, having 6 square faces;

The octaedron, having 8 triangular faces;

The dodecaedron, having 12 pentagonal faces;

The icosaedron, having 20 triangular faces.

Besides these five, there can be no other regular bodies in nature.

Prob. 1. *To construct or form the Regular Solids.*—See the method of describing these figures under the article Body.

2. *To find either the Surface or the Solid Content of any of the Regular Bodies.*—Multiply the proper tabular area or surface (taken from the following table) by the square of the linear edge of the solid, for the superficies. And multiply the tabular solidity, in the last column of the table, by the cube of the linear edge, for the solid content.

Surfaces and Solidities of Regular Bodies, the Side being unity or 1.

No. of sides.	Name.	Surface.	Solidity.
4	Tetraedron	1·7320508	0·1178513
6	Hexaedron	6·0000000	1·0000000
8	Octaedron	3·4641016	0·4714045
12	Dodecaedron	20·6457788	7·6631189
20	Icosaedron	8·6602540	2·1816950

3. The diameter of a sphere being given, to find the side of any of the Platonic bodies, that may be either inscribed in the sphere, or circumscribed about the sphere, or that is equal to the sphere.

Multiply the given diameter of the sphere by the proper or corresponding number, in the following table, answering to the thing sought, and the product will be the side of the Platonic body required.

The diam. of a sphere being 1, the side of a	That may be inscribed in the sphere, is	That may be circumscribed about the sphere, is	That is equal to the sphere, is
Tetraedron	0·816497	2·44948	1·64417
Hexaedron	0·577350	1·00000	0·88610
Octaedron	0·707107	1·22474	1·03576
Dodecaedron	0·525731	0·66158	0·62153
Icosaedron	0·356822	0·44903	0·40883

4. The side of any of the five Platonic bodies being given, to find the diameter of a sphere, that may either be inscribed in that body, or circumscribed about it, or that is equal to it.—As the respective number in the table above, under the title, *inscribed*, *circumscribed*, or *equal*, is to 1, so is the side of the given Platonic body, to the diameter of its inscribed, circumscribed, or equal sphere.

5. The side of any one of the five Platonic bodies being given; to find the side of any of the other four bodies, that may be equal in solidity to that of the given body.—As the number under the title *equal* in the last column of the table above, against the given Platonic body, is to the number under the same title, against the body whose side is sought, so is the side of the given Platonic body, to the side of the body sought.

See demonstrations of many other properties of the Platonic bodies, in my Mensuration, part 3 sect. 2 pa. 183, &c, 4th edition.

REGULAR *Curve*. See CURVE.

REGULATOR *of a Watch*, is a small spring belonging to the balance, serving to adjust the going, and to make it go either faster or slower.

REGULUS, in Astronomy, a star of the first magnitude, in the constellation Leo; called also, from its situation, Cor Leonis, or the Lion's Heart; by the Arabs, Alhabor; and by the Chaldeans, Kalbeleced, or Karbeleceid; from an opinion of its influencing the affairs of the heavens; as Theon observes.—The longitude of Regulus, as fixed by Flamsteed, is 25° 31′ 21″, and its latitude 0° 26′ 38″ north. See LEO.

REINFORCE, in Gunnery, is that part of a gun next the breech, which is made stronger to resist the force of the powder. There are usually two reinforces in each piece, called the first and second reinforce. The second is somewhat smaller than the first, because the inflamed powder in that part is less strong.

REINFORCE *Rings* of a cannon, are flat mouldings, like iron hoops, placed at the breech end of the first and second reinforce, projecting beyond the rest of the metal about a quarter of an inch.

REINHOLD (ERASMUS), an eminent astronomer and mathematician, was born at Salfeldt in Thuringia, a province in Upper Saxony, the 11th of October 1511. He studied mathematics under James Milichi at Wittemberg, in which university he afterwards became professor of those sciences, which he taught with great applause; and after writing a number of useful and learned works, he died the 19th of February 1553, at 42 years of age only. His writings are chiefly the following:

1. Theoriæ novæ Planetarum G. Purbachii, augmented and illustrated with diagrams and Scholia; in 8vo, 1542; and again in 1580.—In this work, among other things worthy of notice, he teaches (pa. 75 and 76) that the centre of the lunar epicycle describes an oval figure in each monthly period, and that the orbit of Mercury is also of the same oval figure.

2. Ptolemy's Almagest, the first book, in Greek, with a Latin version, and Scholia, explaining the more obscure passages; in 8vo, 1549.—At the end of pa. 123 he promises an edition of Theon's Commentaries, which are very useful for understanding Ptolemy's meaning; but his immature death prevented Reinhold from giving this and other works which he had projected.

3. Prutenicæ Tabulæ Cœlestium Motuum, in 4to, 1551; again in 1571; and also in 1585.—Reinhold spent seven years labour on this work, in which he was assisted by the munificence of Albert, duke of Prussia, whence the tables had their name: he compared the observations of Copernicus with those of Ptolemy and Hipparchus, whence he constructed these new tables, the uses of which he has fully explained in a great number of precepts and canons, forming a complete introduction to practical astronomy.

4. Primus liber Tabularum Directionum; to which are added, the Canon Fœcundus, or Table of Tangents, to every minute of the quadrant; and New Tables of Climates, Parallels and Shadows, with an Appendix, containing the second Book of the Canon of Directions; in 4to, 1554.—Reinhold here supplies what was omitted by Regiomontanus in his Table of Directions, &c; showing the finding of the sines, and the construction of the tangents, the sines being found to every minute of the quadrant, to the radius 10,000,000; and he produced the Oblique Ascensions from 60 degrees to the end of the quadrant. He teaches also the use of these tables in the solution of spherical problems.

Reinhold prepared likewise an edition of many other works, which are enumerated in the Emperor's Privilege, prefixed to the Prutenic Tables. Namely, Ephemerides for several years to come, computed from the new tables. Tables of the Rising and Setting of several Fixed Stars, for many different climates and times. The illustration and establishment of Chronology, by the eclipses of the luminaries, and the great conjunctions of the planets, and by the appearance of comets, &c. The Ecclesiastical Calendar. The History of Years, or Astronomical Calendar. Isagoge Spherica, or Elements of the Doctrine of the Primum Mobile. Hypotyposes Orbium Cœlestium, or the Theory of Planets. Construction of a New Quadrant. The Doctrine of Plane and Spherical Triangles. Commentaries on the work of Copernicus. Also Commentaries on the 15 books of Euclid, on Ptolemy's Geography, and on the Optics of Alhazen the Arabian. He also made many Astronomical Observations, but with a wooden quadrant, which observations were seen by Tycho Brahé when he passed through Wittemberg in the year 1575, who wondered that so great a cultivator of astronomy was not furnished with better instruments.

Reinhold left a son, named also Erasmus after himself, an eminent mathematician and physician at Salfeldt. He wrote a small work in the German language, on Subterranean Geometry, printed in 4to at Erfurt 1575.—He wrote also concerning the New Star which appeared in Cassiopeia in the year 1572; with an Astrological Prognostication, published in 1574, in the German language.

RELAIS, in Fortification, a French term, the same with berme.

RELATION, in Mathematics, is the habitude or respect of quantities of the same kind to each other, with regard to their magnitude; more usually called Ratio.—And the equality, identity, or sameness of two such relations, is called proportion.

Relation, *Inharmonical*, in Musical Composition, is that whose extremes form a false or unnatural interval, incapable of being sung.—This is otherwise called a false relation, and stands opposed to a just or true one.

RELATIVE *Gravity*, *Levity*, *Motion*, *Necessity*, *Place*, *Space*, *Time*, *Velocity*, *&c.* See the several substantives.

RELIEVO, in Architecture, denotes the sally or projecture of any ornament.

REMAINDER, is the difference between two quantities, or that which is left after subtracting one from the other.

RENDERING, in Building. See Pargeting.

REPEATING Circle. See Circular *Instruments.*

REPELLING *Power*, in Physics, is a certain power or faculty, residing in the minute particles of natural bodies, by which, under certain circumstances, they mutually fly from each other: being the reverse or opposite to the attractive power. Newton shows, from observation, that such a force does really exist; and he argues, that as in algebra, where positive quantities cease, there negative ones begin; so in physics, where the attractive force ceases, there a repelling force must begin.

As the repelling power seems to arise from the same principle as the attractive, only exercised under different circumstances, it is governed by the same laws. Now the attractive power we find is stronger in small bodies than in great ones, in proportion to the masses; therefore the repelling is so too: and as the rays of light are the most minute bodies we know of; therefore their repelling force must be the greatest. It is computed by Newton, that the attractive force of the rays of light is above 1000000000000000, or one thousand million of millions of times stronger than the force of gravity on the surface of the earth: hence arises that inconceivable velocity with which light must move to reach from the sun to the earth in little more than 7 minutes of time. For the rays emitted from the body of the sun, by the vibrating motion of its parts, are no sooner got without the sphere of attraction of the sun, than they come within the action of the repelling power.

The elasticity or springiness of bodies, or that property by which, after having their figure altered by an external force, they return to their former shape again, follows from the repelling power. See Repulsion.

REPERCUSSION. See Reflection.

REPETEND, in Arithmetic, denotes that part of an infinite decimal fraction, which is continually repeated ad infinitum. Thus in the numbers 2·13 13 13 &c, the figures 13 are the repetend, and marked thus $\dot{1}\dot{3}$.—These repetends chiefly arise in the reduction of vulgar fractions to decimals. Thus, $\frac{1}{3} = 0{\cdot}333\&c = 0{\cdot}\dot{3}$; and $\frac{1}{6} = 0{\cdot}1666\ \&c = 1{\cdot}1\dot{6}$; and $\frac{1}{7} = 0{\cdot}142857\ 142857\ \&c = 0{\cdot}\dot{1}4285\dot{7}$. Where it is to be observed, that a point is set over the figure of a single repetend, and over the first and last figure when there are several that repeat.

Repetends are either Single or Compound.

A *Single* Repetend is that in which only one figure repeats; as $0{\cdot}\dot{3}$, or $0{\cdot}\dot{6}$, &c.

A *Compound* Repetend, is that in which two or more figures are repeated; as ${\cdot}\dot{1}\dot{3}$, or ${\cdot}\dot{2}1\dot{5}$, or ${\cdot}\dot{1}4285\dot{7}$.

Similar Repetends are such as begin at the same place, and consist of the same number of figures: as ${\cdot}\dot{3}$ and ${\cdot}\dot{6}$, or $1{\cdot}\dot{3}4\dot{1}$ and $2{\cdot}\dot{1}5\dot{6}$.

Dissimilar Repetends begin at different places, and consist of an unequal number of figures.

To find the finite Value of any Repetend, or to reduce it to a vulgar fraction. Take the given repeating figure or figures for the numerator; and for the denominator, take as many 9's as there are recurring figures or places in the given repetend.

$$\text{So } {\cdot}\dot{3} = \frac{3}{9} = \frac{1}{3};\ \text{and } {\cdot}0\dot{5} = {\cdot}0\frac{5}{9} = \frac{5}{90} = \frac{1}{18};$$

$$\text{and } {\cdot}\dot{1}2\dot{3} = \frac{123}{999} = \frac{41}{333};\ \text{and } 2{\cdot}\dot{6}\dot{3} = 2\frac{63}{99} = 2\frac{7}{11};$$

$$\text{and } {\cdot}0\dot{5}9440\dot{5} = \frac{594405}{9999990} = \frac{17}{280};\ \&c.$$

Hence it follows, that every such infinite repetend has a certain determinate and finite value, or can be expressed by a terminate vulgar fraction. And consequently, that an infinite decimal which does not repeat or circulate, cannot be completely expressed by a finite vulgar fraction.

It may further be observed, that if the numerator of a vulgar fraction be 1, and the denominator any prime number, except 2 and 5, the decimal which shall be equal to that vulgar fraction, will always be a repetend, beginning at the first place of decimals; and this repetend must necessarily be a submultiple, or an aliquot part of a number expressed by as many 9's as the repetend has figures; that is, if the repetend have six figures, it will be a submultiple of 999999; if four figures, a submultiple of 9999; &c. Whence it follows, that if any prime number be called p, the series 9999&c, produced as far as is necessary, will always be divisible by p, and the quotient will be the repetend of the decimal fraction $1 \div p$.

The same is also true of any odd number whatever that is not divisible by 5; and for any repetend as well as 9. That is, any odd number, not divisible by 5, is a divisor of any repetend digit carried to a sufficient number of places, and these will never exceed the number expressed by the divisor.

It is also a curious circumstance, that all fractions whose denominators are the same, are expressed in decimals by repetends which have the same effective figures, though varied in their position. Thus,

$\frac{1}{7} = {\cdot}142857\ 142857$, &c.
$\frac{2}{7} = {\cdot}285714\ 285714$, &c.
$\frac{3}{7} = {\cdot}428571\ 428571$, &c.
$\frac{4}{7} = {\cdot}571428\ 571428$, &c.
$\frac{5}{7} = {\cdot}714285\ 714285$, &c.
$\frac{6}{7} = {\cdot}857142\ 857142$, &c.

RESIDUAL *Figure*, in Geometry, the figure remaining after subtracting a less from a greater.

Residual *Root*, is a root composed of two parts or members, only connected together with the sign $-$ or minus. Thus, $a-b$, or $5-3$, is a residual root; and is so called, because its true value is no more than the residue, or difference between the parts a and b, or 5 and 3, which in this case is 2.

RESIDUUM *of a Charge*, in Electricity, first discovered by Mr. Gralath, in Germany, in 1746, is that part of the charge that lay on the uncoated part of a Leyden phial, which does not part with all its electricity at once; so that it is afterwards gradually diffused to the coating.

RESISTANCE, or Resisting *Force*, in Physics, any power which acts in opposition to another, so as to destroy or diminish its effect.

There are different kinds of resistance, arising from the various natures and properties of the resisting bodies, and governed by various laws; as, the resistance of solids, the resistance of fluids, the resistance of the air, &c. Of each of these in their order, as below.

Resistance *of Solids*, in Mechanics, is the force with which the quiescent parts of solid bodies oppose the motion of others contiguous to them. Of these, there are two kinds. The first, where the resisting and the resisted parts, i. e. the moving and quiescent bodies, are only contiguous, and do not cohere; constituting separate bodies or masses. This resistance is what Leibnitz calls Resistance of the surface, but which is more properly called Friction: for the laws of which, see the article **Friction**.

The 2d case of resistance, is where the resisting and resisted parts are not only contiguous, but cohere, being parts of the same continued body or mass. This resistance was first considered by Galileo, and may properly be called Renitency.—As to what regards the resistance of bodies when struck by others in motion, see **Percussion**, and **Collision**.

Theory of the Resistance of the Fibres of Solid Bodies.—To conceive an idea of this resistance, or renitency of the parts, suppose a cylindrical body suspended vertically by one end. Here all its parts, being heavy, tend downwards, and endeavour to separate the two contiguous planes or surfaces where the body is the weakest; but all the parts of them resist this separation by the force with which they cohere, or are bound together. Here then are two opposite powers; viz, the weight of the cylinder, which tends to break it; and the force of cohesion of the parts, which resists the fracture.

If now the base of the cylinder be increased, without increasing its length; it is evident that both the resistance and the weight will be increased in the same ratio as the base; and hence it appears that all cylinders of the same matter and length, whatever their bases be, have an equal resistance, when vertically suspended.

But if the length of the cylinder be increased, without increasing its base, its weight is increased, while the resistance or strength continues unaltered; consequently the lengthening has the effect of weakening it, or increases its tendency to break.

Hence, to find the greatest length a cylinder of any matter may have, when it just breaks with the addition of another given weight, we need only take any cylinder of the same matter, and fasten to it the least weight that is just sufficient to break it; and then consider how much it must be lengthened, so that the weight of the part added, together with the given weight, may be just equal to that weight, and the thing is done. Thus, let l denote the first length of the cylinder, c its weight, g the given weight the lengthened cylinder is to bear, and w the least weight that breaks the cylinder l, also x the length sought; then as $l : x :: c : \frac{cx}{l}$ = the weight of the longest cylinder sought; and this, together with the given weight g, must be equal to c together with the weight w; hence then $\frac{cx}{l} + g = c + w$; therefore $x = \frac{c + w - g}{c} l$ = the whole length of the cylinder sought. If the cylinder must just break with its own weight, then is $g = 0$, and in that case $x = \frac{c + w}{c} l$ is the whole length that just breaks by its own weight. By this means Galileo found that a copper-wire, and of consequence any other cylinder of copper, might be extended to 4801 braccios or fathoms of 6 feet each.

If the cylinder be fixed by one end into a wall, with the axis horizontally; the force to break it, and its resistance to fracture will here be both different; as both the weight to cause the fracture, and the resistance of the fibres to oppose it, are combined with the effects of the lever; for the weight to cause the fracture, whether of the beam alone, or combined with an additional weight hung to it, is to be supposed collected into the centre of gravity, where it is considered as acting by a lever equal to the distance of that centre beyond the face of the wall where the cylinder or other prism is fixed; and then the product of the said whole weight and distance, will be the momentum or force to break the prism. Again, the resistance of the fibres may be supposed collected into the centre of the transverse section, and all acting there at the end of a lever equal to the vertical semidiameter of the section, the lowest point of that diameter being immoveable, and about which the whole diameter turns when the prism breaks; and hence the product of the adhesive force of the fibres multiplied by the said semidiameter, will be the momentum of resistance, and which must be equal to the former momentum when the prism just breaks.

Hence, to find the length a prism will bear, fixed so horizontally, before it breaks, either by its own weight, or by the addition of any adventitious weight; take any length of such a prism, and load it with weights till it just break. Then, put

l = the length of this prism,
c = its weight,
w = the weight that breaks it,
a = distance of weight w,
g = any given weight to be borne,
d = its distance,
x = the length required to break.

Then $l : x :: c : \frac{cx}{l}$ the weight of the prism x, and $\frac{cx}{l} \times \frac{1}{2}x = \frac{cx^2}{2l}$ = its momentum; also dg = the momentum of the weight g; therefore $\frac{cx^2}{2l} + dg$ is the momentum of the prism x and its added weight. In like manner $\frac{1}{2}cl + aw$ is that of the former or short prism and the weight that brake it; consequently $\frac{cx^2}{2l} + dg = \frac{1}{2}cl + aw$, and $x = \sqrt{\frac{aw + \frac{1}{2}cl - dg}{c} \times 2l}$ is the length sought, that just breaks with the weight g at the distance d. If this weight g be nothing, then $x = \sqrt{\frac{aw + \frac{1}{2}cl}{c} \times 2l}$ is the length of the prism that just breaks with its own weight.

If two prisms of the same matter, having their bases and lengths in the same proportion, be suspended horizontally; it is evident that the greater has more weight than the lesser, both on account of its length, and of its base; but it has less resistance on account of its length, considered as a longer arm of a lever, and has only more resistance on account of its base; therefore it exceeds the lesser in its momentum more than it does in its resistance, and consequently it must break more easily.

Hence appears the reason why, in making small machines and models, people are apt to be mistaken as to the resistance and strength of certain horizontal pieces, when they come to execute their designs in large, by observing the same proportions as in the small.

When the prism, fixed vertically, is just about to break, there is an equilibrium between its positive and relative weight; and consequently those two opposite powers are to each other reciprocally as the arms of the lever to which they are applied, that is, as half the diameter to half the axis of the prism. On the other hand, the resistance of a body is always equal to the greatest weight which it will just sustain in a vertical position, that is, to its absolute weight. Therefore, substituting the absolute weight for the resistance, it appears, that the absolute weight of a body, suspended horizontally, is to its relative weight, as the distance of its centre of gravity from the fixed point or axis of motion, is to the distance of the centre of gravity of its base from the same.

The discovery of this important truth, at least of an equivalent to it, and to which this is reducible, we owe to Galileo; on whose system of resistance, however, Mariotte made an ingenious remark, which gave birth to a new system. Galileo supposes that where the body breaks, all the fibres break at once; so that the body always resists with its whole absolute force, or the whole force that all its fibres have in the place where it breaks. But Mariotte, finding that all bodies, even glass itself, bend before they break, shows that fibres are to be considered as so many little bent springs, which never exert their whole force, till stretched to a certain point, and never break till entirely unbent. Hence those nearest the fulcrum of the lever, or lowest point of the fracture, are stretched less than those farther off, and consequently employ a less part of their force, and break later.

This consideration only takes place in the horizontal situation of the body: for in the vertical, the fibres of the base all break at once; so that the absolute weight of the body must exceed the united resistance of all its fibres; a greater weight is therefore required here than in the horizontal situation, that is, a greater weight is required to overcome their united resistance, than to overcome their several resistances one after another.

Varignon has improved on the system of Mariotte, and shown that to Galileo's system, it adds the consideration of the centre of percussion: for in each system, the section, where the body breaks, moves on the axis of equilibrium, or line at the lower extremity of the same section; but in the second, the fibres of this section are continually stretching more and more, and that in the same ratio, as they are situated farther and farther from the axis of equilibrium, and consequently are still exerting a greater and greater part of their whole force.

These unequal extensions, like all other forces, must have some common centre where they are united, making equal efforts on each side of it: and as they are precisely in the same proportion as the velocities which the several points of a rod moved circularly would have to one another, the centre of extension of the section where the body breaks, must be the same as its centre of percussion. Galileo's hypothesis, where fibres stretch equally, and break all at once, corresponds to the case of a rod moving parallel to itself, where the centre of extension or percussion does not appear, as being confounded with the centre of gravity.

Hence it follows, that the resistance of bodies in Mariotte's system, is to that in Galileo's, as the distance of the centre of percussion, taken on the vertical diameter of the fracture, is to the whole of that diameter: and hence also, the resistance being less than what Galileo imagined, the relative weight must also be less, and in the ratio just mentioned. So that, after conceiving the relative weight of a body, and its resistance equal to its absolute weight, as two contrary powers applied to the two arms of a lever, in the hypothesis of Galileo, there needs nothing to change it into that of Mariotte, but to imagine that the resistance, or the absolute weight, is become less, in the ratio above mentioned, every thing else remaining the same.

One of the most curious, and perhaps the most useful questions in this research, is to find what figure a body must have, that its resistance may be equal or proportional in every part to the force tending to break it. Now to this end, it is necessary, some part of it being conceived as cut off by a plane parallel to the fracture, that the momentum of the part retrenched be to its resistance, in the same ratio as the momentum of the whole is to its resistance; these four powers acting by arms of levers peculiar to themselves, and are proportional in the whole, and in each part, of a solid of equal resistance; and from this proportion, Varignon easily deduces two solids, which shall resist equally in all their parts, or be no more liable to break in one part than in another: Galileo had found one before. That discovered by Varignon is in the form of a trumpet, and is to be fixed into a wall at its greater end; so that its magnitude or weight is always diminished in proportion as its length, or the arm of the lever by which its weight acts, is increased: and it is remarkable that, however different the two systems may be, the solids of equal resistance are the same in both.

For the resistance of a solid supported at each end, as of a beam between two walls, see **Beam**.

Resistance *of Fluids*, is the force with which bodies, moving in fluid mediums, are impeded and retarded in their motion. A body moving in a fluid is resisted from two causes. The first of these is the cohesion of the parts of the fluid. For a body, in its motion, separating the parts of a fluid, must overcome the force with which those parts cohere. The second is the inertia, or inactivity of matter, by which a certain force is required to move the particles from their places, in order to let the body pass.

The retardation from the first cause is always the same in the same space, whatever the velocity be, the body remaining the same; that is, the resistance is as the space run through in the same time: but the velocity is also in the same ratio of the space run over in the same time: and therefore the resistance, from this cause, is as the velocity itself.

The resistance from the second cause, when a body moves through the same fluid with different velocities, is as the square of the velocity. For, first the resistance increases according to the number of particles or quantity of the fluid struck in the same time; which number must be as the space run through in that time, that is, as the velocity; but the resistance also increases in proportion to the force with which the body strikes against every part; which force is also as the velocity of the body, being double with a double velocity, and triple with a triple one, &c: therefore, on both these accounts, the resistance is as the velocity multiplied by the velocity, or as the square of the velocity. On the whole therefore, on account of both causes, viz, the tenacity and inertia of the fluid, the body is resisted partly as the velocity and partly as the square of the velocity.

But when the same body moves through different fluids with the same velocity, the resistance from the second cause follows the proportion of the matter to be removed in the same time, which is as the density of the fluid.

Hence therefore, if d denote the density of the fluid,
v the velocity of the body,
and a and b constant coefficients:
then $adv^2 + bv$ will be proportional to the whole resist-

ance to the same body, moving with different velocities, in the same direction, through fluids of different densities, but of the same tenacity.

But, to take in the consideration of different tenacities of fluids; if t denote the tenacity, or the cohesion of the parts of the fluid, then $adv^2 + btv$ will be as the whole resistance.

Indeed the quantity of resistance from the cohesion of the parts of fluids, except in glutinous ones, is very small in respect of the other resistance; and it also increases in a much lower degree, being only as the velocity, while the other increases as the square of the velocity, and rather more. Hence then the term btv is very small in respect of the other term adv^2; and consequently the resistance is nearly as this latter term; or nearly as the square of the velocity. Which rule has been employed by most authors, and is very near the truth in slow motions; but in very rapid ones, it differs considerably from the truth, as we shall perceive below; not indeed from the omission of the small term btv, due to the cohesion, but from the want of the full counter pressure on the hinder part of the body, a vacuum, either perfect or partial, being left behind the body in its motion; and also perhaps to some compression or accumulation of the fluid against the fore part of the body. Hence,

To conceive the resistance of fluids to a body moving in them, we must distinguish between those fluids which, being greatly compressed by some incumbent weight, always close up the space behind the body in motion, without leaving any vacuity there; and those fluids which, not being much compressed, do not quickly fill up the space quitted by the body in motion, but leave a kind of vacuum behind it. These differences, in the resisting fluids, will occasion very remarkable varieties in the laws of their resistance, and are absolutely necessary to be considered in the determination of the action of the air on shot and shells; for the air partakes of both these affections, according to the different velocities of the projected body.

In treating of these resistances too, the fluids may be considered either as continued or discontinued, that is, having their particles contiguous or else as separated and unconnected; and also either as elastic or non-elastic. If a fluid were so constituted, that all the particles composing it were at some distance from each other, and having no action between them, then the resistance of a body moving in it would be easily computed, from the quantity of motion communicated to those particles; for instance, if a cylinder moved in such a fluid in the direction of its axis, it would communicate to the particles it met with, a velocity equal to its own, and in its own direction, when neither the cylinder nor the parts of the fluid are elastic: whence, if the velocity and diameter of the cylinder be known, and also the density of the fluid, there would thence be determined the quantity of motion communicated to the fluid, which (as action and reaction are equal) is the same with the quantity lost by the cylinder, and consequently the resistance would thus be ascertained.

In this kind of discontinued fluid, the particles being detached from each other, every one of them can pursue its own motion in any direction, at least for some time, independent of the neighbouring ones; so that, instead of a cylinder moving in the direction of its axis, if a body with a surface oblique to its direction be supposed to move in such a fluid, the motion which the parts of the fluid will hence acquire, will not be in the direction of the resisted body, but perpendicular to its oblique surface; whence the resistance to such a body will not be estimated from the whole motion communicated to the particles of the fluid, but from that part of it only which is in the direction of the resisted body. In fluids then, where the parts are thus discontinued from each other, the different obliquities of that surface which goes foremost, will occasion considerable changes in the resistance; though the transverse section of the solid should in all cases be the same: And Newton has particularly determined that, in a fluid thus constituted, the resistance of a globe is but half the resistance of a cylinder of the same diameter, moving, in the direction of its axis, with the same velocity.

But though the hypothesis of a fluid thus constituted be of great use in explaining the nature of resistances, yet we know of no such fluid existing in nature; all the fluids with which we are conversant being so constituted, that their particles either lie contiguous to each other, or at least act on each other in the same manner as if they did: consequently, in these fluids, no one particle that is contiguous to the resisted body, can be moved, without moving at the same time a great number of others, some of which will be distant from it; and the motion thus communicated to a mass of the fluid, will not be in any one determined direction, but different in all the particles, according to the different positions in which they lie in contact with those from which they receive their impulse; whence, great numbers of the particles being diverted into oblique directions, the resistance of the moving body, which will depend on the quantity of motion communicated to the fluid in its own direction, will be different in quantity from what it would be in the foregoing supposition, and its estimation becomes much more complicated and operose.

If the fluid be compressed by the incumbent weight of its upper parts (as all fluids are with us, except at their very surface), and if the velocity of the moving body be much less than that with which the parts of the fluid would rush into a void space, in consequence of their compression; it is evident, that in this case the space left by the moving body will be instantaneously filled up by the fluid; and the parts of the fluid against which the foremost part of the body presses in its motion, will, instead of being impelled forwards in the direction of the body, in some measure circulate towards the hinder part of it, in order to restore the equilibrium, which the constant influx of the fluid behind the body would otherwise destroy; whence the progressive motion of the fluid, and consequently the resistance of the body, which depends upon it, would in this instance be much less, than in the hypothesis where each particle is supposed to acquire, from the stroke of the resisting body, a velocity equal to that with which the body moved, and in the same direction. Newton has determined, that the resistance of a cylinder, moving in the direction of its axis, in such a compressed fluid as we have here treated of, is but one-fourth part of the resistance to the same cylinder, if it moved with the same velocity in a fluid constituted in the manner described in the first hypothesis, each fluid being supposed of the same density.

But again, it is not only in the quantity of their resistance that these fluids differ, but also in the different manner in which they act upon solids of different forms moving in them. In the discontinued fluid, first described, the obliquity of the foremost surface of the moving body would diminish the resistance; but the same thing does not hold true in compressed fluids, at least not in any considerable degree; for the chief resistance in compressed fluids arises from the greater or less facility with which the fluid, impelled by the fore part of the body, can circulate towards its hinder part; and this being little, if at all, affected by the form of the moving body, whether it be cylindrical, conical, or spherical, it follows, that while the transverse section of the body is the same, and consequently the quantity of impelled fluid also, the change of figure in the body will scarcely affect the quantity of its resistance.

And this case, viz, the resistance of a compressed fluid to a solid, moving in it with a velocity much less than what the parts of the fluid would acquire from their compression, has been very fully considered by Newton, who has ascertained the quantity of such a resistance, according to the different magnitudes of the moving body, and the density of the fluid: but he expressly informs us that the rules he has laid down, are not generally true, being only so on a supposition that the compression of the fluid be increased in the greater velocities of the moving body: however, some unskilful writers, who have followed him, overlooking this caution, have applied this determination to bodies moving with all degrees of velocity, without attending to the different compressions of the fluids they are resisted by; and by this means they have accounted the resistance, for instance, of the air to musket and cannon shot, to be but about one-third part of what it is found to be by experience.

It is indeed evident that the resisting power of the medium must be increased, when the resisted body moves so fast that the fluid cannot instantaneously press in behind it, and fill the deserted space; for when this happens, the body will be deprived of the pressure of the fluid behind it; which in some measure balanced its resistance, or at least the fore pressure, and must support on its fore part the whole weight of a column of the fluid, over and above the motion it gives to the parts of the same; and besides, the motion in the particles driven before the body, is less affected in this case by the compression of the fluid, and consequently they are less deflected from the direction in which they are impelled by the resisted surface; whence it happens that this species of resistance approaches more and more to that described in the first hypothesis, where each particle of the fluid, being unconnected with the neighbouring ones, pursued its own motion, in its own direction, without being interrupted or deflected by their contiguity; and therefore, as the resistance of a discontinued fluid to a cylinder, moving in the direction of its axis, is 4 times greater than the resistance of a fluid sufficiently compressed of the same density, it follows that the resistance of a fluid, when a vacuity is left behind the moving body, may be near 4 times greater than that of the same fluid, when no such vacuity is formed; for when a void space is thus left, the resistance approaches in its nature to that of a discontinued fluid.

This then may probably be the case in a cylinder moving in the same compressed fluid, according to the different degrees of its velocity; so that if it set out with a great velocity, and moves in the fluid till that velocity be much diminished, the resisting power of the medium may be near 4 times greater in the beginning of its motion than in the end.

In a globe, the difference will not be so great, because, on account of its oblique surface, its resistance in a discontinued medium is but about twice as much as in one properly compressed; for its oblique surface diminishes its resistance in one case, and not in the other: however, as the compression of the medium, even when a vacuity is left behind the moving body, may yet confine the oblique motion of the parts of the fluid, which are driven before the body, and as in an elastic fluid, such as our air is, there will be some degree of condensation in those parts; it is highly probable that the resistance of a globe, moving in a compressed fluid with a very great velocity, may greatly exceed the proportion of the resistance to slow motions.

And as this increase of the resisting power of the medium will take place, when the velocity of the moving body is so great, that a perfect vacuum is left behind it, so some degree of augmentation will be sensible in velocities much short of this; for even when, by the compression of the fluid, the space left behind the body is instantaneously filled up; yet, if the velocity with which the parts of the fluid rush in behind, is not much greater than that with which the body moves, the same reasons that have been urged above, in the case of an absolute vacuity, will hold in a less degree in this instance; and therefore it is not to be supposed that, in the increased resistance which has been hitherto treated of, it immediately vanishes when the compression of the fluid is just sufficient to prevent a vacuum behind the resisted body; but we must consider it as diminishing only according as the velocity, with which the parts of the fluid follow the body, exceeds that with which the body moves.

Hence then it may be concluded, that if a globe sets out in a resisting medium, with a velocity much exceeding that with which the particles of the medium would rush into a void space, in consequence of their compression, so that a vacuum is necessarily left behind the globe in its motion; the resistance of this medium to the globe will be much greater, in proportion to its velocity, than what we are sure, from sir I. Newton, would take place in a slower motion: and we may further conclude, that the resisting power of the medium will gradually diminish as the velocity of the globe decreases, till at last, when it moves with velocities which bear but a small proportion to that with which the particles of the medium follow it, the resistance becomes the same with what is assigned by Newton in the case of a compressed fluid.

And from this determination may be seen, how false that position is, which asserts the resistance of any medium to be always in the duplicate ratio of the velocity of the resisted body; for it plainly appears, by what has been said, that this can only be considered as nearly true in small variations of velocity, and can never be applied in comparing together the resistances to all velocities whatever, without incurring the most enormous errors. See Robins's Gunnery, chap. 2, prop. 1, and my Tracts and Course of Mathem. vols. 2 and 3. See also the articles RESISTANCE *of the Air*, PROJECTILE, and GUNNERY.

Resistance and retardation are used indifferently for each other, as being both in the same proportion, and the same resistance always generating the same retardation. But with regard to different bodies, the same resistance frequently generates different retardations; the resistance being as the quantity of motion, and the retardation as that of the celerity. For the difference and measure of the two, see RETARDATION.

The retardations from this resistance may be compared together, by comparing the resistance with the gravity or quantity of matter. It is demonstrated that the resistance of a cylinder, which moves in the direction of its axis, is equal to the weight of a column of the fluid, whose base is equal to that of the cylinder, and its altitude equal to the height through which a body must fall in vacuo, by the force of gravity, to acquire the velocity of the moving body. So that, if a denote the area of the face or end of the cylinder, or other prism, v its velocity, and n the specific gravity of the fluid; then, the altitude due to the velocity v being $\frac{v^2}{4g}$, the whole resistance, or motive force m, will be $a \times n \times \frac{v^2}{4g} = \frac{anv^2}{4g}$; the quantity g being $= 16\frac{1}{12}$ feet, or the space a body falls, in vacuo, in the first second of time. And the resistance to a globe of the same diameter would be the half of this.—Let a ball, for instance, of 3 inches diameter, be moved in water with a celerity of 16 feet per second of time: now from experiments on pendulums, and on falling bodies, it has been found, that this is the celerity which a body acquires in falling from the height of 4 feet; therefore the weight of a cylinder of water of 3 inches diameter, and 4 feet high, that is a weight of about 12lb 4oz, is equal to the resistance of the cylinder; and consequently the half of it, or 6lb 2oz is that of the ball. Or, the formula $\frac{anv^2}{4g}$ gives $\frac{\cdot 7854 \times 9 \times 1000 \times 16 \times 16}{144 \times 4 \times 16} = 196$oz, or 12lb 4oz, for the resistance of the cylinder, or 6lb. 2oz. for that of the ball, the same as before.

Let now the resistance, so discovered, be divided by the weight of the body, and the quotient will show the ratio of the retardation to the force of gravity. So, if the said ball, of 3 inches diameter, be of cast iron, it will weigh nearly 61 ounces, or $3\frac{3}{4}$lb; and the resistance being 6lb 2oz, or 98 ounces; therefore, the resistance being to the gravity as 98 to 61, the retardation, or retarding force, will be $\frac{98}{61}$ or $1\frac{3}{5}$, the force of gravity being 1. Or thus; because a, the area of a great circle of the ball, is $= pd^2$, where d is the diameter, and $p = \cdot 7854$, therefore the resistance to the ball is $m = \frac{pnd^2v^2}{8g}$; and because its solid content is $w = \frac{2}{3}pd^3$, and its weight $\frac{2}{3}Npd^3$, where N denotes its specific gravity; therefore, dividing the resistance or motive force m by the weight w, gives $\frac{m}{w} = \frac{3nv^2}{16Ndg} = f$ the retardation, or retarding force, that of gravity being 1; which is therefore as the square of the velocity directly, and as the diameter inversely; and this is the reason why a large ball overcomes the resistance better than a small one, of the same density. See my Tracts and Course as above.

Resistance *of Fluid Mediums to the Motion of Falling Bodies.*—A body freely descending in a fluid, is accelerated by the relative gravity of the body, (that is, the difference between its own absolute gravity and that of a like bulk of the fluid,) which continually acts upon it, yet not equably, as in a vacuum: for the resistance of the fluid occasions a retardation, or diminution of acceleration, which diminution increases with the velocity of the body. Hence it happens, that there is a certain velocity, which is the greatest that a body can acquire by falling; for if its velocity be such, that the resistance arising from it becomes equal to the relative weight of the body, its motion can be no longer accelerated; for the motion here continually generated by the relative gravity, will be destroyed by the resistance, or the force of resistance is equal to the relative gravity, and the body must then go on equably; for after the velocity is arrived at such a degree, that the resisting force is equal to the weight that urges it, it can increase no longer, and the globe must afterward continue to descend with that velocity uniformly. And a body continually comes nearer and nearer to this greatest celerity, but can never attain it accurately. Now, N and n being the specific gravities of the globe and fluid, $N-n$ will be the relative gravity of the globe in the fluid, and therefore $w = \frac{2}{3}pd^3\,(N-n)$ is the weight by which it is urged downward; also $m = \frac{pnd^2v^2}{8g}$ is the resistance, as above; therefore these two must be equal when the velocity can be no further increased, or $m = w$, that is $\frac{pnd^2v^2}{8g} = \frac{2}{3}pd^3\,(N-n)$, or $nv^2 = \frac{16}{3}dg\,(N-n)$; and hence $v = \sqrt{(\frac{16}{3}dg \times \frac{N-n}{n})}$ is the said uniform or greatest velocity which the body can attain; which is evidently the greater in the subduplicate proportion of d the diameter of the ball. But v is always $= \sqrt{4gfs}$, the velocity generated by any accelerative force f in describing the space s; which being compared with the former, it gives $s = \frac{4}{3}d$, when f is $= \frac{N-n}{n}$; that is, the greatest velocity is that which is generated by the accelerating force $\frac{N-n}{n}$ in passing over the space $\frac{4}{3}d$ or $\frac{4}{3}$ of the diameter of the ball, or it is equal to the velocity generated by gravity in describing the space $\frac{N-n}{n} \times \frac{4}{3}d$. For ex. if the ball be of lead, which is about $11\frac{1}{4}$ times the density of water; then $N = 11\frac{1}{4}$, $n = 1$, $N-n = \frac{N-n}{n} = 10\frac{1}{4} = \frac{41}{4}$, and $\frac{N-n}{n} \times \frac{4}{3}d = \frac{41}{3}d = 13\frac{2}{3}d$; that is, the uniform or greatest velocity of a ball of lead, descending in water, is equal to that which a heavy body acquired by falling in vacuo through a space equal to $13\frac{2}{3}$ of the diameter of the ball, which velocity is $v = 2\sqrt{(\frac{4}{3}dg \times \frac{N-n}{n})} = 2\sqrt{13\frac{2}{3}dg} = 8\sqrt{13\frac{2}{3}d}$ nearly, or 8 times the root of the same space.

Hence it appears, how soon small bodies come to their greatest or uniform velocity in descending in a fluid, as water, and how very small that velocity is: which explains the reason of the slow precipitation of mud, and small particles, in water, as also why, in precipitations, the larger and gross particles descend soonest, and the lowest.

Further, where $N = n$, or the density of the fluid is equal to that of the body, then $N-n = 0$, consequently the velocity and distance descended are each nothing, and the body will just float in any part of the fluid.

Moreover, when the body is lighter than the fluid, then N is less than n, and $N-n$ becomes a negative quantity, or the force and motion tend the contrary way, that is, the ball will ascend up towards the top of the fluid by a motive force which is as $n-N$. In this case then, the body ascending by the action of the fluid, is moved exactly by the same laws as a heavier body falling in the fluid. And wherever the body is placed, it is sustained by the fluid, and carried up with a force equal to the difference of the weight of a quantity of the fluid of the same bulk as the body, from the weight of the body; there is therefore a force which continually acts equably on the body; by which not only the action of gravity of the body is counteracted, so as that it is not to be considered in this case; but the body is also carried upwards by a motion equably accelerated, in the same manner as a body heavier than a fluid descends by its relative gravity: but the equability of acceleration is destroyed in the same manner by the resistance, in the ascent of a body lighter than the fluid, as it is destroyed in the descent of a body that is heavier.

The resistance to a plane surface of 1 foot square, in passing through water with various degrees of velocity, is as below:

Velocities per hour in miles.		Resistances in avoirdupois pounds.
1	- - -	40lbs
$1\frac{1}{2}$	- - -	90
2	- - -	160
$2\frac{1}{2}$	- - -	250
3	- - -	360
$3\frac{1}{2}$	- - -	490
4	- - -	640
&c.		

For the circumstances of the correspondent velocity, space, and time, &c, of a body moving in a fluid in which it is projected with a given velocity, or descending by its own weight, &c, see my Tracts and Course, as beforementioned.

RESISTANCE *of the Air*, in Pneumatics, is the force with which the motion of bodies, particularly of projectiles, is retarded by the opposition of the air or atmosphere. See GUNNERY, PROJECTILES, &c.

The air being a fluid, the general laws of the resistance of fluids obtain in it; subject only to some variations and irregularities from the different degrees of density in the different stations or regions of the atmosphere.

The resistance of the air is chiefly of use in military projectiles, in order to allow for the differences caused by it in their flight and range. Before the time of Mr. Robins, it was thought that this resistance to the motion of such heavy bodies as iron balls and shells, was too inconsiderable to be regarded, and that the rules and conclusions derived from the common parabolic theory, were sufficiently exact for the common practice of gunnery. But that gentleman showed, in his New Principles of Gunnery, that, so far from being inconsiderable, it is in reality enormously great, and by no means to be rejected without incurring the grossest errors; so much so, that balls or shells which range, at the most, in the air, to the distance of two or three miles, would in a vacuum range to 20 or 30 miles, or more. To determine the quantity of this resistance, in the case of different velocities, Mr. Robins discharged musket-balls, with various degrees of known velocity, against his ballistic pendulums, placed at several different distances, and so discovered by experiment the quantity of velocity lost, when passing through those distances or spaces of air, with the several known degrees of celerity. For having thus known the velocity lost or destroyed, in passing over a certain space, in a certain time, (which time is very nearly equal to the quotient of the space divided by the medium velocity between the greatest and least, or between the velocity at the mouth of the gun and that at the pendulum;) that is, knowing the velocity v, the space s, and time t, the resisting force is thence easily known, being equal to $\frac{vb}{2gt}$ or $\frac{vvb}{2gs}$, where b denotes the weight of the ball, and v the medium velocity above-mentioned. The balls employed on this occasion by Mr. Robins, were leaden ones, of $\frac{1}{12}$ of a pound weight, and $\frac{3}{4}$ of an inch diameter; and to the medium velocity of

1600 feet the resistance was 11lb,
1065 feet - - - - - - it was $2\frac{4}{5}$;

but by the theory of Newton, before laid down, the former of these should be only $4\frac{1}{2}$lb, and the latter 2 lb: so that, in the former case the real resistance is more than double of that given by the theory, being increased as 9 to 22; and in the lesser velocity the increase is from 2 to $2\frac{4}{5}$, or as 5 to 7 only.

Mr. Robins also invented another machine, having a whirling or circular motion, by which he measured the resistances to larger bodies, though with much smaller velocities: it is described, and a figure of it given, near the end of the 1st vol. of his works, and in the 3d vol. of my Tracts.

That this resisting power of the air to swift motions is very sensibly increased beyond what Newton's theory for slow motions makes it, seems hence to be evident. By other experiments it appears that the resistance is very sensibly increased, even in the velocity of 400 feet. However, this increased power of resistance diminishes as the velocity of the resisted body diminishes, till at length, when the motion is sufficiently abated, the actual resistance coincides with that supposed in the theory nearly. For these varying resistances Mr. Robins has given a rule, though not correct, extending to 1670 feet velocity.

Mr. Euler has shown, that the common doctrine of resistance answers pretty well when the motion is not very swift, but in swift motions it gives the resistance less than it ought to be, on two accounts. 1. Because in quick motions, the air does not fill up the space behind the body fast enough to press on the hinder parts, to counterbalance the weight of the atmosphere on the fore part. 2. The density of the air before the ball being increased by the quick motion, will press more strongly on the fore part, and so will resist more than lighter air in its natural state. And he has also shown that Mr. Robins has restrained his rule to velocities not exceeding 1670 feet per second; whereas had he extended it to greater velocities, the result must have been erroneous; and he gives another formula himself, and deduces conclusions differing from those of Mr. Robins. See his Principles of Gunnery investigated, translated by Brown in 1777, pa. 224 &c.

Mr. Robins having proved that, in very great changes of velocity, the resistance does not accurately follow the duplicate ratio of the velocity, lays down two positions, which he thought might be of some service in the practice of artillery, till a more complete and accurate theory of resistance, and the changes of its augmentation, could be obtained. The first of these is, that till the velocity of the projectile surpass 1100 or 1200 feet in a second, the resistance may be esteemed to be in the duplicate ratio of the velocity: and the second is, that when the velocity exceeds 1100 or 1200 feet, then the absolute quantity of the resistance will be near 3 times as great as it would be by a comparison with the smaller velocities. On these principles he proceeds in approximating to the actual ranges of pieces with small angles of elevation, viz, such as do not exceed 8° or 10°, which he sets down in a table, compared with their corresponding potential ranges. See his Mathematical Tracts, vol. 1, pa. 179 &c. But we shall see presently that these positions are both without foundation; that there is no such thing as a sudden or abrupt change in the law of resistance, from the square of the velocity to one that gives a quantity three times as much; but that the change is slow and gradual, from the lowest to the highest velocities; and that the increased real resistance no where rises much higher than double of that which Newton's theory gives it.

Mr. Glenie, in his History of Gunnery, 1776, pa. 49, observes, in consequence of some experiments with a rifled piece, properly fitted for experimental purposes, that the resistance of the air to a velocity somewhat less than that mentioned in the first of the above propositions, is considerably greater than in the duplicate ratio of the velocity; and that, to a celerity somewhat greater than that stated in the second, the resistance is considerably less than that which is treble the resistance in the said ratio. Some of Robins's own experiments seem necessarily to make it so; since, to a velocity no quicker than 400 feet in a second, he found the resistance to be somewhat greater than in that ratio. But the true value of the ratio, and other circumstances of this resistance, will more fully appear from what follows.

The subject of the resistance of the air, as begun by Robins, has been prosecuted by myself, to a very great extent and variety, both with the whirling machine, and with cannon balls of all sizes, from 1lb to 6lb weight, as well as with figures of many other different shapes, both on the fore part and hind part of them, and with planes set at all varieties of angles of inclination to the path or motion of the same; from all which I have obtained the real resistance to bodies for all velocities, from 1 up to 2000 feet per second: together with the law of the resistance to the same body for all different velocities, and for different sizes with the same velocity, and also for all angles of inclination; a full account of which is given in the 2d and 3d vols. of my Tracts. Some of the tables and rules are abstracted as below.

TABLE I. *Resistances of several different Bodies.*

Veloc. per sec.	Small Hemis.	Large Hemis.		Cone.		Cylinder	Whole globe	Resist. as the power of the veloc.
	flat side.	flat side	round side	vertex	base			
feet.	oz.	oz.	oz.	oz.	oz.	oz.	oz.	
3	·028	·051	·020	·028	·064	050	·027	
4	·048	·096	·039	·048	·109	·090	·047	
5	·072	·148	·063	·071	·162	·143	·068	
6	·103	·211	·092	·098	·225	·205	·094	
7	·141	·284	·123	·129	·298	·278	·125	
8	·184	·368	·160	·168	·382	·360	·162	
9	·233	·464	·199	·211	·478	·456	·205	
10	·287	·573	·242	·260	·587	·565	·255	
11	·349	·698	·292	·315	·712	·688	·310	2·049
12	·418	·836	·347	·376	·850	·826	·370	2·042
13	·492	·988	·409	·440	1·000	·979	·435	2·036
14	·573	1·154	·478	·512	1·166	1·145	·505	2·031
15	·661	1·336	·552	·589	1·346	1·327	·581	2·031
16	·754	1·538	·634	·673	1·546	1·526	·663	2·033
17	·853	1·757	·722	·762	1·763	1·745	·752	2·038
18	·959	1·998	·818	·858	2·002	1·986	·848	2·044
19	1·073	2·258	·922	·959	2·260	2·246	·949	2·047
20	1·196	2·542	1·033	1·069	2·540	2·528	1·057	2·051
Mean propor. Nos.	140	288	119	126	291	285	124	2·040
1	2	3	4	5	6	7	8	9

In this table are contained the resistances to several forms of bodies, when moved with various degrees of velocity, from 3 feet per second to 20. The names of the bodies are at the tops of the columns, as also which end went foremost through the air; the different velocities are in the first column, and the resistances on the same line, in their several columns, in avoirdupois ounces and decimal parts. So on the first line are contained the resistances when the bodies move with a velocity of 3 feet in a second, viz, in the 2d column for the small hemisphere, of 4¾ inches diameter, its resistance ·028 oz when the flat side went foremost; in the 3d and 4th columns the resistances to a larger hemisphere, first with the flat side, and next the round side foremost, the diameter of this, as well as all the following figures being 6⅜ inches, and therefore the area of the great circle = 32 sq. inches, or $\frac{2}{9}$ of a sq. foot; then in the 5th and 6th columns are the resistances to a cone, first its vertex and then its base foremost, the altitude of the cone being 6⅝ inches, being only ¼ inch more than the diameter of its base; in the 7th column the resistance to the end of the cylinder, and in the 8th that against the whole globe or sphere. All the numbers show the real weights which are equal to the resistances; and at the bottoms of the columns are placed proportional numbers, which show the mean proportions of the resistances of all the figures to one another, with any velocity. Lastly, in the 9th column are placed the exponents of the power of the velocity which the resistances in the 8th column bear to each other, viz, which that of the 10 feet velocity bears to each of the following ones, the medium of all of them being as the 2·04 power of the velocity, that is, very little above the square or second power of the velocity, so far as the velocities in this table extend.—From this table the following inferences are easily deduced.

1. That the resistance is nearly in the same proportion as the surfaces; a small increase only taking place in the greater surfaces, and for the greater velocities. Thus, by comparing together the numbers in the 2d and 3d columns, for the bases of the two hemispheres, the areas of which bases are in the proportion of 17¾ to 32, or 5 to 9 very nearly, it appears that the numbers in those two columns, expressing the resistances, are nearly as 1 to 2 or 5 to 10, as far as the velocity of 12 feet; but after that, the resistances on the greater surface increase gradually more and more above that proportion.

2. The resistance to the same surface, with different velocities, is, in these slow motions, nearly as the square of the velocity; but gradually increases more and more above that proportion as the velocity increases. This is manifest from all the columns; and the index of the power of the velocity is set down in the 9th column, for the resistances in the 8th, the medium being 2·04; by which it appears that the resistance to the same body is, in these slow motions, as the 2·04 power of the velocity, or nearly as the square of it.

3. The round ends, and sharp ends, of solids, suffer less resistance than the flat or plane ends, of the same diameter; but the sharper end has not always the less resistance. Thus, the cylinder, and the flat ends of the hemisphere and cone, have more resistance, than the round or sharp ends of the same; but the round side of the hemisphere has less resistance than the sharper end of the cone.

4. The resistance on the base of the hemisphere, is to that on the round, or whole sphere, as 2⅖ to 1, instead of

2 to 1, as the theory gives that relation. Also the experimented resistance, on each of these, is nearly $\frac{1}{4}$ more than the quantity assigned by the theory.

5. The resistance on the base of the cone, is to that on the vertex, nearly as $2\frac{1}{10}$ to 1; and almost in the same ratio is radius to the sine of the angle of inclination of the side of the cone to its path or axis. So that, in this instance, the resistance is directly as the sine of the angle of incidence, the transverse section being the same.

6. When the hinder parts of bodies are of different forms, the resistances are different, though the fore-parts be exactly alike and equal; owing probably to the different pressures of the air on the hinder parts. Thus, the resistance to the fore part of the cylinder, is less than on the equal flat surface of the cone, or of the hemisphere; because the hinder part of the cylinder is more pressed or pushed, by the following air than those of the other two figures; also, for the same reason, the base of the hemisphere suffers a less resistance than that of the cone, and the round side of the hemisphere less than the whole sphere.

See other deductions in my Tracts, vol. 3, pa. 193 &c.

Table II. *Resistances both by Experiment and Theory, to a Globe of 1·965 Inches Diameter.*

Veloc. per sec. in feet.	Resist. by Exper. oz.	Resist. by Theory. oz.	Ratio of Exper. to Theory.	Resist. as the power of the veloc.
5	0·006	0·005	1·20	
10	0·024½	0·020	1·23	
15	0·055	0·044	1·25	
20	0·100	0·079	1·27	
25	0·158	0·123	1·285	2·050
30	0·229	0·177	1·30	2·048
40	0·413	0·314	1·32	2·046
50	0·651	0·491	1·33	2·044
100	2·675	1·964	1·36	2·042
200	11	7·9	1·40	2·041
300	25	18·7	1·41	2·039
400	45	31·4	1·43	2·039
500	72	49	1·47	2·044
600	107	71	1·51	2·051
700	151	96	1·57	2·059
800	205	126	1·63	2·067
900	271	159	1·70	2·077
1000	350	196	1·78	2·086
1100	442	238	1·86	2·095
1200	546	283	1·90	2·102
1300	661	332	1·99	2·107
1400	785	385	2·04	2·111
1500	916	442	2·07	2·113
1600	1051	503	2·09	2·113
1700	1186	568	2·08	2·111
1800	1319	636	2·07	2·108
1900	1447	709	2·04	2·104
2000	1569	786	2·00	2·098

In the first column of this table are contained the several velocities, from 0 up to the great velocity of 2000 feet per second, with which the ball or globe moved. In the 2d column are the experimented resistances, in avoirdupois ounces. In the 3d column are the correspondent resistances, as computed by the foregoing theory. In the 4th column are the ratios of these two resistances, or the quotients of the former divided by the latter. And in the 5th or last, the indexes of the power of the velocity which is proportional to the experimented resistance; which are found by comparing the resistance of 20 feet velocity with each of the following ones.

From the 2d, 3d and 4th columns it appears, that at the beginning of the motion, the experimented resistance is nearly equal to that computed by theory; but that, as the velocity increases, the experimented resistance gradually exceeds the other more and more, till at the velocity of 1300 feet the former becomes double the latter; after which, the difference increases a little farther, till at the velocity of 1600 or 1700, where that excess is the greatest, and is rather less than $2\frac{1}{10}$; and after this, the difference decreases gradually as the velocity increases, and at the velocity of 2000, the former resistance again becomes just double the latter.

From the last column it appears that, near the beginning, or in slow motions, the resistances are nearly as the square of the velocities; but that the ratio gradually increases, with some small variation, till at the velocity of 1500 or 1600 feet it becomes as the $2\frac{1}{9}$ power of the velocity nearly, which is its highest ascent; and after that it gradually decreases again, as the velocity goes higher. And similar conclusions have also been derived from experiments with larger balls or globes.

And hence we perceive that Mr. Robins's positions are erroneous on two accounts, viz, both in stating that the resistance changes suddenly, or all at once, from being as the square of the velocity, so as then to become as some higher and constant power; and also when he states the resistance as rising to the height of three times that which is given by the theory: since the ratio of the resistance both increases gradually from the beginning, and yet never ascends higher than $2\frac{9}{100}$ of the theory.

Table III. *Resistance to a Plane, set at various Angles of Inclination to its Path.*

Angle with the Path.	Experim. Resistances. oz.	Resist. by this Formula. $\cdot 84s^{1\cdot 842c}$	Sines of the Angles to Radius ·840.
0°	·000	·000	·000
5	·015	·009	·073
10	·044	·035	·146
15	·082	·076	·217
20	·133	·131	·287
25	·200	·199	·355
30	·278	·278	·420
35	·362	·363	·482
40	·448	·450	·540
45	·534	·535	·594
50	·619	·613	·643
55	·684	·680	·688
60	·729	·736	·727
65	·770	·778	·761
70	·803	·808	·789
75	·823	·826	·811
80	·835	·836	·827
85	·839	·839	·838
90	·840	·840	·840

In the second column of this table are contained the actual experimented resistances, in ounces, to a plane of 32 square inches, or $\frac{2}{9}$ of a square foot, moved through the air with a velocity of exactly 12 feet per second, when the plane was set so as to make, with the direction of its path, the corresponding angles in the first column.

And from these has been deduced this formula, or theorem, viz, $\cdot 84s^{1\cdot 842c}$, which brings out very nearly the same numbers, and is a general theorem for every angle, for the same plane of $\frac{2}{9}$ of a foot, and moved with the same velocity of 12 feet in a second of time; where s is the sine, and c the cosine of the angles of inclination in the first column.

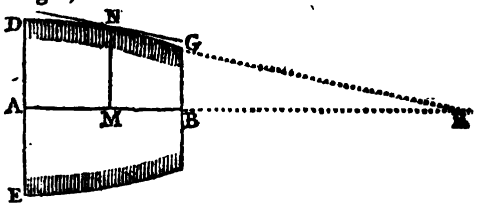
If a theorem be desired for any other velocity v, and any other plane whose area is a, it will be this: $\frac{1}{38}av^2s^{1\cdot 842c}$, or more nearly $\frac{1}{42}av^{2\cdot 04}s^{1\cdot 842c}$; which denotes the resistance nearly to any plane surface whose area is a, moved through the air with the velocity v, in a direction making with that plane an angle, whose sine is s, and cosine c.

If it be water or any other fluid, different from air, this formula will be varied in proportion to the density.

By this theorem were computed the numbers in the 3d column; which it is evident agree very nearly with the experiment resistances in the 2d column, excepting in two or three of the small numbers near the beginning, which are of the least consequence. In all other cases, the theorem gives the true resistance very nearly. In the 4th or last column are entered the sines of the angles of the first column, to the radius ·84, in order to compare them with the resistances in the other columns. Whence it appears, that those resistances are not always proportional to the sines of the angles, nor yet to the squares of the sines, nor to any other power of them whatever. In the beginning of the columns, the sines much exceed the resistances all the way till the angle be between 55 and 60 degrees; after which the sines are less than the resistances all the way to the end, or till the angle become of 90 degrees.

Mr. James Bernoulli gave some theorems for the resistances of different figures, in the Acta Erud. Lips. for June 1693, pa. 252 &c. But as these are deduced from theory only, which we find to be so different from experiment, they cannot be of much use. Messieurs Euler, Dalembert, Gravesande, and Simpson, have also written pretty largely on the theory of resistances, besides what had been done by Newton. Also Mr. Vince, in the Philos. Trans. an. 1795, pa. 24.

Solid of Least RESISTANCE. Sir Isaac Newton, from his general theory of resistance, deduces the figure of a solid which shall have the least resistance, of the same base, height, and content.



The figure is this. Suppose DNG to be a curve of such a nature, that if from any point N the ordinate NM be drawn perpendicular to the axis AB; and from a given point G there be drawn GR parallel to a tangent at N, and meeting the axis produced in R; then if MN be to GR, as GR^3 to $4BR \times BG^2$, a solid described by the revolution of this figure about its axis AB, moving in a medium from A towards B, is less resisted than any other circular solid of the same base, &c.

This theorem, which Newton gave without a demonstration, has been demonstrated by several mathematicians, as Facio, Bernoulli, Hospital, &c. See Maclaurin's Flux. sect. 606 and 607; also Horsley's edit. of Newton, vol. 2, pa. 390. See also Act. Erud. 1699, pa. 514; and Mem. de l'Acad. &c; also Robins's View of Newton's method for comparing the Resistance of Solids, 8vo, 1734; and Simpson's Fluxions, art. 413; or my Principles of Bridges, in my Tracts, prop. 15.

M. Bouguer has resolved this problem in a very general manner; and not in supposing the solid to be formed by the revolutions of any figure whatever. The problem, as enunciated and resolved by M. Bouguer, is this: any base being given, to find what kind of solid must be formed upon it, so that the impulse upon it may be the least possible. Properly however it ought to be the retardive force, or the impulse divided by the weight or mass of matter in the body, that ought to be the minimum.

RESOLUTION, in Physics, the reduction of a body into its original or natural state, by a dissolution or separation of its aggregated parts. Thus, snow and ice are said to be resolved into water; water resolves in vapour by heat; and vapour is again resolved into water by cold; also any compound is resolved into its ingredients, &c.—Some of the modern philosophers, particularly Boyle, Mariotte, Boerhaave, &c, maintain, that the natural state of water is to be congealed, or in ice; inasmuch as a certain degree of heat, which is a foreign and violent agent, is required to make it fluid: so that near the pole, where this foreign agent is wanting, it constantly retains its fixed or icy state.

RESOLUTION, or SOLUTION, in Mathematics, is an orderly enumeration of several things to be done, to obtain what is required in a problem.—Wolfius makes a problem to consist of three parts: the proposition (or what is properly called the problem), the resolution, and the demonstration. As soon as a problem is demonstrated, it is converted into a theorem; of which the resolution is the hypothesis; and the proposition the thesis. For the process of a mathematical resolution, see the following article.

RESOLUTION in *Algebra*, or *Algebraical* RESOLUTION, is of two kinds; the one practised in numerical problems, the other in geometrical ones.

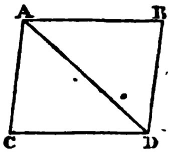
In Resolving a Numerical Problem Algebraically, the method is this. 1st, the given quantities are distinguished from those that are sought; and the former denoted by the initial letters of the alphabet, but the latter by the last letters.—2d, Then as many equations are formed as there are unknown quantities. If that cannot be done from the proposition or data, the problem is indeterminate; and certain arbitrary assumptions must be made, to supply the defect, and which can satisfy the question. When the equations are not contained in the problem itself, they are to be found by particular theorems concerning equations, ratios, proportions, &c.—Since, in an equation, the known and unknown quantities are mixed together, they must be separated in such a manner, that the unknown one remain alone on one side, and the known ones on the other. This reduction, or separation, is made by addition, subtraction, multiplication, division, extraction of roots, and raising of powers; resolving every kind of combination of the quantities, by their counter or reverse ones, and performing the same operation on all the quantities or terms, on both sides of the equation, that the equality may still be preserved.

To Resolve a Geometrical Problem Algebraically.—The same kind of operations are to be performed, as in the former article; besides several others, that depend on the nature of the diagram, and geometrical properties. As, 1st, the thing required or proposed, must be supposed done, the diagram being drawn or constructed in all its parts, both known and unknown. 2. We must then examine the geometrical relations which the lines of the figure have among themselves, without regarding whether they are known or unknown, to find what equations arise from those relations, for finding the unknown quantities. 3. It is often necessary to form similar triangles and rectangles, sometimes by producing of lines, or drawing parallels and perpendiculars, and forming equal angles, &c; till equations can be formed, from them, including both the known and unknown quantities.

If we do not thus arrive at proper equations, the thing is to be tried in some other way. And sometimes the thing itself, that is required, is not to be sought directly, but some other thing, bearing certain relations to it, by means of which it may be found.

The final equation being at last arrived at, the geometrical construction is to be deduced from it, which is performed in various ways according to the different kinds of equations. See ANALYSIS.

RESOLUTION *of Forces*, or *of Motion*, is the resolving or dividing of any one force or motion, into several others, in other directions, but which, taken together, shall have the same effect as the single one; and it is the reverse of the composition of forces or motions. See these articles.



Any single direct force AD, may be resolved into two oblique forces, whose quantities and directions are AB, AC, having the same effect, by describing any parallelogram ABDC, whose diagonal is AD. And each of these may, in like manner, be resolved into two others; and so on, as far as we please. And all these new forces, or motions, so found, when acting together, will produce exactly the same effect as the single original one. See also COLLISION, PERCUSSION, MOTION, COMPOSITION, PARALLELOGRAM and POLYGON *of Forces*, &c.

REST, in Physics, the continuance of a body in the same place; or its continual application or contiguity to the same parts of the ambient and contiguous bodies.—See SPACE.

Rest is either Absolute or Relative, as place is.

Some define Rest to be the state of a thing without motion; and hence again rest becomes either absolute or relative, as motion is.

Newton defines true or absolute rest to be the continuance of a body in the same part of absolute and immoveable space; and relative rest to be the continuance of a body in the same part of relative space. Thus, in a ship under sail, relative rest is the continuance of a body in the same part of the ship. But true or absolute rest is its continuance in the same part of universal space in which the ship itself is contained.

Hence, if the earth were really and absolutely at rest, the body relatively at rest in the ship would really and absolutely move, and that with the same velocity as the ship itself. But as the earth also moves, there arises a real and absolute motion of the body at rest; partly from the real motion of the earth in absolute space, and partly from the relative motion of the ship on the sea. Lastly, if the body be likewise relatively moved in the ship, its real motion will arise partly from the real motion of the earth in immoveable space, and partly from the relative motion of the ship on the sea, and of the body in the ship.

It is an axiom in philosophy, that matter is indifferent as to rest or motion. Hence Newton lays it down, as a law of nature, that every body perseveres in its state, either of rest or uniform motion, except so far as it is disturbed by external causes.—The Cartesians assert, that firmness, hardness, or solidity of bodies, consists in this, that their parts are at rest with regard to each other; and this rest they establish as the great nexus, or principle of cohesion, by which the parts are connected together. On the other hand, they make fluidity to consist in a perpetual motion of the parts, &c. But the Newtonian philosophy furnishes us with much better solutions.—Maupertuis asserts, that when bodies are in equilibrio, and any small motion is impressed on them, the quantity of action resulting will be the least possible. This he calls the law of rest; and from this law he deduces the fundamental proposition of statics. See Berlin Mem. tom. 2, pa. 294. And from the same principle too he deduces the laws of percussion.

RESTITUTION, in Physics, the returning of elastic bodies, forcibly bent, to their natural state; by some called the Motion of Restitution.

RETARDATION, in Physics, the act of retarding, that is, of delaying the motion or progress of a body, or of diminishing its velocity.—The retardation of moving bodies arises from two great causes, the resistance of the medium, and the force of gravity.

RETARDATION *from the Resistance* is often confounded with the resistance itself; because, with respect to the same moving body, they are in the same proportion.

But with respect to different bodies, the same resistance often generates different retardations. For if bodies of equal bulk, but different densities, be moved through the same fluid with equal velocity, the fluid will act equally on each; so that they will have equal resistances, but different retardations; and the retardations will be to each other, as the velocities which might be generated by the same forces in the bodies proposed; that is, they are inversely as the quantities of matter in the bodies, or inversely as the densities.

Suppose then bodies of equal density, but of unequal bulk, to move equally fast through the same fluid; then their resistances increase according to their superficies, that is as the squares of their diameters; but the quantities of matter are increased according to their mass or magnitude, that is as the cubes of their diameters: the resistances are the quantities of motion; the retardations are the celerities arising from them; and dividing the quantities of motion by the quantities of matter, we shall have the celerities; therefore the retardations are directly as the squares of the diameters, and inversely as the cubes of the diameters, that is inversely as the diameters themselves.

If the bodies be of equal magnitude and density, and moved through different fluids, with equal celerity, their retardations are as the densities of the fluids. And when equal bodies are carried through the same fluid with different velocities, the retardations are as the squares of the velocities.

So that, if s denote the superficies of a body, w its weight, d its diameter, v the velocity, and n the density of the fluid medium, and N that of the body; then, in similar bodies, the resistance is as nwv^2 or as nd^2v^2, and the retardation, or retarding force, as $\frac{nwv^2}{w}$, or as $\frac{nd^2v^2}{Nd^3} = \frac{nv^2}{Nd}$.

The RETARDATION *from Gravity* is peculiar to bodies projected upwards. For a body thrown upwards is retarded after the same manner as a falling body is accelerated; only in the one case the force of gravity conspires with the motion acquired, and in the other it acts contrary to it.

As the force of gravity is uniform, the retardation from that cause will be equal in equal times. And hence, as it is the same force which generates motion in the falling body, and diminishes it in the rising one, a body rises till it lose all its motion; which it does in the same time in which a body falling would have acquired a velocity equal to that with which the body was thrown up.

Also, a body thrown up will rise to the same height from which, in falling, it would acquire the same velocity with which it was thrown up: therefore the heights which bodies can rise to, when thrown up with different velocities, are to each other as the squares of the velocities.

Hence, the retardations of motions may be compared together. For they are, first, as the squares of the velocities; 2dly, as the densities of the fluids through which the bodies are moved; 3dly, inversely as the diameters of those bodies; 4thly, inversely as the densities of the bodies themselves; as expressed by the theorem above, viz, $\frac{nv^2}{Nd}$.

The Laws of RETARDATION, are the very same as those for acceleration; motion and velocity being destroyed in the one case, in the very same quantity and proportion as they are generated in the other.

RETICULA, or RETICULE, in Astronomy, a contrivance for very accurately measuring the quantity of eclipses, &c. This instrument, introduced some years since by the Paris Acad. of Sciences, is a little frame, consisting of 13 fine silken threads, parallel to, and equidistant from each other; placed in the focus of object-glasses of telescopes; that is, in the place where the image of the luminary is painted in its full extent. Consequently the diameter of the sun or moon is thus seen divided into 12 equal parts or digits: so that, to find the quantity of the eclipse, there is nothing to do but to number the parts that are dark, or that are luminous.—As a square reticule is only proper for the diameter of the luminary, not for the circumference of it, it is sometimes made circular, by drawing 6 concentric equidistant circles; which represents the phases of the eclipse perfectly.—But it is evident that the reticule, whether square or circular, ought to be perfectly equal to the diameter or circumference of the sun or star, such as it appears in the focus of the glass; otherwise the division cannot be just. And this is no easy matter to effect, because the apparent diameter of the sun and moon differs in each eclipse; nay that of the moon differs from itself in the progress of the same eclipse.—Another imperfection in the reticule is, that its magnitude is determined by that of the image in the focus; and of consequence it will only fit one certain magnitude.

But M. Lahire has found a remedy for all these inconveniences, and contrived that the same reticule shall serve for all telescopes, and all magnitudes of the luminary in the same eclipse. The principle on which his invention is founded, is, that two object-glasses applied against each other, having a common focus, and these forming an image of a certain magnitude, this image will increase in proportion as the distance between the two glasses is increased, as far as to a certain limit. If therefore a reticule be taken of such a magnitude, as just to comprehend the greatest diameter the sun or moon can ever have in the common focus of two object-glasses applied to each other, there needs nothing but to remove them from each other, as the body comes to have a less diameter, to have the image still exactly comprehended in the same reticule.

Further, as the silken threads are subject to swerve from the parallelism, &c, by the different temperature of the air, another improvement is, to make the reticule of a thin looking-glass, by drawing lines or circles upon it with the fine point of a diamond. See MICROMETER.

RETIRED FLANK, in Fortification. See FLANK.

RETROCESSION *of Curves, &c.* See RETROGRADATION.

RETROCESSION *of the Equinox.* See PRECESSION.

RETROGRADATION, or RETROGRESSION, in Astronomy, is an apparent motion of the planets, by which they seem to go backwards in the ecliptic, and to move contrary to the order or succession of the signs.

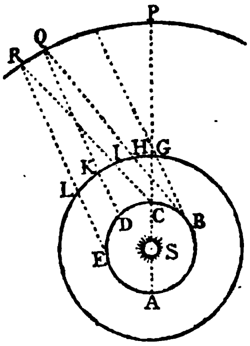
When a planet moves in consequentia, or according to the order of the signs, as from Aries to Taurus, from Taurus to Gemini, &c, which is from west to east, it is said to be Direct.—When it appears for some days in the same place, or point of the heavens, it is said to be Stationary.—And when it goes in antecedentia, or backwards to the foregoing signs, or contrary to the order of the signs, which is from east to west, it is said to be Retrograde. All these different affections or circumstances, may happen in all the planets, except the sun and moon, which are seen to go direct only. But the times of the superior and inferior planets being retrograde are different; the former appearing so about their opposition, and the latter about their conjunction. The intervals of time also between two retrogradations of the several planets, are very unequal.

In Herschel it is	1 year	6 days,
In Saturn - -	1 - - -	13
In Jupiter - -	1 - - -	43
In Mars - - -	2 - - -	50
In Venus - -	1 - -	220
In Mercury -	0 - -	116

Again, Herschel continues retrograde 153 days, Saturn 140, Jupiter 120, Mars 73, Venus 42, and Mercury 22; or nearly so; for the several retrogradations of the same planet are not constantly equal.

These various circumstances however in the motions of the planets are not real, but only apparent; as the inequalities arise from the motion and position of the earth, from which they are viewed; for when they are considered as seen from the sun, their motions appear always uniform and regular. These inequalities are thus explained:

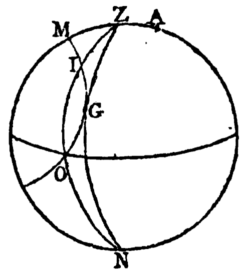
Let S denote the sun; and ABCD&c the path or orbit of the earth, moving from west to east, and in that order; also GK &c the orbit of a superior planet, as Saturn for instance, moving the same way, or in the direction GKLG, but with a much less celerity than the earth's motion. Now when the earth is at the point A of its orbit, let Saturn be at G, in conjunction with the sun, when it will be seen at P in the zodiac, or among the stars; and when the earth has moved from A to B, let Saturn have moved from G to H in its orbit, when it will be seen in the line BHQ, and will appear to have moved from P to Q in the zodiac; also when the earth has got to C, let Saturn be arrived at I, but found at R in the zodiac, where being seen in the line CIR, it appears stationary, or without motion in the zodiac at R. But after this, Saturn will appear for some time in retrogradation, viz, moving backwards, or the contrary way: for when the earth has moved to D, this planet will have got to K, and, being seen in the line DKQ, will appear to have moved retrograde in the zodiac from R to Q; about which place the planet, ceasing to recede any farther, again becomes stationary, and afterwards proceeds forward again; for while the earth moves from D to E, and Saturn from K to L, the latter, being now seen in the line ELR, appears to have moved forward in the zodiac from Q to R: and so on; the superior planets always becoming retrograde a little before they are in opposition to the sun, and continuing so till some time after the opposition: the retrograde motion being swiftest when the planet is in the very opposition itself; and the direct motion swiftest when in the conjunction. The arch RQ, which the planet describes while thus retrograde, is called the arch of retrogradation. These arches are unequal in all the planets, being greatest in the most distant, and gradually less in the nearer ones.



In like manner may be shown the circumstances of the retrogradations of the inferior planets; by which it will appear, that they become stationary a little before their inferior conjunction, and go retrograde till a little time after it; moving the quickest retrograde just at that conjunction, and the quickest direct just at the superior or further conjunction.

Retrogradation *of the Nodes of the Moon*, is a motion of the line of the nodes of her orbit, by which it continually shifts its situation from east to west, contrary to the order of the signs, completing its retrograde circulation in the period of about 19 years: after which time, either of the nodes, having receded from any point of the ecliptic, returns to the same again.—Newton has demonstrated, in his Principia, that the Retrogradation of the moon's nodes is caused by the action of the sun, which continually drawing this planet from her orbit, deflects this orbit from a plane, and causes its intersection with the ecliptic continually to vary; and his determinations on this point have been confirmed by observation.

Retrogradation *of the Sun*, a motion by which in some situations, in the torrid zone, he seems to move retrograde or backwards. When the sun is in the torrid zone, and has his declination AM greater than the latitude of the place AZ, but either northern or southern as that is, the sun will appear to go retrograde, or backwards, both before and after noon. For draw the vertical circle ZGN to be a tangent to the sun's diurnal circle MGO in G, and another ZON through the sun's rising, at O: then it is evident, that all the intermediate vertical circles cut the sun's diurnal circle twice; first in the arc GO, and the second time in the arc GI. So that, as the sun ascends through the arc GO, he continually arrives at farther and farther verticals. But as he continues his ascent through the arc GI, he returns to his former verticals; and therefore is seen retrograde for some time before noon. And in like manner it may be shown that he does the same thing for some time after noon. Hence, as the shadow always tends opposite to the sun, it will be retrograde twice every day in all places of the torrid zone, where the sun's declination exceeds the latitude.



Retrogradation, or Retrogression, in the Higher Geometry, is the same with what is otherwise called Contrary Flexion or Flexure. See Flexure, and Inflexion.

RETROGRADE, denotes backward, or contrary to the forward or natural direction. See Retrogradation.

RETROGRESSION, or Retrocession. The same with Retrogradation.

RETURNING *Stroke*, in Electricity, is an expression used by lord Mahon (now Earl Stanhope) to denote the effect produced by the return of the electric fire into a body from which, in certain circumstances, it has been expelled.

To understand properly the meaning of these terms, it must be premised that, according to the noble author's experiments, an insulated smooth body, immerged within the electrical atmosphere, but beyond the striking distance of another body, charged positively, is at the same time in a state of threefold electricity. The end next to the charged body acquires negative electricity; the farther end is positively electrified; while a certain part of the body, somewhere between its two extremes, is in a natural, unelectrified, or neutral state; so that the two contrary electricities balance each other. It may further be added, that if the body be not insulated, but have a communication with the earth, the whole of it will be in a negative state. Suppose then a brass ball, which may be called A, to be constantly placed at the striking distance of a prime conductor; so that the conductor, the instant when it becomes fully charged, explodes into it. Let another large or second conductor be suspended, in a perfectly insulated state, farther from the prime conductor than the striking distance, but within its electrical atmosphere: let a person standing on an insulated stool touch this second conductor very lightly with a finger of his right hand; while with a finger of his left hand, he communicates with the earth, by touching very lightly a second brass ball fixed at the top of a metallic stand, on the floor, which may be called B. Now while the prime conductor is receiving its electricity, sparks pass (at least if the distance between the two conductors is not too great) from the second conductor to the right hand of the insulated person; while similar and simultaneous sparks pass out from the finger of his left hand into the second metallic ball B, communicating with the earth. At length however the prime conductor, having acquired its full charge, suddenly strikes into the ball A, of the first metallic stand, placed for that purpose at the striking distance. The explosion being made, and the prime conductor suddenly deprived of its elastic atmosphere, its pressure or action on the second conductor, and on the insulated person, as suddenly ceases; and the latter instantly feels a smart returning stroke, though he has no direct or visible communication (except by the floor) with either of the two bodies, and is placed at the distance of 5 or 6 feet from both of them. This returning stroke is evidently occasioned by the sudden re-entrance of the electric fire naturally belonging to his body and to the second conductor, which had before been expelled by the action of the charged prime conductor upon them; and which returns to its former place in the instant when that action or elastic pressure ceases. When the second conductor and the insulated person are placed in the densest part of the electrical atmosphere of the prime conductor, or just beyond the striking distance, the effects are still more considerable; the returning stroke being in that case extremely severe and pungent, and appearing considerably sharper than even the main stroke itself, received directly from the prime conductor. Lord Mahon observes, that persons and animals may be destroyed, and particular parts of buildings may be much damaged, by an electrical returning stroke, occasioned even by some very distant explosion from a thunder-cloud; possibly at the distance of a mile or more. It is certainly not difficult to conceive that a highly charged thunder-cloud must be productive of effects similar to those produced by the prime conductor; but perhaps the effects are not so great, nor the danger so terrible, as it seems have been apprehended. If the quantity of electric fluid naturally contained, for example, in the body of a man, were immense or indefinite, then the estimate between the effects producible by a cloud, and those caused by a prime conductor, might be admitted; but surely no electrical cloud can expel from a body more than the natural quantity of electricity which it contains. On the sudden removal therefore of the pressure by which this natural quantity had been expelled, in consequence of the explosion of the cloud into the earth, no more (at the utmost) than the whole natural stock of electricity can re-enter his body, provided it be so situated, that the returning fire of other bodies must necessarily pass through his body. But perhaps we have no reason to suppose that this quantity is so great, as that its sudden re-entrance into his body should destroy or injure him.

Allowing therefore the existence of the returning stroke, as sufficiently ascertained, and well illustrated, in a variety of circumstances, by the author's experiments, the magnitude and danger of it do not seem to be so alarming as he apprehends. See Lord Mahon's Principles of Electricity, &c. 4to. 1779, pa. 76, 113, and 131. Also Monthly Review, vol. 62, pa. 436.

REVERSION in *Annuities*, or *Reversionary Payments*, are payments that are not to be made till after some stated period; being thus distinguished from payments that are to be made immediately.

Reversions are either certain, or contingent: of the former kind, are all sums payable after a certain number of years, or any other fixed and determinate period of time, as also on the extinction of any lives. And of the latter sort, are all such reversions as depend on any contingency; and particularly the survivorship of any lives beyond, or after, others. See the articles ASSURANCE, ANNUITIES, LIFE ANNUITIES, and SURVIVORSHIP.

REVERSION *of Series*, in Algebra, is the finding the value of the root, or unknown quantity, whose powers enter the terms of an infinite series, by means of another infinite series in which it is not contained. As, in the infinite series $z = ax + bx^2 + cx^3 + dx^4$ &c; then if there be found $x = \mathrm{A}z + \mathrm{B}z^2 + \mathrm{C}z^3$ &c, that series is reverted, or its root x is found in an infinite series of other terms.

This was one of Newton's improvements in analysis, the first specimen of which was given in his Analysis per Æquationes Numero Terminorum Infinitas; and it is of great use in resolving many problems in various parts of the mathematics.

The most usual and general way of reversion, is to assume a series, of a proper form, for the value of the required unknown quantity; then substitute the powers of this value, instead of those of that quantity into the given series; lastly compare the resulting terms with the said given series, and the values of the assumed coefficients will thus be obtained. So, to revert the series $z = ax + bx^2 + cx^3$ &c, or to find the value of x in terms of z; assume it thus, $x = \mathrm{A}z + \mathrm{B}z^2 + \mathrm{C}z^3$ &c; then by involving this series, for the several powers of x, and multiplying the corresponding powers by a, b, c, &c, there results

$$\begin{array}{lllll} z = a\mathrm{A}z & + a\mathrm{B}z^2 & + a\mathrm{C}z^3 & + a\mathrm{D}z^4, \text{ \&c.} \\ & + b\mathrm{A}^2z^2 & + 2b\mathrm{AB}z^3 & + 2b\mathrm{AC}z^4 \\ & & & + b\mathrm{B}^2z^4 \\ & & + c\mathrm{A}^3z^3 & + 3c\mathrm{A}^2\mathrm{B}z^4 \\ & & & + d\mathrm{A}^4z^4 \end{array}$$

Then by comparing the corresponding terms of this last series, or making their coefficients equal, there are obtained these equations, viz,

$a\mathrm{A} = 1$, and $a\mathrm{B} + b\mathrm{A}^2 = 0$, and $a\mathrm{C} + 2b\mathrm{AB} + c\mathrm{A}^3 = 0$, &c, which give these values of the assumed coefficients,

$$\mathrm{A} = \frac{1}{a}; \quad \mathrm{B} = -\frac{b\mathrm{A}^2}{a} = -\frac{b}{a^3};$$

$$\mathrm{C} = -\frac{2b\mathrm{AB} + c\mathrm{A}^3}{a} = \frac{2bb - ac}{a^5};$$

$$\mathrm{D} = -\frac{2b\mathrm{AC} + b\mathrm{B}^2 + 3c\mathrm{A}^2\mathrm{B} + d\mathrm{A}^4}{a}$$

$$= -\frac{5abc - 5b^3 - a^2d}{a^7}; \text{ \&c. consequently}$$

$$x = \frac{1}{a}z - \frac{b}{a^3}z^2 + \frac{2bb - ac}{a^5}z^3 - \frac{5abc - 5b^3 - a^2d}{a^7}z^4, \text{ \&c.}$$

which is therefore a general formula or theorem for every series of the same kind, as to the powers of the quantity x. Thus, for

Ex. Suppose it were required to revert the series $z = x - x^2 + x^3 - x^4$, &c.

Here $a = 1$, $b = -1$, $c = 1$, $d = -1$, &c; which values of these letters being substituted in the theorem, there results $x = z + z^2 + z^3 + z^4$, &c, which is that series reverted, or the value of x in it.

In the same way it will be found that the theorem for reverting the series

$z = ax + bx^3 + cx^5 + dx^7$ &c, is

$$x = \frac{1}{a}z - \frac{b}{a^4}z^3 + \frac{3bb - ac}{a^7}z^5 - \frac{a^2d + 12b^3 - 8abc}{a^{10}}, \text{ \&c.}$$

Various methods of reversion may be seen as given by De Moivre in the Philos. Trans. No. 240; or Maclaurin's Algebra pa. 263; or Stuart's explanation of Newton's Analysis, &c. pa. 455; or Colson's Comment on Newton's Flux. pa. 219; or Horsley's ed. of Newton's works vol. 1, pa. 291; or Simpson's Flux. vol. 2, pa. 302: or most authors on algebra.

REVETEMENT, in Fortification, a strong wall built on the outside of the rampart and parapet, to support the earth, and prevent its rolling into the ditch.

REVOLUTION, in Geometry, the motion of rotation of a line about a fixed point or centre, or of any figure about a fixed axis, or upon any line of surface. Thus, the revolution of a given line about a fixed centre, generates a circle; and that of a right-angled triangle about one side, as an axis, generates a cone; and that of a semicircle about its diameter, generates a sphere or globe, &c.

Revolution, in Astronomy, is the period of a star, planet, or comet, &c; or its course from any point of its orbit, till it return to the same again.

The planets have a twofold revolution. The one about their own axes, usually called their diurnal rotation, which constitutes their day. The other about the sun, called their annual revolution, or period, constituting their year.

REYNEAU (Charles-Rene), commonly called father Reyneau, a noted French mathematician, was born at Brissac in the province of Anjou, in the year 1656. At 20 years of age he entered himself among the Oratorians, a kind of religious order, in which the members lived in community without making any vows, and applied themselves chiefly to the education of youth. He was soon after sent, by his superiors, to teach philosophy at Pezenas, and then at Toulon. This requiring some acquaintance with geometry, he contracted a great affection for that science, which he cultivated and improved to a great extent; in consequence he was called to Angers in 1683, to fill the mathematical chair; and the Academy of Angers elected him a member in 1694.

In this occupation father Reyneau, not content with making himself master of every thing worth knowing, which the modern analysis, so fruitful in sublime speculations and ingenious discoveries, had already produced, undertook to reduce into one body, for the use of his scholars, the principal theories scattered about in Newton, Descartes, Leibnitz, Bernoulli, the Leipsic Acts, the Memoirs of the Paris Academy, and in other works; treasures which by being so widely dispersed, proved much less useful than they otherwise might have been. The fruit of this undertaking, was his Analyse Demontrée, or Analysis Demonstrated, which he published in 2 volumes 4to, 1708.

Reyneau, after thus giving lessons to those who understood something of geometry, thought proper to compose a work also for such as were utterly unacquainted with that science. This was in some measure a condescension in him, but his passion to be useful made it easy and agreeable. Accordingly, in 1714 he published a useful volume in 4to on calculation, under the title of Science du Calcul des Grandeurs.

As soon as the Royal Academy of Sciences at Paris, in consequence of a regulation made in the year 1716, opened its doors to other learned men, under the title of Free Associates, father Reyneau was admitted of the number. The works however which we have already mentioned, besides a small piece upon logic, are the only ones he ever published, or probably ever composed, except most of the materials for a second volume of his Science du Calcul, which he left behind him in manuscript. The last years of his life were attended with too much sickness to admit of any extraordinary application. He died in 1728, at 72 years of age, not more regretted on account of his great learning, than of his many virtues, which all conspired in an eminent degree to make that learning agreeable to those about him, and useful to the world. The first men in France deemed it an honour and a happiness to count him among their friends. Of this number were the chancellor of that kingdom, and father Mallebranche, of whom Reyneau was a zealous and faithful disciple.

RHABDOLOGY, or Rabdology, in Arithmetic, a name given by Napier to a method of performing some of the more difficult operations of numbers by means of certain square little rods. Upon these are inscribed the simple numbers; then by shifting them according to certain rules, those operations are performed by simply adding or subtracting the numbers as they stand upon the rods. See Napier's Rabdologia, printed in 1617. See also the article Napier's *Bones*.

RHEO-STATICS, is used by some for the statics, or the science of the equilibrium of fluids.

RHETICUS (George Joachim), a noted German astronomer and mathematician, who was the colleague of Reinhold in the university of Wittemberg, being joint professors of mathematics there together. He was born at Feldkirk in Tyrol the 15th of February 1514. After studying the elements of the mathematics at Zurich with Oswald Mycone, he went to Wittemberg, where he diligently cultivated that science. Here he was made master of philosophy in 1535, and professor in 1537. He quitted this situation however two years after, and went to Fruenburg to put himself under the assistance of the celebrated Copernicus, being induced to this step by his zeal for astronomical pursuits, and the great fame which Copernicus had then acquired Rheticus assisted this astronomer for some years, and constantly exhorted him to perfect his work, De Revolutionibus, which he published after the death of Copernicus, viz, in 1543, folio, at Norimberg, together with an illustration of the same in a narration, dedicated to Schoner. Here too, to render astronomical calculations more accurate, he began his very elaborate canon of sines, tangents, and secants, to 15 places of figures, and to every 10 seconds of the quadrant, a design which he did not live quite long enough to complete. The canon of sines however to that radius, for every 10 seconds, and for every single second in the first and last degree of the quadrant, computed by him, was published in folio at Francfort 1613 by Pitiscus, who himself added a few of the first sines computed to 22 places of figures. But the larger work, or canon of sines, tangents, and secants, to every 10 seconds, was finished and published after his death, viz, in 1596, by his disciple Valentine Otho, mathematician to the electoral prince Palatine; a particular account and analysis of which work may be seen in the Historical Introduction to my Logarithms.

After the death of Copernicus, Rheticus returned to Wittemberg, viz, in 1541 or 1542, and was again admitted to his office of professor of mathematics. The same year, by the recommendation of Melancthon, he went to Norimberg, where he found certain manuscripts of Werner and Regiomontanus. He afterwards taught mathematics at Leipsic. From Saxony he departed a second time, for what reason is not known, and went to Poland; and from thence to Cassovia in Hungary, where he died December the 4th, 1576, at nearly 63 years of age.

His Narratio de Libris Revolutionum Copernici, was first published at Gedanum in 4to, 1540; and afterwards added to the editions of Copernicus's work. He also composed and published Ephemerides, according to the doctrine of Copernicus, till the year 1551.

Rheticus also projected other works, of various kinds, astronomical, astrological, geographical, chemical, &c, and partly executed them, though they were never published, which are more particularly mentioned in his letter to Peter Ramus in the year 1568, which Adrian Romanus inserted in the preface to the first part of his Idea of Mathematics.

RHOMB-Solid, consists of two equal and right cones joined together at their bases.

RHOMBOID, or Rhomboides, in Geometry, a quadrilateral figure, whose opposite sides and angles are equal; but which is neither equilateral nor equiangular.

RHOMBUS, is an oblique equilateral parallelogram; or a quadrilateral figure, whose sides are equal and parallel, but the four angles not all equal, two of the opposite ones being obtuse, and the other two opposite ones acute. The two diagonals of a rhombus intersect at right angles. As to the area of the rhombus or rhomboides, it is found, like that of all other parallelograms, by multiplying the length or base by the perpendicular breadth.

Rhombus-*Solid*. See Rhomb-*Solid*.

RHUMB, or Rumb, in Navigation, a vertical circle of any given place; or the intersection of a part of such a circle with the horizon. Rhumbs therefore coincide with the points of the horizon. And hence mariners distinguish the rhumbs by the same names as the points and winds. But we may observe, that the rhumbs are denominated from the points of the compass in a different manner from the winds: thus, at sea, the north-east wind is that which blows from the north-east point of the horizon towards the ship in which we are; but we are said to sail upon the north-east rhumb, when we go towards the north-east.—They usually reckon 32 rhumbs, which are represented by the 32 lines in the rose or card of the compass.

Aubin defines a rhumb to be a line on the terrestrial globe, or sea-compass, or sea-chart, representing one of the 32 winds which serve to conduct a vessel. So that the rhumb a vessel pursues is conceived as its route, or course.

Rhumbs are divided and subdivided like points of the compass. Thus, the whole rhumb answers to the cardinal point. The half rhumb to a collateral point, or makes an angle of 45 degrees with the former. And the quarter rhumb makes an angle of 22° 30′ with it. Also the half-quarter rhumb makes an angle of 11° 15′ with the same.

For a table of the rhumbs, or points, and their distances from the meridian, see Wind.

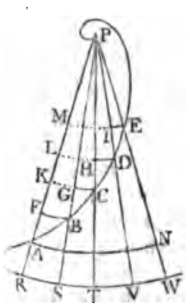
Rhumb-Line, *Loxodromia*, in Navigation, is a line prolonged from any point of the compass in a nautical chart, except the four cardinal points: or it is the line which a ship, keeping in the same collateral point, or rhumb, describes throughout its whole course; being derived from a Portuguese word.

The chief property of the rhumb-line, or loxodromia, and that from which some authors define it, is, that it cuts all the meridians in the same angle. This angle is called the Angle of the Rhumb, or the Loxodromic angle. And the angle which the rhumb-line makes with any parallel to the equator, is called the Complement of the Rhumb.

An idea of the origin and properties of the rhumb-line, the great foundation of navigation, may be conceived thus: A vessel beginning its course, the wind by which it is driven makes a certain angle with the meridian of the place; and as we shall suppose that the vessel runs exactly in the direction of the wind, it makes the same angle with the meridian which the wind makes. Supposing then the wind to continue the same, as each point or instant of the progress may be esteemed the beginning, the vessel always makes the same angle with the meridian of the place where it is each moment, or in each point of its course which the wind makes.—Now a wind, for example, that is north-east, and which consequently makes an angle of 45 degrees with the meridian, is equally north-east wherever it blows, and makes the same angle of 45 degrees with all the meridians it meets. And therefore a vessel, driven by the same wind, always makes the same angle with all the meridians it meets with on the surface of the earth. If the vessel sail north or south, it describes the great circle of a meridian. If it runs east or west, it cuts all the meridians at right angles, and describes either the circle of the equator, or else a circle parallel to it. But if the vessel sails between the two, it does not then describe a circle; since a circle, drawn obliquely to a meridian, would cut all the meridians at unequal angles, which the vessel cannot do. It describes therefore a particular curve, the essential property of which is, that it cuts all the meridians in the same angle, and it is called the Loxodromy, or Loxodromic Curve, or Rhumb-line.

This curve, on the globe, is a kind of spiral, tending continually nearer and nearer to the pole, and making an infinite number of circumvolutions about it, without ever arriving exactly at it. But the spiral rhumbs on the globe become proportional spirals in the stereographic projection on the plane of the equator. The length of a part of this rhumb-line, or spiral, then, is the distance run by the ship while she keeps in the same course. But as such a spiral line would prove very perplexing in the calculation, it was necessary to have the ship's way in a right line; which right line however must have the essential properties of the curve, viz, to cut all the meridians at right angles. The method of effecting which, see under the article Chart.

The arc of the rhumb-line is not the shortest distance between any two places through which it passes; for the shortest distance, on the surface of the globe, is an arc of the great circle passing through those places; so that it would be a shorter course to sail on the arc of this great circle: but the ship cannot be kept in a great circle, because the angle it makes with the meridians is continually varying, more or less.



Let P be the pole, RW the equator, ABCDEP a spiral rhumb, divided into an indefinite number of equal parts at the points B, C, D, &c; through which are drawn the meridians, PS, PT, PV, &c; and the parallels FB, KC, LD, &c, also draw the parallel AN. Then, as a ship sails along the rhumb-line towards the pole, or in the direction ABCD &c, from A to E, the distance sailed AE is made up of all the small equal parts of the rhumb AB + BC + CD + DE; and the sum of all the small differences of latitude AF + BG + CH + DI make up the whole difference of latitude AM or EN; and the sum of all the small parallels FB + GC + HD + IE is what is called the departure in plane sailing; and ME is the meridional distance, or distance between the first and last meridians, measured on the last parallel; also RW is the difference of longitude, measured on the equator. So that these last three are all different, viz, the departure, the meridional distance, and the difference of longitude.

If the ship sail towards the equator, from E to A; the departure, difference of latitude, and difference of longitude, will be all three the same as before; but the meridional distance will then be AN instead of ME; the one of which AN being greater than the departure FB + GC + HD + IE, and the other ME less than the same; and indeed that departure is nearly a mean proportional between the two meridional distances ME, AN. Other properties are as below.

1. All the small elementary triangles ABF, BCG, CDH, &c, are mutually similar and equal in all their parts. For all the angles at A, B, C, D, &c are equal, being the angles which the rhumb makes with the meridians, or the angles of the course; also all the angles F, G, H, I, are equal, being right angles; therefore the third angles are equal, and the triangles all similar. Also the hypothenuses AB, BC, CD, &c, are all equal by the hypothesis; and consequently the triangles are both similar and equal.

2. As radius : distance run AE
: : sine of course $\angle$ A : departure FB + GC &c,
: : cosin. of course $\angle$ A : dif. of lat. AM.

For in any one ABF of the equal elementary triangles, which may be considered as small right-angled plane triangles, it is, as rad. or sin. $\angle$ F : sin. course A : : AB : FB : : (by composition) the sum of all the distances AB + BC + CD &c : the sum of all the departures FB + GC + HD &c.

And, in like manner, as radius : cos. course A : : AB : AF : : AB + BC &c : AF + BG &c.

Hence, of these four things, the course, the difference of latitude, the departure, and the distance run, having any two given, the other two are found by the proportions above in this article.

By means of the departure, the length of the rhumb, or distance run, may be connected with the longitude and latitude, by the following two theorems.

3. As radius : half the sum of the cosines of both the latitudes, of A and E : : dif. of long. RW : departure.

Because RS : FB : : radius : sine of PA or cos. RA,
and VW : IE : : radius : sine of PE or cos. EW.

4. As radius : cos. middle latitude : : dif. of longitude : departure.—For the cosine of middle latitude is nearly equal to half the sum of the cosines of the two extreme latitudes.

RICCI (Michael-Angelo), a learned Italian divine, born at Rome in 1619. He was well skilled in the pure mathematical sciences; and he was created a cardinal in 1681; but did not long enjoy that dignity, as he died in 1683, at 64 years of age. He published at Rome, in 4to, Exercitatio Geometrica, a small tract, which was reprinted at London, and annexed to N. Mercator's Logarithmotechnia; having been thought fit to be so reprinted, partly by reason of its scarceness, but chiefly on account of the excellency of the argument, which is, de maximis et minimis, or the doctrine of limits; where the author shows a deep judgment in exhibiting the means of reducing that lately discovered doctrine to pure geometry. In this tract is demonstrated the doctrine of Caravagio de applicationibus, who affirms, that he who is ignorant of it may mispend his time about equations, in searching for that which cannot be found. He delivers also a method of drawing tangents to all the conic sections, and to several other curves.

RICCIOLI (Joannes-Baptista), a learned Italian astronomer, philosopher, and mathematician, was born in 1598, at Ferrara, a city in Italy, in the dominions of the Pope. At 16 years of age he was admitted into the society of the Jesuits. He was endowed with uncommon talents, which he cultivated with extraordinary application; so that the progress he made in every branch of literature and science was surprising. He was first appointed to teach rhetoric, poetry, philosophy, and scholastic divinity, in the Jesuits' colleges at Parma and Bologna; yet he applied himself in the mean time to making observations in geography, chronology, and astronomy. This was his natural bent, and at length he obtained leave from his superiors to quit all other employment, that he might devote himself entirely to those sciences.

He projected a large work, to be divided into three parts, and to contain as it were a complete system of philosophical, mathematical, and astronomical knowledge. The first of these parts, which regards astronomy, was published at Bologna in 1651, 2 vols. folio, with this title, J. B. Riccioli Almagestum Novum, Astronomiam veterem novamque complectens, observationibus aliorum et propriis, novisque theorematibus, problematibus, ac tabulis promotam. Riccioli imitated Ptolemy in this work, by collecting and digesting into proper order, with observations, every thing ancient and modern, which related to his subject; so that Gassendus very justly called his work, "Promptuarium et thesaurum ingentem Astronomiæ."

In the first volume of this work, he treats of the sphere of the world, of the sun and moon, with their eclipses; of the fixed stars, of the planets, of the comets and new stars, of the several mundane systems, and six sections of general problems serving to astronomy, &c.—In the second volume, he treats of trigonometry, or the doctrine of plane and spherical triangles; proposes to give a treatise of astronomical instruments, and the optical part of astronomy (which part was never published); it also treats of geography, hydrography, with an epitome of chronology.—The third, comprehends observations of the sun, moon, eclipses, fixed stars and planets, with precepts and tables of the primary and secondary motions, and other astronomical tables. Riccioli printed also, two other works, in folio, at Bologna, viz,

2. Astronomia Reformata, 1665: the design of which was, that of considering the various hypotheses of several astronomers, and the difficulty thence arising of concluding any thing certain, by comparing together all the best observations, and examining what is most certain in them, thence to reform the principles of astronomy.

3. Chronologia Reformata, 1669.

Riccioli died in 1671, at 73 years of age.

RICOCHET *Firing*, in the Military Art, is a method of firing with small charges, from pieces of ordnance elevated at small angles, as from 3 to 6 degrees. The word signifies duck-and-drake, or rebounding, because the ball or shot, thus discharged, goes bounding and rolling along, killing or destroying every thing in its way, like the bounding of a flat stone along the surface of water when thrown almost horizontally.

RIDEAU, in Fortification, a small elevation of earth, extending itself lengthways on a plain; serving to cover a camp, or give an advantage to a post.

Rideau is sometimes also used for a trench, the earth of which is thrown up on its side, to serve as a parapet for covering the men.

RIFLE Guns, in the Military Art, are those whose barrels, instead of being straight on the inside, are formed with spiral channels, making each about a turn and a half in the length of the barrel. These carry their balls farther and with more certainty than the common pieces. For the nature and qualities of them, see Robins's Tracts, vol. 1, pa. 328 &c.

RIGEL, in Astronomy. See REGEL.

RIGHT, in Geometry, something that lies evenly or equally, without inclining or bending one way or another. Thus, a right-line is that whose parts all tend the same way. In this sense, right means the same as straight, as opposed to curved or crooked.

RIGHT-*Angle*, is that which one line makes with another upon which it stands, so as to incline neither to one side nor the other. And in this sense the word right stands opposed to oblique.

RIGHT-*angled*, is said of a figure when its sides are at right angles or perpendicular to each other.—This sometimes holds in all the angles of the figure, as in squares and rectangles; sometimes only in part, as in right-angled triangles.

RIGHT-*angled Triangle in Numbers*. It was a favourite speculation with the ancient geometers, to express numerically, or in integer numbers, the sides of a right-angled triangle. The rules which they used for that purpose, are equally simple and ingenious. In symbols they are briefly expressed thus; viz, if n denote any odd number, above 1, then

according to Pythagoras, n, and $\frac{n^2 - 1}{2}$, and $\frac{n^2 + 1}{2}$, or

according to Plato, $2n$, and $n^2 - 1$, and $n^2 + 1$,

will represent the three sides of a right-angled triangle, the last term being the hypothenuse. In the second of these forms, which is only the double of the former, n may be any number, above 1, either odd or even; in the former n must be an odd number, to give integral results. In any case, the results are rational at least; and the proposition is manifest, viz, that the sum of the squares of the first two terms, is equal to the square of the third.

But a more general form of the same property is exhibited in the following terms, $2mn$, $m^2 - n^2$, and $m^2 + n^2$, where m and n may be any two numbers taken at pleasure, so as that m be greater than n. And by taking any particular numbers for m and n, an endless series of right-angled triangles, in rational, or even in whole numbers, will be the result.

Vieta employed another form of the like properties, in his curious work, Canon Mathematicus, seu ad Triangula, in which the three sides are either of the three following forms:

$$\frac{mn}{\frac{1}{4}m^2 + 1},\ \frac{\frac{1}{4}m^2 - 1}{\frac{1}{4}m^2 + 1}n,\ n \ \Big|\ n,\ \frac{mn}{\frac{1}{4}m^2 - 1},\ \frac{\frac{1}{4}m^2 + 1}{\frac{1}{4}m^2 - 1}n,\ \Big|\ n,\ \frac{\frac{1}{4}m^2 - 1}{m}n,\ \frac{\frac{1}{4}m^2 + 1}{m}n;$$

where the 3d or last term, in each of the three sets, denotes the hypothenuse of a triangle, and the two leading terms, the base and perpendicular, as it is evident that, in them all, the square of the 3d term is equal to the sum of the squares of the 1st and 2d. And by these forms, Vieta computed the sides of 4300 right-angled triangles, in rational numbers, and arranged them in tables. See an account of them in the introduction to my Mathematical Tables, pa. 5 and 6; or in my Tracts, vol. 1, pa. 285.

If in the second set, $(2n, n^2 - 1, n^2 + 1)$, above mentioned, we express fractionwise the second term divided by the first, thus, $\frac{n^2 - 1}{2n}$, and expound the value of n successively by the numbers 2, 3, 4, 5, &c, we shall obtain a series of fractions, of which the numerators and denominators will be the two less sides of a series of right-angled triangles, in whole numbers.

Or, if for n we substitute $2m + 1$, the aforesaid fraction $\frac{n^2 - 1}{2n}$ becomes $\frac{2m^2 + 2m}{2m + 1} = m + \frac{m}{2m + 1}$, in the form of a mixt number, in which the value of m may be any number whatever, the denominator $2m + 1$ denoting the least side, the numerator $2m^2 + 2m$ the other side, and the same increased by 1, viz, $2m^2 + 2m + 1$ the hypothenuse. So, if m be expounded successively by the numbers 1, 2, 3, 4, &c, the series of fractions will be $\frac{4}{3}$, $\frac{12}{5}$, $\frac{24}{7}$, $\frac{40}{9}$, &c, the denominators being the least sides, the numerators the greater, and these increased by 1, the hypothenuses, of the series of triangles. Or if we employ the form $m + \frac{m}{2m + 1}$, the same numbers 1, 2, 3, &c, will give the mixed numbers $1\frac{1}{3}$, $2\frac{2}{5}$, $3\frac{3}{7}$, $4\frac{4}{9}$, $5\frac{5}{11}$, $6\frac{6}{13}$, &c, in which the law of continuation is manifest; the denominator of each being the least side, the integer multiplied by the denominator and the numerator added is the greater side, and 1 more is the hypothenuse.

Again, if instead of n, in the first fraction $\frac{n^2 - 1}{n}$, be substituted $2m + 2$, that fraction then becomes $\frac{4m^2 + 8m + 3}{4m + 4} = m + \frac{4m + 3}{4m + 4}$. Then, taking for m successively the numbers 1, 2, 3, 4, &c, this other series of mixed numbers results, viz, $1\frac{7}{8}$, $2\frac{11}{12}$, $3\frac{15}{16}$, $4\frac{19}{20}$, $5\frac{23}{24}$, $6\frac{27}{28}$, &c, expressing another series of the triangles, in the same way as the former one.—And in like manner for any other series.

RIGHT *Cone*, *Cylinder*, *Prism*, or *Pyramid*, one whose axis is at right-angles to the base.

RIGHT-*lined Angle*, one formed by right lines.

RIGHT *Sine*, one that stands at right-angles to the diameter; as opposed to versed sine.

RIGHT *Sphere*, is that where the equator cuts the horizon at right-angles; or that which has the poles in the horizon, and the equinoctial in the zenith. Such is the position of the sphere with regard to those who live at the equator, or under the equinoctial. The consequences of which are; that they have no latitude, nor elevation of the pole; they see both poles of the world, and all the stars rise, culminate and set; also the sun always rises and descends at right-angles, and make their days and nights equal. In a right sphere, the horizon is a meridian; and if the sphere be supposed to revolve, all the meridians successively become horizons, one after another.

RIGHT *Ascension*, *Descension*, *Parallax*, &c. See the respective articles.

RIGHT *Circle*, in the Stereographic Projection of the Sphere, is a circle at right-angles to the plane of projection, or that which is projected into a right line.

RIGHT *Sailing*, is that in which a voyage is performed on some one of the four cardinal points, east, west, north, or south.—If the ship sail on a meridian, that is, north or south, she does not alter her longitude, but only changes the latitude, and that just as much as the number of degrees she has run.—But if she sail on the equator, directly east or west, she varies not her latitude, but only changes the longitude, and that just as much as the number of degrees she has run.—And if she sail directly east or west upon any parallel, she again does not change her latitude, but only the longitude; yet not the same as the number of degrees of a great circle she has sailed, as on the equator, but more, according as the parallel is remoter from the equinoctial towards the pole. For the less any parallel is, the greater is the difference of longitude answering to the distance run.

RIGIDITY, a brittle hardness; or that kind of hardness which is supposed to arise from the mutual indentation of the component particles within one another. Rigidity is opposed to ductility, malleability, &c.

RING, in Geometry, is a figure returning into itself, the axis being bent round into a circular form.—This is either plane or solid. In the former case, it is the space or figure contained between the circumferences of two concentric circles. In the latter, or solid ring, it resembles a cylinder, or other prism, bent round into a circular form. And, in either of them, the transverse section, perpendicular to the axis, is the same quantity; in the plane ring, it is the same line, or difference of the two radii; in the solid ring, it is the same plane figure.

For the measures of the surface and solidity of rings, multiply the axis by the transverse section perpendicular to it. See my Mensuration, pa. 110 and 194, edit. 4.

Ring, in Astronomy and Navigation, an instrument used for taking the sun's altitude &c. It is usually of brass, about 9 inches diameter, suspended by a little swivel, at the distance of 45° from the point of which is a perforation, which is the centre of a quadrant of 90° divided in the inner concave surface.

To use it, let it be held up by the swivel, and turned round to the sun, till his rays, falling through the hole, make a spot among the degrees, which marks the altitude required. This instrument is preferred to the astrolabe, because the divisions are here larger than on that instrument.

Ring, of Saturn, is a thin, broad, opaque circular arch, encompassing the body of that planet, like the wooden horizon of an artificial globe, without touching it, and appearing double, when seen through a good telescope.

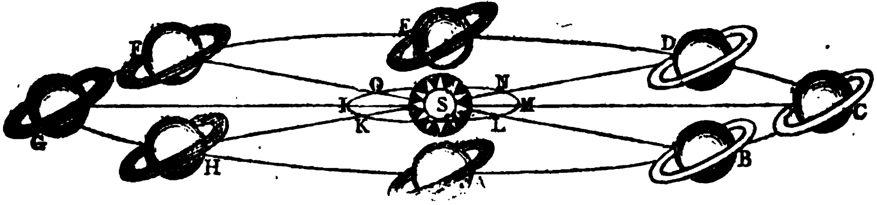
This ring was first discovered by Huygens, who, after frequent observation of the planet, perceived two lucid points, like ansæ or handles, arising out from the body in a right line. Hence, as in subsequent observations he always found the same appearance, he concluded that Saturn was encompassed with a permanent ring; and accordingly produced his New System of Saturn, in 1659. It was however, Galileo who first discovered that the figure of Saturn was not round.

Huygens estimates the space between the globe of Saturn and the ring as equal to the breadth of the ring, or rather more, being about 22000 miles broad; and the greatest diameter of the ring, in proportion to that of the globe, as 9 to 4. But Mr. Pound, by an excellent micrometer applied to the Huygenian glass of 123 feet, determined this proportion, more exactly, to be as 7 to 3.

Observations have also determined, that the plane of the ring is inclined to the plane of the ecliptic in an angle of 30 degrees; that the ring probably turns, in the direction of its plane, round its axis, because when it is almost edgewise to us, it appears rather thicker on one side of the planet than on the other; and the thickest edge has been seen on different sides at different times: the sun shines almost 15 of our years together on one side of Saturn's ring without setting, and as long on the other in its turn; so that the ring is visible to the inhabitants of that planet for almost 15 of our years, and as long invisible, by turns, if its axis has no inclination to its ring; but if the axis of the planet be inclined to the ring, ex. gr. about 30 degrees, the ring will appear and disappear once every natural day to all the inhabitants within 30 degrees of the equator, on both sides, frequently eclipsing the sun in a Saturnian day. Moreover, if Saturn's axis be so inclined to his ring, it is perpendicular to his orbit; by which the inconvenience of different seasons to that planet is avoided.

This ring, seen from Saturn, appears like a large luminous arch in the heavens, as if it did not belong to the planet.

When we see the ring most open, its shadow upon the planet is broadest; and from that time the shadow grows narrower, as the ring appears to do to us; until, by Saturn's annual motion, the sun comes to the plane of the ring, or even with its edge; which, being then directed towards us, becomes invisible, on account of its thinness.



The phenomena of Saturn's ring are illustrated by a view of this figure. Let s be the sun, ABCDEFGH Saturn's orbit, and IKLMNO the earth's orbit. Both Saturn and the earth move according to the order of the letters; and when Saturn is at A, his ring is turned edgewise to the sun S, and he is then seen from the earth as if he had lost his ring, let the earth be in any part of its orbit whatever, except between N and O; for while it describes that space, Saturn is apparently so near the sun as to be hid in his beams. As Saturn goes from A to C, his ring appears more and more open to the earth; at C the ring appears most open; but seems to become more and more narrower as Saturn goes from C to E; and when arrived at this point the ring is again turned edgewise both to the sun and earth; and as neither of its sides is illuminated, it is invisible to us, because its edge is too thin to be perceptible; and Saturn appears again as if he had lost his ring. But as he goes from E to G, his ring opens more and more to our view on the under side; and seems just as open at G as it was at C, and may be seen in the night-time from the earth in any part of its orbit, except about M, when the sun hides the planet from our view.

As Saturn goes from G to A, his ring turns more and more edgewise to us, and therefore, it seems to be narrower; till at A it quite disappears as before. Hence, while Saturn goes from A to E, the sun shines on the upper side of his ring, and the under side is dark; and while he goes from E to A, the sun shines on the under side of his ring, and the upper side is dark. The ring disappears twice in every annual revolution of Saturn, viz, when he is in the 19th degree of Pisces and of Virgo, and when Saturn is in the middle between these points, or in the 19th degree either of Gemini or of Sagittarius, his ring appears most open to us; and then its longest diameter is to its shortest, as 9 to 4. Ferguson's Astr. sect. 204.

There are various hypotheses concerning this ring. Kepler, in his Epitom. Astron. Copern., and after him Dr. Halley, in his Enquiry into the Causes of the Variation of the Needle, Phil. Trans. No. 195, supposes our earth may be composed of several crusts or shells, one within another, and concentric to each other. If this be the case, it is possible the ring of Saturn may be the fragment or remaining ruin of his formerly exterior shell, the rest of which is broken or fallen down upon the body of the planet. And some have supposed that the ring may be a congeries or series of moons revolving about the planet.

Later observations have thrown much more light on this curious phenomenon, especially respecting its dimensions, and rotation, and division into two or more parts. Lalande and Laplace say, that Cassini saw the breadth of the ring divided into two separate parts that are equal, or nearly so. Mr. Short assured M. Lalande, that he had seen many divisions on the ring, with his 12 feet telescope. And Mr. Hadley, with an excellent 5¼ feet reflector, saw the ring divided into two parts. Several excellent theories have been given in the French Memoirs, particularly by Laplace, contending for the division of the ring into many parts. But finally, the observations of Dr. Herschel, in several volumes of the Philos. Trans., seem to confirm the division into two concentric parts only. The dimensions of these two rings, and the space between them, he states in the following proportions to each other.

Inner diam. of smaller ring - -	146345 miles.
Outside diam. of ditto - -	184393
Inner diam. of larger ring - -	190248
Outside diam. of ditto - -	204883
Breadth of the inner ring - -	20000
Breadth of the outer ring - -	7200
Breadth of the vacant space - -	2839

Ring revolves in its own plane, in $10^h\ 32'\ 15''\cdot4$.

So that the outside diameter of the larger ring is almost 26 times the diameter of the earth.

Dr. Herschel adds, " Some theories and observations, of other persons, lead us to consider the question, whether the construction of this ring is of a nature so as permanently to remain in its present state? or whether it be liable to continual and frequent changes, in such a manner as in the course of not many years, to be seen subdivided into narrow slips, and then again as united into one or two circular planes only. Now, without entering into a discussion, the mind seems to revolt, even at first sight, against an idea of the chaotic state in which so large a mass as the ring of Saturn must needs be, if phenomena like these can be admitted. Nor ought we to indulge a suspicion of this being a reality, unless repeated and well-confirmed observations had proved, beyond a doubt, that this ring was actually in so fluctuating a condition." But from his own observations he concludes, " It does not appear to me, that there is a sufficient gound for admitting the ring of Saturn to be of a very changeable nature; and I guess that its phenomena will hereafter be so fully explained, as to reconcile all observations. In the mean-while, we must withhold a final judgment of its construction, till we can have more observations. Its division however into two very unequal parts, can admit of no doubt." See Philos. Trans. vol. 80, pa. 4, 481 &c, and the vol. for 1792, pa. 1 &c, also Hist. de l'Acad. des Scienc. de Paris, 1787, pa. 249 &c.

Rings *of Colours*, in Optics, a phenomenon first observed in thin plates of various substances, by Boyle, and Hooke, but afterwards more fully explained by Newton.

Mr. Boyle having exhibited a variety of colours in colourless liquors, by shaking them till they rose in bubbles, as well as in bubbles of soap and water, and also in turpentine, procured glass blown so thin as to exhibit similar colours; and he observes, that a feather of a proper shape and size, and also a black ribband, held at a proper distance between his eye and the sun, showed a variety of little rainbows, as he calls them, with very vivid colours. Boyle's Works by Shaw, vol. 2, pa. 70. Dr. Hooke, about nine years after the publication of Mr. Boyle's Treatise on Colours, exhibited the coloured bubbles of soap and water, and observed, that though at first it appeared white and clear, yet as the film of water became thinner, there appeared upon it all the colours of the rainbow. He also described the beautiful colours that are seen in thin plates of Muscovy glass; which appeared, through the microscope, to be ranged in rings surrounding the white specks or flaws in them, and with the same order of colours as those of the rainbow, and which were often repeated ten times. He also took two thin pieces of glass, ground plane and polished, and putting them one upon another, pressed them till there began to appear a red coloured spot in the middle; and pressing them closer, he observed several rings of colours encompassing the first place, till, at last, all the colours disappeared out of the middle of the circles, and the central spot appeared white. The first colour that appeared was red, then yellow, then green, then blue, then purple; then again red, yellow, green, blue, and purple; and again in the same order; so that he sometimes counted nine or ten of these circles, the red immediately next to the purple; and the last colour that appeared before the white was blue; so that it began with red, and ended with purple. These rings, he says, would change their places, by changing the position of the eye, so that, the glasses remaining the same, that part which was red in one position of the eye, was blue in a second, green in the third, &c. Birch's Hist. of the Royal Society, vol. 3, pa. 54.

Newton, having demonstrated that every different colour consists of rays which have a different and specific degree of refrangibility, and that natural bodies appear of this or that colour, according to their disposition to reflect this or that species of rays (see Colour), pursued the hint suggested by the experiments of Dr. Hooke, already recited, and casually noticed by himself, with regard to thin transparent substances. On compressing two prisms hard together, in order to make their sides touch one another, he observed, that in the place of contact they were perfectly transparent, which appeared like a dark spot, and when it was looked through, it seemed like a hole in that air, which was formed into a thin plate, by being impressed between the glasses. When this plate of air, by turning the prisms about their common axis, became so little inclined to the incident rays, that some of them began to be transmitted, there arose in it many slender arcs of colours, which increased, as the motion of the prisms was continued, and bended more and more about the transparent spot, till they were completed into circles, or rings, surrounding it; and afterwards they became continually more and more contracted.

By another experiment, with two object-glasses, he was enabled to observe distinctly the order and quality of the colours from the central spot, to a very considerable distance. Next to the pellucid central spot, made by the contact of the glasses, succeeded blue, white, yellow, and red. The next circuit immediately surrounding these, consisted of violet, blue, green, yellow, and red. The third circle of colours was purple, blue, green, yellow, and red. The fourth circle consisted of green and red. All the succeeding colours became more and more imperfect, till, after three or four revolutions, they ended in perfect whiteness.

When these rings were examined in a darkened room, by the coloured light of a prism cast on a sheet of white paper, they became more distinct, and visible to a far greater number than in the open air. He sometimes saw more than twenty of them, whereas in the open air he could not discern above eight or nine.

From other curious observations on these rings, made by different kinds of light thrown upon them, he inferred, that the thicknesses of the air between the glasses, where the rings are successively made, by the limits of the seven colours, red, orange, yellow, green, blue, indigo, and violet, in order, are one to another as the cube roots of the squares of the eight lengths of a chord, which sound the notes in an octave, sol, la, fa, sol, la, mi, fa, sol; that is, as the cube roots of the squares of the numbers 1, $\frac{8}{9}$, $\frac{5}{6}$, $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{16}$, $\frac{1}{2}$. These rings appeared of that prismatic colour, with which they were illuminated, and by projecting the prismatic colours immediately upon the glasses, he found that the light, which fell on the dark spaces between the coloured rings, was transmitted through the glasses without any change of colour. From this circumstance he thought that the origin of these rings is manifest; because the air between the glasses is disposed according to its various thickness, in some places to reflect, and in others to transmit the light of any particular colour, and in the same place to reflect that of one colour, where it transmits that of another.

In examining the phenomena of colours made by a denser medium surrounded by a rarer, such as those which appear in plates of Muscovy glass, bubbles of soap and water, &c, the colours were found to be much more vivid than the others, which were made with a rarer medium surrounded by a denser.

From the preceding phenomena it is an obvious deduction, that the transparent parts of bodies, according to their several series, reflect rays of one colour and transmit those of another; on the same account that thin plates, or bubbles, reflect or transmit those rays; and this Newton supposed to be the reason of all their colours. Hence also he has inferred, that the size of those component parts of natural bodies that affect the light, may be conjectured by their colours. See COLOUR, and REFLECTION.

Newton, pursuing his discoveries concerning the colours of thin substances, found that the same were also produced by plates of a considerable thickness, divisible into lesser thicknesses. The rings formed in both cases have the same origin, with this difference, that those of the thin plates are made by the alternate reflections and transmissions of the rays at the second surface of the plate, after one passage through it; but that, in the case of a glass speculum, concave on one side, and convex on the other, and quicksilvered over on the convex side, the rays go through the plate and return before they are alternately reflected and transmitted. Newton's Optics, pa. 169, &c. or Newtoni Opera, Horsley's edit. vol 4, pa. 121, &c. p. 184, &c.

The abbé Mazeas, in his experiments on the rings of colours that appear in thin plates, has discovered several important circumstances attending them, which were overlooked by the sagacious Newton, and which tend to invalidate his theory for explaining them. In rubbing the flat side of an object-glass against another piece of flat and smooth glass, he found that they adhered very firmly together after this friction, and that the same colours were exhibited between these plane glasses, which Newton had observed between the convex object-glass of a telescope, and another that was plane; and that the colours were in proportion to their adhesion. When the surfaces of pieces of glass, that are transparent and well polished, are equally pressed, a resistance will be perceived; and wherever this is felt, two or three very fine curve lines will be discovered, some of a pale red, and others of a faint green. If the friction be continued, the red and green lines increase in number at the place of contact; the colours being sometimes mixed without any order, and sometimes disposed in a regular manner; in which case the coloured lines are generally concentric circles, or ovals, more or less elongated, as the surfaces are more or less united.

When the colours are formed, the glasses adhere with considerable force; but if the glasses be separated suddenly, the colours will appear immediately upon their being put together, without the least friction. Beginning with the slightest touch, and increasing the pressure by insensible degrees, there first appears an oval plate of a faint red, and in the centre of it a spot of light green, which enlarges by the pressure, and becomes a green oval, with a red spot in the centre; and this enlarging in its turn, discovers a green spot in its centre. Thus the red and green succeed one another in turns, assuming different shades, and having other colours mixed with them. The greatest difference between these colours exhibited between plane surfaces, and those by curve ones, is, that, in the former, pressure alone will not produce them, except in the case above-mentioned.

In rubbing together two prisms, with very small refracting angles, which were joined so as to form a parallelopiped, the colours appeared with a surprizing lustre at the places of contact, and differently coloured ovals appeared. In the centre there was a black spot, bordered by a deep purple; next to this appeared violet, blue, orange, red tinged with purple, light green, and faint purple.

The other rings appeared to the naked eye to consist of nothing but faint reds and greens. When these coloured glasses were suspended over the flame of a candle, the colours disappeared suddenly, though they still adhered; but being suffered to cool, the colours returned to their former places, in the same order as before. At first the abbé Mazeas had no doubt but that these colours were owing to a thin plate of air between the glasses, to which Newton has ascribed them; but the remarkable difference in the circumstances attending those produced by the flat plates and those produced by the object-glasses of Newton, convinced him that the air was not the cause of this appearance. The colours of the flat plates vanished at the approach of flame, but those of the object-glasses did not. Nor was this difference owing to the plane glasses being less compressed than the convex ones; for though the former were compressed ever so much by a pair of forceps, it did not in the least hinder the effect of the flame. He then put both the plane glasses and the convex ones into the receiver of an air-pump, suspending the former by a thread, and keeping the latter compressed by two strings; but he observed no change in the colours of either of them, in the most perfect vacuum that he could make. Suspecting still that the air adhered to the surface of the glasses, so as not to be separated from them by the force of the pump, he had recourse to other experiments, which rendered it still more improbable that the air should be the cause of these colours. Having laid the coloured plates, after warming them gradually, on burning coals; and thus, when they were nearly red, rubbing them together, he observed the same coloured circles and ovals as before. When he ceased to press upon them, the colours seemed to vanish; but they returned, as he renewed the friction. In order to determine whether the colours were owing to the thickness of some matter interposed between the glasses, he rubbed them together with suet and other soft substances between them; yet his endeavour to produce the colours had no effect. However by continuing the friction with some degree of violence, he observed, that a candle appeared through them encompassed with two or three concentric greens, and with a lively red inclining to yellow, and a green like that of an emerald, and at length the rings assumed the colours of blue, yellow, and violet. The abbé was thus confirmed in his opinion that there must be some error in Newton's hypothesis, by considering that, according to his measures, the colours of the plates varied with the difference of a millionth part of an inch; whereas he was satisfied that there must have been much greater differences in the distance between his glasses, when the colours remained unchanged. From other experiments he concluded, that the plate of water introduced between the glasses was not the cause of their colours, as Newton apprehended; and that the coloured rings could not be owing to the compression of the glasses. After all, he adds, that the theory of light, thus reflected from thin plates, is too delicate a subject to be completely ascertained by a small number of observations. Berlin Mem. for 1752, or Memoires Presentes, vol. 2, pa. 28—43. M. du Tour repeated the experiments of the abbé Mazeas, and added some observations of his own. See Mem. Pres. vol. 4, pa. 288.

Musschenbroeck is also of opinion, that the colours of thin plates do not depend upon the air; but as to the cause of them, he acknowledges that he could not satisfy himself about it. Introd. ad Phil. Nat. vol. 2, pa. 738. See on this subject Priestley's Hist. of Light, &c, per. 6, sect. 5, pa. 498, &c.

For an account of the rings of colours produced by electrical explosions, see COLOURS *of Natural Bodies*, CIRCULAR *Spots*, and FAIRY *Circles*.

RISING, in Astronomy, the appearence of the sun, or a star, or other luminary, above the horison, which before was hid beneath it. By reason of the refraction of the atmosphere, the heavenly bodies always appear to rise before their time; that is, they are seen above the horison, while they are really below it, by about 33′ of a degree.

There are three poetical kinds of rising of the stars. See ACRONICAL, COSMICAL, and HELIACAL.

RITTENHOUSE (Dr. DAVID), President of the American Philosophical Society, died July 10, 1796, in the 64th year of his age. He was a native of Pennsylvania; and, in the early part of life, mixed the pursuits of science with the active employments of farming and watch-making. In 1769, he was invited by the American Philosophical Society, in association with other gentlemen, for making astronomical observations, particularly of the transit of Venus, that year; when he greatly distinguished himself by the accuracy of his observations and calculations. He afterwards constructed an observatory, which he superintended in person, and which became the source of many important discoveries, as well as greatly tended to the general diffusion of science in the western world. During the American war he was an active asserter of the cause of independence. After the conclusion of the peace, he successively filled the offices of treasurer of the state of Pennsylvania, and director of the national mint. He succeeded the illustrious Franklin in the office of President of the Philosophical Society; a situation which the bent of his mind, and the course of his studies, had rendered him eminently adequate to fill. Towards the close of his life he had retired from active occupations. He was the author of several excellent papers, chiefly on astronomical subjects, inserted in the Transactions of the American Philosophical Society.

RIVER, in Geography, a stream or current of fresh water, flowing in a bed or channel, from a source or spring, into the sea.—When the stream is not large enough to bear boats, or small loaden vessels, it is properly called by the diminutive, Rivulet or Brook; but when it is considerable enough to carry larger vessels, it is called by the general name River.—Rivulets have their rise sometimes from great rains, or great quantities of thawed snow, especially in mountainous places; but they more usually arise from springs.—Rivers themselves all arise either from the confluence of several rivulets, or from lakes.

RIVER, in Physics, denotes a stream of water running by its own gravity, from the more elevated parts of the earth towards the lower parts, in a natural bed or channel open above.—When the channel is artificial, or cut by art, it is called a canal; of which there are two kinds, viz, that whose channel is every where open, without sluices, called an artificial river, and that whose water is kept up and let off by means of sluices, which is properly a canal.

Modern philosophers endeavour to reduce the motion and flux of rivers to precise laws; and with this view they have applied geometry and mechanics to this subject; so that the doctrine of rivers is become a part of the new philosophy. The authors who have most distinguished themselves in this branch, are the Italians, the French, and the Dutch, but especially the first, and among them more particularly Gulielmini, and Ximenes.

Rivers, says Gulielmini, usually have their sources in mountains or elevated grounds; in the descent from which it is, that they mostly acquire the velocity, or acceleration, which maintains their future current. In proportion as they advance farther, this velocity diminishes, on account of the continual friction of the water against the bottom and sides of the channel; as also from the various obstacles they meet with in their progress, and from their arriving at length in plains where the descent is less. Thus the Reno, a river in Italy, which he says gave occasion, in some measure, to his speculations, is found to have near its mouth a declivity of scarce 52 seconds, being only 1 foot in 4000.

When the acquired velocity is quite spent, by means of the many obstacles that the water meets with, so that the current becomes horizontal, there will then remain nothing to propagate the motion, and continue the stream, but the depth, or the perpendicular pressure of the water, which is always proportional to the depth. And, happily for us, this resource increases, as the occasion for it increases; for in proportion as the water loses of the velocity acquired by the descent, it rises and increases in its depth.—It appears from the laws of motion pertaining to bodies moving on inclined planes, that when water flows freely upon an inclined bed, it acquires a velocity, which is always as the square root of the quantity of descent of the bed. But in an horizontal bed, opened by sluices or otherwise, at one or both ends, the water flows out by its gravity alone.

The upper parts of the water of a river, and those at a distance from the banks, may continue to flow, from the simple cause or principle of declivity, how small soever it be; for not being detained by any obstacle, the minutest difference of level will have its effect; but the lower parts, which roll along the bottom, will scarcely be sensible of so small a declivity; and will only have what motion they receive from the pressure of the superincumbent waters.—The greatest velocity of a river is about the middle of its depth and breadth, or that point which is the farthest possible from the surface of the water, and from the bottom and sides of the bed or channel. Whereas, on the contrary, the least velocity of the water is at the bottom and sides of the bed, because there the resistance arising from friction is the greatest, which is communicated to the other parts of the section of the river inversely as the distances from the bottom and sides.—To find whether the water of a river, almost horizontal, flows by means of the velocity acquired in its descent, or by the pressure of its depth; set up an obstacle perpendicular to it; then if the water rise and swell immediately against the obstacle, it runs by virtue of its fall; but if it first stop a little while, in virtue of its pressure.

Rivers, according to this author, almost always make their own beds. If the bottom have originally been a large declivity, the water, hence falling with a great force, will have swept away the most elevated parts of the soil, and carrying them lower down, will gradually render the bottom more nearly horizontal.—The water having made its bed horizontal, becomes so itself, and consequently rakes with the less force against the bottom, till at length that force becomes only equal to the resistance of the bottom, which is now arrived at a state of permanency, at least for a considerable time; and the longer according to the quality of the soil, clay and chalk resisting longer than sand or mud.

On the other hand, the water is continually wearing away the brims of its channel, and this with the more force, as, by the direction of its stream, it impinges more directly against them. By this means it has a continual tendency to render them parallel to its own course. At the same time that it has thus rectified its edges, it has widened its own bed, and thence becoming less deep, it loses part of its force and pressure: this it continues to do till there is an equilibrium between the force of the water and the resistance of its banks, and then they will remain without farther change. And it appears by experience that these equilibriums are all real, as we find that rivers only deepen and widen to a certain pitch.

The very reverse of all these things does also on some occasions happen. Rivers, whose waters are thick and muddy, raise their bed, by depositing part of the heterogeneous matters contained in them: they also contract their banks, by a continual opposition of the same matter, in brushing over them. This matter, being thrown aside far from the stream of water, might even serve, by reason of the dullness of the motion, to form new banks. If these various causes of resistance to the motion of flowing waters did not exist, viz, the attraction and continual friction of the bottom and sides, the inequalities in both, the windings and angles that occur in their course, and the diminution of their declivity the farther they recede from their springs, the velocity of their currents would be accelerated to 10, 15, or even 20 times more than it is at present in the same rivers, by which they would become absolutely unnavigable.

The union of two rivers into one, makes the whole flow the swifter, because, instead of the friction of four shores, they have only two to overcome, and one bottom instead of two; also the stream, being farther distant from the banks, goes on with the less interruption, besides, that a greater quantity of water, moving with a greater velocity, digs deeper in the bed, and of course retrenches its former width. Hence also it is, that rivers, by being united, take up less space on the surface of the earth, and are more advantageous to low grounds, which drain their superfluous moisture into them, and have also less occasion for dykes to prevent their overflowing.

A very good and simple method of measuring the velocity of the current of a river, or canal, is the following. Take a cylindrical piece of dry, light wood, and of a length something less than the depth of the water in the river; about one end of it let there be suspended as many small weights, as may keep the cylinder in a vertical or upright position, with its head just above water. To the centre of this end fix a small straight rod, precisely in the direction of the cylinder's axis; in order that, when the instrument is suspended in the water, the deviations of the rod from a perpendicularity to the surface of it, may indicate which end of the cylinder goes foremost, and by which may be discovered the different velocities of the water at different depths; for when the rod inclines forward, according to the direction of the current, it is a proof that the surface of the water has the greatest velocity; but when it reclines backward, it shows that the swiftest current is at the bottom; and when it remains perpendicular, it is a sign that the velocities at the top and bottom are equal.

This instrument, being placed in the current of a river or canal, receives all the percussions of the water throughout the whole depth, and will have an equal velocity with that of the whole current from the surface to the bottom at the place where it is put in, and by that means may be found, both with exactness and ease, the mean velocity of that part of the river for any determinate distance and time.

But to obtain the mean velocity of the whole section of the river, the instrument must be put successively both in the middle and towards the sides, because the velocities at those places are often very different from each other. Having by this means found the several velocities, from the spaces run over in certain times, the arithmetical mean proportional of all these trials, which is found by dividing the common sum of them all by the number of the trials, will be the mean velocity of the river or canal. And if this medium velocity be multiplied by the area of the transverse section of the waters at any place, the product will be the quantity running through that place in a second of time.

If it be required to find the velocity of the current only at the surface, or at the middle, or at the bottom, a sphere of wood loaded, or a common bottle corked with a little water in it, of such a weight as will remain suspended in equilibrium with the water at the surface or depth which we want to measure, will be better for the purpose than the cylinder, because it is only affected by the water of that sole part of the current where it remains suspended.

It follows from what has been said in the former part of this article, that the deeper the waters are in their bed in proportion to its breadth, the more their motion is accelerated; so that their velocity increases in the inverse ratio of the breadth of the bed, and also of the magnitude of the section; whence, in order to augment the velocity of water in a river or canal, without increasing the declivity of the bed, we must increase the depth of the channel, and diminish its breadth. And these principles are agreeable to observation; as it is well known, that the velocity of flowing waters depends much more on the quantity and depth of the water, and on the compression of the upper parts on the lower, than on the declivity of the bed; and therefore the declivity of a river must be made much greater in the beginning than toward the end of its course; where it should be almost insensible. If the depth or volume of water in a river or canal be considerable, it will suffice, in the part next the mouth, to allow one foot of declivity through 6000, or 8000, or even (according to Dechales, De Fontibus et Fluviis, prop. 49) 10,000 feet in horizontal extent; at most it need not be above 1 in 6 or 7 thousand. From hence the quantity of declivity in equal spaces must slowly and gradually increase as far as the current is to be made fit for navigation; but in such a manner, that at this upper end there may not be above one foot of perpendicular declivity in 4000 feet of horizontal extent.

To conclude this article, M. de Buffon observes, that people accustomed to rivers can easily foretell when there is going to be a sudden increase of water in the bed from floods, produced by sudden falls of rain in the higher countries through which the rivers pass. This they perceive by a particular motion in the water, which they express by saying, that the river's bottom moves, that is, the water at the bottom of the channel runs off faster than usual; and this increase of motion at the bottom of a river always announces a sudden increase of water coming down the stream. Nor, says he, is their opinion ill grounded; because the motion and weight of the waters coming down, though not yet arrived, must act upon the waters in the lower parts of the river, and communicate by impulsion part of their motion to them, within a certain distance.

On the subject of this article, see an elaborate treatise on rivers and canals, in the Philos. Trans. vol. 69, pa. 555 &c, by Mr. Mann, who has availed himself of the observations of Gulielmini, and most other writers.

ROBERTSON (John), F. R. S. was born in the year 1712; and though he was at first placed out in a trade, yet he must soon have quitted it, as in the title of his first book, a Complete Treatise on Mensuration, in 1739, he is styled Teacher of the mathematics. In this line, as a private teacher, he continued several years, till in 1754 he was appointed Master of the Royal Mathematical School in Christ's Hospital; in which year also he published the first edition of his Elements of Navigation. The year following, however, he left Christ's Hospital, in consequence of an Admiralty appointment to be first master of the Royal Naval Academy at Portsmouth; soon after which he published his Treatise on Mathematical Instruments. In 1766, through the petty cabals of the second master, they were both dismissed from that service by the first lord of the Admiralty; on which Mr. R. returned to London, where he was soon appointed clerk and librarian to the Royal Society; an office which he respectably held to the time of his death, in December 1776, at 64 years of age.

Besides the three works above-mentioned, which were all excellent of their kind, particularly the Navigation, he had many ingenious papers inserted in the Philos. Trans. from the 46th to the 60th volume. Mr. R. was a person of very honourable character and conduct, being greatly respected by the more learned and best characters among the members of the Royal Society; on most occasions his advice in the council was much regarded; and he had the honour to be one of the committee chosen to inspect and report on the government's powder-magazine at Purfleet, concerning its damage and security from lightning. In his mode of teaching, and arranging the matter in his publications, Mr. R. was remarkably neat and methodical; a habit which he probably in some measure acquired in imitation of his good friend and master, William Jones, esq. many of whose papers, on his decease, came into the possession of Mr. R. which were sold by auction, along with the valuable library of the latter, after his death, on which occasion many of them were purchased by myself.

ROBERVAL (Giles-Personne), an eminent French mathematician, was born in 1602, at Roberval, a parish in the diocese of Beauvais. He was first professor of mathematics at the College of Maitre-Gervais, and afterwards at the College-royal. A similarity of taste connected him with Gassendi and Morin; the latter of whom he succeeded in the mathematical chair at the Royal College, without quitting however that of Ramus.

Roberval made experiments on the Torricellian vacuum: he invented two new kinds of balance, one of which was proper for weighing air; and made many other curious experiments. He was one of the first members of the ancient Academy of Sciences of 1666; but died in 1675, at 73 years of age. His principal works are,

I. A Treatise on Mechanics.

II. A work entitled Aristarchus Samos.

He had several memoirs inserted in the volumes of the Academy of Sciences of 1666, viz, 1. Experiments concerning the Pressure of the Air. 2. Observations on the Composition of Motion, and on the Tangents of Curve Lines. 3. The Recognition of Equations. 4. The Geometrical Resolution of Plane and Cubic Equations. 5. Treatise on Indivisibles. 6. On the Trochoid, or Cycloid. 7. A Letter to Father Mersenne. 8. Two Letters from Torricelli. 9. A new kind of Balance.

ROBERVALLIAN *Lines*, a name given to certain lines, used for the transformation of figures: thus called from their inventor Roberval. These lines bound spaces that are infinitely extended in length, which are nevertheless equal to other spaces that are terminated on all sides.

The abbot Gallois, in the Memoirs of the Royal Academy, anno 1693, observes, that the method of transforming figures, explained at the latter end of Roberval's treatise of Indivisibles, was the same with that afterwards published by James Gregory, in his Geometria Universalis, and also by Barrow in his Lectiones Geometricæ; and that, by a letter of Torricelli, it appears, that Roberval was the inventor of this method of transforming figures, by means of certain lines, which Torricelli therefore called Robervallian Lines. He adds, that it is highly probable, that J. Gregory first learned the method in the journey he made to Padua in 1668, the method itself having been known in Italy from the year 1646, though the book was not published till the year 1692.

This account David Gregory has endeavoured to refute, in vindication of his uncle James. His answer is inserted in the Philos. Trans. of 1694, and the abbot rejoined in the French Memoirs of the Academy of 1703.

ROBINS (Benjamin), an English mathematician and philosopher of great genius and eminence, was born at Bath in Somersetshire, 1707. His parents were Quakers of low condition; and consequently neither able from their circumstances, nor willing from their religious profession, to have him much instructed in that kind of learning which they are taught to despise as human. Nevertheless, he made an early and surprising progress in various branches of science and literature, particularly in the mathematics; and his friends being desirous that he might continue his pursuits, and that his merit might not be buried in obscurity, wished that he could be properly recommended to teach that science in London. Accordingly, a specimen of his abilities in this way was sent up thither, and shown to Dr. Pemberton, the author of the "View of Sir Isaac Newton's Philosophy;" who, thence conceiving a good opinion of the writer, for a farther trial of his skill sent him some problems, which Robins resolved very much to his satisfaction. He then came to London, where he confirmed the opinion which had been preconceived of his abilities and knowledge.

But though Robins was possessed of much more skill than is usually required in a common teacher; yet being very young, it was thought proper that he should employ some time in perusing the best writers upon the sublimer parts of the mathematics, before he should publicly undertake the instruction of others. In this interval, besides improving himself in the modern languages, he had opportunities of reading in particular the works of Archimedes, Apollonius, Fermat, Huygens, De Witt, Slusius, Gregory, Barrow, Newton, Taylor, and Cotes. These authors he readily understood without any assistance, of which he gave frequent proofs to his friends: one was, a demonstration of the last proposition of Newton's treatise on Quadratures, which was thought not undeserving a place in the Philosophical Transactions for 1727.

Not long after, an opportunity occurred of exhibiting to the public a specimen also of his knowledge in Natural Philosophy. The Royal Academy of Sciences at Paris had proposed, among their prize questions in 1724 and 1726, to demonstrate the laws of motion in bodies impinging on one another. John Bernoulli here condescended to be a candidate; and as his dissertation lost the reward, he appealed to the learned world by printing it in 1727. In this piece he endeavoured to establish Leibnitz's opinion of the force of bodies in motion from the effects of their striking against springy materials; as Poleni had before attempted to evince the same thing from experiments of bodies falling on soft and yielding substances. But as the insufficiency of Poleni's arguments had been demonstrated in the Philosophical Transactions, for 1722; so Robins published in the Present State of the Republic of Letters, for May 1728, a Confutation of Bernoulli's performance, which was allowed to be unanswerable.

Robins now began to take scholars; and about this time he quitted the garb and profession of a Quaker; for, having neither enthusiasm nor superstition in his nature, as became a mathematician, he soon shook off the prejudices of such early habits. But though he professed to teach the mathematics only, he would frequently assist particular friends in other matters; for he was a man of universal knowledge: and the confinement of this way of life not suiting his disposition, which was active, he gradually declined it, and went into other courses, that required more exercise. Hence he tried many laborious experiments in gunnery; believing that the resistance of the air had a much greater effect on swift projectiles, than was generally supposed. And hence he was led to consider those mechanic arts that depend upon mathematical principles, in which he might employ his invention: as, the constructing of mills, the building of bridges, draining of fens, rendering of rivers navigable, and making of harbours. Among other arts of this kind, fortification very much engaged his attention; in which he met with opportunities of perfecting himself, by a view of the principal strong places of Flanders, in some journeys he made abroad with persons of distinction.

On his return home from one of these excursions, he found the learned here amused with Dr. Berkeley's treatise, printed in 1734, entitled, "The Analyst;" in which an examination was made into the grounds of the doctrine of Fluxions, and occasion thence taken to explode that method. Robins was therefore advised to clear up this affair, by giving a full and distinct account of Newton's doctrines, in such a manner, as to obviate all the objections, without naming them, which had been advanced by Berkeley; and accordingly he published, in 1735, A Discourse concerning the Nature and Certainty of Sir Isaac Newton's Method of Fluxions, and of Prime and Ultimate Ratios. This is a very clear, neat, and elegant performance: and yet some persons, even among those who had written against The Analyst, taking exception at Robins's manner of defending Newton's doctrine, he afterwards wrote two or three additional discourses.

In 1738, he defended Newton against an objection, contained in a note at the end of a Latin piece, called "Matho, sive Cosmotheoria puerilis," written by Baxter, author of the "Inquiry into the Nature of the Human Soul:" and the year after he printed Remarks on Euler's Treatise of Motion, on Smith's System of Optics, and on Jurin's Discourse of Distinct and Indistinct Vision, annexed to Dr. Smith's work.

In the mean time Robins's performances were not confined to mathematical subjects: for, in 1739, he published three pamphlets on political affairs, which did him great honour. The first was entitled, Observations on the present Convention with Spain: the second, A Narrative of what passed in the Common Hall of the Citizens of London, assembled for the Election of a Lord Mayor: the third, An Address to the Electors and other free Subjects of Great Britain, occasioned by the late Succession; in which is contained a Particular Account of all our Negociations with Spain, and their Treatment of us for above ten years past. These were all published without our author's name; and the first and last were so universally esteemed, that they were generally reputed to have been the production of the great man himself, who was at the head of the opposition to Sir Robert Walpole. They proved of such consequence to Mr. Robins, as to occasion his being employed in a very honourable post; for, the patriots at length gaining ground against Sir Robert, and a committee of the House of Commons being appointed to examine into his past conduct, Robins was chosen their secretary. But after the committee had presented two reports of their proceedings, a sudden stop was put to their farther progress, by a compromise between the contending parties.

In 1742, being again at leisure, he published a small treatise, entitled, New Principles of Gunnery; containing the result of many experiments that he had made, by which are discovered the force of gunpowder, and the difference in the resisting power of the air to swift and slow motions. To this treatise was prefixed a full and learned account of the progress which modern fortification had made from its first rise; as also of the invention of gunpowder, and of what had already been performed in the theory of gunnery. It seems that the occasion of this publication, was the disappointment of a situation at the Royal Military Academy at Woolwich. On the new modelling and establishing of that Academy, in 1741, our author and the late Mr. Muller were competitors for the place of professor of fortification and gunnery. Mr. Muller held then some post in the Tower of London, under the Board of Ordnance, so that, notwithstanding the great knowledge and abilities of our author, the interest which Mr. Muller had with the Board of Ordnance carried the election in his favour. On this disappointment Mr. Robins, indignant at the affront, determined to show them, and the world, by his military publications, what sort of a man he was that they had rejected.

On a discourse containing certain experiments being published in the Philosophical Transactions, with a view to invalidate some of Robins's opinions, he thought proper, in an account he gave of his book in the same Transactions, to take notice of those experiments: and in consequence of this, several dissertations of his on the resistance of the air were read, and the experiments exhibited before the Royal Society, in 1746 and 1747; for which he was presented with the annual gold medal by that Society.

In 1748 came out Anson's Voyage round the World; which, though it bears Walter's name in the title-page, was in reality written by Robins. Of this voyage the public had for some time been in expectation of seeing an account, composed under that commander's own inspection: for which purpose the reverend Richard Walter was employed, as having been chaplain on board the Centurion the greatest part of the expedition. Walter had accordingly almost finished his task, having brought it down to his own departure from Macao for England; when he proposed to print his work by subscription. It was thought proper that an able judge should first review and correct it, and Robins was appointed; when, on examination, it was resolved, that the whole should be written entirely by Robins, and that what Walter had done, being mostly taken verbatim from the journals, should serve as materials only. Hence it was that the whole of the introduction, and many dissertations in the body of the work, were composed by Robins, without receiving the least hint from Walter's manuscript; and what he had transcribed from it regarded chiefly the wind and weather, the currents, courses, bearings, distances, offings, soundings, moorings, the qualities of the ground they anchored on, and such particulars as usually fill up a seaman's account. No production of this kind ever met with a more favourable reception, four large impressions having been sold off within a year: it was also translated into most of the European languages; and it still supports its reputation, having been repeatedly reprinted in various sizes. The fifth edition at London in 1749 was revised and corrected by Robins himself; and the 9th edition was printed there in 1761.

Thus becoming famous for his elegant talents in writing, he was requested to compose an apology for the unfortunate affair at Prestonpans in Scotland. This was added as a preface to the Report of the Proceedings and Opinion of the Board of General Officers on their Examination into the Conduct of Lieutenant-General Sir John Cope, &c, printed at London in 1749; which preface was esteemed a master-piece of its kind.

Robins had afterwards, by the favour of lord Anson, opportunities of making farther experiments in Gunnery; which have been published since his death, in the edition of his works by his friend Dr. Wilson. He also not a little contributed to the improvements made in the Royal Observatory at Greenwich, by procuring for it, through the interest of the same noble person, a second mural quadrant, and other instruments; by which it became perhaps the most complete of any observatory in the world.

His reputation being now arrived at its meridian, he was offered the choice of two very considerable employments. The first was to go to Paris, as one of the commissaries for adjusting the limits in Acadia; the other, to be engineer general to the East India Company, whose forts, being in a most ruinous condition, wanted an able person to put them into a proper state of defence. He accepted the latter, as it was suitable to his genius, and as the Company's terms were both advantageous and honourable. He designed, if he had remained in England, to have written a second part of the Voyage round the World; as appears by a letter from lord Anson to him, dated Bath, Oct. 22, 1749, as follows.

"Dear sir—When I last saw you in town, I forgot to ask you, whether you intended to publish the second volume of my Voyage before you leave us; which I confess I am very sorry for. If you should have laid aside all thoughts of favouring the world with more of your works, it will be much disappointed, and no one in it more than your very obliged humble servant,

"ANSON."

Robins said, a little before his death, that the only thing he had to regret during his life, was writing lord Anson's voyage. Hence it has been supposed the *expectation* induced him to *heighten* the narrative; but it seems that his principal reward consisted in *promises*.

Robins was also preparing an enlarged edition of his New Principles of Gunnery: but, having provided himself with a complete set of astronomical and other instruments, for making observations and experiments in the Indies, he departed hence at Christmas in 1749; and after a voyage, in which the ship was near being cast away, he arrived there in July following. He immediately set about his proper business with the greatest diligence, and formed complete plans for Fort St. David and Madras: but he did not live to put them into execution. For the great difference of the climate from that of England being beyond his constitution to support, he was attacked by a fever in September the same year; and though he recovered out of this, yet about eight months after he fell into a languishing condition, in which he continued till his death, which happened the 29th of July 1751, at only 44 years of age.

By his last will, Mr. Robins left the publishing of his Mathematical Works to his honoured and intimate friend Martin Folkes, esq. president of the Royal Society, and to Dr. James Wilson; but the former of these gentlemen being incapacitated by a paralytic disorder, for some time before his death, they were afterwards published by the latter, in 2 volumes 8vo, 1761. To this collection, which contains his mathematical and philosophical pieces only, Dr. Wilson has prefixed an account of Mr. Robins, from which this memoir is chiefly extracted. He added also a large appendix at the end of the second volume, containing a great many curious and critical matters in various interesting parts of the mathematics. As to Mr. Robins's own papers in these two volumes, they are as follow: viz, in vol. I,

1. New Principles of Gunnery. First printed in 1742.
2. An account of that book. Read before the Royal Society, April the 14th and 21st, 1743.
3. Of the Resistance of the Air. Read the 12th of June, 1746.
4. Of the Resistance of the Air; together with the Method of computing the Motions of Bodies projected in that Medium. Read June 19, 1746.
5. Account of Experiments relating to the Resistance of the Air. Read the 4th of June, 1747.
6. Of the Force of Gunpowder, with the Computation of the Velocities thereby communicated to military Projectiles. Read the 25th of June, 1747.
7. A Comparison of the Experimental Ranges of Cannon and Mortars, with the Theory contained in the preceding papers. Read the 27th of June, 1751.
8. Practical Maxims relating to the Effects and Management of Artillery, and the Flight of Shells and Shot.
9. A Proposal for increasing the Strength of the British Navy. Read the 2d of April, 1747.
10. A Letter to Martin Folkes, esq. President of the Royal Society. Read the 7th of January, 1748.
11. A Letter to Lord Anson. Read the 26th of October, 1749.
12. On Pointing, or Directing of Cannon to strike distant Objects.
13. Observations on the Height to which Rockets ascend. Read the 4th of May 1749.
14. An Account of some Experiments on Rockets, by Mr. Ellicott.
15. Of the Nature and Advantage of Rifled Barrel Pieces, by Mr. Robins. Read the 2d of July, 1747.

In volume II are,

16. A Discourse concerning the Nature and Certainty of Sir Isaac Newton's Methods of Fluxions, and of Prime and Ultimate Ratios.
17. An Account of the preceding Discourse.
18. A Review of some of the principal Objections, that have been made to the Doctrine of Fluxions and Ultimate Proportions, with some Remarks on the different Methods, that have been taken to obviate them.
19. A Dissertation, showing that the Account of the Doctrines of Fluxions and of Prime and Ultimate Ratios, delivered in Mr. Robins's Discourse, is agreeable to the real Meaning of their great Inventor.
20. A Demonstration of the Eleventh Proposition of Sir Isaac Newton's Treatise of Quadratures.
21. Remarks on Bernoulli's Discourse upon the Laws of the Communication of Motion.
22. An Examination of a Note concerning the Sun's Parallax, published at the end of Baxter's Matho.
23. Remarks on Euler's Treatise of Motion; Dr. Smith's System of Optics; and Dr. Jurin's Essay on Distinct and Indistinct Vision.
24. Appendix by the Publisher.

It is but justice to say, that Mr. Robins was one of the most accurate and elegant mathematical writers that our language can boast of; and that he made more real improvements in Artillery, the flight and the resistance of projectiles, than all the preceding writers on that subject. His New Principles of Gunnery were translated into several other languages, and commented on by several eminent writers. The celebrated Euler translated the work into the German, accompanied with a large and critical commentary; and this work of Euler's was again translated into English in 1774, by Mr. Hugh Brown, with Notes, in one volume 4to.

ROBINS, or ROBYNS (JOHN), an English mathematician, was born in Staffordshire about the close of the 15th century, as he was entered a student at Oxford in 1516, where he was educated for the church. But the bent of his genius lay to the sciences, and he soon made such a progress, says Wood, in "the pleasant studies of mathematics and astrology, that he became the ablest person in his time for those studies, his friend Record not excepted, whose learning was of a nature more general. At length, taking the degree of bachelor of divinity in 1531, he was the year following made by King Henry the 8th (to whom he was chaplain), one of the canons of his college in Oxon, and in December 1543 canon of Windsor, and finally chaplain to Queen Mary, who held him in great veneration for his learning. Among several things that he wrote relating to astrology (or astronomy) I find these following:

"De Culminatione Fixarum Stellarum, &c.
De Ortu & Occasu Stellarum Fixarum, &c.
Annotationes Astrologicæ, &c. lib. 3.
Annotationes Edwardo VI.
Tractatus de Prognosticatione per Eclipsin.

"All which books, that are in MS, were some time in the choice library of Mr. Thomas Allen, of Glocester Hall. After his death, coming into the hands of Sir Kenelm Digby, they were by him given to the Bodleian library, where they yet remain. It is also said, that he, the said Robyns, hath written a book intitled, De Portentosis Cometis, but such a thing I have not yet seen, nor do I know any thing else of the author, only that, paying his last debt to nature the 25th of August 1558, he was buried in the chappel of St. George at Windsore."

ROBISON (JOHN), LL.D. an eminent philosopher, was born in Scotland, about 1733. After receiving a liberal education in his native country, he went to Russia, on the appointment of director of the marine cadet academy, at Cronstadt, a situation which he held for several years. On his return to Scotland he was appointed professor of chemistry in the university of Glasgow; and shortly afterwards he was invited to fill the chair of natural philosophy professor at Edinburgh; an office which he held with much honour to himself, and benefit to the students of that university, till his death, which happened early in the year 1805.

Though Dr. Robison laboured under a most painful and distressing complaint during the last 18 years of his life, still his mind was always active, and generally directed to the most useful purposes. He was well known as the author of most of the mathematical and philosophical articles in the third edition of the Encyclopedia Britannica, and the Supplement to that valuable work. Those articles are of established character; and, though several of them are written in a very desultory manner, yet they are rich in important remarks and useful information. They appear to have been the substance of his lectures delivered in the college; and some of them were afterwards thrown into an improved form, and published under the title of Elements of Mechanical Philosophy, of which it seems only one large 8vo volume has been published. In 1797 Dr. R. astonished the world by a publication which he called, Proofs of a Conspiracy against all Religions and Governments of Europe, carried on in the Secret Meetings of Free-Masons, Illuminati, and Reading-Societies; and in 1803 he published a valuable edition of Dr. Black's Lectures on the Elements of Chemistry, accompanied with much interesting disquisition and history, by the editor.

Altogether, Dr. R. may justly be considered as one of the most eminent philosophers ever produced in Scotland; though perhaps inferior to Gregory and Maclaurin. He certainly possessed a very extensive acquaintance with chemistry, as well as with both pure and mixed mathematics; and he doubtless enjoyed a most happy talent at converting and applying the knowledge he possessed to important practical purposes.

ROCKET, in Pyrotechny, an artificial firework, usually consisting of a cylindrical case of paper filled with a composition of certain combustible ingredients; which being tied to a rod, mounts into the air to a considerable height, and there bursts. These are called Sky Rockets. Besides which, there are others called Water Rockets, from their acting in water.

The composition with which rockets are filled, consists of the three following ingredients, viz, saltpetre, charcoal, and sulphur, all well ground; and in the smaller sizes, gunpowder dust is also added. But the proportions of all the ingredients vary with the weight of the rocket, as in the following table.

Composition for Rockets of various Sizes.

The general composition for rockets is,

Saltpetre	4lb.
Sulphur	1lb.
Charcoal	1lb.

But for large rockets,

Saltpetre	4lb.
Sulphur	1lb.
Mealpowder	1lb.

For rockets of a middle size,

Saltpetre	3lb.
Sulphur	2lb.
Mealpowder	1lb.
Charcoal	1lb.

When rockets are intended to mount upwards, they have a long slender rod fixed to the lower end, to direct their motion.

Theory of the Flight of Rockets.—Mariotte takes the rise of rockets to be owing to the impulse or resistance of the air against the flame. Desaguliers accounts for it thus. Conceive the rocket to have no vent at the choke, and to be set on fire in the conical bore; the consequence would be, either that the rocket would burst in the weakest place, or that, if all parts were equally strong, and able to sustain the impulse of the flame, the rocket would burn out immoveable. Now, as the force of the flame is equable, suppose its action downwards, or that upwards, sufficient to lift 40 pounds; as these forces are equal, but their directions contrary, they will destroy each other's action.

Imagine then the rocket opened at the choke; by this means the action of the flame downwards is taken away, and there remains a force equal to 40 pounds acting upwards, to carry up the rocket, and the stick or rod it is tied to. Accordingly we find that if the composition of the rocket be very weak, so as not to give an impulse greater than the weight of the rocket and stick, it does not rise at all; or if the composition be slow, so that a small part of it only kindles at first, the rocket will not rise.

The stick serves to keep it perpendicular; for if the rocket should begin to tumble, moving round a point in the choke, as being the common centre of gravity of rocket and stick, there would be so much friction against the air, by the stick between the centre and the point, and the point would beat against the air with so much velocity, that the reaction of the medium would restore it to its perpendicularity. When the composition is burnt out, and the impulse upwards has ceased, the common centre of gravity is brought lower towards the middle of the stick; by which means the velocity of the point of the stick is decreased, and that of the point of the rocket is increased; so that the whole will fall down, with the rocket end foremost.

During the time the rocket burns, the common centre of gravity is shifting and getting downwards, and still the faster and the lower as the stick is lighter; so that it sometimes begins to tumble before it is quite burnt out: but when the stick is too heavy, the common centre of gravity will not get so low, but that the rocket will rise straight, though not so fast.

From the experiments of Mr. Robins, and other gentlemen, it appears that the rockets of 2, 3, or 4 inches diameter, rise the highest; and they found them rise to all heights in the air, from 400 to 1254 yards, which is about ¾th of a mile. See Robins's Tracts, vol. 2, pa. 317, and the Philos. Trans. vol. 46, pa. 578.

ROD, or *Pole*, is a long measure, of 16½ feet, or 5½ yards, or the 4th part of a Gunter's chain, for land-measuring.

ROEMER (OLAUS), a noted Danish astronomer and mathematician, was born at Arhusen in Jutland, 1644;

and at 18 years of age was sent to the university of Copenhagen. He applied assiduously to the study of the mathematics and astronomy, and became so expert in those sciences, that when Picard was sent by Lewis the 14th, in 1671, to make observations in the north, he was greatly surprised and pleased with him. He engaged him to return with him to France, and had him presented to the king, who honoured him with the dauphin as a pupil in mathematics, and settled a pension on him. He was joined with Picard and Cassini, in making astronomical observations; and in 1672 he was admitted a member of the academy of sciences.

During the ten years he resided at Paris, he gained great reputation by his discoveries; yet it is said he complained afterwards, that his coadjutors ran away with the honour of many things which belonged to him. Here it was that Roemer, first of any one, found out the velocity with which light moves, by means of the eclipses of Jupiter's satellites. He had observed for many years that, when Jupiter was at his greatest distance from the earth, the emersions of his first satellite happened constantly 15 or 16 minutes later than the calculation gave them. Hence he concluded that the light reflected by Jupiter took up this time in running over the excess of distance, and consequently that it took up 16 or 18 minutes in running over the diameter of the earth's orbit, and 8 or 9 in coming from the sun to us, provided its velocity was nearly uniform. This discovery had at first many opposers; but it was afterwards confirmed by Dr. Bradley in the most ingenious and beautiful manner. See ABERRATION.

In 1681 Roemer was recalled back to his own country by Christian the 5th, king of Denmark, who made him professor of astronomy at Copenhagen. The king employed him also in reforming the coin and the architecture, in regulating the weights and measures, and in measuring and laying out the high roads throughout the kingdom; offices which he discharged with the greatest credit and satisfaction. In consequence he was honoured by the king with the appointment of chancellor of the exchequer and other dignities. Finally he became counsellor of state and burgomaster of Copenhagen, under Frederic the 4th, the successor of Christian. Roemer was preparing to publish the result of his observations, when he died the 19th of September 1710, at 66 years of age: but this loss was supplied by Horrebow, his disciple, then professor of astronomy at Copenhagen, who published, in 4to, 1735, various observations of Roemer, with his method of observing, under the title of Basis Astronomiæ.—He had also printed various astronomical observations and pieces, in several volumes of the Memoirs of the Royal Academy of Sciences at Paris, of the institution of 1666, particularly vols. 1 and 10 of that collection.

ROHAULT (JAMES), a French philosopher, was the son of a rich merchant at Amiens, where he was born in 1620. He cultivated the languages and belles-lettres in his own country, and then was sent to Paris to study philosophy. He read the ancient and modern philosophers, but Descartes was the author who most engaged his notice. Accordingly he became a zealous follower of that great man, and drew up an abridgment and explanation of his philosophy with great clearness and method. In the preface to his Physics, for so his work is called, he makes no scruple to say, that "the abilities and accomplishments of this philosopher must oblige the whole world to confess, that France is at least as capable of producing and raising men versed in all arts and branches of knowledge, as ancient Greece." Clerselier, well known for his translation of many pieces of Descartes, conceived such an affection for Rohault, on account of his attachment to this philosopher, that he gave him his daughter in marriage, against all the remonstrances of his family.

Rohault's Physics were written in French, but have been translated into Latin by Dr. Samuel Clarke, with notes, in which the Cartesian errors are corrected on the Newtonian system. The 4th and best edition of Rohault's Physica, by Clarke, is that of 1718, in 8vo. He wrote also,

Elemens de Mathematiques,
Traité de Mechanique, and
Entretiens sur la Philosophie.

But these dialogues are founded and carried on upon the principles of the Cartesian philosophy, which has now little other merit, than that of having corrected the errors of the ancients. Rohault died in 1675, and left behind him the character of an amiable, as well as a learned and philosophic man.

His posthumous works were collected and printed in two neat little volumes, first at Paris, and then at the Hague in 1690. The contents of which are, 1. The first 6 books of Euclid. 2. Trigonometry. 3. Practical Geometry. 4. Fortification. 5. Mechanics. 6. Perspective. 7. Spherical Trigonometry. 8. Arithmetic.

ROLLE (MICHEL), a French mathematician, was born at Ambert, a small town in Auvergne, the 21st of April 1652. His first studies and employments were under notaries and attorneys; occupations but little suited to his genius. He went to Paris in 1675, with the only resource of fine penmanship, and subsisted by giving lessons in writing. But as his inclination for the mathematics had drawn him to that city, he attended the masters in this science, and soon became one himself. Ozanam proposed a question in arithmetic to him, to which Rolle gave so clear and good a solution, that the minister Colbert made him a handsome gratuity, which at last grew into a fixed pension. He then abandoned penmanship, and gave himself up entirely to algebra and other branches of the mathematics. His conduct in life gained him many friends; in which his scientific merit, his peaceable and regular behaviour, with an exact and scrupulous probity of manners, were his only solicitors.

Rolle was chosen a member of the ancient academy of Sciences in 1685, and named second geometrical-pensionary on its renewal in 1699; an honour which he enjoyed till his death, which happened the 5th of July 1719, at 67 years of age.

The works published by Rolle, were,

I. A Treatise of Algebra; in 4to, 1690.

II. A Method of resolving Indeterminate Questions in Algebra; in 1699. Besides a great many curious pieces inserted in the Memoirs of the Academy of Sciences, as follow:

1. A Rule for the Approximation of Irrational Cubes: an. 1666, vol. 10.—2. A Method of resolving Equations of all Degrees which are expressed in General Terms: an. 1666, vol. 10.—3. Remarks on Geometric Lines: 1702 and 1703.—4. On the New System of Infinity: 1703.—5. On the Inverse Method of Tangents: 1705, pa. 25, 171, 222.—6. Method of finding the Foci of Geometric Lines of all kinds: 1706, pa. 284.—7. On Curves, both Geometrical and Mechanical, with their Radii of Curvature: 1707, pa. 370.—8. On the Construction of Equa-

tions: 1708, and 1709.—9. On the Extermination of the Unknown Quantities in the Geometrical Analysis: 1709, pa. 419.—10. Rules and Remarks for the Construction of Equations: 1711, pa. 86.—11. On the Application of Diophantine Rules to Geometry: 1712.—12. On a Paradox in Geometric Effections: 1713, pa. 243.—13. On Geometric Constructions: 1713, pa. 261, and 1714, pa. 5.

ROLLING, or *Rotation*, in Mechanics, a kind of circular motion, by which the moveable body turns round its own axis, or centre, and continually applies new parts of its surface to the body it moves upon. Such is that of a wheel, a sphere, a garden roller, or the like.

The motion of rolling is opposed to that of sliding; in which latter motion the same surface is continually applied to the plane it moves along.

In a wheel, it is only the circumference that properly and simply rolls; the rest of the wheel proceeds in a compound angular kind of motion, and partly rolls, partly slides. The want of distinguishing between which two motions, occasioned the difficulty of that celebrated problem of Aristotle's Wheel. See ROTA *Aristotelica*.

The friction of a body in rolling, is much less than the friction in sliding. And hence arises the great use of wheels, rolls, &c, in machines; as much of the action as possible being laid upon it, to make the resistance the less. See ROTATION.

ROMAN *Order*, in Architecture, is the same as the Composite. It was invented by the Romans, in the time of Augustus: it is composed of the Ionic and Corinthian orders, being more ornamental than either.

RONDEL, in Fortification, a round tower, sometimes erected at the foot of a bastion.

ROOD, a square measure, being a quantity of land equal to the 4th part of an acre, or equal to 40 perches or square poles.

ROOF, in Architecture, the uppermost part of a building; being that which forms the covering of the whole. In this sense, the roof comprises the timber work, together with its furniture, of slate, or tile, or lead, or whatever else serves for a covering: though the carpenters usually restrain roof to the timber-work only.

The form of a roof is various: viz, 1. Pointed, when the ridge, or angle formed by the two sides, is an acute angle.—2. Square, when the pitch or angle of the ridge is a right angle, called the true pitch.—3. Flat or pediment roof, being only pediment pitch, or the angle very obtuse. There are also various other forms, as hip roofs, valley roofs, hopper roofs, double ridges, platforms, round, &c.—In the true pitch, when the sides form a square or right angle, the girt over both sides of the roof, is accounted equal to the breadth of the building and the half of the same.

ROOKE (LAWRENCE), an English astronomer and geometrician, was born at Deptford in Kent, 1623, and educated at Eton school. Hence he removed to King's College, Cambridge, in 1639; and after taking the degree of master of arts in 1647, he retired into the country. But in the year 1650 he went to Oxford, and settled in Wadham College, that he might have the company of, and receive improvement from Dr. Wilkins, and Mr. Seth Ward the Astronomy Professor; and that he might attend Mr. Boyle in his chemical operations.

After the death of Mr. Foster, he was chosen Astronomy Professor in Gresham College, London, in the year 1652. He made some observations on the comet at Oxford, which appeared in the month of December that year; which were printed by Dr. Seth Ward the year following. And, in 1655, Dr. Wallis publishing his treatise on Conic Sections, he dedicated that work to those two gentlemen.

In 1657, Mr. Rooke was permitted to exchange the astronomy professorship for that of geometry. This step might seem strange, as astronomy still continued to be his favourite study; but it was thought to have been from the convenience of the lodgings, which opened behind the reading hall, and therefore were proper for the reception of those gentlemen after the lectures, who in the year 1660 formed the Royal Society there.

Mr. Rooke having thus successively enjoyed those two places some years before the restoration in 1658, most of those gentlemen who had been accustomed to assemble with him at Oxford, coming to London, joined with other philosophical men, and usually met at Gresham College to hear Mr. Rooke's lectures, and afterwards withdrew into his apartment; till their meetings were interrupted by the quartering of soldiers in the college that year. And after the Royal Society was formed and settled into a regular body, Mr. Rooke was very zealous and serviceable in promoting that great and useful institution; though he did not live till it received its establishment by the Royal charter.

The Marquis of Dorchester, who was not only a patron of learning, but learned himself, used to entertain Mr. Rooke at his seat at Highgate after the restoration, and bring him every Wednesday in his coach to the Royal Society, which then met on that day of the week at Gresham College. But the last time Mr. Rooke was at Highgate, he walked from thence; and it being in the summer, he overheated himself, and taking cold after it, was thrown into a fever, which cost him his life. He died at his apartments at Gresham College the 27th of June 1662, in the 40th year of his age.

Another very unfortunate circumstance attended his death, which was, that it happened the very night that he had for some years expected to finish his accurate observations on the satellites of Jupiter. When he found his illness prevented him from making that observation, Dr. Pope says, he sent to the Society his request, that some other person, properly qualified, might be appointed for that purpose; so intent was he to the last on making those curious and useful discoveries, in which he had been so long engaged.

Mr. Rooke made a nuncupatory will, leaving what he had to Dr. Ward, then lately made bishop of Exeter: whom he permitted to receive what was due upon bond, if the debtors offered payment willingly, otherwise he would not have the bonds put in suit: "for," said he, "as I never was in law, nor had any contention with any man, in my life-time; neither would I be so after my death."

Few persons have left behind them a more agreeable character than Mr. Rooke, from every person that was acquainted with him, or with his qualifications; and in nothing more than for his veracity: for what he asserted positively, might be fully relied on: but if his opinion was asked concerning any thing that was dubious, his usual answer was, "I have no opinion." Mr. Hooke has given this copious, though concise character of him: "I never was acquainted with any person who knew more, and spoke less, being indeed eminent for the knowledge and improvement of astronomy." Dr. Wren and Seth Ward describe him, as a man of profound judgment, a vast comprehension, prodigious memory, and solid experience. His skill in the mathematics was reverenced by all the lovers of those studies, and his perfection in many other kinds of learning deserves no less admiration; but above all, as another writer characterises him, his extensive knowledge had a right influence on the temper of his mind, which had all the humility, goodness, calmness, strength, and sincerity, of a sound and unaffected philosopher. These accounts give us his picture only in miniature; but his successor, Dr. Isaac Barrow, has drawn it in full proportion, in his oration at Gresham College; which is too long to be inserted in this place.

His writings were chiefly;

1. Observations on the Comet of Dec. 1652. This was printed by Dr. Seth Ward, in his Lectures on Comets, 4to, 1653.

2. Directions for Seamen going to the East and West Indies. Published in the Philosophical Transactions for Jan. 1665.

3. A Method of Observing the Eclipses of the Moon &c. In the Philos. Trans. for Feb. 1666.

4. A Discourse concerning the Observations of the Eclipses of the Satellites of Jupiter. In the History of the Royal Society, pa. 183.

5. An Account of an Experiment made with Oil in a long Tube. Read to the Royal Soc. April 23, 1662.—By this experiment it was found, that the oil sunk when the sun shone out, and rose when he was clouded; the proportions of which are set down in the account.

ROOT, in Arithmetic and Algebra, denotes a quantity which being multiplied by itself produces some higher power; or a quantity considered as the basis or foundation of a higher power, out of which this arises and grows, like as a plant from its root.

In the involution of powers, from a given root, the root is also called the first power; when this is once multiplied by itself, it produces the square or second power; this multiplied by the root again, makes the cube or 3d power; and so on. And hence the denominations square-root, cube-root, &c, or 2d root, or 3d root, &c, according as the given power or quantity is considered as the square, or cube, or 2d power, or 3d power, &c. Thus, 2 is the square-root or 2d root of 4, and the cube-root or 3d root of 8, and the 4th root of 16, &c.

Hence, the square-root is the mean proportional between 1 and the square or given power; and the cube-root is the first of two mean proportionals between 1 and the given cube; and so on.

Root is also applied sometimes in a different sense; thus we say the root or radix of any system of notation, or the radix of a system of logarithms. The radix of our present scale of notation is 10, and this is also the radix of the modern or Brigges's logarithms. The advantages of which consist in this equality between the roots of the system of notation and logarithms, by which means the tables of the latter are much contracted, and are also much readier in their application.

For the method of extracting the roots of numbers, and algebraic quantities, see the articles EXTRACTION, and BINOMIAL *Theorem*.

Finite approximating rules for the extraction of roots have been given by several authors, as Raphson, De Lagney, Halley, &c. See the articles APPROXIMATION and EXTRACTION. See also Newton's Universal Arith. the Appendix; Philos. Trans. numb. 210; Maclaurin's Alg. pa. 242; Simpson's Alg. pa. 155; or his Essays, pa. 82, or his Dissertations, pa. 102, or his Select Exerc. pa. 215: where various general theorems for approximating to the roots of pure powers are given. See also EQUATION and REDUCTION of Equations, APPROXIMATION, and CONVERGING.

But the most commodious and general rule of any, for such approximations, I believe, is that which has been invented by myself, and explained in my Tracts, vol. 1, pa. 210: which theorem is this;

$\frac{\overline{n+1}.N + \overline{n-1}.a^n}{\overline{n-1}.N + \overline{n+1}.a^n}a = \sqrt[n]{N}$. That is, having to extract the nth root of the given number N; take a^n the nearest rational power to that given quantity N, whether greater or less, its root of the same kind being a; then the required root, or $\sqrt[n]{N}$, will be as is expressed in this formula above; or the same expressed in a proportion will be thus:

$(n-1).N+(n+1).a^n : (n+1).N+(n-1).a^n :: a : \sqrt[n]{N}$ the root sought very nearly.

This rule includes all the particular rational formulas of De Lagney, and Halley, which were separately investigated by them;[1] and yet this general formula is perfectly simple and easy to apply, and more easily kept in mind than any one of the said particular formulas.

Ex. Suppose it be required to double the cube, or to extract the cube root of the number 2.

Here $N = 2$, $n = 3$, the nearest root $a = 1$, also $a^3 = 1$; hence, for the cube root the formula becomes $\frac{4N+2a^3}{2N+4a^3}a$ or $\frac{2N+a^3}{N+2a^3}a = \sqrt[3]{N}$.

But $N + 2a^3 = 4$, and $2N + a^3 = 5$; therefore as $4 : 5 :: 1 : \frac{5}{4} = 1{\cdot}25$ = the root nearly by a first approximation.

Again, for a second approximation, take $a = \frac{5}{4}$, and consequently $a^3 = \frac{125}{64}$;

hence $2N + a^3 = 4 + \frac{125}{64} = \frac{381}{64}$,

and $N + 2a^3 = 2 + \frac{250}{64} = \frac{378}{64}$;

therefore as $378 : 381$, or as $126 : 127 :: \frac{5}{4} : \frac{635}{504} =$ $1{\cdot}259921$ &c, for the required cube root of 2, which is true even in the last place of decimals.

ROOT *of an Equation*, denotes the value of the unknown quantity in an equation; which is such a quantity, as being substituted instead of that unknown letter, into the equation, shall make all the terms to vanish, or both sides equal to each other. Thus, of the equation $3x+5 = 14$, the root or value of x is 3, because substituting 3 for x, makes it become $9+5 = 14$. And the root of the equation $2x^2 = 32$ is 4, because $2 \times 4^2 = 32$. Also the root of the equation $x^2 = a^2+c^2$ is $x = \sqrt{(a^2+c^2)}$.

For the nature of roots, and for extracting the several roots of equations, see EQUATION.

Every equation has as many roots, or values of the unknown quantity, as are units in the dimensions or highest power in it. So a simple equation has one root, a quadratic two, a cubic three, and so on.

Roots are positive or negative, real or imaginary, rational or radical, &c. See EQUATION.

Cubic ROOT. This is threefold, even for a simple cubic. So the cube root of a^3, is either

a, or $\frac{-1+\sqrt{-3}}{2}a$, or $\frac{-1-\sqrt{-3}}{2}a$.

And even the cube Root of 1 itself is either

1, or $\frac{-1+\sqrt{-3}}{2}$, or $\frac{-1-\sqrt{-3}}{2}$.

Real and Imaginary ROOTS. The odd roots, as the 3d, 5th, 7th, &c roots, of all real-quantities, whether positive or negative, are real, and are respectively positive or negative. So the cube root of a^3 is a, and of $-a^3$ is $-a$.

But the even roots, as the 2d, 4th, 6th, &c, are only real when the quantity is positive; being imaginary or impossible when the quantity is negative. So the square root of a^2 is a, which is real; but the square root of $-a^2$, that is, $\sqrt{-a^2}$, is imaginary or impossible; because there is no quantity, neither $+a$ nor $-a$, which by squaring will make the given negative square $-a^2$.

The large Table of Roots, Squares, and Cubes, at the end of vol. 1 of my Tracts, is very useful in many calculations, and will serve to find square roots and cube roots, as well as square and cubic powers, &c.

ROTA, in Mechanics. See WHEEL.

ROTA *Aristotelica*, or *Aristotle's Wheel*, denotes a celebrated problem in mechanics, concerning the motion or rotation of a wheel about its axis; so called because first noticed by Aristotle. The difficulty is this. While a circle makes a revolution on its centre, advancing at the same time in a right line along a plane, it describes, on that plane, a right line which is equal to its circumference. Now if this circle, which may be called the deferent, carry with it another smaller circle, concentric with it, like the nave of a coach wheel; then this little circle, or nave, will describe a line in one revolution, which is equal to that of the large wheel or circumference itself; because its centre advances in a right line as fast as that of the wheel, being in reality the same with it.—The solution given by Aristotle, is no more than a good explication of the difficulty.

Galileo, who next attempted it, had recourse to an infinite number of infinitely little vacuities in the right line described by the two circles: and imagines that the little circle never applies its circumference to those vacuities; but in reality only applies it to a line equal to its own circumference; though it appears to have applied it to a much larger. But all this is nothing to the purpose.

Tacquet says, that the little circle, making its rotation more slowly than the great one, does on that account describe a line longer than its own circumference; yet without applying any point of its circumference to more than one point of its base. But this is no more satisfactory than the former.

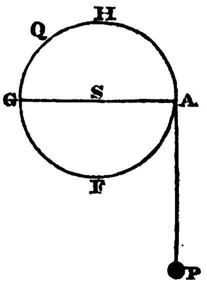
After the fruitless attempts of so many great men, M. Dortous de Meyran, a French gentleman, had the good fortune to hit upon a solution, which he sent to the Academy of Sciences; where being examined by Mess. de Louville and Soulmon, appointed for that purpose, they made their report that it was satisfactory. The solution is to this effect:

The wheel of a coach is only acted on, or drawn in a right line; its rotation or circular motion arises purely from the resistance of the ground upon which it is applied. Now this resistance is equal to the force which draws the wheel in the right line, inasmuch as it defeats that direction; consequently the causes of the two motions, the one right and the other circular, are equal. And hence the wheel describes a right line on the ground equal to its circumference.

As for the nave of the wheel, the case is otherwise. It is drawn in a right line by the same force as the wheel; but it only turns round because the wheel does so, and can only turn in the same time with it. Hence it follows, that its circular velocity is less than that of the wheel, in the ratio of the two circumferences; and therefore its circular motion is less than the rectilinear one. Since then it necessarily describes a right line equal to that of the wheel, it can only do it partly by sliding, and partly by revolving, the sliding part being more or less as the nave itself is smaller or larger. See CYCLOID.

ROTATION, or ROTARY *Motion*, in Mechanics, is the motion of a body, or system of bodies, about a fixed axis; being thus distinguished from rectilinear motion, in which bodies are supposed to describe spaces in the direction of the impelling force, which is always considered as acting in a right line passing through the centre of gravity of the body moved; and therefore, that every particle of such body must partake of the same degree of velocity as that with which the centre of gravity moves. But in numerous instances which occur in practice, a body, or system of bodies, is so situated, that when any force or number of forces are impressed upon it, it cannot take any other motion than one of rotation about a fixed axis, which may either pass through the body or system, or be at an extremity of it: so that the velocity of the constituent moleculæ of the system shall be greater or less according to the greater or less distance of any individual particle from the axis about which the motion is performed. And in such cases, it is necessary to call to our aid other considerations than what are required in discussing the properties of acceleration and retardation.

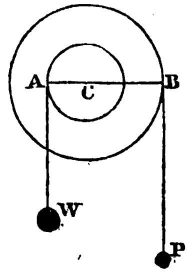
In these considerations, two things are principally to be attended to, i. e. the moving force by which the revolving motion is generated, and the inertia of the parts that compose the system: the moving force exerted on any given particle in the system, as well as its inertia, depends on its distance from the axis of motion, every thing else being the same, and if both these be ascertained, the absolute acceleration will be determined, and consequently the absolute velocity generated in it in a given time. Thus,



Let AFGH represent the circumference of a wheel, which turns in its own plane round an horizontal axis, passing through S its centre, and let a weight P, fixed at the extremity of a line AP, communicate motion to the wheel. Also, let the whole weight of the wheel be Q, and suppose this weight to be collected uniformly into the circumference AFGH; then during the descent of the weight P, each point of the circumference must move with a velocity equal to that with which P descends; and consequently, since the moving force is the weight P, and the mass moved P + Q, the force which accelerates P in its descent, will be that part of the accelerating force of gravity which is expressed by the fraction $\frac{P}{P+Q}$ (see ACCELERATION). The velocity therefore which is generated in P, in any given time, is found by proportion, namely, it will be to the velocity generated by gravity in a falling body in the same time, as this fraction to unity; so, if $Q = P$, then its velocity to that of gravity, is as 1 to 2. And this is universally true while the axis of the body, or system of bodies, passes through their centre of gravity. But if, instead of this, we suppose all the matter of the wheel to be collected into one point as at Q; then it is manifest, that if the mass Q be acted on by gravity, the force which communicates motion to the system round S, will be variable, it being the greatest when SQ is horizontal, and gradually diminishing till Q has descended to its lowest point. But if, instead of supposing Q to be acted on by gravity, we consider it as destitute of weight, and to possess inertia only, then the moving force will be constant, being equal to P, and the bodies moved will be $P + Q$, and therefore the accelerating force of the weight P will be represented by $\frac{P}{P + Q}$, the same as before; which ought to be the result, because in the former case the parts of the weight Q being uniformly disposed over the circumference, balance each other round the common centre of gravity S, and their weight therefore has no effect in accelerating or retarding the descent of the weight P.

In general, the accelerating force of the body P will be represented by the motive force divided by the inertia of the bodies moved; and therefore, if the body P be destitute of inertia, the accelerating force will be expressed simply by the fraction $\frac{P}{Q}$.

In what has been said above, we have supposed all the matter of the wheel to be uniformly disposed throughout the circumference of it; but supposing the wheel of uniform thickness and density, or in any other way constituted, before we enter upon the investigation of the law of acceleration, we must first determine the centre of gyration, or that point of it into which, if all the matter of the body be collected, the same angular velocity would be produced, which in a uniform circle is at $r\sqrt{\frac{1}{2}}$ distance from the centre, r being the radius of the wheel. All the matter of the wheel being supposed to be collected in that circumference whose radius is $r\sqrt{\frac{1}{2}}$, we shall have the moving force as rP, because the weight of the wheel, being uniformly distributed, will balance it on its centre, and therefore can neither tend to accelerate nor retard the descent of the body P. But the inertia of bodies being as the square of their distances from the axis of motion, we shall have $\frac{1}{2}r^2Q$ for the inertia of the wheel, and r^2P for the inertia of the weight P, and therefore $\frac{rP}{r^2(\frac{1}{2}Q + P)}$ for the accelerating force of the wheel, or of the lever AH; and as the acceleration of any point of a lever must, (besides the accelerating force with which the lever itself is made to revolve) be in proportion to the distance of that point from the axis of suspension, therefore the acceleration of the point P will be as $\frac{r^2P}{r^2(\frac{1}{2}Q + P)} = \frac{P}{\frac{1}{2}Q + P}$.



Let now ABC represent a wheel and axle, the diameters of which are given, W and P two given weights; the former, being fixed to the axle, is drawn up by the descent of the latter attached to the circumference of the wheel; and let it be required to determine the accelerative force of the descending body, the wheel and axle being supposed of no weight.

Put $BC = b$, $AC = a$, then from what has been before observed, the moving force will be as bP, and the retarding force as aW, and therefore the motive force will be expressed by $bP - aW$; also the inertia of the bodies will be as $b^2P + a^2W$, and hence the accelerating force of the lever will be as $\frac{bP - aW}{b^2P + a^2W}$; also, the acceleration of any point of the lever being as its distance from the axis, we have for the accelerative force of P, $\frac{bP - aW}{b^2P + a^2W} \times b = \frac{b^2P - abW}{b^2P + a^2W}$; and if P be a power of that kind which is not possessed of inertia, the expression becomes simply $\frac{b^2P - abW}{a^2W}$. See Atwood on the Rectilinear Motion and Rotation of Bodies, pa. 183, and Gregory's Mechanics, vol. 1, pa. 257; see also the articles GYRATION, OSCILLATION, CENTRE *of Spontaneous Rotation*, &c, in this Dictionary.

ROTATION, in Geometry, the circumvolution of a surface round an immoveable line, called the Axis of Rotation. By such rotation of planes, the figures of certain regular solids are formed or generated. Such as, a cylinder by the rotation of a rectangle, a cone by the rotation of a triangle, a sphere or globe by the rotation of a semicircle, &c.

The method of cubing solids that are generated by such rotation, is laid down by Demoivre, in his specimen of the use of the doctrine of fluxions, Philos. Trans. numb. 216; and indeed by most of the writers on fluxions. In every such solid, all the sections perpendicular to the axis are circles, and therefore the fluxion of the solid, at any section, is equal to that circle multiplied by the fluxion of the axis. So that, if x denote an absciss of that axis, and y an ordinate to it in the revolving plane, which will also be the radius of that circle; then, putting $n = 3{\cdot}1416$, the area of the circle is ny^2, and consequently the fluxion of the solid is $ny^2\dot{x}$; the fluent of which will be the content.

Such solid may also be expressed in terms of the generating plane and its centre of gravity; for the solid is always equal to the product arising from the generating plane multiplied by the path of its centre of gravity, or by the line described by that centre in the rotation of the plane. And this theorem is general, by whatever kind of motion the plane is moved, in describing a solid.

ROTATION, *Revolution*, in Astronomy. See REVOLUTION.

Diurnal ROTATION. See DIURNAL, and EARTH.

ROTONDO, or ROTUNDO, in Architecture, a popular term for any building that is round both withinside and without, whether it be a church, hall, a saloon, a vestibule, or the like.

ROUND, ROUNDNESS, ROTUNDITY, the property of a circle and sphere or globe, &c.

ROWNING (JOHN), an ingenious English mathematician and philosopher, was fellow of Magdalen College, Cambridge, and afterwards rector of Anderby in Lincolnshire, in the gift of that society. He was a constant attendant at the meetings of the Spalding Society, and was a man of a great philosophical turn of mind, though of a cheerful and sociable disposition. He had a good genius for mechanical contrivances in particular. In 1738 he printed at Cambridge, in 8vo, A Compendious System of Natural Philosophy, in 2 vols 8vo; a very ingenious work, which has gone through several editions. He had also two pieces inserted in the Philosophical Transactions, viz, 1. A Description of a Barometer wherein the Scale of Variation may be increased at pleasure; vol. 38, pa. 39. And 2. Direction for making a Machine for finding the Roots of Equations universally, with the Manner of using it; vol. 60, pa. 240.—Mr Rowning died at his lodgings in Carey-street near Lincoln's-Inn Fields, London, the latter end of November 1771, at 72 years of age.

Though a very ingenious and pleasant man, he had rather an unpromising and forbidding appearance: he was tall, stooping in the shoulders, and of a sallow and gloomy countenance.

ROYAL *Oak*, *Robur Carolinum*, in Astronomy, one of the new southern constellations, the stars of which, according to Sharp's catalogue, annexed to the Britannic, are 12.

RUDOLPHINE *Tables*, a set of astronomical tables that were published by the celebrated Kepler, and so called from the emperor Rudolph or Rudolphus.

RULE, *The Carpenter's*, a folding ruler generally used by carpenters and other artificers; and is otherwise called the sliding rule.—This instrument consists of two equal pieces of box-wood, each one foot in length, connected together by a folding joint. One side or face, of the rule, is divided into inches, and half-quarters, or eighths. On the same face also are several plane scales, divided into 12th parts by diagonal lines; which are used in planning dimensions that are taken in feet and inches. The edge of the rule is commonly divided decimally, or into 10ths; viz, each foot into 10 equal parts, and each of these into 10 parts again, or 100th parts of the foot: so that by means of this last scale, dimensions are taken in feet and tenths and hundredths, and multiplied together as common decimal numbers, which is the best way.

On the one part of the other face are four lines, marked A, B, C, D, the two middle ones B and C being on a slider, which runs in a groove made in the stock. The same numbers serve for both these two middle lines, the one line being above the numbers, and the other below them.—These four are logarithmic lines, and the three A, B, C, which are all equal to one another, are double lines, as they proceed twice over from 1 to 10. The lowest line D is a single one, proceeding from 4 to 40. It is also called the girt line, from its use in computing the contents of trees and timber: and on it are marked WG at 17·15, and AG at 18·95, the wine and ale gauge points, to make this instrument serve the purpose of a gauging-rule.—On the other part of this face is a table of the value of a load, or 50 cubic feet, of timber, at all prices, from 6d. to 2s. a foot.

When 1 at the beginning of any line is accounted only 1, then the 1 in the middle is 10, and the 10 at the end 100; but when the 1 at the beginning is accounted 10, then 1 in the middle is 100, and the 10 at the end 1000; and so on. All the smaller divisions being also altered proportionally.

By means of this rule all the usual operations of arithmetic may be easily and quickly performed, as multiplication, division, involution, evolution, finding mean proportionals, 3d and 4th proportionals, or the rule-of-three, &c. For all which, see my Mensuration, part 5, sect. 3.

RULES *of Philosophizing*. See PHILOSOPHIZING.

RULE, in Arithmetic, denotes a certain mode of operation with figures, to find sums or numbers unknown, and to facilitate computations.—Each rule in arithmetic has its particular name, according to the use for which it is intended. The first four, which serve as a foundation of the whole art, are called addition, subtraction, multiplication, and division.

From these arise numerous other rules, which are indeed only applications of these to particular purposes and occasions; as the Rule-of-three, or Golden Rule, or Rule of Proportion; also the Rules of Fellowship, Interest, Exchanges, Position, Progressions, &c, &c. For which, see each article severally.

RULE-*of-Three*, or *Rule of Proportion*, commonly called the *Golden Rule* from its great use, is a rule that teaches how to find a 4th proportional number to three others that are given.

As, if 3 degrees of the equator contain 208 miles, how many are contained in 360 degrees, or the whole circumference of the earth? The rule is this: State, or set the three given terms down in the form of the first three terms of a proportion, stating them proportionally, thus:

$$\begin{array}{rcccccccc} & \text{deg.} & & \text{mil.} & & \text{deg.} & & \text{miles.}\\ \text{as} & 3 & : & 208 & :: & 360 & : & 24960\\ & & & 360 & & & & \\ \cline{4-4} & & & 12480 & & & & \\ & & & 624 & & & & \\ \cline{3-4} & & 3) & 74880 & & & & \\ \cline{3-4} & & & 24960 & & & & \end{array}$$

Then multiply the 2d and 3d terms together, and divide the product by the 1st term, so shall the quotient be the 4th term in proportion, or the answer to the question, which in this example is 24960, or nearly 25,000 miles, for the circumference of the earth.

This rule is often considered as of two kinds, viz, *Direct*, and *Inverse*.

Rule-of-Three Direct, is that in which more requires more, or less requires less. As in this; if 3 men mow 21 yards of grass in a certain time, how much will 6 men mow, in the same time? Here more requires more, that is, 6 men, which are more than 3 men, will also perform more work, in the same time. Or if it were thus: if 6 men mow 42 yards, how much will 3 men mow in the same time? Here then less requires less, or 3 men will perform proportionally less work, in the same time. In both these cases then, the rule, or the proportion, is direct; and the stating must be

thus, as 3 : 6 : : 21 : 42,
or thus, as 6 : 3 : : 42 : 21.

Rule-of-Three Inverse, is when more requires less, or less requires more. As in this; if 3 men mow a certain quantity of grass in 14 hours, in how many hours will 6 men mow the like quantity? Here it is evident that 6 men, being more than 3, will perform the same work in less time, or fewer hours; hence then more requires less, and the rule or question is inverse, and must be stated by making the number of men change places, thus, as 6 : 3 : : 14 : 7 hours, the time in which 6 men will perform the work; still multiplying the 2d and 3d terms together, and dividing by the 1st.

For various abbreviations, and other particulars relating to these rules, see my books of arithmetic.

Double RULE-*of-Three*, or *Compound Proportion*, is where two statings are required to be wrought, or to be combined together, to find out the number sought.

This rule may be performed, either by working the two statings or proportions separately, making the result or 4th term of the 1st operation to be the 3d term of the last proportion; or else by reducing the two statings into one, by multiplying the two first terms together, and the two third terms together, and using the products as the 1st and 3d terms of the compound stating. As, if the question be this: If 100*l.* in 2 years yield 9*l.* interest, how much will 500*l.* yield in 6 years. Here, the two statings are,

$$\left.\begin{matrix}100\\2\end{matrix}\right\} : 9 :: \left\{\begin{matrix}500\\6\end{matrix}\right.$$

Then, to work the two statings separately,

as 100 : 9 : : 500 : 45*l.*
and 2 : 6 : : 45 : 135*l.*

so that 135l. is the interest or answer sought. But to work by one stating, it will be thus,

$$\begin{array}{llll} 100 & & 500 & \\ \underline{\ \ 2\ \ } & & \underline{\ \ \ 6\ \ } & \\ 200 : 9 :: & & \overline{3000} : 135l. & \text{the answer.} \end{array}$$

$$2{\cdot}00\ )\ 270{\cdot}00\ (\ 135l.$$

See the books of arithmetic for more particulars.

Central RULE. See CENTRAL *Rule.*

Parallel RULER. See PARALLEL *Ruler.*

RUMB. See RHUMB.

RUMB-*Line*, or *Loxodromie*. See RHUMB-*Line*.

RUSTIC, in Architecture, denotes a manner of building in imitation of simple or rude nature, rather than according to the rules of art.

RUSTIC *Quoin*. See QUOIN.

RUSTIC *Work* is where the stones in the face &c of a building, instead of being smooth, are hatched or picked with the point of an instrument.

Regular RUSTICS, are those in which the stones are chamfered off at the edges, and form angular or square recesses of about an inch deep at their jointings, or beds, and ends.

RUSTIC *Order*, is an order decorated with rustic quoins, or rustic work, &c.

RUTHERFORD (THOMAS, D. D.), an ingenious English philosopher, was the son of the Rev. Thomas Rutherford, rector of Papworth Everard in the county of Cambridge, who had made large selections for the history of that county. He was born the 13th of October 1712. He studied at Cambridge, and became fellow of St. John's college, and regius professor of divinity, in that university; afterwards rector of Shenfield in Essex, and of Barley in Hertfordshire, and archdeacon of Essex. He died October 5, 1771, at 59 years of age.

Dr. Rutherford, besides a number of theological writings, published, at Cambridge,

1. Ordo Institutionum Physicarum, 1743, in 4to.
2. A System of Natural Philosophy, in 2 vols, 4to, 1748. A work which has been much esteemed.
3. He communicated also to the Gentleman's Society at Spalding, a curious correction of Plutarch's description of the instrument used to renew the vestal fire, as relating to the triangle with which the instrument was formed. It was nothing more, it seems, than a concave speculum, whose principal focus, which collected the rays, is not in the centre of concavity, but at the distance of half a diameter from its surface. But some of the ancients thought otherwise, as appears from prop. 31 of Euclid's Catoptrics.

S.

S, IN books of Navigation, &c, denotes south. So also S. E. is south-east; S. W. south-west; and S. S. E. south-south-east, &c. See COMPASS.

SACROBOSCO. See HOLYWOOD.

SAGITTA, in Astronomy, the *Arrow* or *Dart*, a constellation of the northern hemisphere near the eagle, and one of the 48 old asterisms. The Greeks say that this constellation owes its origin to one of the arrows of Hercules, with which he killed the eagle or vulture that gnawed the liver of Prometheus. The stars in this constellation, in the catalogues of Ptolemy, Tycho, and Hevelius, are only 5, but in Flamsteed's they are extended to 18.

SAGITTA, in Geometry, is a term used by some writers for the absciss of a curve.

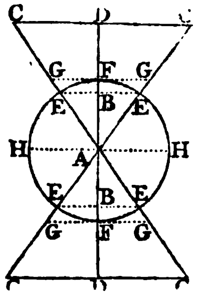
SAGITTA, in Trigonometry &c, is the same as the versed sine of an arch; being so called because it is like a dart or arrow, standing on the chord of the arch.

SAGITTARIUS, SAGITTARY, the *Archer*, one of the signs of the zodiac, being the 9th in order, and marked with the character ♐ of a dart or arrow. This constellation is drawn in the figure of a Centaur, or an animal half man and half horse, in the act of shooting an arrow from a bow. This figure the Greeks feign to be Crotus, the son of Eupheme, the nurse of the muses. Among more ancient nations the figure was probably meant for a hunter, to denote the hunting season, when the sun enters this sign. The stars in this constellation are, in Ptolemy's catalogue 31, in Tycho's 14, in Hevelius's 22, and in the Britannic catalogue 69.

SAILING, in a general sense, denotes the movement by which a vessel is wafted along the surface of the water, by the action of the wind upon her sails. Sailing is also used for the art or act of navigating; or of determining all the cases of a ship's motion, by means of sea-charts &c. These charts are constructed either on the supposition that the earth is a large extended flat surface, whence we obtain those that are called plane charts; or on the supposition that the earth is a sphere, whence are derived globular charts. Accordingly, sailing may be distinguished into two general kinds, viz, Plane Sailing, and Globular Sailing. Sometimes indeed a third sort is added, viz, Spheroidical Sailing, which proceeds on the supposition of the spheroidical figure of the earth.

Plane SAILING is that which is performed by means of a plane chart; in which case the meridians are considered as parallel lines, the parallels of latitude are at right angles to the meridians, the lengths of the degrees on the meridians, equator, and parallels of latitude, are every where equal. In plane sailing, the principal terms and circumstances made use of, are, course, distance, departure, difference of latitude, rhumb, &c; for as to longitude, that has no place in plane sailing, but belongs properly to globular or spherical sailing. The explanation of all which terms, are given under the respective articles.

If a ship sails either due north or south, she sails on a meridian, her distance and difference of latitude are the same, and she makes no departure: but where the ship sails either due east or west, she runs on a parallel of latitude, making no difference of latitude, and her departure and distance are the same. It may further be observed, that the departure and difference of latitude always make the legs of a right-angled triangle, whose hypothenuse is the distance the ship has sailed; and the angles are the course, its complement, and the right angle; therefore, among these four things, course, distance, difference of latitude, and departure, any two of them being given, the rest may be found by plane trigonometry. Thus, in the annexed figure, suppose the circle FHFH to represent the horizon of the place A, from whence a ship sails; AC the rhumb she sails upon, and C the place arrived at: then HH represents the parallel of latitude she sailed from, and CC the parallel of the latitude arrived in: so that



AD becomes the dif. of lat.
DC the departure,
AC the distance sailed,
∠DAC is the course, and
∠DCA the comp. of the course.

And all these particulars will be alike represented, whether the ship sails in the NE, or NW, or SE, or SW quarter of the horizon.

From the same figure, in which
AB or AF or AH represents the rad. of the tables,
EB the sine of the course,
AB the cosine of the course,
we may easily deduce all the proportions or canons, as they are usually called by mariners, that can arise in plane sailing; because the triangles ADC and ABE and AFG are evidently similar. These proportions are exhibited in the following table, which consists of 6 cases, according to the varieties of the two parts that can be given.

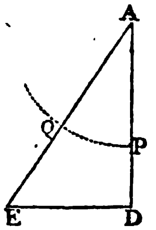
Case.	Given.	Required.	Solutions.
1	∠A and AC, i. e. course and distance.	AD and DC, i. e. difference of latitude and departure.	AE : AB : : AC : AD, i. e. rad. : s. course : : dist. : dif. lat. AE : EB : : AC : DC, i. e. rad. : cos. course : : dist. : depart.
2	∠A and AD, i. e. course and difference of latitude.	AC and DC, i. e. distance and departure.	AB : AE : : AD : AC, i. e. cos. cour. : rad. : : dif. lat. : dist. AB : BE : : AD : DC, i. e. cos. cour. : s. cour. : : dif. lat. : dep.
3	∠A and DC, i. e. course and departure.	AC and AD, i. e. distance and difference of latitude.	BE : AE : : DC : AC, i. e. s. cour. : rad. : : depart. : dist. BE : AB : : DC : AD, i. e. s. cour. : cos. cour. : : dep. : dif. lat.
4	AC and AD, i. e. distance and difference of latitude.	∠A and DC, i. e. course and departure.	AC : AD : : AE : AB, i. e. dist. : dif. lat. : : rad. : cos. course. AE : EB : : AC : DC, i. e. rad. : s. course : : dist. : depart.
5	AC and DC, i. e. distance and departure.	∠A and AD, i. e. course and difference of latitude.	AC : DC : : AE : EB, i. e. dist. : dep. : : rad. : s. course. AE : AB : : AC : AD, i. e. rad. : cos. cour. : : dist. : dif. lat.
6	AD and DC, i. e. difference of latitude and departure.	∠A and AC i. e. course and distance.	AD : DC : : AF : FG, i. e. dif. lat. : dep. : : rad. : tang. course. BE : AE : : DC : AC, i. e. s. cour. : rad. : : dep. : distance.

For the ready working of any single course, there is a table, called a Traverse Table, usually annexed to books of navigation; which is so contrived, that by finding the given course in it, and a distance not exceeding 100 or 120 miles, the usual extent of the table; then the difference of latitude and the departure are had by inspection. And the same table will serve for greater distances, by doubling, or trebling, or quadrupling, &c, or taking proportional parts. See TRAVERSE *Table*.

An ex. to the first case may suffice to show the method. Thus, a ship from the latitude 47° 30′ N, has sailed SW by S 98 miles; required the departure made, and the latitude arrived in.

1. *By the Traverse Table.* In the column of the course, viz. 3 points, against the distance 98, stands the number 54·45 miles for the departure, and 81·5 miles for the diff. of lat; which is 1° 21½′; and this being taken from the given. lat. 47° 30′, leaves 46° 8½′ for the lat. come to.

2. *By Construction.* Draw the meridian AD; and drawing an arc, with the chord of 60, make PQ or angle A equal to 3 points; through Q draw the distance AQE = 98 miles, and through E the departure ED perp. to AD. Then, by measuring, the diff. of lat. AD measures about 81½ miles, and the departure DE about 54½ miles.



3. *By Computation.*

First, as radius - - - - - - 10·00000
to sin. course 33° 45′ - - - 9·74474
so dist. 98 - - - - - - - - 1·99123
to depart. 54·45 - - - - - 1·73597

Again, as radius - - - - - 10·00000
to cos. course - - - - - - 9·91985
so dist. 98 - - - - - - - - 1·99123
to diff. of lat. 81·48 - - - 1·91108

4. *By Gunter's Scale.* The extent from radius, or 8 points, to 3 points, on the line of sine rhumbs, applied to the line of numbers, will reach from 98 to 54½ the departure. And the extent from 8 points to 5 points, of the rhumbs, reaches from 98 to 81½ on the line of numbers, for the difference of latitude.

And in like manner for other cases.

Traverse SAILING, or *Compound Courses,* is the uniting of several cases of plane sailing into one; as when a ship sails in a zigzag manner, certain distances upon several different courses, to find the whole difference of latitude and departure made good on all of them. This is done by working all the cases separately, by means of the traverse table, and constructing the figure as in the following example.

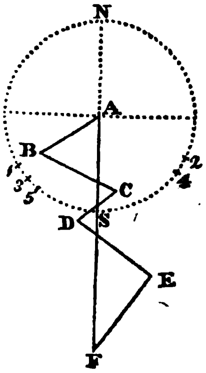
Ex. A ship sailing from a place in latitude 24° 32′ N, has run five different courses and distances, as set down in the 1st and 2d columns of the following traverse table; required her present latitude, with the departure, and the direct course and distance, between the place sailed from, and the place come to.

Traverse Table.

Courses.	Dist.	N.	S.	E.	W.
SWbS	45		25·0		37·4
ESE	50		19 1	46·2	
SW	30		21·2		21·2
SEbE	60		33 3	49·9	
SWbS¼W	63		50·6		37·5
			149·2	96·1	96·1

Here, by finding, in the general traverse table, the difference of latitude and departure answering to each course and distance, they are set down on the same lines with each course, and in their proper columns of northing, southing, easting, or westing, according to the quarter of the compass the ship sails in, at each course. As here, there is no northing, the differences of latitude are all southward, also two departures are eastward, and three are westward. Then, adding up the numbers in each column, the sum of the eastings appears to be exactly equal to the sum of the westings, consequently the ship is arrived in the same meridian, without making any departure; and the southings, or difference of latitude being 149·2 miles or minutes,

that is - - - - - - - - - - 2° 29′,
which taken from - - - - 24 32,
the latitude dep. from,
leaves - - - - - - - - - 22 3N,
the latitude come to.

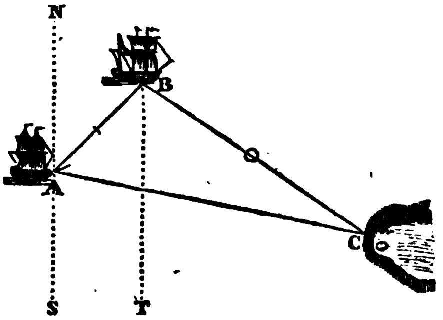


To Construct this Traverse. With the chord of 60 degrees describe the circle N S &c, and quarter it by the two perpendicular diameters; then from S set upon it the several courses, to the points marked 1, 2, 3, 4, 5, through which points draw lines from the centre A, or conceive them to be drawn; lastly, upon the first line lay off the first distance 45 from A to B, also draw BC = 50 and parallel to A 2, and CD = 30 parallel to A 3, and DE = 60 parallel to A 4, and EF = 63, parallel to A 5; then it is found that the point F falls exactly on the meridian NAF produced, thereby showing that there is no departure; and by measuring AF, it gives 149 miles for the difference of latitude.

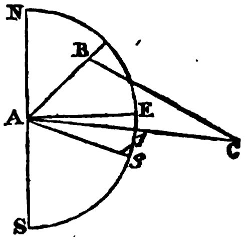
Oblique SAILING, is the resolution of certain cases and problems in sailing by oblique triangles, or in which oblique triangles are concerned. In this kind of sailing, it may be observed, that *to set an object,* means to observe what rhumb or point of the nautical compass is directed to it. And the *bearing* of an object is the rhumb on which it is seen; also the bearing of one place from another, is reckoned by the name of the rhumb passing through those two places.

In every figure relating to any case of plane sailing, the bearing of a line, not running from the centre of the circle or horizon, is found by drawing a line parallel to it, from the centre, and towards the same quarter.

Ex. A ship sailing at sea, observed a point of land to bear E by S; and then after sailing NE 12 miles, its bearing was found to be SE by E. Required the place of that point, and its distance from the ship at the last observation.



Construction. Draw the meridian line NAS, and, assuming A for the first place of the ship, draw AC the E by S rhumb, and AB the NE one, upon which lay off 12 miles from A to B; then draw the meridian BT parallel to NS, from which set off the SE by E point BC, and the point C will be the place of the land required; then the distance BC measures 26 miles.



Or thus, describe the circle NES &c, and draw NS and AE cutting each other at right angles in the centre A; which is supposed to be the place of the ship. Draw also AB the NE line, A 1 the E by S, and A 3, the SE by E line. Take AB = 12, and draw BC parallel to A 3, then produce A 1 till it cuts BC in C, so shall C be the point of land, which measures 26 miles as above.

By Computation. Here are given the side AB, and the two angles A and B, viz, the ∠A = 5 points or 56° 15′, and the ∠B = 9 points or 101° 15′; consequently the ∠C = 2 points or 22° 30′. Then, by plane trigonometry,

As sin. ∠C 22° 30′ - - - - 9·58284
To sin. ∠B 56 15 - - - - 9·91985

So is AB 12 miles - - - - 1·07918
To BC 26·073 miles - - - - 1·41619

SAILING *to Windward*, is working the ship towards that quarter of the compass from which the wind blows.

For rightly understanding this part of navigation, it will be necessary to explain the terms that occur in it, though most of them may be seen in their proper places in this work. When the wind is directly, or partly, against a ship's direct course for the place she is bound to, she reaches her port by a kind of zigzag or z like course; which is made by sailing with the wind first on one quarter, and then on the other.

In a ship, when you look towards the head,
Starboard denotes the right hand side;
Larboard, the left hand side;
Forwards, or afore, is towards the head;
Aft, or abaft, is towards the stern. The Beam means athwart or across the middle of the ship.

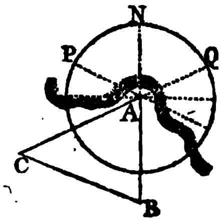
When a ship sails the same way that the wind blows, she is said to sail or run before the wind; and the wind is said to be right aft, or right astern; and her course is then 16 points, or the farthest possible, from the wind, that is, from the point the wind blows from.—When the ship sails with the wind blowing directly across her, she is said to have the wind on the beam; and her course is 8 points from the wind.—When the wind blows obliquely across the ship, the wind is said to be abaft the beam when it pursues her, or blows more on the hinder part, but before the beam when it meets or opposes her course, her course being more than 8 points from the wind in the former case, but less than 8 points in the latter case.—When a ship endeavours to sail towards that point of the compass from which the wind blows, she is said to sail on a wind, or to ply to windward.—And a vessel sailing as near as she can to the point from which the wind blows, is said to be close hauled. Most ships will lie within about 6 points of the wind; but sloops, and some other vessels, will lie much nearer. To know how near the wind a ship will lie; observe the course she goes on each tack, when she is close hauled; then half the number of points between the two courses, will show how near the wind the ship will lie.

The windward, or weather side, is that side of the ship on which the wind blows; and the other side is called the leeward, or lee-side.—Tacks and sheets are large ropes fastened to the lower corners of the fore and main sails; by which either of these corners is hauled fore or aft.—When a ship sails on a wind, the windward tacks are always hauled forwards, and the leeward sheets aft.—The starboard tacks are aboard, when the starboard side is to windward, and the larboard side to leeward. And the larboard tacks are aboard, when the larboard side is to windward, and the starboard to leeward.

The most common cases in turning to windward may be constructed by the following precepts. Having drawn a circle with the chord of 60°, to represent the horizon of the place, quarter it by drawing the meridian and parallel of latitude perpendicular to each other, and both through the centre; mark the place of the wind in the circumference; draw the rhumb passing through the place bound to, and lay on it, from the centre, the distance of that place. On each side of the wind lay off, in the circumference, the points or degrees showing how near the wind the ship can lie; and draw these rhumbs. Now the first course will be on one of these rhumbs, according to the tack the ship leads with. Draw a line through the place bound to, parallel to the other rhumb, and meeting the first; and this will show the course and distance on the other tack.

Ex. The wind being at north, and a ship bound to a port 25 miles directly to windward; beginning with the starboard tacks, what must be the course and distance on each of two tacks to reach the port?

Construction. Having drawn the circle &c, as above described, where A is the port, AP and AQ the two rhumbs, each within 6 points of AN; in NA produced take AB = 25 miles, then B is the place of the ship; draw BC parallel to AP, and meeting QA produced in C; so shall BC and CA be the distances on the two tacks; the former being WNW, and the latter ENE.



Computation.

Here $\angle B = NAP = 6$ points,
and $\angle A = NAQ = 6$ points,
theref. $\angle C = 4$ points.

So that all the angles are given, and the side AB, to find the other two sides AC and BC, which are equal to each other, because their opposite angles A and B are equal. Hence, as sin. C : AB : : sin. A : BC,
i. e. $s.\,45° : 25 :: s.\,67°\ 30' : 32\frac{2}{3}$ = BC or AC, the distance to be run on each tack.

SAILING *in Currents*, is the method of determining the true course and distance of a ship when her own motion is affected and combined with that of a current.

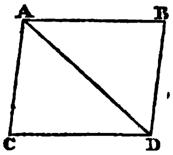
A current or tide is a progressive motion of the water, causing all floating bodies to move that way towards which the stream is directed.—The setting of a tide, or current, is that point of the compass towards which the waters run; and the drift of the current is the rate at which it runs per hour.

The drift and setting of the most remarkable tides and currents, are pretty well known; but for unknown currents, the usual way to find the drift and setting, is thus: Let three or four men take a boat a little way from the ship; and by a rope, fastened to the boat's stem, let down a heavy iron pot, or loaded kettle, into the sea, to the depth of 80 or 100 fathoms, when it can be done: by which means the boat will ride almost as steady as at anchor. Then heave the log, and the number of knots run out in half a minute will give the rate of the current, or the miles which it runs per hour; and the bearing of the log shows the setting of the current.

A body moving in a current, may be considered in three cases: viz,

1. Moving with the current, or the same way it sets.
2. Moving against it, or the contrary way it sets.
3. Moving obliquely to the current's motion.

In the 1st case, or when a ship sails with a current, its velocity will be equal to the sum of its proper motion, and the current's drift. But in the 2d case, or when a ship sails against a current, its velocity will be equal to the difference of her own motion and the drift of the current: so that if the current drives stronger than the wind, the ship will drive astern, or lose way. In the 3d case, when the current sets oblique to the course of the ship, her real course, or that made good, will be somewhere between that in which the ship endeavours to go, and the direction of the current; and indeed it will always be along the diagonal of a parallelogram, of which one side represents the ship's course set, and the other adjoining side the current's drift.



Let AB be the direction of the wind, or the direction of the vessel when acted on by wind only, and AB the distance the ship would run in any given time, by the action of this force; also let AC be the direction of the current, and the distance the ship would be carried, in the same time as above, by this force only. Draw BD parallel to AC, and CD parallel to AB, meeting BD in D, and join AD; then will AD represent the real course of the vessel when acted on by those two forces conjointly. For the wind neither accelerates nor retards the motion of the ship towards the line CD, the current therefore will bring her there in the same time as if the wind did not act. And in the same manner, the current will have no effect on the motion of the ship in the direction AB, the wind therefore will bring her to the line BD in the same time as if the current did not act. Therefore the ship at the end of that time, will be found in both those lines, that is, in their point of meeting D. Consequently the ship must have passed from A to D in the diagonal AD.

Hence, drawing the rhumbs for the proper course of the ship and of the current, and setting the distances off upon them, according to the quantity run by each in the given time; then forming a parallelogram of these two, and drawing its diagonal, this will be the real course and distance made good by the ship.

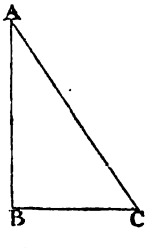
Ex. 1. A ship sails E. 5 miles an hour, in a tide setting the same way 4 miles an hour: required the ship's course, and the distance made good.

The ship's motion is 5m. E.
The current's motion is 4m. E.
Theref. the ship's run is 9m. E.

Ex. 2. A ship sails SSW. with a brisk gale, at the rate of 9 miles an hour, in a current setting NNE. 2 miles an hour: required the ship's course, and the distance made good.

The ship's motion is SSW. 9m.
The current's motion is NNE. 2m.
Theref. ship's true run is SSW. 7m.

Ex. 3. A ship running south at the rate of 5 miles an hour, in 10 hours crosses a current, which all that time was setting east at the rate of 3 miles an hour; required the ship's true course and distance sailed.



Here the ship is first supposed to be at A, her imaginary course is along the line AB, which is drawn south, and equal to 50 miles, the run in 10 hours; then draw BC east, and equal to 30 miles, the run of the current in 10 hours. Then the ship is found at C, and her true path is in the line AC = 58·31 her distance, and her course is the angle at A = 30° 58′ from the south towards the east.

Globular SAILING is the estimating the ship's motion and run on principles derived from the globular figure of the earth, viz. her course, distance, and difference of latitude and longitude.

The principles of this method are explained under the articles RHUMB-*line*, *Mercator's* CHART, and MERIDIONAL *Parts*; which see.

Globular Sailing, in the extensive sense here applied to the term, comprehends Parallel Sailing, Middle-latitude Sailing, and Mercator's Sailing; to which may be added Circular Sailing, or Great-circle Sailing. Of each of which it may be proper to give a brief account in this place.

Parallel SAILING is the art of finding what distance a ship should run due east or west, in sailing from the meridian of one place to that of another, in any parallel of latitude.

The computations in parallel sailing depend on the following rule:

As radius,
To cosine of the lat. of any parallel;
So are the miles of long. between any two meridians,
To the dist. of these meridians in that parallel.

Also, for any two latitudes,
As the cosine of one latitude,
Is to the cosine of another latitude;
So is a given meridional dist. in the 1st parallel,
To the like meridional dist. in the 2d parallel.

Hence, counting 60 nautical miles to each degree of longitude, or on the equator; then, by the first rule the number of miles in each degree on the other parallels, will be found as in the following table.

Table of Meridional Distances.

Lat.	Miles.	Lat.	Miles.	Lat.	Miles.
1	59·99	31	51·43	61	29·09
2	59·96	32	50·88	62	28·17
3	59·92	33	50·32	63	27·24
4	59·85	34	49·74	64	26·30
5	59.77	35	49·15	65	25·36
6	59·67	36	48·54	66	24·41
7	59·56	37	47·92	67	23·44
8	59·42	38	47·28	68	22·48
9	59·26	39	46·63	69	21·50
10	59·09	40	45·96	70	20·52
11	58·89	41	45·28	71	19·53
12	58·69	42	44·59	72	18·54
13	58·46	43	43·88	73	17·54
14	58·22	44	43·16	74	16·54
15	57·95	45	42·43	75	15·53
16	57·67	46	41·68	76	14·51
17	57·38	47	40·92	77	13·50
18	57·06	48	40·15	78	12·48
19	56·73	49	39·36	79	11·45
20	56·38	50	38·57	80	10·42
21	56·01	51	37·76	81	9·38
22	55·63	52	36·94	82	8·35
23	55·23	53	36·11	83	7·32
24	54·81	54	35·27	84	6·28
25	54·38	55	34·41	85	5·23
26	53·93	56	33·55	86	4·18
27	53·46	57	32·68	87	3·14
28	52·97	58	31·79	88	2·09
29	52·47	59	30·90	89	1·05
30	51·96	60	30·00	90	0·00

See another table of this kind, allowing $69\frac{1}{11}$ English miles to one degree, under the article DEGREE.

To find the meridional distance to any number of minutes between any of the whole degrees in the table, as for instance in the parallel of 48° 26′; take out the tabular distances for the two whole degrees between which the parallel or the odd minutes lie, as for 48° and 49°; subtract the one from the other, and take the proportional part of the remainder for the odd minutes, by multiplying it by those minutes, and dividing by 60; and lastly, subtract this proportional part from the greater tabular number. Thus,

Lat. 48° - - 40·15
Lat. 49. - - 39·36
As 60′ : 26′ : : 0·79 rem. : 0·34
26
474
158
60) 20·54
0·34 pro. part
Taken from - - 40·15 for lat. 48°
Leaves merid. dist. 39·81 for lat. 48° 26′.

And, in like manner, by the counter operation, to find what latitude answers to a given meridional distance. As, for ex. in what latitude 46·08 miles answer to a degree of longitude.

From 46·63 for 39°	from 46·63 for 39°
Take 45·96 for 40°	take 46·08 given number.
Then as 0·67 : 60′ : :	0·55 : 49′
	60
	67) 3300
	49′ pro. part.

Therefore the latitude sought is 39° 49′.

Ex. 3. Given the latitude and meridional distance; to find the corresponding difference of longitude. As, if a ship, in latitude 53° 36′, and longitude 10° 18′ east, sail due west 236 miles; required her present longitude.

Here, by the first rule,

As cos. lat. 53° 36′ comp. 0·22664
To radius - 90 00 - - 10·00000
So merid. dist. 236 m. - 2·37291
To diff. long. 397·7 - - 2·59955
Its 60th gives 6° 38′ w. diff. long.
Taken from 10 18 E. long. from
Leaves - - 3 40 E. long. come to.

By the table; the length of a degree on the parallel of 53° 36′ is 35·6.
Then as 35·6 : 60 : : 236 : 397·7, the diff. of long. the same as before.

Middle-latitude SAILING, is a method of resolving the cases of globular sailing by means of the middle latitude between the latitude departed from, and that come to. This method is not quite accurate, being only an approximation to the truth, and it makes use of the principles of plane sailing and parallel sailing conjointly.

The method is founded on the supposition that the departure is reckoned as a meridional distance in that latitude which is a middle parallel between the latitude sailed from, and that arrived at. But the method is not quite accurate, because the arithmetical mean, or half sum of the cosines of two distant latitudes, is not exactly the cosine of the middle latitude, or half the sum of those latitudes; nor is the departure between two places, on an oblique rhumb, equal to the meridional distance in the middle latitude; as is presumed in this method. Yet when the parallels are near the equator, or near to each other, in any latitude, the error is not considerable.

This method seems to have been invented on account of the easy manner in which the several cases may be resolved by the traverse table, and when a table of meridional parts is wanting. The computations depend on the following rules:

1. Take half the sum, or the arithmetical mean, of the two given latitudes, for the middle latitude. Then,

2. As cosine of middle latitude,
Is to the radius;
So is the departure,
To the diff. of longitude. And,

3. As cosine of middle latitude,
Is to the tangent of the course;
So is the difference of latitude,
To the difference of longitude.

Mercator's SAILING, is the art of resolving the several cases of globular sailing, by plane trigonometry, with the assistance of a table of meridional parts, or of logarithmic tangents. And the computations are performed by the following rules:

1. As meridional diff. lat.
To diff. of longitude;
So is the radius,
To tangent of the course.

2. As the proper diff. lat.
To the departure;
So is merid. diff. lat.
To diff. of longitude.

3. As diff. log. tang. half colatitudes,
To tang. of 51° 38′ 09″;
So is a given diff. longitude,
To tangent of the course.

The manner of working with the meridional parts and logarithmic tangents, will appear from the two following cases.

1. Given the latitudes of two places; to find their meridional difference of latitude.

By the Merid. Parts. When the places are both on the same side of the equator, take the difference of the meridional parts answering to each latitude; but when the places are on opposite sides of the equator, take the sum of the same parts, for the meridional difference of latitude sought.

By the Log. Tangents. In the former case, take the difference of the log. tangents of the half colatitudes; but in the latter case, take the sum of the same; then the said difference or sum divided by 12·63, will give the meridional difference of latitude sought.

2. Given the latitude of one place, and the meridional difference of latitude between that and another place; to find the latitude of this latter place.

By the Merid. Parts. When the places have like names, that is both north or both south, take the sum of the merid. parts of the given lat. and the given diff.; but take the difference between the same when they have unlike names; then the result, being found in the table of meridional parts, will give the latitude sought.

By the Log. Tangents. Multiply the given meridional diff. of lat. by 12·63; then in the former case subtract the product from the log. tangent of the given half colatitude, but in the latter case add them; then seek the degrees and minutes answering to the result among the log.

tangents, and these degrees, &c. doubled, will be the colatitude sought.

Circular SAILING, or *Great-circle* SAILING, is the art of finding what places a ship must go through, and what courses to steer, that her track may be in the arc of a great circle on the globe, or nearly so, passing through the place sailed from and the place bound to.

This method of sailing has been proposed, because the shortest distance between two places on the sphere, is an arc of a great circle intercepted between them, and not the spiral rhumb passing through them, unless when that rhumb coincides with a great circle, which can only be on a meridian, or on the equator.

The solutions of the cases in Mercator's sailing are performed by plane triangles, but in great-circle sailing they are resolved by means of spherical triangles. A great variety of cases might be here proposed, but those that are the most useful, and more commonly occur, pertain to the following problem.

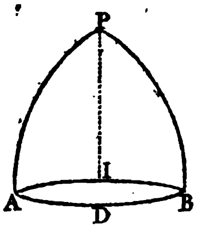
Problem I. Given the latitudes and longitudes of two places on the earth; to find their nearest distance on the surface, together with the angles of position from either place to the other.

This problem comprehends 6 cases.

Case 1. When the two places lie under the same meridian; then their difference of latitude will give their distance, and the position of one from the other will be directly north and south.

Case 2. When the two places lie under the equator; their distance is equal to their difference of longitude, and the angle of position is a right angle, or the course from one to the other is due east or west.

Case 3. When both places are in the same parallel of latitude. Ex. gr. The places both in 37° north, but the longitude of the one 25° west, and of the other 76° 23' west.



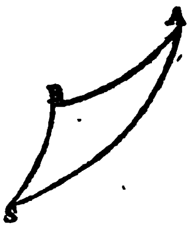
Let P denote the north pole, and A and B the two places on the same parallel BDA, also BIA their distance asunder, or the arc of a great circle passing through them. Then is the angle A or B that of position, and the angle BPA = 51° 23' the difference of longitude, and the side PA or PB = 53° the colatitude.

Draw PI perp. to AB, or bisecting the angle at P. Then in the triangle API, right-angled at I, are given the hypothenuse AP = 53°, and the angle APT = 25° 41' 30''; to find the angle of position A or B = 73° 51'; and the half distance AI = 20° 15'½; this doubled gives 40° 31' for the whole distance AB, or 2431 nautical miles, which is 31 miles less than the distance along ADB, or by parallel sailing.

Case 4. When one place has latitude, and the other has none, or is under the equator. For example, suppose the Island of St. Thomas, lat. 0°, and long. 1° 0' east, and Port St. Julian, in lat. 48° 51' south, and long. 65° 10' west.

Port St. Julian, lat. 48° 51' S. - long. 65° 10' W.
Isle St. Thomas - 0 00 - - - - 1 00 E.
Julian's colat. - 41 09 Diff. long. 66 10

Hence, if S denote the south pole, A the Isle St. Thomas at the equator, and B St. Julian; then in the triangle are given SA a quadrant or 90°, BS = 41° 9' the colat. of St. Julian, and the ∠S = 66° 10' the dif. of longitude; to find AB = 74° 35' = 4475 miles, which is less by 57 miles than th distance found by Mercator's sailing; also the angle of position at A = 51° 22', and the angle of position B = 108° 24.

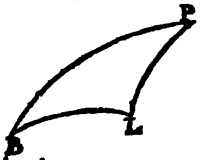


Case 5. When the two given places are both on the same side of the equator; for example the Lizard, and the island of Bermudas.

The Lizard, lat. 49° 57' N. - long. 5° 21' W.
Bermudas, 32 35 N. - - 63 32 W.
58 11

Here, if P be the north pole, L the Lizard, and B Bermudas; there are given,

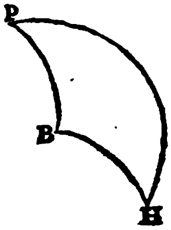
PL = 40° 03' colat. of the Lizard,
PB = 57 25 colat. of Bermudas,
∠P = 58 11 diff. of longitude; to find BL = 45° 44' = 2744 miles the distance, and
∠ of position B = 49° 27', also
∠ of position L = 90° 31'.



Case 6. When the places lie on different sides of the equator; as suppose St. Helena and Bermudas. Here

PB = 57° 25' polar dist. Bermudas,
PH = 105 55 polar dist. St. Helena,
∠P = 57 43 diff. long.

To find BH = 73° 26' = 4406 miles, the distance, also the angle of position H = 48° 0', and the angle of position B = 121° 59'.



From the solutions of the foregoing cases it appears, that to sail on the arc of a great circle, the ship must continually alter her course; but as this is a difficulty too great to be admitted into the practice of navigation, it has been thought sufficiently exact to employ a kind of approximation, that is, by a method which nearly approaches to the sailing on a great circle: namely, on this principle, that in small arcs, the difference between the arc and its chord or tangent is so small, that they may be taken for each other in any nautical operations: and accordingly it is supposed that the great circles on the earth are made up of short right lines, each of which is a segment of a rhumb line. On this supposition the solution of the following problem is deduced.

Problem II. Having given the latitudes and longitudes of the places sailed from and bound to; to find the successive latitudes on the arc of a great circle in those places where the alteration in longitude shall be a given quantity; together with the courses and distances between those places.

1. Find the angle of position at each place, and their distance, by one of the preceding cases.

2. Find the greatest latitude the great circle runs through, i. e. find the perpendicular from the pole to that circle; and also find the several angles at the pole, made by the given alterations of longitude between this perpendicular and the successive meridians come to.

3. With this perpendicular and the polar angles severally, find as many corresponding latitudes, by saying, as radius : tang. greatest lat. : : cos. 1st polar angle : tang. 1st lat. : : cos. 2d polar angle : tang. of 2d lat. &c.

4. Having now the several latitudes passed through, and the difference of longitude between each, then by Merca-

tor's sailing find the courses and distances between those latitudes. And these are the several courses and distances the ship must run, to keep nearly on the arc of a great circle.

The smaller the alterations in longitude are taken, the nearer will this method approach to the truth; but it is sufficient to compute to every 5 degrees of difference of longitude; as the length of an arc of 5 degrees differs from its chord, or tangent, only by 0·002.

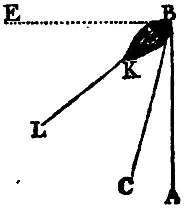
The track of a ship, when thus directed nearly in the arc of a great circle, may be delineated on the Mercator's chart, by marking on it, by means of the latitudes and longitudes, the successive places where the ship is to alter her course; then those places or points, being joined by right lines, will show the path along which the ship is to sail, under the proposed circumstances.—On the subject of these articles, see Robertson's Elements of Navigation, vol. 2.

Spheroidical SAILING, is computing the cases of navigation on the supposition or principles of the spheroidical figure of the earth. See Robertson's Navigation, vol. 2, b. 8, sect. 8.

Sailing, *in a more confined sense*, is the art of conducting a ship from place to place, by the working or handling of her sails and rudder.—To bring sailing to certain rules, M. Renau computes the force of the water, against the ship's rudder, stem, and side; and the force of the wind against her sails. In order to this, he first considers all fluid bodies, such as the air, water, &c, to be composed of little particles, which when they act upon any surface, all move parallel to one another, or strike against the surface after the same manner. Secondly, that the motion of any body, with regard to the surface it strikes, must be either perpendicular, parallel, or oblique. From these principles he computes, that the force of the air or water, striking perpendicularly upon a sail or rudder, is to the force of the same striking obliquely, in the duplicate ratio of radius to the sine of the angle of incidence: and consequently that all oblique forces of the wind against the sails, or of the water against the rudder, will be to each other in the duplicate ratio of the sines of the angles of incidence.—Such are the conclusions from theory; but it is very different in real practice, or experiments, as appears from the tables inserted in the article RESISTANCE.

Further, when the different degrees of velocity are considered, it is also found that the forces are as the squares of the velocities of the moving air or water nearly; that is, a wind that blows twice as swift, as another, will act with 4 times the force upon the sail; and when 3 times as swift, 9 times the force, &c. And it being also indifferent, whether we consider the motion of a solid in a fluid at rest, or of the fluid against the solid at rest; therefore, the reciprocal impressions being always the same, if a solid be moved with different velocities in the same fluid matter, as water, the different resistances which it will receive from that water, will be in the same proportion as the squares of the velocities of the moving body.

He then applies these principles to the motions of a ship, both forwards and sideways, through the water, when the wind, with certain velocities, strikes the sails in various positions. After which, the author proceeds to demonstrate, that the best position or situation of a ship, so that she may make the least lee-way, or side motion, but go to windward as much as possible, is this: that, let the sail have what situation it will, the ship must be always in a line bisecting the complement of the wind's angle of incidence on the sail. That is, supposing the sail in the position BC, and the wind blowing from A to B, and consequently the angle of the wind's incidence on the sail is ABC, the complement of which is CBE: then must the ship be put in the position BK, or move in the line BL, bisecting the ∠CBE.



He shows further, that the angle which the sail ought to make with the wind, i. e. the angle ABC, ought to be but 24 degrees; that being the most advantageous situation for working to windward.

To this might be added many curious particulars from Borelli de Vi Percussionis, concerning the different directions given to a vessel by the rudder, when sailing with a wind, or floating without sails in a current: in the former case, the head of the ship always coming to the rudder, and in the latter always flying off from it; as also from Euler, Bouguer, and Juan, who have all written learnedly on this subject.

SALIANT, in Fortification, is said of an angle that projects its point outwards; in opposition to a re-entering angle, which has its point turned inwards. Instances of both kinds of these occur in tenailles and star-works.

SALON, or SALOON, in Architecture, a grand, lofty, spacious kind of hall, vaulted at top, and usually comprehending two stories, with two ranges of windows: and may be either square, round, oval, or octagonal.

SAP, or SAPP, in Building, as to sap a wall, &c, is to dig out the ground from beneath it, so as to bring it down all at once for want of support.

SAP, in the Military Art, denotes a work carried on under cover of gabions and fascines on the flank, and mantlets or stuffed gabions on the front, to gain the descent of a ditch, or the like. It is performed by digging a deep trench, descending by steps from top to bottom, under a corridor, carrying it as far as the bottom of the ditch, when that is dry; or as far as the surface of the water, when wet.

SAROS, in Chronology, a period of 223 lunar months. The etymology of the word is said to be Chaldean, signifying restitution, or return of eclipses; that is, conjunctions of the sun and moon in nearly the same place of the ecliptic. The Saros was a cycle like to that of Meto.

SARRASIN, or SARRAZIN, in Fortification, a kind of port cullis, otherwise called a herse, which is hung with ropes over the gate of a town or fortress, to be let fall in case of a surprise.

SATELLITES, in Astronomy, are certain secondary planets, moving round the other planets, as the moon does round the earth. They are so called because always found attending them, from rising to setting, and making the tour about the sun together with them. The words moon and satellite are sometimes used indifferently: thus we say, either Jupiter's moons, or Jupiter's satellites; but usually we restrain the term moon to the earth's attendant, and apply that of satellite to the little moons discovered about Jupiter, Saturn, and Uranus, by the assistance of the telescope, which is necessary to render them visible.

The satellites move about their primary planets, as a centre, by the same laws as those primary ones do round their centre the sun; viz, in such a manner that, in the satellites of the same planet, the squares of the periodic times are proportional to the cubes of their distances from the primary planet. For the physical cause of their motions, see Gravity. See also Planets.

We know not of any satellites besides those above mentioned; what other discoveries may be made by further improvements in telescopes, time only can bring to light.

Satellites *of Jupiter*. There are 4 little moons, or satellites now known to perform their evolutions about Jupiter, as that planet does about the sun.

Simon Marius, mathematician of the elector of Brandenburg, about the end of November 1609, is said to have observed three little stars moving round Jupiter's body, and proceeding along with him; and in January 1610, he found a 4th. In January 1610 Galileo also observed the same in Italy, and in the same year published his observations. And indeed Montucla gives the honour of the first discovery entirely to Galileo. These satellites were also observed in the same month of January 1710, by Thomas Harriot, the celebrated author of a work on algebra, and who made constant observations of them, from that time till the 26th of February 1612; as appears by his curious astronomical papers, lately discovered by Dr. Zach, at the seat of the earl of Egremont, at Petworth in Sussex.

When Jupiter is in a line with any of his satellites and the sun, the satellite disappears, being then eclipsed, or involved in his shadow.—When the satellite goes behind the body of Jupiter, with respect to an observer on the earth, it is then said to be occulted, being hid from our sight by his body, whether in his shadow or not.—And when the satellite comes into a position between Jupiter and the sun, it casts a shadow upon the face of that planet, which we see as an obscure round spot.—Lastly, when the satellite is in a line between Jupiter and us, it is said to transit the disc of the planet, upon which it appears as a round black spot.

The periods or revolutions of Jupiter's satellites, are found out from their conjunctions with that planet; after the same manner, as those of the primary planets are discovered from their oppositions to the sun. And their distances from the body of Jupiter are measured by a micrometer, and estimated in semidiameters of that planet, and thence in miles. By the latest and most exact observations, the periodical times and distances of these satellites, and the angles under which their orbits are seen from the earth, at its mean distance from Jupiter, are as below:

Satellites *of* Jupiter.

Satellites.	Periodic Time.	Distances in		Angles of Orbit.
		Semidiameters.	Miles.	
1	$1^d\ 18^h\ 27^m\ 34^s$	$5\frac{2}{3}$	266,000	3′ 55″
2	3 13 13 42	$9\frac{1}{59}$	423,000	6 14
3	7 3 42 33	$14\frac{1}{13}$	676,000	9 58
4	16 16 31 50	$25\frac{3}{10}$	1,189,000	17 30

The eclipses of the satellites, especially of those of Jupiter, are of very great use in astronomy. First, in determining pretty exactly the distance of Jupiter from the earth. A second advantage still more considerable, which is drawn from these eclipses, is the proof which they give of the progressive motion of light. It is demonstrated by these eclipses, that light does not come to us in an instant, as the Cartesians pretended, though its motion is extremely rapid. For if the motion of light were infinite, or came to us in an instant, it is evident that we should see the commencement of an eclipse of a satellite at the same moment, at whatever distance we might be from it; but, on the contrary, if light move progressively, then it is as evident, that the farther we are from a planet, the later we shall be in seeing the moment of its eclipse, because the light will take up a longer time in arriving at us; and so it is found in fact to happen, the eclipses of these satellites appearing always later and later than the true computed times, as the earth removes farther and farther from the planet. When Jupiter and the earth are nearest to each other; that is, when they are in conjunction on the same side of the sun; then the eclipses are observed to happen about $7\frac{1}{2}$ minutes before the computed time for the mean distance; and when those two planets are at their greatest distance, being then in opposition, the eclipses happen about $7\frac{1}{2}$ minutes after the time predicted by calculation. Now the difference between the least and greatest distance being equal to the diameter of the earth's orbit, it therefore follows that light takes up a quarter of an hour in travelling across the orbit of the earth, or near 8 minutes in passing from the sun to the earth; which gives about 12 millions of miles per minute, or 200,000 miles per second, for the velocity of light. A discovery that was first made by M. Roemer.

The third and greatest advantage derived from the eclipses of Jupiter's satellites, (and which was hinted at by Galileo on the first discovery of them,) is the knowledge of the longitudes of places on the earth. Suppose two observers of an eclipse, the one, for example, at London, the other at the Canaries; it is certain that the eclipse will appear at the same moment to both observers; but as they are situated under different meridians, they count different hours, being perhaps 9 o'clock to the one, when it is only 8 to the other; by which observations of the true time of the eclipse, on communication, they find the difference of their longitude to be one hour in time, which answers to 15 degrees of longitude.

To the above we may also add, that this discovery had a very considerable influence in eradicating the errors of the ancient astronomers, and consequently in firmly establishing the Copernican system; as a supporter of which, the venerable discoverer, Galileo, was, at this time, smarting under the recollection of the condemnation which had been passed upon him, by that most detestable of all tyrannies, the inquisition.

Satellites *of Saturn*, are 7 in number revolving about him. One of them, which till lately was reckoned the 4th in order from Saturn, was discovered by Huygens, the 25th of March 1655, by means of a telescope 12 feet long, and the 1st, 2d, 3d, and 5th, at different times, by Cassini; viz, the 5th in October 1671, by a telescope of 17 feet; the 3d in December 1672, by a telescope of Campani's, 35 feet long; and the first and second in March 1684, by help of Campani's Glasses, of 100 and 136 feet. Finally, the 6th and 7th satellites have lately been discovered by Dr. Herschel, with his 40 feet reflecting telescope, viz, the 6th on the 19th of August 1787, and the 7th on the 17th of September 1788. These two he has called the 6th and 7th satellites, though they are nearer to the planet Saturn than any of the former five, that the names or numbers of these might not be mistaken or confounded, with regard to former observations of them.

Moreover, the great distance between the 4th and 5th satellite gave occasion to Huygens to suspect that there might be some intermediate one, or else that the 5th might have some other satellite moving round it, as its centre. Dr. Halley, in the Philos. Trans. (No. 145,) gives a correction of the theory of the motions of the 4th or Huygenian satellite. Its true period he makes 11d 22h 41m 6s.

The periodical revolutions, and distances of these satellites from the body of Saturn, expressed in semidiameters of that planet, and in miles, are as follow.

SATELLITES *of* SATURN.

Satellites.	Periods.	Distances in		Diameter of Orbit.
		Semidiameters.	Miles.	
1	$1^d\ 21^h\ 18'\ 26''$	$4\frac{3}{8}$	170,000	1′ 27″
2	2 17 44 51	$5\frac{1}{2}$	217,000	1 52
3	4 12 25 11	8	303,000	2 36
4	15 22 41 14	18	704,000	6 18
5	79 7 54 37	54	2,050,000	17 4
6	1 8 53 9	$3\frac{5}{6}$	135,000	1 14
7	0 22 37 30	2[illegible]	107,000	0 57

The first four describe ellipses like to those of the ring, and are in the same plane. Their inclination to the ecliptic is from 30 to 31 degrees. The 5th describes an orbit inclined from 17 to 18 degrees with the orbit of Saturn; his plane lying between the ecliptic and those of the other satellites, &c. Dr. Herschel observes that the 5th satellite turns once round its axis exactly in the time in which it revolves about the planet Saturn; in which respect it resembles our moon, which does the same thing. And he makes the angle of its distance from Saturn, at his mean distance, 17′ 2″. Philos. Trans. 1792, pa. 22. See a long account of observations of these satellites, with tables of their mean motions, by Dr. Herschel, Philos. Trans. 1790, pa. 427 &c.

SATELLITES *of Herschel*, or *Uranus*, are 6 little moons that revolve about him, like those of Jupiter and Saturn. These satellites were discovered by Dr. Herschel, who gave an account of them in the Philos. Trans, from which it appears that their synodical periods, and angular distances from their primary, are as follow:

Satellite.	Sidereal Revolution.	Mean Dist.
1	$5^d\ 21^h\ 25^m\ 21^s$	0′ 25·5″
2	8 16 57 48	0 33·0
3	10 23 3 59	0 38·6
4	13 10 56 30	0 44·2
5	38 1 48 0	0 88·4
6	107 16 39 56	0 176·8

The orbits of these satellites are nearly perpendicular to the ecliptic, and contrary to the order of the signs. In magnitude they are probably not less than those of Jupiter.

SATELLITE *of Venus*. Cassini thought he saw one, and Mr. Short and other astronomers have suspected the same thing. (Hist. de l'Acad. 1741, Philos. Trans. No. 459.) But the many fruitless searches that have been since made to discover it, leave room to suspect that it has been only an optical illusion, formed by the glasses of telescopes; as appears to be the opinion of F. Hell, at the end of his Ephemeris for 1766, and Boscovich, in his 5th Optical Dissertation.—Neither has it been discovered that either of the other planets have any satellites revolving about them. It is remarkable that our moon, and some of the other satellites, as far as they have been observed, keep always the same face or side towards their respective primaries; around which they appear to be moved as a stone is whirled round in a sling.

SATURDAY, the 7th or last day of the week, so called, as supposed, from the idol Seater, worshipped on this day by the ancient Saxons, and thought to be the same as the Saturn of the Latins. In astronomy, every day of the week is denoted by some one of the planets, and this day is marked with the planet ♄ Saturn. Saturday answers to the Jewish sabbath.

SATURN, one of the primary planets, being the 10th in order of distance from the sun, and the outermost of all, except the planet Herschel, is marked with the character ♄, denoting an old man supporting himself with a staff, representing the ancient god Saturn.

Saturn shines with but a feeble light, partly on account of his great distance, and partly from his dull red colour. This planet is perhaps one of the most engaging objects that astronomy offers to our view; it is surrounded with a double ring, one without the other, and beyond these by 7 satellites, most of them in the plane of the rings; the rings and planets being all dark and dense bodies, like Saturn himself, these bodies casting their shadows mutually upon each other; though the reflected light of the rings is usually brighter than that of the planet itself.

Saturn has also certain obscure zones, or belts, appearing at times across his disc, like those of Jupiter, which are changeable, and are probably obscurations in his atmosphere. Dr. Herschel, Philos. Trans. 1790, shows that Saturn has a dense atmosphere; that he revolves about an axis, which is perpendicular to the plane of the rings; that his figure is, like the other planets, the oblate spheroid, being flatted at the poles, the polar diameter being to the equatorial one as 10 to 11; that his ring has a motion of rotation in its own plane, its axis of motion being the same as that of Saturn himself, and its periodical time equal to 10h 32m $15^s{\cdot}4$. See also RING, and SATELLITE.

Concerning the discovery of the ring and figure of Saturn, we find that Galileo first perceived that his figure is not round: but Huygens showed, in his Systema Saturniana 1659, that this was owing to the positions of his ring; for his spheroidical form could only be seen by Herschel's telescope; though indeed Cassini, in an observation made June 19, 1692, saw the oval figure of Saturn's shadow upon his ring.

Mr. Bugge determines (Philos. Trans. 1787, pa. 42) the heliocentric longitude of Saturn's descending node to be $9^s\ 21^\circ\ 5'\ 8''\frac{1}{2}$; and that the planet was in that node August 21, 1784, at $18^h\ 20^m\ 10^s$ time, at Copenhagen.

The annual period of Saturn about the sun, is 10759 days 7 hours, or almost 30 years; and his diameter is about 67000 miles, or near $8\frac{1}{2}$ times the diameter of the earth; also his distance is about $9\frac{1}{2}$ times that of the earth. Hence some have concluded that his light and heat are entirely unfit for rational inhabitants. But that their light is not so weak as we imagine, is evident from its brightness in the night-time. Besides, allowing the sun's light to be 45000 times as strong, with respect to us, as the light of the moon when full, the sun will afford 500 times as much light to Saturn as the full moon does to us, and 1600 times as much to Jupiter. So that these two planets, even without any moon, would be much more enlightened than we at first imagine; and by having so many, they may be very comfortable places of residence. Their heat, so far as it depends on the force of the sun's rays, is certainly much less than ours; to which no doubt the bodies of their inhabitants are as well adapted as ours are to the seasons we enjoy. And if it be considered that Jupiter never has any winter, even at his poles, which probably is also the case with Saturn, the cold cannot be so intense on these two planets as is generally imagined. To this may be added, that there may be something in the nature of their soil warmer than in that of our earth; and we find that all our heat does not depend on the rays of the sun; for if it did, we should always have the same months equally hot or cold at their annual return, which is very far from being the case.

See the articles PLANET, PERIOD, RING, SATELLITE.

SAUCISSE, in Artillery, a long train of powder inclosed in a roll or pipe of pitched cloth, and sometimes of leather, about 2 inches in diameter; serving to set fire to mines or caissons. It is usually placed in a wooden pipe, called an auget, to prevent its growing damp.

SAUCISSON, in Fortification, a kind of faggot, made of thick branches of trees, or of the trunks of shrubs, bound together, for the purpose of covering the men, and to serve as epaulements; and also to repair breaches, stop passages, make traverses over a wet ditch, &c. The saucisson differs from the fascine, which is only made of small branches; and by its being bound at both ends, and in the middle.

SAVILLE (Sir HENRY), a very learned Englishman, the second son of Henry Saville, esq. was born at Bradley, near Halifax, in Yorkshire, November the 30th, 1549. He was entered of Merton-college, Oxford, in 1561, where he took the degree B. A., and was chosen fellow. He became master of arts in 1570, having read for that degree on the Almagest of Ptolemy, which procured him the reputation of a man eminently skilled in mathematics and the Greek language; in the former of which he gratuitously read a public lecture in the university for some time.

In 1578 he travelled into France and other countries; where, diligently improving himself in all useful learning, in languages, and the knowledge of the world, he became a most accomplished gentleman. At his return, he was made tutor in the Greek tongue to queen Elizabeth, who had a great esteem for him.

In 1585 he was made warden of Merton-college, which he governed six-and-thirty years with great honour, and improved it by all the means in his power.—In 1596 he was chosen provost of Eton-college; which he filled with many learned men.—James the First, on his accession to the crown of England, expressed a great regard for him, and would have preferred him either in church or state; but Saville declined these offers, and only accepted the ceremony of knighthood from the king at Windsor in 1604. His only son Henry dying about that time, he thenceforth devoted his fortune to the promoting of learning. Among other things, in 1619, he founded, in the university of Oxford, two lectures, or professorships, one in geometry, the other in astronomy; which he endowed with a salary of 160l. a year each, besides a legacy of 600l. to purchase more lands for the same use. He also furnished a library with mathematical books near the mathematical school, for the use of his professors; and gave 100l. to the mathematical chest of his own appointing: adding afterwards a legacy of 40l. a year to the same chest, to the university, and to his professors jointly. He likewise gave 120l. towards the new building of the schools, besides several rare manuscripts and printed books to the Bodleian library; and a good quantity of Greek types to the printing-press at Oxford.

After a life thus spent in the encouragement and promotion of science and literature in general, he died at Eton-college the 19th of February 1622, in the 73d year of his age, and was buried in the chapel there. On this occasion, the university of Oxford paid him the greatest honours, by having a public oration and verses made in his praise, which were published soon after in 4to, under the title of Ultima Linea Savilii.

As to the character of Saville, the highest encomiums are bestowed on him by all the learned of his time: by Casaubon, Mercerus, Meibomius, Joseph Scaliger, and especially the learned bishop Montague; who, in his Diatribæ upon Selden's History of Tythes, styles him, "that magazine of learning, whose memory shall be honourable amongst not only the learned, but the righteous for ever."

Several noble instances of his munificence to the republic of letters have already been mentioned; in the account of his publications many more, and even greater, will appear. These are,

1. Four Books of the Histories of Cornolius Tacitus, and the Life of Agricola; with Notes upon them, in folio, dedicated to Queen Elizabeth, 1581.

2. A View of certain Military Matters, or Commentaries concerning Roman Warfare, 1598.

3. Rerum Anglicarum Scriptores post Bedam, &c. 1596. This is a collection of the best writers of our English history; to which he added chronological tables at the end, from Julius Cæsar to William the Conqueror.

4. The Works of St. Chrysostom, in Greek, in 8 vols. folio, 1613. This is a very fine edition, and composed with great cost and labour. In the preface he says, "that having himself visited, about 12 years before, all the public and private libraries in Britain, and copied out thence whatever he thought useful to this design, he then sent some learned men into France, Germany, Italy, and the East, to transcribe such parts as he had not already, and to collate the others with the best manuscripts." At the same time, he makes his acknowledgments to several eminent men for their assistance; as Thuanus, Velserus, Schottus, Casaubon, Ducæus, Gruter, Hoeschelius, &c. In the 8th volume are inserted Sir Henry Saville's own notes, with those of other learned men. The whole charge of this edition, including the several sums paid to learned men, at home and abroad, employed in finding out, transcribing, and collating the best manuscripts, is said to have amounted to no less than 8000l. A still more sumptuous and voluminous edition was afterwards printed at Paris, in 13 folio volumes, by the Benedictines and the learned Montfaucon, the 1st vol. in 1718, and the last in 1738.

5. In 1618 he published a Latin work, written by Thomas Bradwardin, abp. of Canterbury, against Pelagius, intitled, De Causa Dei contra Pelagium, et de vir-

tute causarum; to which he prefixed the life of Bradwardin.

6. In 1621 he published a collection of his own Mathematical Lectures on Euclid's Elements; in 4to.

7. Oratio coram Elizabetha Regina Oxoniæ habita, anno 1592. Printed at Oxford in 1658, in 4to.

8. He translated into Latin king James's Apology for the Oath of Allegiance. He also left several manuscripts behind him, written by order of king James; all which are in the Bodleian library. He wrote notes also on the margin of many books in his library, particularly Eusebius's Ecclesiastical History; which were afterwards used by Valesius, in his edition of that work in 1659.—Four of his letters to Camden are published by Smith, among Camden's Letters, 1691, 4to.

Sir Henry Saville had a younger brother, **THOMAS SAVILLE**, who was admitted probationer fellow of Merton-college, Oxford, in 1580. He afterwards travelled abroad into several countries. On his return he was chosen fellow of Eton-college; but he died at London in 1593. Thomas Saville was also a man of great learning, and an intimate friend of Camden; among whose letters, just mentioned, there are 15 of Mr. Saville's to him.

SAUNDERSON (Dr. NICHOLAS), an eminent professor of mathematics in the university of Cambridge, and a fellow of the Royal Society, was born at Thurlston in Yorkshire in 1682. When he was but twelve months old, he lost not only his eye-sight, but his very eye-balls, by the small-pox; so that he could retain no more ideas of vision than if he had been born blind. At an early age, however, being of very promising parts, he was sent to the free-school at Penniston, and there laid the foundation of that knowledge of Greek and Latin languages, which he afterwards improved so far, by his own application to the classic authors, as to hear the works of Euclid, Archimedes, and Diophantus read in their original Greek.

Having acquired a grammatical education, his father, who was in the excise, instructed him in the common rules of arithmetic. And here it was that his excellent mathematical genius first appeared: for he very soon became able to work the common questions, to make long calculations by the strength of his memory, and to form new rules to himself for the better resolving of such problems as are often proposed to learners as trials of skill.

At the age of 18, our author was introduced to the acquaintance of Richard West, of Underbank, Esq. a lover of mathematics, who, observing Mr. Saunderson's uncommon capacity, took the pains to instruct him in the principles of algebra and geometry, and gave him every encouragement in his power to the prosecution of these studies. Soon after this he became acquainted also with Dr. Nettleton, who took the same pains with him. And it was to these two gentlemen that Mr. Saunderson owed his first institution in the mathematical sciences: they furnished him with books, and often read and expounded them to him. But he soon surpassed his masters, and became fitter to teach, than to learn any thing from them.

His father, otherwise burdened with a numerous family, finding a difficulty in supporting him, his friends began to think of providing both for his education and maintenance. His own inclination led him strongly to Cambridge, and it was at length determined that he should try his fortune there, not as a scholar, but as a master: or, if this design should not succeed, they promised themselves success in opening a school for him at London. Accordingly he went to Cambridge in 1707, being then 25 years of age, and his fame in a short time filled the university. Newton's Principia, Optics, and Universal Arithmetic, were the foundations of his lectures, and afforded him a noble field for the display of his genius; and great numbers came to hear a blind man give lectures on optics, discourse on the nature of light and colours, explain the theory of vision, the effect of glasses, the phenomenon of the rainbow, and other objects of sight.

As he instructed youth in the principles of the Newtonian philosophy, he soon became acquainted with its incomparable author, though he had several years before left the university; and frequently conversed with him on the most difficult parts of his works: he also held a friendly communication with the other eminent mathematicians of the age, Halley, Cotes, Demoivre, &c.

Mr. Whiston was during this time in the mathematical professor's chair, and read lectures in the manner proposed by Mr. Saunderson on his settling at Cambridge; so that an attempt of this kind looked like an encroachment on the privilege of his office; but, as a good-natured man, and an encourager of learning, he readily consented to the application of friends made in behalf of so uncommon a person.

On the removal of Mr. Whiston from his professorship, Mr. Saunderson's merit was thought so much superior to that of any other competitor, that an extraordinary step was taken in his favour, to qualify him with a degree, which the statute requires: in consequence he was chosen, in 1711, Mr. Whiston's successor in the Lucasian professorship of mathematics, Sir Isaac Newton interesting himself greatly in his favour. His first performance, after he was seated in the chair, was an inaugural speech made in very elegant latin, and a style truly Ciceronian; for he was well versed in the writings of Tully, who was his favourite in prose, as Virgil and Horace were in verse. From this time he applied himself closely to the reading of lectures, and gave up his whole time to his pupils. He continued to reside among the gentlemen of Christ-college till the year 1723, when he took a house in Cambridge, and soon after married a daughter of Mr. Dickens, rector of Boxworth in Cambridgeshire, by whom he had a son and a daughter.

In the year 1728, when king George visited the university, he expressed a desire of seeing so remarkable a person; and accordingly our professor attended his majesty in the senate, and by his favour was there created doctor of laws.

Dr. Saunderson was naturally of a strong healthy constitution; but being too sedentary, and constantly confining himself to the house, he became a valetudinarian: and in the spring of the year 1739 he complained of a numbness in his limbs, which ended in a mortification in his foot, of which he died the 19th of April that year, in the 57th year of his age.

There was scarcely any part of the mathematics on which Dr. Saunderson had not composed something for the use of his pupils. But he discovered no intention of publishing any thing till, by the persuasion of his friends, he prepared his Elements of Algebra for the press, which after his death were published by subscription in 2 vols 4to, 1740.

He left many other writings, though none perhaps prepared for the press. Among these were some valuable comments on Newton's Principia, which not only explain the more difficult parts, but often improve upon the doctrine itself. These are published in Latin at the end of his posthumous Treatise on Fluxions, a valuable work, published in 8vo, 1756.—His manuscript lectures too, on most parts of natural philosophy, which I have seen, might make a considerable volume, and prove an acceptable present to the public if printed.

Dr. Saunderson, as to his character, was a man of much wit and vivacity in conversation, and esteemed an excellent companion. He was endued with a great regard to truth; and was such an enemy to disguise, that he thought it his duty to speak his thoughts at all times with unrestrained freedom. Hence his sentiments on men and opinions, his friendship or disregard, were expressed without reserve; a sincerity which raised him many enemies.

A blind man, moving in the sphere of a mathematician, seems a phenomenon difficult to be accounted for, and has excited the admiration of every age in which it has appeared. Tully mentions it as a thing scarce credible in his own master in philosophy, Diodotus; that he exercised himself in it with more assiduity after he became blind; and what he thought next to impossible to be done without sight, that he professed geometry, describing his diagrams so exactly to his scholars, that they could draw every line in its proper direction. St. Jerome relates a still more remarkable instance in Didymus of Alexandria, who, though blind from his infancy, and therefore ignorant of the very letters, not only learned logic, but geometry also to very great perfection, which seems most of all to require sight. But, if we consider that the ideas of extended quantity, which are the chief objects of mathematics, may as well be acquired by the sense of feeling as that of sight, that a fixed and steady attention is the principal qualification for this study, and that the blind are by necessity more abstracted than others (for which reason it is said that Democritus put out his eyes, that he might think more intensely), we shall perhaps find reason to suppose that there is no branch of science so much adapted to their circumstances.

At first, Dr. Saunderson acquired most of his ideas by the sense of feeling; and this, as is commonly the case with the blind, he enjoyed in great perfection. Yet he could not, as some are said to have done, distinguish colours by that sense; for, after having made repeated trials, he used to say, it was pretending to impossibilities. But he could with great nicety and exactness observe the smallest degree of roughness or defect of polish in a surface. Thus, in a set of Roman medals, he distinguished the genuine from the false, though they had been counterfeited with such exactness as to deceive a connoisseur who had judged by the eye. By the sense of feeling also, he distinguished the least variation; and he has been seen in a garden, when observations have been making on the sun, to take notice of every cloud that interrupted the observation almost as justly as they who could see it. He could also tell when any thing was held near his face, or when he passed by a tree at no great distance, merely by the different impulse of the air on his face.

His ear was also equally exact. He could readily distinguish the 5th part of a note. By the quickness of this sense he could judge of the size of a room, and of his distance from the wall. And if ever he walked over a pavement, in courts or piazzas which reflected a sound, and was afterwards conducted thither again, he could tell in what part of the walk he stood, merely by the note it sounded.

Dr. Saunderson had a peculiar method of performing arithmetical calculations, by an ingenious machine and method, which is particularly described in a piece prefixed to the first volume of his Algebra. That he was able to make long and intricate calculations, both arithmetical and algebraical, is a thing as certain as it is wonderful. He had contrived for his own use, a commodious notation for any large numbers, which he could express on his abacus, or calculating table, and with which he could readily perform any arithmetical operations, by the sense of feeling only, for which reason it was called his Palpable Arithmetic.

His calculating table was a smooth thin board, a little more than a foot square, raised upon a small frame so as to lie hollow; which board was divided into a great number of little squares, by lines intersecting one another perpendicularly, and parallel to the sides of the table, and the parallel ones only one-tenth of an inch from each other; so that every square inch of the table was thus divided into 100 little squares. At every point of intersection the board was perforated by small holes, capable of receiving a pin; for it was by the help of pins, stuck up to the head through these holes, that he expressed his numbers. He used two kinds of pins, a larger and a smaller sort; at least their heads were different, and might easily be distinguished by feeling. Of these pins he had a large quantity in two boxes, with their points cut off, which always stood ready before him when he calculated. The writer of that account describes particularly the whole process of using the machine, and concludes, "He could place and displace his pins with incredible nimbleness and facility, much to the pleasure and surprise of all the beholders. He could even break off in the middle of a calculation, and resume it when he pleased, and could presently know the condition of it, by only drawing his fingers gently over the table."

SAURIN (Joseph), an ingenious French mathematician, was born in 1659, at Courtaison, in the principality of Orange. His father, minister at Grenoble, was a man of a very studious disposition, and was the first preceptor or instructor to our author; who made a rapid progress in his studies, and at a very early age was admitted a minister at Eure in Dauphiny: but preaching an offensive sermon, he was obliged to quit France in 1683. On this occasion he retired to Geneva; whence he went into the State of Berne, and was appointed to a living at Yverdun. He was no sooner established in this his situation, than certain theologians raised a clamour against him. Saurin, disgusted with the controversy, and still more with the Swiss, where his talents were buried, passed into Holland, and from thence into France, where he put himself under the protection of the celebrated Bossu, to whom he made his abjuration in 1690, as it is suspected, that he might find protection, and have an opportunity of cultivating the sciences at Paris. And in this he was not disappointed: he met with many flattering encouragements; was even much noticed by the king, had a pension from the court, and was admitted of the Academy of Sciences in 1707, in the quality of geometrician. This science was now his chief study and delight; with many writings upon which he enriched the volums of the Journal des Savans, and the Memoirs of the Academy of Sciences. These were the only works of this kind that he published:

he was author of several other pieces of a controversial nature, against the celebrated Rousseau, and other antagonists, over whom, with the assistance of government, he was enabled to triumph. The latter part of his life was spent more peaceably in cultivating the mathematical sciences. He died the 29th of December 1737, of a lethargic fever, at 78 years of age.

The character of Saurin was lively and impetuous, endued with a considerable degree of that noble independence and loftiness of manner, which is apt to be mistaken for haughtiness or insolence; in consequence of which, his memory was attacked after his death, as his reputation had been during his life; and it was even said he had been guilty of crimes, by his own confession, that ought to have been punished with death.

Saurin's mathematical and philosophical papers, printed in the Memoirs of the Academy of Sciences, which are pretty numerous, are to be found in the volumes for the years following; viz, 1709, 1710, 1713, 1716, 1718, 1720, 1722, 1723, 1725, 1727.

SAUSSURE (Horace Benedict de), an ingenious philosopher, who was born at Geneva in 1740, and died in 1799. At the age of 21 he was elected philosophical professor at Geneva, where he taught for 25 years, with great public benefit. He first visited Paris in 1768, and next examined the discoveries of Montgolfier at Lyons; he then travelled through Holland, Belgium, England, and Italy. He visited the island of Elba, examined Vesuvius, and measured the crater of Ætna. He invented several instruments, in scientific operations. In his excursions among the Alps, he crossed them 14 times, at 8 different places; and he ascended to the summit of Mont Blanc, where he could hardly breathe. He was made member of the Academy of Sciences at Paris, &c. In the French revolution he was elected, on the union of his country to France, to the National Assembly; but the disorders of the times ruined his little fortune, and broke his heart.

Saussure was author of an Eulogy on his friend Bonnet, 8vo: Dissertatio Physica de Igne: Inquiry on the Bark of Leaves, &c: Dissertatio Physica de Electricitate, 8vo: Plan of Reform for the College of Geneva: Description of the Electrical Effects of Thunder: Essay on Hygrometry, 4to: Travels in the Alps, 4 vols. 4to, a valuable work: and other pieces.

SAUVEUR (Joseph), an eminent French mathematician, was born at La Fleche the 24th of March 1653. He was absolutely dumb till he was seven years of age; and then the organs of speech did not disengage so effectually, but that he was ever after obliged to speak very slowly and with difficulty. He very early discovered a great turn for mechanics, and was always inventing and constructing something or other in that way.

He was sent to the college of the Jesuits to study polite literature, but made very little progress in poetry and eloquence. Virgil and Cicero had no charms for him; but he read with eagerness books of arithmetic and geometry. However, he was prevailed on to go to Paris in 1670, and, being intended for the church, there he applied himself for a time to the study of philosophy and theology; but still succeeded no better. In short, mathematics was the only study he had any relish for, and this he cultivated with extraordinary success; for during his course of philosophy, he learned the first six books of Euclid in the space of a month, without the help of a master.

As he had an impediment in his voice, though otherwise endued with extraordinary abilities, he was advised by M. Bossuet, to give up all designs for the church, and to apply himself to the study of physic: but this being utterly against the inclination of his uncle, from whom he drew his principal resources, Sauveur determined to devote himself to his favourite science, and to perfect himself in it, so as to teach it for his support; and in effect he soon became the fashionable preceptor in mathematics, so that at 23 years of age he had prince Eugene for his scholar.—He had not yet read the geometry of Descartes; but a foreigner of the first quality desiring to be taught it, he made himself master of it in an inconceivably small space of time.—Basset being a fashionable game at that time, the marquis of Dangeau asked him for some calculations relating to it, which gave such satisfaction, that Sauveur had the honour to explain them to the king and queen.

In 1681 he was sent with M Mariotte to Chantilli, to make some experiments upon the waters there, which he did with much applause. The frequent visits he made to this place inspired him with the design of writing a treatise on fortification; and, in order to join practice with theory, he went to the siege of Mons in 1691, where he continued all the while in the trenches. With the same view also he visited all the towns of Flanders; and on his return he became the mathematician in ordinary at the court, with a pension for life.—In 1680 he had been chosen to teach mathematics to the pages of the Dauphiness. In 1686 he was appointed mathematical professor in the Royal College. And in 1696 admitted a member of the Academy of Sciences, where he was in high esteem with the members of that society.—He became also particularly acquainted with the prince of Condé, from whom he received many marks of favour and affection. Finally, M. Vauban having been made marshal of France, in 1703, he proposed Sauveur to the king as his successor in the office of examiner of the engineers; to which his majesty agreed, and honoured him with a pension, which our author enjoyed till his death, which happened the 9th of July 1716, in the 64th year of his age.

Sauveur, in his character, was of a kind obliging disposition, of a sweet, uniform, and unaffected temper; and though his fame was pretty generally spread abroad, it did not alter his humble deportment, and the simplicity of his manners. He used to say, that what one man could accomplish in mathematics, another might do also, if he chose it.

An extraordinary part of Sauveur's character is, that though he had neither a musical voice nor ear, yet he studied no science more than music, of which he composed an entire new system. And though he was obliged to borrow other people's voice and ears, yet he amply repaid them with such demonstrations as were unknown to former musicians. He also introduced a new diction in music, more appropriate and extensive. He invented a new doctrine of sounds; and was the first that discovered, by theory and experiment, the velocity of musical strings, and the spaces they describe in their vibrations, under all circumstances of tension and dimensions. It was he also who first invented for this purpose the monochord and the echometer. In short, he pursued his researches even to the music of the ancient Greeks and Romans, to the Arabs, and to the very Turks and Persians; so jealous was he, lest any thing should escape him in the science of sounds.

Sauveur's writings, which consist of pieces rather than of set works, are all inserted in the volumes of the Memoirs of the Academy of Sciences, from the year 1700 to the year 1716, on various geometrical, mathematical, philosophical, and musical subjects.

SCALE, a mathematical instrument, consisting of certain lines drawn on wood, metal, ivory, &c, divided into various parts, either equal or unequal. It is of great use in laying down distances in proportion, or in measuring distances already laid down. There are scales of various kinds, accommodated to the several uses: the principal are the Plane Scale, the Diagonal Scale, Gunter's Scale, and the Plotting Scale.

Plane or Plain SCALE, a mathematical instrument of very extensive use and application; which is commonly made of 2 feet in length; and the lines usually drawn upon it are the following, viz,

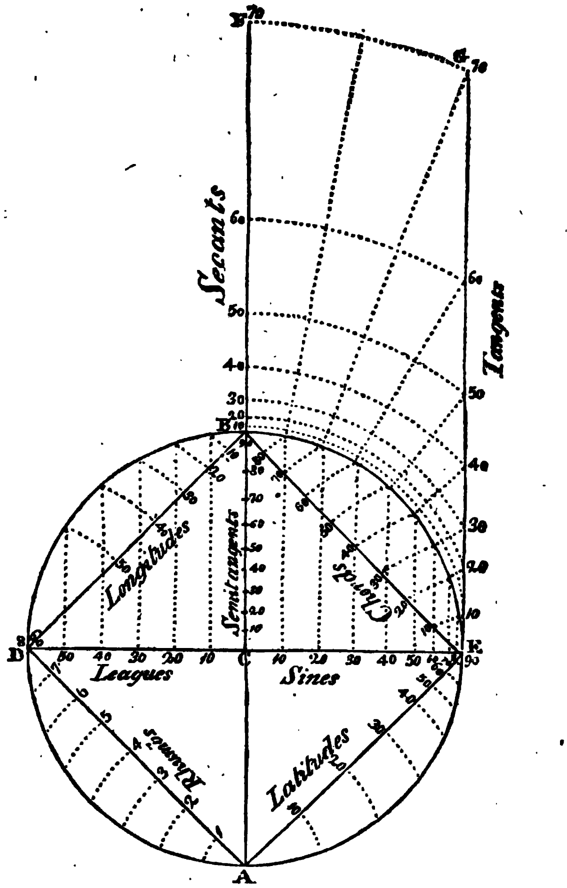
1 Lines of	Equal Parts, and marked	E. P.
2 - -	Chords - - - - - -	Cho.
3 - -	Rhumbs - - - - -	Ru.
4 - -	Sines - - - - - -	Sin.
5 - -	Tangents - - - - - -	Tan.
6 - -	Secants - - - - -	Sec.
7 - -	Semitangents - - -	S. T.
8 - -	Longitude - - - -	Long.
9 - -	Latitude - - - -	Lat.

1. The lines of equal parts are of two kinds, viz, simply divided, and diagonally divided. The first of these are formed by drawing three lines parallel to one another, and dividing them into any number of equal parts by short lines drawn across them, and in like manner subdividing the first division or part into 10 other equal small parts; by which, numbers or dimensions of two figures may be taken off. On some rulers, several of these scales of equal parts are ranged parallel to each other, with figures set to them to show into how many equal parts they divide the inch; as 20, 25, 30, 35, 40, 45, &c. The 2d or diagonal divisions are formed by drawing eleven long parallel and equidistant lines, which are divided into equal parts, and crossed by other short lines, as the former; then the first of the equal parts have the two outermost of the eleven parallels divided into 10 equal parts, and the points of division being connected by lines drawn diagonally, the whole scale is thus divided into dimensions or numbers of three places of figures.

The other lines upon the scales are such as are commonly used in trigonometry, navigation, astronomy, dialling, projection of the sphere, &c, &c; and their constructions are mostly taken from the divisions of a circle, as follow:

Describe a circle with any convenient radius, and quarter it by drawing the diameters AB and DE at right angles to each other; continue the diameter AB out towards F, and draw the tangent line EG parallel to it; also draw the chords AD, DB, BE, EA. Then,

2. For the line of chords, divide a quadrant BE into 90 equal parts; on E as a centre, with the compasses transfer these divisions to the chord line EB, which mark with the corresponding numbers, and it will become a line of chords, to be transferred to the ruler.



3. For the line of rhumbs, divide the quadrant AD into 8 equal parts: then with the centre A transfer the divisions to the chord AD, for the line of rhumbs.

4. For the line of sines, through each of the divisions of the arc BE, draw right lines parallel to the radius BC, which will divide the radius CE into the sines, or versed sines, numbering it from C to E for the sines, and from E to C for the versed sines.

5. For the line of tangents, lay a ruler on C, and the several divisions of the arc BE, and it will intersect the line EG, which will become a line of tangents, and numbered from E to G with 10, 20, 30, 40, &c.

6. For the line of secants, transfer the distances between the centre C and the divisions on the line of tangents to the line BF, from the centre C, and these will give the divisions of the line of secants, which must be numbered from B towards F, with 10, 20, 30, &c.

7. For the line of semitangents, lay a ruler on D and the several divisions of the arc EB, which will intersect the radius CB in the divisions of the semitangents, which are to be marked with the corresponding figures of the arc EB.

The chief uses of the sines, tangents, secants, and semitangents, are to find the poles and centres of the several circles represented in projections of the sphere.

8. For the line of longitude, divide the radius CD into 60 equal parts; through each of these, parallels to the radius BC will intersect the arc BD in as many points: from D as a centre the divisions of the arc BD being trans-

ferred to the chord BD, will give the divisions of the line of longitude.

If this line be laid upon the scale close to the line of chords, both inverted, so that 60° in the scale of longitude be against 0° in the chords, &c; and any degree of latitude be counted on the chords, there will stand opposite to it, in the line of longitude, the miles contained in one degree of longitude, in that latitude; the measure of 1 degree under the equator being 60 geographical miles.

9. For the line of latitude, lay a ruler on B, and the several divisions on the sines on CE, and it will intersect the arc AE in as many points; on A as a centre transfer the intersections of the arc AE to the chord AE, for the line of latitude.

See also Robertson's Description and Use of Mathematical Instruments.

Diagonal SCALE. See the article 1, above.

Decimal, or *Gunter's*, or *Plotting*, or *Proportional*, or *Reducing* SCALE. See the several articles.

SCALE, in Architecture and Geography, a line divided into equal parts, placed at the bottom of a map or draught, to serve as a common measure to all the parts of the building, or all the distances and places of the map

In maps of large tracts, as kingdoms and provinces, &c, the scale usually consists of miles: whence it is denominated a scale of miles.—In more particular maps, as those of manors, &c, the scale is usually of chains &c.—The scales used in draughts of buildings mostly consist of modules, feet, inches, palms, fathoms, or the like.

To find the distance between two towns &c, in a map, the interval is taken in the compasses, and set off in the scale; and the number of divisions it includes gives the distance. The same method serves to find the height of a story, or other part in a design.

Front SCALE, in Perspective, is a right line in the draught, parallel to the horizontal line; divided into equal parts, representing feet, inches, &c.

Flying SCALE, is a right line in the draught, tending to the point of view, and divided into unequal parts, representing feet, inches, &c.

Differential SCALE, is used for the scale of relation subtracted from unity. See SERIES.

SCALE *of Notation*, is the order of progression on which any system of arithmetic is founded; as the Binary Scale, Quartenary, Sexenary, Denary, Duodenary, &c.

The denary, or decimal scale, is that on which our present notation is established, and by which the value of our numerical characters increase in a tenfold proportion, from the right hand towards the left, the number of characters employed being ten. In the binary scale, there are only two characters, namely 1 and 0; and generally, for any scale of notation, the number of characters necessary for expressing a given quantity, will never exceed the radix of that system.

The following examples will give some idea of the use of the different scales.

Scale.	*Numb.*	*Progressions.*	*Common Expression.*
Binary - -	101110	$= 1 \times 2^5 + 0 \cdot 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2 + 0$	= 46
Ternary -	121201	$= 1 \times 3^5 + 2 \times 3^4 + 1 \times 3^3 + 2 \times 3^2 + 0 \times 3 + 1$	= 451
Quartenary	123013	$= 1 \times 4^5 + 2 \times 4^4 + 3 \times 4^3 + 0 \times 4^2 + 1 \times 4 + 3$	= 1735
Quinary -	413402	$= 4 \times 5^5 + 1 \times 5^4 + 3 \times 5^3 + 4 \times 5^2 + 0 \times 5 + 2$	=13602
Sexenary -	532412	$= 5 \times 6^5 + 3 \times 6^4 + 2 \times 6^3 + 4 \times 6^2 + 1 \times 6 + 2$	=43352
Decimal -	17844	$= 1 \times 10^4 + 7 \times 10^3 + 8 \times 10^2 + 4 \times 10 + 4$	=17844
Duodenary	7846	$= 7 \times 12^3 + 8 \times 12^2 + 4 \times 12 + 6$	=13302

For more on this subject, see NOTATION.

SCALE *of Relation*, in Algebra, an expression denoting the relation of the terms of recurring series to each other. See SERIES.

Hour SCALE. See HOUR.

SCALE, in Music, is a denomination given to the arrangement of the six syllables, invented by Guido Aratino, *ut re mi fa sol la;* called also gammut. It is called scale, or ladder, because it represents a kind of ladder, by means of which the voice rises to acute, or sinks to grave; each of the six syllables being as it were one step of the ladder.

SCALE is also used for a series of sounds rising or falling towards acuteness or gravity, from any given pitch of tune to the greatest distance that is fit or practicable, through such intermediate degrees as to make the succession most agreeable and perfect, and in which we have all the harmonical intervals most commodiously divided.—The scale is otherwise called an Universal System, as including all the particular systems belonging to music. See SYSTEM.—There were three different scales in use among the ancients, which had their denominations from the three several kinds of music, viz, the Diatonic, Chromatic, and Inharmonic; which see.

SCALENE, or SCALENOUS *Triangle*, is a triangle whose sides and angles are all unequal.—A cylinder or cone, whose axis is oblique or inclined to its base, is also said to be scalenous: though more frequently it is called oblique.

SCALIGER (JOSEPH JUSTUS), a celebrated French chronologer and critic, was the son of Julius Cæsar Scaliger, and born at Agen in France, in 1540. He studied in the college of Bourdeaux; after which his father took him under his own care, and employed him in transcribing his poems; by which means he obtained such a taste for poetry, that before he was 17 years of age he wrote a tragedy on the subject of Oedipus, in which he introduced all the poetical ornaments of style and sentiment.

His father dying in 1558, he went to Paris the year following, with a design to apply himself to the Greek tongue; for which purpose he for two months attended the lectures of Turnebus; but finding that in the usual course he should be a long time in gaining his point, he shut himself up in his closet, and by constant application for two years gained a perfect knowledge of the Greek language. After which he applied himself to the Hebrew, which he learned by himself with great facility. And in like manner he ran through many other languages, till he could speak, it is said, no less than 13 ancient and modern tongues. He made no less progress in the sciences; and his writings procured him the reputation of one of the greatest men of that or any other age. He embraced the reformed religion at 22 years of age: and in 1563, he attached himself to Lewis Casteignier de la Roch Pazay, whom he attended in several journeys. And, in 1593, the curators of the university of Leyden invited him to an honorary professorship in that university, where he lived 16 years, till he died of a dropsy in 1609, at 69 years of age.

Scaliger was a man of great temperance; was never married; and was so close a student, that he often spent whole days in his study without eating: and though his circumstances were always very narrow, he constantly refused the presents that were offered him.

He was author of many ingenious works on various subjects. His elaborate work, De Emendatione Temporum; his exquisite animadversions on Eusebius; with his Canon Isagogicus Chronologiæ; and his accurate comment upon Manilius's Astronomicon, sufficiently evince his knowledge in astronomy, and other branches of learning, among the ancients, and who, according to the opinion of the celebrated Vieta, was far superior to any of that age. And he had no less a character given him by the learned Casaubon.—He wrote Cyclometrica et Diatriba de Equinoctiorum Anticipatione. Also notes upon Seneca, Varro, and Ausonius's Poems. But that which above all things renders the name of Scaliger memorable to posterity, is the invention of the Julian period, which consists of 7980 years, being the continued product of the three cycles, of the sun 28, the moon 19, and Roman indiction 15. This period had its beginning fixed to the 764th year before the creation, and is not yet completed, and comprehends all other cycles, periods, and epochas, with the times of all memorable actions and histories. The collections intitled Scaligeriana, were made from his conversations by one of his friends; and being ranged in alphabetical order, were published by Isaac Vossius.

SCANTLING, a measure, size, or standard, by which the dimensions &c of things are to be determined. The term is particularly applied to the dimensions of any piece of timber, with regard to its breadth and thickness.

SCAPEMENT, in Clock-work, a general term for the manner of communicating the impulse of the wheels to the pendulum. The ordinary scapements consist of the swing-wheel and pallets only; but modern improvements have added other levers or detents, chiefly for the purposes of diminishing friction, or for detaching the pendulum from the pressure of the wheels during part of the time of its vibration. Notwithstanding the very great importance of the scapement to the performance of clocks, no material improvement was made in it from the first application of the pendulum to clocks, to the days of Mr. George Graham; nothing more was attempted before his time, than to apply the impulse of the swing-wheel, in such manner as was attended with the least friction, and would give the greatest motion to the pendulum. Dr. Halley discovered, by some experiments made at the Royal Observatory at Greenwich, that by adding more weight to the pendulum, it was made to vibrate larger arcs, and the clock went faster; by diminishing the weight of the pendulum, the vibrations became shorter, and the clock went slower; the result of these experiments being diametrically opposite to what ought to be expected from the theory of the pendulum, probably first roused the attention of Mr. Graham, and led him to such further trials as convinced him, that this seeming paradox was occasioned by the retrograde motion, which was given to the swing-wheel by every construction of scapement that was at that time in use; and his great sagacity soon produced a remedy for this defect, by constructing a scapement which prevented all recoil of the wheels, and restored to the clock pendulum, wholly in theory, and nearly in practice, all its natural properties in its detached simple state; this scapement was named by its celebrated inventor the Dead Beat, and its great superiority was so universally acknowledged, that it was soon introduced into general use, and still continues in universal esteem. The importance of the scapement to the accurate going of clocks, was by this improvement rendered so unquestionable, that artists of the first rate all over Europe, were forward in producing each his particular construction, as may be seen in the works of Thiout l'ainé, M. J. A. Lepante, M. le Roy, M. Ferdinand Bertoud, and Mr. Cummings' Elements of Clock and Watchwork, in which we have a minute description of several new and ingenious constructions of scapements, with an investigation of the principles on which their claim to merit is founded; also a comparative view of the advantages or defects of the several constructions. Besides the scapements described in the above works, many curious constructions have been produced by eminent artists, who have not published any account of them, nor of the motives which have induced each to prefer his favourite construction: Mr. Harrison, Mr. Hindley of York, Mr. Ellicot, Mr. Mudge, Mr. Arnold, Mr. Whitehurst, and many other ingenious artists of this country, have made scapements of new and peculiar constructions, of which we are unable, for the above reason, to give any further account than that those of Mr. Harrison and Mr. Hindley had scarce any friction, with a certain mode and quantity of recoil; those of all the other gentlemen, we believe, have been on the principle of the dead beat, with such other improvements as they severally judged most conducive to a good performance.

Count Bruhl published, in 1794, a small pamphlet, "On the Investigation of Astronomical Circles," to which he has annexed, "A Description of the Scapement in Mr. Mudge's first Timekeeper, drawn up in August 1771." Before entering upon the description, the count premises a few observations, in one of which he recognizes a hint concerning the nature of Mr. Mudge's scapement, thrown out by this artist in a small tract printed by him in the year 1763, which is this: "The force derived from the mainspring should be made as equal as possible, by making the mainspring wind up another smaller spring at a less distance from the balance, at short intervals of time. I think it would not be impracticable to make it wind up at every vibration, a small spring similar to the pendulum spring, that should immediately act on the balance, by which the whole force acting on the balance would be reduced to the greatest simplicity, with this advantage, that the force would increase in proportion to the arch." From this hint, Count Bruhl is surprised that no other artists have taken up Mr. Mudge's invention. He then gives the description of that invention, in the pamphlet above-mentioned.

For a detailed account of Mudge's scapement, and other inventions of this kind, see Gregory's Mechanics, vol. 2, pa. 329.

SCARP, in Fortification, the interior slope of the ditch of a place; that is, the slope of that side of a ditch which is next to the place, or on the outside of the rampart at its foot, facing the champaign or open country. The slope on the outer side of the ditch is called the Counterscarp.

SCENOGRAPHY, in Perspective, the perspective representation of a body on a plane; or a description and view of it in all its parts and dimensions, such as it appears to the eye in any oblique view. This differs essentially from the ichnography and the orthography. The ichnography of a building, &c, represents the plan or ground work of the building, or section parallel to it; and the orthography the elevation, or front, or one side, also in its natural dimensions; but the scenography exhibits the whole of the building that appears to the eye, front, sides, height, and all, not in their real dimensions or extent, but raised on the geometrical plan in perspective.—In architecture and fortification, scenography is the manner of delineating the several parts of a building or fortress, as they are represented in perspective.

To exhibit the Scenography *of any body.* 1. Lay down the basis, ground-plot, or plan, of the body, in the perspective ichnography, that is, draw the perspective appearance of the plan or basement, by the proper rules of perspective. 2. On the several points of the said perspective plan, raise the perspective heights, and connect the tops of them by the proper slope or oblique lines. So will the scenography of the body be completed, when a proper shade is added. See Perspective.

SCHEINER (Christopher), a considerable German mathematician and astronomer, was born at Mundelheim in Schwaben in 1575. He entered into the society of the Jesuits at 20 years of age; and afterwards taught the Hebrew tongue and the mathematics at Ingolstadt, Friburg, Brisac, and Rome. At length he became confessor to the archduke Charles, and rector of the college of the Jesuits at Neisse in Silesia, where he died in 1650, at 75 years of age.

Scheiner was chiefly remarkable for being one of the first, though not the very first, who observed the spots in the sun with the telescope; for his observations of those spots were first made, at Ingolstadt, in the latter part of the year 1611, whereas Galileo and Harriot both observed them in the latter part of the year before, or 1610. Scheiner continued his observations on the solar phenomena for many years afterwards at Rome, with great assiduity and accuracy, constantly making drawings of them on paper, describing their places, figures, magnitude, revolutions, and periods, so that Riccioli delivered it as his opinion that there was little reason to hope for any better observations of those spots. Descartes and Hevelius also say, that, in their judgment, nothing can be expected of that kind more satisfactory. These observations were published in one volume folio, 1630, under the title of Rosa Ursina, &c; almost every page of which is adorned with an image of the sun with the spots. He wrote also several smaller pieces relating to mathematics and philosophy, the principal of which are,

2. Oculus, sive Fundamentum Opticum, &c; which was reprinted at London, in 1652, in 4to.
3. Sol Eclipticus, Disquisitiones Mathematicæ.
4. De Controversiis et Novitatibus Astronomicis.

SCHEME, a draught or representation of any geometrical or astronomical figure, or problem, by lines sensible to the eye; or of the celestial bodies in their proper places for any moment; otherwise called a diagram.

Scheme *Arches.* See Arch.

SCHOLIUM, a note, remark, or annotation, occasionally made on some passage, proposition, &c.

The term is much used in geometry, and other parts of the mathematics; where, after demonstrating a proposition, it is used to point out how it might be done some other way; or to give some advice or precaution, in order to prevent mistakes; or to add some particular use or application of it.

Wolfius has given abundance of curious and useful arts and methods, and a good part of the modern philosophy, with the description of mathematical instruments, &c; all by way of scholia to the respective propositions in his Elementa Matheseos.

SCHONER (John), a noted German philosopher and mathematician, was born at Carolostadt in the year 1477, and died in 1547, at 70 years of age.—His early propensity to those sciences may be deemed a just prognostication of the great progress which he afterwards made in them. From his uncommon acquirements, he was chosen mathematical professor at Nuremburg when he was but a young man. He wrote a great many works, and was particularly celebrated for his astronomical tables, which he published after the manner of those of Regiomontanus, and to which he gave the title of Resolutæ, on account of their clearness. But notwithstanding his great knowledge, he was, after the custom of the times, much addicted to judicial astrology, which he took great pains to improve. The list of his writings is chiefly as follows:

1. Three Books of Judicial Astrology.
2. The Astronomical Tables named Resolutæ.
3. De Usu Globi Stelliferi; De Compositione Globi Cœlestis; De Usu Globi Terrestris, et de Compositione ejusdem.
4. Æquatorium Astronomicum.
5. Libellus de Distantiis Locorum per Instrumentum et Numeros Investigandis.
6. De Compositione Torqueti.
7. In Constructionem et Usum Rectanguli sive Radii Astronomici Annotationes.
8. Horarii Cylindri Canones.
9. Planisphærium, seu Meteoriscopium.
10. Organum Uranicum.
11. Instrumentum Impedimentorum Lunæ.

All printed at Nuremburg, in folio, 1551.

Of these, the large treatise of dialling rendered him more known in the learned world than all his other works besides; in which he discovers a surprising genius and fund of learning of that kind.

SCHOOL, a place where languages, or arts and sciences, &c, are taught.

School is also used for a whole faculty, university, or sect; as Plato's school, the school of Epicurus, the school of Paris, &c. The school of Tiberias was celebrated among the ancient Jews; and it is to this we owe the Massora, and Massoretes.

School *Philosophy*, &c. the same with *scholastic.*

SCHOOTEN (Francis), was a professor of mathematics at Leyden, being a very acute and respectable proficient in that science. He published, in 1649, an edition of Descartes's Geometry, with learned and elaborate annotations on that work, as also those of Beaume Hudde, and Van Heauralt. Schooten published also two very useful and learned works of his own compositions, viz,

1. Principia Matheseos Univers. &c, 4to, 1651.
2. Exercitationes Mathematicæ, 4to, 1657.

SCIAGRAPHY, or Sciography, the profile or vertical section of a building; used to show the inside of it.

Sciagraphy, in Astronomy &c, is a term used by some authors for the art of finding the hour of the day or night, by the shadow of the sun, moon, stars, &c. See Dial.

SCIENCE, a clear and certain knowledge of any thing, founded on demonstration, or on self-evident principles. In this sense, doubting is opposed to science; and opinion is the middle between the two.

Science is more particularly used for a formed system of any branch of knowledge, comprehending the doctrine, reason, and theory of the thing, without any immediate application of it to any uses or offices of life. And in this sense, the word is used in opposition to art.

Science may be divided into three classes: First, the knowledge of things, their constitutions, properties, and operations, whether material or immaterial. And this, in a little more enlarged sense of the word, may be called physics, or natural philosophy. Secondly, the skill of rightly applying our own powers and actions for the attainment of good and useful things, as Ethics. Thirdly, the doctrine of signs; as words, logic, &c.

SCIENTIFIC, or Scientifical, something relating to the pure and subliminer sciences; or that abounds in science, or knowledge. A work, or method, &c, is said to be scientifical, when it is founded on the pure reason of things, or conducted wholly on the principles of them. In which sense the word stands opposed to narrative, arbitrary, opinionative, positive, tentative, &c.

SCIOPTIC, or Scioptric *Ball*, a sphere or globe of wood, with a circular hole or perforation, where a lens is placed. It is so fitted that, like the eye of an animal, it may be turned round every way, to be used in making experiments of the darkened room, or camera obscura.

SCIOPTRICS. See Camera Obscura.

SCIOTHERICUM *Telescopium*, is an horizontal dial, adapted with a telescope for observing the true time both by day and night, to regulate and adjust pendulum clocks, watches, and other time-keepers. It was invented by Mr. Molyneux, who published a book with this title, which contains an accurate description of this instrument, with all its uses and applications.

SCLEROTICA, one of the common membranes of the eye, on its hinder part. It is a large, thick, firm, hard, opaque membrane, extended from the external circumference of the cornea to the optic nerve, and forms much of the greater part of the external globe of the eye. The sclerotica and the cornea compose the case in which all the internal coats of the eye and its humours are contained.

SCONCES, small forts, built for the defence of some pass, river, or other place. Some sconces are made regular, of four, five, or six bastions; others are of smaller dimensions, fit for passes, or rivers; and others for the field.

SCORE, in Music, denotes partition, or the original draught of the whole composition, in which the several parts, viz, the treble, second treble, bass, &c, are distinctly scored, and marked.

SCORPIO, the *Scorpion*, the 8th sign of the zodiac, denoted by the character ♏, being a rude design of the animal of that name.

The Greeks, who would be supposed the founders of astronomy, and who have, with that intent, applied some story or other of their own to every one of the constellations, give a very singular account of the origin of this sign. They tell us that this is the creature which killed Orion; and according to them the famous hunter of that name boasted to Diana and Latona, that he would destroy every animal that was upon the earth; the earth, they say, enraged at this, sent forth the poisonous reptile the scorpion, which insignificant creature stung him, that he died. Jupiter then raised the scorpion to the heavens, giving him this place among the constellations; and that afterwards Diana requested of him to do the same honour to Orion, which he at last consented to, but placed him in such a situation, that when the scorpion rises, he sets.

But the Egyptians, or whatever early nation it was that framed the zodiac, probably placed this poisonous reptile in that part of the heavens to denote that when the sun arrived at it, fevers and sicknesses, the maladies of autumn, would begin to rage. This they represented by an animal whose sting was of such a nature as to occasion some of them; and it was thus they formed all the constellations.

The ancients allotted one of the twelve principal among their deities to be the guardian for each of the 12 signs of the zodiac. The scorpion, as their history of it made it a fierce and fatal animal that had killed the great Orion, fell naturally to the protection of the god of war; Mars is therefore its tutelary deity; and to this single circumstance is owing all that jargon of the astrologers, who tell us that there is a great analogy between the planet Mars and the constellation scorpio. To this also is owing the doctrine of the alchymists, that iron, which they call Mars, is also under the dominion of the same constellation, and that the transmutation of that metal into gold can only be performed when the sun is in this sign.

The stars in scorpio, in Ptolemy's catalogue, are 24; in that of Tycho 10, in that of Hevelius 20, but in that of Flamsteed and Sharp 44.

Scorpion is also the name of an ancient military engine, used chiefly in the defence of the walls, &c. Marcellinus describes the scorpion, as consisting of two beams bound together by ropes; from the middle of which rose a third beam, so disposed, as to be pulled up and let down at pleasure; and on the top of this were fastened iron hooks, where a sling was hung, either of iron or hemp; and under the third beam lay a piece of haircloth full of chaff, tied with cords. It had its name Scorpio, because when the long beam or tiller was erected, it had a sharp top like a sting.

To use the engine, a round stone was put into the sling, and four persons on each side, loosening the beams bound by the ropes, drew back the erect beam to the hook; then the engineer, standing on an eminence, gave a stroke with a hammer on the chord to which the beam was fastened with its hook, which set it at liberty; so that hitting against the soft hair-cloth, it struck out the stone with a great force.

SCOTIA, in Architecture, a semicircular cavity or channel between the tores, in the bases of columns; and sometimes under the larmier or drip, in the cornice of the Doric order. The workmen often call it the casement, and it is also otherwise called the Trochilus.

SCOTT (George Lewis), a learned and respectable member of the Royal Society, and of the Board of Longitude. He was the eldest son of Mr. Scott of Bristow, in Scotland, who married Miss Stewart, daughter of sir James Stewart, who was lord advocate of Scotland in the time of K. William and Q. Anne. That lady was also his cousin-german, their mothers being sisters, and both daughters of Mr. Robert Trail, one of the ministers of Edinburgh, of the same family as the rev. Dr. Wm. Trail, the learned author of the Account of the Life and Writings of Dr. Rob. Simson, professor of mathematics at Glasgow.

Mr. Scott (the father), with his family, lived many years abroad, in a public character; and he had three sons born while residing at the court of Hanover. The eldest of these was our author George Lewis, named (in both these names) after his godfather the Elector, who was afterwards George the 1st of Britain. Geo. Lewis Scott was a gentleman of respectable talents and general learning; he was well skilled also in all the mathematical sciences; for which he manifested at times a fine and critical taste, as may be particularly seen in some letters which, in the year 1764, passed in a literary correspondence between him and Dr. Simson of Glasgow, and inserted in Dr. Trail's Account of the Life and Writings of Dr. Simson, pa. 113, &c. Mr. G. L. Scott was the author of the Supplement to Chambers's Dictionary, in 2 large folio volumes, which was much esteemed, and for which he received 1500l. from the booksellers, a considerable price at the time of that publication. Mr. Scott was sub-preceptor, for the Latin language, to his present Majesty, George the 3d, when Prince of Wales. After that he was appointed a commissioner of excise; a situation which his friends have considered as not adequate to his past deserts, and inferior to what he probably would have had, but for the freedom of his political opinions. Mr. Scott died the 7th of December, 1780.

SCREW, one of the six mechanical powers; chiefly used in pressing bodies close, though sometimes also in raising weights.

The screw is a spiral thread or groove cut round a cylinder, and everywhere making the same angle with the length of it. So that, if the surface of the cylinder, with this spiral thread upon it, were unfolded and stretched into a plane, the spiral thread would form a straight inclined plane, whose length would be to its height, as the circumference of the cylinder is to the distance between two threads of the screw; as is evident by considering, that in making one round, the spiral rises along the cylinder the distance between the two threads.

Hence the threads of a screw may be traced upon the smooth surface of a cylinder thus: cut a sheet of paper into the form of a right-angled triangle, having its base to its height in the above proportion, viz, as the circumference of the cylinder of the screw is to the intended distance between two threads; then wrap this paper triangle about the cylinder, and the hypothenuse of it will trace out the line of the spiral thread.

When the spiral thread is upon the outside of a cylinder, the screw is said to be a male or convex one. But if the thread be cut along the inner surface of a hollow cylinder, or a round perforation, it is said to be female or concave. And this latter is also sometimes called the box or nut.

When motion is to be given to something, the male and female screw are necessarily conjoined; that is, whenever the screw is to be used as a simple engine, or mechanical power. But when joined with an axis in peritrochio, there is no occasion for a female screw; but in that case it becomes part of a compound engine.

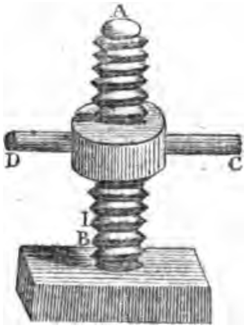
The screw cannot properly be called a simple machine, because it is never used without the application of a lever, or winch, to assist in turning it.

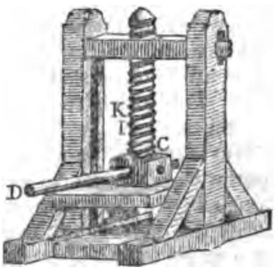
Of the Force and Power of the Screw.

1. The force of a power applied to turn a screw round, is to the force with which it presses upwards or downwards, setting aside the friction, as the distance between two threads, is to the circumference where the power is applied.—For, the screw being only an inclined plane, or half wedge, whose height is the distance between two threads, and its base the said circumference; and the force in the horizontal direction being to that in the vertical one as the lines perpendicular to them, viz, as the height of the plane, or distance of the two threads, is to the base of the plane, or circumference at the place where the power is applied; therefore the power is to the pressure, as the distance of two threads, is to that circumference.

Or the same may be otherwise shown thus. Since the momentum which any power generates, is equal to the momentum of that power; therefore the momentum of the screw, is equal to the momentum of the force applied to move it; which last is measured by the space passed over by the power in a given time. But this space is the circumference of a circle of which the lever is the radius, while the space passed over by the screw, in the same time, is only equal to the breadth of the threads, therefore the force or power of the screw, is to the power applied to move it, as the space passed over by the screw, to the space passed over by the power; that is, as the breadth of the threads to the circumference where the power is applied.

2. Hence, when the screw is put in motion; then the power is to the weight which would keep it in equilibrio, as the velocity of the latter is to that of the former. And hence their two momenta are equal, which are produced by multiplying each weight or power by its own velocity. Two different forms of screw-presses, are as below.



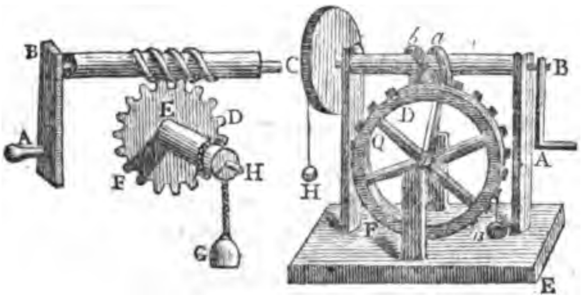


3. Hence we can easily compute the force of any machine turned by a screw. Let the annexed figure represent a press driven by a screw, whose threads are each a quarter of an inch asunder; and let the screw be turned by a handle of 4 feet long from C to D; then if the natural force of a man, by which he can lift, or pull, or draw, be 150 pounds; and it be required to determine with what force the screw will press on the board, when the man turns the handle at C and D with his whole force: the diameter CD of the power being 4 feet, or 48 inches, its circumference is 48 × 3·1416 or 150⅘ nearly; and the distance of the threads being ¼ of an inch; therefore the power is to the pressure, as ¼ to 150⅘, or as 1 to 603⅕: but the power is equal to 150lb; therefore as 1 : 603⅕ : : 150 : 90480; and consequently the pressure at the bottom of the screw, is equal to a weight of 90480 pounds, independent of friction.

But the power has to overcome, not only the weight, or other resistance, but also the friction of the screw, which in this machine is very great, in some cases equal to the weight itself, since it is sometimes sufficient to sustain the weight when the power is taken off.

Mr. Hunter has described a new method of applying the screw with advantage in particular cases, in the Philos. Trans. vol. 71, pa. 58 &c. A brief account of which may also be seen in Gregory's Mechanics, vol. 1, pa. 99.

The Endless Screw, or *Perpetual* Screw, is one which works in, and turns, a dented wheel DF, without a concave screw; being so called because it may be turned for ever, without coming to an end. From the following figures it is evident, that while the screw turns once round, the wheel only advances the distance of one tooth.



1. If the power applied to the lever, or handle, of an endless screw AB, be to the weight, in a ratio compounded of the periphery of the axis of the wheel EH, to the periphery described by the power in turning the handle, and of the revolutions of the wheel DF to the revolutions of the screw CB, the power will balance the weight. Hence,

2. As the motion of the wheel is very slow, a small power may raise a very great weight, by means of an endless screw. And therefore the chief use of such a screw is, either where a great weight is to be raised through a small space; or where only a slow gentle motion is wanted. For which reason it is very useful in clocks and watches.

3. Having given the number of teeth, the distance of the power from the centre of the screw B, the radius of the axis HE, and the power; to find the weight it will raise. Multiply the distance of the power from the centre of the screw AB, by the number of the teeth, and the product will be the space passed through by the power, while the weight passes through a space equal to the periphery of the axis: then say, as the radius of the axis is to the space of the power just found, so is the power to a 4th proportional, which will be the weight the power is able to sustain. Thus, if AB = 3, the radius of the axis HE = 1, the power 150 pounds, and the number of teeth of the wheel DF 48; then the weight will be found = 21600 = 3 × 150 × 48. Whence it appears that the endless screw exceeds all others in increasing the force of a power.

Again, if the endless screw AB be turned by the handle AC of 20 inches, the threads of the screw being each $\frac{1}{2}$ an inch distant; and the screw turns a toothed wheel E, whose pinion L turns another wheel F, and pinion M of this another wheel G, to the pinion or barrel of which is hung a weight W; it is required to determine what weight a man will be able to raise, with this machine, working at the handle C; supposing the diameters of the wheel to be 18 inches, and those of the pinion and barrel 2 inches; the teeth and pinions being all of a size; and the man supposed, as in the former case, to be able to lift 150lbs, by his natural strength.

Here $20 \times 3{\cdot}1416 \times 2 = 125{\cdot}664$, is the circumference of the power.

And $125{\cdot}664 : \frac{1}{2}$, or $251{\cdot}328 : 1$, is the force of the screw alone.

Also 18 : 2, or 9 : 1, being the proportion of wheels to the pinions, and as there are three of them, therefore $9^3 : 1^3$ or 729 : 1 is the power gained by the wheels.

Plate 38, fig. 5.

Consequently $(251{\cdot}328 \times 729) : 1$ or 183218 : 1 nearly, is the ratio of the power to the weight, arising from the advantage both of the screw and the wheels. Hence $150 \times 183218 = 27482700$ lbs, is the weight which the man may sustain.

This must, however, only be considered as theoretical; for in practice the friction, which is very great, must of course enter into consideration.

4. A machine for showing the power of the screw may be contrived in the following manner. Let the wheel C (last fig. former column), have a screw *ab* on its axis, working in the teeth of the wheel D, which suppose to be 48 in number. It is plain that for every revolution of the wheel C, and screw *ab*, by the winch A, the wheel D will be moved one tooth by the screw; and therefore in 48 revolutions of the winch, the wheel D will be turned once round. Then if the circumference of a circle, described by the handle of the winch, be equal to the circumference of a groove *e* round the wheel D, the velocity of the handle will be 48 times as great as the velocity of any given point in the groove. Consequently when a line G goes round the groove *e*, and has a weight of 48lb hung to it below the pedestal EF, a power equal to one pound at the handle will balance and support the weight.

Archimedes's Screw, is a spiral pump, being a machine for raising water, first invented by him. Its structure and use will be understood by the following description of it. ABCD (Pl. 28, fig. 6) is a wheel, which is turned round, according to the order of those letters, by the fall of water EF, which need not be more than 3 feet. The axis G of the wheel is raised so as to make an angle of about 44° with the horizon; and on the top of that axle is a wheel H, which turns such another wheel I of the same number of teeth; the axle K of this last wheel being parallel to the axle G of the two former wheels. The axle G is cut into a double threaded screw, as in the annexed figure (fig. 7), exactly resembling the screw on the axis of the fly of a common jack, which must be what is called a right-handed screw, if the first wheel turns in the direction ABCD; but a left-handed screw, if the stream turns the wheel the contrary way; and the screw on the axle G must be cut in a contrary way to that on the axle K, because these axles turn in contrary directions. These screws must be covered close over with boards, like those of a cylindrical cask; and then they will be spiral tubes. Or they may be made of tubes or pipes of lead, and wrapt round the axles in shallow grooves cut in it, as in figure 8. The lower end of the axle G turns constantly in the stream that turns the wheel, and the lower ends of the spiral tubes are open into the water. So that, as the wheel and axle are turned round, the water rises in the spiral tubes, and runs out at L through the holes M, N, as they come about below the axle. These holes, of which there may be any number, as 4 or 6, are in a broad close ring on the top of the axle, into which ring the water is delivered from the upper open ends of the screw tubes, and falls into the open box N. The lower end of the axle K turns on a gudgeon in the water in N; and the spiral tubes in that axle take up the water from N, and deliver it into another such box under the top of K; on which there may be such another wheel as I, to turn a third axle by such a wheel upon it. And in this manner may water be raised to any proposed height, when there is a stream sufficient for that purpose to act on the broad float boards of the first wheel. Archimedes's screw, or a still simpler form of it, is also represented in fig. 9.

SCROLLS, or SCROWLS, or *Volutes*, a term in Architecture. See VOLUTES.

SCRUPLE, the least of the weights used by the ancients. Among the Romans it was the 24th part of an ounce, or the third part of a drachm.

SCRUPLE is still a small weight among us, equal to 20 grains, or the 3d part of a drachm. Among goldsmiths the scruple is 24 grains.

SCRUPLE, in Chronology, a small portion of time much used by the Chaldeans, Jews, Arabs, and other eastern people, in computations of time. It is the 1080th part of an hour, and by the Hebrews called Helakin.

SCRUPLES, in Astronomy. As

SCRUPLES *Eclipsed*, denote that part of the moon's diameter which enters the shadow, expressed in the same measure in which the apparent diameter of the moon is expressed. See DIGIT.

SCRUPLES *of Half Duration*, an arch of the moon's orbit, described by her from the beginning of an eclipse to its middle.

SCRUPLES *of Immersion*, or *Incidence*, an arch of the moon's orbit, which her centre describes from the beginning of the eclipse, to the time when the centre falls into the shadow. See IMMERSION.

SCRUPLES *of Emersion*, an arch of the moon's orbit, which her centre describes in the time from the first emersion of the moon's limb, to the end of the eclipse.

SEA, in Geography, is frequently used for that vast tract of water encompassing the whole earth, more properly called ocean. But

SEA is more properly used for a particular part or division of the ocean, denominated from the countries it washes, or from other circumstances. Thus we say, the Irish sea, the Mediterranean sea, the Baltic sea, the Red sea, &c.

SEA among sailors is variously applied, to a single wave, or to the agitation produced by a multitude of waves in a tempest, or to their particular progress and direction. Thus they say, a heavy sea broke over our quarter, or we shipped a heavy sea; there is a great sea in the offing; the sea sets to the southward. Hence a ship is said to head the sea, when her course is opposed to the setting or direction of the surges. A Long Sea implies a steady and uniform motion of long and extensive waves. On the contrary, a Short Sea is when they run irregularly, broken, and interrupted, so as frequently to burst over a vessel's side or quarter.

Properties and Affections of the SEA.

1. *General Motion of the Sea.* M. Dassie of Paris, in a work long since published, has been at great pains to prove that the sea has a general motion, independent of winds and tides, and of more consequence in navigation than is usually supposed. He affirms that this motion is from east to west, inclining toward the north when the sun is on the north side of the equinoctial, but toward the south when he is on the south side of it. Philos. Trans. No. 135.

2. *Bason or Bottom of the Sea*, or Fundus Maris, a term used to express the bed or bottom of the sea in general. Mr. Boyle has published a treatise on this subject, in which he has given an account of its irregularities and various depths, founded on the observations communicated to him by mariners.

Count Marsigli has, since Boyle's time, given a more accurate account of this part of the globe. The materials which compose the bottom of the sea, may reasonably be supposed, in some degree, to influence the taste of its waters; and this author has made many experiments to prove that fossil coal, and other bituminous substances, which are found in plenty at the bottom of the sea, may communicate in a great measure its bitterness to it.

It is a general rule among sailors, and is found to hold true in many instances, that the more the shores of any place are steep and high, forming perpendicular cliffs, the deeper the sea is below; and that on the contrary, level shores denote shallow seas. Thus the deepest part of the Mediterranean is generally allowed to be under the height of Malta. And the observation of the strata of earth and other fossils, on and near the shores, may serve to form a good judgment as to the materials to be found in its bottom. For the veins of salt and of bitumen doubtless run on the same, and in the same order, as we see them at land; and the strata of rocks that serve to support the earth of hills and elevated places on shore, serve also, in the same continued chain, to support the immense quantity of water in the bason of the sea.

The coral fisheries have given occasion to observe that there are many, and those very large caverns or hollows in the bottom of the sea, especially where it is rocky; and the same caverns are sometimes found in the perpendicular rocks which form the steep sides of those fisheries. These caverns are often of great depth, as well as extent, and have sometimes wide mouths, and sometimes only narrow entrances into large and spacious hollows.

The bottom of the sea is covered with a variety of matters, such as could not be imagined by any but those who have examined into it, especially in deep water, where the surface only is disturbed by tides and storms, the lower part, and consequently its bed at the bottom, remaining for ages perhaps undisturbed. The soundings, when the plummet first touches the ground on approaching the shores, give some idea of this. The bottom of the plummet is hollowed, and in that hollow there is placed a lump of tallow; which being the part that first touches the ground, the soft nature of the fat receives into it some part of those substances which it meets with at the bottom: this matter, thus brought up, is sometimes pure sand, sometimes a kind of sand made of the fragments of shells, beaten to a kind of powder, sometimes it is made of a like powder of the several sorts of corals, and sometimes it is composed of fragments of rocks; but besides these appearances, which are natural enough, and are what might well be expected, it brings up substances which are of the most beautiful colours. Marsigli Hist. Phys. de la Mer.

Dr. Donati, in an Italian work, containing an essay towards a natural history of the Adriatic sea, printed at Venice in 1750, has related many curious observations on this subject, and which confirm the observations of Marsigli. Having carefully examined the soil and productions of the various countries that surround the Adriatic sea, and compared them with those which he took up from the bottom of the sea; he found that there is very little difference between the former and the latter. At the bottom of the water there are mountains, plains, valleys, and caverns, similar to those upon land. The soil consists of different strata placed one upon another, and mostly parallel and correspondent to those of the rocks, islands, and neighbouring continents. They contain stones of different kinds, minerals, metals, various putrefied bodies, pumice stones, and lavas formed by volcanos.

One of the objects which most excited his attention, was a crust, which he discovered under the water, composed of crustaceous and testaceous bodies, and beds of polypes of different kinds, confusedly blended with earth, sand, and gravel; the different marine bodies which form this crust, are found at the depth of a foot or more, entirely petrified and reduced into marble; these he supposes are naturally placed under the sea when it covers them, and not by means of volcanos and earthquakes, as some have conjectured. On this account he imagines that the bottom of the sea is constantly rising higher and higher, with which other obvious causes of increase concur; and from this rising of the bottom of the sea, that of its level or surface naturally results; in proof of which this writer cites a great number of facts. Philos. Trans. vol. 49, pa. 585.

3. *Luminousness of the Sea.* This is a phenomenon that has been noticed by many nautical and philosophical writers. Mr. Boyle ascribes it to some cosmical law or custom of the terrestrial globe, or at least of the planetary vortex.

Father Bourzes, in his voyage to the Indies, in 1704, took particular notice of this phenomenon, and very minutely describes it, without assigning the true cause.

The Abbé Nollet was long of opinion, that the light of the sea proceeded from electricity; and others have had recourse to the same principle, and shown that the luminous points in the surface of the sea are produced merely by friction.

There are however two other hypotheses, which have been advanced to account for this phenomenon; the one of these ascribes it to the shining of luminous insects or animalcules, and the other to the light proceeding from the putrefaction of animal substances. The Abbé Nollet, who at first considered this luminousness as an electrical phenomenon, having had an opportunity of observing the circumstances of it, when he was at Venice in 1749, relinquished his former opinion, and concluded that it was occasioned either by the luminous aspect, or by some liquor or effluvia of an insect which he particularly describes, though he does not altogether exclude other causes, and especially the spawn or fry of fish.

The same hypothesis had also occurred to M. Vianelli; and both he and Grizellini, a physician in Venice, have given drawings of the insects from which they imagined this light to proceed.

A similar conjecture is proposed by a correspondent of Dr. Franklin, in a letter read at the Royal Society in 1756; the writer of which apprehends, that this appearance may be caused by a great number of little animals, floating on the surface of the sea. And Mr. Forster, in his account of a voyage round the world with captain Cook, in the years 1772, 3, 4, and 5, describes this phenomenon as a kind of blaze of the sea; and, having attentively examined some of the shining water, expresses his conviction that the appearance was occasioned by innumerable minute animals of a round shape, moving through the water in all directions, which appear separately as so many luminous sparks when taken up on the hand: he imagines that these small gelatinous luminous specks may be the young fry of certain species of some medusæ, or blubber. And M. Dagelat and M. Rigaud observed several times, and in different parts of the ocean, such luminous appearances by vast masses of different animalcules; and a few days after the sea was covered, near the coasts, with whole banks of small fish in innumerable multitudes, which they supposed had proceeded from the shining animalcules.

But M. le Roi, after giving much attention to this phenomenon, concludes that it is not occasioned by any shining insects, especially as, after carefully examining with a microscope some of the luminous points, he found them to have no appearance of an animal; and he also found that the mixture of a little spirit of wine with water just drawn from the sea, would give the appearance of a great number of little sparks, which would continue visible longer than those in the ocean: the same effect was produced by all the acids, and various other liquors. M. le Roi is far from asserting that there are no luminous insects in the sea; for he allows that several gentlemen have found them; but he is satisfied that the sea is luminous chiefly on some other account, though he does not so much as offer a conjecture with respect to the true cause.

Other authors, equally dissatisfied with the hypothesis of luminous insects, for explaining the phenomenon which is the subject of this article, have ascribed it to some substance of the phosphoric kind, arising from putrefaction. The observations of F. Bourzes, above referred to, render it very probable, that the luminousness of the sea arises from slimy and other putrescent matter, with which it abounds, though he does not mention the tendency to putrefaction, as a circumstance of any consequence to the appearance. But the experiments of Mr. Canton, which have the advantage of being easily made, seem to leave no room to doubt that the luminousness of the sea is chiefly owing to putrefaction. And his experiments confirm an observation of Sir John Pringle's, that the quantity of salt contained in sea water hastens putrefaction; but since that precise quantity of salt which promotes putrefaction the most, is less than that which is found in sea water, it is probable, Mr. Canton observes, that if the sea were less salt, it would be more luminous. See Philos. Trans. vol. 59, pa. 446, and Franklin's Exper. and Observ. pa. 274.

4. *Of the Depth of the Sea, its Surface, &c.*

What proportion the superficies of the sea bears to that of the land, is not accurately known, though it is said to be somewhat more than two to one. This ratio of the surface of the sea to the land, has been found by experiment thus: taking the printed paper map or covering of a terrestrial globe, with a pair of scissors clip out the parts that are land, and those that are water; then weighing these parts separately in a pair of fine scales, the land is found to be near $\frac{1}{3}$, and the water rather more than $\frac{2}{3}$ of the whole.

With regard to the profundity or depth of the sea, Varenius affirms, that it is in some places unfathomable, and in others very various, being in certain places from $\frac{1}{20}$th of a mile to $4\frac{1}{2}$ miles in depth, in other places deeper, but much less in bays than in oceans. In general, the depths of the sea bear a great analogy to the height of mountains on the land, so far as is hitherto discovered.

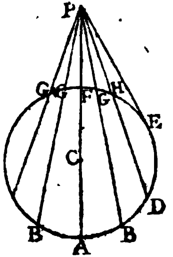
There is very good reason why the sea does not increase by means of rivers, &c, running every where into it; viz, because the vapours raised from the sea, and falling in rain upon the land, only cause a circulation of the water, but no increase of it. It has been found by calculations, founded on experiments, that in a summer's day, there may be raised in vapours from the surface of the Mediterranean sea, 528 millions of tuns of water; and yet this sea does not, from all its nine great rivers, receive more than 183 millions of tuns per day, which is but about a third part of what is exhausted in vapours; and this defect in the supply by the rivers, may serve to account for the continual influx of a current by the mouth or straits at Gibraltar. Indeed it is rather probable, that the waters of the sea suffer a continual slow decrease as to their quantity, by sinking always deeper into the earth, by filtering through the fissures in the strata and component parts; as also by the slow increase and raising of the earth's surface.

SEASONS, certain portions or quarters of the year, distinguished by the signs which the sun then enters at those periods.

The year is divided into four seasons, spring, summer, autumn, winter, which take their beginnings when the sun enters the first point of the signs Aries, Cancer, Libra, Capricorn.

The seasons are well illustrated by fig. 1, plate x; where the candle at I represents the sun in the centre, about which the earth moves in the ecliptic ABCD, which cuts the equinoctial *abcd* in the two equinoxes E and G. When the earth is in these two points, it is evident that the sun equally illuminates both the poles, and makes the days and nights equal in all parts of the earth. But while the earth moves from G by C to ♑, the upper or north pole becomes more and more enlightened, the days become longer, and the nights shorter; so that when the earth is at ♑, or the sun at ♋, our days are at the longest, as at midsummer. While the earth moves from ♑ by D to E, our days continually decrease, by the north pole gradually declining from the sun, till at E or autumn they become equal to the nights, or 12 hours long. Again, while the earth moves from E by A to F, the north pole beomes always more and more involved in darkness, and the days become shorter and shorter, till at F or ♋, when it is midwinter to the inhabitants of the northern sphere. Lastly, while the earth moves from ♋ by B to G, the north parts emerge more out of darkness, and the days grow continually longer, till at G the two poles are equally enlightened, and the days equal to the nights again. And so on continually year after year.

SECANT, in Geometry, a line that cuts another, whether right or curved: Thus the line PA or PB, &c, is a secant of the circle ABD, because of their cutting it in the point F, or G, &c. Properties of such secants to the circle are as follow:



1. Of several secants PA, PB, PD, &c, drawn from the same point P, that which passes through the centre C is the greatest; and from thence they decrease more and more as they recede farther from the centre; viz, PB less than PA, and PD less than PB, and so on, till they arrive at the tangent at E, which is the limit of all the secants.

2. Of these secants, the external parts PF, PG, PH, &c, are in the reverse order, increasing continually from F to E, the greater secant having the less external part, and in such proportion, that any secant and its external part are reciprocals, or the whole is reciprocally as its external part, and consequently that the rectangle of every secant and its external part is equal to a constant quantity, viz, the square of a tangent. Thus,

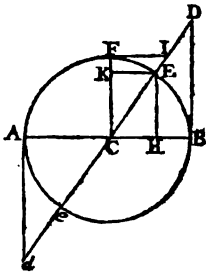
$$\mathrm{PA} : \frac{1}{\mathrm{PF}} :: \mathrm{PB} : \frac{1}{\mathrm{PG}} :: \mathrm{PD} : \frac{1}{\mathrm{PH}} \ \&c,$$

or $\mathrm{PA} \times \mathrm{PF} = \mathrm{PB} \times \mathrm{PG} = \mathrm{PD} \times \mathrm{PH} = \mathrm{PE}^2$.

3. The tangent PE is a mean proportional between any secant and its external part; as between PA and PF, or PB and PG, or PD and PH, &c.

4. The angle DPB, formed by two secants, is measured by half the difference of its intercepted arcs DB and GH.

SECANT, in Trigonometry, denotes a right line drawn from the centre of a circle, and, cutting the circumference, proceeds till it meets with a tangent to the same circle. Thus, the line CD, drawn from the centre C, till it meets the tangent BD, is called a secant; and particularly the secant of the arc BE, to which BD is a tangent. In like manner, by producing DC to meet the tangent A*d* in *d*, then C*d*, equal to CD, is the secant of the arch AE, which is the supplement of the arch BE. So that an arch and its supplement have their secants equal, only the latter one is negative to the former, being drawn the contrary way. And thus the secants in the 2d and 3d quadrant are negative, while those in the 1st and 4th quadrants are positive.



The secant CI of the arc EF, which is the complement of the former arc BE, is called the Cosecant of BE, or the secant of its complement. The cosecants in the 1st and 2d quadrants are affirmative, but in the 3d and 4th negative.

The secant of an arc is reciprocally as the cosine, and the cosecant reciprocally as the sine; or the rectangle of the secant and cosine, and the rectangle of the cosecant and sine, are each equal to the square of the radius.

For $\mathrm{CD} : \mathrm{CE} :: \mathrm{CB} : \mathrm{CH}$, or $\mathrm{S} : r :: r : c$,

and $\mathrm{CI} : \mathrm{CE} :: \mathrm{CF} : \mathrm{CK}$, or $\sigma : r :: r : s$;

and consequently $r^2 = c\mathrm{S} = s\sigma$; where r denotes the radius, s the sine, c the cosine, S the secant, and σ the cosecant.

Some of the most useful trigonometrical formulæ, into which the secants and cosecants enter, are the following.

$$\mathrm{Sec} = \sqrt{(r^2+\tan^2)} = \frac{r^2}{\cos} = \frac{r.\tan}{\sin} = \frac{\cot.\tan}{\cos}$$

$$= r\,\frac{\sqrt{(r^2+\cot^2)}}{\cot} = \frac{r^3}{\sin.\cot} = \frac{r.\mathrm{cosec}}{\cot}$$

$$= \frac{\tan.\mathrm{cosec}}{r} = \frac{\sin.\mathrm{cosec}}{\cos} = \frac{r.\mathrm{cosec}}{\sqrt{(\mathrm{cosec}^2-r^2)}}.$$

The secant of the sum or difference of any two arcs a and b may be expressed as follows:

$$\mathrm{Sec}\ (a \pm b) = \frac{r.\sec a.\sec b}{r^2 \mp \tan a.\tan b}.$$

$$\mathrm{Cosec}\ (a \pm b) = \frac{\mathrm{cosec}\ a.\mathrm{cosec}\ b}{\cot b \pm \cot a}.$$

The secants of the multiple arcs are exhibited in the following formulæ:

$$\mathrm{Sec}\ a = \sec a$$

$$\mathrm{Sec}\ 2a = \frac{\sec^2 a}{2-\sec^2 a}$$

$$\text{Sec } 3a = \frac{\sec^3 a}{4-3\sec^2 a}.$$

$$\text{Sec } 4a = \frac{\sec^4 a}{8-8\sec^2 a+\sec^4 a}.$$

$$\text{Sec } 5a = \frac{\sec^5 a}{16-20\sec^2 a+5\sec^4 a}, \text{ \&c, \&c.}$$

Or generally $\sec na =$

$$\frac{\sec^n a}{2^{n-1}\left(r^{n-1} - \frac{nr^{n-3}}{1.2^2}\sec^2 a + \frac{n(n-3)r^{n-5}}{1.2.2^4}\sec.^4 a - \frac{n(n-4)(n-5)r^{n-7}}{1.2.3.2^6}\sec.^6 a, \text{ \&c.}\right)}$$

Also, an arc a, to the radius r, being given, the secant s, and cosecant σ, and their logarithms, or the logarithmic secant and cosecant, may be expressed in infinite series, as follows, viz,

$$s = r + \frac{a^2}{2r} + \frac{5a^4}{24r^3} + \frac{61a^6}{720r^5} + \frac{277a^8}{8064r^7} \text{ \&c.}$$

$$\sigma = \frac{r^2}{a} + \frac{a}{6} + \frac{7a^3}{360r^2} + \frac{31a^5}{15120r^4} + \frac{127a^7}{604800r^6} \text{ \&c.}$$

$$\log. s = m \times \left(\frac{a^2}{2} + \frac{a^4}{12} + \frac{a^6}{45} + \frac{17a^8}{2520} \text{ \&c.}\right)$$

$$\log. \sigma = -\log. a + m \times \left(\frac{a^2}{6} + \frac{a^4}{180} + \frac{a^6}{2835} + \frac{a^8}{37800} \text{ \&c}\right)$$

where m is the modulus of the system of logarithms.

SECANTS, *Figure of*. See FIGURE *of Secants*.

SECANTS, *Line of*. See SECTOR, and SCALE.

SECOND, in Geometry, or Astronomy, &c, the 60th part of a prime or minute: either in the division of circles, or in the measure of time. A degree, or an hour, are each divided into 60 minutes, marked thus ′; a minute is subdivided into 60 seconds, marked thus ″; a second into 60 thirds, marked thus ‴; &c.

We sometimes say a second minute, a third minute, &c, but more usually only second, third, &c.

The seconds pendulum, or pendulum that vibrates seconds, in the latitude of London, is 39⅛ inches long.

SECONDARY *Circles of the Ecliptic*, are circles of longitude of the stars; or circles which, passing through the poles of the ecliptic, are at right angles to it.

By means of these secondary circles, all points in the heavens are referred to the ecliptic; that is, any star, planet, or other phenomenon, is understood to be in that point of the ecliptic, which is cut by the secondary circle that passes through such star, &c.

If two stars be thus referred to the same point of the ecliptic, they are said to be in conjunction; if in opposite points, they are in opposition; if they are referred to two points at a quadrant's distance, they are said to be in a quartile aspect, if the points differ a 6th part of the ecliptic, they are in sextile aspect, &c.

In general, all circles that intersect one of the six greater circles of the sphere at right angles, may be called secondary circles. As the azimuth or vertical circles in respect of the horizon, &c; the meridian in respect of the equator, &c.

SECONDARY *Planets*, or *Satellites*, are those moving round other planets as the centres of their motion, and along with them round the sun.

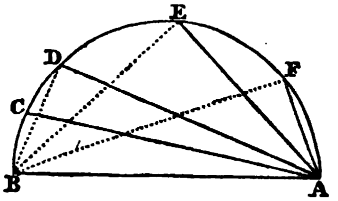
SECTION, in Geometry, denotes the intersection of two planes, or the surface made by a body's being cut by a plane, &c.

The common section of two planes is always a right line: being the line supposed to be drawn by one plane in its cutting or entering the other. If a sphere be cut in any manner by a plane, the figure of the section will be a circle; also the common intersection of the surfaces of two spheres, is the circumference of a circle; and the two common sections of the surfaces of a right cone and a sphere, are the circumferences of circles if the axis of the cone pass through the centre of the sphere, otherwise not; moreover, of the two common sections of a sphere and a cone, whether right or oblique, if the one be a circle the other will be a circle also, otherwise not. See my Tracts, vol. 1, tract 13, prop. 7, 8, 9.

The sections of a cone by a plane, are five; viz, a triangle, circle, ellipse, hyperbola, and parabola. See each of these terms, as also CONIC SECTION.

Sections of buildings and bodies, &c, are either vertical, or horizontal, &c. The

Angular SECTIONS, is a term given by Vieta to the analytical investigation of the law of increase, or decrease, of the sines and chords of multiple and submultiple arcs. Vieta first published this ingenious theory in 1579, with his Canon Mathematicus, which is nothing more than a table of sines constructed according to this principle. He there shows that, if in the semicircle BCD &c, there be taken any number of equal arcs, BD, DE, EF, FG, &c; and if we make the radius equal to 1, and AD = x, we shall have the series of supplementary chords AD, AE, AF, &c; which, according to the modern method of expression, will be represented as follows:



AB = 2
AD = x
AE = $x^2 - 2$
AF = $x^3 - 3x$
AG = $x^4 - 4x^2 + 2$
AH = $x^5 - 5x^3 + 5x$
AI = $x^6 - 6x^4 + 9x^2 - 2$
AK = $x^7 - 7x^5 + 14x^3 - 7x$
&c.

Vieta has also pointed out the law of this progression, by which it may be continued to infinity; that of the powers and signs is evident; and as to the co-efficients, he observes that, the coefficients of the second column are the series of natural numbers, beginning at 2; those of the third columns are triangular numbers, beginning at 2, instead of 1, as in the common form of those numbers; that is, 2, (2+3), (2+3+4), (2+3+4+5), &c: in the fourth column, they are the pyradical numbers, &c.

The ratio of the chords themselves as BD, BE, BF, &c, Vieta has also shown may be expressed in the following manner, by calling the first chord x, and radius = 1, as before, then the

2d chord - - - - - $2 - x^2$
3d - - - - - - - $3x + x^3$
4th - - - - - - $2 + 4x^2 - x^4$
5th - - - - - - $5x - 5x^3 + x^5$, &c.

The law of the progression being the same as in the former case.

Various other curious and useful formulæ and observations, on the doctrine of angular sections, may be seen in the work above alluded to, and in the Opuscula of Oughtred, first published in 1667.

Vertical SECTION, or simply the SECTION, of a building, denotes its profile, or a delineation of its heights and depths raised on the plan; as if the fabric had been cut asunder by a vertical plane, to discover the inside. And

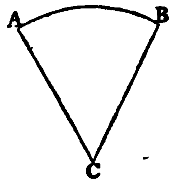
Horizontal Section is the ichnography or ground plan, or a section parallel to the horizon.

Section *of a Ratio*, or Proportional Section, one of the last works of Apollonius, in 2 books, restored by Snell, in 1607, and by Halley, in 1706, 8vo.

Section *of a Space*, another of the last works of Apollonius, in 2 books, restored by Snell in 1607.

Section, *Determinate*. See Determinate *Section*.

SECTOR, of a Circle, is a portion of the circle comprehended between two radii and their included arc. Thus, the figure ABC, contained between the two radii AC and BC, and the arc AB, is a sector of the circle.



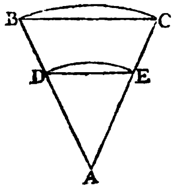
The sector of a circle, as ABC, is equal to a triangle, whose base is the arc AB, and its altitude the radius AC or BC. And therefore the radius being drawn into the arc, half the product gives the area.

Similar Sectors, are those which have equal angles included between their radii. These are to each other as the squares of their bounding arcs, or as their whole circles.

Sector also denotes a mathematical instrument, which is of great use in geometry, trigonometry, surveying, &c, in measuring and laying down and finding proportional quantities of the same kind: as, between lines and lines, surfaces and surfaces, &c: whence it was called the Compass of Proportion, by the French and the Germans, &c.

The great advantage of the sector above the common scales, &c, is, that it is adapted to all radii, and all scales. By the lines of chords, sines, &c, on the sector, we have lines of chords, sines, &c, to any radius between the length and breadth of the sector when open.

The sector is founded on the 4th proposition of the 6th book of Euclid; where it is demonstrated, that similar triangles have their like sides proportional. An idea of the theory of its construction may be conceived thus. Let the lines AB, AC represent the legs of the sector; and AD, AE, two equal sections from the centre: then if the points BC and DE be connected, the lines BC and DE will be parallel; therefore the triangles ABC, ADE will be similar, and consequently the sides AB, BC, AD, DE proportional, that is, as AB : BC : : AD : : DE; so that if AD be the half, 3d, or 4th part of AB, then DE will be a half, 3d, or 4th part of BC: and the same holds of all the rest. Hence, if DE be the chord, sine or tangent, of any arc, or of any number of degrees, to the radius AD, then BC will be the same to the radius AB.



The sector, it is supposed, was the invention of Guido Baldo or Ubaldo, about the year 1568. The first printed account of it was in 1584, by Gaspar Mordente at Antwerp, who indeed says that his brother Fabricius Mordente invented it, in the year 1554. It was next treated of by Daniel Speckle, at Strasburgh, in 1589; after that by Dr. Thomas Hood, at London, in 1598; then by Lewin Hulse, at Frankfort on the Maine, 1603, who says it was invented long before by Justus Byrgius, an engineer in the service of the Landgrave of Hesse. But that honour was claimed, and even contended for, by Galileo and by Balthasar Capra of Milan. The former published a Tract on that useful instrument in 1607; and it doubtless received improvements from him, as well as from our countrymen Gunter, Foster, and others. See Wolfii Elem. Math. tom. 5, pa. 49; also Saverien Diction. art. Compass, and Cunn on the Sector, published by Stone, Preface. It was treated on afterwards by many other writers on practical geometry, in all the nations of Europe.

Description of the Sector. This instrument consists of two rules or legs, the longer the better, made of box, or ivory, or brass, &c, representing the radii, moveable round an axis or joint, the middle of which represents the centre; from whence several scales are drawn on the faces. See the fig. 1, plate xxxii.

The scales usually set upon sectors, may be distinguished into single and double. The single scales are such as are set upon plane scales: the double scales are those which proceed from the centre; each of these being laid twice on the same face of the instrument, viz, once on each leg. From these scales, dimensions or distances are to be taken, when the legs of the instrument are set in an angular position.

The scales set upon the best sectors are

Single	1	A line of	Inches, each divided into 8 and 10 parts,			
	2		Decimals, containing 100 parts.			
	3		Chords		marked	Cho.
	4		Sines			Sin.
	5		Tangents			Tan.
	6		Rhumbs			Rhum.
	7		Latitude			Lat.
	8		Hours			Hou.
	9		Longitude			Lon.
	10		Inclin. Merid.			In. mer.
	11		the logarithms of	Numbers		Num.
	12			Sines		Sin.
	13			Versed sines		V. Sin.
	14			Tangents		Tan.
Double	1	a line of	Lines, or equal parts		marked	Lin.
	2		Chords			Cho.
	3		Sines			Sin.
	4		Tangents to 45°			Tan.
	5		Secants			Sec.
	6		Tangents to above 45°.			Tan.
	7		Polygons			Pol.

The manner in which these scales are disposed on the sector, is best seen in the figure.

The scales of lines, chords, sines, tangents, rhumbs, latitudes, hours, longitude, incl. merid. may be used, with the instrument either shut or open, each of these scales being contained on one of the legs only. The scales of inches, decimals, log. numbers, log. sines, log. versed sines, and log. tangents, are to be used with the sector quite open, with the two rulers or legs stretched out in the same direction, part of each scale lying on both legs.

The double scales of lines, chords, sines, and lower tangents, or tangents under 45°, are all of the same radius or length; they begin at the centre of the instrument, and are terminated near the other extremity of each leg; viz, the lines at the division 10, the chords at 60, the sines at 90, and the tangents at 45; the remainder of the tangents, or those above 45°, are on other scales beginning at ¼ of the length of the former, counted from the centre, where they are marked with 45, and run to about 76 degrees.

The secants also begin at the same distance from the centre, where they are marked with 10, and are from thence continued to as many degrees as the length of the sector will allow, which is about 75°.

The angles made by the double scales of lines, of chords, of sines, and of tangents to 45 degrees, are always equal. And the angles made by the scales of upper tangents, and of secants, are also equal.

The scales of polygons are set near the inner edge of the legs; and where these scales begin, they are marked with 4, and from thence are figured backwards, or towards the centre, to 12.

From this disposition of the double scales, it is plain, that those angles that are equal to each other while the legs of the sector were close, will still continue to be equal, though the sector be opened to any distance.

The scale of inches is laid close to the edge of the sector, and sometimes on the very edge; it contains as many inches as the instrument will receive when opened; each inch being usually divided into 8, and also into 10 equal parts. The decimal scale lies next to this: it is of the length of the sector when opened, and is divided into 10 equal parts, or primary divisions, and each of these into 10 other equal parts; so that the whole is divided into 100 equal parts: and by this decimal scale, all the other scales, that are taken from tables, may be laid down. The scales of chords, rhumbs, sines, tangents, hours, &c, are such as are described under Plane Scale.

The scale of logarithmic or artificial numbers, called Gunter's scale, or Gunter's line, is a scale expressing the logarithms of common numbers, taken in their natural order.

The construction of the double scale will be evident by inspecting the instrument. As to the scale of polygons, it usually comprehends the sides of the polygons from 6 to 12 sides inclusive: the divisions are laid down by taking the lengths of the chords of the angles at the centre of each polygon, and laying them down from the centre of the instrument. When the polygons of 4 and 5 sides are also introduced, this line is constructed from a scale of chords, where the length of 90° is equal to that of 60° of the double scale of chords on the sector.

In describing the use of the sector, the terms *lateral distance* and *transverse distance* often occur. By the former is meant the distance taken with the compasses on one of the scales only, beginning at the centre of the sector; and by the latter, the distance taken between any two corresponding divisions of the scales of the same name, the legs of the sector being in an angular position.

Uses of the SECTOR.

Of the Line of Lines. This is useful, to divide a given line into any number of equal parts, or in any proportion, or to make scales of equal parts, or to find 3d and 4th proportionals, or mean proportionals, or to increase or decrease a given line in any proportion. Ex. 1. To divide a given line into any number of equal parts, as suppose 9: make the length of the given line a transverse distance to 9 and 9, the number of parts proposed; then will the transverse distance of 1 and 1 be one of the equal parts, or the 9th part of the whole; and the transverse distance of 2 and 2 will be 2 of the equal parts, or $\frac{2}{9}$ of the whole line; and so on. 2. Again, to divide a given line into any number of parts that shall be in any assigned proportion, as suppose three parts, in the proportion of 2, 3, and 4. Make the given line a transverse distance to 9, the sum of the proposed numbers 2, 3, 4; then the transverse distances of these numbers severally will be the parts required.

Of the Scale of Chords. 1. To open the sector to any angle, as suppose 50 degrees: Take the distance from the joint, or centre, to 50 on the chords, the number of degrees proposed; then open the sector till the transverse distance from 60 to 60, on each leg, be equal to the said lateral distance of 50; so shall the scale of chords make the proposed angle of 50 degrees.—By the converse of this operation, may be known the angle the sector is opened to; viz, taking the transverse distance of 60, and applying it laterally from the joint.

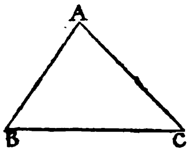
2. To protract or lay down an angle of any given number of degrees. At any opening of the sector, take the transverse distance of 60°, with which extent describe an arc; then take the transverse distance of the number of degrees proposed, and apply it to that arc; and through the extremities of this distance on the arc draw two lines from the centre, and they will form the angle as proposed. When the angle exceeds 60°, lay it off at twice or thrice. —By the converse operation any angle may be measured; viz, With any radius describe an arc from the angular point; set that radius transversely from 60 to 60; then take the distance of the intercepted arc, and apply it transversely to the chords, which will show the degrees in the given angle.

Of the Line of Polygons. 1. In a given circle to inscribe a regular polygon, for example an octagon. Open the legs of the sector till the transverse distance from 6 to 6 be equal to the radius of the circle; then will the transverse distance of 8 and 8 be the side of the inscribed octagon. 2. Upon a line given to describe a regular polygon. Make the given line a transverse dis. to 5 and 5; and at that opening of the sector take the transverse distance of 6 and 6; with which as a radius, from the extremities of the given line describe arcs to intersect each other, and this intersection will be the centre of a circle in which the proposed polygon may be inscribed; then from that centre describe the said circle through the extremities of the given line, and apply this line continually round the circumference, for the several angular points of the polygon. 3. On a given right line as a base, to describe an isosceles triangle, having the angles at the base double the angle at the vertex. Open the sector till the length of the given line fall transversely on 10 and 10 on each leg; then take the transverse distance to 6 and 6, and it will be the length of each of the equal sides of the triangle.

Of the Sines, Tangents, and Secants. By the several lines disposed on the sector, we have scales of several radii. So that, 1. Having a length or radius given, not exceeding the length of the sector when opened, we can find the chord, sine, &c, to the same: for ex. suppose the chord, sine, or tangent of 20 degrees to a radius of 3 inches be required. Make 3 inches the opening or transverse distance to 60 and 60 on the chords; then will the same extent reach from 45 to 45 on the tangents, and from 90 to 90 on the sines; so that to whatever radius the line of chords is set, to the same are all the others set also. In this disposition therefore, if the transverse distance between 20 and 20 on the chords be taken with the compasses, it will give the chord of 20 degrees; and if the transverse of 20 and 20 be in like manner taken on the sines, it will be the sine of 20 degrees; and lastly, if the transverse distance of 20 and 20 be taken on the tangents, it will be the tangent of 20 degrees, to the same radius.—2. If the chord or tangent of 70 degrees were required. For the chord, the transverse distance of half the arc, viz 35, must be taken, as before; which distance taken twice gives the chord of 70 degrees. To find the tangent of 70 degrees, to the same radius, the scale of upper tangents must be used, the under one only reaching to 45: making therefore 3 inches the transverse distance to 45 and 45 at the beginning of that scale, the extent between 70 and 70 degrees on the same, will be the tangent of 70 degrees to 3 inches radius.—3. To find the secant of an arc; make the given radius the transverse distance between 0 and 0 on the secants; then will the transverse distance of 20 and 20, or 70 and 70, give the secant of 20 or 70 degrees.—4. If the radius, and any line representing a sine, tangent, or secant, be given, the degrees corresponding to that line may be found by setting the sector to the given radius, according as a sine, tangent, or secant is concerned; then taking the given line between the compasses, and applying the two feet transversely to the proper scale, and sliding the feet along till they both rest on like divisions on both legs; then the divisions will show the degrees and parts corresponding to the given line.

Use of the Sector in Trigonometry, or in working any other proportions.

By means of the double scales, which are the parts more peculiar to the sector, all proportions are worked by the property of similar triangles, making the sides proportional to the bases, that is, on the sector, the lateral distances proportional to the transverse ones; thus, taking the distance of the first term, and applying it to the 2d, then the distance of the 3d term, properly applied, will give the 4th term: observing that the sides of triangles are taken off the line of numbers laterally, and the angles are taken transversely, off the sines or tangents or secants, according to the nature of the proportion. For example, in a plane triangle ABC, given two sides and an angle opposite to one of them, to find the rest; viz, given AB = 56, AC = 64 and ∠B = 46° 30′, to find BC and the angles A and C. In this case, the sides are proportional to the sines of their opposite angles; hence these proportions,



as AC (64) : sin. ∠B (46° 30′) : : AB (56) : sin. ∠C, and as sin. B : AC : : sin. A : : BC.

Therefore, to work these proportions by the sector, take the lateral distance of 64 = AC from the lines, and open the sector to make this a transverse distance of 46° 30′ = ∠B, on the sines; then take the lateral distance of 56 = AB on the lines, and apply it transversely on the sines, which will give 39° 24′ = ∠C. Hence, the sum of the angles B and C, which is 85° 54′, taken from 180°, leaves 94° 6′ = ∠A. Then, to work the 2d proportion, the sector remaining set at the same opening as before, take the transverse distance of 94° 6′ = ∠A, on the sines, or, which is the same thing, the transverse distance of its supplement 85° 54′; then this applied laterally to the lines, gives 88 = the side BC sought.

For the complete history of the sector, with its more ample and particular construction and uses, see the Introduction to Robertson's Treatise of such Mathematical Instruments, as are usually put into a Portable Case.

SECTOR *of a Sphere*, is the solid generated by the revolution of the sector of a circle about one of its radii; the other radius describing the surface of a cone, and the circular arc a circular portion of the surface of the sphere of the same radius. So that the spherical sector consists of a right cone, and of a segment of the sphere having the same common base with the cone. And hence the solid content of it will be found by multiplying the base or spherical surface by the radius of the sphere, and taking a 3d part of the product.

SECTOR *of an Eclipse*, or *of an Hyperbola*, &c, is a part resembling the circular sector, being contained by three lines, two of which are radii, or lines drawn from the centre of the figure to the curve, and the intercepted arc or part of that curve.

Astronomical SECTOR, an instrument invented by Mr. George Graham, for finding the difference in right ascension and declination between two objects, whose distance is too great to be observed through a fixed telescope, by means of a micrometer. This instrument (fig. 2, pl. 32,) consists of a brass plate, called the sector, formed like a T, having the shank CD, as a radius, about 2½ feet long, and 2 inches broad at the end D, and an inch and a half at C; and the cross-piece AB, as an arch, about 6 inches long, and one and a half broad; upon which, with a radius of 30 inches, is described an arch of 10 degrees, each degree being divided in as many parts as are convenient. Round a small cylinder C, containing the centre of this arch, and fixed in the shank, moves a plate of brass, to which is attached a telescope CE, having its line of collimation parallel to the plane of the sector, and passing over the centre C of the arch AB, and the index of a Vernier's dividing plate, whose length, being equal to 16 quarters of a degree, is divided into 15 equal parts, fixed to the eye end of the telescope, and made to slide along the arch; which motion is performed by a long screw, G, at the back of the arch, communicating with the Vernier through a slit cut in the brass, parallel to the divided arch. Round the centre F of a circular brass plate *abc*, of 5 inches diameter, moves a brass cross KLMN, having the opposite ends O and P of one bar turned up perpendicularly about 3 inches, to serve as supporters to the sector, and screwed to the back of its radius; so that the plane of the sector is parallel to the plane of the circular plate, and can revolve round the centre of that plate in this parallel position. A square iron axis HIF, 18 inches long, is screwed flat to the back of the circular plate along one of its diameters, so that the axis is parallel to the plane of the sector. The whole instrument is supported on a proper pedestal, so that the said axis shall be parallel to the earth's axis, and proper contrivances are annexed to fix it in any position. The instrument, thus supported, can revolve round its axis HI, parallel to the earth's axis, with a motion like that of the stars, the plane of the sector being always parallel to the plane of some hour-circle, and consequently every point of the telescope describing a parallel of declination; and if the sector be turned round the joint F of the circular plate, its graduated arch may be brought parallel to an hour-circle; and consequently any two stars, whose difference of declination does not exceed the degrees in that arch, will pass over it.

To observe their passage, direct the telescope to the preceding star, and fix the plane of the sector a little to the westward of it; move the telescope by the screw G, and observe at the transit of each over the cross wires the time shown by the clock, and also the division upon the arch AB, shown by the index; then is the difference of the arches the difference of the declination; and that of the times shows the difference of the right ascension of those stars. For a more particular description of this instrument, see Smith's Optics, book iii, chap. 9.

SECULAR *Equations*, or Century Equations, in Astronomy, are corrections required to compensate such inequalities, in the celestial motions, as occur in the course of a century or 100 years. Thus, there are secular inequalities in the moon's motion, which require for their correction as many distinct secular equations. For which, see the books on astronomy.

SECULAR *Year*, the same with Jubilee.

SECUNDANS, an infinite series of numbers, beginning from nothing, and proceeding according to the squares of numbers in arithmetical progression, as 0, 1, 4, 9, 16, 25, 36, 49, 64, &c.

SEEING, the act of perceiving objects by the organ of sight; or the sense we have of external objects by means of the eye.

For the apparatus, or diposition of the parts necessary to seeing, see EYE. And for the manner in which seeing is performed, and the laws of it, see VISION.

Our best anatomists differ greatly as to the cause why we do not see double with the two eyes. Galen, and others after him, ascribe it to a coalition, or decussation, of the optic nerve, behind the os sphenoides. But whether they decussate or coalesce, or only barely touch one another, is not well agreed on.—The Bartholines and Vesalius say expressly, they are united by a perfect confusion of their substance; Dr. Gibson allows them to be united by the closest conjunction, but not by a confusion of their fibres.—Alhazen, an Arabian philosopher of the 12th century, accounts for single vision by two eyes, by supposing that when two corresponding parts of the retina are affected, the mind perceives but one image.

Descartes and others account for the effect another way; viz, by supposing that the fibrillæ constituting the medullary part of those nerves, being spread in the retina of each eye, have each of them corresponding parts in the brain, so that when any of those fibrillæ are struck by any part of an image, the corresponding parts of the brain are affected by it. Somewhat like which is the opinion of Dr. Briggs, who takes the optic nerves of each eye to consist of homologous fibres, having their rise in the thalamus nervorum opticorum, and being thence continued to both the retinæ, which are composed of them; and further, that those fabrillæ have the same parallelism, tension, &c, in both eyes; consequently when an image is painted on the same corresponding sympathizing parts of each retina, the same effects are produced, the same notice carried to the thalamus, and so imparted to the mind. Hence it is, that double vision ensues upon an interruption of the parallelism of the eyes; as when one eye is depressed by the finger, or their symphony is interrupted by disease: but Dr. Briggs maintains, that it is but in few subjects there is any decussation; and in none any conjunction more than mere contact; though his notion is by no means consonant to facts, and it is attended with many improbable circumstances.

It was the opinion of Sir Isaac Newton, and of many others, that objects appear single, because the two optic nerves unite before they reach the brain. But Dr. Porterfield shows, from the observation of several anatomists, that the optic nerves do not mix or confound their substance, being only united by a close cohesion; and objects have appeared single, where the optic nerves were found to be disjoined. To account for this phenomenon, this ingenious writer supposes, that, by an original law in our natures, we imagine an object to be situated somewhere in a right line drawn from the picture of it upon the retina, through the centre of the pupil; consequently the same object appearing to both eyes to be in the same place, we cannot distinguish it into two. In answer to an objection to this hypothesis, from objects appearing double when one eye is distorted, he says, the mind mistakes the position of the eye, imagining, that it had moved in a manner corresponding to the other, in which case the conclusion would have been just: in this he seems to have recourse to the power of habit, though he disclaims that hypothesis. This principle however has been thought sufficient to account for this appearance.

Originally, every object making two pictures, one in each eye, is imagined to be double; but, by degrees, we find that when two corresponding parts of the retina are impressed, the object is but one; but if those corresponding parts be changed by the distortion of one of the eyes, the object must again appear double as at the first. This seems to be verified by Mr. Cheselden, who informs us, that a gentleman, who, from a blow on his head, had one eye distorted, found every object to appear double, but by degrees the most familiar ones came to appear single again, and in time all objects did so without amendment of the distortion. A similar case is mentioned by Dr. Smith.—On the other hand, Dr. Reid is of opinion, that the correspondence of the centres of two eyes, on which single vision depends, does not arise from custom, but from some natural constitution of the eye, and of the mind. M. du Tour adopts an opinion, long before suggested by Gassendi, that the eye attends to no more than the image made in one eye at a time; in support of which, he produces several curious experiments; but as M. Buffon observes, it is a sufficient answer to this hypothesis, that we see more distinctly with two eyes than with one; and that when a round object is near us, we plainly see more of the surface in one case than in the other.

With respect to single vision with two eyes, Dr. Hartley observes, that it deserves particular attention, that the optic nerves of man, and such other animals as look the same way with both eyes, unite in the sella turrica in a ganglion, or little brain, as it may be called, peculiar to themselves, and that the associations between synchronous impressions on the two retinas, must be made sooner and cemented stronger on this account; also that they ought to have a much greater power over one another's image, than in any other part of the body. And thus an impression made on the right eye alone by a single object, propagates itself into the left, and there raises up an image almost equal in vividness to itself; and, consequently, when we see with one eye only, we may however have pictures in both eyes.

It is a common observation, says Dr. Smith, that objects seen with both eyes appear more vivid and stronger than they do to a single eye, especially when both of them are equally good. Porterfield on the Eye, vol. ii, pa. 285, 315. Smith's Optics, Remarks, pa. 31. Reid's Inquiry, pa. 267. Mem. Présentes, pa. 514. Acad. Par. 1747. Mem. Pr. 334. Hartley on Man, vol. i, pa. 207. Priestley's Hist. of Light and Colours, pa. 663, &c.

Whence it is that we see objects erect, when it is certain that the images thereof are painted invertedly on the retina, is another difficulty in the theory of seeing. Descartes accounts for it hence, that the notice which the soul takes of the object, does not depend on any image, nor any action coming from the object, but merely on the situation of the minute parts of the brain, whence the nerves arise; ex. gr. the situation of a capillament brain, which occasions the soul to see all those places lying in a right line with it.

But Mr. Molyneux gives another account of this matter. The eye, he observes, is only the organ, or instrument; it is the soul that sees. To enquire then, how the soul perceives the object erect by an inverted image, is to enquire into the soul's faculties. Again, imagine that the eye receives an impulse on its lower part, by a ray from the upper part of an object; must not the visive faculty be hereby directed to consider this stroke as coming from the top, rather than the bottom of the object, and consequently be determined to conclude it the representation of the top?

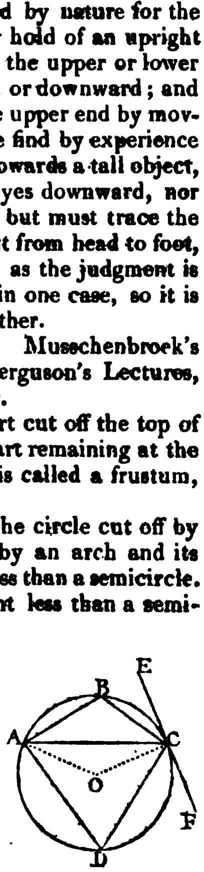
On these principles, we are to consider, that inverted is only a relative term, and that there is a very great difference between the real object, and the means or image by which we perceive it. When all the parts of a distant prospect are painted on the retina (supposing that to be the seat of vision), they are all right with respect to one another, as well as the parts of the prospect itself; and we can only judge of an object being inverted, when it is turned reverse to its natural position with respect to other objects which we see and compare it with.

The eye or visive faculty (says Molyneux) takes no notice of the internal surface of its own parts, but uses them as an instrument only, contrived by nature for the exercise of such a faculty. If we lay hold of an upright stick in the dark, we can tell which is the upper or lower part of it, by moving our hand upward or downward; and very well know that we cannot feel the upper end by moving our hand downward. Just so, we find by experience and habit, that by directing our eyes towards a tall object, we cannot see its top by turning our eyes downward, nor its foot by turning our eyes upward; but must trace the object the same way by the eye to see it from head to foot, as we do by the hand to feel it; and as the judgment is informed by the motion of the hand in one case, so it is also by the motion of the eye in the other.

Molyneux's Dioptr. pa. 105, &c. Musschenbroek's Int. ad Phil. Nat. vol. ii, pa. 762. Ferguson's Lectures, pa. 132. See VISIBLE, VISION, &c.

SEGMENT, in Geometry, is a part cut off the top of a figure by a line or plane; and the part remaining at the bottom, after the segment is cut off, is called a frustum, or a zone. So, a

SEGMENT *of a Circle*, is a part of the circle cut off by a chord, or a portion comprehended by an arch and its chord; and may be either greater or less than a semicircle. Thus, the portion ABCA is a segment less than a semicircle; and ADCA a segment greater.



The angle formed by lines drawn from the extremities of a chord to meet in any point of the arc, is called an angle *in* the segment. So the angle ABC is an angle *in* the segment ABCA; and the angle ADC, an angle *in* the segment ADCA.

Also the angle B is said to be the angle *upon* the segment ADC, and D the angle *on* the segment ABC.

The angle which the chord AC makes with a tangent EF, is called the angle *of* a segment; and it is equal to the angle in the alternate or supplemental segment, or equal to the supplement of the angle in the same segment. So the angle ACE is the angle *of* the segment ABC, and is equal to the angle ADC, or to the supplement of the angle B; also the angle ACF is the angle *of* the segment ADC, and is equal to the angle B, or to the supplement of the angle D.

The area of a segment ABC, is evidently equal to the difference between the sector OABC of the same arc, and the triangle OAC on the same chord; the triangle being subtracted from the sector, to give the segment when less than a semicircle; but to be added when greater. See more rules for the segment in my Mensuration, pa. 99, &c, 4th edition.

Similar SEGMENTS, are those that have their chords directly proportional to their radii or diameters, or that have similar arcs, or such as contain the same number of degrees.

SEGMENT *of a Sphere*, is a part cut off by a plane.

The base of a segment is always a circle. And the convex surfaces of different segments, of the same sphere, are to each other as their altitudes, or versed sines. And as the whole convex surface of the sphere is equal to 4 of its great circles, or 4 circles of the same diameter; so the surface of any segment, is equal to 4 circles on a diameter equal to the chord of half the arc of the segment. So that if d denote the diameter of the sphere, or the chord of half the circumference, and c the chord of half the arc of any other segment, also a the altitude or versed sine of the same; then,

$3{\cdot}1416d^2$ is the surface of the whole sphere, and

$3{\cdot}1416c^2$, or $3{\cdot}1416ad$, the surface of the segment.

For the solid content of a segment, there are two rules usually given; viz, 1. To 3 times the square of the radius of its base, add the square of its height; multiply the sum by the height, and the product by ·5236. Or, 2dly, From 3 times the diameter of the sphere, subtract twice the height of the frustum; multiply the remainder by the square of the height, and the product by ·5236. That is, in symbols, the solid content is either

$= {\cdot}5236a \times (3r^2 + a^2)$, or $= {\cdot}5236a^2 \times (3d - 2a)$; where a is the altitude of the segment, r the radius of its base, and d the diameter of the whole sphere.

Line of SEGMENTS, are two particular lines, so called, on Gunter's sector. They lie between the lines of sines and superficies, and are numbered with 5, 6, 7, 8, 9, 10. They represent the diameter of a circle, so divided into 100 parts, that a right line drawn through those parts, and perpendicular to the diameter, shall cut the circle into two segments, the greater of which will have the same proportion to the whole circle, as the parts cut off have to 100.

SELENOGRAPHY, the description and representation of the moon, with all the parts and appearances of her disc or face; as geography does those of the earth. Since the invention of the telescope, selenography is very much improved. We have now distinct names for most of the supposed regions, seas, lakes, mountains, &c, visible in the moon's body. Hevelius, a celebrated astronomer of Dantzic, and who published the first selenography, named the several places of the moon from those of the earth. But Riccioli afterwards called them by the names of the most celebrated astronomers and philosophers. Thus, what the one calls Mons Porphyrites, the other calls Aristarchus; what the one calls Ætna, Sinai, Athos, Apenninus, &c, the other calls Copernicus, Posidonius, Tycho, Gassendus, &c.—M. Cassini has published a work called Instructions Seleniques, and has published the best map of the moon.

SELEUCIDÆ, in Chronology, the era of the Seleucidæ, or the Syro-Macedonian era, which is a computation of time, commencing from the establishment of the Seleucidæ, a race of Greek kings, who reigned as successors of Alexander the Great, in Syria, as the Ptolemies did in Egypt. According to the best accounts, the first year of this era falls in the year 311 before Christ, which was 12 years after the death of Alexander.

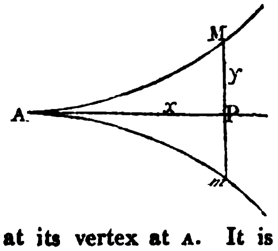
SELL, in Building, is of two kinds, viz, Ground-Sell, which denotes the lowest piece of timber in a wooden building, and that upon which the whole superstructure is raised. And sell of a window, or of a door, which is the bottom piece in the frame of them, upon which they rest.

SEMICIRCLE, in Geometry, is half a circle, or a figure comprehended between the diameter of a circle, and half the circumference.

SEMICIRCLE is also an instrument in Surveying, sometimes called the Graphometer. It consists of a semicircular limb or arch, as FIG (fig. 3, pl. 32) divided into 180 degrees, and sometimes subdivided diagonally or otherwise into minutes. This limb is subtended by a diameter FG, having two sights erected at its extremities. In the centre of the semicircle, or the middle of the diameter, is fixed a box and needle; and on the same centre an alidade, or moveable index, carrying two other sights, as H, I: the whole being set on a staff, with a ball and socket, &c.

Hence it appears, that the semicircle is nothing but half a theodolite; with this only difference, that whereas the limb of the theodolite, being an entire circle, takes in all the 360° successively; while in the semicircle the degrees only going from 1 to 180, it is usual to have the remaining 180°, or those from 180° to 360°, graduated in another line on the limb within the former.

To take an Angle with a Semicircle.—Place the instrument in such manner, as that the radius CG may hang over one leg of the angle to be measured, with the centre C over the vertex of the same. The first is done by looking through the sights F and G, at the extremities of the diameter, to a mark fixed up in one extremity of the leg; and the latter is had by letting fall a plummet from the centre of the instrument. This done, turn the moveable index HI on its centre towards the other leg of the angle, till, through the sights fixed in it, you see a mark in the extremity of the leg. Then the degree which the index cuts on the limb, is the quantity or measure of the angle. Other uses are the same as in the theodolite.



SEMICUBICAL PARABOLA, a curve of the 2d order, of such a nature that the cubes of the ordinates are proportional to the squares of the abscisses, its equation being $ay^2 = x^3$. This curve, AMm, is one of Newton's five diverging parabolas, being his 70th species; having a cusp at its vertex at A. It is otherwise named the Neilian parabola, from the name of the author who first treated of it, or squared it.

The area of the space APM, is $= \frac{4}{15}xy = \frac{4}{15}$AP × PM, or $\frac{4}{15}$ of the circumscribing rectangle.

The content of the solid generated by the revolution of the space APM about the axis AP, is $\frac{1}{4}pxy^2 = \cdot7854$AP × PM², or $\frac{1}{4}$ of the circumscribing cylinder. And a circle equal to the surface of that solid may be found from the quadrature of an hyperbolic space.

Also the length of any arc AM of the curve may be easily obtained from the quadrature of a space contained under part of the curve of the common parabola, two semiordinates to the axis, and the part of the axis contained between them.

This curve may be described by a continued motion, viz, by fastening the angle of a square in the vertex of a common parabola; and then carrying the intersection of one side of this square and a long ruler (which ruler always moves perpendicularly to the axis of the parabola) along the curve of that parabola. For the intersection of the ruler, and the other side of the square will describe a semicubical parabola. Maclaurin performs this without a common parabola, in his Geometria Organica.

SEMIDIAMETER, the Radius, or half-diameter of a circle or sphere, is a line drawn from the centre to the circumference. And in any curve that has diameters and a centre, it is the radius, or half-diameter, or a line drawn from the centre to some point in the curve.

The distances, diameters, &c, of the heavenly bodies, are usually estimated by astronomers in semidiameters of the earth; the number of which terrestrial semidiameters, contained in that of each of those planets, is as below.

	Semidiam.		Semidiam.
The Earth - -	1	Juno - - -	
The Sun - -	111·25	Pallas - - -	
The Moon - -	0·27	Ceres - - -	
Mercury - -	0·38	Jupiter - -	11·81
Venus - - -	1·15	Saturn - -	9·77
Mars - - -	0·65	Uranus - -	4·32
Vesta - - -			

SEMIDIAPASON, in Music, a defective or imperfect octave; or an octave diminished by a lesser semitone, or 4 commas.

SEMIDIAPENTE, in Music, a defective or imperfect fifth, called usually by the Italians, falsa quinta, and by us a false fifth.

SEMIDIATESSARON, in Music, a defective fourth, called also a false fourth.

SEMIDIATONE, in Music, is the lesser third, having its terms as 6 to 5.

SEMIORDINATES, in Geometry, the halves of the ordinates or applicates, being the lines applied between the absciss and the curve.

SEMIPARABOLA, &c, in the higher geometry, a curve defined by the equation $ax^{n-1} = y^n$; as $ax^2 = y^3$, or $ax^3 = y^4$, &c. In semiparabolas, $y^n : v^n :: ax^{n-1} : az^{n-1} :: x^{n-1} : z^{n-1}$; or the powers of the semiordinates are as the powers of the abscisses one degree lower: for instance, in cubical semiparabolas, the cubes of the ordinates are as the squares of the abscisses; that is, $y^3 : v^3 :: x^2 : z^2$.

SEMIQUADRATE, or SEMIQUARTILE, is an aspect of the planets, when distant from each other one sign and a half, or 45 degrees.

SEMIQUAVER, in Music, the half of a quaver.

SEMIQUINTILE, is an aspect of the planets when distant from each other the half of a 5th of the circle, or by 36 degrees.

SEMISEXTILE, an aspect of two planets, when they are distant from each other 30 degrees, or the half of a sextile, which is 2 signs or 60°. The semisextile is marked *s. s.*

SEMITONE, in Music, a half tone or half note, one of the degrees or intervals of concords. There are three degrees, or less intervals, by which a sound can move upwards and downwards, successively from one extreme of any concord to the other, and yet produce true melody. These degrees are the greater tone, the less tone, and the semitone. The ratios defining these intervals are these, viz, the greater tone 8 to 9, the less tone 9 to 10, and the semitone 15 to 16. Its compass is 5 commas, and it has its name from being nearly half a whole, though it is really somewhat more.

There are several species of semitones; but those that usually occur in practice are of two kinds, distinguished by the addition of greater and less. The first is expressed by the ratio of 16 to 15, or $\frac{16}{15}$; and the second by 25 to 24, or $\frac{25}{24}$. The octave contains 10 semitones major, and 2 dieses, nearly, or 17 semitones minor, nearly; for the measure of the octave being expressed by the log. 100,000, the semitone major will be measured by - 0,09311, and the semitone minor by - - 0,05889. These two differ by a whole enharmonic diesis; which is an interval practicable by the voice. It was much in use among the ancients, and is not unknown among modern practitioners. Euler Tent. Nov. Theor. Mus. pa. 107. See INTERVAL.

These semitones are called fictitious notes; and, with respect to the natural ones, they are expressed by characters called flats and sharps. The use of them is to remedy the defects of instruments, which, having their sounds fixed, cannot always be made to answer to the diatonic scale. By means of these, we have a new kind of scale, called the

SEMITONIC *Scale*, or the *Scale of Semitones*, which is a scale or system of music, consisting of 12 degrees, or 13 notes, in the octave, being an improvement on the natural or diatonic scale, by inserting between each two notes of it, another note, which divides the interval or tone into two unequal parts, called semitones.

The use of this scale is for instruments that have fixed sounds, as the organ, harpsichord, &c, which are exceedingly defective on the foot of the natural or diatonic scale. For the degrees of the scale being unequal, from every note to its octave there is a different order of degrees; so that from any note we cannot find every interval in a series of fixed sounds; which yet is necessary, that all the notes of a piece of music, carried through several keys, may be found in their just tune, or that the same song may be begun indifferently at any note, as may be necessary for accommodating some instrument to others, or to the voice, when they are to accompany each other in unison.

The diatonic scale, beginning at the lowest note, being first settled on an instrument, and the notes of it distinguished by their names *a, b, c, d, e, f, g*; the inserted notes, or semitones, are called fictitious notes, and take the name or letter below with a ♯, as $c^{\sharp}$ called *c* sharp; signifying that it is a semitone higher than the sound of *c* in the natural series; or this mark ♭ called a flat, with the name of the note above signifying it to be a semitone lower.

Now $\frac{15}{16}$ and $\frac{128}{135}$ being the two semitones the greater tone is divided into, and $\frac{15}{16}$ and $\frac{24}{25}$, the semitones the less tone is divided into, the whole octave will stand as in the following scheme, where the ratios of each term to the next are written fraction-wise between them below.

$c.\quad c^{\sharp}.\quad d.\quad d^{\sharp}.\quad e.\quad f.\quad f^{\sharp}.\quad g.\quad g^{\sharp}.\quad a^{\flat}.\quad b.\quad cc.$

$\frac{15}{16}\quad \frac{128}{135}\quad \frac{15}{16}\quad \frac{24}{25}\quad \frac{15}{16}\quad \frac{128}{135}\quad \frac{15}{16}\quad \frac{15}{16}\quad \frac{24}{25}\quad \frac{15}{16}\quad \frac{128}{135}\quad \frac{15}{16}$

For the names of the intervals in this scale, it may be considered, that as the notes added to the natural scale are not designed to alter the species of melody, but leave it still diatonic, and only correct certain defects arising from something foreign to the office of the scale of music, viz, the fixing and limiting the sounds; we see the reason why the names of the natural scale are continued, only making a distinction of each into a greater and less. Thus an interval of one semitone, is called a less second; of two semitones, a greater second; of three semitones, a less third; of four, a greater third, &c.

A second kind of semitonic scale we have from another division of the octave into semitones, which is performed by taking an harmonical mean between the extremes of the greater and less tone of the natural scale, which divides it into two semitones nearly equal. Thus, the greater tone 8 to 9 is divided into two semitones, which are 16 to 17, and 17 to 18; where 16, 17, 18, is an arithmetical division, the numbers representing the lengths of the chords; but if they represent the vibration, the lengths of the chords are reciprocal; viz as 1, $\frac{16}{17}$, $\frac{8}{9}$; which puts the greater semitone $\frac{16}{17}$ next the lower part of the tone, and the lesser $\frac{17}{18}$ next the upper, which is the property of the harmonical division. And after the same manner the less tone 9 to 10 is divided into two semitones, 18 to 19, and 19 to 20; and the whole octave stands thus:

$c.\quad c^{\sharp}.\quad d.\quad d^{\sharp}.\quad e.\quad f.\quad f^{\sharp}.\quad g.\quad g^{\sharp}.\quad a^{\flat}.\quad b.\quad c.$

$\frac{16}{17}\quad \frac{17}{18}\quad \frac{18}{19}\quad \frac{19}{20}\quad \frac{15}{16}\quad \frac{16}{17}\quad \frac{17}{18}\quad \frac{18}{19}\quad \frac{19}{20}\quad \frac{16}{17}\quad \frac{17}{18}\quad \frac{15}{16}.$

This scale, Mr. Salmon tells us, in the Philosophical Transactions, he made an experiment of before the Royal Society, on chords, exactly in these proportions, which yielded a perfect concert with other instruments, touched by the best hands. Mr. Malcolm adds, that, having calculated the ratios of them, for his own satisfaction, he found more of them false than in the preceding scale, but then their errors were considerably less, which made amends. Malcolm's Music, chap. 10. § 2.

SENSIBLE *Horizon*, or *Point*, or *Quality*, &c. See the substantives.

SEPTUAGESIMA, in the Calendar, is the 9th Sunday before Easter, so called, as some have supposed, because it is near 70 days, though in reality it is only 63 days, before it.

SERIES, in Algebra, denotes a rank or progression of quantities or terms, which usually proceed according to some certain law.

As the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ &c,
or the series, $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ &c,

where the former is a geometrical series, proceeding by the constant division by 2, or the denominators multiplied by 2; and the latter is an harmonical series, being the reciprocals of the arithmetical series 1, 2, 3, 4, &c, or the denominators being continually increased by 1.

The only traces of the doctrine of series found among the ancients are in the works of Archimedes and Pappus. Thus, in comparing the spheroid with the cone and cylinder, Archimedes supposes the terms of a progression to increase constantly by the same difference, and demonstrates several properties of such a progression relating to the sum of the terms, and the sum of their squares; by means of which he compares the parabolic conoid, the spheroid, and hyperbolic conoid, with the cone, and the area of his spiral line with the area of the circle. Again, in his Treatise on the Quadrature of the Parabola, he mentions a progression whose terms decrease constantly in the ratio of 4 to 1: but he does not suppose this progression to be continued to infinity, or mention the sum of an infinite number of terms; though it is plain that all that can be understood by those who assign that sum was fully known to him. He contents himself however, with demonstrating this plain property of such a series, that the sum of the terms continued at pleasure, added to the $\frac{1}{3}$ part of the last term, amounts always to $\frac{4}{3}$ of the first term. See pr. 23 quad. parab.

Pappus touches on a subject nearly allied to the modern doctrine of series, in the 4th book of his mathematical collections, where he treats of the general problem relating to an infinite series of circles inscribed in the space called arbelon, contained between the circumferences of two circles touching inwardly. But both of these authors investigate their respective problems geometrically, and without any reference to the algebraic method.

With regard to series considered algebraically, the first notices are found in the works of Dr. Wallis. Thus, in his arithmetical works, published in 1657, he for the first time reduced the fraction $\frac{A}{1-R}$, by a continued division, into the infinite series $A+AR+AR^2+AR^3+AR^4+$&c. This, and a few other deductions of similar import, gave the idea to Nic. Mercator, who made some advances in the doctrine. It was afterwards taken up by Brouncker, James Gregory, &c; but the genius of Newton first gave it body and form.

This method is chiefly useful in the quadrature of curves; where, as we often meet with quantities which cannot be expressed by any precise definite numbers, such as is the ratio of the diameter of a circle to the circumference, we are glad to express them by a series, which, infinitely continued, is the value of the quantity sought, and which is called an infinite series.

The Nature, Origin, &c. of Series.

Infinite series commonly arise, either from a continued division, as was practised by Mercator, or the extraction of roots, as first performed by Newton, who also explained other general ways for the expanding of quantities into infinite series as by the binomial theorem. Thus, to divide 1 by 3, or to expand the fraction $\frac{1}{3}$ into an infinite series; by division in decimals in the ordinary way, the series is 0·3333 &c, or $\frac{3}{10}+\frac{3}{100}+\frac{3}{1000}+\frac{3}{10000}$ &c, where the law of continuation is manifest. Or, if the same fraction $\frac{1}{3}$ be set in this form $\frac{1}{2+1}$, and division be performed algebraically, the quotient will be

$$\frac{1}{3}=\frac{1}{2+1}=\frac{1}{2}-\frac{1}{4}+\frac{1}{8}-\frac{1}{16}+\frac{1}{32}\text{ \&c.}$$

Or, if it be expressed in this form $\frac{1}{3}=\frac{1}{4-1}$, by a like division there will arise the series,

$$\frac{1}{3}=\frac{1}{4}+\frac{1}{16}+\frac{1}{64}\text{ \&c}=\frac{1}{4}+\frac{1}{4^2}+\frac{1}{4^3}\text{ \&c.}$$

Again, by dividing 1 by 5−2, or 6−3, or 7−4, &c, the series answering to the fraction $\frac{1}{3}$, may be found in an endless variety of infinite series. The finite quantity $\frac{1}{3}$ is called the value or radix of the series, as also its sum, being the number or sum to which the series would amount, or the limit to which it would tend or approximate, by summing up its terms, or by collecting them together one after another.

In like manner, by dividing 1 by the algebraic sum $a+c$, or by $a-c$, the quotient will be in these two cases, as below, viz,

$$\frac{1}{a+c}=\frac{1}{a}-\frac{c}{a^2}+\frac{c^2}{a^3}-\frac{c^3}{a^4}\text{ \&c,}$$
$$\frac{1}{a-c}=\frac{1}{a}+\frac{c}{a^2}+\frac{c^2}{a^3}+\frac{c^3}{a^4}\text{ \&c;}$$

where the terms of each series are the same, and they differ only in this, that the signs are alternately positive and negative in the former, but all positive in the latter.

And hence, by expounding a and c by any numbers whatever, we obtain an endless variety of infinite series, whose sums or values are known. So, by taking a or c equal to 1 or 2 or 3 or 4, &c, we obtain these series, and their values or roots:

$$\frac{1}{1+1}=\frac{1}{2}=1-1+1-1+1-1\text{ \&c,}$$
$$\frac{1}{3-1}=\frac{1}{2}=\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}+\frac{1}{3^4}\text{ \&c,}$$
$$\frac{1}{2+1}=\frac{1}{3}=\frac{1}{2}-\frac{1}{2^2}+\frac{1}{2^3}-\frac{1}{2^4}\text{ \&c,}$$
$$\frac{1}{1+2}=\frac{1}{3}=1-2+2^2-2^3\text{ \&c,}$$
$$\frac{1}{3+1}=\frac{1}{4}=\frac{1}{3}-\frac{1}{3^2}+\frac{1}{3^3}-\frac{1}{3^4}\text{ \&c,}$$

And hence it appears, that the same quantity or radix may be expressed by a great variety of infinite series, or that many different series may have the same radix or value.

Another way in which an infinite series arises, is by the extraction of roots. Thus, by extracting the square root of the number 3 in the common way, we obtain its value in a series as follows, viz, $\sqrt{3}=1·73205$ &c $=1+\frac{7}{10}+\frac{3}{100}+\frac{2}{1000}+\frac{5}{100000}$ &c; in which way of resolution the law of the progression of the series is not visible, as it is when found by division. Thus, the square root of the algebraic quantity a^2+c^2 gives

$$\sqrt{(a^2+c^2)}=a+\frac{c^2}{2a}-\frac{c^4}{8a^3}+\frac{c^6}{16a^5}\text{ \&c.}$$

And a 3d way is by Newton's binomial theorem, which is a universal method, that serves for all kinds of quantities, whether fractional or radical ones: and by this means the same root of the last given quantity becomes

$$\sqrt{(a^2+c^2)}=a+\frac{c^2}{2a}-\frac{1.c^4}{2.4a^3}+\frac{1.3c^6}{2.4.6a^5}\text{ \&c,}$$

where the law of continuation is evident.

See Extraction *of Roots*, and Binomial *Theorem*.

From the specimens above given, it appears that the signs of the terms may be either all plus, or alternately plus and minus. Though they may be varied in many other ways. It also appears that the terms may be either continually smaller and smaller, or larger and larger, or they may be all equal. In the first case therefore the series is said to be a decreasing one, in the 2d case an increasing one, and in the 3d case an equal one. Also the first series is called a converging one, because that by collecting its terms successively, taking in always one term more, the successive sums approximate or converge to the value or sum of the whole infinite series. So, in the series $\frac{1}{3-1}=\frac{1}{2}=\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}$, &c, the first term $\frac{1}{3}$ is too little, or below $\frac{1}{2}$ which is the value or sum of the whole infinite series proposed; the sum of the first two terms $\frac{1}{3} + \frac{1}{9}$ is $\frac{4}{9}$ = ·4444 &c, is also too little, but nearer to $\frac{1}{2}$ or ·5 than the former; and the sum of three terms $\frac{1}{3} + \frac{1}{9} + \frac{1}{27}$ is $\frac{13}{27}$ = ·481481 &c, is nearer than the last, but still too little; and the sum of four terms $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$, is $\frac{40}{81}$ = ·493827 &c, which is again nearer than the former, but still too little; and this is always the case when the terms are all positive. But when the converging series has its terms alternately positive and negative, then the successive sums are alternately too great and too little, though still approaching nearer and nearer to the final sum or value. Thus in in the series

$$\frac{1}{3+1} = \frac{1}{4} = 0{\cdot}25 = \frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} \text{ \&c,}$$

the 1st term $\frac{1}{3}$ = ·333 &c, is too great,
two terms $\frac{1}{3} - \frac{1}{9}$ = ·222 &c, are too little,
three terms $\frac{1}{3} - \frac{1}{9} + \frac{1}{27}$ = ·259259 &c, are too great,
four terms $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81}$ = ·246913 &c, are too small, and so on, alternately too great and too small, but every succeeding sum still nearer than the former, or converging.

In the second case, or when the terms become larger and larger, the series is called a diverging one, because that by collecting the terms continually, the successive sums diverge, or go always farther and farther from the true value or radix of the series; being all too great when the terms are all positive, but alternately too great and too little when they are alternately positive and negative. Thus, in the series

$$\frac{1}{1+2} = \frac{1}{3} = 1 - 2 + 4 - 8 \text{ \&c.}$$

the first term + 1 is too great.
two terms $1 - 2 = -1$ are too little,
three terms $1 - 2 + 4 = +3$ are too great,
four terms $1 - 2 + 4 - 8 = -5$ are too little,

and so on continually, after the 2d term, diverging more and more from the true value or radix $\frac{1}{3}$, but alternately too great and too little, or positive and negative. But the alternate sums would be always more and more too great if the terms were all positive, and always too little if negative.

But in the third case, or when the terms are all equal, the series of equals, with alternate signs, is called a neutral one, because the successive sums, found by a continual collection of the terms, are always at the same distance from the true value or radix, but alternately positive and negative, or too great and too little. Thus, in the series

$$\frac{1}{1+1} = \frac{1}{2} = 1 - 1 + 1 - 1 + 1 - 1 \text{ \&c,}$$

the first term 1 is too great,
two terms $1 - 1 = 0$ are too little,
three terms $1 - 1 + 1 = 1$ too great,
four terms $1 - 1 + 1 - 1 = 0$ too little,

and so on continually, the successive sums being alternately 1 and 0, which are equally different from the true value or radix $\frac{1}{2}$, the one as much above it, as the other below it.'

A series may be terminated and rendered finite, and accurately equal to its radix, by assuming the supplement or remainder, after any particular term, and combining it with the foregoing terms. So, in the series $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16}$ &c, which is equal to $\frac{1}{3}$, and found by dividing 1 by 2 + 1, after the first term, $\frac{1}{2}$, of the quotient, the remainder is $-\frac{1}{2}$, which divided by 2 + 1, or 3, gives $-\frac{1}{6}$ for the supplement, which being combined with the first term $\frac{1}{2}$, gives $\frac{1}{2} - \frac{1}{6} = \frac{1}{3}$ the true sum of the series. Again, after the first two terms $\frac{1}{2} - \frac{1}{4}$, the remainder is $+\frac{1}{4}$, which divided by the same divisor 3, gives $\frac{1}{12}$ for the supplement, and this combined with those two terms $\frac{1}{2} - \frac{1}{4}$, makes $\frac{1}{2} - \frac{1}{4} + \frac{1}{12} = \frac{1}{4} + \frac{1}{12} = \frac{4}{12}$ or $\frac{1}{3}$ the same sum or value as before. And in general, by dividing 1 by $a + c$, there is obtained

$$\frac{1}{a+c} = \frac{1}{a} - \frac{c}{a^2} + \frac{c^2}{a^3} \cdots\cdots \pm \frac{c^n}{a^{n+1}} \mp \frac{c^{n+1}}{a^{n+1}\,(a+c)};$$

where, stopping the division at any term as $\frac{c^n}{a^{n+1}}$, the remainder after this term is $\frac{c^{n+1}}{a^{n+1}}$, which being divided by the same divisor $a + c$, gives $\frac{c^{n+1}}{a^{n+1}\,(a+c)}$ for the supplement as above.

The Law of Continuation.—A series being proposed, one of the chief questions concerning it, is to find the law of its continuation. Indeed, no universal rule can be given for this; but it often happens that the terms, taken two and two, or three and three, or in greater numbers, have an obvious and simple relation, by which the series may be determined and produced indefinitely. Thus, if 1 be divided by $1 - x$, the quotient will be a geometrical progression, viz, $1 + x + x^2 + x^3$ &c, where the succeeding terms are produced by the continual multiplication by x. In like manner, in other cases of division, other progressions are produced.

But in most cases the relation of the terms of a series is not constant, as it is in those that arise by division. Yet their relation often varies according to a certain law, which is sometimes obvious on inspection, and sometimes it is found by dividing the successive terms one by another, &c. Thus, in the series $1 + \frac{2}{3}x + \frac{8}{15}x^2 + \frac{16}{35}x^3 + \frac{128}{315}x^4$ &c, by dividing the 2d term by the 1st, the 3d by the 2d, the 4th by the 3d, and so on, the quotients will be $\frac{2}{3}x$, $\frac{4}{5}x$, $\frac{6}{7}x$, $\frac{8}{9}x$, &c; and therefore the terms may be continued indefinitely by the successive multiplication by these fractions. Also in the following series $1 + \frac{1}{6}x + \frac{3}{40}x^2 + \frac{5}{112}x^3 + \frac{35}{1152}x^4$ &c, by dividing the adjacent terms successively by each other, the series of quotients is $\frac{1}{6}x$, $\frac{9}{20}x$, $\frac{25}{42}x$, $\frac{49}{72}x$, &c, or $\frac{1.1}{2.3}x$, $\frac{3.3}{4.5}x$, $\frac{5.5}{6.7}x$, $\frac{7.7}{8.9}x$, &c; and therefore the terms of the series may be continued by the multiplication of these fractions.

Another method of expressing the law of a series, is one that defines the series itself, by its *general term*, showing the relation of the terms generally by their distances from the beginning, or by differential equations. To do this, Mr. Stirling conceives the terms of the series to be placed as so many ordinates on a right line given by position, taking unity as the common interval between these ordinates. The terms of the series he denotes by the initial letters of the alphabet, A, B, C, D, &c; A being the first, B the 2d, C the 3d, &c: and he denotes any term in general by the letter T, and the rest following it in order by T', T'', T''', T'''', &c; also the distance of the term T from any given term, or from any given intermediate point between two terms, he denotes by the indeterminate quantity z: so that the distances of the terms T', T'', T''', &c, from the said term or point, will be $z + 1$, $z + 2$, $z + 3$, &c; because the increment of the absciss is the common interval of the ordinates, or terms of the series, applied to the absciss.

These things being premised, let this series be proposed, viz, 1, $\frac{1}{2}x$, $\frac{3}{8}x^2$, $\frac{5}{16}x^3$, $\frac{35}{128}x^4$, $\frac{63}{256}x^5$, &c; in which it is found, by dividing the terms by each other, that the relations of the terms are,

$\text{B} = \frac{1}{2}\text{A}x$, $\text{C} = \frac{3}{4}\text{B}x$, $\text{D} = \frac{5}{6}\text{C}x$, $\text{E} = \frac{7}{8}\text{D}x$, &c: then the relation in general will be defined by the equation $\text{T}' = \frac{2z+1}{2z+2}\text{T}x$ or $\frac{z+\frac{1}{2}}{z+1}\text{T}x$, where z denotes the distance of T from the first term of the series. For by substituting 0, 1, 2, 3, 4, &c, successively instead of z, the same relations will arise as in the proposed series above. In like manner, if z be the distance of T from the 2d term of the series, the equation will be $\text{T}' = \frac{2z+3}{2z+4}\text{T}x$ or $\frac{z+\frac{3}{2}}{z+2}\text{T}x$, as will appear by substituting the numbers − 1, 0, 1, 2, 3, &c, successively for z. Or, if z denote the place or number of the term T in the series, its successive values will be 1, 2, 3, 4, &c, and the equation or general term will be $\text{T}' = \frac{2z-1}{2z}\text{T}x$.

It appears therefore, that innumerable differential equations may define one and the same series, according to the different points from whence the origin of the absciss z is taken. And, on the contrary, the same equation defines innumerable different series, by taking different successive values of z. For in the equation $\text{T}' = \frac{2z-1}{2z}\text{T}x$, which defines the foregoing series when 1, 2, 3, 4, &c are the successive values of the abscisses; if $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, $4\frac{1}{2}$, &c, be successively substituted for z, the relations of the terms arising will be, $\text{B} = \frac{2}{3}\text{A}x$, $\text{C} = \frac{4}{5}\text{B}x$, $\text{D} = \frac{6}{7}\text{C}x$, &c, from whence will arise the series A, $\frac{2}{3}\text{A}x$, $\frac{8}{15}\text{A}x^2$, $\frac{16}{35}\text{A}x^3$, $\frac{128}{315}\text{A}x^4$, &c, which is different from the former.

And thus the equation will always determine the series from the given values of the absciss and of the first term, when the equation includes but two terms of the series, as in the last example, where the first term being given, all the rest will be given.

But when the equation includes three terms, then two must be given; and three must be given, when it includes four; and so on. So, if there be proposed the series x, $\frac{1}{6}x^3$, $\frac{3}{40}x^5$, $\frac{5}{112}x^7$, $\frac{35}{1152}x^9$, &c, where the relations of the terms are, $\text{B} = \frac{1.1}{2.3}\text{A}x^2$, $\text{C} = \frac{3.3}{4.5}\text{B}x^2$, $\text{D} = \frac{5.5}{6.7}\text{C}x^2$, &c, the equation defining this series will be $\text{T}' = \frac{(2z-1).(2z-1)}{2z.(2z+1)}\text{T}x^2 = \frac{4zz-4x+1}{4zz+2z}\text{T}x^2$, where the successive values of z are 1, 2, 3, 4, &c. See Stirling's Methodus Differentialis, in the introduction.

This may suffice to give a notion of these differential equations, defining the nature of series. But as to the application of these equations in interpolations, and finding the sums of series, it would require a treatise to explain it. We must therefore refer to that excellent one just quoted, as also to Demoivre's Miscellanea Analytica; and several curious papers by Euler in the Acta Petropolitana.

A series often converges so slowly, as to be of no use in practice. Thus, if it were required to find the sum of the series $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \frac{1}{7.8} + \frac{1}{9.10}$ &c, which lord Brouncker found for the quadrature of the hyperbola, true to 9 figures, by the mere addition of the terms of the series; Mr. Stirling computes that it would be necessary to add a thousand millions of terms for that purpose; for which the life of man would be too short. But by that gentleman's method, the sum of the series may be found by a very moderate computation. See Method. Differ. pa. 26.

Series are of various kinds or descriptions. So,

An *Ascending* SERIES, is one in which the powers of the indeterminate quantity increase; as $1 + ax + bx^2 + cx^3$ &c. And a

Descending SERIES, is one in which the powers decrease, or else increase in the denominators, which is the same thing; as

$$1 + ax^{-1} + bx^{-2} + cx^{-3} \text{ \&c, or } 1 + \frac{a}{x} + \frac{b}{x^2} + \frac{c}{x^3} \text{ \&c.}$$

A Circular SERIES, which denotes a series whose sum depends on the quadrature of the circle. Such is the series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9}$ &c: See Demoivre Miscel. Analyt. pa. 111, or my Mensur. pa. 119. Or the sum of the series $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25}$ &c, continued ad infinitum, according to Euler's discovery.

Continued Fraction or *Series*, is a fraction of this kind, to infinity,

$$\cfrac{a}{b + \cfrac{c}{d + \cfrac{e}{f + \cfrac{g}{h \text{ \&c.}}}}}$$

The first series of this kind was given by lord Brouncker, first president of the Royal Society, for the quadrature of the circle, as related by Dr. Wallis, in his Algebra, pa. 317. His series is,

$$1 + \cfrac{1}{2 + \cfrac{9}{2 + \cfrac{25}{2 + \cfrac{49}{2 + \cfrac{81}{2 + \text{\&c,}}}}}}$$

which denotes the ratio of the square of the diameter of a circle to its area. Mr. Euler has treated on this kind of series, in the Petersburg Commentaries, vol. 11, and in his Analys. Infinit. vol. 1, pa. 295, where he shows various uses of it, and how to transform ordinary fractions and common series into continued fractions. A common fraction is transformed into a continued one, after the manner of seeking the greatest common measure of the numerator and denominator, by dividing the greater by the less, and the last divisor always by the last remainder. Thus to change $\frac{1461}{59}$ to a continued fraction.

```
59 ) 1461 ( 24
     118
     ---
      281
      236
     ----
       45 ) 59 ( 1
            45
           ---
           14 ) 45 ( 3
                42
               ---
                3 ) 14 ( 4
                    12
                   ---
                     2 ) 3 ( 1
                         2
                        ---
                         1 ) 2 ( 2
                             2
```

$$\text{Theref.}\frac{1461}{59} = 24 + \cfrac{1}{1 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{+\cfrac{1}{2.}}}}}$$

It is not, however, in this form of the series that they are applied to any useful purposes; they must first be reduced to a series of converging fractions, which will be finite when the radix is rational, but infinite when the radix is a surd. The rule for performing this is as follows.

Having divided one number by another, as above directed, till nothing remains, place all the quotients thus arising in one horizontal line, in the order in which they were obtained, proceeding from left to right. Then the first fraction will have 1 for its numerator, and the first quotient figure for its denominator. The second fraction will have the second quotient figure for its numerator; and for its denominator, the product of the first denominator, and the said quotient plus 1.

And all the other terms will then be found as follows. For the numerator, multiply the numerator already found by the next quotient figure, and to the product add the preceding numerator, which will form the new numerator. And the denominators are obtained in exacly the same manner.

Thus, to reduce the above continued fraction to a series of converging fractions, we have

$$\begin{array}{llllll} \text{Quotients} & 24, & 1, & 3, & 4, & 1, & 2. \\ \left.\begin{array}{r}\text{Converging}\\ \text{fractions}\end{array}\right\} & \frac{24}{1}, & \frac{25}{1}, & \frac{99}{4}, & \frac{421}{17}, & \frac{520}{21}, & \frac{1461}{59}. \end{array}$$

A surd quantity, such as $\sqrt{19}$, is reduced to a series of converging fractions, which will go on to infinity, in a similar manner, after the series of quotients $9, 9', 9'', 9'''$, &c is obtained, which is done by taking out the greatest square, then the next greatest, and so on, thus,

$$\begin{array}{lllll}
q & = & \sqrt{19} & = 4 + & \frac{\sqrt{19}-4}{1} \\
q^{\text{i}} & = \frac{1}{\sqrt{19}-4} & = \frac{\sqrt{19}+4}{3} & = 2 + & \frac{\sqrt{19}-2}{3} \\
q^{\text{ii}} & = \frac{3}{\sqrt{19}-2} & = \frac{\sqrt{19}+2}{5} & = 1 + & \frac{\sqrt{19}-3}{5} \\
q^{\text{iii}} & = \frac{5}{\sqrt{19}-3} & = \frac{\sqrt{19}+3}{2} & = 3 + & \frac{\sqrt{19}-3}{2} \\
q^{\text{iv}} & = \frac{2}{\sqrt{19}-3} & = \frac{\sqrt{19}+3}{5} & = 1 + & \frac{\sqrt{19}-2}{3} \\
q^{\text{v}} & = \frac{5}{\sqrt{19}-2} & = \frac{\sqrt{19}+2}{3} & = 2 + & \frac{\sqrt{19}-4}{3} \\
q^{\text{vi}} & = \frac{3}{\sqrt{19}-4} & = \frac{\sqrt{19}+4}{1} & = 8 + & \frac{\sqrt{19}-4}{1} \\
q^{\text{vii}} & = \frac{1}{\sqrt{19}-4} & = \frac{\sqrt{19}+4}{3} & = 2 + & \text{\&c.}
\end{array}$$

So that the quotients are

$$4 \quad 2 \quad 1 \quad 3 \quad 1 \quad 2 \quad 8 \quad 2;\ 4,\ 2,\ 1, \text{\&c.}$$

$\left.\begin{array}{r}\text{Converging}\\ \text{fractions}\end{array}\right\} \frac{4}{1}, \frac{9}{2}, \frac{13}{3}, \frac{48}{11}, \frac{61}{14}$, &c, each of which fractions approximates towards the $\sqrt{19}$; differing from the common series in this, that in those, it is the sum of all the terms that gives the approximate value of the radix, whereas in this each term, singularly, approaches towards the value of the radical.

Converging SERIES, is a series whose terms continually decrease, or the successive sums of whose terms approximate or converge always nearer to the ultimate sum of the whole series. And, on the contrary, a

Diverging SERIES, is one whose terms continually increase, or that has the successive sums of its terms diverging, or going off always the farther, from the sum or value of the series.

Determinate SERIES, is a series whose terms proceed by the powers of a determinate quantity; as $1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \text{\&c.}$ If that determinate quantity be unity, the series is said to be determined by unity. Demoivre, Miscel. Analyt. pa. 111.

Indeterminate SERIES is one whose terms proceed by the powers of an indeterminate quantity x; as $x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4$ &c; or sometimes also with indeterminate exponents, or indeterminate coefficients.

The *Form of a* SERIES, is used for that affection of an indeterminate series, such as

$ax^n + bx^{n+r} + cx^{n+2r} + dx^{n+3r}$ &c, which arises from the different values of the indices of x. Thus,

If $n = 1$, and $r = 1$, the series will take the form

$$ax + bx^2 + cx^3 + dx^4 \text{ \&c.}$$

If $n = 1$, and $r = 2$, the form will be

$$ax + bx^3 + cx^5 + dx^7 \text{ \&c.}$$

If $n = \frac{1}{2}$, and $r = 1$, the form is

$$ax^{\frac{1}{2}} + bx^{\frac{3}{2}} + cx^{\frac{5}{2}} + dx^{\frac{7}{2}} \text{ \&c.}$$ And

If $n = 0$, and $r = -1$, the form will be

$$a + bx^{-1} + cx^{-2} + dx^{-3} \text{ \&c.}$$

When the value of a quantity cannot be found exactly, it is of use in algebra, as well as in common arithmetic, to seek an approximate value of that quantity, which may be useful in practice. Thus, in arithmetic, as the true value of the square root of 2 cannot be assigned, a decimal fraction is found to a sufficient degree of exactness in any particular case; which decimal fraction is in reality, no more than an infinite series of fractions converging or approximating to the true value of the root sought. For the expression $\sqrt{2} = 1{\cdot}414213$ &c, is equivalent to this $\sqrt{2} = 1 + \frac{4}{10} + \frac{1}{100} + \frac{4}{1000}$ &c; or supposing $x = 10$, to this

$$\sqrt{2} = 1 + \frac{4}{x} + \frac{1}{x^2} + \frac{4}{x^3} + \frac{2}{x^4} \text{ \&c.}$$

$$\text{or} = 1 + 4x^{-1} + x^{-2} + 4x^{-3} + 2x^{-4} \text{ \&c,}$$

which last series is a particular case of the more general indeterminate series $ax^n + bx^{n+r} + cx^{n+2r}$ &c, viz, when $n = 0$, $r = -1$, and the coefficients $a = 1$, $b = 4$, $c = 1$, $d = 4$, &c.

But the application of the notion of approximations in numbers, to species, or to algebra, is not so obvious. Newton, with his usual sagacity, took the hint, and prosecuted it; by which were discovered general methods in the doctrine of infinite series, which had before been treated only in a particular manner, though with great acuteness, by Wallis and a few others. See Newton's Method of Fluxions and Infinite Series, with Colson's Comment; as also the Analysis per Æquationes Numero Terminorum Infinitas, published by Jones in 1711, and since translated and explained by Stewart, together with Newton's Tract on Quadratures, in 1745. To these may be added Maclaurin's Algebra, part. 2, chap. 10, pa. 244; and Cramer's Analyse des Lignes Courbes Algebraiques, chap. 7, pa. 148; and many other authors.

Among the various methods for determinating the value of a quantity by a converging series, that seems preferable to the rest, which consists in assuming an indeterminate series as equal to the quantity whose value is sought, and afterwards determining the values of the terms of this assumed series. For instance, suppose a logarithm were given, to find the natural number answering to it. Suppose the logarithm to be z, and the corresponding number sought $1 + x$: then by the nature of logarithms and fluxions, $\dot{z} = \frac{\dot{x}}{1+x}$, or $\dot{z} + x\dot{z} = \dot{x}$. Now assume a series for the value of the unknown quantity x, and substitute it and its fluxion instead of x and $\dot{x}$ in the last equation, then determine the assumed coefficients, by comparing or equating the like terms of the equation. Thus,

assume $x = az + bz^2 + cz^3 + dz^4$ &c,
then $\dot{x} = a\dot{z} + 2bz\dot{z} + 3cz^2\dot{z} + 4dz^3\dot{z}$ &c;
and $\dot{x} = (\dot{z} + x\dot{z}) = \dot{z} + az\dot{z} + bz^2\dot{z} + cz^3\dot{z}$ &c;

hence, comparing the like terms of these two values of $\dot{x}$, there arises $a = 1$, $b = \frac{1}{2}$, $c = \frac{1}{6}$, $d = \frac{1}{24}$, &c; which values being substituted for a, b, c, &c, in the assumed series $ax + bx^2 + cx^3$ &c, it gives

$x = z + \frac{1}{2}z^2 + \frac{1}{6}z^3 + \frac{1}{24}z^4 + \frac{1}{120}z^5$, &c, or

$x = z + \frac{1}{1.2}z^2 + \frac{1}{1.2.3}z^3 + \frac{1}{1.2.3.4}z^4 + \frac{1}{1.2.3.4.5}z^5$ &c;

and consequently the number sought will be

$$1 + x = 1 + z + \tfrac{1}{2}z^2 + \tfrac{1}{6}z^3 \text{ \&c.}$$

But the indeterminate series $az + bz^2 + cz^3$ &c, was here assumed arbitrarily, with regard to its exponents 1, 2, 3, &c, which will not succeed in all cases, because some quantities require other forms for the exponents. For instance, if from a given arc, it were required to find the tangent. Making x = the tangent, and z = arc, the radius being = 1. Then, from the nature of the circle we shall have $\frac{\dot{x}}{1 + x^2} = \dot{z}$, or $\dot{x} = \dot{z} + x^2\dot{z}$. Now if, to find the value of x, we suppose $x = az + bz^2 + cz^5$ &c, and proceed as before, we shall find all the alternate coefficients b, d, f, &c, or those of the even powers of z, to be each = 0; and therefore the series assumed is not of a proper form. But making $x = az + bz^3 + cz^5 + dz^7$, &c, then we find $a = 1$, $b = \frac{1}{3}$, $c = \frac{2}{15}$, $d = \frac{17}{315}$, &c, and consequently $x = z + \frac{1}{3}z^3 + \frac{2}{15}z^5 + \frac{17}{315}z^7$ &c. And other quantities require other forms of series.

Now to find a proper indeterminate series in all cases, tentatively, would often be very laborious, and even impracticable. Mathematicians have therefore endeavoured to find out a general rule for this purpose; though till lately the method has been but imperfectly understood and delivered. Most authors indeed have explained the manner of finding the coefficients a, b, c, d, &c, of the indeterminate series $ax^n + bx^{n+r} + cx^{n+2r}$ &c, which is easy enough; but the values of n and r, in which the chief difficulty lies, have been assigned by many in a manner as if they were self-evident, or at least discoverable by an easy trial or two, as in the last example.

As to the number n, Newton himself has shown the method of determining it, by his rule for finding the first term of a converging series, by the application of his parallelogram and ruler. For the particulars of this method, see the authors above cited; see also PARALLELOGRAM.

Taylor, in his Methodus Incrementorum, investigates the number r; but Stirling observes that his rule sometimes fails. Lineæ Tert. Ordin. Newton. pa. 28. Mr. Stirling gives a correction of Taylor's rule, but says he cannot affirm it to be universal, having only found it by chance. And again,

Gravesande observes, that though he thinks Stirling's rule never leads into an error, yet that it is not perfect. See Gravesande, De Determin. Form. Seriei Infinit. printed at the end of his Matheseos Universalis Elementa. This learned professor has endeavoured to rectify the rule. But Cramer has shown that it is still defective in several respects; and he himself, to avoid the inconveniences to which the methods of former authors are subject, has had recourse to the first principles of the method of infinite series, and has entered into a more exact and instructive detail of the whole method, than is to be met with elsewhere; for which reason, and many others, his treatise deserves to be particularly recommended to beginners. See also my Tracts, v. 3, p. 369, for an easy method of determining the exponents in the assumed indeterminate series.

But it is to be observed, that in determining the value of a quantity by a converging series, it is not always necessary to have recourse to an indeterminate series: for it is often better to find it by division, or by extraction of roots. See Newton's Meth. of Flux. and Inf. Series, above cited. Thus, if it were required to find the arc of a circle from its tangent being given, that is, to find the value of $\dot{z}$ in the given fluxional equation, $\dot{z} = \frac{\dot{x}}{1 + xx}$, by an infinite series: dividing $\dot{x}$ by $1 + xx$, the quotient will be the series $\dot{x} - x^2\dot{x} + x^4\dot{x} - x^6\dot{x}$ &c $= \dot{z}$; and taking the fluents of the terms, there results $z = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7$ &c, which is the series often used for the quadrature of the circle. If $x = 1$, or the tangent of 45°, then will $z = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$ &c = the length of an arc of 45°, or $\frac{1}{8}$ of the circumference, to the radius 1, or $\frac{1}{4}$ of the circumference to the diameter 1. Consequently, if 1 be the diameter, then $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7}$ &c will be the area of the circle, because $\frac{1}{4}$ of the circumference multiplied by the diameter, gives the area of the circle. This series was first given by Leibnitz and James Gregory.

See the form of the series for the binomial theorem, determined, both as to the coefficients and exponents, in my Tracts, vol. 1, pa. 228.

Harmonical SERIES, the reciprocal of arithmeticals. See HARMONICAL.

Hyperbolic SERIES, is used for a series whose sum depends on the quadrature of the hyperbola. Such is the series $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ &c. Demoivre's Miscel. Analyt. pa. 111.

Interpolation of SERIES, the inserting of some terms between others, &c. See INTERPOLATION.

Interscendent SERIES. See INTERSCENDENT.

Mixt SERIES, one whose sum depends partly on the quadrature of the circle, and partly on that of the hyperbola. Demoivre, Miscel. Analyt. pa. 111.

Recurring SERIES, is used for a series which is so constituted, that having taken at pleasure any number of its terms, each following term shall be related to the same number of preceding terms by some constant law of relation. Thus, in the following series,

$$\begin{array}{cccccc} a & b & c & d & e & f \\ 1 + & 2x + & 3x^2 + & 10x^3 + & 34x^4 + & 97x^5 \text{ \&c,} \end{array}$$

in which the terms being respectively represented by the letters a, b, c, &c, set over them, we shall have

$$d = 3cx - 2bx^2 + 5ax^3,$$
$$e = 3dx - 2cx^2 + 5bx^3,$$
$$f = 3ex - 2dx^2 + 5cx^3,$$
&c, &c,

where it is evident that the law of relation between d and e, is the same as between e and f, each being formed in the same manner from the three terms which precede it in the series.

The quantities $3x - 2x^2 + 5x^3$, taken together and connected by their proper signs, form what Demoivre calls the index, or the scale of relation; though sometimes the bare coefficients $3 - 2 + 5$ are called the scale of relation. And the scale of relation subtracted from unity, is called the differential scale. On the subject of Recurring Series, see Demoivre's Miscel. Analyt. pa. 27 and 72, and his Doctrine of Chances, 3d edit. pa. 220; also Euler's Analys. Infinit. tom. 1, pa. 175.

Having given a recurring series, with its scale of relation, the sum of the whole infinite series will also be given. For instance, suppose a series $a + bx + cx^2 + dx^3$ &c, where the relation between the coefficient of any term and the coefficients of any two preceding terms may be expressed by $f - g$; that is, $e = fd - gc$, and $d = fc - gb$, &c; then will the sum of the series, infinitely continued, be $\frac{a + (b - fa)x}{1 - fx + gx^2}$.

Thus, for example, assume 2 and 5 for the coefficients of the first two terms of a recurring series; and suppose f and g to be respectively 2 and 1; then the recurring series will be

$$2 + 5x + 8x^2 + 11x^3 + 14x^4 + 17x^5 \text{ \&c},$$

and its sum $= \frac{2 + 5x - 4x}{1 - 2x + xx} = \frac{2 + x}{(1 - x)^2}$. For the proof of which divide $2 + x$ by $(1 - x)^2$, and there arises the said series $2 + 5x + 8x^2 + 11x^3$ &c. And similar rules might be derived for more complex cases.

Demoivre's general rule is this: 1. Take as many terms of the series as there are parts in the scale of relation. 2. Subtract the scale of relation from unity, and the remainder is the differential scale. 3. Multiply the terms taken in the series by the differential scale, beginning at unity, and so proceeding orderly, remembering to leave out what would naturally be extended beyond the last of the terms taken. Then will the product be the numerator, and the differential scale will be the denominator of the fraction expressing the sum required.

But it must here be observed, that when the sum of a recurring series extended to infinity, is found by Demoivre's rule, it ought to be supposed that the series converges indefinitely, that is, that the terms may become less than any assigned quantity. For if the series diverge, that is, if its terms continually increase, the rule does not give the true sum. For the sum in such case is infinite, or greater than any given quantity, whereas the sum exhibited by the rule, will often be finite. The rule therefore in this case only gives a fraction expressing the radix of the series, by the expansion of which the series is produced. Thus $\frac{1}{(1 - x)^2}$ by expansion becomes the recurring series $1 + 2x + 3x^2$ &c, whose scale of relation is $2 - 1$, and its sum by the rule will be $\frac{a + bx - fax}{1 - fx + gxx} = \frac{1 + 2x - 2x}{1 - 2x + xx} = \frac{1}{(1 - x)^2}$, the quantity from which the series arose. But this quantity cannot in all cases be deemed equal to the infinite series $1 + 2x + 3x^2$ &c: for stop where you will, there will always want a supplement to make the product of the quotient by the divisor equal to the dividend. Indeed when the series converges infinitely, the supplement, diminishing continually, becomes less than any assignable quantity, or equal to nothing; but in a diverging series, this supplement becomes infinitely great, and the series deviates indefinitely from the truth. See Colson's Comment on Newton's Method of Fluxions and Infinite Series, pa. 152; Stirling's Method. Differ. pa. 36; Bernoulli de Serieb. Infin. pa. 249; and Cramer's Analyse des Lignes Courbes, pa. 174.

A recurring series being given, the sum of any finite number of the terms of that series may be found. This is prob. 3, pa. 73, Demoivre's Miscel. Analyt. and prob. 5, pa. 223 of his Doctrine of Chances. The solution is effected, by taking the difference between the sums of two infinite series, differing by the terms answering to the given number; viz, from the sum of the whole infinite series, commencing from the beginning, subtract the sum of another infinite number of terms of the same series, commencing after so many of the first terms whose sum is required; and the difference will evidently be the sum of that number of terms of the series. For example, to find the sum of n terms of the infinite geometrical series $a + ax + ax^2 + ax^3$ &c. Here are two infinite series; the one beginning with a, and the other with ax^n, which is the next term after the first n terms of the original series. By the rule, the sum of the first infinite progression will be $\frac{a}{1 - x}$, and the sum of the second $\frac{ax^n}{1 - x}$; the difference of which is $\frac{a - ax^n}{1 - x}$, which is therefore the sum of the first n terms of the series. This quantity $\frac{a - ax^n}{1 - x}$ is equal to $\frac{ax^n - a}{x - 1}$, which last expression, putting $ax^{n-1} = l$, will be equivalent to this, $\frac{lx - a}{x - 1}$, which is the common rule for finding the sum of any geometric progression, having given the first term a, the last term l, and the ratio x. See Miscel. Analyt. pa. 167, 168.

In a recurring series, any term may be obtained whose place is assigned. For after having taken so many terms of the series as there are terms in the scale of relation, the series may be protracted till it reach the place assigned. But when that place is very distant from the beginning of the series, the continuing the terms is very laborious; and therefore other methods have been contrived. See Miscel. Analyt. pa. 33, and Doctrine of Chances, pa. 224.

These questions have been resolved in many cases, besides those of recurring series. But as there is no universal method for the quadrature of curves, neither is there one for the summation of series; indeed there is a great analogy between these things, and similar difficulties arise in both. See the authors above cited.

The investigation of Daniel Bernoulli's method for finding the roots of algebraic equations, which is inserted in the Petersburg Acts, tom. 3, pa. 92, depends on the doctrine of recurring series. See Euler's Analysis Infinitorum, tom. 1, pa. 276.

Reversion of SERIES. See REVERSION *of Series*.

Summable SERIES, is one whose sum can be accurately found. Such is the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ &c, the sum of which is said to be unity, or to speak more accurately, the limit of its sum is unity or 1.

An indefinite number of summable infinite series may be assigned: such are, for instance, all infinite recurring converging series, and many others, for which, consult Demoivre, Bernoulli, Stirling, Euler, and Maclaurin; viz, Miscel. Analyt. pa. 110; De Serieb. Infinit. passim; Method. Different. pa. 34; Acta Petrop. passim; Fluxions, art. 350.

The obtaining the sums of infinite serieses of fractions has been one of the principal objects of the modern method of computation; and these sums may often be found, and sometimes not. Thus the sums of the two following series of geometrical progressionals are easily found to be

1 and $\frac{1}{2}$, viz, $1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}$ &c,
and $\frac{1}{2} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81}$ &c.

But the series of fractions that occur in the solution of problems, can seldom be reduced to geometric progressions; nor can any general rule, in cases so infinitely various, be given. The art here, as in most other cases, is only to be acquired by examples, and by a careful observation of the arts used by great authors in the investigation of such series of fractions as they have considered. And the general methods of infinite series, which have been carried so far by Demoivre, Stirling, Euler, &c, are often found necessary to determine the sum of a very simple series of fractions. See the quotations above.

The sum of a series of fractions, though decreasing continually, is not always finite. This is the case of the series $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ &c, which is the harmonic series, consisting of the reciprocals of arithmeticals, the sum of which exceeds any given number whatever; and this is shown from the analogy between this progression and the space comprehended by the common hyperbola and its asymptote; though the same may be shown also from the nature of progressions. See James Bernoulli, de Seriebus Infin. But, what is curious, the sum of the squares of its terms is finite; for if the same terms of the harmonic series, $\frac{1}{1} + \frac{1}{2} + \frac{1}{3}$ &c, be squared, forming the series $\frac{1}{1} + \frac{1}{4} + \frac{1}{9}$ &c, being the reciprocals of the squares of the natural series of numbers; the sum of this series of fractions will not only be limited, but it is remarkable that this sum will be precisely equal to the 6th part of the number which expresses the ratio of the square of the circumference of a circle to the square of its diameter. That is, if c denote 3·14159 &c, the ratio of the circumference to the diameter, then is $\frac{1}{6}c^2 = \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25}$ &c. This property was first discovered by John Bernoulli; and his investigation may be seen in the Acta Petrop. vol. 7. And Maclaurin has since observed, that this may easily be deduced from his Fluxions, art. 822. Philos. Trans. numb. 469.

It would require a whole volume to enumerate the various kinds of series of fractions which may or may not be summed. Sometimes the sum cannot be assigned, either because it is infinite, as in the harmonic series $\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ &c, or, though its sum be finite (as in the series $\frac{1}{1} + \frac{1}{4} + \frac{1}{9}$ &c), yet its sum cannot be assigned in finite terms, or by the quadrature of the circle or hyperbola, which was the case of this series before Euler's discovery; but yet the sum of any given number of the terms of the series may be expeditiously found, and the whole sum may be assigned by approximation, independent of the circle. See Stirling's Method. Different. and De Moivre's Miscel. Analyt. Also the works of John Bernoulli, who first summed this series.

Besides the series of fractions, the sums of which converge to a certain quantity, there sometimes occur others, which converge by a continued multiplication. Of this kind is the series found by Wallis, for the quadrature of the circle, which he expresses thus,

$$\square = \frac{3 \times 3 \times 5 \times 5 \times 7 \times 7 \times 9 \times 9 \times \text{&c}}{2 \times 4 \times 4 \times 6 \times 6 \times 8 \times 8 \times 10 \times \text{&c}},$$

where the character $\square$ denotes the ratio of the square of the diameter to the area of the circle. Hence the denominator of this fraction, is to its numerator, both infinitely continued, as the circle is to the square of the diameter. It may further be observed that this series is equivalent to $\frac{9}{8} \times \frac{25}{24} \times \frac{49}{48} \times$ &c, or to $\frac{3^2}{3^2 - 1} \times \frac{5^2}{5^2 - 1} \times \frac{7^2}{7^2 - 1} \times$ &c, that is, the product of the squares of all the odd numbers 3, 5, 7, 9, &c, is to the produce of the same squares severally diminished by unity, as the square of the diameter is to the area of the circle. See Arithmet. Infinit. prop. 191. Oper. vol. 1, pa. 469. Id. Oper. vol. 2, pa. 819. And these products of fractions, and the like quantities arising from the continued multiplication of certain factors, have been particularly considered by Euler, in his Analysis Infinit. vol. 1, chap. 15, pa. 221.

For an easy and general method of summing all alternate series, such as $a - b + c - d$ &c, see my Tracts, vol. 1, pa. 176; and in the same vol. may be seen many other curious tracts on infinite series.

Summation of Infinite SERIES, is the finding the value of them, or the radix from which they may be raised. For which consult the authors upon this science, particularly Stirling, and Clark's translation of Lorgna.

To find an infinite series by extracting of roots; and to find an infinite series by a presupposed series; see QUADRATURE *of the Circle*.

To extract the roots of an infinite series, see EXTRACTION *of Roots*.

To raise an infinite series to any power, see INVOLUTION, and POWER.

Transcendental SERIES. See TRANSCENDENTAL.

There are many other important writings on the subject of Infinite series, besides those above quoted. A very good elementary tract on this science is that of James Bernoulli, intituled, Tractatus de Seriebus Infinitis, and annexed to his Ars Conjectandi, published in 4to, 1713.

SERPENS, in Astronomy, a constellation in the northern hemisphere, being one of the 48 old constellations mentioned by all the ancients, and is called more particularly Serpens Ophiuchi, being grasped in the hands of the constellation Ophiuchus. The Greeks, in their fables, have ascribed it sometimes to one of Triptolemus's dragons, killed by Carnabos; and sometimes to the serpent of the river Segaris, destroyed by Hercules. This is by some supposed to be the same as the author of the book of Job calls the Crooked Serpent; but this expression more probably meant the constellation Draco, near the north pole.—The stars in the constellation Serpens, in Ptolemy's catalogue are 18, in Tycho's 13, in Hevelius's 22, and in the Britannic catalogue 64.

SERPENTARIUS, a constellation of the northern hemisphere, being one of the 48 old constellations mentioned by all the ancients. It is called also Ophiuchus, and anciently Æsculapius. It is in the figure of a man grasping the serpent. The Greeks had different fables about this, and other constellations, because they were ignorant of the true meaning of them. Some of them say, it represents Carnabos, who killed one of the dragons of Triptolemus. Others say, it was Hercules, killing the serpent at the river Segaris. And others again say, it represents the celebrated physician Æsculapius, to denote his skill in medicine in curing the bite of the serpent.

The stars in the constellation Serpentarius, in Ptolemy's catalogue are 29, in Tycho's 15, in Hevelius's 40, and in the Britannic catalogue they are 74.

SERPENTINE *Line*, the same with spiral.

SESQUI, an expression of a certain ratio, viz, the second ratio of inequality, called also superparticular ratio; being that in which the greater term contains the less once, and some certain part over; as 3 to 2, where the first term contains the second once, and unity over, which is a quota part of 2. Now if this part remaining be just half the less term, the ratio is called sesquialtera; if the remaining part be a 3d part of the less term, as 4 to 3, the ratio is called sesquitertia, or sesquiterza; if a 4th part, as 5 to 4, the ratio is called sesquiquarta; and so on continually, still adding to sesqui the ordinal number of the smaller term. In English we sometimes say, sesquialteral, or sesquialterate, sesquithird, sesquifourth, &c. As to the kinds of triples expressed by the particle sesqui, they are these:

SESQUILATERATE, the greater perfect, which is a triple, where the breve is three measures, or semibreves.

SESQUIALTERATE, greater imperfect, which is where the breve, when pointed, contains three measures, and without any point, two.

SESQUIALTERATE, less imperfect, a triple, where the semibreve with a point contains three measures, and two without.

SESQUIALTERATE, in Arithmetic and Geometry, is a ratio between two numbers, or lines, &c, where the greater is equal to once and a half of the less. Thus 6 and 9 are in a sesquialterate ratio, as also 20 and 30.

SESQUIDITONE, in Music, a concord resulting from the sounds of two strings whose vibrations, in equal times, are to each other in the ratio of 5 to 6.

SESQUIDUPLICATE *Ratio*, is that in which the greater term contains the less, twice and a half; as the ratio of 15 to 6, or 50 to 20.

SESQUIQUADRATE, an aspect or position of the planets, when they are distant by 4 signs and a half, or 135 degrees.

SESQUIQUINTILE, is an aspect of the planets when they are distant $\frac{1}{5}$ of the circle and a half, or 108 degrees.

SESQUITERTIONAL *Proportion*, is that in which the greater term contains the less once and one third; as 4 to 3, or 12 to 9.

SETTING, in Astronomy, the sinking of a star or planet below the horizon. Astronomers and poets count three different kinds of setting of the stars, viz, ACHRONICAL, COSMICAL, and HELIACAL. See these terms respectively.

SETTING, in Navigation, Surveying, &c, denotes the observing the bearing or situation of any distant object by the compass, &c, to discover the angle it makes with the nearest meridian, or with some other line. See BEARING. Thus, to *set the land*, or *the sun*, by the compass, is to observe how the land bears on any point of the compass, or on what point of the compass the sun is. Also, when two ships come in sight of each other, to mark on what point the chace bears, is termed Setting the chace by the compass.

SETTING also denotes the direction of the wind, current, or sea, particularly of the two latter.

SEVEN STARS, a common denomination given to the cluster of stars in the neck of the sign Taurus, the bull, properly called the pleiades. They are so called from their number seven which appear to the naked eye, though some persons can discover only 6 of them; but by the help of telescopes there appears to be a great multitude of them.

SEVENTH, *Septima*, an interval in Music, called by the Greeks heptachordon.

SEXAGENARY, something relating to the number 60.

SEXAGENARY *Arithmetic*. See SEXAGESIMAL.

SEXAGENARY *Tables*, are tables of proportional parts, showing the product of two sexagenaries that are to be multiplied, or the quotient of two that are to be divided.

SEXAGESIMA, the eighth Sunday before Easter; being so called because near 60 days before it.

SEXAGESIMAL or SEXAGENARY *Arithmetic*, a method of computation proceeding by 60ths. Such is that used in the division of a degree into 60 minutes, of the minute into 60 seconds, of the second into 60 thirds, &c.

The Greeks performed many of their calculations by means of the sexagesimal division of quantities, particularly their divisions and extraction of roots. This method, though very laborious, was certainly preferable to what these rules would have been in their common notation, as they appear to have had no idea, nor indeed did their notation admit, of finding one figure at a time in the quotient as we do. The Greeks therefore were under the necessity of finding either by trials, or otherwise, the whole quotient for the first period, then the whole quotient again for the second period, and so on. See NOTATION.

SEXAGESIMALS, or SEXAGESIMAL *Fractions*, are fractions whose denominators proceed in a sexagecuple ratio; that is, a prime, or the first minute $= \frac{1}{60}$, a second $= \frac{1}{3600}$, and third $= \frac{1}{216000}$. Anciently there were no other than sexagesimals used in astronomical operations, for which reason they are sometimes called astronomical fractions, and they are still retained in many cases, as in the divisions of time and of a circle; but decimal arithmetic is now much used in the calculations, and the French have entirely discarded the sexagesimal division, and employed only the decimal, an improvement in astronomy which may in time be adopted by other nations. See DEGREE.—Sexagesimals were probably first used for the divisions of a circle, 360, or 6 times 60 making up the whole circumference, on account that 360 days made up the year of the ancients, in which time the sun was supposed to complete his course in the circle of the ecliptic.—In these fractions, the denominator being always 60, or a multiple of it, it is usually omitted, and the numerator only set down: thus, $3^\circ\ 45'\ 24''\ 40'''$ &c, is to be read, 3 degrees, 45 minutes, 24 seconds, 40 thirds, &c.

SEXANGLE, in Geometry, a figure having 6 angles, and consequently 6 sides also.

SEXENARY or SEXTUPLE *Scale* of Notation, is that in which the local value of the digits increase in a sixfold proportion. See SCALE, and NOTATION.

SEXTANS, a sixth part of certain things. The Romans divided their *as*, which was a pound of brass, into 12 ounces, called uncia, from unum; and the quantity of 2 ounces was called sextans, as being the 6th part of the pound.

SEXTANS was also a measure, which contained 2 ounces of liquor, or 2 cyathi.

SEXTANS, the Sextant, in Astronomy, a new constellation, placed across the equator, but on the south side of the ecliptic, and by Hevelius made up of some unformed stars, or such as were not included in any of the 48 old constellations. In Hevelius's catalogue it contains 11 stars, but in the Britannic catalogue 41.

SEXTANT, denotes the 6th part of a circle, or an arch containing 60 degrees.

SEXTANT is more particularly used for an astronomical instrument. It is made like a quadrant, excepting that its limb only contains 60 degrees. Its use and application are the same with those of the QUADRANT; which see.

SEXTARIUS, an ancient Roman measure, containing 2 cotylæ, or 2 heminæ.

SEXTILE, the aspect or position of two planets, when they are distant the 6th part of the circle, viz, 2 signs or 60 degrees; and it is marked thus ✱.

SEXTUPLE, denotes 6 fold in general. But in music it denotes a mixed sort of triple time, which is beaten in double time.

SHADOW, *Shade*, in Optics, a certain space deprived of light, or where the light is weakened by the interposition of some opaque body before the luminary. The doctrine of shadows makes a considerable article in optics, astronomy, and geography; and is the general foundation of dialling. As nothing is seen but by light, a mere shadow is invisible; and therefore when we say we see a shadow, we mean, partly that we see bodies placed in the shadow, and illuminated by light reflected from collateral bodies, and partly that we see the confines of the light.

When the opaque body, that projects the shadow, is perpendicular to the horizon, and the plane it is projected on is horizontal, the shadow is called a right one: such as the shadows of men, trees, buildings, mountains, &c. But when the body is placed parallel to the horizon, it is called a versed shadow; as the arms of a man when stretched out, &c.

Laws of the Projection of Shadows.

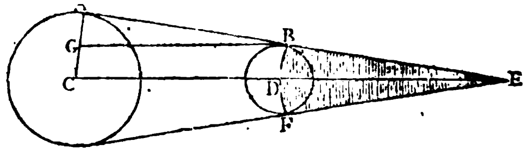
1. Every opaque body projects a shadow in the same direction with the rays of light; that is, towards the part opposite to the light. Hence, as either the luminary or the body changes place, the shadow likewise changes its place.

2. Every opaque body projects as many shadows as there are luminaries to enlighten it.

3. As the light of the luminary is more intense, the shadow is the deeper. Hence, the intensity of the shadow is measured by the degrees of light that space is deprived of. In reality, the shadow itself is not deeper; but it appears so, because the surrounding bodies are more vividly illuminated.

4. When the luminous body and opaque one are equal, the shadow is always of the same breadth with the opaque body. But when the luminous body is the larger, the shadow becomes always less and less, the farther it is from the body. And when the luminous body is the smaller of the two, the shadow increases always the wider, the farther from the body. Hence, the shadow of an opaque globe is, in the first case a cylinder, in the second case it is a cone verging to a point, and in the third case a truncated cone that enlarges still the more the farther it is from the body. Also, in all these cases, a transverse section of the shadow, by a plane, is a circle, respectively, in the three cases, equal, less, or greater than a great circle of the globe.

5. To find the length of the shadow, or the axis of the shady cone, projected by a sphere, when it is illuminated by a larger one; the diameters and distance of the two spheres being known. Let C and D be the centres of the two spheres, CA the semidiameter of the larger, and DB that of the smaller, both perpendicular to the side of the conical shadow BEF, whose axis is DE, continued to C; and draw BG parallel to the same axis. Then, the two triangles AGB and BDE being similar, it will be AG : GB or CD : : BD : DE, that is, as the difference of the semidiameters is to the distance of the centres, so is the semidiameter of the opaque sphere to the axis of the shadow, or the distance of its vertex from the said opaque sphere.



Ex. gr. If BD = 1 be the semidiameter of the earth, and AC = 101 the mean semidiameter of the sun, also their distance CD or GB = 24000; then as 100 : 24000 : : 1 : 240 = DE, which is the mean height of the earth's shadow, in semidiameters of the base.

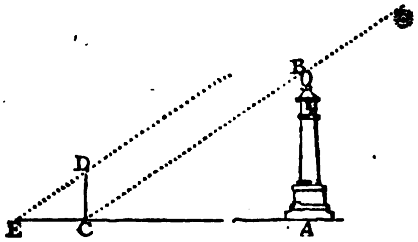
6. To find the length of the shadow AC projected by an opaque body AB; having given the altitude of the luminary, for ex. of the sun, above the horizon, viz, the angle C, and the height of the object AB. Here the proportion is, as tang. ∠C : radius : : AB : AC.

Or, if the length of the shadow AC be given, to find the height AB, it will be,

as radius : tang. ∠C : : AC : AB.

Or, if the length of the shadow AC, and of the object AB, be given, to find the sun's altitude above the horizon, or the angle at C. It will be,

as AC : AB : : radius : tang. ∠C sought.

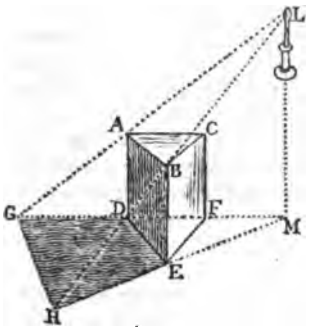


7. To measure the height of any object, ex. gr. a column AB, by means of its shadow projected on an horizontal plane.—At the extremity of the shadow, at C, erect a stick or pole CD, and measure the length of its shadow CE; also measure the length of the shadow AC of the tower. Then, by similar triangles, it will be, as EC : CD : : CA : AB. So if EC = 10 feet, CD = 6 feet, and CA = 95 feet; then as 10 : 6 : : 95 : 57 feet = AB, the height of the tower sought.

SHADOW, in Geography. The inhabitants of the earth are divided, with respect to their shadows, into ASCII, AMPHISCII, HETEROSCII, and PERISCII. See these several terms.

SHADOW, in Perspective, is of great use in this art.—Having given the appearance of an opaque body, and a luminous one, whose rays diverge, as a candle, or lamp, &c; to find the exact appearance of the shadow, according to the laws of perspective. The method is this: From the luminous body, which is here considered as a point, let fall a perpendicular to the perspective plane or table; and from the several angles, or raised points of the body, let fall perpendiculars to the same plane; then connect the points on which these latter perpendiculars fall, by right lines, with the point on which the first falls; continuing these lines beyond the side opposite to the luminary, till they meet with asmany other lines drawn from the centre of the luminary through the said angles or raised points; so shall the points of intersection of these lines be the extremes or bounds of the shadow.

For example, to project the appearance of the shadow of a prism ABCDEF, scenographically delineated. Here M is the place of the perpendicular of the light L, and D, E, F those of the raised points A, B, C, of the prism; therefore, draw MEH, MDG, &c, and LBH, LAG, &c, which will give DEGH &c for the appearance of the shadow.



As for those shadows that are intercepted by other objects, it may be observed, that when the shadow of a line falls upon any object, it must necessarily take the form of that object. If it fall upon another plane, it will be a right line; if upon a globe, it will be circular; and if upon a cylinder or cone, it will be circular, or oval, &c. If the body intercepting it be a plane, whatever be the situation of it, the shadow falling upon it might be found by producing that plane till it intercepted the perpendicular let fall upon it from the luminous body; for then a line drawn from that point would determine the shadow, just as if no other plane had been concerned. But the appearance of all these shadows may be drawn with less trouble, by first drawing it through these intercepted objects, as if they had not been in the way, and then making the shadow to ascend perpendicularly up every perpendicular plane, and obliquely on those that are situated obliquely, in the manner described by Dr. Priestley, in his Perspective, pa. 73 &c.

Here we may observe in general, that since the shadows of all objects which are cast upon the ground, will vanish into the horizontal line; so, for the same reason, the vanishing points of all shadows, which are cast upon any inclined or other plane, will be somewhere in the vanishing line of that plane.

When objects are not supposed to be viewed by the light of the sun, or of a candle &c, but only in the light of a cloudy day, or in a room into which the sun does not shine, there is no sensible shadow of the upper part of the object, and the lower part only makes the adjacent objects, or plane of the ground or floor on which it stands, a little darker than the rest. This imperfect obscure kind of shadow is easily made, being nothing more than a shade on the ground, opposite to the side on which the light comes; and it may be continued to a greater or less distance, according to the supposed brightness of the light by which it is made. It is in this manner (in order to save trouble, and sometimes to prevent confusion) that the shadows in most drawings are made. On this subject, see Priestley's Perspect. above quoted; also Kirby's Persp. book 2, ch. 4.

SHAFT *of a Column*, in Building, is the body of it; thus called from its straightness: but by architects more commonly the Fust.

SHAFT is also used for the spire of a church steeple; and for the shank or tunnel of a chimney.

SHARP (ABRAHAM), an ingenious mathematician, mechanist, and astronomer, was descended from an ancient family at Little-Horton, near Bradford, in the West Riding of Yorkshire, where he was born about the year 1651. At a proper age he was put apprentice to a merchant at Manchester; but his genius led him so strongly to the study of mathematics, both theoretical and practical, that he soon became uneasy in that situation of life. By the mutual consent therefore of his master and himself, though not altogether with that of his father, he quitted the business of a merchant. On this he removed to Liverpool, where he gave himself up wholly to the study of mathematics, astronomy, &c; and where, for a subsistance, he opened a school, and taught writing and accounts, &c.

He had not been long at Liverpool when he accidentally fell in company with a merchant or tradesman visiting that town from London, in whose house it seems the astronomer Mr. Flamsteed then lodged. With the view therefore of becoming acquainted with this eminent man, Mr. Sharp engaged himself with the merchant as a bookkeeper. In consequence he soon contracted an intimate acquaintance and friendship with Mr. Flamsteed, by whose interest and recommendation he obtained a more profitable employment in the dock-yard at Chatham; where he continued till his friend and patron, knowing his great merit in astronomy and mechanics, called him to his assistance, in contriving, adapting, and fitting up the astronomical apparatus in the Royal Observatory at Greenwich, which had been lately built, namely about the year 1676; Mr. Flamsteed being then 30 years of age, and Mr. Sharp 25.

In this situation he continued to assist Mr. Flamsteed in making observations (with the mural arch, of 80 inches radius, and 140 degrees on the limb, contrived and graduated by Mr. Sharp) on the meridional zenith distances of the fixed stars, sun, moon, and planets, with the times of their transits over the meridian; also the diameters of the sun and moon, and their eclipses, with those of Jupiter's satellites, the variation of the compass, &c. He assisted him also in making a catalogue of near 3000 fixed stars, as to their longitudes and magnitudes, their right ascensions and polar distances, with the variations of the same while they change their longitude by one degree.

But from the fatigue of continually observing the stars at night, in a cold thin air, joined to a weakly constitution, he was reduced to a bad state of health; for the recovery of which he desired leave to retire to his house at Horton; where, as soon as he found himself on the recovery, he began to fit up an observatory of his own; having first made an elegant and curious engine for turning all kinds of work in wood or brass, with a maundril for turning irregular figures, as ovals, roses, wreathed pillars, &c. Besides these, he made himself most of the tools used by joiners, clockmakers, opticians, mathematical instrument-makers, &c. The limbs or arcs of his large equatorial instrument, sextant, quadrant, &c, he graduated with the nicest accuracy, by diagonal divisions into degrees and minutes. The telescopes he used were all of his own making, and the lenses ground, figured, and adjusted with his own hands.

It was at this time that he assisted Mr. Flamsteed in calculating most of the tables in the second volume of his Historia Cœlestis, as appears by their letters, to be seen in the hands of Mr. Sharp's friends at Horton. Likewise the curious drawings of the charts of all the constellations visible in our hemisphere, with the still more excellent drawings of the planispheres both of the northern and southern constellations. And though these drawings of the constellations were sent to be engraved at Amsterdam by a masterly hand, yet the originals far exceeded the engravings in point of beauty and elegance: these were published by Mr. Flamsteed, and both copies may be seen at Horton.

The mathematician meets with something extraordinary in Sharp's elaborate treatise of Geometry Improved (in 4to 1717, signed A. S. Philomath.), 1st, by a large and accurate table of segments of circles, its construction and various uses in the solution of several difficult problems, with compendious tables for finding a true proportional part; and their use in these or any other tables exemplified in making logarithms, or their natural numbers, to 60 places of figures; there being a table of them for all primes to 1100, true to 61 figures. 2d, His concise treatise of Polyedra, or solid bodies of many bases, both the regular ones and others: to which are added twelve new ones, with various methods of forming them, and their exact dimensions in surds, or species, and in numbers: illustrated with a variety of copper-plates, neatly engraved by his own hands. Also the models of these polyedra he cut out in boxwood with amazing neatness and accuracy. Indeed few or none of the mathematical instrument-makers could exceed him in exactly graduating or neatly engraving any mathematical or astronomical instrument, as may be seen in the equatorial instrument above-mentioned, or in his sextant, quadrants and dials of various kinds; also in a curious armillary sphere, which, besides the common properties, has moveable circles &c, for exhibiting and resolving all spherical triangles; also his double sector, with many other instruments, all contrived, graduated and finished, in a most elegant manner, by himself. In short, he possessed at once a remarkably clear head for contriving, and an extraordinary hand for executing any thing, not only in mechanics, but also in drawing, writing, and making the most exact and beautiful schemes or figures in all his calculations and geometrical constructions.

The quadrature of the circle was undertaken by him for his own private amusement in the year 1699, deduced from two different series, by which the truth of it was proved to 72 places of figures; that is, if the diameter of a circle be 1, the circumference will be found equal to 3·141592653589793238462643383279502884197169399375105820974944592307816405&c. He gave also ingenious improvements on the making of logarithms, and the constructing of the natural sines, tangents, and secants.

He also calculated the natural and logarithmic sines, tangents, and secants, to every second in the first minute of the quadrant: the laborious investigation of which may probably be seen in the archives of the Royal Society, as they were presented to Mr. Patrick Murdoch for that purpose; exhibiting his very neat and accurate manner of writing and arranging his figures, not to be equalled perhaps by the best penman now living.

The late ingenious Mr. Smeaton says (Philos. Trans. an. 1786, pa. 5, &c):—" In the year 1689, Mr. Flamsteed completed his mural arc at Greenwich; and, in the Prolegomena to his Historia Cœlestis, he makes an ample acknowledgment of the particular assistance, care, and industry of Mr. Abraham Sharp; whom, in the month of August 1688, he brought into the observatory, as his amanuensis; and being, as Mr. Flamsteed tells us, not only a very skilful mathematician, but exceedingly expert in mechanical operations, he was principally employed in the construction of the mural arc; which in the compass of 14 months he finished, so greatly to the satisfaction of Mr. Flamsteed, that he speaks of him in the highest terms of praise.

" This celebrated instrument, of which he also gives the figure at the end of the Prolegomena, was of the radius of 6 feet 7½ inches; and, in like manner as the sextant, it was furnished both with screw and diagonal divisions, all performed by the accurate hand of Mr. Sharp. Yet, whoever compares the different parts of the table for conversion of the revolutions and parts of the screw belonging to the mural arc into degrees, minutes, and seconds, with each other, at the same distance from the zenith on different sides; and with their halves, quarters, &c, will find as notable a disagreement of the screw-work from the hand divisions, as had appeared before in the work of Mr. Tompion: and hence we may conclude, that the method of Dr. Hooke, being executed by two such masterly hands as Tompion and Sharp, and found defective, is in reality not to be depended upon in nice matters.

" From the account of Mr. Flamsteed it appears also, that Mr. Sharp obtained the zenith point of the instrument, or line of collimation, by observation of the zenith stars, with the face of the instrument on the east and on the west side of the wall: and that having made the index stronger (to prevent flexure) than that of the sextant, and thereby heavier, he contrived, by means of pulleys and balancing weights, to relieve the hand that was to move it from a great part of its gravity. Mr. Sharp continued in strict correspondence with Mr. Flamsteed as long as he lived, as appeared by letters of Mr. Flamsteed's found after Mr. Sharp's death; many of which I have seen.

" I have been the more particular relating to Mr. Sharp, in the business of constructing this mural arc; not only because we may suppose it the first good and valid instrument of the kind, but because I look upon Mr. Sharp to have been the first person that cut accurate and delicate divisions upon astronomical instruments; of which, independent of Mr. Flamsteed's testimony, there still remain considerable proofs: for, after leaving Mr. Flamsteed, and quitting the department above-mentioned, he retired into Yorkshire, to the village of Little Horton, near Bradford, where he ended his days about the year 1742; and where I have seen not only a large and very fine collection of mechanical tools, the principal ones being made with his own hands, but also a great variety of scales and instruments made with them, both in wood and brass, the divisions of which were so exquisite, as would not discredit the first artists of the present times: and I believe there is now remaining a quadrant, of 4 or 5 feet radius, framed of wood, but the limb covered with a brass plate; the subdivisions being done by diagonals, the lines of which are as finely cut as those upon the quadrants at Greenwich. The delicacy of Mr. Sharp's hand will indeed permanently appear from the copper-plates in a quarto book, published in the year 1718, intituled Geometry Improved by A. Sharp, Philomath." (or rather 1717, by A. S. Philomath.) " whereof not only the geometrical lines upon the plates, but the whole of the engraving of letters and figures, were done by himself, as I was told by a person in the mathematical line, who very frequently attended Mr. Sharp in the latter part of his life. I therefore look upon Mr. Sharp as the first person that brought the affair of hand division to any degree of perfection."

Mr. Sharp kept up a correspondence by letters with most of the eminent mathematicians and astronomers of his time, as Mr. Flamsteed, Sir Isaac Newton, Dr. Halley, Dr. Wallis, Mr. Hodgson, Mr. Sherwin, &c, the answers to which letters are all written upon the backs, or empty spaces, of the letters he received, in a short-hand of his own contrivance. From a great variety of letters (of which a large chest full remain with his friends) from these and many other celebrated mathematicians, it is evident that Mr. Sharp spared neither pains nor time to promote real science. Indeed, being one of the most accurate and indefatigable computers that ever existed, he was for many years the common resource for Mr. Flamsteed, Sir Jonas Moore, Dr. Halley, and others, in all sorts of troublesome and delicate calculations.

Mr. Sharp continued all his life a bachelor, and spent his time as recluse as a hermit. He was of a middle stature, but very thin, being of a weakly constitution; he was remarkably feeble the last three or four years before he died, which was on the 18th of July 1742, in the 91st year of his age.

In his retirement at Little Horton, he employed four or five rooms or apartments in his house for different purposes, into which none of his family could possibly enter at any time without his permission. He was seldom visited by any persons, except two gentlemen of Bradford, the one a mathematician, and the other an ingenious apothecary: these were admitted, when he chose to be seen by them, by the signal of rubbing a stone against a certain part of the outside wall of the house. He duly attended the dissenting chapel at Bradford, (of which he was a member,) every Sunday; at which time he took care to be provided with plenty of halfpence, which he very charitably suffered to be taken singly out of his hand, held behind him during his walk to the chapel, by a number of poor people who followed him, without his ever looking back, or asking a single question.

Mr. Sharp was very irregular as to his meals, and remarkably sparing in his diet, which he frequently took in the following manner. A little square hole, something like a window, made a communication between the room where he was usually employed in calculations, and another chamber or room in the house where a servant could enter; and before this hole he had contrived a sliding board: the servant always placed his victuals in this hole, without speaking or making any the least noise; and when he had a little leisure he visited his cupboard to see what it afforded to satisfy his hunger or thirst. But it often happened, that the breakfast, dinner, and supper have remained untouched by him, when the servant has gone to remove what was left—so deeply engaged had he been in calculations. Cavities might easily be perceived in an old English oak table where he sat to write, by the frequent rubbing and wearing of his elbows.—*Gutta cavat lapidem, &c.*

By Mr. Sharp's epitaph it appears that he was related to archbishop Sharp. And Mr. Sharp the eminent surgeon, who it seems has lately retired from business, is the nephew of our author. Another nephew was the father of Mr. Ramsden, the late celebrated instrument-maker, who says that his grand uncle Abraham, our author, was some time in his younger days an exciseman; which occupation he quitted on coming to a patrimonial estate of about 200l. a-year.

Sharp, in Music, a kind of artificial note, or character, thus formed ♯: this being prefixed to any note, shows that it is to be sung or played a semitone or half note higher than the natural note is. When a sharp is placed at the beginning of a stave or movement, it shows that all notes that are found on the same line, or space, throughout, are to be raised half a tone above their natural pitch, unless a natural intervene. When a sharp occurs accidentally, it only affects as many notes as follow it on the same line or space, without a natural, in the compass of a bar.

SHEAVE, in Mechanics, a solid cylindrical wheel, fixed in a channel, and moveable about an axis, as being used to raise or increase the mechanical powers applied to remove any body.

SHEERS, aboard a ship, an engine used to hoist or displace the lower masts of a ship.

SHEKEL, or **Shekle**, an ancient Hebrew coin and weight, equal to 4 Attic drachmas, or 4 Roman denarii, or 2s. 9½d. sterling. According to father Mersenne, the Hebrew shekel weighs 268 grains, and is composed of 20 oboli, each obolus weighing 16 grains of wheat.

SHERBURNE (Edward), an ingenious scholar, was born in London in 1616, and died in 1702. After completing his education, he travelled abroad; but returned in 1641, and succeeded, on his father's death, to the office of clerk of the ordnance. He was imprisoned for some time by the parliament, and on recovering his liberty joined the king, whom he served with great bravery, by which he suffered considerably in his estate. After the battle of Edgehill he went to Oxford, where he was created master of arts. At the restoration he recovered his place, was knighted, and made commissary-general of the artillery.

Sherburne published a volume of poems, and a translation of Seneca's tragedies. But his chief work was a translation of The Sphere of M. Manilius, made an English poem, with annotations and an astronomical appendix: London 1675, in folio. Of the parts of this poem, their distribution and order, and of the interpreter's labours in explaining it, both in his learned notes and considerable appendix, he observes, that the poem begins with a succinct indication of the origin and progress of arts and sciences, particularly of astronomy; of which last, besides what the translator has noted in his marginal illustrations, he has added, for the satisfaction of the more curious, a compendious history, continued down to the age of Manilius; with a very instructive catalogue of the most eminent astronomers, from the first parent of all arts, and mankind itself, to the editor's time. A more particular and satisfactory account of this work may be seen in the Philos. Trans. vol. 9, pa. 228, or in my Abridg. vol. 2, pa. 185.

SHILLING, an English silver coin, equal to 12 pence, or the 20th part of a pound sterling. This was a Saxon coin, being the 48th part of their pound weight. Its value at first was 5 pence; but it was reduced to 4 pence about a century before the conquest. After the conquest, the French solidus of 12 pence, which was in use among the Normans, was called by the English name of shilling; and the Saxon shilling of 4 pence took a Norman name, and was called the groat, or great coin, because it was the largest English coin then known. From this time, the shilling underwent many alterations.

In the time of Edward the 1st, the pound troy was the same as the pound sterling of silver, consisting of 20 shillings; so that the shilling weighed the 20th part of a pound, or more than half an ounce troy. But some are of opinion, there were no coins of this denomination, till Henry the 7th, in the year 1504, first coined silver pieces of 12 pence value, which we call shillings. Since the reign of Elizabeth, a shilling weighs the 62nd part of a pound troy, or 3 dwts. $20\frac{28}{31}$ grs, the pound weight of silver making 62 shillings. And hence the ounce of silver is worth $5\frac{1}{6}$ shillings, or 5s. 2d.

Many other nations have also their shillings. The English shilling is worth about 23 French sols; those of Holland and Germany about half as much, or $11\frac{1}{2}$ sols; those of Flanders, about 9. The Dutch shillings are also called sols de gros, because equal to 12 gross. The Danes have copper shillings, worth about one fourth of a farthing sterling.

SHIVERS, in a ship, the seamen's term for those little round wheels, in which the rope of a pulley or block runs. They turn with the rope, and have pieces of brass in their centres, into which the pin of the block goes, and on which they turn.

SHORT (James), a very eminent optician and telescope-maker, was the son of a joiner at Edinburgh, where James was born in 1710. At ten years of age, his parents being both dead, he was placed as a poor boy, in Heriot's charity hospital at that place. Two years after however, having shown uncommon talents, he was sent to the high-school of that city, where he so much distinguished himself in classical learning, that his friends thought of qualifying him for a learned profession. After 4 years spent at the high-school, in 1726 he was entered a student in the university of Edinburgh; where he passed through a regular course of study; took his degree of master of arts; and, at the earnest entreaties of his relations, attended the divinity hall; after which, in 1731, he passed his trials to fit him for a preacher in the church of Scotland.

Soon after this, however, the mind of our young artist began to revolt against the idea of a profession so little suited to his talents; and having had occasion to attend a course of Mr. Maclaurin's mathematical class in the college, he there so much distinguished himself, that the professor took great notice of him, and invited him often to his house, where he had opportunities of knowing more fully the extent of the young man's capacity. In 1732, Mr. M. kindly permitted his pupil to make use of his rooms in the college, for his apparatus, where he began to work in his new profession of telescope-making, under the eye of his eminent master and patron; who, in a letter about two years after to Dr. Jurin, mentions the proficiency made by Mr. Short, in constructing reflecting telescopes, in these words: "Mr. Short, who had begun with making glass specula, is now employing himself to improve the metallic. By taking care of the figure, he is enabled to give them larger apertures than others have done; and, upon the whole, they surpass in perfection all that I have seen of other workmen." The figure which Mr. S. gave to his great specula, was parabolic: which he did however not by any rule or canon, but by practice and mechanical devices.

Mr. S. continued from this time to practise his art as a regular profession, with much success; so that when, in the year 1736, he was called up to London, at the desire of queen Caroline, to give mathematical instructions to Wm. duke of Cumberland, he had cleared the sum of 500l. by the profits of his business. Towards the end of the same year he returned to Edinburgh; and having made several useful improvements in his art, during his stay in England, he now prosecuted it with fresh vigour and success. In 1739, being then again at London, the earl of Morton took Mr. S. with him on a tour to the Orkney isles, and engaged him there to adjust the geography of that part of Scotland. He returned to London with the earl, and finally established himself there, in the line of his profession. In 1743, he was employed by lord Thos. Spencer, to make a reflector of 12 feet focus, being the largest that he ever constructed, except those for the king of Spain, and some others of the same focal distance, with great improvements and higher magnifiers. The telescope for the king of Spain was finished in the year 1752, which, with its whole apparatus, cost 1200*l*. But the instrument made for lord Thomas Spencer, having fewer accompaniments, was purchased for 600 guineas. Mr. Short died at Newington Butts, near London, in 1768, at 58 years of age; and, from the great profits and success of his trade, left at his death a fortune of 20 thousand pounds.

Mr. S. was a good general scholar, besides well skilled in optics and mathematical learning. He was a very useful member of the Royal Society, and wrote a great multitude of excellent papers in the Philos. Trans. from the year 1736 till the time of his death. Among them, his determination of the sun's parallax at about $8\frac{2}{3}''$, from his ingenious calculations on the transit of Venus, has been pretty generally adopted by astronomers.

SHORT-SIGHTEDNESS, *myopia*, a defect in the conformation of the eye, when the crystalline &c being too convex, the rays that enter the eye are refracted too much, and made to converge too fast, so as to unite before they reach the retina, by which means vision is rendered dim and confused.

It is commonly thought that short-sightedness wears off in old age, on account of the eye becoming flatter; but Dr. Smith questions whether this be matter of fact, or only hypothesis. It is remarkable that short-sighted persons commonly write a small hand, and affect a small print, because they can see more of it at one view: that it is customary with them not to look at the person they converse with, because they cannot well see the motion of his eyes and features, and are therefore attentive to his words only: that they see more distinctly, and somewhat further off, by a strong light, than by a weak one; because a strong light causes a contraction of the pupil, and consequently of the pencils, both here and at the retina, which lessens their mixture, and consequently the apparent confusion; and therefore, to see more distinctly, they almost close their eye-lids, for which reason they were anciently called myopes. Smith's Optics, vol. 2, Rem. pa. 10.

Dr. Jurin observes, that persons who are much and long accustomed to view objects at small distances, as students in general, watchmakers, engravers, painters in miniature, &c, see better at small distances, and worse at great distances, than other people. And he gives the reasons, from the mechanical effect of habit in the eye. Essay on Dist. and Indist. Vision.

The ordinary remedy for short-sightedness is a concave lens, held before the eye; for this causing the rays to diverge, or at least diminishing much of their convergency, it makes a compensation for the too great convexity of the crystalline. Dr. Hooke suggests another remedy; which is to employ a convex glass, in a position between the object and the eye, by means of which, the object may be made to appear at any distance from it, and so the eye be made to contemplate the picture in the same manner as if the object itself were in its place. But here unfortunately the image will appear inverted: for this however he has some whimsical expedients; viz, in reading to turn the book upside down, and to learn to write upside down. As to distant objects, the doctor asserts, from his own experience, that with a little practice in contemplating inverted objects, one gets as good an idea of them as if seen in their natural posture.

SHOT, in the Military Art, includes all kinds of balls or bullets for fire arms, from the cannon to the pistol. As to those for mortars, they are usually called shells. Shot are mostly of a round form, though there are other shapes. Those for cannon are of iron; but those for muskets and pistols are of lead. Cannon shot and shells are usually set up in piles, or heaps, tapering from the base towards the top; the base being either a triangle, a square, or a rectangle; from which the number in the pile is easily computed. See PILE.

The weight and dimensions of balls may be found, the one from the other, whether they are of iron or of lead. Thus, the weight of an iron ball of 4 inches diameter, is 9lb, and because the weight is as the cube of the diameter, therefore as $4^3 : 9 :: d^3 : \frac{9}{64} d^3 = w$, the weight of the iron ball whose diameter is d; that is, $\frac{9}{64}$ of the cube of its diameter. And, conversely, if the weight be given, to find the diameter, it will be $\sqrt[3]{\frac{64}{9} w} = d$; that is, take $\frac{64}{9}$ or $7\frac{1}{9}$ of the weight, and the cube root of that will be the diameter of the iron ball.

For leaden balls; one of $4\frac{1}{4}$ inches diameter weighs 17 pounds; therefore as the cube of $4\frac{1}{4}$ is to 17, or nearly as $9 : 2 :: d^3 : \frac{2}{9} d^3 = w$, the weight of the leaden ball whose diameter is d, that is, $\frac{2}{9}$ of the cube of the diameter. On the contrary, if the weight be given, to find the diameter, it will be $\sqrt[3]{\frac{9}{2} w} = d$; that is, $\frac{9}{2}$ or $4\frac{1}{2}$ of the weight, and the cube root of the product. See my Conic Sections and Select Exercises, pa. 141; or my Math. Course, vol. 2, p. 269

SHOULDER *of a Bastion*, in Fortification, is the angle where the face and the flank meet.

SHOULDERING, in Fortification. See EPAULEMENT.

SHUCKBURGH-EVELYN (Sir GEORGE A. W. bart.) died at his seat in Warwickshire, Sept. 1804, in the 54th year of his age. He had represented that county in three successive parliaments; where his integrity, and independent conduct as a British senator, procured him the respect of all wise and good men. Sir G. was an elegant classical scholar, and had improved his knowledge of men and science by profitable travels through Europe. He was a considerable mathematician and philosopher, and well skilled in astronomy both theoretical and practical; in which sciences his deep and laborious researches gave him a distinguished rank in the Royal and Antiquarian Societies, whose publications are adorned with several of his learned and ingenious compositions, particularly his paper on the Barometrical Measurements of Altitudes. Sir Geo. carried his mathematical and logical habits into every purpose in life, in every circumstance of which, he was one of the most correct and methodical of men. Of men, and motives of action, Sir Geo. was a most accurate judge, and was always attentive to guard himself against the impositions of the designing. In matters of science too, no man was more wary of making hasty inferences, or of forming general conclusions from partial or inaccurate observations. Truth was his darling object; which he endeavoured to discover, and to detect error, by the most patient vigilance. Had Sir Geo. devoted more of his time to those pursuits, he would probably have had few superiors in philosophical celebrity. The pains he took to adjust a regular and uniform standard of weights and measures, the tardy cautiousness of his experiments, the accuracy of his calculations, and the practicability of his schemes, entitle him to the highest praise, among such as have laboured for the public benefit.

SHWAN-*pan*, a Chinese instrument, composed of a number of wires, with beads upon them, which they move backwards, and forwards, and which serves to assist them in their computations. See ABACUS.

SIDE, *latus*, in Geometry. The side of a figure is a line making part of the periphery of any superficial figure. In triangles, the sides are also called legs. In a right-angled triangle, the two sides that include the right angle, are called catheti, or sometimes the base and perpendicular; and the third side, the hypothenuse.

SIDE *of a Polygonal Number*, is the number of terms in the arithmetical progression that are summed up to form the number.

SIDE *of a Power*, is what is usually called the root.

SIDES of *Horn-works*, *Crown-works*, *Double-tenailles*, &c, are the ramparts and parapets which inclose them on the right and left, from the gorge to the head.

SIDEREAL, something relating to the stars. As sidereal year, day, &c, being those marked out by the stars.

SIDEREAL *Year*. See YEAR.

SIDEREAL *Day*, is the time in which any star appears to revolve from the meridian to the meridian again; or the time in which the earth makes one complete revolution on its axis, which is 23 hours $56' 4'' 6'''$ of mean solar time; there being 366 sidereal days in a year; that is, the earth makes 366 revolutions on its axis, though we only see the sun rise 365 times; so that 366 terrestrial revolutions would be exactly equal to 365 diurnal revolutions of the sun, if the equinoctial points were at rest in the heavens. But these points go backward, with respect to the stars, at the rate of $50''$ of a degree in a Julian year; which causeth the stars to have an apparent progressive motion eastward $50''$ in that time. And as the sun's mean motion in the ecliptic is only 11 signs $29° 45' 40'' 15'''$ in 365 days, it follows, that at the end of that time he will be $14' 19'' 45'''$ short of that point of the ecliptic from which he set out at the beginning; and the stars will be advanced $50''$ of a degree with respect to that point.

Consequently, if the sun's centre be on the meridian with any star on any given day of the year, that star will be $14' 19'' 45''' + 50''$ or $15' 9'' 45'''$ east of the sun's centre, on the 365th day afterward, when the sun's centre is on the meridian; and therefore that star will not come to the meridian on that day till the sun's centre has passed it by $1' 0'' 38''' 57''''$ of mean solar time; for the sun takes so much time to go through an arc of $15' 9'' 45'''$; and then, in $365^d\, 0^h\, 1' 0'' 38''' 57''''$ the star will have just completed its 366th revolution to the meridian.

In the following table, of sidereal revolutions, the first column contains the number of revolutions of the stars; the others exhibit the times in which these revolutions are made, as shown by a well regulated clock; those on the right hand show the daily accelerations of the stars, that is, how much any star gains upon the time shown by such a clock, in the corresponding revolutions.

Revol. of the stars.	Times in which the revolutions are made.						Accelerations of the stars.				
	da.	ho.	m.	sec.	th.	fo.	ho.	m.	sec.	th.	fo.
1	0	23	56	4	6	0	0	3	55	54	0
2	1	23	52	8	12	1	0	7	51	47	59
3	2	23	48	12	18	1	0	11	47	41	59
4	3	23	44	16	24	2	0	15	43	35	58
5	4	23	40	20	30	2	0	19	39	29	58
6	5	23	36	24	36	3	0	23	35	23	57
7	6	23	32	28	42	3	0	27	31	17	57
8	7	23	28	32	48	4	0	31	27	11	56
9	8	23	24	36	54	4	0	35	23	5	56
10	9	23	20	41	0	5	0	39	18	59	55
11	10	23	16	45	6	5	0	43	14	53	55
12	11	23	12	49	12	6	0	47	10	47	54
13	12	23	8	53	18	6	0	51	6	41	54
14	13	23	4	57	24	7	0	55	2	35	53
15	14	23	1	1	30	7	0	58	58	29	53
16	15	22	57	5	36	8	1	2	54	23	52
17	16	22	53	9	42	8	1	6	50	17	52
18	17	22	49	13	48	9	1	10	46	11	51
19	18	22	45	17	54	9	1	14	42	5	51
20	19	22	41	22	0	10	1	18	37	59	50
21	20	22	37	26	6	10	1	22	33	53	50
22	21	22	33	30	12	11	1	26	29	47	49
23	22	22	29	34	18	11	1	30	25	41	49
24	23	22	25	38	24	12	1	34	21	35	48
25	24	22	21	42	30	12	1	38	17	29	48
26	25	22	17	46	36	13	1	42	13	23	47
27	26	22	13	50	42	13	1	46	9	17	47
28	27	22	9	54	48	14	1	50	5	11	46
29	28	22	5	58	54	14	1	54	1	5	46
30	29	22	2	3	0	15	1	57	56	59	45
40	39	21	22	44	0	19	2	37	15	59	41
50	49	20	43	25	0	24	3	16	34	59	36
100	99	17	26	50	0	48	6	33	9	59	12
200	199	10	53	40	1	37	13	6	19	58	23
300	299	4	20	30	2	25	19	39	29	57	35
360	359	0	24	36	2	54	23	35	23	57	6
365	364	0	4	56	32	56	23	55	3	27	4
366	365	0	1	0	38	57	23	58	59	21	3

This table will not differ the 279,936,000,000th part of a second of time from the truth in a whole year. It was calculated by Mr. Ferguson; and it is the only table of the kind in which the recession of the equinoctial points has been taken into the calculation.

SIGN, in Algebra, a symbol or character, employed to denote some particular operation. Those most commonly used are, + for addition, − for subtraction, × or · for multiplication, ÷ for division, $\sqrt{}$ for the square root, $\sqrt[3]{}$ for the cube root, and $\sqrt[n]{}$ for the *n*th root; also = for equality, &c.

Signs, *like, positive, negative, radical,* &c. See the adjectives.

Sign, in Astronomy, a 12th part of the ecliptic, or zodiac; or a portion containing 30 degrees of the same.

The ancients divided the zodiac into 12 segments, called signs; commencing at the point where the ecliptic and equinoctial intersect, and so counting forward from west to east, according to the course of the sun; these signs they named from the 12 constellations which possessed those segments in the time of Hipparchus. But the constellations have since so changed their places, by the precession of the equinox, that Aries is now found in the sign called Taurus, and Taurus in that of Gemini, &c.

The names, and characters, of the 12 signs, and their order, are as follow: Aries ♈, Taurus ♉, Gemini ♊, Cancer ♋, Leo ♌, Virgo ♍, Libra ♎, Scorpio ♏, Sagittarius ♐, Capricornus ♑, Aquarius ♒, Pisces ♓; each of which, with the stars in them, see under its proper article, **Aries**, **Taurus**, &c.

The signs are distinguished, with regard to the season of the year when the sun is in them, into vernal, æstival, autumnal, and brumal.

Vernal or *Spring* **Signs**, are Aries, Taurus, Gemini.

Æstival or *Summer* **Signs**, are Cancer, Leo, Virgo.

Autumnal **Signs**, are Libra, Scorpio, Sagittary.

Brumal or *Winter* **Signs**, are Capricorn, Aquarius, Pisces. The vernal and summer signs are also called northern signs, because they are on the north side of the equinoctial; and the autumnal and winter signs are called southern ones, because they are on the south side of the same.

The signs are also distinguished into ascending and descending, according as they are ascending toward the north, or descending toward the south. Thus, the

Ascending **Signs**, are the winter and spring signs, or those six from the winter solstice to the summer solstice, viz, the signs Capricorn, Aquarius, Pisces, Aries, Taurus, Gemini. And the

Descending **Signs** are the summer and autumn signs, or the signs Cancer, Leo, Virgo, Libra, Scorpio, Sagittary.

Signs, *Fixed, Masculine,* &c; see the adjectives.

SILLON, in Fortification, an elevation of earth, made in the middle of the moat; to fortify it, when too broad. It is more usually called the Envelope.

SIMILAR, in Arithmetic and Geometry, the same with like. Similar things have the same disposition or conformation of parts, and differ in nothing but as to their quantity or magnitude; as two squares, or two circles, &c. In Mathematics, similar parts, as A, *a*, have the same ratio to their wholes B, *b*; and if the wholes have the same ratio to the parts, the parts are similar.

Similar *angles*, are also equal angles.

Similar *arcs*, of circles, are such as are like parts of their whole peripheries. And, in general, similar arcs of any like curves, are the like parts of the wholes.

Similar *bodies*, in Natural Philosophy, are such as have their particles of the same kind and nature one with another.

Similar *Curves*. Two segments of two curves are said to be similar when, any right-lined figure being inscribed within one of them, we can inscribe always a similar rectilineal figure in the other.

Similar *Conic Sections*, are such as are of the same kind, and have their principal axes and parameters proportional. So, two ellipses are figures of the same kind, but they are not similar unless the axes of the one have the same ratio as the axes of the other. And the same of two hyperbolas, or two parabolas. And generally, those curves are similar, that are of the same kind, and have their corresponding dimensions in the same ratio.—All circles are similar figures.

Similar *Diameters of Conic Sections*, are such as make equal angles with their ordinates.

Similar *Figures*, or plane figures, are such as have all their angles equal respectively, each to each, and their sides about the equal angles proportional. And the same of similar polygons.—Similar plane figures have their areas or contents, in the duplicate ratio of their like sides, or as the squares of those sides.

Similar *Plane Numbers*, are such as may be ranged into the form of similar rectangles; that is, into rectangles whose sides are proportional. Such are 12 and 48; for the sides of 12 are 6 and 2, and the sides of 48 are 12 and 4, which are in the same proportion, viz, 6 : 2 : : 12 : 4.

Similar *Polygons*, are polygons of the same number of angles, and the angles in the one equal severally to the angles in the other, also the sides about those angles proportional.

Similar *Rectangles*, are those that have their sides about the like angles proportional.—All squares are similar.

Similar *Segments of circles*, are such as contain equal angles.

Similar *Solids*, are such as are contained under the same number of similar planes, alike situated.—Similar solids are to each other as the cubes of their like linear dimensions.

Similar *Solid Numbers*, are those whose little cubes may be so ranged, as to form similar parallelopipedons.

Similar *Triangles*, are such as are equiangular ones, or have all their three angles respectively equal in each triangle. For it is sufficient for triangles to be similar, that they be equiangular; because, being equiangular, they necessarily have their sides proportional, which is a condition of similarity in all figures. As to other figures, having more sides than three, they may be equiangular, without having their sides proportional, and therefore without being similar.—Similar triangles are as the squares of their like sides.

SIMILITUDE, in Arithmetic and Geometry, denotes the relation of things that are similar to each other. Euclid and, after him, most other authors, demonstrate every thing in geometry from the principle of congruity. Wolfius, instead of it, substitutes that of similitude, which, he says, was communicated to him by Leibnitz, and which he finds of very considerable use in geometry, as serving to demonstrate many things directly, which are only demonstrable from the principle of congruity in a very tedious manner.

SIMPLE, something not mixed, or not compounded; in which sense it stands opposed to compound. The elements are simple bodies, from the composition of which there result all sorts of mixed bodies.

Simple *Equation, Fraction,* and *Surd.* See the substantives.

Simple *Quantities*, in Algebra, are those that consist of one term only; as a, or $-ab$, or $3abc$: in opposition to compound quantities, which consist of two or more terms; as $a + b$, or $a + 2b - 3ac$.

Simple *Flank*, and *Tenaille*, in Fortification. See the substantives.

Simple *Machine, Motion, Pendulum,* and *Wheel*, in Mechanics. See the substantives. The simplest machines are always the most esteemed. And in geometry, the most simple demonstrations are the best.

Simple *Problem*, in Mathematics. See Linear *Problem*.

Simple *Vision*, in Optics. See Vision.

SIMPSON (Thomas), F. R. S. a very eminent mathematician, and professor of mathematics in the Royal Military Academy at Woolwich, was born at Market Bosworth, in the county of Leicester, the 20th of August 1710. His father was a stuff weaver in that town; and though in tolerable circumstances, yet, intending to bring up his son Thomas to his own business, he took so little care of his education, that he was only taught to read English. But nature had furnished him with talents and a genius for far other pursuits; which led him afterwards to the highest rank in the mathematical and philosophical sciences.

Young Simpson very soon gave indications of his turn for study in general, by eagerly reading all books he could meet with, teaching himself to write, and embracing every opportunity he could find of deriving knowledge from other persons. His father observing him thus to neglect his business, by spending his time in reading what he thought useless books, and following other similar pursuits, used all his endeavours to check such proceedings, and to induce him to follow his profession with steadiness and better effect. And after many struggles for this purpose, the differences thus produced between them at length rose to such a height, that our author quitted his father's house entirely.

On this occasion he repaired to Nuneaton, a town at a small distance from Bosworth, where he went to lodge at the house of a tailor's widow, of the name of Swinfield, who had been left with two children, a daughter and a son, by her husband, of whom the son, who was the younger, being but about two years older than Simpson, had become his intimate friend and companion. And here he continued some time, working at his trade, and improving his knowledge by reading such books as he could procure.

Among several other circumstances which, long before this, gave occasion to show our author's early thirst for knowledge, as well as proving a fresh incitement to acquire it, was that of a large solar eclipse, which took place on the 11th day of May, 1724. This phenomenon, so awful to many who are ignorant of the cause of it, struck the mind of young Simpson with a strong curiosity to discover the reason of it, and to be able to predict the like surprising events. It was however several years before he could obtain his desire, which at length was gratified by the following accident. After he had been some time at Mrs. Swinfield's, at Nuneaton, a travelling pedlar came that way, and took a lodging at the same house, according to his usual custom. This man, to his profession of an itinerant merchant, had joined the more profitable one of a fortune-teller, which he performed by means of judicial astrology. Every one knows with what regard persons of such a cast are treated by the inhabitants of country villages; it cannot be surprising therefore that an untutored lad of 19 should look upon this man as a prodigy, and, regarding him in this light, should endeavour to ingratiate himself into his favour; in which he succeeded so well, that the sage was no less taken with the quick natural parts and genius of his new acquaintance. The pedlar, intending a journey to Bristol fair, left in the hands of young Simpson an old edition of Cocker's Arithmetic, to which was subjoined a short Appendix on Algebra, and a book upon Genitures, by Partridge the almanac-maker. These books he had perused to so good purpose, during the absence of his friend, as to excite his amazement upon his return; in consequence of which he set himself about erecting a genethliacal figure, in order to a presage of Thomas's future fortune.

This position of the heavens having been maturely considered secundum artem, the wizard, with great confidence, pronounced, that, "within two years time Simpson would turn out a greater man than himself!"

In fact, our author profited so well by the encouragement and assistance of the pedlar, afforded him from time to time when he occasionally came to Nuneaton, that, by the advice of his friend, he at length made an open profession of casting nativities himself; from which, together with teaching an evening school, he derived a pretty pittance, so that he greatly neglected his weaving, to which indeed he had never manifested any great attachment, and soon became the oracle of Nuneaton, Bosworth, and the environs. Scarce a courtship advanced to a match, or a bargain to a sale, without previously consulting the infallible Simpson about the consequences. But as to helping people to stolen goods, he always declared that above his skill; and over life and death he declared he had no power: all those called lawful questions he readily resolved, provided the persons were certain as to the horary data of the horoscope: and, he has often declared, with such success, that if from very cogent reasons he had not been thoroughly convinced of the vain foundation and fallaciousness of his art, he never should have dropt it, as he afterwards found himself in conscience bound to do.

About this time he married the widow Swinfield, in whose house he lodged, though she was then almost old enough to be his grandmother, being upwards of fifty years of age. After this the family lived comfortably enough together for some short time, Simpson occasionally working at his business of a weaver in the day-time, and teaching an evening school or telling fortunes at night; the family being also further assisted by the labours of young Swinfield, who had been brought up in the profession of his father.

But this tranquillity was soon interrupted, and our author driven at once from his home and the profession of astrology, by the following accident. A young woman in the neighbourhood had long wished to hear or know something of her lover, who had been gone to sea; but Simpson had put her off from time to time, till the girl grew at last so importunate, that he could deny her no longer. He asked her if she would be afraid if he should raise the devil, thinking to deter her; but she declared she feared neither ghost nor devil: so he was obliged to comply. The scene of action pitched on was a barn, and young Swinfield was to act the devil or ghost; who being concealed under some straw in a corner of the barn, was, at a signal given, to rise slowly out from among the straw, with his face marked so that the girl might not know him. Every thing being in order, the girl came at the time appointed; when Simpson, after cautioning her not to be afraid, began muttering some mystical words, and chalking round about them, till, on the signal given, up rises the tailor slow and solemn, to the great terror of the poor girl, who, before she had seen half his shoulders, fell into violent fits, crying out, it was the very image of her lover; and the effect upon her was so dreadful, that it was thought either death or madness must be the consequence. So that poor Simpson was obliged immediately to abandon at once both his home and the profession of a conjuror.

On this occasion it would seem he fled to Derby, where he remained about two or three years, viz, from 1733 till 1735 or 1736; instructing pupils in an evening school, and working at his trade by day.

It would seem that Simpson had an early turn for versifying, both from the circumstance of a song written here in favour of the Cavendish family, on occasion of the parliamentary election at that place, in the year 1733; and from his first two mathematical questions that were published in the Ladies Diary, which were both in a set of verses, not ill written for the occasion. These were printed in the Diary for 1736, and therefore must at latest have been written in the year 1735. These two questions, being at that time pretty difficult ones, show the great progress he had even then made in the mathematics; and from an expression in the first of them, viz, where he mentions his residence as being in latitude 52°, it appears he was not then come up to London, though he must have done so very soon after.

Together with his astrology, he had soon furnished himself with arithmetic, algebra, and geometry sufficient to be qualified for looking into the Ladies Diary (of which he had afterwards for several years the direction), by which he came to understand that there was a still higher branch of the mathematical knowledge than any he had yet been acquainted with; and this was the method of Fluxions. But our young analyst was quite at a loss to discover any English author who had written on the subject, except Mr. Hayes; and his work being a folio, and then pretty scarce, exceeded his ability of purchasing: however an acquaintance lent him Mr. Stone's Fluxions, which is a translation of the Marquis de l'Hospital's Analyse des Infinimens Petits: by this one book, and his own penetrating talents, he was, as we shall see presently, enabled in a very few years to compose a much more accurate treatise on this subject than any that had before appeared in our language.

After he had quitted astrology and its emoluments, he was driven to hardships for the subsistence of his family, while at Derby, notwithstanding his other industrious endeavours in his own trade by day, and teaching pupils at evenings. This determined him to repair to London, which he did in 1735 or 1736.

On his first coming to London, Mr. Simpson wrought for some time at his business in Spitalfields, and taught mathematics at evenings, or any spare hours. His industry turned to so good account, that he returned down into the country, and brought up his wife and three children, she having produced her first child to him in his absence. The number of his scholars increasing, and his abilities becoming in some measure known to the public, he was encouraged to make proposals for publishing by subscription, "A new Treatise of Fluxions: wherein the Direct and Inverse Methods are demonstrated after a new, clear, and concise Manner, with their Application to Physics and Astronomy: also the Doctrine of Infinite Series and Reverting Series universally, are amply explained, Fluxionary and Exponential Equations solved: together with a variety of new and curious Problems."

The book was published in 4to, in the year 1737, though the author had been frequently interrupted from furnishing the press so fast as he could have wished, through his unavoidable attention to his pupils for his immediate support. The principles of fluxions treated of in this work, are demonstrated in a method accurately true and genuine, not different from that of their great inventor, being entirely expounded by finite quantities.

In 1740, Mr. Simpson published a Treatise on The Nature and Laws of Chance, in 4to. To which are annexed, Full and clear Investigations of two important Problems added in the 2d edition of Mr. Demoivre's Book on Chances, as also two New Methods for the Summation of Series.

Our author's next publication was a 4to volume of Essays on several curious and interesting Subjects in Speculative and Mixed Mathematics; printed in the same year 1740. Soon after the publication of this book, he was chosen a member of the Royal Academy at Stockholm.

Our author's next work was, The Doctrine of Annuities and Reversions, deduced from general and evident Principles: with useful Tables, showing the Values of Single and Joint Lives, &c, in 8vo, 1742. This was followed, in 1743, by an Appendix containing some Remarks on a late book on the same Subject (by Mr. Abr. Demoivre, F. R. S.) with Answers to some personal and malignant Representations in the Preface thereof. To this answer Mr. Demoivre never thought fit to reply. A new edition of this work has lately been published, augmented with the tract on the same subject that was printed in our author's Select Exercises.

In 1743 also was published his Mathematical Dissertations on a variety of Physical and Analytical Subjects, in 4to; containing, among other particulars,

A Demonstration of the true Figure which the Earth, or any Planet, must acquire from its Rotation about an Axis. A general Investigation of the Attraction at the Surfaces of Bodies nearly spherical. A Determination of the Meridional Parts, and the Lengths of the several Degrees of the Meridian, according to the true Figure of the Earth. An Investigation of the Height of the Tides in the Ocean. A new Theory of Astronomical Refractions, with exact Tables deduced from the same. A new and very exact Method for approximating the Roots of Equations in Numbers; which quintuples the Number of Places at each Operation. Several new Methods for the Summation of Series. Some new and very useful Improvements in the Inverse Method of Fluxions. The work being dedicated to Martin Folkes, esq. president of the Royal Society.

His next book was A Treatise of Algebra, wherein the fundamental Principles are demonstrated, and applied to the Solution of a Variety of Problems. To which he added, The Construction of a great Number of Geometrical Problems, with the Method of resolving them numerically.

This work, which was designed for the use of young beginners, was printed in 8vo, 1745. A new edition appeared in 1755, with additions and improvements; among which was a new and general method of resolving all biquadratic equations, that are complete, or having all their terms. The work has gone through several other editions since that time: the 6th, or last, was in 1790.

His next work was, "Elements of Geometry, with their Application to the Mensuration of Superficies and Solids, to the Determination of Maxima and Minima, and to the Construction of a great Variety of geometrical Problems:" first published in 1747, in 8vo. And a second edition of the same came out in 1760, with great alterations and additions, being in a manner a new work, designed for young beginners, particularly for the gentlemen educated at the Royal Military Academy at Woolwich, and other editions have appeared since.

Mr. Simpson met with some trouble and vexation in consequence of the first edition of his Geometry. First, from some reflexions made upon it, as to the accuracy of certain parts of it, by Dr. Robert Simson, the learned professor of mathematics in the university of Glasgow, in the notes subjoined to his edition of Euclid's Elements. This brought an answer to those remarks from Mr. Simpson, in the notes added to the 2d edition as above; to some parts of which Dr. Simson again replied in his notes on the next edition of the said Elements of Euclid.— The second was by an illiberal charge of having stolen his Elements from Mr. Muller, the professor of fortification and artillery at the same academy at Woolwich, where our author was professor of geometry and mathematics. This charge was made at the end of the preface to Mr. Muller's Elements of Mathematics, in two volumes, printed in 1748; which was fully refuted by Mr. Simpson in the preface to the 2d edition of his Geometry.

In 1748 came out Mr. Simpson's Trigonometry, Plane and Spherical, with the Construction and Application of Logarithms, 8vo. This little book contains several things new and useful.

In 1750 came out, in two volumes, 8vo. The Doctrine and Application of Fluxions, containing, besides what is common on the Subject, a Number of new Improvements in the Theory, and the Solution of a Variety of new and very interesting Problems in different Branches of the Mathematics.—In the preface the author offers this to the world as a new book, rather than a second edition of that which was published in 1737, in which he acknowledges, that, besides errors of the press, there are several obscurities and defects, for want of experience, and the many disadvantages he then laboured under, in his first sally.

The idea and explanation here given of the first principles of fluxions, are not essentially different from what they are in his former treatise, though expressed in other terms. The consideration of time introduced into the general definition, will, he says, perhaps be disliked by those who would have fluxions to be mere velocities: but the advantage of considering them otherwise, viz, not as the velocities themselves, but as magnitudes they would uniformly generate in a given time, appears to obviate any objection on that head. By taking fluxions as mere velocities, the imagination is confined as it were to a point, and without proper care insensibly involved in metaphysical difficulties. But according to this other mode of explaining the matter, less caution in the learner is necessary, and the higher orders of fluxions are rendered much more easy and intelligible. Besides, though sir Isaac Newton defines fluxions to be the velocities of motions, yet he has recourse to the increments or moments generated in equal particles of time, in order to determine those velocities; which he afterwards teaches to expound by finite magnitudes of other kinds. This work was dedicated to George earl of Macclesfield.

In 1752 appeared, in 8vo, the Select Exercises for young Proficients in the Mathematics. This neat volume contains, A great Variety of algebraical Problems, with their Solutions. A select Number of Geometrical Problems, with their Solutions, both algebraical and geometrical. The Theory of Gunnery, independent of the Conic Sections. A new and very comprehensive Method for finding the Roots of Equations in Numbers. A short Account of the first Principles of Fluxions. Also the Valuation of Annuities for single and joint Lives, with a Set of new Tables, far more extensive than any extant. This last part was designed as a supplement to his Doctrine of Annuities and Reversions; but being thought too small to be published alone, it was inserted here at the end of the Select Exercises; from which however it has been removed in the last editions, and referred to its proper place, the end of the annuities, as before mentioned. The examples that are given to each problem in this last piece, are according to the London bills of mortality; but the solutions are general, and may be applied with equal facility and advantage to any other table of observations.

Mr. Simpson's Miscellaneous Tracts, printed in 4to, 1757, were his last legacy to the public: a most valuable bequest, whether we consider the dignity and importance of the subjects, or his sublime and accurate manner of treating them. The first of these papers is concerned in determining the Precession of the Equinox, and the different Motions of the Earth's Axis, arising from the Attraction of the Sun and Moon. It was drawn up about the year 1752, in consequence of another on the same subject, by M. de Sylvabelle, a French mathematician. Though this gentleman had gone through one part of the subject with success and perspicuity, and his conclusions were perfectly conformable to Dr. Bradley's observations; it nevertheless appeared to Mr. Simpson, that he had greatly failed in a very material part, and that indeed the only very difficult one; that is, in the determination of the momentary alteration of the position of the earth's axis, caused by the forces of the sun and moon; of which forces, the quantities, but not the effects, are truly investigated. The second paper contains the Investigation of a very exact Method or Rule for finding the Place of a Planet in its Orbit, from a Correction of Bishop Ward's circular Hypothesis, by Means of certain Equations applied to the Motion about the upper Focus of the Ellipse. By this Method the Result, even in the Orbit of Mercury, may be found within a Second of the Truth, and that without repeating the Operation. The third shows the Manner of transferring the Motion of a Comet from a parabolic Orbit, to an elliptic one; being of great Use, when the observed Places of a new Comet are found to differ sensibly from those computed on the Hypothesis of a parabolic Orbit. The fourth is an Attempt to show, from mathematical Principles, the Advantage arising from taking the Mean of a Number of Observations, in practical Astronomy; wherein the Odds that the Result in this Way, is more exact than from one single Observation, is evinced, and the Utility of the Method in Practice clearly made appear. The fifth contains the Determination of certain Fluents, and the Resolution of some very useful Equations, in the higher Orders of Fluxions, by Means of the Measures of Angles and Ratios, and the right and versed Sines of circular Arcs. The 6th treats of the Resolution of algebraical Equations, by the Method of Surd-divisors; in which the Grounds of that Method, as laid down by Sir Isaac Newton, are investigated and explained. The 7th exhibits the Investigation of a general Rule for the Resolution of Isoperimetrical Problems of all Orders, with some Examples of the Use and Application of the said Rule. The 8th, or last part, comprehends the Resolution of some general and very important Problems in Mechanics and Physical Astronomy; in which, among other Things, the principal Parts of the 3d and 9th Sections of the first Book of Newton's Principia are demonstrated in a new and concise Manner. But what may perhaps best recommend this excellent tract, is the application of the general equations, thus derived, to the determination of the Lunar Orbit.

According to what Mr. Simpson had intimated at the conclusion of his Doctrine of Fluxions, the greatest part of this arduous undertaking was drawn up in the year 1750. About that time M. Clairaut, a very eminent mathematician of the French Academy, had started an objection against Newton's general law of gravitation. This was a motive to induce Mr. Simpson, among some others, to endeavour to discover whether the motion of the moon's apogee, on which that objection had its whole weight and foundation, could not be truly accounted for, without supposing a change in the received law of gravitation, from the inverse ratio of the squares of the distances. The success answered his hopes, and induced him to look farther into other parts of the theory of the moon's motion, than he had at first intended: but before he had completed his design, M. Clairaut arrived in England, and paid Mr. Simpson a visit; from whom he learnt, that he had a little before printed a piece on that subject, a copy of which Mr. Simpson afterwards received as a present, and found in it the same things demonstrated, to which he himself had directed his enquiry, besides several others.

The facility of the method Mr. Simpson fell upon, and the extensiveness of it, will in some measure appear from this, that it not only determines the motion of the apogee, in the same manner, and with the same ease, as the other equations, but utterly excludes all that dangerous kind of terms that had embarrassed the greatest mathematicians, and would, after a great number of revolutions, entirely change the figure of the moon's orbit: whence this important consequence is derived, that the moon's mean motion, and the greatest quantities of the several equations, will remain unchanged, unless disturbed by the intervention of some foreign or accidental cause.

Besides the foregoing, which are the whole of the regular books or treatises that were published by Mr. Simpson, he wrote and composed several other papers and fugitive pieces, as follow:

Several papers of his were read at the meetings of the Royal Society, and printed in their Transactions; but as most, if not all of them, were afterwards inserted, with alterations or additions, in his printed volumes, it is needless to take any farther notice of them here.

He proposed, and resolved many questions in the Ladies Diaries, &c; sometimes under his own name, as in the years 1735 and 1736; and sometimes under feigned or fictitious names; such as, it is thought, Hurlothrumbo, Kubernetes, Patrick O'Cavenah, Marmaduke Hodgson, Anthony Shallow, Esq. and probably several others; see the Diaries for the years 1735, 1736, 42, 43, 53, 54, 55, 56, 57, 58, 59, and 60. Mr. Simpson was also the editor or compiler of the Diaries from the year 1754 till the year 1760, both inclusive, during which time he raised that work to the highest degree of respect. He was succeeded in the editorship by Mr. Edw. Rollinson, who continued till his death in the year 1773. See my Diarian Miscellany, vol. 3.

It has also been commonly supposed that he was the real editor of, or had a principal share in, two other periodical works of a miscellaneous mathematical nature; viz, the Mathematician, and Turner's Mathematical Exercises, two volumes, in 8vo, which came out in periodical numbers, in the years 1750 and 1751, &c. The latter of these seems especially to have been set on foot to afford a proper place for exposing the errors and absurdities of Mr. Robert Heath, the then conductor of the Ladies Diary and Palladium; and which controversy between them ended in the disgrace of Mr. Heath, and expulsion from his office of editor to the Ladies Diary, and the substitution of Mr. Simpson in his stead, in the year 1753.

In the year 1760, when the plans proposed for erecting a new bridge at Blackfriars were in agitation, Mr. Simpson, among other gentlemen, was consulted on the best form for the arches, by the New-bridge Committee. On this occasion he gave a preference to the semicircular form; and, besides his report to the Committee, some letters also appeared, by himself and others, on the same subject, in the public newspapers, particularly in the Daily Advertiser, and in Lloyd's Evening Post. The same were also collected in the Gentleman's Magazine for that year, page 143 and 144.

It is probable that this reference to him, gave occasion to the turning his thoughts more seriously to this subject, so as to form the design of composing a regular treatise upon it: for his family have often informed me, that he laboured hard upon this work for some time before his death, and was very anxious to have completed it, frequently remarking to them, that this work, when published, would procure him more credit than any of his former publications. But he lived not to put the finishing hand to it. Whatever he wrote upon this subject, probably fell, together with all his other remaining papers, into the hands of Major Henry Watson, of the engineers, in the service of the India Company, being in all a large chest full of papers. This gentleman had been a pupil of Mr. Simpson's, and had lodged in his house. After Mr. Simpson's death, Mr. Watson prevailed upon the widow to let him have the papers, promising either to give her a sum of money for them, or else to print and publish them for her benefit. But neither of these was ever done; this gentleman always declaring, when urged on this point by myself and others, that no use could be made of any of the papers, owing to the very imperfect state in which he said they were left. And yet he persisted in his refusal to give them up again.

From Mr. Simpson's writings, I now return to himself. Through the interest and solicitations of William Jones, Esq. he was, in 1743, appointed professor of mathematics, then vacant by the death of Mr. Derham, in the Royal Academy at Woolwich; his warrant bearing date August 25th. And in 1745 he was admitted a fellow of the Royal Society, having been proposed as a candidate by Martin Folkes, esq. president, William Jones, esq. Mr. George Graham, and Mr. John Machin, secretary; all very eminent mathematicians. The president and council, in consideration of his very moderate circumstances, were pleased to excuse his admission fees, and likewise his giving bond for the settled future payments.

At the academy he exerted his faculties to the utmost, in instructing the pupils who were the immediate objects of his duty, as well as others, whom the superior officers of the ordnance permitted to be boarded and lodged in his house. In his manner of teaching, he had a peculiar and happy address; a certain dignity and perspicuity, tempered with such a degree of mildness, as engaged both the attention, esteem, and friendship of his scholars; of which the good of the service, as well as of the community, was a necessary consequence.

In the latter of stage of his existence, when his life was in danger, exercise and a proper regimen were prescribed him, but to little purpose; for he sank gradually into such a lowness of spirits, as often in a manner deprived him of his mental faculties, and at last rendered him incapable of performing his duty, or even of reading the letters of his friends; and so trifling an accident as the dropping of a tea-cup would flurry him as much as if a house had tumbled down.

The physicians advised his native air for his recovery; and in February, 1761, he set out, with much reluctance (believing he should never return), for Bosworth, along with some relations. The journey fatigued him to such a degree, that on his arrival he betook himself to his chamber, where he grew continually worse and worse, to the day of his death, which happened the 14th of May, in the fifty-first year of his age.

SIMSON (Dr. Robert), professor of mathematics in the university of Glasgow, was the eldest son of Mr. John Simson, of Kirtonhall in Ayrshire, and was born on the 14th of Oct. 1687. Being designed by his father for the church, after having got the usual school education, he was sent to the university of Glasgow about the year 1701, where he was distinguished by his proficiency in classical learning, and in the sciences. At this time, from temporary circumstances, it happened, that no mathematical lectures were given in the college; but young Simson's inquisitive mind, from some fortunate incident, having been directed to geometry, he soon found the study of that science to be congenial to his taste and capacity. This taste however, from an apprehension that it might obstruct his application to subjects more connected with the study of theology, was anxiously discouraged by his father, though it would seem with little effect.

Having procured a copy of Euclid's Elements, with the aid only of a few preliminary explanations from some more advanced students, he entered on the study of that oldest and best introduction to mathematics. In a short time he read and understood the first six, with the 11th and 12th books, and afterwards proceeding still farther in his mathematical pursuits, by his progress in the more difficult branches he laid the foundation of his future eminence. He did not however neglect the other sciences then taught in the college; but in proceeding through the regular course of academical study, he acquired the principles of that variety of knowledge, which he retained through life, and which contributed much to the estimation of his conversation and manners in society. His chief attention, however, was directed to his favourite science; so that his reputation as a mathematician in a few years became so high, and his general character so much respected, that in 1710, when he was only 22 years of age, the members of the college voluntarily made him an offer of the mathematical chair, in which a vacancy in a short time was expected to take a place. From his natural modesty however, he felt much reluctance, at so early an age, to advance abruptly from the state of a student, to that of a professor in the same college; and therefore he solicited permission to spend one year at least in London, where, besides other obvious advantages, he might have opportunities of becoming acquainted with some of the eminent mathematicians of England, who were then the most distinguished in Europe. In this proper request he was readily indulged; and without delay he proceeded to London, where he remained about a year, diligently employed in the improvement of his mathematical knowledge.

This journey turned out very favourable to his views; and he had much satisfaction in the acquaintance of some respectable mathematicians, particularly of Mr. Jones, Mr. Caswell, Dr. Jurin, and Mr. Ditton. With the latter, indeed, who was then mathematical master of Christ's-

Hospital, and well esteemed for his learning &c, he was more particularly connected. It appears from Mr. S.'s own account, in his letter, dated London, Nov. 1710, that he expected to have had an assistant in his studies chosen by Mr. Caswell; but, from some mistake, it was omitted, and Mr. S. himself applied to Mr. Ditton. He went to him not as a scholar (his own words), but to have general information and advice, about his mathematical studies. Mr. Caswell afterwards mentioned to Mr. S. that he meant to have procured Mr. Jones's assistance, if he had not been engaged.

When the vacancy in the professorship of mathematics at Glasgow did occur, in the following year, by the resignation of Dr. Robert Sinclair, or Sinclar (a descendant or other relative probably of Mr. George Sinclar, who died in that office in 1696), the university, while Mr. Simson was still in London, appointed him to fill it; and the minute of election, which is dated March 11, 1711, concluded with this very proper condition, "That they will admit the said Mr. Robert Simson, providing always, that he give satisfactory proof of his skill in mathematics, previous to his admission." He returned to Glasgow before the ensuing session of the college, and having gone through the form of a trial, by resolving a geometrical problem proposed to him, and also by giving "a satisfactory specimen of his skill in mathematics, and dexterity in teaching geometry and algebra;" having produced also respectable certificates of his knowledge of the science, from Mr. Caswell and others, he was duly admitted professor of mathematics, on the 20th of November of that year.

Mr. Simson, immediately after his admission, entered on the duties of his office, and his first occupation necessarily was the arrangement of a proper course of instruction for the students who attended his lectures, in two distinct classes. Accordingly he prepared elementary sketches of some branches on which there were not suitable treatises in general use. Both from a sense of duty and from inclination, he now directed the whole of his attention to the study of mathematics; and though he had a decided preference for geometry, which continued through life, yet he did not devote himself to it to the exclusion of the other branches of mathematical science, in most of which there is sufficient evidence of his being well skilled. From 1711, he continued near 50 years to teach mathematics to two separate classes, at different hours, five days in the week, during a continued session of seven months. His manner of teaching was uncommonly clear and successful; and among his scholars, several rose to distinction as mathematicians; among which may be mentioned the celebrated names of Dr. Matthew Stewart, professor of mathematics at Edinburgh; the two rev. Dr. Williamsons, one of whom succeeded Dr. Simson at Glasgow; the rev. Dr. Trail, formerly professor of mathematics at Aberdeen; Dr. James Moor, Greek professor at Glasgow: and professor Robison, of Edinburgh, with many others of distinguished merit. In the year 1758, Dr. S. being then 71 years of age, found it necessary to employ an assistant in teaching; and in 1761, on his recommendation, the rev. Dr. Williamson was appointed his assistant and successor.

During the remaining ten years of his life, he enjoyed a pretty equal share of good health; and continued to occupy himself in correcting and arranging some of his mathematical papers, and occasionally for amusement, in the solution of problems, and demonstration of theorems, which occurred from his own studies, or from the suggestions of others. His conversation on mathematical and other subjects continued to be clear and accurate; yet he had some strong impressions of the decline of his memory, of which he frequently complained; and this probably protracted, and finally prevented his undertaking the publication of some of his works, which were in so advanced a state, that with little trouble they might have been completed for the press. So that his only publication, after resigning his office, was a new and improved edition of Euclid's Data, which in 1762 was annexed to the 2d edition of the Elements. But from that period, though much solicited to bring forward some of his other works on the ancient geometry, though he knew well how much it was desired, and though he was fully apprised of the universal curiosity excited respecting his discovery of Euclid's Porisms, he resisted every importunity on the subject.

A life like Dr. Simson's, purely academical and perfectly uniform, seldom contains occurrences, the recording of which could be either interesting or useful. But his mathematical labours and inventions form the important part of his character; and with respect to them, there are abundant materials of information in his printed works; and some circumstances also may be gathered from a number of MS. papers which he left; and which, by the direction of his executor, are deposited in the library of the college of Glasgow. It is to be regretted, that, of the extensive correspondence which he carried on through life, with many distinguished mathematicians, a small portion only is preserved. Through Dr. Jurin, then secretary of the Royal Society, he had some intercourse with Dr. Halley, and other distinguished members of that Society. And both about the same time, and afterwards, he had frequent correspondence with Mr. Maclaurin, with Mr. James Stirling, Dr. James Moor, Dr. Matthew Stewart, Dr. Wm. Trail, and Mr. Williamson of Lisbon. In the latter part of his life, his mathematical correspondence was chiefly with that eminent geometer the late earl Stanhope, and with George Lewis Scott, esq.

As to his character, Dr. S. was originally possessed of great intellectual powers, an accurate and distinguishing understanding, an inventive genius, and a retentive memory: and these powers, being excited by an ardent curiosity, produced a singular capacity for investigating the truths of mathematical science. By such talents, with a correct taste, formed by the study of the Greek geometers, he was also peculiarly qualified for communicating his knowledge, both in his lectures and in his writings, with perspicuity and elegance. He was at the same time modest and unassuming; and, though not indifferent to literary fame, he was cautious, and even reserved, in bringing forward his own discoveries, but always ready to do justice to the merits and inventions of others. Though his powers of investigation, in the early part of life, were admirable, yet before any decline of his health appeared, he felt strong impressions of the decay both of his memory and other faculties; occasioned probably by the continued exertion of his mind, in those severe studies, which for a number of years he pursued with unremitting ardour.

Besides his mathematical attainments, from his liberal education he acquired a considerable knowledge of other sciences, which he preserved through life, by occasional reading, and, in some degree, by his constant intercourse with many learned men in his college. He was esteemed a good classical scholar; and, though the simplicity of geometrical demonstration does not admit of much variety of style, yet in his works a good taste in that respect may be distinguished. In his Latin prefaces also, in which there is some history and discussion, the purity of language has been generally approved. It is to be regretted, indeed, that he had not had an opportunity of employing, in early life, his Greek and mathematical learning, in giving an edition of Pappus in the original language.

Dr. Simson never was married; and the uniform regularity of a long life, spent within the walls of his college, naturally produced fixed and peculiar habits, which however, with the sincerity of his manners, were unoffending, and became even interesting to those with whom he lived. The strictness of these habits, which indeed pervaded all his occupations, probably had an influence also on the direction and success of some of his scientific pursuits. His hours of study, of amusement, and of exercise, were all regulated with uniform precision. The walks even in the squares or garden of the college were all measured by his steps, and he took his exercises by the hundreds of paces, according to his time or inclination.

It has been mentioned, that an ardent curiosity was an eminent feature in his character. It contributed essentially to his success in the mathematical investigations, and it displayed itself in the small and even trifling occurrences of common life. Almost every object and event excited it, and suggested some problem which he was impatient to resolve. This disposition, when opposed, as it often necessarily was, to his natural modesty, and to the formal civility of his manners, occasionally produced an embarrassment, which was amusing to his friends, and sometimes a little distressing to himself.

In his disposition, Dr. S. was both chearful and sociable; and his conversation, when he was at ease among his friends, was animated and various, enriched with much anecdote, especially of the literary kind, but always unaffected. It was enlivened also by a certain degree of natural humour; and even the slight fits of absence, to which in company he was occasionally liable, contributed to the entertainment of his friends, without diminishing their affection and respect, which his excellent qualities were calculated to inspire. One evening (Friday) in the week he devoted to a club, chiefly of his own selection, which met in a tavern near the college. The first part of the evening was employed in playing the game of whist, of which he was particularly fond; but, though he took no small trouble in estimating chances, it was remarked that he was often unsuccessful. The rest of the evening was spent in chearful conversation; and, as he had some taste for music, he did not scruple to amuse his party with a song; and it is said that he was rather fond of singing some Greek odes, to which modern music had been adapted. On Saturdays he usually dined in the village of Anderston, then about a mile distant from Glasgow, with some of the members of his regular club, and with a variety of other respectable visitors, who wished to cultivate the acquaintance, and enjoy the society of so eminent a person. In the progress of time, from his age and character, it became the wish of his company that every thing in these meetings should be directed by him; and though his authority, growing with his years, was somewhat absolute, yet the good humour with which it was administered, rendered it pleasing to every body. He had his own chair and place at table; he gave instructions about the entertainment, regulated the time of breaking up, and adjusted the expense. These parties, in the years of his severe study, were a desirable and useful relaxation to his mind, and they continued to amuse him till within a few months of his death.

Strict integrity and private worth, with corresponding purity of morals, gave the highest value to a character, which, from other qualities and attainments, was much respected and esteemed. On all occasions, even in the gayest hours of social intercourse, the Doctor maintained a constant attention to propriety. He had serious and just impressions of religion; but he was uniformly reserved in expressing particular opinions about it; and, from his sentiments of decorum, he never introduced religion as a subject of conversation in mixed society, and all attempts to do so in his clubs were checked with gravity and decision.

In his person, Dr. S. was tall and erect; and his countenance, which was handsome, conveyed a pleasing expression of the superior character of his mind. His manner had always somewhat of the fashion which prevailed in the early part of his life, but was uncommonly graceful. He was seriously indisposed only for a few weeks before his death, and through a very long life had enjoyed a uniform state of good health. He died on the first of October 1768, when his 81st year was almost completed; having bequeathed his small paternal estate in Ayrshire to the eldest son of his next brother, probably of his brother Thomas, who was professor of medicine in the university of St. Andrews, and who is known by some works of reputation, particularly a Dissertation on the Nervous System, occasioned by the Dissection of a Brain completely Ossified.

The preceding account of Dr. S. has been abridged and extracted from some other accounts of him; as, the Account of his Life and Writings by the rev. Dr. William Trail, lately published (1812); and from the account of Dr. S. and his works, by the late professor Robison, in the Encyclopædia Britannica; and partly from an ingenious MS. account of his life and writings, written and communicated by Mr. James Miller, the present mathematical professor of Glasgow; but more closely from Dr. Trail's work, where a very learned and critical account of Dr. Simson's writings is to be seen.

The writings and publications of Dr. S. were almost exclusively of the pure geometrical kind, after the genuine manner of the ancients. He has only two pieces printed in the volumes of the Philosophical Transactions: viz,

1. Two General Propositions of Pappus, in which many of Euclid's Porisms are included, vol. 32, ann. 1723.—These two propositions were afterwards incorporated into the author's large posthumous Works, published in 1776, by Philip, earl Stanhope.

2. On the Extraction of the Approximate Roots of Numbers by Infinite Series: vol. 48, ann. 1753.

The separate publications in his life-time, were:

3. Conic Sections, in 1735, 4to.

4. The Loci Plani of Apollonius, restored; in 1749, 4to.

4. Euclid's Elements; in 1756, 4to, and since that time, many editions in 8vo, with the addition of Euclid's Data.

5. After his death, earl Stanhope was at the expense of a publication, in 1776, of several of Dr. S.'s posthumous pieces; which were (1) Apollonius's Determinate Section: (2) A Treatise on Porisms: (3) A Tract on Logarithms: (4) On the Limits of Quantities and Ratios: (5) Some Select Geometrical Problems.

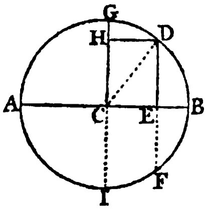
Besides the tracts published in these posthumous works, Dr. S.'s manuscripts contained a great variety of geometrical propositions, and other interesting observations on different parts of the mathematics; though not in a state fit for publication. Among other things, was an edition of the works of Pappus, in a state of considerable advancement, and which, had he lived, he perhaps might have published. The copy of Pappus, with all Dr. S.'s notes and explanations, it seems were, soon after his death, sent by his executor to the University of Oxford, with a view to publication; but which however it does not appear has yet been accomplished. It is true indeed, Dr. S.'s copy contains a large collection of materials, from which to make a proper selection would probably require considerable labour, as well as judgment.

SINCLAR (George), was professor of philosophy in the university of Glasgow, and author of several works on mathematical and physical subjects. He was dismissed from his professorship soon after the restoration, on account of his political principles; but was recalled to it on the change of government at the revolution in 1688; he died in 1696. Mr. Sinclar's publications were, 1. Tyrocinia Mathematica, 12mo. Glasc. 1661; 2. Ars Nova et Magna, &c, 4to. Roterod. 1669; 3. Hydrostatics, 4to, Edinb. 1672; 4. Hydrostatical Experiments, with a Discourse on Coal, 8vo. Edinb. 1680; 5. Principles of Astronomy and Navigation, 12mo. Edinb. 1688. Besides which, a very extravagant production, called, "Satan's Invisible World discovered" has been ascribed to him; it bears the initials, G. S. of his name.

Mr. Sinclar's writings are not destitute of ingenuity and research; though they may contain some erroneous and excentric views. His work on Hydrostatics, and his Ars Nova et Magna Gravitatis et Levitatis, and perhaps also his political principles, provoked the indignation of some persons; on which occasion Mr. James Gregory, author of the Optica Promota, &c, and then professor of mathematics at Saint Andrews, animadverted on him rather severely in a treatise entitled, "The Great and New Art of weighing Vanity, &c. under the name of Patrick Mathers, Archbedal of St. Andrews."

Considerable attention seems to have been paid by Mr. Sinclar to such branches of hydrostatics as were of a practical nature; and it has been said he was the first person who suggested the proper method of draining the water from the numerous coal mines in the south-west of Scotland. During the period he was deprived of his office, he resided about the southern and border counties, collecting and affording useful information on the subjects of mining, engineering, &c: particularly he was employed by the magistrates of Edinburgh on the then new plan for supplying the city with water, &c.

SINE, of an arc, in Trigonometry, a right line drawn from one extremity of the arc, perpendicular to the radius drawn to the other extremity of it: Or, it is half the chord of double the arc. Thus the line DE is the sine of the arc BD; being drawn from one end D of that arc, perpendicular to CB the radius drawn to the other extremity B. For the same reason also DE is the sine of the arc AD, which is the supplement of BD to a semicircle or 180 degrees; so that every sine is common to two arcs, which are supplements to each other, or whose sum is equal to 180 degrees.



Hence the sines increase always from nothing at B, till they become the radius CG, which is the greatest, being the sine of the quadrant BG. From hence they decrease throughout the second quadrant from G to A, till they quite vanish at the point A; thereby showing that the sine of the semicircle BGA, or 180 degrees, is nothing. After this they are negative in the next semicircle, or 3d and 4th quadrants AFB, being drawn on the opposite side, or downwards from the diameter AB.

Whole Sine, or *Sinus Totus*, is the sine of the quadrant BG, or of 90 degrees; that is, the whole sine is the same with the radius CG.

Sine-*Complement*, or *Cosine*, is the sine of an arc DG, which is the complement of another arc BD, to a quadrant. That is, the line DH is the cosine of the arc BD; because it is the sine of DG which is the complement of BD. And for the same reason DE is the cosine of DG. Hence the sine and cosine and radius, of any arc, form a right-angled triangle CDE or CDH, of which the radius CD is the hypothenuse; and therefore the square of the radius is equal to the sum of the squares of the sine and cosine of any arc, that is, $CD^2 = CE^2 + ED^2$ or $= CH^2 + DH^2$. It is evident that the cosine of 0 or nothing, is the whole radius CB. From B, where this cosine is greatest, the cosine decreases as the arc increases from B along the quadrant BDG, till it become 0 for the complete quadrant BG. After this, the cosines, decreasing, become negative from hence to the complete semicircle at A. Then the cosines increase again all the way from A through I to B; at I the negation is destroyed, and the cosine is equal to 0 or nothing; from I to B it is positive, and at B it is again become equal to the radius. So that, in general, the cosines in the 1st and 4th quadrants are positive, but in the 2d and 3d negative.

Versed-Sine, is the part of the diameter between the sine and the arc. So BE is the versed sine of the arc BD, and AE the versed sine of AD, also GH the versed sine of DG, &c. All versed sines are affirmative. The sum of the versed sine and cosine, of any arc or angle, is equal to the radius, that is, $BE + EC = AC$.

The sines &c, of every degree and minute in a quadrant, are calculated to the radius 1, and ranged in tables for use. But because operations with these natural sines require much labour in multiplying and dividing by them, the logarithms of them are taken, and ranged in tables also; and these logarithmic sines are commonly used in practice, instead of the natural ones, as they require only additions and subtractions, instead of multiplications and divisions. For the method of constructing the scales of sines &c, see the article Scale.

The sines were introduced into trigonometry by the Arabians. And for the etymology of the word *Sine*, see Introduction to my Logarithms, pa. 17 &c. Also the various ways of calculating tables of the sines, may be seen in the same place, pa. 13 &c.

The relation which subsists between the sines and cosines of any arcs of a circle, and those of their sums, differences, and multiples, constitute what is sometimes termed the arithmetic of sines. This branch of calculation has its origin in the application of algebra to geometry, and is of great importance in the more difficult parts of the mathematics, as well as in their application to physics. The following theorems are those of the greatest utility, and of the most extensive application; the investigation of which may be seen in my Course of Mathematics, and other works on trigonometry.

Previous however to exhibiting those formulæ, we may make the following connected observations, which are immediately deduced from what has been before said in the definition of the sine of an arc: namely, in the

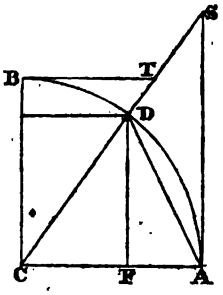
	1st quad.	2d quad.	3d quad.	4th quad.	
Sin is	+	+	−	−	
Cos	+	−	−	+	
Of	0°	90°	180°	270°	360°
Sin is	0	rad.	0	− rad.	0
Cos is	rad.	0	− rad.	0	r.

Again, if p be made to represent the semicircumference of a circle, the radius of which is r, and n be 0, or any whole number, then we shall have the following general results.

$\text{Sin } np = 0$	$\text{Cos } 2np = r$
$\text{Sin } \frac{4n+1}{2} p = r$	$\text{Cos } \frac{4n+1}{2} p = 0$
$\text{Sin } \frac{4n+2}{2} p = 0$	$\text{Cos } \frac{4n+2}{2} p = -r$
$\text{Sin } \frac{4n+3}{2} p = -r$	$\text{Cos } \frac{4n+3}{2} p = 0$
$\text{Sin } \frac{4n+4}{2} p = 0$	$\text{Cos } \frac{4n+4}{2} p = r$
$\text{Sin } \frac{4n-1}{2} p = -r$	$\text{Cos } \frac{4n-1}{2} p = 0.$

Also, $\sin a = \sin(p - a) = \sin(2p - a) = \sin(3p - a)$ &c.

Now, from the annexed figure, and what has been already observed of the similar triangles, we easily deduce the following formulæ, expressing the sine and cosine of any arc a, in terms of the tangent, secant, radius, &c.



$$\text{Sin } a = \sqrt{(r^2 - \cos^2 a)} = \frac{r \cdot \cos a}{\cot a} = \frac{\cos a \cdot \tan a}{r} = \frac{r \cdot \tan a}{\sqrt{(r^2 + \tan^2 a)}}$$

$$= \frac{r^2}{\sqrt{(r^2 + \cot^2 a)}} = \frac{r^2}{\text{cosec } a} = \frac{r \cdot \tan a}{\sec a} = \frac{\cos a \cdot \sec a}{\text{cosec } a}$$

$$= \frac{\tan a \cdot \cot a}{\text{cosec } a} = \frac{r\sqrt{(\sec^2 a - r^2)}}{\sec a}.$$

$$\text{Cos } a = \sqrt{(r^2 - \sin^2 a)} = \frac{r \sin a}{\tan a} = \frac{\sin a \cdot \cot a}{r} = \frac{r \cdot \cot a}{\sqrt{(r^2 + \cot^2 a)}} = \frac{r^2}{\sqrt{(r^2 + \tan^2 a)}} = \frac{r^2}{\sec a} = \frac{r \cdot \cot a}{\text{cosec } a}$$

$$= \frac{\sin a \cdot \text{cosec } a}{\sec a} = \frac{\tan a \cdot \cot a}{\sec a} = \frac{r\sqrt{(\text{cosec}^2 a - r^2)}}{\text{cosec } a}.$$

Vers $a = r - \cos a$; covers $a = r - \sin a$; supvers $a = r + \cos a$.

Also, for the sine of the sum and difference of any two arcs to radius 1, we have the following theorems:

1. $\text{Sin}(a + b) = \sin a \cdot \cos b + \cos a \cdot \sin b$
2. $\text{Sin}(a - b) = \sin a \cdot \cos b - \cos a \cdot \sin b$
3. $\text{Cos}(a + b) = \cos a \cdot \cos b - \sin a \cdot \sin b$
4. $\text{Cos}(a - b) = \cos a \cdot \cos b + \sin a \cdot \sin b.$

Hence again,

5. $\text{Sin}(a + b) + \sin(a - b) = 2 \sin a \cdot \cos b$
6. $\text{Sin}(a + b) - \sin(a - b) = 2 \cos a \cdot \sin b$
7. $\text{Cos}(a - b) + \cos(a + b) = 2 \cos a \cdot \cos b$
8. $\text{Cos}(a - b) - \cos(a + b) = 2 \sin a \cdot \sin b.$

And if in these last four formulæ, we substitute na for a, and a for b, we obtain,

9. $2 \text{ Cos } a \cdot \sin na = \sin (n + 1)a + \sin (n - 1) a$
10. $2 \text{ Sin } a \cdot \cos na = \sin (n + 1)a - \sin (n - 1) a$
11. $2 \text{ Cos } a \cdot \cos na = \cos (n + 1)a + \cos (n -) a$
12. $2 \text{ Sin } a \cdot \sin na = -\cos(n + 1)a + \cos (n - 1) a$

And from these we may deduce the powers of the sines and cosines of arcs, in terms of the sum and difference of certain multiples of those arcs, thus:

$\text{Sin } a = \sin a$

$2 \text{ Sin}^2 a = -\cos 2a + 1$

$4 \text{ Sin}^3 a = -\sin 3a + 3 \sin a$

$8 \text{ Sin}^4 a = \cos 4a - 4 \cos 2a + 3$

$16 \text{ Sin}^5 a = \sin 5a - 5 \sin 3a + 10 \sin a$

$32 \text{ Sin}^6 a = -\cos 6a + 6 \cos 4a - 15 \cos 2a + 10$

$64 \text{ Sin}^7 a = -\sin 7a + 7 \sin 5a - 21 \sin 3a + 35 \sin a.$

And generally

$$2^{n-1} \sin^n a = \pm \sin na \pm n \sin (n - 2) a \pm \frac{n \cdot (n - 1)}{2} \sin (n - 4) a \text{ \&c; Or}$$

$$2^{n-1} \sin^n a = \pm \cos na \mp n \cos (n - 2)a \pm \frac{n \cdot n (n - 1)}{2} \cos (n - 4) a \text{ \&c.}$$

In the first of which series, the upper sign must be used when n is an odd number of the form $4m + 1$, and the lower sign, when n is of the form $4m - 1$.

In the second series, the upper sign must be used when n is of the form $4m$, and the lower sign when n is of the form $2(2m + 1)$.

Similar formulæ may also be found for the successive powers of the cosines of any simple arc, which are as follows:

$\text{Cos } a = \cos a$

$2 \text{ Cos}^2 a = \cos 2a + 1$

$4 \text{ Cos}^3 a = \cos 3a + 3 \cos a$

$8 \text{ Cos}^4 a = \cos 4a + 4 \cos 2a + 3$

$16 \text{ Cos}^5 a = \cos 5a + 5 \cos 3a + 10 \cos a$

$32 \text{ Cos}^6 a = \cos 6a + 6 \cos 4a + 15 \cos 2a + 10$

$64 \text{ Cos}^7 a = \cos 7a + 7 \cos 5a + 21 \cos 3a + 35 \cos a.$

And generally

$$2^{n-1} \text{ Cos}^n a = \cos na + n \cos (n - 2) a + \frac{n \cdot n - 1}{2} \cos (n - 4)a \text{ \&c.}$$

Again, the sines and cosines of the multiple arcs, may be expressed in terms of the sines and cosines of the inferior arcs, as below.

$\text{Sin } a = \sin a$

$\text{Sin } 2a = 2 \cos a \cdot \sin a$

$\text{Sin } 3a = 2 \cos a \cdot \sin 2a - \sin a$

$\text{Sin } 4a = 2 \cos a \cdot \sin 3a - \sin 2a$

$\text{Sin } 5a = 2 \cos a \cdot \sin 4a - \sin 3a$ &c

$\text{Sin } na = 2 \cos a \cdot \sin (n - 1) a - \sin (n - 2) a$

$\text{Cos } a = \cos a$

$\text{Cos } 2a = 2 \cos a \cdot \cos a - 1$

$\text{Cos } 3a = 2 \cos a \cdot \cos 2a - \cos a$

$\text{Cos } 4a = 2 \cos a \cdot \cos 3a - \cos 2a$

$\text{Cos } 5a = 2 \cos a \cdot \cos 4a - \cos 3a$ &c

$\text{Cos } na = 2 \cos a \cdot \cos (n - 1) a - \cos (n - 2) a.$

And if now, in order to abbreviate, we make cos $a = c$, and sine $a = s$, and observing at the same time that $c^2 = 1 - s^2$, we derive

$$\text{Sin } a = s$$
$$\text{Sin } 2a = 2cs$$
$$\text{Sin } 3a = 4c^2s - s = -4s^3 + 3s$$
$$\text{Sin } 4a = 8c^3s - 4sc = c(-8s^3 + 4s)$$
$$\text{Sin } 5a = 16c^4s - 12c^2s + s = 16s^5 - 20s^3 + 5s$$
$$\text{Sin } na = nsc^{n-1} - \frac{n.n-1.n-2}{1.2.3}s^3c^{n-3} + \frac{n.n-1.n-2.n-3.n-4}{1.2.3.4.5}s^5c^{n-5} \text{ \&c}$$

$$\text{Cos } a = c$$
$$\text{Cos } 2a = 2c^2 - 1$$
$$\text{Cos } 3a = 4c^3 - 3c$$
$$\text{Cos } 4a = 8c^4 - 8c^2 + 1$$
$$\text{Cos } 5a = 16c^5 - 20c^3 + 5c$$
$$\text{Cos } na = 2^{n-1}c^n - \frac{n.2^{n-1}}{1.2}c^{n-2} + \frac{n.n-3.2^{n-1}}{1.2.2^4}c^{n-4} - \frac{n.n-4.n-5.2^{n-1}}{1.2.3.2^6}c^{n-6} \text{ \&c.}$$

Or if, instead of the above substitution, we make 2 cos $a = y + \frac{1}{y}$, we readily deduce the following elegant formulæ for the cosines of the multiple arcs.

$$2 \text{ Cos} \quad y + \frac{1}{y}$$
$$2 \text{ Cos } 2a = y^2 + \frac{1}{y^2}$$
$$2 \text{ Cos } 3a = y^3 + \frac{1}{y^3}$$
$$2 \text{ Cos } 4a = y^4 + \frac{1}{y^4}$$
$$2 \text{ Cos } 5a = y^5 + \frac{1}{y^5}$$
$$2 \text{ Cos } na = y^n + \frac{1}{y^n}.$$

We might pursue this subject to a much greater length, but the above are the principal formulæ which occur in the doctrine of sines &c. We shall conclude this article with the following formulæ, expressing the log. of the sine of an arc, the arc in terms of the sine, the sine in terms of the arc, &c; where it is to be observed, that s is the sine, c the cosine, a the arc, and r the radius:

$$s = a - \frac{a^3}{2.3r^2} + \frac{a^5}{2.3.4.5r^4} - \frac{a^7}{2.3.4.5.6.7r^6} \text{ \&c.}$$
$$a = s + \frac{s^3}{2.3r^2} + \frac{1.3s^5}{2.4.5r^4} + \frac{1.3.5s^7}{2.4.6.7r^6} \text{ \&c.}$$
$$\text{Log. } s = \text{log. } a - \text{M}\left(\frac{a^2}{6} + \frac{a^4}{180} + \frac{a^6}{2835} + \frac{a^8}{37800} \text{ \&c}\right)$$
$$\text{or Log. } s = -\tfrac{1}{2}\text{M}\left(c^2 + \tfrac{1}{2}c^4 + \tfrac{1}{3}c^6 + \tfrac{1}{4}c^8 \text{ \&c}\right)$$
$$\text{or Log. } s = -2\text{M}\left(z + \tfrac{1}{3}z^3 + \tfrac{1}{5}z^5 + \tfrac{1}{7}z^7 \text{ \&c.}\right)$$

where $z = \frac{1-s}{1+s}$, radius 1, and $\text{M} = \cdot 43429448$ &c.

If $h = 2{\cdot}718281828$ &c, the number whose hyp. log. is 1; then

$$\text{Sin } a = s = \frac{h^{a\sqrt{-1}} - h^{-a\sqrt{-1}}}{2\sqrt{-1}}.$$
$$\text{Cos } a = c = \frac{h^{a\sqrt{-1}} + h^{-a\sqrt{-1}}}{2}.$$

See many other curious expressions of this kind in Bougainville's Calcul Integral, and in Bertrand's mathematics.

From theorems 1, 2, &c, the sines of a great variety of angles, or number of degrees, may be computed. Ex. gr. as below.

Angles.	*Sines.*
90°	r
75	$\frac{1}{2}r\sqrt{2+\sqrt{3}} = r \times \frac{\sqrt{6}+\sqrt{2}}{4}$
72	$\frac{1}{2}r\sqrt{\frac{5+\sqrt{5}}{2}}$
67½	$\frac{1}{2}r\sqrt{2+\sqrt{2}}$
60	$\frac{1}{2}r\sqrt{3}$
54	$\frac{1}{2}r\sqrt{\frac{3+\sqrt{5}}{2}} = r \times \frac{\sqrt{5}+1}{4}$
45	$\frac{1}{2}r\sqrt{2}$
36	$\frac{1}{2}r\sqrt{\frac{5-\sqrt{5}}{2}}$
30	$\frac{1}{2}r$
22½	$\frac{1}{2}r\sqrt{2-\sqrt{2}}$
18	$\frac{1}{2}r\sqrt{\frac{3-\sqrt{5}}{2}} = r \times \frac{\sqrt{5}-1}{4}$
15	$\frac{1}{2}r\sqrt{2-\sqrt{3}} = r \times \frac{\sqrt{6}-\sqrt{2}}{4}$

Of the Tables of Sines, &c.

In estimating the quantity of the sines &c, we assume radius for unity; and then compute the quantity of the sines, tangents, and secants, in fractions of it. From Ptolemy's Almagest we learn, that the ancients divided the radius into 60 parts, which they called degrees, and thence determined the chords in minutes, seconds, and thirds; that is, in sexagesimal fractions of the radius, which they likewise used in the resolution of triangles. As to the sines, tangents and secants, they are modern inventions; the sines being introduced by the Moors or Saracens, and the tangents and secants afterwards by the Europeans. See Introd. to my Logs. pa. 1 to 19.

Regiomontanus, at first, with the ancients, divided the radius into 60 degrees; and determined the sines of the several degrees in decimal fractions of it. But he afterwards found it would be more convenient to assume 1 for radius, or 1 with any number of ciphers, and take the sines in decimal parts of it; and thus he introduced the present method in trigonometry. In this way, different authors have divided the radius into more or fewer decimal parts; but in the common tables of sines and tangents, the radius is conceived to be divided into 10000000 parts; by which all the sines are estimated.

An idea of some of the modes of constructing the tables of sines, may be conceived from what follows: First, by common geometry the sides of some of the regular polygons inscribed in the circle are computed, from the given radius, which will be the chords of certain portions of the circumference, denoted by the number of the sides; viz, the side of the triangle the chord of the 3d part, or 120 degrees; the side of the pentagon the chord of the 5th part, or 72 degrees; the side of the hexagon the chord of the 6th part, or 60 degrees; the side of the octagon the chord of the 8th part, or 45 degrees; and so on. By this means there are obtained the chords of several of such arcs; and the halves of these chords will be the sines of the halves of the same arcs. Then the theorem $c = \sqrt{(1 - s^2)}$ will give the cosines of the same half arcs. Next, by bisecting these arcs continually, there will be found the sines and cosines of a continued series as far as we please by these two theorems,

$$\text{Sin } \tfrac{1}{2}a = \sqrt{\frac{1-c}{2}}; \text{ and cos. } \tfrac{1}{2}a = \sqrt{\frac{1+c}{2}}.$$

Then, by the formulæ for the sums and differences of arcs, from the foregoing series will be derived the sines and cosines of various other arcs, till we arrive at length at the arc of 1′, or 1″, &c, whose sine and cosine thus become known. Or, rather, the sine of 1 minute will be much more easily found from the series $s = a - \frac{1}{6}a^3 + \frac{1}{120}a^5$ &c, because the arc may be considered as equal to its sine in small arcs; whence $s = a$ only in such small arcs. But the length of the arc of 180° or 10800′ is known to be 3·14159265 &c; therefore, by proportion, as 10800′ : 1′ : : 3·14159265 : 0·0002908882 $= a$ the arc or s the sine of 1′, which number is true to the last place of decimals. Then, for the cosine of 1′, it is $c = \sqrt{(1 - s^2)} =$ 0·9999999577 the cosine of the same 1′.

Hence we shall readily obtain the sines and cosines of all the multiples of 1′, as of 2′, 3′, 4′, 5′, &c, by the application of these two theorems,

$$\sin (n+1)a = 2c \times \sin na - \sin (n-1)a;$$
$$\cos (n+1)a = 2c \times \cos na - \cos (n-1)a;$$

for supposing $a =$ the arc of 1, then $c = 0{\cdot}9999999577$, and taking n successively equal to 1, 2, 3, 4, &c, the theorems for the sines and cosines give severally the sines and cosines of 1′, 2′, 3′, 4′, &c; viz, the sines thus:

$$\begin{array}{l}
\sin 1' = s \text{ - - - - - - - } = {\cdot}0002908882 \\
\sin 2' = 2c \times \sin 1' - \sin 0' = {\cdot}0005817764 \\
\sin 3' = 2c \times \sin 2' - \sin 1' = {\cdot}0008726645 \\
\sin 4' = 2c \times \sin 3' - \sin 2' = {\cdot}0011635526 \\
\sin 5' = 2c \times \sin 4' - \sin 3' = {\cdot}0014544406 \\
\quad\&\text{c.}
\end{array}$$

And the cosines thus,

$$\begin{array}{l}
\cos 1' = c \text{ - - - - - - - } = {\cdot}9999999577 \\
\cos 2' = 2c \times \cos 1' - \cos 0' = {\cdot}9999998308 \\
\cos 3' = 2c \times \cos 2' - \cos 1' = {\cdot}9999996192 \\
\cos 4' = 2c \times \cos 3' - \cos 2' = {\cdot}9999993231 \\
\cos 5' = 2c \times \cos 4' - \cos 3' = {\cdot}9999989423 \\
\quad\&\text{c.}
\end{array}$$

In this manner then all the sines and cosines are made, by only one constant multiplication and a subtraction, up to 30 degrees, forming thus the sines of the first and last 30 degrees of the quadrant, or from 0 to 30° and from 60° to 90°; or, which will be much the same thing, the sines only may be thus computed all the way up to 60°.

Then the sines of the remaining 30°, from 60 to 90, will be found by one addition only for each of them, by means of this theorem, viz, $\sin. (60 + a) = \sin. (60 - a) + \sin. a$; that is, to the sine of any arc below 60°, add the sine of its defect below 60, and the sum will be the sine of another arc which is just as much above 60.

The sines of all arcs being thus found, they give also very easily the versed sines, the tangents, and the secants. The versed sines are only the arithmetical complements to 1, that is, each cosine taken from the radius 1.

The tangents are found by these three theorems:

1. As cosine to sine, so is radius to tangent.
2. Radius is a mean proportional between the tangent and cotangent.
3. Half the difference between the tangent and cotangent, is equal to the tangent of the difference between the arc and its complement. Or, the sum arising from the addition of double the tangent of an arc with the tangent of half its complement, is equal to the tangent of the sum of that arc and the said half complement.

By the 1st and 2d of these theorems, the tangents are to be found for one half of the quadrant: then the other half of them will be found by one single addition, or subtraction, for each, by the 3d theorem.

This done, the secants will be all found by addition or subtraction only, by these two theorems: 1st. The secant of an arc, is equal to the sum of its tangent and the tangent of half its complement, 2nd. The secant of an arc, is equal to the difference between the tangent of that arc and the tangent of the arc added to half its compliment.

Sines, *&c, by a new Division of the Quadrant.* In the 2d vol. of my Tracts, pa. 122, &c, is described a new mode of dividing the quadrantal arc, for which to construct new tables of sines, tangents, and secants, which would be very useful, and is different from all other methods of dividing it, and of constructing the tables. In this method the arcs of the quadrant are divided into, and expressed by the equal parts of the radius, the same as the sines and tangents themselves; being divisions in the common decimal scale of numbers. In this project I have made many thousands of calculations for the sines; and a specimen of the tables may be seen inserted in the volume above mentioned.

The French have also made and printed very extensive tables of sines &c, on a plan of divisions differing both from the old sexagesimal way, and from mine, above mentioned: those being to decimal divisions of arcs, the quadrant being divided into 10000 equal parts, each part being nearly equal 30″ in the sexagesimal division.

Artificial Sines, are the logarithmic sines, or the logarithms of the sines.

Arithmetic of Sines. See chap. iii, p. 53, vol. 3, of my Course of Mathematics; also the foregoing article in this dictionary.

Curve or *Figure of the* Sines. See Figure *of the Sines, &c.* To what is there said of the figure of the sines, may be here added as follows, from a property just given above, viz, if x denote the absciss of this curve, or the corresponding circular arc, and y its ordinate, or the sine of that arc; then the equation of the curve will be this,

$$y = \sin x = \frac{h^{x\sqrt{-1}} - h^{-x\sqrt{-1}}}{2\sqrt{-1}};$$

where $h = 2{\cdot}718281828$ &c, the number whose hyp. log. is 1.

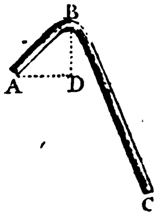
Line of Sines, is a line on the sector, or Gunter's scale, &c, divided according to the sines, or expressing the sines. See those articles.

Sine *of Incidence*, or *of Reflection*, or *of Refraction*, is used for the sine of the *angle* of incidence, &c.

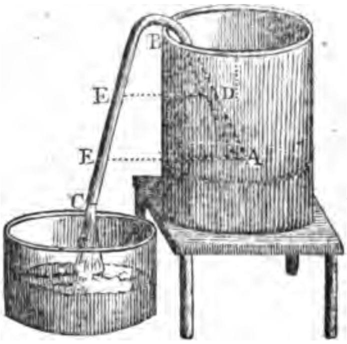
SINICAL *Quadrant*, is a quadrant, made of wood or metal, with lines drawn from each side intersecting one another, with an index, divided by sines, also with 90 degrees on the limb, and two sights at the edge. Its use is to take the altitude of the sun. Instead of the sines, it is sometimes divided all into equal parts; and then it is used by seamen to resolve, by inspection, any problem of plane sailing.

SIPHON, or Syphon, in Hydraulics, a crooked pipe or tube used in the raising of fluids, emptying of vessels, and in various hydrostatical experiments. It is otherwise called a crane. Wolfius describes two vessels under the name of Siphons; the one cylindrical in the middle and conical at the two extremes; the other globular in the middle, with two narrow tubes fitted to it axis-wise; both serving to take up a quantity of liquid, and to retain it when up.

But the most usual syphon is that which is here represented; where ABC is any crooked tube, having two legs of unequal lengths; but such however that, in any position, the perpendicular altitude BD, of B above A, when AB is filled with any fluid, the weight of that fluid may not be more than about 15lb. upon every square inch of the base, or equal to the pressure of the atmosphere, because the pressure of the atmosphere will raise or suspend the fluid so high, when the tube is exhausted of air. This height is about 30 inches when the fluid is quicksilver, and about 34 feet when it is water; and so on for other fluids, according to the rarity of them.



To use the siphon, in drawing off any fluid; immerse the shorter end A into the fluid, then suck or draw the air out by the other or lower end C, and the fluid will presently follow, and run out by the siphon, from the vessel at A to the vessel at C; till such time as the surface of the fluid sink as low as the orifice at A, when the decanting will cease, and the siphon will empty itself of the fluid, the whole of that which is in it running out at C. The principle upon which the siphon acts, is this: when the tube is exhausted of air, the pressure of the atmosphere upon the surface of the fluid at D, forces it into the tube by the orifice at A, as in the barometer tube, and down the leg BC, if B is not above the surface at D more than 34 feet for water, or 30 inches for quicksilver, &c. Here, if the external leg of the siphon terminate at E, on a horizontal level with the immersed end at A, or rather on a level with the water at D, the perpendicular pressures of the fluid in each leg, and of the external air, against each orifice, being alike in both, the fluid will be at rest in the siphon, completely filling it, but without running or preponderating either way. But if the external end be the lower, terminating at C, then the fluid in this end being the heavier, or having more pressure, will preponderate and run out by the orifice at C; this would leave a vacuum at B but for the continual pressure of the atmosphere at D, which forces the fluid up by A to B, and so producing a continued motion of it through the tube, and a discharge or stream at C.



Instead of sucking out the air at C, another method is, first to fill the tube completely with the fluid, in an inverted position with the angle B downward; and, stopping the two orifices with the fingers, revert the tube again, and immerge the end A in the fluid; then take off the fingers, and immediately the stream commences from the end C.

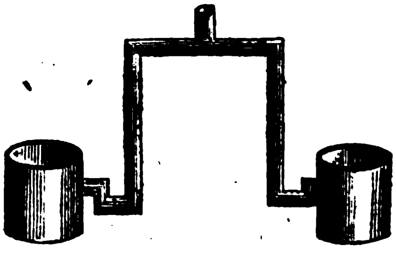
Either of the two foregoing methods can be conveniently practised when the siphon is small, and easily managed by the hand; as in decanting off liquors from casks, &c. But when the siphon is very large, and many feet in height, as in exhausting water from a valley or a pit, the following method is then recommended: Stop the orifice C; and, by means of an opening made in the top at B, fill the tube completely with water; then stop the opening at B with a plug, and open that at C; upon which the water will presently flow out there, and so continue till that at A is exhausted. And this method of conveying water over a hill, from one valley to another, is described by Hero, the chief author of any consequence on this subject among the ancients. But in this experiment it must be noted, that the effect will not be produced when the hill at B is more than 33 or 34 feet above the surface of the water at A.

In an experiment of this kind, it is even said that the water in the legs, unless it be purged of its air, will not rest at a height of quite 30 feet above the water in the vessels; because air will extricate itself out of the water, and getting above the water in the legs, press it downward, so that its height will be less to balance the pressure of the atmosphere. But with very fine, or capillary tubes, the experiment will succeed to a height somewhat greater; because the attraction of the matter of the very fine tube will raise the fluid, and support it at some certain height, independent of the pressure of the atmosphere. For which reason also it is, that the experiment succeeds for small heights in the exhausted receiver; as has been tried both with water and mercury, by Desaguliers and many other philosophers. Exper. Philos. vol. 2, pa. 168.

The figure of the vessel may be varied at pleasure, provided the orifice C be but below the level of the surface of the water to be drawn off, but still the farther it is below it, the quicker will the fluid run off. And if, in the course of the efflux, the orifice A be drawn out of the fluid; all the liquor in the siphon will issue out at the lower orifice C; the fluid in the leg BC dragging, as it were, that in the shorter leg AB after it.

But if a filled siphon be so disposed, as that both orifices, A and C, be in the same horizontal line; the fluid will remain pendant in each leg, how unequal soever the length of the legs may be. So that fluids in siphons seem, as it were, to form one continued body; the heavier part descending like a chain, and drawing the lighter after it.

The *Wirtemberg* SIPHON, which is represented in the annexed figure, is a very extraordinary machine, performing several things which the common siphon will not reach. This siphon was projected by Jordan Pelletier, and executed at the expence of prince Frederic Charles, admi-



nistrator of Wirtemberg, by his mathematician Shahackard, who made each branch 20 feet long, and set them 18 feet apart; and the description of it was published by Reiselius, the duke's physician. This gave occasion to Papin to invent another; which performed the same things, and is described in the Philos. Trans. vol. 14. Reselius, in another paper in the same volume, ingenuously owns that this is the same with the Wirtemberg siphon.

In this engine, though the legs be on the same level, yet the water rises up the one, and descends through the other. The water rises even through the aperture if the less leg be only half immerged in water. The siphon has its effect after continuing dry a long time. Either of the apertures being opened, the other remaining shut for a whole day, and then opened, the water flows out as usual. Lastly, the water rises and falls indifferently through either leg.

Musschenbroek, in accounting for the operation of this siphon, observes that no discharge could be made by it, if the water applied to either leg did not cause the one to be shorter, and the other longer by its own weight. Introd. ad Phil. Nat. tom. 2, pa. 853, ed. 4to. 1762.

SIRIUS, the *Dog-star*; a very bright star of the first magnitude, in the mouth of the constellation *Canis Major*, or the Great Dog. This is the brightest of all the stars in our firmament, and therefore probably, says Dr. Maskelyne, the nearest to us of them all, in a paper recommending the discovery of its parallax, Philos. Trans. vol. 51, pa. 889. Some however suppose Arcturus to be the nearest. The Arabs call it Aschers, Elschecre, Scera; the Greeks, Sirius; and the Latins, Canicula, or Canis candens. See CANICULA.

This is one of the earliest named stars in the heavens. Hesiod and Homer mention only four or five constellations, or stars, and this is one of them. Sirius and Orion, the Hyades, Pleiades, and Arcturus are almost the whole of the old poetical astronomy. The three last the Greeks formed of their own observation, as appears by the names; the two others were Egyptian. Sirius was so called from the Nile, one of the names of that river being Siris; and the Egyptians, seeing that river begin to swell at the time of a particular rising of this star, paid divine honours to the star, and called it by a name derived from that of the river, expressing the star of the Nile.

SITUS, in Algebra and Geometry, denotes the situation of lines, surfaces, &c. Wolfius delivers some things in geometry, which are not deduced from the common analysis, particularly matters depending on the situs of lines and figures. Leibnitz has even founded a particular kind of analysis upon it, called Calculus Situs.

SKY, the blue expanse of the air or atmosphere. The azure colour of the sky is attributed, by Newton, to vapours beginning to condense, having attained consistence enough to reflect the most reflexible rays, viz, the violet ones; but not enough to reflect any of the less reflexible ones.

Lahire attributes it to our viewing a black object, viz, the dark space beyond the regions of the atmosphere, through a white or lucid one, viz, the air illuminated by the sun; a mixture of black and white always appearing blue. But this hypothesis is not originally his; being as old as Leonardo da Vinci.

SLIDING, in Mechanics, is when the same point of a body, moving along a surface, describes a line on that surface. Such is the motion of a parallelopipedon moved along a plane.—From sliding arises friction.

SLIDING *Rule*, a mathematical instrument serving to perform computations in gauging, measuring, &c, without the use of compasses; merely by the sliding of the parts of the instrument one by another, the lines and divisions of which give the answer or amount by inspection. This instrument is variously contrived and applied by different authors, particularly Gunter, Partridge, Hunt, Everard, and Coggeshall; but the most usual and useful ones are those of the two latter.

Everard's SLIDING *Rule* is chiefly used in cask gauging. It is commonly made of box, 12 inches long, 1 inch broad, and $\frac{6}{10}$ of an inch thick. It consists of three parts; viz, the stock just mentioned, and two thin slips, of the same length, sliding in small grooves in two opposite sides of the stock: consequently, when both these pieces are drawn out to their full extent, the instrument is 3 feet long.

On the first broad face of the instrument are four logarithmic lines of numbers; for the properties &c, of which, see GUNTER's *Line*. The first, marked A, consisting of two radii numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 1; and then, 2, 3, 4, 5, &c, to 10. On this line are four brass centre-pins, two in each radius; one in each of them being marked MB, for malt-bushel, is set at 2150·42 the number of cubic inches in a malt-bushel; the other two are marked with A, for ale-gallon, at 282, the number of cubic inches in an ale gallon. The 2d and 3d lines of numbers are on the sliding pieces, and are exactly the same with the first; but they are distinguished by the letter B. In the first radius is a dot, marked *Si*, at ·707, the side of a square inscribed in a circle whose diameter is 1. Another dot, marked *Se.* stands at ·886, the side of a square equal to the area of the same circle. A third dot, marked W, is at 231, the cubic inches in a wine gallon. And a fourth, marked C, at 3·14, the circumference of the same circle whose diameter is 1. The fourth line of numbers, marked MD, to signify malt-depth, is a broken line of two radii, numbered 2, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 9, 8, 7, &c; the number 1 being set directly against MB on the first radius.

On the second broad face, marked *cd*, are several lines: as 1st, a line marked D, and numbered 1, 2, 3, &c, to 10. On this line are four centre pins: the first, marked WG, for wine-gauge, is at 17·15, the gauge-point for wine gallons, being the diameter of a cylinder whose height is one inch, and content 231 cubic inches, or a wine gallon: the second centre-pin, marked AG, for ale-gauge, is at 18·95, the like diameter for an ale gallon: the 3d, marked MS, for malt-square, is at 46·3, the square root of 2150·42, or the side of a square whose content is equal to the number of inches in a solid bushel: and the fourth, marked MR, for malt-round, is at 52·32, the diameter of a cylinder, or bushel, the area of whose base is the same 2150·42, the inches in a bushel. 2dly, Two lines of numbers on the sliding piece, on the other side, marked C. On these are two dots; the one, marked *c*, at ·0795, the area of a circle whose circumference is 1; and the other, marked *d*, at ·785, the area of the circle whose diameter is 1. 3dly, Two lines of segments, each numbered 1, 2, 3, to 100; the first for finding the ullage of a cask, taken as the middle frustum of a spheroid, lying with its axis parallel to the horizon; and the other for finding the ullage of a cask standing.

Again, on one of the narrow sides, noted *c*, are, 1st, a line of inches, numbered 1, 2, 3, &c to 12, each subdivided into 10 equal parts. 2dly, A line by which, with that of inches, is found a mean diameter for a cask, in the figure of the middle frustum of a spheroid: it is marked Spheroid, and numbered 1, 2, 3, &c to 7. 3dly, A line for finding the mean diameter of a cask, in the form of the middle frustum of a parabolic spindle, which gaugers call the second variety of casks; it is therefore marked Second Variety, and is numbered 1, 2, 3, &c. 4thly, A line by which is found the mean diameter of a cask of the third variety, consisting of the frustums of two parabolic conoids, abutting on a common base; it is therefore marked Third Variety, and is numbered 1, 2, 3, &c.

On the other narrow face, marked *f*, are, 1st, a line of a foot divided into 100 equal parts, marked FM. 2dly, A line of inches, like that before mentioned, marked IM. 3dly, A line for finding the mean diameter of the fourth variety of casks, which is formed of the frustums of two cones, abutting on a common base. It is numbered 1, 2, 3, &c; and marked FC, for frustum of a cone.

On the back side of the two sliding pieces is a line of inches, from 12 to 36, for the whole extent of the 3 feet, when the pieces are put endwise; and against that, the correspondent gallons, and 100th parts, that any small tub, or the like open vessel, will contain at 1 inch deep. For the various uses of this instrument, see the authors mentioned above, and most other writers on gauging.

Coggeshall's SLIDING *Rule* is chiefly used in measuring the superficies and solidity of timber, masonry, brickwork, &c. This consists of two rulers, each a foot long, which are united together in various ways. Sometimes they are made to slide by one another, like glaziers' rules: sometimes a groove is made in the side of a common two-foot joint rule, and a thin sliding piece in one side, and Coggeshall's lines added on that side; thus forming the common or carpenter's rule: and sometimes one of the two rulers is made to slide in a groove made in the side of the other.

On the sliding side of the rule are four lines of numbers, three of which are double, that is, are lines to two radii, and the fourth is a single broken line of numbers. The first three, marked A, B, C, are figured 1, 2, 3, &c to 9; then 1, 2, 3, &c to 10; the construction and use of them being the same as those on Everard's Sliding rule. The single line, called the girt line, and marked D, whose radius is equal to the two radii of any of the other lines, is broken for the easier measuring of timber, and figured 4, 5, 6, 7, 8, 9, 10, 20, 30, &c. From 4 to 5 it is divided into 10 parts, and each 10th subdivided into 2; and so on from 5 to 10, &c.

On the back side of the rule are, 1st, a line of inch measures, from 1 to 12; each inch being divided and subdivided. 2dly, A line of foot measure, consisting of one foot divided into 100 equal parts, and figured 10, 20, 30, &c. The backside of the sliding piece is divided into inches, halves, &c, and figured from 12 to 24; so that when the slide is out, there may be a measure of 2 feet.

In the carpenter's rule, the inch measure is on one side, continued all the way from 1 to 24, when the rule is unfolded, and subdivided into 8ths or half-quarters: on this side are also some diagonal scales of equal parts. And upon the edge, the whole length of 2 feet is divided into 200 equal parts, or 100ths of a foot.

SLING, a string instrument, serving for the casting of stones &c with the greater violence. Pliny, lib. 76, chap. 5, atributes the invention of the sling to the Phœnicians; but Vegetius ascribes it to the inhabitants of the Balearic islands, who were celebrated in antiquity for the dextrous management of it. Florus and Strabo say, those people bore three kinds of slings; some longer, others shorter, which they used according as their enemies were more remote or nearer hand. Diodorus adds, that the first served them for a headband, the 2d for a girdle, and that the third they constantly carried with them in the hand. But it must be impossible to tell who were the first inventors of the sling, as the instrument is so simple, and has been in general use by almost all nations. The instrument is much spoken of in the wars and history of the Israelites. David was so expert a slinger, that he ventured to go out, with one in his hand, against the giant and champion Goliah, and at a distance struck him on the forehead with the stone. And there were a number of left-handed men of one of the tribes of Israel, who, it is said, could sling a stone at an hair's breadth. Judges, ch. 20, v. 16.

The motion of a stone discharged from a sling arises from its centrifugal force, when whirled round in a circle. The velocity with which it is discharged, is the same as that which it had in the circle, being much greater than what can be given to it by the hand alone. And the direction in which it is discharged, is that of the tangent to the circle at the point of discharge. Whence its motion and effect may be computed as a projectile.

SLUICE, a watergate, a floodgate, a vent for water.

SLUSE, or SLUSIUS (*René Francis Walter*), of Vise, a small town in the county of Liege, where he enjoyed honours and preferment. He became abbé of Amas, canon, counsellor and chancellor of Liege, and made his name famous for his knowledge in theology, physics, and mathematics. The Royal Society of London elected him one of their members, and inserted several of his compositions in their Transactions. This very ingenious and learned man died at Liege in 1685, at 63 years of age.—Of Slusius's works there have been published, some learned letters, and a work intitled, Mesolabium et Problemata solida; besides the following pieces in the Philosophical Transactions, viz,

1. Short and Easy Method of drawing Tangents to all Geometrical Curves; vol. 7, pa. 5143.
2. Demonstration of the same; vol. 8, pa. 6059.
3. On the Optic Angle of Alhazen; vol. 8, pa. 6139.

SMEATON (JOHN), F. R. S. and a very celebrated civil engineer, was born 1724, at Austhorpe, near Leeds, in a house built by his grandfather, where the family have resided ever since, and where our author died the 28th of October 1792, in the 69th year of his age.

Mr. Smeaton seems to have been born an engineer. The originality of his genius and the strength of his understanding appeared at a very early age. His playthings were not those of children, but the tools men work with; and he had always more amusement in observing artificers work, and asking them questions, than in any thing else. Thus had Mr. Smeaton, by the strength of his genius, and indefatigable industry, acquired, at 18 years of age, an extensive set of tools, and the art of working in most of the mechanical trades; which he continued to work with occasionally to the end of his life, part of every day when at the place where his tools were: and few men could work better.

Mr. Smeaton's father was an attorney, and was desirous of bringing his son up to the same profession. He was therefore sent up to London in 1742, where for some time he attended the courts in Westminster Hall. But, finding that the profession of the law did not suit the bent of his genius, as his usual expression was, he wrote a strong memorial to his father on the subject, whose good sense from that moment left Mr. Smeaton to pursue the bent of his genius in his own way.

Mr. Smeaton after this continued to reside in London, and about the year 1750 he commenced philosophical instrument maker, which he continued for some time, and became acquainted with most of the ingenious men of that time. This same year he made his first communication to the Royal Society, being an account of Dr. Knight's improvements of the mariner's compass. Continuing his very useful labours, and making experiments, he communicated to that learned body, the two following years, a number of other ingenious improvements, in the arts and sciences.

In 1753 he was elected a member of the Royal Society; and in 1759 he was honoured with their gold medal, for his paper concerning the natural powers of water and wind to turn mills, and other machines depending on a circular motion. This paper, he says, was the result of experiments made on working models in the years 1752 and 1753, but not communicated to the Society till 1759, having in the interval found opportunities of putting the result of these experiments into real practice, in a variety of cases, and for various purposes, so as to assure the Society he had found them to answer.—In 1754 his great thirst after experimental knowledge led him to undertake a voyage to Holland and the Low Countries, where he made himself acquainted with most of the curious works of art so frequent in those places.

In December 1755, the Edystone lighthouse was burnt down, and the proprietors, being desirous of rebuilding it in the most substantial manner, inquired of the earl of Macclesfield, then president of the Royal Society, who he thought might be the fittest person to rebuild it, when he immediately recommended our author. Mr. Smeaton accordingly undertook the work, which he completed with stone in the summer of 1759. Of this work he gives an ample description in a folio volume, with plates, published in 1791; a work which contains, in a great measure, the history of four years of his life; in which the originality of his genius is fully displayed, as well as his activity, industry, and perseverance.

In 1764 Mr. S. was appointed one of the receivers of the forfeited Derwentwater estates, which were applied to the benefit of Greenwich Hospital; which office he held till 1777, when he was prevailed on to resign it, in favour of Sir John Turner, said to be a son of Earl Sandwich, who was then governor of that hospital, and first lord of the admiralty. After this, Mr. S. going into full employment as an engineer, it would be endless to attempt to particularize all the great works he so ably conducted; as mills, wheels, engines, levels, canals, bridges, harbours, &c, in all of which he was equally eminent. Particularly, he saved from destruction London Bridge, after the opening of its great arch. Indeed, as a civil engineer, Mr. S. was perhaps unrivalled, certainly not excelled by any one.

Astronomy was also, for amusement, a favourite pursuit of Mr. S., and he made several curious instruments of this kind for his friends, as well as for himself; with which, to the time of his death, he continued to make many observations. The chief of Mr. S.'s publications, was his History of Edystone lighthouse. Besides which, many of his reports and memorials, on the different works he was concerned in, were occasionally printed in his lifetime, which have since been collected and printed, in 3 vols. 4to, to which is prefixed a pretty full account of his life and labours. He had also inserted in the Philos. Trans. a considerable number of valuable papers, both mechanical and astronomical, in most of the volumes from the year 1750 to 1776.

In 1771, he became, jointly with his friend Mr. Holmes, proprietor of the works for supplying Deptford and Greenwich with water; which by their united endeavours they brought to be of general use to those they were made for, and moderately beneficial to themselves. About the year 1785, Mr. Smeaton's health began to decline; in consequence of which he took the resolution to avoid any new undertakings in business as much as he could, that he might thereby also have the more leisure to publish some account of his inventions and works. Of this plan however he got no more executed than the account of the Edystone lighthouse, and some preparations for his intended treatise on mills. It had for many years been the practice of Mr. Smeaton to spend part of the year in London, and the remainder in the country, at his house at Austhorpe; on one of these excursions in the country, while walking in his garden, on the 16th of September 1792, he was struck with the palsy, which put an end to his useful life the 28th of October following, to the great regret of a numerous set of friends and acquaintances.

In his person, Mr. Smeaton was of a middle stature, but broad and strong made, and possessed of an excellent constitution. He had a great simplicity and plainness in his manners: he had a warmth of expression that might appear, to those who did not know him well, to border on harshness; but such as were more closely acquainted with him, knew it arose from the intense application of his mind, which was always in the pursuit of truth, or engaged in the investigation of difficult subjects. He would sometimes break out hastily, when any thing was said that was contrary to his ideas of the subject; and he would not give up any thing he argued for, till his mind was convinced by sound reasoning. As a companion, he was always entertaining and instructive, and none could spend their time in his company without improvement.

As to the list of his writings; besides the large work above mentioned, being the History of Edystone Lighthouse, and numbers of reports and memorials, which have been printed in 3 vols. 4to, as before-mentioned, his communications to the Royal Society, and inserted in their Transactions, are as follow:

1. Account of Dr. Knight's Improvements of the Mariner's Compass; an. 1750, pa. 513.
2. Some Improvements in the Air-pump; an. 1752.
3. An Engine for raising Water by Fire; being an improvement on Savary's construction, to render it capable of working itself; an. 1752.
4. Description of a new Combination of Pulleys. Ib.
5. Experiments on a machine for measuring the Way of a Ship at Sea. An. 1754.
6. Description of a new Pyrometer. Ib.
7. Effects of Lightning on the Steeple and Church of Lestwithial in Cornwall. An. 1757.
8. Remarks on the different Temperature of the Air at Edystone Light-house, and at Plymouth. An. 1758.

9. Experimental inquiry concerning the natural powers of Water and Wind to turn mills and other machines depending on a circular motion. An. 1759.

10. On the Menstrual Parallax arising from the mutual gravitation of the earth and moon, its influence on the observation of the sun and planets, &c. An. 1768.

11. Description of a new method of observing the heavenly bodies out of the meridian. An. 1768.

12. Observations on a Solar Eclipse. An. 1769.

13. Description of a new Hygrometer. An. 1771.

14. An Experimental Examination of the quantity and proportion of Mechanical Power, necessary to be employed in giving different degrees of velocity to heavy bodies from a state of rest. An. 1776. pa. 450.

In two of those articles, viz, the experiments of 1759 and of 1776, it may be remarked that Mr. Smeaton has manifested several inconsistencies and inaccuracies, apparently from erroneous notions concerning the Newtonian doctrine of the force of bodies in motion. Hence, though the experiments are good in themselves, from reasoning wrongly upon them, he fallaciously infers that their results are contrary to the theory, which, rightly managed, they tend to confirm. He does not properly distinguish between what he calls Mechanical Power, and the Newtonian term Momentum, or quantity of motion. These two powers are, from their very definitions, as well as from their nature, of different kinds. The one being measured or estimated by its momentary or instantaneous action; the other by its action during some certain time. The one, by its definition, is in the compound ratio of the mass of a body and its velocity; or as the product of the body and its velocity, and therefore simply as the velocity in a given body: whereas the other, by its definition, is estimated by the mass or weight compounded with the space it has fallen, or described, in acquiring its velocity: and since, as is well known, the space fallen by a body, is as the square of the velocity acquired; it follows, that this force must needs be as the square of the velocity in a given body. The Newtonian momentum or force, therefore, and Mr. Smeaton's mechanical force or power, are two things that are quite different in their nature or measure, and in their mode of action; though both may produce true results when applied to their proper objects.

SMITH (Robert) d. d. and f. r. s. It seems not a little remarkable that I have not met with any account of the life, or death, or works of Dr. Smith, a man who, from his connections and situation and works, has so well deserved of the literary world. It barely appears, that he was the maternal cousin of the celebrated Roger Cotes, whom he succeeded, in the year 1716, as Plumian professor at Cambridge; that he became master of Trinity College there; that he published some of the works of his cousin Cotes; as, his Hydrostatical and Pneumatical Lectures, in 8vo, 1737; also a collection of Mr. Cotes's pieces from the Philosophical Transactions, and elsewhere, and his Harmonia Mensurarum, with a large commentary, &c, in one vol. 4to, 1722: that Dr. Smith published also two excellent works of his own, viz, his complete System of Optics, in 2 vols. 4to, 1728; and his Harmonics, or the Philosophy of Musical Sounds, &c.

SMOKE, or *Smoak*, a humid matter exhaled in form of vapour by the action of heat, either external or internal; or smoke consists of palpable particles, elevated by means of the rarefying heat, or by the force of the ascending current of air, from certain bodies exposed to heat; which particles vary much in their properties, according to the substances from which they are produced.

Sir Isaac Newton observes, that smoke ascends in the chimney by the impulse of the air it floats in: for that air, being rarefied by the heat of the fire underneath, has its specific gravity diminished; and thus, being disposed to ascend itself, it carries up the smoke along with it. The tail of a comet, the same author supposes, ascends from the nucleus after the same manner.—Smoke of fat unctuous woods, as fir, beech, &c, makes what is called lamp-black.

There are various inventions for preventing and curing smokey chimneys: as the æolipiles of Vitruvius, the ventiducts of Cardan, the windmills of Bernard, the capitals of Serlio, the little drums of Paduanus, and several artifices of De Lorme. See also the philosophical works of Dr. Franklin. Pans, resembling sugar pans, placed over the tops of chimneys, are useful to make them draw better; and the fire-grates called register-stoves, are always a sure remedy.

In the Philosophical Transactions is the description of an engine, invented by M. Dalesme, which consumes the smoke of all kinds of wood so effectually, that the eye cannot discover it in the room, nor the nose distinguish the smell of it, though the fire be made in the middle of the floor. It consists of several iron hoops, 4 or 5 inches in diameter, which shut into one another, and is placed on a trevet.

The late invention called Argand's lamp, also consumes the smoke, and gives a very strong light. Its principle is a thin broad cotton wick, rolled into the form of a hollow cylinder; the air passes up the hollow of it, and the smoke is almost all consumed.

Smoke *Jack*, is a jack for turning a spit, turned by the smoke of the kitchen fire, by means of thin iron sails set obliquely on an axis in the flue of the chimney. See Jack.

SNELL (Rodolph), a respectable Dutch philosopher, was born at Oudenwater in 1546. He was some time professor of Hebrew and mathematics at Leyden, where he died in 1613, at 67 years of age. He was author of several works on geometry, and on all parts of the philosophy of his time.

Snell (Willebrord), son of Rodolph above mentioned, an excellent mathematician, was born at Leyden in 1591, where he succeeded his father in the mathematical chair in 1613, and where he died in 1626, at only 35 years of age. Willebrord Snell was author of several ingenious works and discoveries. Thus, it was he who first discovered the true law of the refraction of the rays of light; a discovery which he made before it was announced by Descartes, as Huygens assures us. Though the work which Snell prepared on this subject, and on optics in general, was never published, yet the discovery was very well known to belong to him, by several authors about his time, who had seen it in his manuscripts.—He undertook also to measure the earth. This he effected by measuring a space between Alcmaer and Bergen-op-zoom, the difference of latitude between these places being 1° 11′ 30″. He also measured another distance between the parallels of Alcmaer and Leyden; and from the mean of both these measurements, he made a degree to consist of 55021 French toises or fathoms. These measures were afterwards repeated and corrected by Muschenbroek, who found the degree to contain 57033 toises.—He was author of a great many learned mathematical works, the principal of which are,

1. Apollonius Batavus; being the restoration of some lost pieces of Apollonius, concerning Determinate Section, with the Section of a Ratio and Space: in 4to, 1608, published in his 17th year.

2. A curious tract, De Re Nummariâ; in 12mo, 1613.

3. Eratosthenes Batavus; in 4to, 1617. Being the work in which he gives an account of his operations in measuring the earth.

4. A translation out of the Dutch language, into Latin, of Ludolph van Collen's book De Circulo & Adscriptis, &c; in 4to, 1619.

5. Cyclometricus, De Circuli Dimensione &c; 4to, 1621. In this work, the author gives several ingenious approximations to the measure of the circle, both arithmetical and geometrical.

6. Tiphis Batavus; being a treatise on Navigation and Naval Affairs; in 4to, 1624; a very well-written work.

7. A posthumous treatise, being four books Doctrinæ Triangulorum Canonicæ; in 8vo, 1627. In which are contained the canon of secants; and in which the construction of sines, tangents, and secants, with the dimension or calculation of triangles, both plane and spherical, are briefly and clearly treated.

8. Hessian and Bohemian Observations; with his own notes.

9. Libra Astronomica & Philosophica; in which he undertakes the examination of the principles of Galileo concerning comets.

10. Concerning the Comet which appeared in 1618, &c.

SNOW, a well known meteor, formed by the freezing of the vapours in the atmosphere. It differs from hail and hoar-frost in being as it were chrystallized, which they are not. This appears on examination of a flake of snow by a magnifying glass; when the whole of it appears to be composed of fine shining spicula diverging like rays from a centre. As the flakes descend through the atmosphere, they are continually joined by more of these radiated spicula, and thus increase in bulk like the drops of rain or hailstones; so that it seems as if the whole body of snow were an infinite mass of icicles irregularly figured. The lightness of snow, though it is firm ice, is owing to the excess of its surface, in comparison to the matter contained under it; as even gold itself may be extended in surface, till it will float upon the least breath of air.

According to Beccaria, clouds of snow differ in nothing from clouds of rain, but in the circumstance of cold that freezes them. Both the regular diffusion of the snow, and the regularity of the structure of its parts, show that clouds of snow are acted on by some uniform cause like electricity; and he endeavours to show how electricity is capable of forming these figures. He was confirmed in his conjectures by observing, that his apparatus for showing the electricity of the atmosphere, never failed to be electrified by snow as well as by rain. Professor Wintrop sometimes found his apparatus electrified by snow when driven about by the wind, though it had not been affected by it when the snow itself was falling. A more intense electricity, according to Beccaria, unites the particles of hail more closely than the more moderate electricity does those of snow, in the same manner as we see that the drops of rain which fall from the thunder-clouds, are larger than those which fall from others, though the former descend through a less space.

In the northern countries, the ground is covered with snow for several months; which proves excedingly favovourable for vegetation, by preserving the plants from those intense frosts which are common in such countries, and which would certainly destroy them. Bartholin ascribes great virtues to snow-water, but experience does not seem to warrant his assertions. Snow-water, or ice-water, is always deprived of its fixed air: and those nations who live among the Alps, and use it for their constant drink, are subject to affections of the throat, which it is thought are occasioned by it.

From some late experiments on the quantity of water yielded by snow, it appears that the latter gives only about one-tenth of its bulk in water.

SOCIETY, an assemblage or union of several learned persons, for their mutual assistance, improvement, or information, and for the promotion of philosophical or other knowledge. There are various philosophical societies instituted in different parts of the world. See ROYAL *Society*.

Royal SOCIETY *of England*, is an academy or body of persons, supposed to be eminent for their learning, instituted by king Charles the 2d, for promoting natural knowledge. This society originated from an assembly of ingenious men, residing in London, who, being inquisitive into natural knowledge, and the new and experimental philosophy, agreed, about the year 1645, to meet weekly on a certain day, to discourse upon such subjects. These meetings, it is said, were suggested by Mr. Theodore Haak, a native of the Palatinate in Germany; and they were held sometimes at Dr. Goddard's lodgings in Wood-street, sometimes at a convenient place in Cheapside, and sometimes in or near Gresham College. This assembly seems to be that mentioned under the title of the Invisible, or Philosophical College, by Mr. Boyle, in some letters written in 1646 and 1647. About the years 1648 and 1649, the company which formed these meetings, began to be divided, some of the gentlemen removing to Oxford, as Dr. Wallis, and Dr. Goddard, where, in conjunction with other gentlemen, they held meetings also, and brought the study of natural and experimental philosophy into fashion there; meeting first in Dr. Petty's lodgings, afterwards at Dr. Wilkins's apartments in Wadham College, and, on his removal, in the lodgings of Mr. Robert Boyle; while those gentlemen who remained in London continued their meetings as before. The greater part of the Oxford Society coming to London about the year 1659, they met once or twice a week in term-time at Gresham College, till they were dispersed by the public distractions of that year, the place where they met being made a quarter for soldiers. On the restoration, in 1660, their meetings were revived, and attended by many gentlemen, eminent for their character and learning.

They were at length noticed by the government, and probably by the advice of Sir Jonas Moore, the king granted them a charter, first the 15th of July 1662, then a more ample one on the 22d of April 1663, and thirdly the 8th of April 1669; by which they were erected into a corporation, "consisting of a president, council, and fellows, for promoting natural knowledge," and endued with various privileges and authorties.

Their manner of electing members is by balloting; and two-thirds of the members present are necessary to carry the election in favour of the candidate. The council consists of 21 members, including the president, vice-president, treasurer, and two secretaries; ten of whom go out annually, and ten new members are elected instead of them, all chosen on St. Andrew's day. They had formerly also two curators, whose business it was to perform experiments before the society.

Each member, at his admission, subscribes an engagement, that he will endeavour to promote the good of the society; from which he may be freed at any time, by signifying to the president that he desires to withdraw.

The charges are five guineas paid to the treasurer at admission; and one shilling per week, or 52s. per year, as long as the person continues a member; or, in lieu of the annual subscription, a composition of 25 guineas in one payment.

The ordinary meetings of the society, are once a week, from November till the end of Trinity term the next summer. At first, the time of meeting was from 3 o'clock till 6 in the afternoon. Afterwards, it was from 6 till 7 in the evening, to allow more time for dinner, which continued for a long series of years, till the hour of meeting was removed, by the present president, to between 8 and 9 at night, that gentlemen of fashion, as was alleged, might have the opportunity of coming to attend the meetings after dinner; which has not been found to answer the purpose; besides that many members, especially elderly persons, find it inconvenient to be so late out as 9 or 10 o'clock at night.

Their design is to "make faithful records of all the works of nature or art, which come within their reach; so that the present, as well as after ages, may be enabled to put a mark on errors which have been strengthened by long prescription; to restore truths that have been long neglected; to push those already known to more various uses; to make the way more passable to what remains unrevealed, &c."

To this purpose they have made a great number of experiments and observations on most of the works of nature; as eclipses, comets, planets, meteors, mines, plants, earthquakes, inundations, springs, damps, fires, tides, currents, the magnet, &c: their motto being Nullius in Verba. They have registered experiments, histories, relations, observations, &c, and reduced them into one common stock. They have, from time to time, published some of the most useful of these, under the title of Philosophical Transactions, &c, usually one volume each year. Those papers that are not printed, are laid up in their registers.

They have a good library of books, which has been formed, and continually augmenting, by numerous donations. They had also a museum of curiosities in nature, kept in one of the rooms of their own house in Crane Court, Fleet-street, where they held their meetings, with the greatest reputation, for many years, keeping registers of the weather, and making other experiments; for all which purposes those apartments were well adapted. But, disposing of these apartments, in order to remove into those allotted them in Somerset Place, where having neither room nor convenience for such purposes, the museum was obliged to be disposed of, and their useful meteorological registers discontinued for many years.

Sir Godfrey Copley, bart. left 5 guineas to be given annually to the person who should write the best paper in the year, under the head of experimental philosophy: this reward, which is now changed to a gold medal, is the highest honour the society can bestow; and it is conferred on St. Andrew's day: but the communications of late years have been thought of so little importance, that the prize medal remains sometimes for years undisposed of.

Indeed, this society now consists of a great proportion of honorary members, who do not usually communicate papers; and many scientific members being discouraged from making their usual communications, by what is deemed the present arbitrary government of the society, the annual volumes have in consequence become of much less importance, both in respect of their bulk and the quality of their contents. The number of home members has increased to about 600; the foreign members are about 44 in number.

American Philosophical SOCIETY, was established at Philadelphia in the year 1769, for promoting useful knowledge, under the direction of a patron, a president, three vice-presidents, a treasurer, four secretaries, and three curators. The first volume of their Transactions comprehends a period of two years, viz, from Jan. 1, 1769, to Jan. 1, 1771. Their labours seem to have been interrupted during the troubles in America, which commenced soon after; but since their termination, other volumes have been published, containing a number of very ingenious and useful memoirs.

American Academy of Arts and Sciences, was established by a law of the Commonwealth of Massachusetts in North America, in the year 1780.

Boston Academy of Arts and Sciences. This is a society similar to the former, which has lately been established at Boston in New England, under the title of the Academy of Arts and Sciences &c.

Berlin SOCIETY. The Society of Natural Historians at Berlin, was founded by Dr. Martini. There is also a philosophical society in the same place.

Brussels SOCIETY. The Imperial and Royal Academy of Sciences and Belles Lettres of Brussels was founded in 1773. Several volumes of their Transactions have since been published.

Dublin SOCIETY. This is an experimental society, for promoting natural knowledge, which was instituted in 1777: the members meet once a week, and distribute three honarary gold medals annually for the most approved discovery, invention, or essay, on any mathematical or philosophical subject. The society is under the direction of a president, two vice-presidents, and a secretary.

Edinburgh Royal or *Philosophical* SOCIETY, succeeded the Medical Society, and was formed upon the plan of including all the different branches of natural knowledge and the antiquities of Scotland. The meetings of this society, interrupted in 1745, were revived in 1752; and in 1754 the first volume of their collection was published, under the title of Essays or Observations Physical and Literary, which has been succeeded by other volumes. This society has been lately incorporated by royal charter, under the name of the Royal Society of Scotland, instituted for the advancement of learning and useful knowledge. The members are divided into two classes, physical and literary; and those who are near enough to Edinburgh to attend the meetings, pay a guinea on admission, and the same sum annually. The first meeting was held on the first Monday of August 1783; when there were chosen, a president, two vice-presidents, a secretary, treasurer, and a council of 12 persons. Several of the volumes of their Transactions have been published, which are very respectable both for their magnitude and contents.

In *France* there have been several institutions of this kind for the improvement of science, besides those recounted under the word Academy: As, the Royal Academy at Soissons, founded in 1674; at Villefranche, Beaujolois, in 1679; at Nismes, in 1682; at Angers, in 1685; the Royal Society at Montpelier, in 1706, which is so intimately connected with the Royal Academy of Sciences of Paris, as to form with it, in some respects, one body: the literary productions of this society are published in the memoirs of the academy: the Royal Academy of Sciences and Belles Lettres at Lyons, in 1700; at Bourdeaux, in 1703; at Marseilles, in 1726; at Rochelle, in 1734; at Dijon, in 1740; at Pau in Bern, in 1721; at Beziers, in 1723; at Montauban, in 1744; at Rouen, in 1744; at Amiens, in 1750; at Toulouse, in 1750; at Besançon, in 1752; at Metz, in 1760; at Arras, in 1773; and at Chalons sur Maine, in 1775. And the National Institute, established at Paris in 1794. For other institutions of a similar nature, and their literary productions, see the articles ACADEMY, JOURNAL, and TRANSACTIONS.

Manchester Literary and Philosophical SOCIETY, is of considerable reputation, and has been lately established there, under the direction of two presidents, four vice-presidents, and two secretaries. The number of members is limited to 50; besides these there are several honorary members, all of whom are elected by ballot; and the officers are chosen annually in April. Several valuable essays have been already read at the meetings of this society.

Newcastle-upon-Tyne Literary and Philosophical SOCIETY. This society was instituted the 7th of February 1793, under the direction of a president, four vice-presidents, two secretaries, a treasurer, which together with four of the ordinary members form a committee, all annually elected at a general meeting. The subjects proposed for the consideration and improvement of this society, comprehend the mathematics, natural philosophy and history, chemistry, polite literature, antiquity, civil history, biography, questions of general law and policy, commerce, and the arts. From such ample scope in the objects of the society, with the known respectability, zeal, and talents of the members, the greatest improvements and discoveries may be expected to be made in those important branches of useful knowledge.

Several other similar societies have been since instituted at other places.

SOCRATES, the chief of the ancient philosophers, was born at Alopece, a small village of Attica, in the 4th year of the 77th olympiad, or about 467 years before Christ. Sophroniscus, his father, being a statuary or carver of images in stone, our author followed the same profession for some time, for a subsistence. But being naturally averse to this employment, he only followed it when necessity compelled him; and on getting a little before-hand, would for a while lay it aside. These intermissions of his trade were bestowed upon philosophy, to which he was naturally addicted; and this being observed by Crito, a rich philosopher of Athens, Socrates was at length taken from his shop, and put into a condition of philosophising at his ease and leisure.

He had various instructors in the sciences, as Anaxagoras, Archylaus, Damon, Prodicus, to whom may be added the two learned women Diotyma and Aspasia, of the last of whom he learned rhetoric: of Euenus he learned poetry; of Ichomachus, husbandry; and of Theodorus, geometry.

At length he began himself to teach; and was so eloquent, that he could lead the mind to approve or disapprove whatever he pleased; but never used this talent for any other purpose than to conduct his fellow-citizens into the path of virtue. The academy of the Lycæum, and a pleasant meadow without the city on the side of the river Ilyssus, were places where he chiefly delivered his instructions, though it seems he was never out of his way in that respect, as he made use of all times and places for that purpose.

He is represented by Xenophon as excellent in all kinds of learning, and particularly instances arithmetic, geometry, and astrology or astronomy: Plato mentions natural philosophy; Idomeneus, rhetoric; Laertius, medicine. Cicero affirms, that by the testimony of all the learned, and the judgment of all Greece, he was, as well in wisdom, acuteness, politeness, and subtlety, as in eloquence, variety, and richness, in whatever he applied himself to, without exception, the prince of all.

It has been observed by many, that Socrates little affected travel; his life being wholly spent at home, excepting when he went out upon military services. In the Peloponnesian war he was thrice personally engaged: on which occasions it is said he outwent all the soldiers in hardiness: and if at any time, saith Alcibiades, as it often happens in war, the provisions failed, there were none who could bear the want of meat and drink like Socrates: yet, on the other hand, in times of feasting, he alone seemed to enjoy them; and though of himself he would not drink, yet being invited, he far outdrank every one, though he was never seen intoxicated.

To this great philosopher Greece was principally indebted for her glory and splendor. He formed the manners of the most celebrated persons of Greece, as Alcibiades, Xenophon, Plato, &c. But his great services and the excellent qualities of his mind could not secure him from envy, persecution, and calumny. The thirty tyrants forbad his instructing youth; and as he derided the plurality of the Pagan deities, he was accused of impiety. The day of trial being come, Socrates made his own defence, without procuring an advocate, as the custom was, to plead for him. He did not defend himself with the tone and language of a suppliant or guilty person, but, as if he were master of the judges themselves, with freedom, firmness, and some degree of contumacy. Many of his friends also spoke in his behalf; and lastly, Plato went up into the chair, and began a speech in these words: "Though I, Athenians, am the youngest of those that come up into this place"—but they stopped him, crying out, "of those that go down," which he was thereupon constrained to do; and then proceeding to vote, they condemned Socrates to death, which was effected by means of poison, when he was 70 years of age. Plato gives an affecting account of his imprisonment and death, and concludes, "This was the end of the best, the wisest, and the justest of men." And that account of it by Plato, Tully professes, he could never read without tears.

As to the person of Socrates, he is represented as very homely; he was bald, had a dark complexion, a flat nose, eyes sticking out, and a severe downcast look. But the defects of his person were amply compensated by the virtues and accomplishments of his mind. Socrates was indeed a man of all virtues; and so remarkably frugal, that how little soever he had, it was always enough. When he was amidst a great variety of rich and expensive objects, he would often say to himself, "How many things are there which I do not want!"

Socrates had two wives, one of which was the noted Xantippe; whom Aulus Gellius describes as an accursed froward woman, always chiding and scolding, by day and by night, and whom it was said he made choice of as a trial and exercise of his temper. Several instances are recorded of her impatience and his forbearance. One day, before some of his friends, she fell into the usual extravagances of her passion; when he, without answering a word, went abroad with them: but on his going out of the door, she ran up into the chamber, and threw down water upon his head; upon which, turning to his friends, "Did not I tell you (says he), that after so much thunder we should have rain?" Another time she pulled his cloak from his shoulders in the open forum; and some of his friends advising him to beat her, "Yes (says he), that while we two fight, you may all stand by, and cry, Well done, Socrates; to him, Xantippe."

They who affirm that Socrates wrote nothing, mean only in respect to his philosophy; for it is attested and allowed, that he assisted Euripides in composing tragedies, and was the author of some pieces of poetry. Dialogues also and epistles are ascribed to him: but his philosophical disputations were committed to writing only by his scholars: and that chiefly by Plato and Xenophon. The latter set the example to the rest in doing it first, and also with the greatest punctuality; as Plato did it with the most liberty, intermixing so much of his own, that it is hardly possible to know what part belongs to each. Hence Socrates, hearing him recite his Lysis, cried out, "How many things doth this young man feign of me!" Accordingly, the greatest part of his philosophy is to be found in the writings of Plato. To Socrates is ascribed the first introduction of moral philosophy. Man having a twofold relation to things divine and human, his doctrines were with regard to the former metaphysical, to the latter moral. His metaphysical opinions were chiefly, that, There are three principles of all things, God, matter, and idea. God is the universal intellect; matter the subject of generation and corruption; idea, an incorporeal substance, the intellect of God; God the intellect of the world. God is one, perfect in himself, giving the being and well-being of every creature.—That God, not chance, made the world and all creatures, is demonstrable from the reasonable disposition of their parts, as well for use as defence; from their care to preserve themselves, and continue their species.—That he particularly regards man in his body, appears from his noble upright form, and from the gift of speech; in his soul, from the excellency of it above others.—That God takes care of all creatures, is demonstrable from the benefit he gives them of light, water, fire, and fruits of the earth in due season. That he hath a particular regard of man; from the destination of all plants and creatures for his service; from their subjection to man, though they may exceed him ever so much in strength; from the variety of man's sense, accommodated to the variety of objects, for necessity, use, and pleasure; from reason, by which he discourseth through reminiscence from sensible objects; from speech, by which he communicates all he knows, gives laws, and governs states. Finally, that God, though invisible himself, at once sees all, hears all, is every where, and orders all.

As to the other great object of metaphysical research, the soul, Socrates taught, that it is pre-existent to the body, endued with the knowledge of eternal ideas, which in its union to the body it loseth, as stupefied, until awakened by discourse from sensible objects; on which account, all its learning is only reminiscence, a recovery of its first knowledge. That the body, being compounded, is dissolved by death; but that the soul, being simple, passeth into another life, incapable of corruption. That the souls of men are divine. That the souls of the good after death are in a happy state, united to God in a blessed inaccessible place; that the bad in convenient places suffer condign punishment.

All the Grecian sects of philosophers refer their origin to the discipline of Socrates; particularly the Platonics, Peripatetics, Academics, Cyrenaics, Stoics, &c.

SOLAR, something relating to the sun. Thus, we say solar fire in contradistinction to culinary fire.

Solar *Civil Month*. See Month.

Solar *Cycle*. See Cycle.

Solar *Comet*. See Discus.

Solar *Eclipse*, is a privation of the light of the sun, by the interposition of the opake body of the moon. See Eclipse.

Solar *Month*, *Rising*, *Spots*. See the substantives.

Solar *System*, the order and disposition of the several heavenly bodies, which revolve round the sun as the centre of their motion; viz, the planets, primary and secondary, and the comets. See System.

Solar *Year*. See Year.

SOLID, in Physics, a body whose minute parts are connected together, so as not to give way, or slip from each other, on the smallest impression. The word is used in this sense, in contradistinction to fluid.

Solid, in Geometry, is a magnitude extended in every possible direction. Though it is commonly said to be endued with three dimensions only, length, breadth, and depth or thickness. Hence, as all bodies have these three dimensions, and nothing but bodies, solid and body are often used indiscriminately. The extremes of solids are surfaces. That is, solids are terminated either by one surface, as a globe, or by several, either plane or curved. And from the circumstances of these, solids are distinguished into regular or irregular.

Regular **Solids**, are those that are terminated by regular and equal planes. These are the tetraedron, hexaedron, or cube, octaedron, dodecaedron, and icosaedron; nor can there possibly be more than these five regular solids or bodies, unless perhaps the sphere or globe be considered as one of an infinite number of sides. See these articles severally, also the article *Regular* Body.

Irregular **Solids**, are all such as do not come under the definition of regular ones: such as cylinder, cone, prism, pyramid, &c. Similar solids are to one another in the triplicate ratio of their like sides, or as the cubes of the same. And all sorts of prisms, as also pyramids, are to one another in the compound ratio of their bases and altitudes.

Solid *Angle*, is that formed by three or more plane angles meeting in a point; like an angle of a die, or the point of a diamond well cut. The sum of all the plane angles forming a solid angle, is always less than 360°;

otherwise they would constitute the plane of a circle, and not a solid. See a disquisition on the nature and measure of solid angles in my Course of Mathematics, vol. 3.

Atmosphere of SOLIDS. See ATMOSPHERE.

SOLID *Bastion.* See BASTION.

Cubature of SOLIDS. See CUBATURE and SOLIDITY.

Measure of a SOLID. See MEASURE.

SOLID *Foot.* See FOOT.

SOLID *Numbers*, are those which arise from the multiplication of a plane number, by any other number whatever. Thus, 18 is a solid number, produced from the plane number 6 and 3, or from 9 and 2.

SOLID *Place.* See LOCUS.

SOLID *Problem*, is one which cannot be constructed geometrically; but by the intersection of a circle and a conic section, or by the intersection of two conic sections. Thus, to describe an isosceles triangle on a given base, so that either angle at the base shall be triple of that at the vertex, is a solid problem, resolved by the intersection of a parabola and circle, and it serves to inscribe a regular heptagon in a given circle.

In like manner, to describe an isosceles triangle having its angles at the base each equal to 4 times that at the vertex, is a solid problem, effected by the intersection of an hyperbola and a parabola, and serves to inscribe a regular nonagon in a given circle. And such a problem as this has four solutions, and no more; because two conic sections can intersect in 4 points only.—How all such problems are constructed, is shown by Dr. Halley, in the Philos. Trans. num. 188.

SOLID *of Least Resistance.* See RESISTANCE.

Surfaces of SOLIDS. See AREA, and SUPERFICIES.

SOLID *Theorem.* See THEOREM.

SOLIDITY, in Physics, a property of matter or body, by which it excludes every other body from that place which is possessed by itself. Solidity in this sense is a property common to all bodies, whether solid or fluid. It is usually called impenetrability; but solidity expresses it better, as carrying with it somewhat more of positive than the other, which is a negative idea.

The idea of solidity, Mr. Locke observes, arises from the resistance we find one body makes to the entrance of another into its own place. Solidity, he adds, seems the most extensive property of body, as being that by which we conceive it to fill space; it is distinguished from mere space, by this latter not being capable of resistance or motion.—It is distinguished from hardness, which is only a firm cohesion of the solid parts.

The difficulty of changing situation gives no more solidity to the hardest body than to the softest; nor is the hardest diamond properly a jot more solid than water. By this we distinguish the idea of the extension of body, from that of the extension of space: that of body is the continuity or cohesion of solid, separable, moveable parts; that of space the continuity of unsolid, inseparable, immoveable parts.

The Cartesians however will, by all means, deduce solidity, or, as they call it, impenetrability, from the nature of extension; they contend, that the idea of the former is contained in that of the latter; and hence they argue against a vacuum. Thus, say they, one cubic foot of extension cannot be added to another without having two cubic feet of extension; for each has in itself all that is required to constitute that magnitude. And hence they conclude, that every part of space is solid, or impenetrable, as of its own nature it excludes all other. But the conclusion is false, and the instance they give follows from this, that the parts of space are immoveable, not from their being impenetrable or solid. See MATTER.

SOLIDITY is also used for hardness, or firmness; as opposed to fluidity; viz, when body is considered either as fluid or solid, or hard or firm.

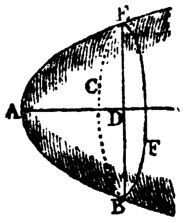
SOLIDITY, in Geometry, denotes the quantity of space contained in a solid body, or occupied by it; called also the solid content, or the cubical content; for all solids are measured by cubes, whose sides are inches, feet, or yards, &c; and hence the solidity of a body is said to be so many cubic inches, feet, yards, &c, as will fill its capacity or space, or another of an equal magnitude.

The solidity of a cube, parallelopipedon, cylinder, or any other prismatic body, i. e. one whose parallel sections are all equal and similar throughout, is found by multiplying the base by the height or perpendicular altitude. And of any cone or other pyramid, the solidity is equal to one-third part of the same prism, because any pyramid is equal to the 3d part of its circumscribing prism. Also, because a sphere or globe may be considered as made up of an infinite number of pyramids, whose bases form the surface of the globe, and their vertices all meet in the centre, or having their common altitude equal to the radius of the globe; therefore the solid content of it is equal to one-third part of the product of its radius and surface. For the solidity of other figures, see each figure separately.

The foregoing rules are such as are derived from common geometry. But there are in nature numberless other forms, which require the aid of other methods and principles, as follows.

Of the SOLIDITY *of Bodies formed by a Plane revolving about any Axis, either within or without the Body.*—Concerning such bodies, there is a remarkable property or relation between their solidity and the path or line described by the centre of gravity of the revolving plane; viz, the solidity of the body generated, whether by a whole revolution, or only a part of one, is always equal to the product arising from the generating plane drawn into the path or line described by its centre of gravity, during its motion in describing the body. And this rule holds true for figures generated by all kinds of motion whatever, whether rotatory, or direct or parallel, or irregularly zigzag, &c, provided the generating plane does not vary, but continue the same throughout. And the same law holds true also for all surfaces any how generated by the motion of a right line. This is called the Centrobaric method. See CENTROBARIC, and my Mensuration, sect. 3, part 4.

Of the SOLIDITY *of Bodies by the Method of Fluxions.*—This method applies very advantageously in all cases also in which a body is conceived to be generated by the revolution of a plane figure about an axis, or, which is much the same thing, by the parallel motion of a circle, gradually expanding and contracting itself, according to the nature of the generating plane. It is also particularly useful for the solids generated by any curvilineal plane figures. Thus, let the plane AED revolve about the axis AD; then it will generate the solid ABFEC. But as every ordinate DE, perpendicular to the axis AD, describes a circle BCBF in the revolution, therefore the same solid may be conceived as generated by a circle BCBF, gradually expanding itself larger and larger, and moving



perpendicularly along the axis AD. Consequently the area of that circle being drawn into the fluxion of the axis, will produce the fluxion of the solid; and therefore the fluent, when taken, will give the solidity of that body. That is, AD × circle BCF, (whose radius is DE, or diameter BE) is the fluxion of the solidity.

Hence then, putting $AD = x$, $DE = y$, $c = 3\cdot1416$; because cy^2 is equal to the area of the circle BCF; therefore $cy^2\dot{x}$ is the fluxion of the solid. Consequently if the value of either y^2 or $\dot{x}$ be found in terms of each other, from the given equation expressing the nature of the curve, and that value be substituted for it in the fluxional expression $cy^2\dot{x}$, the fluent of the resulting quantity, being taken, will be the required solidity of the body.

For Ex. Suppose the figure to be a parabolic conoid, generated by the rotation of the common parabola ADE about its axis AD. In this case, the equation of the curve of the parabola is $px = y^2$, where p denotes the parameter of the axis. Substituting therefore px instead of y^2, in the fluxion $cy^2\dot{x}$, it becomes $cpx\dot{x}$; and the fluent of this is $\frac{1}{2}cpx^2 = \frac{1}{2}cxy^2$ for the solidity; that is, half the product of the base of the solid drawn into its altitude; for cy^2 is the area of the circular base BCF, and x is the altitude. And so on for other such figures. See the content of each solid under its proper article.

For the SOLIDITY *of Irregular Solids*, or such as cannot be considered as generated by any regular motion or description; they must either be considered as cut or divided into several parts of known forms, as prisms, or pyramids, or wedges, &c, and the contents of these parts found separately. Or, in the case of the smaller bodies, of forms so irregular as not to be easily divided in that way, put them into some hollow regular vessel, as a hollow cylinder or parallelopipedon, &c: then pour in water or sand so that it may fill the vessel exactly to the top of the inclosed irregular body, noting the height it rises to; then take out the body, and note the height the fluid again stands at; the difference of these two heights is to be considered as the altitude of a prism of the same base and form as the hollow vessel; and consequently the product of that altitude and base will be the accurate solidity of the immerged body, be it ever so irregular.

SOLSTICE, in Astronomy, is the time when the sun is in one of the solstitial points, that is, when he is at the greatest distance from the equator, which is now nearly 23° 28′ on either side of it. It is so called, because the sun then seems to stand still, and not to change his place, as to declination, either way. There are two solstices, in each year, when the sun is at the greatest distance on the north and south sides of the equator; viz, the estival or summer solstice, and the hyemal or winter solstice.

The summer solstice is when the sun is in the tropic of Cancer; which is about the 21st of June, when he makes the longest day. And the winter solstice is when he enters the first degree of Capricorn; which is about the 22d of December, when he makes the shortest day.—This is to be understood, as in our northern hemisphere; for in the southern, the sun's entrance into Capricorn makes their summer solstice, and that into Cancer the winter one. So that it is more precise and determinate, to say the northern and southern solstice.

SOLSTITIAL *Points*, are those points of the ecliptic the sun is in at the times of the two solstices, being the first points of Cancer and Capricorn, which are diametrically opposite to each other.

SOLSTITIAL *Colure*, is that which passes through the solstitial points.

SOLUTION, in Mathematics, is the answering or resolving of a question or problem that is proposed. See RESOLUTION, and REDUCTION *of Equations*.

SOLUTION, in Physics, is the reduction of a solid or firm body, into a fluid state, by means of some menstruum.—Solution is often confounded with what is called dissolution, though there is a difference.

SOSIGENES, was an Egyptian mathematician, whose principal studies were chronology and the mathematics in general, and who flourished in the time of Julius Cæsar. He is represented as well versed in the mathematics and the astronomy of the ancients; particularly of those celebrated mathematicians, Thales, Archimedes, Hipparchus, Calippus, and many others, who had undertaken to determine the quantity of the solar year; which they had ascertained much nearer the truth than one can well imagine they should, with instruments so very imperfect; as may appear by reference to Ptolemy's Almagest.

It seems that Sosigenes made great improvements, and gave proofs of his being able to demonstrate the certainty of his discoveries; by which means he became popular, and obtained repute with those who had a genius to understand and relish such inquiries. Hence he was sent for by Julius Cæsar, who being convinced of his capacity, employed him in reforming the calendar; and it was he who formed the Julian year which begins 45 years before the birth of Christ. His other works are lost since that period.

SOUND, in Geography, denotes a strait or inlet of the sea, between two capes or head-lands.

The SOUND is used, by way of eminence, for that celebrated strait which connects the German sea to the Baltic. It is situated between the island of Zealand and the coast of Schonen. It is about 16 leagues in length, and in general about 5 in breadth, except near the castle of Cronenberg, where it is but one; so that there is no passage for vessels but under the cannon of the fortress.

SOUND, in Physics, a perception of the mind, communicated by means of the ear; being an effect of the collision of bodies, and their consequent tremulous motion, communicated to the ambient fluid, and so propagated through it to the organs of hearing.

To illustrate the cause of sound, it is to be observed, 1st, That a motion is necessary in the sonorous body for the production of sound. 2dly, That this motion exists first in the small and insensible parts of the sonorous bodies, and is excited in them by their mutual collision against each other, which produces the tremulous motion so observable in bodies that have a clear sound, as bells, musical chords, &c. 3dly, That this motion is communicated to, or produces a like motion in the air, or such parts of it as are fit to receive and propagate it. Lastly, That this motion must be communicated to those parts that are the proper and immediate instruments of hearing.

Now that motion of a sonorous body, which is the immediate cause of sound, may be owing to two different causes; either the percussion between it and other hard bodies, as in drums, bells, chords, &c; or the beating and dashing of the sonorous body and the air immediately against each other, as in flutes, trumpets, &c.

But in both these cases, the motion, which is the consequence of the mutual action, as well as the immediate cause of the sonorous motion which the air conveys to the ear, is supposed to be an invisible, tremulous or undulating motion, in the small and insensible parts of the body. Perrault adds, that the visible motion of the grosser parts contributes no otherwise to sound, than as it causes the invisible motion of the smaller parts, which he calls particles, to distinguish them from the sensible ones, which he calls parts, and from the smallest of all, which are called corpuscles.

The sonorous body having made its impression on the contiguous air, that impression is propagated from one particle to another, according to the laws of pneumatics. A few particles, for instance, driven from the surface of the body, push or press their adjacent particles into a less space; and the medium, as it is thus rarefied in one place, becomes condensed in the other; but the air thus compressed in the second place, is, by its elasticity, returned back again, both to its former place and its former state; and the air contiguous to that is compressed; and the like obtains when the air less compressed, expanding itself, a new compression is generated. Therefore from each agitation of the air there arises a motion in it, analogous to the motion of a wave on the surface of the water; which is called a wave or undulation of air. In each wave, the particles go and return back again, through very short equal spaces; the motion of each particle being analogous to the motion of a vibrating pendulum while it performs two oscillations; most of the laws of the pendulum, with very little alteration, being applicable to the former.

Sounds are as various as are the means that concur in producing them. The chief varieties result from the figure, constitution, quantity, &c, of the sonorous body; the manner of percussion, with the velocity, &c, of the consequent vibrations; the state and constitution of the medium; the disposition, distance, &c, of the organ; the obstacles between the organ and the sonorous object and the adjacent bodies. The most notable distinction of sounds, arising from the various degrees and combinations of the conditions above mentioned, are into loud and low (or strong and weak); into grave and acute (or sharp and flat, or high and low); and into long and short. The management of which is the office of music.

Euler is of opinion, that no sound making fewer vibrations than 30 in a second, or more than 7520, is distinguishable by the human ear. According to this doctrine, the limit of our hearing, as to acute and grave, is an interval of 8 octaves. Tentam. Nov. Theor. Mus. cap. 1, sect. 13.

The velocity of sound is the same with that of the aerial waves, and does not vary much, whether it go with the wind or against it. By the wind indeed a certain quantity of air is carried from one place to another; and the sound is accelerated while its waves move through that part of the air, if their direction be the same as that of the wind. But as sound moves vastly swifter than the wind, the acceleration it will hereby receive is but inconsiderable; and the chief effect we can perceive from the wind is, that it increases and diminishes the space of the waves, so that by help of it the sound may be heard to a greater distance than otherwise it would.

That the air is the usual medium of sound, appears from various experiments in rarefied and condensed air. In an unexhausted receiver, a small bell may be heard to some distance; but when much exhausted, it can scarce be heard at the smallest distance, not at all in a perfect vacuum. When the air is condensed, the sound is louder in proportion to the condensation, or quantity of air crowded in; of which there are many instances in Hauksbee's experiments, in Dr. Priestley's, and others. Besides, sounding bodies communicate tremors to distant bodies; for example, the vibrating motions of a musical string put others in motion, whose tension and quantity of matter dispose their vibrations to keep time with the pulses of air, propagated from the string that was struck. Galileo explains this phenomenon by observing, that a heavy pendulum may be put in motion by the least breath of the mouth, provided the blasts be repeated, so as to keep time exactly with the vibrations of the pendulum; and also by attending to the same circumstances of vibration, the raising a large bell is easily effected.

It is not air alone that is capable of the impressions of sound, but water also; as is manifest by striking a bell under water, the sound of which may plainly enough be heard, only not so loud, and also a fourth deeper, according to good judges in musical notes. And Mersenne says, a sound made under water is of the same tone or note, as if made in air, and heard under the water.

The real vehicle of sound, or that by which it is transmitted from the sonorous body to the ear, is a subject that has much engaged the attention of philosophers. From the above-mentioned experiments in an exhausted receiver some have concluded, rather hastily, that air is this vehicle; but though air will convey sound, and even though it cannot be transmitted through a vacuum, yet it does not follow that air is the only medium of transmission: this indeed is proved by the experiment of striking a bell in water, which, as above observed, may be heard nearly as well as when sounded in the air. Again, solid bodies transmit sound much more readily than the air: it has been lately determined, by some experiments accurately made and frequently repeated in France by Mr. Biot, that sound is transmitted through a solid body in $0''\cdot29$, which in open air would require $2''\cdot79$ to be conveyed to the same distance, according to the experiments of the Academy of Sciences.

The velocity of sound, or the space through which it is propagated in a given time, has been very differently estimated by authors who have written on this subject. Roberval states it at the rate of 560 feet in a second; Gassendus at 1473; Mersenne at 1474; Duhamel, in the History of the Academy of Sciences at Paris, at 1338; Newton at 968; Derham, in whose measure Flamsteed and Halley acquiesce, at 1142.—The reason of this variety is ascribed by Derham, partly to some of those gentlemen using strings and plummets instead of regular pendulums; and partly to the too small distance between the sonorous body and the place of observation; and partly to no regard being had to the winds.

But by the accounts since published by M. Cassini de Thury, in the Memoirs of the Royal Acad. of Sciences at Paris, 1738, where cannon were fired at various great distances, under many varieties of weather, wind, and other circumstances, and where the measures of the different places had been settled with the utmost exactness, it was found that sound was propagated, on a medium, at the rate of 1038 French feet in a second of time. But the French foot is in proportion to the English as 15 to 16; and consequently 1038 French feet are equal to 1107 English feet. Therefore the difference of the measures of Derham and Cassini is 35 English feet, or 33 French feet, in a second. Whence the medium velocity of sound is nearly at the rate of a mile, or 5280 feet, in 4⅔ seconds, or a league in 14 seconds, or 13 miles in a minute. But geographical miles are to English miles nearly as 7. to 6; and therefore sound moves over a geographical mile in 5⅓ seconds nearly, or a sea league in 16 seconds.

Farther, it is a common observation, that persons in good health have about 75 pulsations, or beats of the artery at the wrist, in a minute: consequently in 75 pulsations, sound flies about 13 English miles, or 11⅐ sea miles, which is about 1 English mile in 6 pulses, or a league in 20 pulses. And hence the distance of objects may be found, by knowing the time employed by sound in moving from those objects to an observer. For ex. On seeing the flash of a gun at sea, if 54 beats of the pulse at the wrist were counted before the report was heard; the distance of the gun will easily be found by dividing 54 by 20, which gives 2·7 leagues, or about 8 miles.

On the nature, production, &c, of sound, see the article PHONICS and ECHO; also the Memoirs of the Acad. and the Philos. Trans. in many places; Newton, Principia; Kircher, Mesurgia Universalis; Mersenne; Borelli, Del Suono; Bernoulli and Euler, &c, in the Petersburg Memoirs; Priestley, Exper. and Observ. vol. 5; Hales, Sonorum Doctrina rationalis et experimentalis, 4to, 1778; Dr. Matthew Young on Sounds and Musical Strings; see also an ingenious treatise published 1790, by Mr. Geo. Saunders, on Theatres; in which he relates many experiments made by himself, on the nature and propagation of sound. In this work, he shows the great effect of water, and some other bodies, in conducting of sound. Some of his conclusions and observations are as follow:

Earth may be supposed to have a twofold property with respect to sound. Being very porous, it absorbs sound, which is counteracted by its property of conducting sound, and occasions it to pass on a plane, in an equal proportion to its progress in air, unencumbered by any body. If a sound be sufficiently intense to impress the earth in its tremulous quality, it will be carried to a considerable distance, as when the earth is struck with any thing hard, as by the motion of a carriage, horses feet, &c. Plaster is proportionally better than loose earth for conducting sound, as it is more compact. Clothes of every kind, particularly woollen cloths, are very prejudicial to sound: their absorption of sound may be compared to that of water, which they greedily imbibe.

A number of people seated before others, as in the pit or gallery of a theatre, do considerably prevent the voice reaching those behind; and hence it is, that we hear so much better in the front of the galleries, or of any situation, than behind others, though we may be nearer to the speaker. Our seats, rising so little above each other, occasion this defect, which would be remedied, could we have the seats to rise their whole height above each other, as in the ancient theatres. Paint has generally been thought unfavourable to sound, from its being so to musical instruments, whose effects it quite destroys.

Musical instruments mostly depend on the vibrative or tremulous property of the material, which a body of colour hardened in oil must very much alter; but we should distinguish that this regards the formation of sound, which may not altogether be the case in the progress of it. Water has been little noticed, with respect to its conducting sound; but it will be found to be of the greatest consequence. I had often perceived in newly-finished houses, that while they were yet damp, they produced echoes; but that the echoing abated as they dried.

Exp. When I made the following experiment there was a gentle wind; consequently the water was proportionally agitated. I chose a quiet part of the river Thames, near Chelsea Hospital, and with two boats tried the distance the voice would reach. On the water we could distinctly hear a person read at the distance of 140 feet, on land at that of 76. It should be observed, that on land no noise intervened; but on the river some noise was occasioned by the flowing of the water against the boats; so that the difference on land and on water must be much more.

Watermen observe, that when the water is still, and the weather quite calm, if no noise intervene, a whisper may be heard across the river; and that with the current it will be carried to a much greater distance, and vice versa against the current.—Mariners well know the difference of sound on sea and land. When a canal of water was laid under the pit floor of the theatre of Argentino, at Rome, a surprising difference was observed; the voice has since been heard at the end very distinctly, where it was before scarce distinguishable. It is observable that, in this part, the canal is covered with a brick arch, over which there is a quantity of earth, and the timber floor over all.

The villa Simonetta near Milan, so remarkable for its echoes, is entirely over arcades of water. Another villa near Rouen, remarkable for its echo, is built over subterraneous cavities of water. A reservoir of water domed over, near Stanmore, has a strong echo. I do not remember ever being under the arches of a stone bridge that did not echo; which is not always the case with similar structures on land. A house in Lambeth Marsh, inhabited by Mr. Turtle, is very damp during winter, when it yields an echo which abates as the house becomes dry in summer. Kircher observes, that echoes repeat more by night than during the day: he makes the difference to be double. Dr. Plott says, the echo in Woodstock park, repeated 17 times by day, and 20 by night. And Addison's experiment at the Villa Simonetta was in a fog, when it produced 56 repetitions.

After all these instances, I think little doubt can remain of the influence water has on sound; and I conclude that it conducts sound more than any other body whatever. After water, stone may be reckoned the best conductor of sound. To what cause it may be attributed, I leave to future enquiries: I have confined myself to speak of facts only as they appear. Stone is sonorous, but gives a harsh disagreeable tone, unfavourable to music. Brick, in respect to sound, has nearly the same properties as stone. Part of the garden wall of the late

W. Pitt, esq. of Kingston in Dorsetshire, conveys a whisper to the distance of near 200 feet. Wood is sonorous, conductive, and vibrative; of all materials it produces a tone the most agreeable and melodious; and it is therefore the fittest for musical instruments, and for lining of rooms and theatres.

The common notion that whispering at one end of a long piece of timber would be heard at the other end, I found by experiment to be erroneous. A stick of timber 65 feet long being slightly struck at one end, a sound was heard at the other, and the tremor very perceptible: which is easily accounted for, when we consider the number or length of the fibres that compose it, each of which may be compared to a string of catgut.

For the Reflection, Refraction, &c, of SOUND; see ECHO, and PHONICS.

Articulate SOUND. See ARTICULATE.

SOUND, in Music, denotes a quality of the several agitations of the air, so as to make music or harmony.—Sound is the object of music; which is nothing but the art of applying sounds, under such circumstances of tone and time, as to raise agreeable sensations. The principal affection of sound, by which it becomes fitted to have this end, is that by which it is distinguished into acute and grave. This difference depends on the nature of the sonorous body; the particular figure and quantity of it; and even in some cases, on the part of the body where it is struck: and it is this that constitutes what are called different tones.

The cause of this difference appears to be no other than the different velocities of the vibrations of the sounding body. Indeed the tone of a sound is found, by numerous experiments, to depend on the nature of those vibrations, whose differences we can conceive no otherwise than as having different velocities: and since it is proved that the small vibrations of the same chord are all performed in equal times, and that the tone of a sound, which continues for some time after the stroke, is the same from first to last, it follows, that the tone is necessarily connected with a certain quantity of time in making each vibration, or each wave; or that a certain number of vibrations or waves, made in a given time, constitute a certain and determinate tone. From this principle are all the phenomena of tune deduced.

If the vibrations be isochronous, or performed in the same time, the sound is called musical, and is said to continue at the same pitch; and it is also accounted acuter, sharper, or higher than any other sound, whose vibrations are slower, and therefore graver, flatter, or lower, than any other whose vibrations are quicker. See UNISON.

From the same principle arise what are called concords, &c; which result from the frequent unions and coincidences of the vibrations of two sonorous bodies, and consequently of the pulses or the waves of the air occasioned by them. On the contrary, the result of less frequent coincidences of those vibrations, is what is called discord.

Another considerable distinction of musical sounds, is that by which they are called long and short, owing to the continuation of the impulse of the efficient cause on the sonorous body for a longer or shorter time, as in the notes of a violin &c, which are made longer or shorter by strokes of different length or quickness. This continuity is properly a succession of several sounds, or the effect of several distinct strokes, or repeated impulses, on the sonorous body, so quick, that we judge it one continued sound, especially where it is continued in the same degree of strength; and hence arises the doctrine of measure and time.

Musical sounds are also divided into simple and compound; and that in two different ways. In the first, a sound is said to be compound, when a number of successive vibrations of the sonorous body, and the air, come so fast upon the ear, that we judge them the same continued sound; like as in the phenomenon of the circle of fire, caused by putting the lighted end of a stick in a quick circular motion; where supposing the end of the stick in any point of the circle, the idea we receive of it there continues till the impression is renewed by a sudden return.

A *Simple* SOUND then, with regard to this composition, should be the effect of a single vibration, or of as many vibrations as are necessary to raise in us the idea of sound. In the second sense of composition, a simple sound is the product of one voice, or one instrument, &c.

A *Compound* SOUND consists of the sounds of several distinct voices or instruments all united in the same individual time, and measure of duration, that is, all striking the air together, whatever their other differences may be. But in this sense again, there is a twofold composition; a natural and an artificial one. The natural composition is that proceeding from the manifold reflections of the first sound from adjacent bodies, where the reflections are not so sudden as to occasion echoes, but are all in the same tone with the first note.

The artificial composition, which alone comes under the musician's province, is that mixture of several sounds, which being made by art, the ingredient sounds are separable, and distinguishable from one another. In this sense the distinct sounds of several voices or instruments, or several notes of the same instrument, are called simple sounds, in contradistinction from the compound ones, which, in order to answer the end of music, the simples must have such an agreement in all relations, chiefly as to acuteness and gravity, as that the ear may receive the mixture with pleasure.

Another distinction of sounds, with regard to music, is that by which they are said to be smooth or even, and rough or harsh, also clear and hoarse: the cause of which difference depends on the disposition and state of the sonorous body, or the circumstances of the place; but the ideas of the differences must be sought from observation.

Smooth and rough sounds depend chiefly on the sounding body; of which we have a remarkable instance in strings that are uneven, and not of the same dimension and constitution throughout.

As to clear and hoarse sounds, they depend on circumstances that are accidental to the sonorous body. Thus, a voice or instrument will be hollow and hoarse if sounded within an empty hogshead, that yet is clear and bright out of it: the effect is owing to the mixture of different sounds, raised by reflections, which corrupt and change the species of the primitive sound.

For sounds to be fit to obtain the end of music, they ought to be smooth and clear, especially the first; since, without this, they cannot have one certain and discernible tone, capable of being compared to others, in a certain relation of acuteness, which the ear may judge of. So that, with Malcolm, we call that an harmonic or musical sound which, being clear and even, is agreeable to the ear, and gives a certain and discernible tune (hence called tunable sound), which is the subject of the whole theory of harmony.—Wood has a particular vibrating quality, owing to its elasticity; and all musical instruments made of this matter, are of a thickness proportioned to the superficies of the wood, and the tone they are to produce.—Metals are sonorous and vibrative, producing a harsh tone, very serviceable to some parts of music. Most wind instruments are made of metal, which is acted on in its elastic and tremulous quality, being capable of being reduced very thin for that purpose. Instruments of this kind are such as horns, trumpets, &c. Some instruments however depend more on the form than the material; as flutes, for instance, which, if their lengths and bore be the same, have very little difference in their sounds, whatever the matter of them may be. See HARMONICAL.

SOUND-BOARD, the principal part of an organ, and that which makes the whole machine play. This sound-board, or summer, is a reservoir into which the air, drawn in by the bellows, is conducted by a port-vent, and thence distributed into the pipes placed over the holes of its upper part. This wind enters them by valves, which open by pressing upon the stops or keys, after drawing the registers, which prevent the air from going into any of the other pipes besides those it is required in.

SOUND-*board* denotes also a thin broad board placed over the head of a public speaker, to enlarge and extend or strengthen his voice. Sound-boards, in theatres, are found by experience to be of no service; their distance from the speaker being too great, to be impressed with sufficient force. But sound-boards immediately over a pulpit have often a good effect, when the case is made of a just thickness, and according to certain principles.

SOUND-*Post*, is a post placed withinside of a violin, &c, as a prop between the back and the belly of the instrument, and nearly under the bridge.

SOUNDING, in Navigation, the act of trying the depth of the water, and the quality of the bottom, by a line and plummet, or other artifice. At sea, there are two plummets used for this purpose, both shaped like the frustum of a cone or pyramid. One of these is called the hand-lead, weighing about 8 or 9lb; and the other the deep-sea-lead, weighing from 25 to 30lb. The former is used in shallow waters, and the latter at great distances from the shore. The line of the hand-lead, is about 25 fathoms in length, and marked at every two or three fathoms, in this manner, viz, at 2 and 3 fathoms from the lead there are marks of black leather; at 5 fathoms a white rag, at 7 a red rag, at 10 and at 13 black leather, at 15 a white rag, and at 17 a red one.

Sounding with the hand-lead, which the seaman call heaving the lead, is generally performed by a man who stands in the main-chains to windward. Having the line all ready to run out, without interruption, he holds it nearly at the distance of a fathom from the plummet, and having swung the latter backwards and forwards three or four times, in order to acquire the greater velocity, he swings it round his head, and thence as far forward as is necessary; so that, by the lead's sinking whilst the ship advances, the line may be almost perpendicular when it reaches the bottom. The person sounding then proclaims the depth of the water in a kind of song resembling the cries of hawkers in a city; thus, if the mark of 5 be close to the surface of the water, he calls, 'by the mark 5,' and as there is no mark at 4, 6, 8, &c, he estimates those numbers, and calls, 'by the dip four, &c.' If he judges it to be a quarter or a half more than any particular number, he calls, 'and a quarter 5,' 'and a half 4' &c. If he conceives the depth to be three quarters more than a particular number, he calls it a quarter less than the next: thus, at 4 fathom ¾, he calls, 'a quarter less 5,' and so on.

The deep-sea-lead line is marked with 2 knots at 20 fathom, 3 at 30, 4 at 40, &c, to the end. It is also marked with a single knot at the middle of each interval, as at 25, 35, 45 fathoms, &c. To use this lead more effectually at sea, or in deep water on the sea-coast, it is usual previously to bring-to the ship, in order to retard her course: the lead is then thrown as far as possible from the ship on the line of her drift, so that, as it sinks, the ship drives more perpendicularly over it. The pilot feeling the lead strike the bottom, readily discovers the depth of the water by the mark on the line nearest its surface. The bottom of the lead, which is a little hollowed there for the purpose, being also well rubbed over with tallow, retains the distinguishing marks of the bottom, as shells, ooze, gravel, &c, which naturally adhere to it.

The depth of the water, and the nature of the ground, which are called the soundings, are carefully marked in the log-book, as well to determine the distance of the place from the shore, as to correct the observations of former pilots. Falconer. For a machine to measure unfathomable depths of the sea, see ALTITUDE.

SOUNDING *the pump*, at sea, is done by letting fall a small line, with some weight at the end, down into the pump, to know what depth of water there is in it.

SOUTH, one of the four cardinal points of the wind, or compass, being that which is directly opposite to the north.

SOUTH *Direct Dials*. See PRIME *Verticals*.

SOUTHERN *Hemisphere, Signs, &c*, those in the south side of the equator.

SOUTHING, in Navigation, the difference of latitude made by a ship in sailing to the southward.

SPACE, denotes room, place, distance, capacity, extension, duration, &c. When space is considered barely in length between any two bodies, it gives the same idea as that of distance. When it is considered in length, breadth, and thickness, it is properly called capacity. And when considered between the extremities of matter, which fills the capacity of space with something solid, tangible, and moveable, it is then called extension. So that extension is an idea belonging to body only; but space may be considered without it. Therefore space, in the general signification, is the same thing with distance considered every way, whether there be any matter in it or not.

Space is usually divided into absolute and relative.

Absolute SPACE is that which is considered in its own nature, without regard to any thing external, which always remains the same, and is infinite and immoveable.

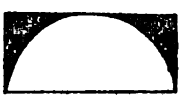
Relative SPACE is that moveable dimension, or measure of the former, which our senses define by its positions to bodies within it; and this is the vulgar use for immoveable space. Relative space, in magnitude and figure, is always the same with absolute: but it is not necessary it should be so numerically. Thus, when a ship is perfectly at rest, then the places of all things within her are the same both absolutely and relatively, and nothing changes its place: but, or the contrary, when the ship is under sail, or in motion, she continually passes through new parts of absolute space; though all things on board, considered relatively, in respect to the ship, may yet be in the same places, or have the same situation and position, in regard to one another.'

The Cartesians, who make extension the essence of matter, assert, that the space any body takes up, is the same thing with the body itself; and that there is no such thing in the universe as mere space, void of all matter; thus making space or extension a substance. See this disproved under Vacuum. Among those too who admit a vacuum, and consequently an essential difference between space and matter, there are some who assert that space is a substance. Among these we find Gravesande, Introd. ad Philos. sect. 19.

Others again put space in the same class of beings as time and number; thus making it to be no more than a notion of the mind. So that according to these authors, absolute space, of which the Newtonians speak, is a mere chimera. See the writings of the late bishop Berkeley. Space and time, according to Dr. Clarke, are attributes of the Deity; and the impossibility of annihilating these, even in idea, is the same with that of the necessary existence of the Deity.

Space, in Geometry, denotes the area of any figure; or that which fills the interval or distance between the lines that terminate or bound it. Thus, the parabolic space is that included in the whole parabola. The conchoidal space, or the cissoidal space, is what is included within the cavity of the conchoid or cissoid. And the asymptotic space, is what is included between an hyperbolic curve and its asymptote. By the application of algebra to geometry, it is demonstrated that the conchoidal and cissoidal spaces, though infinitely extended in length, are yet only finite magnitudes or spaces.

Space, in Mechanics, is the line a moveable body, considered as a point, is conceived to describe by its motion.



SPANDREL, or **Spandril**, with builders, is the space included between the curve of an arch and the straight or right lines which inclose it; as the space *a*, or *b*.

SPEAKING *Trumpet*. See *Speaking* Trumpet.

SPECIES, in Algebra, are the letters, symbols, marks, or characters, which represent the quantities in any operation or equation. This short and advantageous way of notation was chiefly introduced by Vieta, about the year 1590; and by means of which he made many discoveries in algebra, and the theory of numbers. The reason why Vieta gave this name of species to the letters of the alphabet used in algebra, and hence called Arithmetica Speciosa, seems to have been in imitation of the civilians, who call cases in law that are put abstractedly, between John a Nokes and Tom a Stiles, between A and B; supposing those letters to stand for any persons indefinitely. Such cases they call species: whence, as the letters of the alphabet will also as well represent quantities, as persons, and that also indefinitely, one quantity as well as another, they are properly enough called species; that is, general symbols, marks, or characters. Whence the literal algebra has since been often called Specious Arithmetic, or Algebra in Species.

Species, in Optics, the image painted on the retina by the rays of light reflected from the several points of the surface of an object, received in by the pupil, and collected in their passage through the crystalline, &c. Philosophers have been in great doubt, whether the species of objects, which give the soul an occasion of seeing, are an effusion of the substance of the body; or a mere impression which they make on all ambient bodies, and which these all reflect, when in a proper disposition and distance; or lastly, whether they are not some other more subtile body, as light, which receives all these impressions from bodies, and is continually sent and returning from one to another, with the different impressions and figures it has taken. But the moderns have decided this point by their invention of artificial eyes, in which the species of objects are received on a paper, in the same manner as they are received in the natural eye.

SPECIFIC, in Philosophy, that which is proper and peculiar to any thing; or that characterises it, and distinguishes it from every other thing. Thus, the attracting of iron is specific to the loadstone, or is a specific property of it. A just definition should contain the specific notion of the thing defined, or that which specifies and distinguishes it from every thing else.

Specific *Gravity*, in Hydrostatics, is the relative proportion of the weight of bodies of the same bulk. See *Specific* Gravity.

Specific *Gravity of living men*. Mr. John Robertson, late librarian to the Royal Society, in order to determine the specific gravity of men, prepared a cistern 78 inches long, 30 inches wide, 30 inches deep; and having procured 10 men for his purpose, the height of each was taken and his weight; and afterwards they plunged successively into the cistern. A ruler or scale, graduated to inches and decimal parts, was fixed to one end of the cistern, and the height of the water shown by it was noted before each man went in, and to what height it rose when he immersed himself under its surface. The following table contains the several results of his experiments:

No. of Men.	Height. Ft.	In.	Weight. lbs.	Water raised Inches.	Solidity. Feet.	Weight of water. lbs.	Specific gravity. (Wat. 1.)
1	6	2	161	1·90	2·573	160·8	1·001
2	5	$10\frac{3}{8}$	147	1·91	2·586	161·6	0·901
3	5	$9\frac{1}{4}$	156	1·85	2·505	156·6	0·991
4	5	$6\frac{3}{4}$	140	2·04	2·763	172·6	0·801
5	5	$5\frac{7}{8}$	158	2·08	2·817	176·0	0·900
6	5	$5\frac{1}{2}$	158	2·17	2·939	183·7	0·849
7	5	$4\frac{3}{8}$	140	2·01	2·722	170·1	0·823
8	5	$4\frac{1}{8}$	121	1·79	2·424	151·5	0·800
9	5	$3\frac{1}{4}$	146	1·73	2·343	146·4	0·997
10	5	$3\frac{1}{8}$	132	1·85	2·505	156·6	0·843
medium of all	5	$6\frac{2}{3}$	146	1·933	2·618	163·6	0·891

One of the reasons, Mr. Robertson says, that induced him to make these experiments, was a desire of knowing what quantity of timber would be sufficient to keep a man afloat in water, thinking that most men were specifically heavier than river or common fresh water; but the contrary appears from the trials above recited; for, except the first, every man was lighter than an equal bulk of fresh water, and much more so than that of sea-water. So that if persons who fall into water had presence of mind enough to avoid the fright usual on such occasions, they might be preserved from drowning; and a piece of wood not larger than an oar, would buoy a man partly above water as long as he had strength or spirits to keep his hold. Philos. Trans. vol. 50, art. 5.—From the last line of the table appears the medium of all the circumstances of height, weight, &c; particularly the mean specific gravity, 0·891, which is about $\frac{1}{9}$ less than common water.

SPECTACLES, an optical machine, consisting of two lenses set in a frame, and applied on the nose, to assist in correcting defects of the organ of sight.—Old people, and all presbytæ, use spectacles of convex lenses, to make amends for the flatness of the eye, which does not make the rays converge enough to have them meet in the retina. Short-sighted people, or myopes, use concave lenses, to prevent the rays from converging so fast, on account of the greater roundness of the eye, or smallness of the sphere, which is such as to make them meet before they reach the retina.—F. Cherubin, a capuchin, describes a kind of spectacle telescopes, for viewing remote objects with both eyes; and hence called binoculi. Though F. Rheita had mentioned the same before him, in his Oculus Enoch et Eliæ. See BINOCLE. The same author invented a kind of spectacles, with three or four glasses, which performed very well.

The invention of spectacles has been much disputed. They were certainly not known to the ancients. Francisco Redi, in a learned treatise on spectacles, contends that they were first invented between the years 1280 and 1311, probably about 1290; and adds, that Alexander de Spina, a monk of the order of Predicants of St. Catharine, at Pisa, first communicated the secret, which was of his own invention, on learning that another person had it as well as himself. The author tells us, that in an old manuscript still preserved in his library, composed in 1299, spectacles are mentioned as a thing invented about that time: and that a celebrated Jacobin, one Jourdon de Rivalto, in a treatise composed in 1305, says expressly, that it was not yet 20 years since the invention of spectacles. He likewise quotes Bernard Gordon in his Lilium Medicinæ, written the same year, where he speaks of a collyrium, proper to enable an old man to read without spectacles.

Musschenbroek observes, (Introd. vol. 2, pa. 786) that it is inscribed on the tomb of Salvinus Armatus, a nobleman of Florence, who died in 1317, that he was the inventor of spectacles. Du Cange, however, carries the invention of spectacles further back; assuring us, that there is a Greek poem in manuscript in the French king's library, which shows that spectacles were in use in the year 1150; however the dictionary of the Academy Della Crusca, under the word Occhiale, inclines to Redi's side; and quotes a passage from Jourdon's sermons, which says that spectacles had not been 20 years in use; and Salvati has observed that those sermons were composed between the years 1330 and 1336.

It is probable that the first hint of the construction and use of spectacles, was derived from the writings either of Alhazen, who lived in the 12th century, or of our own countryman Roger Bacon, who was born in 1214, and died in 1292, or 1294. The following remarkable passage occurs in Bacon's Opus Majus by Jebb, pa. 352. Si vero homo aspiciat literas et alias res minutas per medium crystalli, vel vitri, vel alterius perspicui suppositi literis, et sit portio minor sphæræ, cujus convexitas sit versus oculum et oculus sit in aëre, longe melius videbit literas, et apparebunt ei majores.—Et ideo hoc instrumentum est utile senibus et habentibus oculos debiles: nam literam quantumcunque parvam possunt videre in sufficienti magnitudine. Hence, and from other passages in his writings, much to the same purpose, Molyneux, Plott, and others, have attributed to him the invention of reading-glasses. Dr. Smith indeed, observing that there are some mistakes in his reasoning on this subject, has disputed his claim. See Molyneux's Dioptr. pa. 256. Smith's Optics, Rem. 86—89. Also the article BACON, R. in this dictionary.

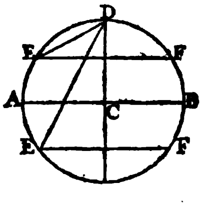
SPECULATIVE *Geometry*, *Mathematics*, *Music*, and *Philosophy*. See the SUBSTANTIVES.

SPECULUM, or *Mirror*, in Optics, any polished body, impervious to the rays of light: such as polished metals, and glasses lined with quicksilver, or any other opaque matter, popularly called Looking-glasses; or even the surface of mercury or of water, &c. For the several kinds and forms of specula, plane, concave, and convex, with their theory and phenomena, see MIRROR. And for their laws and effects, see REFLECTION and BURNING-*Glass*.

As for the specula of reflecting telescopes, it may here be observed, that the perfection of the metal of which they should be made, consists in its hardness, whiteness, and compactness; for upon these properties the reflective powers and durability of the specula depend. There are various compositions recommended for these specula, in Smith's Optics, book 3, ch. 2, sect. 787; also by Mr. Mudge in the Philos. Trans. vol. 67; and in various other places, as by Mr. Edwards, in the Naut. Alm. for 1787, whose metal is the whitest and best of any that I have seen.—For the method of grinding, see GRINDING.

Mr. Hearne's method of cleaning a tarnished speculum was this: get a little of the strongest soap ley from the soap-makers, and having laid the speculum on a table with its face upwards, put on as much of the ley as it will hold, and let it remain about an hour: then rub it softly with a silk or muslin, till the ley is all gone; then put on some spirit of wine, and rub it dry with another part of the silk or muslin. If the speculum will not perform well after this, it must be new polished. A few faint spots of tarnish may be rubbed off with spirit of wine only, without the ley. Smith's Optics, Rem. p. 107.

SPHERE, in Geometry, a solid body contained under one single uniform surface, every point of which is equally distant from a certain point in the middle called its centre. The sphere may be supposed to be generated by the revolution of a semicircle ABD about its diameter AB, which is also called the axis of the sphere, and the extreme points of the axis, A and B, the poles of the sphere; also the middle of the axis C is the centre, and half the axis, AC, the radius.



Properties of the SPHERE, are as follow.—1. A sphere may be considered as made up of an infinite number of pyramids, whose common altitude is equal to the radius of the sphere, their bases forming the surface of the sphere. Therefore the solid content of the sphere is equal to that of a pyramid whose altitude is the radius, and its base is equal to the surface of the sphere, that is, the solid content is equal to $\frac{1}{3}$ of the product of its radius and surface.

2. A sphere is equal to $\frac{2}{3}$ of its circumscribing cylinder, or of the cylinder of the same height and diameter, and therefore equal to the cube of the diameter multi-

plied by ·5236, or ⅔ of ·7854; or equal to double a cone of the same base and height. Hence also different spheres are to one another as the cubes of their diameters. And their surfaces as the squares of the same diameters.

3. The surface or superficies of any sphere, is equal to 4 times the area of its great circle, or of a circle of the same diameter as the sphere. Or,

4. The surface of the whole sphere is equal to the area of a circle whose radius is equal to the diameter of the sphere. And, in like manner, the curve surface of any segment EDF, whether greater or less than a hemisphere, is equal to a circle whose radius is the chord line DE, drawn from the vertex D of the segment to the circumference of its base, or the chord of half its arc.

5. The curve surface of any segment or zone of a sphere, is also equal to the curve surface of a cylinder of the same height with that portion, and of the same diameter with the sphere. Also the surface of the whole sphere, or of a hemisphere, is equal to the curve surface of its circumscribing cylinder. And the curve surfaces of their corresponding parts are equal, that are contained between any two planes parallel to the base. And consequently the surface of any segment or zone of a sphere, is as its height or altitude.

Most of these properties are contained in Archimedes's treatise on the sphere and cylinder. And many other rules for the surfaces and solidities of spheres, their segments, zones, frustums, &c, may be seen in my Mensuration, part 3, sect. 1, prob. 10, &c. Hence, if d denote the diameter or axis of a sphere, s its curve surface, c its solid content, and $a = \cdot 7854$ the area of a circle whose diameter is 1; then we shall, from the foregoing properties, have these following general values or equations, viz,

$$s = 4ad^2 = \frac{6c}{d} = 6\sqrt[3]{\tfrac{2}{3}ac^2}.$$

$$c = \tfrac{1}{6}ds = \tfrac{2}{3}ad^3 = \tfrac{1}{12}\sqrt{\frac{s^3}{a}}.$$

$$d = \frac{6c}{s} = \sqrt{\frac{s}{4a}} = \sqrt[3]{\frac{3c}{2a}}.$$

Doctrine of the SPHERE. See SPHERICS.

Projection of the SPHERE. See PROJECTION.

SPHERE *of Activity*, of any body, is that determinate space or extent all around it, to which, and no farther, the effluvia or the virtue of that body reaches, and in which it operates according to the nature of the body. See ACTIVITY.

SPHERE, in Astronomy, that concave orb or expanse which invests our globe, and in which the heavenly bodies, the sun, moon, stars, planets, and comets, appear to be fixed at an equal distance from the eye. This is also called the sphere of the world; and it is the subject of spherical astronomy.

This sphere, as it includes the fixed stars, whence it is sometimes called the sphere of the fixed stars, is immensely great. So much so, that the diameter of the earth's orbit is incomparably small in respect of it; and consequently the centre of the sphere is not sensibly changed by any alteration of the spectator's place in the several parts of the orbit: but still in all points of the earth's surface, and at all times, the inhabitants have the same appearance of the sphere; that is, the fixed stars seem to possess the same points in the surface of the sphere. For, our way of judging of the places &c of the stars, is to conceive right lines drawn from the eye, or from the centre of the earth, through the centres of the stars, and thence continued till they cut the sphere; and the points where these lines so meet, are the apparent places of those stars. The better to determine the places of the heavenly bodies in the sphere, several circles are conceived to be drawn in the surface of it, which are called circles of the sphere.

SPHERE, in Geography, &c, denotes a certain disposition of the circles on the surface of the earth, with regard to one another, which varies in the different parts of it. The circles originally conceived on the surface of the sphere of the world, are almost all transferred, by analogy, to the surface of the earth, where they are conceived to be drawn directly underneath those of the sphere, or in the same positions with them; so that, if the planes of those of the earth were continued to the sphere of the stars, they would coincide with the respective circles on it. Thus, we have an horizon, meridian, equator, &c, on the earth. And as the equinoctial, or equator, in the heavens, divides the sphere, into two equal parts, the one north and the other south, so does the equator on the surface of the earth divide its globe in the same manner. And as the meridians in the heavens pass through the poles of the equinoctial, so do those on the earth, &c. With regard then to the position of some of these circles in respect of others, we have a right, an oblique, and a parallel sphere.

A Right or Direct SPHERE, (fig. 4, plate 32), is that which has the poles of the world PS in its horizon, and the equator EQ in the zenith and nadir. The inhabitants of this sphere live exactly at the equator of the earth, or under the line. They have therefore no latitude, nor no elevation of the pole. They can see both poles of the world; all the stars rise, culminate, and set to them; and the sun always rises at right angles to their horizon, making their days and nights of equal length at all times of the year, because the horizon bisects the circle of the diurnal revolution.

An Oblique SPHERE, (fig. 5, plate 32), is that in which the equator EQ, as also the axis PS, cuts the horizon HO obliquely. In this sphere, one pole P is above the horizon, and the other below it; and therefore the inhabitants of it see always the former pole, but never the latter; the sun and stars &c all rise and set obliquely; and the days and nights are always varying, becoming alternately longer and shorter.

A Parallel SPHERE, (fig. 6, plate 32), is that which has the equator in or parallel to the horizon, as well as all the sun's parallels of declination. Hence, the poles are in the zenith and nadir; the sun and stars move always quite around parallel to the horizon, the inhabitants, if any, being just at the two poles, having 6 months continual day, and 6 months night, in each year; and the greatest height to which the sun rises to them, is 23° 28′, or equal to his greatest declination.

Armillary or Artificial SPHERE, is an astronomical instrument, representing the several circles of the sphere in their natural order; serving to give an idea of the office and position of each of them, and to resolve various problems relating to astronomy. It is thus called, as consisting of a number of rings of brass, or other matter, called by the Latins *armillæ*, from their resembling bracelets or rings for the arm. By this, it is distinguished from the globe, which, though it has all the circles of the sphere on its surface, yet is not cut into armillæ or rings, to represent the circles simply and alone; but exhibits also the intermediate spaces between the circles.

Armillary spheres are of different kinds, with regard to the position of the earth in them; whence they become distinguished into Ptolemaic and Copernican spheres: in the first of which, the earth is in the centre, and in the latter near the circumference, according to the position which that planet obtains in those systems.

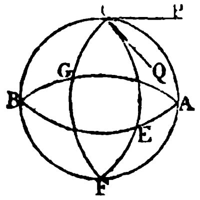
The Ptolemaic SPHERE, is that commonly in use, and is represented in fig. 6, plate 2, vol. 1, with the names of the several circles, lines, &c of the sphere inscribed upon it. In the middle, on the axis of the sphere, is a ball T, representing the earth, on the surface of which are the circles &c of the earth. The sphere is made to revolve about the said axis, which remains at rest; by which means the sun's diurnal and annual courses about the earth are represented according to the Ptolemaic hypothesis: and even by means of this, all problems relating to the phenomena of the sun and earth are resolved, as upon the celestial globe, and after the same manner; which see described under GLOBE.

Copernican SPHERE, fig. 7, plate 32, is very different from the Ptolemaic, both in its constitution and use; and is more intricate in both. Indeed the instrument is in the hands of so few people, and its use so inconsiderable, except what we have in the other more common instruments, particularly the globe and the Ptolemaic sphere, that any further account of it is unnecessary.

Dr. Long had an armillary sphere of glass, of a very large size, which is described and represented in his astronomy. And Mr. Ferguson constructed a similar one of brass, which is exhibited in his Lectures, p. 194 &c.

SPHERICAL, something relating to the sphere.

SPHERICAL *Angle*, is the angle formed on the surface of a sphere or globe by the circumferences of two great circles. This angle, formed by the circumferences, is equal to that formed by the planes of the same circles, or equal to the inclination of those two planes; or equal to the angle made by their tangents at the angular point. Thus, the inclination of the two planes CAF, CEF, forms the spherical angle ACE, equal to the tangential angle PCQ.



The measure of a spherical angle, ACE, is an arc of a great circle AE, described from the vertex C, as from a pole, and intercepted between the legs CA and CE. Hence, 1st, Since the inclination of the plane CEF to the plane CAF, is every where the same, the angles in the opposite intersections, C and F, are equal.—2d, Hence the measure of a spherical angle ACE, is an arc described at the interval of a quadrant CA or CE, from the vertex C between the legs CA, CE.—3d, If a circle of the sphere CEFG cut another AEBG, the adjacent angles AEC and BEC are together equal to two right angles; and the vertical angles AEC, BEF are equal to one another. Also all the angles formed at the same point, on the same side of a circle, are equal to 2 right angles, and all those quite around any point equal to 4 right angles.

SPHERICAL *Triangle*, is a triangle formed on the surface of a sphere, by the intersecting arcs of three great circles; as the triangle ACB.

Spherical triangles are either right-angled, oblique, equilateral, isosceles, or scalene, in the same manner as plane triangles. They are also said to be quadrantal, when they have one side a quadrant. Two sides or two angles are said to be of the same affection, when they are at the same time either both greater, or both less than a quadrant or a right angle or 90°; and of different affections, when one is greater and the other less than 90 degrees.

Properties of SPHERICAL *Triangles*.—1. Spherical triangles have many properties in common with plane ones: such as, That, in a triangle, equal sides subtend equal angles, and equal angles are subtended by equal sides: That the greater angles are subtended by the greater sides, and the less angles by the less sides.

2. In every spherical triangle, each side is less than a semicircle: any two sides taken together are greater than the third side: and all the three sides taken together are less than the whole circumference of a circle.

3. In every spherical triangle, any angle is less than 2 right angles; and the sum of all the three angles taken together, is greater than 2, but less than 6, right angles.

4. In an oblique spherical triangle, if the angles at the base be of the same affection, the perpendicular from the other angle falls within the triangle; but if they be of different affections, the perpendicular falls without the triangle.

Dr. Maskelyne's remarks on the properties of spherical triangles, are as follow: (See the Introd. to my Logs. pa. 171, 5th edition.)

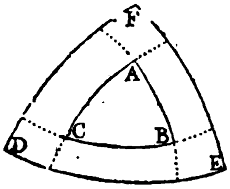
5. "A spherical triangle is equilateral, isoscelar, or scalene, according as it has its three angles all equal, or two of them equal, or all three unequal; and vice versa.

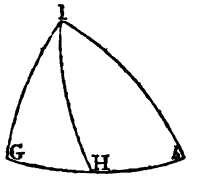
6. The greatest side is always opposite the greatest angle, and the smallest side opposite the smallest angle.

7. The sum of any two sides is greater, and their difference less, than the third side.

8. If the three angles are all acute, or all right, or all obtuse; the three sides will be, accordingly, all less than 90°, or equal to 90°, or greater than 90°; and vice versa.

9. If from the three angles A, B, C, of a triangle ABC, as poles, there be described, on the surface of the sphere, three arches of a great circle DE, DF, FE, forming by their intersections a new spherical triangle DEF; each side of the new triangle will be the supplement of the angle at its pole; and each angle of the same triangle, will be the supplement of the side opposite to it in the triangle ABC.





10. In any triangle GHI, or GhI, right angled at G, 1st, The angles at the hypothenuse are always of the same kind as their opposite sides; 2dly, The hypothenuse is less or greater than a quadrant, according as the sides including the right angle, are of the same or different kinds; that is to say, according as these same sides are either both acute, or both obtuse, or as one is acute and the other obtuse. And, vice versa, 1st, The sides including the right angle, are always of the same kind as their opposite angles; 2dly, The sides including the right angle will be

of the same or different kinds, according as the hypothenuse is less or greater than 90°; but one at least of them will be of 90°, if the hypothenuse is so."

Of the Area of a SPHERICAL *Triangle.* The mensuration of spherical triangles and polygons was first found out by Albert Girard, about the year 1600, and is given at large in his Invention Nouvelle en l'Algebre, pa. 50, &c; 4to, Amst. 1629. In any spherical triangle, the area or surface inclosed by its three sides upon the surface of the globe, will be found by this proportion:

As 8 right angles or 720°,
Is to the whole surface of the sphere;
Or, as 2 right angles or 180°,
To one great circle of the sphere;
So is the excess of the 3 angles above 2 right angles,
To the area of the spherical triangle.

Hence, if a denote ·7854,
$d =$ diam. of the globe, and
$s =$ sum of the 3 angles of the triangle;

then $add \times \frac{s-180}{180} =$ area of the spherical triangle.

Hence also, if r denote the radius of the sphere, and c its circumference; then the area of the triangle will be thus variously expressed; viz, area =

$$ad^2 \times \frac{s-180}{180} = cd \times \frac{s-180}{720} = cr \times \frac{s-180}{360};$$

or barely $= r \times (s-180°)$, in square degrees, when the radius r is estimated in degrees; for then the circumference c is $= 360°$.

Further, because the radius r, of any circle, when estimated in degrees, is, $= \frac{180}{3·14159 \text{ \&c.}} - = 57·2957795$, the last rule $r \times (s-180)$, for the area A of the spherical triangle, in square degrees, will be barely

$$A = 57·2957795s - 10313·24, \text{ or}$$
$$A = 57\tfrac{50}{169}s - 10313\tfrac{1}{4} \text{ very nearly.}$$

Hence may be found the sums of the three angles in any spherical triangle, having its area A known; for the last equation gives the sum

$$s = \frac{A}{r} + 180 = \frac{A}{57·29 \text{ \&c.}} + 180 = \frac{169A}{9683} + 180.$$

So that, for a triangle on the surface of the earth, whose three sides are known; if it be but small, as of a few miles extent, its area may be found from the known lengths of its sides, considering it as a plane triangle, which gives the value of the quantity A; and then the last rule above will give the value of s, the sum of the three angles; which will serve to prove whether those angles are nearly exact, that have been taken with a very nice instrument, as in large and extensive measurements on the surface of the earth. Hence A ÷ 57·29 &c is the spherical excess.

Resolution of SPHERICAL *Triangles.* See TRIANGLE, and TRIGONOMETRY.

SPHERICAL *Polygon*, is a figure of more than three sides, formed on the surface of a globe by the intersecting arcs of great circles.

The area of any spherical polygon will be found by the following proportion; viz,

As 8 right angles or 720°,
To the whole surface of the sphere;
Or, as 2 right angles or 180°,
To a great circle of the sphere;
So is the excess of all the angles above the product of 180 and 2 less than the number of angles,
To the area of the spherical polygon.

That is, putting $n =$ the number of angles,
$s =$ sum of all the angles,
$d =$ diam. of the sphere,
$a =$ ·78539 &c;

Then $A = ad^2 \times \frac{s-(n-2)180}{180} =$ the area of the spherical polygon.—Hence other rules might be found, similar to those for the area of the spherical triangle. Hence also, the sum s of all the angles of any spherical polygon, is always less than $180n$, but greater than $180(n-2)$, that is less than n times 2 right angles, but greater than $n-2$ times 2 right angles.

SPHERICAL *Astronomy*, that part of astronomy which considers the universe such as it appears to the eye. See ASTRONOMY. Under spherical astronomy are included all the phenomena and appearances of the heavens and heavenly bodies, such as we perceive them, without any inquiry into the reason, the theory, or truth of them. By which it is distinguished from theorical astronomy, which considers the real structure of the universe, and the causes of those phenomena. In spherical astronomy, the world is conceived to be a concave spherical surface, in whose centre is the earth, or rather the eye, about which the visible frame revolves, with stars and planets fixed in its circumference. And on this supposition all the other phenomena are determined. Theorical astronomy teaches us, from the laws of optics, &c, to correct this scheme and reduce the whole to a juster system.

SPHERICAL *Compasses.* See COMPASSES.

SPHERICAL *Excess.* See EXCESS.

SPHERICAL *Geometry*, the doctrine of the sphere; particularly of the circles described on its surface, with the method of projecting the same on a plane; and measuring their arches and angles when projected.

SPHERICAL *Numbers.* See CIRCULAR *Numbers.*

SPHERICAL *Trigonometry.* See TRIGONOMETRY.

SPHERICITY, the quality of a sphere; or that by which a thing becomes spherical or round.

SPHERICS, the doctrine of the sphere, particularly of the several circles described on its surface; with the method of projecting the same on a plane. See PROJECTION *of the Sphere.*

A circle of the sphere is that which is made by a plane cutting it. If the plane pass through the centre, it is a great circle: if not, it is a small circle. The pole of a circle, is a point on the surface of the sphere equidistant from every point of the circumference of the circle. Hence every circle has two poles, which are diametrically opposite to each other; and all circles that are parallel to each other have the same poles.

Properties of the Circles of the Sphere.—1. If a sphere be cut in any manner by a plane, the section will be a circle; and a great circle when the section passes through the centre, otherwise it is a small circle. Hence, all great circles are equal to each other: and the line of section of two great circles of the sphere, is a diameter of the sphere: therefore two great circles intersect each other in points diametrically opposite; and make equal angles at those points; and divide each other into two equal parts; also any great circle divides the whole sphere into two equal parts.

2. If a great circle be perpendicular to any other circle, it passes through its poles. And if a great circle pass through the pole of any other circle, it cuts it at right angles, and into two equal parts.

3. The distance between the poles of two circles, is equal to the angle of their inclination.

4. Two great circles passing through the poles of another great circle, cut all the parallels to this latter into similar arcs. Hence, an angle made by two great circles of the sphere, is equal to the angle of inclination of the planes of these great circles. And hence also the lengths of those parallels are to one another as the sines of their distances from their common pole, or as the cosines of their distances from their parallel great circle. Consequently, as radius is to the cosine of the latitude of any point on the globe, so is the length of a degree at the equator, to the length of a degree in that latitude.

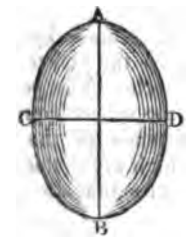
5. If a great circle pass through the poles of another; this latter also passes through the poles of the former; and the two circles cut each other perpendicularly.

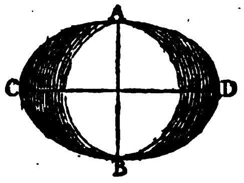
6. If two or more great circles intersect each other in the poles of another great circle; this latter will pass through the poles of all the former.

7. All circles of the sphere that are equally distant from the centre, are equal; and the further they are distant from the centre, the less they are.

8. The shortest distance on the surface of a sphere, between any two points on that surface, is the arc of a great circle passing through those points. And the smaller the circle is that passes through the same points, the longer is the arc of distance between them. Hence the proper measure, or distance, of two places on the surface of the globe, is an arc of a great circle intercepted between the same. See Theodosius and other writers on spherics.

SPHEROID, a solid body approaching to the figure of a sphere, though not exactly round, but having one of its diameters longer than the other. This solid is usually considered as generated by the rotation of an oval plane figure about one of its axis. If that be the longer or transverse axis, the solid so generated is called an oblong spheroid, and sometimes prolate, which resembles an egg, or a lemon; but if the oval revolve about its shorter axis, the solid will be an oblate spheroid, which resembles an orange, which is the figure of the earth, and the other planets.





The axis about which the oval revolves, is called the fixed axis, as AB; and the other CD is the revolving axis: whichever of them happens to be the longer.

When the revolving oval is a perfect ellipse, the solid generated by the revolution is properly called an ellipsoid, as distinguished from the spheroid, which is generated from the revolution of any oval whatever, whether it be an ellipse or not. But generally speaking, in the common acceptation of the word, the term spheroid is used for an ellipsoid; and therefore, in what follows, they are considered as one and the same thing.

Any section of a spheroid, by a plane, is an ellipse (except the sections perpendicular to the fixed axe, which are circles); andall parallel sections are similar ellipses, or have their transverse and conjugate axes in the same constant ratio; and the sections parallel to the fixed axe are similar to the ellipse from which the solid was generated. See my Tracts, vol. 2, pa. 134.

For the Surface of a Spheroid, whether it be oblong or oblate. Let f denote the fixed axe, r the revolving axe; and $a = \cdot 7854$, and $q = \frac{f^2 \backsim r^2}{f^2}$; then will the surface s be expressed by the following series, using the upper signs for the oblong spheroid, and the under signs for the oblate one; viz,

$$s = 4arf \times \left(1 \mp \frac{1}{2.3}q - \frac{1}{2.4.5}q^2 \mp \frac{3}{2.4.6.7}q^3 \,\&c\right);$$

where the signs of the terms, after the first, are all negative for the oblong spheroid, but alternately positive and negative for the oblate one. Hence, because the factor $4arf$ is equal to 4 times the area of the generating ellipse, it appears that the surface of the oblong spheroid is less than 4 times the generating ellipse, but the surface of the oblate spheroid is greater than 4 times the same: while the surface of the sphere falls in between the two, being just equal to 4 times its generating circle.

Huygens, in his Horolog. Oscillat. prop. 9, has given two elegant constructions for describing a circle equal to the superficies of an oblong and an oblate spheroid, which he says he discovered towards the latter end of the year 1657. As he gave no demonstrations of these, I have demonstrated them, and also rendered them more general, by extending and adapting them to the surface of any segment or zone of the spheroid. See my Mensuration, pa. 226 &c, 4th ed. where also are several other rules and constructions for the surfaces of spheroids, besides those of their segments, and frustums.

Of the Solidity of a Spheroid. Every spheroid, whether oblong or oblate, is, like the sphere, exactly equal to two-thirds of its circumscribing cylinder. So that, if f denote the fixed axe, r the revolving axe, and $a = \cdot 7854$; then $\frac{2}{3}afr^2$ denotes the solid content of either spheroid. Or, which comes to the same thing, if t denote the transverse, and c the conjugate axe of the generating ellipse;

then $\frac{2}{3}ac^2t$ is the content of the oblong spheroid,
and $\frac{2}{3}act^2$ is the content of the oblate spheroid.

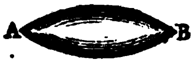
Consequently, the ratio of the former solid to the latter, is as c to t, or as the less axis to the greater.

Further, if about the two axes of an ellipse, there be generated two spheres and two spheroids, the four solids will be continued proportionals, and the common ratio will be that of the two axes of the ellipse; that is, as the greater sphere, or the sphere upon the greater axe, is to the oblate spheroid, so is the oblate spheroid to the oblong spheroid, and so is the oblong spheroid to the less sphere, and so is the transverse axis to the conjugate. See my Mensuration, pa. 248 &c, 4th ed. where may be seen many other rules for the solid contents of spheroids, and their various parts. See also Archimedes on spheroids and conoids.

Dr. Halley has demonstrated, that in a sphere, Mercator's nautical meridian line is a scale of logarithmic tangents of the half complements of the latitudes. But as it has been found that the shape of the earth is spheroidal, this figure will make some alteration in the numbers resulting from Dr. Halley's theorem. Maclaurin has therefore given a rule, by which the meridional parts to any spheroid may be found with the same exactness as in a sphere. There is also an ingenious tract by Mr. Murdoch on the same subject. See Philos. Trans. No. 219. Mr. Cotes has also demonstrated the same proposition, Harm. Mens. pa. 20, 21. See MERIDIONAL *Parts.*

Universal SPHEROID, a name given to the solid generated by the rotation of an ellipse about some other diameter, which is neither the transverse nor conjugate axis. This produces a figure resembling a heart. See my Mensuration, pa. 266, 4th ed.

SPINDLE, in Geometry, a solid body generated by the revolution of some curve line about its base or double ordinate AB; in opposition to a conoid, which is generated by the rotation of the curve about its axis or absciss, perpendicular to its ordinate. The spindle is denominated circular, elliptic, hyperbolic, or parabolic, &c, according to the figure of its generating curve. See my Mensur. in several places.

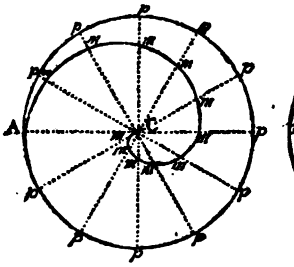


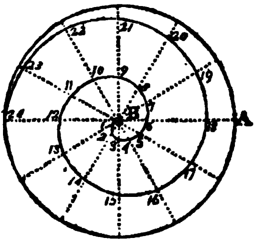
SPINDLE, in Mechanics, sometimes denotes the axis of a wheel, or roller, &c; and its ends are the pivots.

See also *Double* CONE.

SPIRAL, in Geometry, a curve line of the circular kind, which, in its progress, recedes always more and more from a point within, called its centre; or beginning its motion at a distance from the centre; it approaches nearer and nearer to that point. A spiral may be supposed to be thus generated.

Divide the circumference of a circle A*pp* &c into any number of equal parts, by a continual bisection at the points *pp* &c. Divide also the radius AC into the same number of equal parts, and make C*m*, C*m*, C*m*, &c, equal to 1, 2, 3, &c of these equal parts; then a line drawn, with a steady hand, through all the points *m*, *m*, *m*, &c, will trace out the spiral. This is more particularly called the first spiral, when it has made one complete revolution to the point A; and the space included between the spiral and the radius CA, is the spiral space. The first spiral may be continued to a second, by describing another circle with double the radius of the first; and the second may be continued to a third, by a third circle; and so on.





Hence it follows, that the parts of the circumference A*p*, are as the parts of the radii C*m*; or A*p* is to the whole circumference, as C*m* is to the whole radius. Consequently, if c denote the circumference, r the radius, $x = \text{C}m$, and $y = \text{A}p$; then there arises this proportion $r : c :: x : y$, which gives $ry = cx$ for the equation of this spiral; and which therefore it has in common with the quadratrix of Dinostrates, and that of Tschirnhausen: so that $r^n y^m = c^n x^m$ will serve for infinite spirals and quadratrices.

The first treatise on the spiral was by Archimedes, who thus gives the description of it, by a continued uniform motion. If a right line, as AB (*last fig. above*) having one end moveable about a fixed point at B, be uniformly turned round, so as the other end A may describe the circumference of a circle; and at the same time a point be conceived to move uniformly forward from B towards A, in the right line or radius AB, so that the point may describe that line, while the line generates the circle; then will the point, with the double motion, describe the curve B, 1, 2, 3, 4, 5, &c, of the same spiral as before.

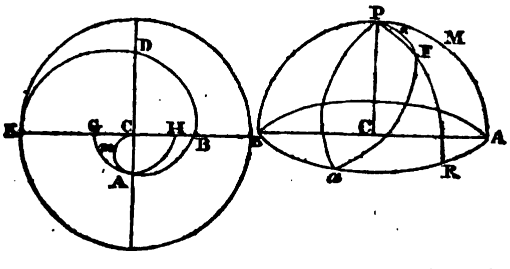
Again, if the point B be conceived to move only half as fast as the line AB revolves, so that it shall get but half way along BA, when that line shall have formed the circle; and if then you imagine a new revolution to be made of the line carrying the point, so that they shall end their motion at last together, there will be formed a double spiral line, as in the last figure. From the manner of this description may easily be drawn these corollaries:

1. That the lines B12, B11, B10, &c, making equal angles with the first and second spiral (as also B12, B10, B8), &c, are in arithmetical progression.

2. The lines B7, B10, &c, drawn any how to the first spiral, are to one another as the arcs of the circle intercepted between BA and those lines; because whatever parts of the circumference the point A describes, as suppose 7, the point B will also have run over 7 parts of the line AB.

3. Any lines drawn from B to the second spiral, as B18, B22, &c, are to each other as the aforesaid arcs, together with the whole circumference added on both sides: for at the same time that the point A runs over 12, or the whole circumference, or perhaps 7 parts more, shall the point B have run over 12, and 7 parts of the line AB, which is now supposed to be divided into 24 equal parts.

4. The first spiral space is equal to $\frac{1}{3}$ of the first or circumscribing circle. That is, the area CABDE of the spiral, is equal to $\frac{1}{3}$ part of the circle described with the radius CE. In like manner, the whole spiral area, generated by the ray drawn from the point C to the curve, when it makes two revolutions, is $\frac{2}{3}$ of the circle described with the radius 2CE.



And, generally, the whole area generated by the ray from the beginning of the motion, till after any number n of revolutions, is equal to $\frac{n}{3}$ of the circle whose radius is $n \times$ CE, that is equal to the 3d part of the space which is the same multiple of the circle described with the greatest ray, as the number of revolutions is of unity.

In like manner also, any sector or portion of the area of the spiral, terminated by the curve C*m*A and the right line CA, is equal to $\frac{1}{3}$ of the circular sector CAG terminated by the right lines CA and CG, this latter being the situation of the revolving ray when the point that describes the curve sets out from C. See Maclaurin's Flux. Introd. pa. 30, 31; also QUADRATURE of the Spiral of Archimedes; and Emerson's neat tract on spirals, added to his Conic Sections.

A brief synopsis of the first treatise on spirals, by Archimedes, is as follows:—Propositions 1 and 2 are of the nature of lemmas, and are employed to demonstrate the ratios of lines that are described by the equable motion of points.—Props. 3, 4, 5, 6, 7, 8, 9 demonstrate the possibility of taking, in a circle, chords, tangents, secants, &c, as well as certain parts of them, in a given ratio.—Prop. 10 shows that, in a series of quantities proceeding from 0, and equally exceeding one another, (viz, a continued arithmetical series,) the sum of the rectangles of the least term drawn into all the terms, together with as many times the square of the greatest term as is denoted by one more than the number of the terms, is equal to 3 times the sum of the squares of all the terms: that is,

$a(a + b + c + d + \&c, \text{ to } z) + (n + 1)z^2 = 3(a^2 + b^2 + c^2 + d^2 + \&c, \ldots . z^2)$;

where a, b, c, &c, are the terms of series whose common difference is a, the greatest term z, and number of terms n.—Prop. 11 is also employed about the squares of the terms of such a progression.

Having delivered these preparatory propositions, the author comes to the definitions of the helix or spiral, and of the several parts, lines, and circles attending it; in particular, his helix is the curve described by a point moving uniformly through a right line revolving equably about the end from which the point sets out.—The next 6 props. are employed about the proportions of the several parts and radii, &c, of the helix, till, in the 18th prop. it is shown that the circumference of the first circle, is equal to a line drawn from the centre perpendicular to the radius, and bounded by a tangent to the spiral at the extremity of the said radius.

Prop. 19 shows that such a perpendicular, as above, from the centre to the end of the 2d, 3d, 4th, &c spiral, and bounded by the tangent at the same point, is equal to double, triple, quadruple, &c, of the circumference of the circle described through the same tangent point.—Prop. 20, in like manner shows that such a perpendicular to a radius at any point, not at the end of the spiral, is as multiplex less by one of the circumference, together with as much more as is contained between that point and the beginning. So that here we have the rectification of the circular arc by means of the construction of the spiral.

Props. 21, 22, 23, are employed in showing that figures may be described in, and about spirals, that shall differ from them by less than any assignable quantity.—And then prop. 24 shows that the 1st spiral space is equal to $\frac{1}{3}$ of the 1st, or its circumscribing circle. And prop. 25 shows the ratio of the 2d, 3d, 4th, &c spiral space, to the 2d, 3d, 4th, &c circle.

Then the remaining three props. show the ratios of different parts of spirals to their corresponding sectors of the circles. After which is added a theorem showing the proportions of different sectors of a spiral, viz, that they are as the cubes of their respective radii. To which is subjoined a problem, to cut an angle, or a circular arc, in any ratio, by means of the spiral.

SPIRAL, *Logistic*, or *Logarithmic*. See LOGISTIC, and QUADRATURE.

SPIRAL *of Pappus*, a spiral formed on the surface of a sphere, by a motion similar to that by which the Spiral of Archimedes is described on a plane. This spiral is so called from its inventor Pappus. Collect. Mathem. lib. 4 prop. 30. Thus, if C be the centre of the sphere, ARBA a great circle, P its pole; and while the quadrant PMA revolves about the pole P with an uniform motion, if a point proceeding from P move with a given velocity along the quadrant, it will trace upon the spherical surface the spiral PZF*a*.

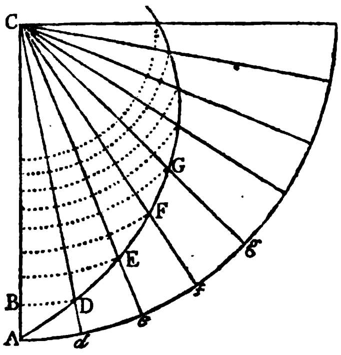
Now if we suppose the quadrant PMA to make a complete revolution in the same time that the point, which traces the spiral on the surface of the sphere, describes the quadrant, which is the case considered by Pappus; then the portion of the spherical surface terminated by the whole spiral, and the circle ARBA, and the quadrant PMA, will be equal to the square of the diameter AB. In any other case, the area PMA*a*FZP is to the square of that diameter AB, as the arc A*a* is to the whole circumference ARBA. And this area is always to the spherical triangle PA*a*, as a square is to its circumscribing circle, or as the diameter of a circle is to half its circumference, or as 2 is to 3·14159 &c. See Maclaurin's Fluxions, Introd. pa. 31—33.

The portion of the spherical surface, terminated by the quadrant PMA, with the arches AR, FR, and the spiral PZF, admits of a perfect quadrature, when the ratio of the arch A*a* to the whole circumference can be assigned. See Maclaurin, ibid. pa. 33.

Parabolic SPIRAL. See HELICOID.

Proportional SPIRAL, is generated by supposing the radius to revolve uniformly, and a point from the circumference to move towards the centre with a motion decreasing in geometrical progression. See LOGISTIC.

From the nature of a decreasing geometrical progression, it is easy to conceive that the radius CA may be continually divided; and though each successive division becomes shorter than the next preceding one, yet there must be an infinite number of divisions or terms before the last of them become of no finite magnitude. Whence it follows, that this spiral winds continually round the centre, without ever falling into it in any finite number of revolutions.



It is also evident that any proportional spiral cuts the intercepted radii at equal angles: for if the divisions A*d*, *de*, *ef*, *fg*, &c, of the circumference be very small, the several radii will be so close to one another, that the intercepted parts AD, DE, EF, FG, &c, of the spiral may be taken as right lines; and the triangles CAD, CDE, CEF, &c, will be similar, having equal angles at the point C, and the sides about those angles proportional; therefore the angles at A, D, E, F, &c, are equal, that is, the spiral cuts the radii at equal angles. Robertson's Elem. of Navig. book 2, pa. 87.

Proportional spirals are such spiral lines as the rhumb lines on the terraqueous globe; which, because they make equal angles with every meridian, must also make equal angles with the meridians in the stereographic projection on the plane of the equator, and therefore will be, as Dr. Halley observes, Proportional spirals about the polar point. Whence he demonstrates, that the meridian line is a scale of log. tangents of the half complements of the latitudes. See RHUMB, LOXODROMY, and MERIDIONAL *Parts*.

SPIRAL *Pump*. See *Archimedes's* SCREW.

Spiral, in Architecture and Sculpture, denotes a curve that ascends, winding about a cone, or spire, so that all the points of it continually approach the axis. By this it is distinguished from the helix, which winds in the same manner about a cylinder.

SPORADES, in Astronomy, a name by which the ancients distinguished such stars as were not included in any constellation. These the moderns more usually call unformed, or extraconstellary stars. Many of the sporades of the ancients have been since formed into new constellations: thus, of those between Ursa Major and Leo, Hevelius has formed a constellation named Leo Minor; and of those between Ursa Minor and Auriga, he also formed the Lynx; and of those under the tail of Ursa Minor, another called Canis Venaticus; &c.

SPOTS, in Astronomy, are dark places observed on the disks or faces of the sun, moon, and planets. The spots on the sun are seldom if ever visible, except through a telescope. I have indeed met with persons whose eyes were so strong that they have declared they could distinguish the solar spots; and it is mentioned in Josephus à Costa's Natural and Moral History of the West Indies, book 1, ch. 2, before the use of telescopes, that in Peru there are spots to be seen in the sun, which are not to be seen in Europe. See a memoir by Dr. Zach, in the Astronomical Ephemeris of the Acad. of Berlin for 1788, relating to the discoveries and unpublished papers of Thomas Harriot the celebrated algebraist. In that memoir it is shown, for the first time, that Harriot was also an excellent astronomer, both theoretical and practical; that he made innumerable observations with telescopes from the year 1610, and, among them, 199 observations of the solar spots, with their drawings, calculations, and the determinations of the sun's revolution round his axis. These spots were also discovered near about the same time by Galileo and Scheiner. See Joh. Fabricius Phrysius De Maculis in Sole observatis & apparente eorum cum sole conversione narratio, 1611; also Galileo's Istoria e Demonstrazioni intorne alle Machie Solare e loro accidenti, 1613.

Some distinguish the spots into Maculæ, or dark spots; and Faculæ, or bright spots. They are very changeable as to number, form, &c; and are sometimes in a multitude, and sometimes none at all. Some imagine they may become so numerous, as to hide the whole face of the sun, or at least the greater part of it; and to this they ascribe what Plutarch mentions, viz, that in the first year of the reign of Augustus, the sun's light was so faint and obscure, that one might look steadily at it with the naked eye. To which Kepler adds, that in 1547, the sun appeared reddish, as when viewed through a thick mist; and hence he conjectures that the spots in the sun are a kind of dark smoke, or clouds, floating on his surface.

Some again will have them stars, or planets, passing over the body of the sun: but others, with more probability, think they are opaque bodies, in the manner of crusts, formed like the scums on the surface of liquors.

Mr. Gascoigne, the inventor of the micrometer, and some others, fancied them to be planets revolving very near the sun. But his friend Mr. Crabtrie explained to him very good reasons against such a notion; stating, from his observations, that they are no stars; but mere "fading bodies, unconstant (in regard of their generation) and irregular excrescences arising out of, or proceeding from the sun's body." Abridg. Philos. Trans. vol. 5, pa. 630, &c.

Dr. Derham, from a variety of particulars, which he has recited, concerning the solar spots, and their congruity to what we observe in our own globe, infers, that they are caused by the eruption of some new volcano in the sun, which, pouring out at first a prodigious quantity of smoke and other opaque matter, causeth the spots: and as that fuliginous matter decays and spends itself, and the volcano at last becomes more torrid and flaming, so the spots decay and become umbræ, and at last faculæ: which faculæ he supposes to be no other than more flaming lighter parts than any other parts of the sun. Philos. Trans. vol. 23, pa. 1504, and vol. 27, pa. 270; or my Abridg. vol. 5, pa. 79 and 622.

Dr. Franklin (in his Exper. and Observ. pa. 266.) suggests a conjecture, that the parts of the sun's sulphur separated by fire, rise into the atmosphere, and there being freed from the immediate action of the fire, they collect into cloudy masses, and gradually becoming too heavy to be longer supported, they descend to the sun, and are burnt over again. Hence, he says, the spots appearing on his face, which are observed to diminish daily in size, their consuming edges being of particular brightness.

Dr. Alex. Wilson, of Glasgow, from observations and a train of reasoning, is of opinion that all spots, small as well as great, which consist of a dark nucleus and surrounding umbra, are excavations in the luminous matter of the sun. He has also endeavoured to give a general idea of the production, changes, and decay of the solar spots, considered as excavations in the body of the sun. But concerning the nature of that mighty agency, which occasions those amazing commotions in the luminous matter, or concerning the density, viscidity, and other qualities of the matter, and many other questions, he freely confesses that they far surpass his knowledge. Abridg. Ph. Tr. v. 13, pa. 366, and v. 15, pa. 482.

To this opinion of Dr. Wilson several persons exhibit objections; among others M. Lalande, in the Memoirs of the French Acad. 1776, contends on the contrary, that the spots are phenomena arising from dark bodies like rocks, which, by an alternate flux and reflux of the liquid igneous matter of the sun, sometimes raise their heads above the general surface. That part of the opaque rock, which at any time thus stands above, gives the appearance of the nucleus, while those parts which lie only a little under the igneous matter, appear to us as the surrounding umbra.

Some other respectable remarks on these phenomena are given by Mr. H. Marshall and the Rev. F. Wollaston. See the Abr. Ph. Tr. v. 13, pa. 529, 532.

Dr. Herschel's explanation of these phenomena is different from all the rest. The sun, he supposes, has an atmosphere resembling that of the earth; and this atmosphere consists of various elastic fluids, some of which exhibit a shining brilliancy, while others are merely transparent. Whenever the lucid fluid is removed, the body of the sun may be seen through those that are transparent, as a dark spot. Like as an observer, placed on the moon, sees the solid body of our earth only in those places where the transparent fluids of our atmosphere permit him. In others, the opaque vapours reflect the light of the sun, without permitting his view to penetrate to the surface of our globe. By changes in the atmosphere of Jupiter, Dr. H. accounts for the phenomena of his belts; and on the same principle he illustrates the various appearances of a spot, which he observed on the sun in 1779. This spot extended more than 50 thousand miles; and he says that, 'the idea of its being occasioned by a volcanic explosion, violently driving away a fiery fluid, which on its return would gradually fill up the vacancy, and thus restore the sun in that place to its former splendour, ought to be rejected on many accounts.' Dr. H. apprehends there are considerable inequalities in the surface of the sun; and that there may be elevations not less than 5 or 600 miles high. 'A very high country, or chain of mountains, may oftener become visible, by the removal of the obstructing fluid, than the lower regions, on account of its not being so deeply covered with it; and some of the solar mountains may be high enough occasionally to project above the shining elastic fluid, when, by some agitation, or other cause, it is not of the usual height. And this opinion is much strengthened by the return of some remarkable spots, which served Cassini to ascertain the period of the sun's rotation.—According to Dr. H's hypothesis, the black spots are the opaque ground or body of the sun; and the luminous part is an atmosphere, which, being interrupted or broken, gives us a transient glimpse of the sun itself. These spots appear, with a 7-feet reflector, much depressed below the surface of the luminous part. The faculæ, as Hevelius calls them, are elevated bright places, which appear at different times, and in different circumstances, of very various figures, which, with the lower opaque parts, gives the sun at times a kind of motled appearance. Philos. Trans. Abr. v. 17, pa. 478.

In short, Dr. Herschel, who has paid great attention to the spots of the sun, considers that luminary as similar to the planets, and not a flaming body.—It contains mountains, some of which he supposes to be 200 leagues in height. Its atmosphere is composed of different elastic fluids, some of which are luminous or phosphoric, and others only transparent. The former make the sun appear like a mass of light or fire; but the parts of that atmosphere which are only transparent, suffer his body to be seen. These are the spots. He believes the sun to be inhabited like the other planets.

Lalande, on the other hand, thinks that the sun is really a solid body, but that his surface and part of his mass are composed of an incandescent fluid. This fluid, by any movement, leaves uncovered sometimes a portion of the body of the sun or his mountains, and these are the spots. —— Wilson considers the spots of the sun as eruptions or volcanoes.

For another solution of these phenomena, see MACULÆ. Various other accounts and hypotheses of these spots may be seen in many of the other volumes of the Philos. Trans. In one of these, viz. vol. 57, pa. 398, Dr. Horsley attempts to determine the height of the sun's atmosphere from the height of the solar spots above his surface.

By means of the observations of these spots, has been determined the period of the sun's rotation about his axis, viz, by observing their periodical return.

The lunar spots are fixed: and astronomers reckon about 48 of them on the moon's face; to each of which they have given names. The 21st, called *Tycho*, is one of the most considerable.

Circular SPOTS, in Electricity. See CIRCULAR *Spots* and COLOURS.

Lucid SPOTS, *in the heavens*, are several little whitish spots, that appear magnified, and more luminous when seen through telescopes; and yet without any stars in them. One of these is in Andromeda's girdle, and was first observed in 1612, by Simon Marius: it has some whitish rays near its middle, is liable to several changes, and is sometimes invisible. Another is near the ecliptic, between the head and bow of Sagittarius; it is small, but very luminous. A third is in the back of the Centaur, which is too far south to be seen in Britain. A fourth, of a smaller size, is before Antinous's right foot, having a star in it, which makes it appear more bright. A fifth is in the constellation Hercules, between the stars ζ and η, which is visible to the naked eye, though it is but small, when the sky is clear and the moon absent. It is probable that with more powerful telescopes these lucid spots will be found to be congeries of very minute fixed stars. See NEBULOUS.

Planetary SPOTS, are those of the planets. Astronomers find that the planets are not without their spots. Jupiter, Mars, and Venus, when viewed through a telescope, show several very remarkable ones: and it is by the motion of these spots that the rotation of the planets about their axes is concluded, in the same manner as that of the sun is deduced from the apparent motion of his maculæ.

SPOUT, or *Water* SPOUT, an extraordinary meteor, or appearance, consisting of a moving column or pillar of water; called by the Latins typho, and sipho; and by the French trompe, from its shape, which resembles a speaking trumpet, the widest end uppermost. Its first appearance is in form of a deep cloud, the upper part of which is white, and the lower black. From the lower part of this cloud there hangs, or rather falls down, what is properly called the spout, representing a conical tube, largest at top. Under this tube is always a great agitation of the water of the sea, as in a jet d'eau. For some yards above the surface of the sea, the water stands like a column, or pillar; from the extremity of which it spreads, and goes off, as in a kind of smoke. Frequently the cone descends as low as the middle of this column, and continues for some time contiguous to it; though sometimes it only points to it at some distance, either in a perpendicular, or in an oblique line.

It frequently happens that it can scarcely be distinguished, whether the cone or the column appears the first, both rushing as it were to each other instantaneously. But sometimes the water boils up from the sea to a great height, without any appearance of a spout pointing to it, either perpendicularly or obliquely. Indeed, generally, the boiling or flying up of the water has the priority, this always preceding its being formed into a column. It more commonly happens that the cone does not appear hollow till towards the end, when the sea water is violently thrown up along its middle, as smoke up a chimney: soon after this, the spout or canal breaks and disappears; the boiling up of the water, and even the pillar, continuing to the last, and for some time afterwards; sometimes till the spout form itself again, and appear anew, which it will do several times in a quarter of an hour. See a description of several water-spouts by Mr. Gordon, and by Dr. Stuart, in Phil. Trans. Abr. vol. iv, pa. 564, and 647.

M. de la Pryme, from a near observation of two or three spouts in Yorkshire, described in the Philosophical Transactions, num. 281, or Abr. vol. iv, pa. 709, concludes, that the water spout is nothing but a gyration of clouds by contrary winds meeting in a point, or centre; and there, where the greatest condensation and gravitation is, falling down into a pipe, or great tube, somewhat like Archimedes's spiral screw; and, by its working and whirling motion, absorbs and raises the water, in the same manner as the spiral screw does; thus destroying even the largest ships &c.

In the month of June he observed the clouds very much agitated above, and driven together; upon which they became very black, and were hurried round; whence proceeded a most audible whirling noise like that usually heard in a mill. Soon after there issued a long tube, or spout, from the centre of the congregated clouds, in which he observed a spiral motion, like that of a screw, by which the water was raised up.

Again, August 15, 1687, the wind blowing at the same time out of the several quarters, created a great vortex and whirling among the clouds, the centre of which every now and then dropt down, in the shape of a long thin black pipe, in which he could distinctly behold a motion like that of a screw, continually drawing upwards, and screwing up, as it were, wherever it touched.

In its progress it moved slowly over a grove of trees, which bent under it like wands, in a circular motion. Proceeding farther, it tore off the thatch from a barn, bent a huge oak tree, broke one of its greatest branches, and threw it to a considerable distance. He adds, that whereas it is commonly said, the water works and rises in a column, before the tube comes to touch it, this is doubtless a mistake, owing to the fineness and transparency of the tubes, which do most certainly touch the surface of the sea, before any considerable motion can be raised in it; but which do not become opaque and visible, till after they have imbibed a considerable quantity of water.

The dissolution of water spouts he ascribes to the great quantity of water they have gathered: which, by its weight, impeding their motion, upon which their force, and even existence depends, they break, and let go their contents; which frequently proves fatal to whatever is found underneath.

A remarkable instance of this may be seen in the Philosophical Transactions (num. 363, or Abr. vol. vi. pa. 440) related by Dr. Richardson. A spout, in 1718, breaking on Emmotmoor, nigh Coln, in Lancashire, the country was immediately inundated; a brook, in a few minutes, rose six feet perpendicularly high; and the ground upon which the spout fell, which was 66 feet over, was torn up to the very rock, which was no less than 7 feet deep; and a deep gulf was made for above half a mile, the earth being raised in vast heaps on each side. See a description and figure of a water-spout, with an attempt to account for it in Franklin's Exp. and Obs. pa. 226, &c.

Signor Beccaria has taken pains to show that water-spouts have an electrical origin. To make this the more evident, he first describes the circumstances attending their appearance, which are the following.

They generally appear in calm weather. The sea seems to boil, and to send up a smoke under them, rising in a hill towards the spout. At the same time, persons who have been near them have heard a rumbling noise. The form of a water-spout is that of a speaking trumpet, the wider end being in the clouds, and the narrower end towards the sea.

The size is various, even in the same spout. The colour is sometimes inclining to white, and sometimes to black. Their position is sometimes perpendicular to the sea, sometimes oblique; and sometimes the spout itself forms a curve. Their continuance is very various, some disappearing as soon as formed, and some continuing a considerable time. One that he had heard of continued for an hour. But they often vanish, and presently appear again in the same place. The very same things that water-spouts are at sea, are some kinds of whirlwinds and hurricanes by land. They have been known to tear up trees, to throw down buildings, and make caverns in the earth; and in all these cases, to scatter earth, bricks, stones, timber, &c, to great distances in every direction. Great quantities of water have been left, or raised by them, so as to make a kind of deluge; and they have always been attended by a prodigious rumbling noise.

That these phenomena depend upon electricity cannot but appear very probable from the nature of several of them; but the conjecture is made more probable from the following additional circumstances. They generally appear in months peculiarly subject to thunder-storms, and are commonly preceded, accompanied, or followed by lightning, rain, or hail, the previous state of the air being similar. Whitish or yellowish flashes of light have sometimes been seen moving with prodigious swiftness about them. And lastly, the manner in which they terminate exactly resembles what might be expected from the prolongation of one of the uniform protuberances of electrified clouds, mentioned before, towards the sea; the water and the cloud mutually attracting each other: for they suddenly contract themselves, and disperse almost at once; the cloud rising, and the water of the sea under it falling to its level. But the most remarkable circumstance, and the most favourable to the supposition of their depending on electricity, is, that they have been dispersed by presenting to them sharp pointed knives or swords. This, at least, is the constant practice of mariners, in many parts of the world, where these water-spouts abound, and he was assured by several of them, that the method has often been undoubtedly effectual.

The analogy between the phenomena of water-spouts and electricity, he says, may be made visible by hanging a drop of water to a wire communicating with the prime conductor, and placing a vessel of water under it. In these circumstances, the drop assumes all the various appearances of a water-spout, both in its rise, form, and manner of disappearing. Nothing is wanting but the smoke, which may require a great force of electricity to become visible.

Mr. Wilcke also considers the water-spout as a kind of great electrical cone, raised between the cloud strongly electrified, and the sea or the earth, and he relates a very remarkable appearance which occurred to himself, and which strongly confirms his supposition. On the 20th of July 1758, at three o'clock in the afternoon, he observed a great quantity of dust rising from the ground, and covering a field, and part of the town in which he then was. There was no wind, and the dust moved gently towards the east, where appeared a great black cloud, which, when it was near its zenith, electrified his apparatus positively, and to as great a degree as ever he had observed it to be done by natural electricity. This cloud passed his zenith, and went gradually towards the west, the dust then following it, and continuing to rise higher and higher till it composed a thick pillar, in the form of a sugar-loaf, and at length seemed to be in contact with the cloud. At some distance from this, there came, in the same path, another great cloud, together with a long stream of smaller clouds, moving faster than the preceding. These clouds electrified his apparatus negatively, and when they came near the positive cloud, a flash of lightning was seen to dart through the cloud of dust, the positive cloud, the large negative cloud, and, as far as the eye could distinguish, the whole train of smaller negative clouds which followed it. Upon this, the negative clouds spread very much, and dissolved in rain, and the air was presently clear of all the dust. The whole appearance lasted not above half an hour. See Priestley's Electr. vol. i. pa. 438, &c.

This theory of water spouts has been farther confirmed by the account which Mr. Forster gives of one of them, in his Voyage round the World, vol. i, pa. 191, &c. On the coast of New Zealand he had an opportunity of seeing several, one of which he has particularly described. The water, he says, in a space of 50 or 60 fathoms, moved towards the centre, and there rising into vapour, by the force of the whirling motion, ascended in a spiral form towards the clouds. Directly over the whirlpool, or agitated spot in the sea, a cloud gradually tapered into a long slender tube, which seemed to descend to meet the rising spiral, and soon united with it into a straight column of a cylindrical form. The water was whirled upwards with the greatest violence in a spiral, and appeared to leave a hollow space in the centre; so that the water seemed to form a hollow tube, instead of a solid column; and that this was the case, was rendered still more probable by the colour, which was exactly like that of a hollow glass tube. After some time, this last column was incurvated, and broke like the others; and the appearance of a flash of lightning which attended its disjunction, as well as the hail-stones which fell at the time, seemed plainly to indicate, that water-spouts either owe their formation to the electric matter, or, at least, that they have some connexion with it.

In Pliny's time, the seamen used to pour vinegar into the sea, to assuage and lay the spout when it approached them: our modern seamen think to keep it off, by making a noise with filing and scratching violently on the deck; or by discharging great guns to disperse it.

See the figure of a water-spout, fig. 1, plate 33.

SPRING, in Natural History, a fountain or source of water, rising out of the ground.—The most general and probable opinion among philosophers, on the formation of springs, is, that they are formed from the rain-water which penetrates the earth till such time as it meets a clayey soil, or stratum; which proving a bottom sufficiently solid to sustain and stop its descent, it glides along it that way to which the earth declines, till, meeting with a place or aperture on the surface, through which it may escape, it forms a spring, and perhaps the head of a stream or brook. Now, that the rain is sufficient for this effect, appears from hence, that upon calculating the quantity of rain and snow which falls yearly on the tract of ground that is to furnish, for instance, the water of the Seine, it is found that this river does not take up above one-sixth part of it.

Springs commonly rise at the bottom of mountains: the reason is, that mountains collect the most waters, and give them the greatest descent the same way. And if we sometimes see springs on high grounds, and even on the tops of mountains, they must come from other remoter places, considerably higher, along beds of clay, or clayey ground, as in their natural channels. So that if there happen to be a valley between a mountain on whose top is a spring, and the mountain which is to furnish it with water, the spring must be considered as water conducted from a reservoir of a certain height, through a subterraneous channel, to make a jet of an almost equal height.

As to the manner in which this water is collected, so as to form reservoirs of the different kinds of springs, it seems to be this: the tops of mountains usually abound with cavities and subterraneous caverns, formed by nature to serve as reservoirs: and their pointed summits, which seem to pierce the clouds, stop those vapours which float in the atmosphere; which being thus condensed, they precipitate in water, and by their gravity and fluidity easily penetrate through beds of sand and the lighter earth, till they become stopped in their descent by the denser strata, such as beds of clay, stone, &c, where they form a bason or cavern, and working a passage horizontally, or a little declining, they issue out at the sides of the mountains. Many of these springs discharge water, which running down between the ridges of hills, unite their streams, and form rivulets or brooks, and many of these uniting again on the plain, become a river.

The perpetuity of some springs, always yielding the same quantity of water, as well when the least rain or vapour is afforded as when they are the greatest, furnish, in the opinion of some persons, considerable objections to the universality or sufficiency of the above theory. Dr. Derham mentions a spring in his own parish of Upminster, which he could never perceive by his eye was diminished in the greatest droughts, even when all the ponds in the country, as well as an adjoining brook, had been dry for several months together; nor ever to be increased in the most rainy seasons, excepting perhaps for a few hours, or at most for a day, from sudden and violent rains. Had this spring, he thought, derived its origin from rain or vapours, there would be found an increase and decrease of its water corresponding to those of its causes; as we actually find in such temporary springs, as have undoubtedly their rise from rain and vapour.

Some naturalists therefore have recourse to the sea, and derive the origin of springs immediately from thence. But how the sea-water should be raised up to the surface of the earth, and even to the tops of the mountains, is a difficulty, in the solution of which they cannot agree. Some fancy a kind of hollow subterranean rocks to receive the watery vapours raised from channels communicating with the sea, by means of an internal fire, and to act the part of alembics, in freeing them from their saline particles, as well as condensing and converting them into water. This kind of subterranean laboratory, serving for the distillation of sea-water, was the invention of Descartes: see his Princip. part 4, § 64. Others, as Lahire &c (Mem. de l'Acad. 1703) set aside the alembics, and think it enough that there be large subterranean reservoirs of water at the height of the sea, from whence the warmth of the bottom of the earth, &c, may raise vapours; which pervade not only the intervals and fissures of the strata, but the bodies of the strata themselves, and at length arrive near the surface; where, being condensed by the cold, they glide along on the first bed of clay they meet with, till they issue forth by some aperture in the ground. Lahire adds, that the salts of stone and minerals may contribute to the detaining and fixing the vapours, and converting them into water. Again, it is urged by others, that there is a still more natural and easy way of exhibiting the rise of the sea-water up into mountains &c, viz, by putting a little heap of sand, or ashes, or the like, into a bason of water; in which case the sand &c will represent the dry land, or an island; and the bason of water, the sea about it. Here, say they, the water in the bason will rise to the top of the heap, or nearly so, in the same manner, and from the same principle, as the waters of the sea, lakes, &c, rise in the hills. The principle of ascent in both is accordingly supposed to be the same with that of the ascent of liquids in capillary tubes, or between contiguous planes, or in a tube filled with ashes; all which are now generally accounted for by the doctrine of attraction.

Against this last theory, Perrault and others have urged several unanswerable objections. It supposes a variety of subterranean passages and caverns, communicating with the sea, and a complicated apparatus of alembics, with heat and cold, &c, of the existence of all which we have no sort of proof. Besides, the water that is supposed to ascend from the depths of the sea, or from subterranean canals proceeding from it, through the porous parts of the earth, as it rises in capillary tubes, ascends to no great height, and in much too small a quantity to furnish springs with water, as Perrault has sufficiently shown. And though the sand and earth through which the water ascends may acquire some saline particles from it, they are nevertheless incapable of rendering it so fresh as the water of our fountains is generally found to be. Not to add, that in process of time the saline particles of which the water is deprived either by subterranean distillation or filtration, must clog and obstruct those canals and alembics, by which it is supposed to be conveyed to our springs, and the sea must likewise gradually lose a considerable quantity of its salt.

Different kinds of SPRINGS. Springs are either such as run continually, called perennial; or such as run only for a time, and at certain seasons of the year, and therefore called temporary springs. Others again are called intermitting springs, because they flow and then stop, and flow and stop again; and reciprocating springs, whose waters rise and fall, or flow and ebb, by regular intervals.

In order to account for these differences in springs, let ABCDE (fig. 2, pl. 33) represent the declivity of a hill, along which the rain descends; passing through the fissures or channels BF, CG, DH, and LK, into the cavity or reservoir FGHKMI; from this cavity let there be a narrow drain or duct KE, which discharges the water at E. As the capacity of the reservoir is supposed to be large in proportion to that of the drain, it will furnish a constant supply of water to the spring at E. But if the reservoir FGHKMI be small, and the drain large, the water contained in the former, unless it is supplied by rain, will be wholly discharged by the latter, and the spring will become dry: and so it will continue, even though it rains, till the water has had time to penetrate through the earth, or to pass through the channels into the reservoir; and the time necessary for furnishing a new supply to the drain KE will depend on the size of the fissures, the nature of the soil, and the depth of the cavity with which it communicates. Hence it may happen, that the spring at E may remain dry for a considerable time, and even while it rains; but when the water has found its way into the cavity of the hill, the spring will begin to run. Springs of this kind, it is evident, may be dry in wet weather, especially if the duct KE be not exactly level with the bottom of the cavity in the hill, and discharge water in dry weather; and the intermissions of the spring may continue several days. But if we suppose XOP to represent another cavity, supplied with water by the channel NO, as well as by fissures and clefts in the rock, and by the draining of the adjacent earth; and another channel STV, communicating with the bottom of it at S, ascending to T, and terminating on the surface at V, in the form of a siphon; this disposition of the internal cavities of the earth, which we may reasonably suppose that nature has formed in a variety of places, will serve to explain the principle of reciprocating springs; for it is plain, that the cavity XOP must be supplied with water to the height QPT, before it can pass over the bend of the channel at T, and then it will flow through the longer leg of the siphon TV, and be discharged at the end V, which is lower than S. Now if the channel STV be considerably larger than NO, by which the water is principally conveyed into the reservoir XOP, the reservoir will be emptied of its water by the siphon; and when the water descends below its orifice S, the air will drive the remaining water out of the channel STV, and the spring will cease to flow. But in time the water in the reservoir will again rise to the height QPT, and be discharged at V as before. It is easy to conceive, that the diameters of the channels NO and STV may be so proportioned to one another, as to afford an intermission and renewal of the spring V at regular intervals. Thus, if NO communicates with a well supplied by the tide, during the time of flow, the quantity of water conveyed by it into the cavity XOP may be sufficient to fill it up to QPT; and STV may be of such a size as to empty it, during the time of ebb. It is easy to apply this reasoning to more complicated cases, where several reservoirs and siphons communicating with each other, may supply springs with circumstances of greater variety. See Musschenbroek's Introd. ad Phil. Nat. tom. ii. pa. 1010. Desagu. Exp. Phil. vol. ii, pa. 173, &c. And Nicholson's Philos. Journal, v. 35, p. 178, &c.

We shall here observe, that Desaguliers calls those reciprocating springs which flow constantly, but with a stream subject to increase and decrease; and thus he distinguishes them from intermitting springs, which flow or stop alternately. It is said that in the diocese of Paderborn, in Westphalia, there is a spring which disappears after twenty-four hours, and always returns at the end of six hours with a great noise, and with so much force, as to turn three mills, not far from its source. It is called the Bolderborn, or boisterous spring. Phil. Trans. No. 7. There are many springs of an extraordinary nature in our own country, which it is needless to recite, as they are explicable by the general principles already illustrated.

SPRING, *Ver*, in Astronomy and Cosmography, denotes one of the seasons of the year; commencing, in the northern parts of the earth, on the day the sun enters the first degree of Aries, which is about the 21st day of March, and ending when the sun enters Cancer, at the summer solstice, about the 21st of June; spring ending when the summer begins. Or, more strictly and generally, for any part of the earth, or on either side of the equator, the spring season begins when the meridian altitude of the sun, being on the increase, is at a medium between the greatest and least; and ends when the meridian altitude is at the greatest. Or the spring is the season, or time, from the moment of the sun's crossing the equator till he rise to the greatest height above it.

Elater SPRING, in Physics, denotes a natural faculty, or endeavour, of certain bodies, to return to their first state, after having been violently put out of the same by compressing, or bending them, or the like. This faculty is usually called by philosophers, elastic force, or elasticity.

SPRING, in Mechanics, is used to signify a body of any shape, perfectly elastic, or nearly so.

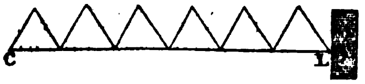
Elasticity of a SPRING. See ELASTICITY.

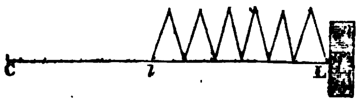
Length of a SPRING, may, from its etymology, signify the length of any elastic body; but it is particularly used by Dr. Jurin to signify the greatest length to which a spring can be forced inwards, or drawn outwards, without prejudice to its elasticity. He observes, this would be the whole length, were the spring considered as a mathematical line; but in a material spring, it is the difference between the whole length, when the spring is in its natural situation, or the situation it will rest in when not disturbed by any external force, and the length or space it takes up when wholly compressed and closed, or when drawn out.

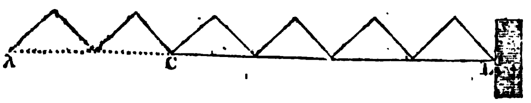
Strength or Force of a SPRING, is used for the force or weight which, when the spring is wholly compressed or closed, will just prevent it from unbending itself. Also the force of a spring partly bent or closed, is the force or weight which is just sufficient to keep the spring in that state, by preventing it from unbending itself any farther.

The theory of springs is founded on this principle, ut intensio, sic vis: that is, the intensity is as the compressing force; or if a spring be any way forced or put out of its natural situation, its resistance is proportional to the space by which it is removed from that situation. This principle has been verified by the experiments of Dr. Hooke, and since him by those of others, particularly by the accurate hand of Mr. George Graham. Lectures De Potentia Restitutiva, 1678.

For elucidating this principle, on which the whole theory of springs depends, suppose a spring CL, resting at L against any immoveable support, but otherwise lying in its natural situation, and at full liberty. If this spring be pressed inwards by any force p, or from C towards L, through the space of one inch, and can be there detained by that force p, the resistance of the spring, and the force p, exactly counterbalancing each other; then will the double force $2p$ bend the spring through the space of 2 inches, and the triple force $3p$ through 3 inches, and the quadruple force $4p$ through 4 inches, and so on. The space CL through which the spring is bent, or by which its end C is removed from its natural situation, being always proportional to the force which will bend it so far, and will just detain it when so bent. On the other hand, if the end C be drawn outwards to any place λ, and be there detained from returning back by any force p, the space Cλ, through which it is so drawn outwards, will be also proportional to the force p, which is just able to retain it in that situation.

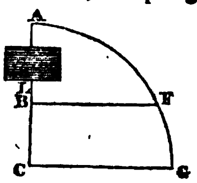






It may here be observed, that the spring of the air, or its elastic force, is a power of a different nature, and governed by different laws, from that of a palpable rigid spring. For supposing the line LC to represent a cylindrical volume of air, which by compression is reduced to L*l*, or by dilation is extended to Lλ, its elastic force will be reciprocally as L*l* or Lλ; whereas the force or resistance of a spring is directly as C*l* or Cλ.

This principle being premised, Dr. Jurin lays down a general theorem concerning the action of a body striking on one end of a spring, while the other end is supposed to rest against an immoveable support. Thus, if a spring of the strength P, and the length CL, lying at full liberty upon an horizontal plane, rest with one end L against an immoveable support; and a body of the weight M, moving with the velocity V, in the direction of the axis of the spring, strike directly on the other end C, and so force the spring inwards, or bend it through any space CB; and if a mean proportional CG be taken between $\frac{M}{P} \times$ CL and $2a$, where a denotes the height to which a body would ascend in vacuo with the velocity V; and further, if upon the radius R = CG be described the quadrant of a circle GFA: then,



1. When the spring is bent through the right sine CB of any arc GF, the velocity v of the body M is to the original velocity V, as the cosine BF is to the radius CG; that is $v : V :: BF : CG$, or $v = \frac{BF}{R} \times V$.

2. The time t of bending the spring through the same sine CB, is to T, the time of a heavy body's ascending in vacuo with the velocity V, as the corresponding arc is to $2a$; that is $t : T :: GF : 2a$, or $t = \frac{GF}{2a} \times T$.

The doctor gives a demonstration of this theorem, and deduces a great many curious corollaries from it; which he divides into three classes. The first contains such corollaries as are of more particular use when the spring is wholly closed before the motion of the body ceases: the second comprehends those relating to the case, when the motion of the body ceases before the spring is wholly closed: and the third when the motion of the body ceases at the instant that the spring is wholly closed.

3. We shall here mention some of the last class, as being the most simple; having first premised, that P = the strength of the spring, L = its length, V = the initial velocity of the body closing the spring, M = its mass, t = time spent by the body in closing the spring, A = height from which a heavy body will fall in vacuo in a second of time, a = the height to which a body would ascend in vacuo with the velocity V, C = the velocity gained by the fall, m = the circumference of a circle, whose diameter is 1. Then, the motion of the striking body ceasing when the spring is wholly closed, it will be, 1st, 2d, and 3d,

$V = C\sqrt{\frac{PL}{2MA}}$; $Vt = \frac{mCa}{4A} \times 1''$; $MV = C\sqrt{\frac{PLM}{2A}}$; the first momentum.

4. If a quantity of motion MV bend a spring through its whole length, and be destroyed by it; no other quan-

tity of motion equal to the former, as $nM \times \frac{V}{n}$, will close the same spring, and be wholly destroyed by it.—5. But a quantity of motion, greater or less than MV, in any given ratio, may close the same spring, and be wholly destroyed in closing it; and the time spent in closing the spring will be respectively greater or less, in the same given ratio.—6. The initial vis viva, or MV^2 is $= \frac{c^2PL}{2A}$; and $2aM = PL$; also the initial vis viva is as the rectangle under the length and strength of the spring, that is, MV^2 is as PL.—7. If the vis viva MV^2 bend a spring through its whole length, and be destroyed in closing it; any other vis viva, equal to the former, as $n^2M \times \frac{V^2}{n^2}$, will close the same spring, and be destroyed by it.—8. But the time of closing the spring by vis viva $n^2M \times \frac{V^2}{n^2}$, will be to the time of closing it by the vis viva MV^2, as n to 1.—9. If the vis viva MV^2 be wholly consumed in closing a spring, of the length L, and strength P; then the vis viva n^2MV^2 will be sufficient to close, 1st, Either a spring of the length L and strength n^2P. 2d, Or a spring of the length nL and strength nP. 3d, Or of the length n^2L and strength P. 4th, Or, if n be a whole number, the number n^2 of springs, each of the length L and strength P.—It may be added, that it appears from hence, that the number of similar and equal springs a given body in motion can wholly close, is always proportional to the squares of the velocity of that body. And it is from this principle that the chief argument, to prove that the force of a body in motion is as the square of its velocity, is deduced. See FORCE.

The theorem given above, and its corollaries, will equally hold good, if the spring be supposed to have been at first bent through a certain space, and by unbending itself to press upon a body at rest, and thus to drive that body before it, during the time of its expansion: only V, instead of being the initial velocity with which the body struck the spring, will now be the final velocity with which the body parts from the spring when totally expanded.

It may also be observed, that the theorem, &c, will equally hold good, if the spring, instead of being pressed inward, be drawn outward by the action of the body. The like may be said, if the spring be supposed to have been already drawn outward to a certain length, and in restoring itself draw the body after it. And lastly, the theorem extends to a spring of any form whatever, provided L be the greatest length it can be extended to from its natural situation, and P the force which will confine it to that length. See Philos. Trans. num. 472, sect. 10, or vol. 43, art. 10.

SPRING is more particularly used, in the Mechanic Arts, for a piece of tempered steel, put into various machines to give them motion, by the endeavour it makes to unbend itself. In watches, it is a fine piece of well-beaten steel, coiled up in a cylindrical case, or frame; which by stretching itself forth, gives motion to the wheels, &c.

SPRING *Arbor*, in a Watch, is that part in the middle of the spring-box, about which the spring is wound or turned, and to which it is hooked at one end.

SPRING *Box*, in a Watch, is the cylindrical case, or frame, containing within it the spring of the watch.

SPRING *Compasses*. See COMPASSES.

SPRING *of the Air*, or its elastic force. See AIR, and ELASTICITY.

SPRING-*Tides*, are the higher tides, about the times of the new and full moon. See TIDE.

SPRINGY, or *Elastic Body*. See ELASTIC *Body*.

SQUARE, in Geometry, a quadrilateral figure, whose angles are right, and sides equal. Or it is an equilateral rectangular parallelogram. A square, and indeed any other parallelogram, is bisected by its diagonal; but the side of a square is incommensurable with its diagonal, being in the ratio of 1 to $\sqrt{2}$.

To find the Area of a SQUARE. Multiply the side by itself, and the product is the area. So, if the side be 10, the area is 100; and if the side be 12, the area is 144.

SQUARE *Foot*, is a square each side of which is equal to a foot, or 12 inches; and the area, or square foot is equal to 144 square inches.

Geometrical SQUARE, a compartment often added on the face of a quadrant. See *Line* of SHADOWS, and QUADRANT.

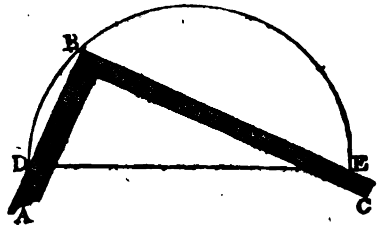
Gunner's SQUARE. See QUADRANT.

Magic SQUARE. See MAGIC *Square*.

SQUARE *Measures*, the squares of the lineal measures; as in the following table of square measures:

Squa. Inches.	Sq. Feet.	Sq. Yards.	Sq Poles.	S.Chs.	Acres.	Sq.Miles.
144	1					
1296	9	1				
39204	272¼	30¼	1			
627264	4356	484	16	1		
6272640	43560	4840	160	10	1	
4014489600	27878400	3097600	102400	6400	640	1

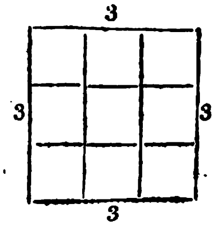
Normal SQUARE, is an instrument, made of wood or metal, serving to describe and measure right angles; such is ABC. It consists of two rulers or branches fastened together perpendicularly.—When the two legs are moveable on a joint, it is called a bevel.—To examine whether the square be exact or not. Describe a semicircle DBE, with any radius at pleasure; in the circumference of which apply the angle of the square to any point as B, and the edge of one leg to one end of the diameter as D, then if the other leg pass just by the other extremity at E, the square is true; otherwise not.



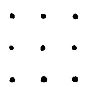
SQUARE *Number*, is the product arising from a number multiplied by itself. Thus, 4 is the square of 2, and 16 the square of 4.

The series of square integers, is 1, 4, 9, 16, 25, 36, &c;
which are the squares of - 1, 2, 3, 4, 5, 6, &c.
Or the square fractions - - $\frac{1}{4}, \frac{4}{9}, \frac{9}{16}, \frac{16}{25}, \frac{25}{36}, \frac{36}{49}$, &c,
which are the squares of - $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$, &c.

A square number is so called, either because it denotes the area of a square, whose side is expressed by the root of the square number; as in the annexed square, which consists of 9 little squares, the side being equal to 3; or else, which is much the same thing, because the points in the number may be ranged



in the form of a square, by making the root, or factor, the side of it, thus,



Some properties of squares are as follow:

1. Of the

Natural series of squares,	1^2, 2^2, 3^2, 4^2, &c,
which are equal to	1, 4, 9, 16, &c;

The mean proportional mn between any two of these squares m^2 and n^2, is equal to the less square plus its root multiplied by the difference of the roots; or also equal to the greater square minus its root multiplied by the said difference of the roots. That is,

$$mn = m^2 + dm = n^2 - dn;$$

where $d = n - m$ is the difference of their roots.

2. An arithmetical mean between any two squares m^2 and n^2, exceeds their geometrical mean, by half the square of the difference of their roots.

That is, $\frac{1}{2}m^2 + \frac{1}{2}n^2 = mn + \frac{1}{2}d^2$.

3. Of three equidistant squares in the series, the geometrical mean between the extremes, is less than the middle square by the square of their common distance in the series, or of the common difference of their roots.

That is, $mp = n^2 - d^2$;

where m, n, p, are in arithmetical progression, the common difference being d.

4. The difference between the two adjacent squares m^2, and n^2, is $n^2 - m^2 = 2m + 1$;

in like manner, $p^2 - n^2 = 2n + 1$, the difference between the next two adjacent squares n^2 and p^2; and so on, for the next following squares. Hence the difference of these differences, or the second difference of the squares, is $2n - 2m = 2 \times (n-m) = 2$ only, because $n - m = 1$; that is, the second differences of the squares are each the same constant number 2: therefore the first differences will be found by the continual addition of the number 2; and then the squares themselves will be found by the continual addition of the first differences; and thus the whole series of squares is constructed by addition only, as here below:

2d Diff.		2	2	2	2	2	2	&c.
1st Diff.	1	3	5	7	9	11	13	&c.
Squares	1	4	9	16	25	36	49	&c.

And this method of constructing the table of square numbers was first noticed by Peletarius, in his Algebra.

5. Another curious property, also noted by the same author, is, that the sum of any number of the cubes of the natural series 1, 2, 3, 4, &c, taken from the beginning, always makes a square number; and that the series of squares, so formed, have for their roots the numbers

1, 3, 6, 10, 15, 21, &c,

the diffs. of which are 1, 2, 3, 4, 5, 6, &c,

viz, $1^3 = 1^2$,

$1^3 + 2^3 = 3^2$,

$1^3 + 2^3 + 3^3 = 6^2$,

$1^3 + 2^3 + 3^3 + 4^3 = 10^2$; and in general

$1^3 + 2^3 + 3^3 + n^3 = (1+2+3+n)^2 = \frac{1}{2}n(n+1)$;

where n is the number of the terms or cubes.

6. Every odd square number, when divided by 8, leaves a remainder 1. Or every odd square is of the form $8n+1$.

7. Every even square is of the form $4n$. Therefore no number of the form $4n+2$, or $4n+3$, is a square number.

8. No number of the form $2t^2 \pm 3n^2$, can be a square. No number of the form $2t^2 \sim 5n^2$ can be a square. The following table shows many of the impossible forms for square number, arranged according to every modulus from 2 to 11; that is, no number contained in any one of these forms can be a square.

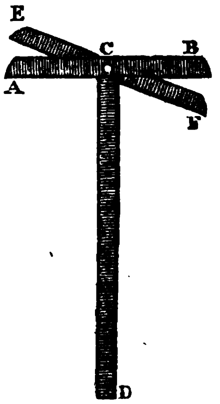
TABLE

Of the impossible Forms for square numbers.

Modulus 3.	Modulus 4.	Modulus 5.
$2t^2 \pm 3qn^2$	$2t^2 \backsim 4qn^2$	$2t^2 \backsim 5qn^2$
$3t^2 \pm 3qn^2$	$3t^2 \pm 4qn^2$	$3t^2 \backsim 5qn^2$
$5t^2 \pm 3qn^2$	$6t^2 \backsim 4qn^2$	$7t^2 \backsim 5qn^2$
$8t^2 \pm 3qn^2$	$7t^2 \pm 4qn^2$	$8t^2 \backsim 5qn^2$
$11t^2 \pm 3qn^2$	$10t^2 \backsim 4qn^2$	$12t^2 \backsim 5qn^2$
$14t^2 \pm 3qn^2$	$11t^2 \pm 4qn^2$	$13t^2 \backsim 5qn^2$
General Forms.	General Forms.	General Form.
$(3p+2)t^2 \pm 6qn^2$ $3pt^2 \pm 3qn^2$	$(4p \pm 2)t^2 \backsim 4qn^2$ $(4p+3)t^2 \pm 4qn^2$	$(5p \pm 2)t^2 \backsim 5qn^2$
Modulus 6.	**Modulus 7.**	**Modulus 8.**
$2t^2 \pm 6qn^2$	$3t^2 \pm 7qn^2$	$2t^2 \pm 8qn^2$
$3t^2 \pm 6qn^2$	$5t^2 \pm 7qn^2$	$3t^2 \pm 8qn^2$
$5t^2 \pm 6qn^2$	$6t^2 \pm 7qn^2$	$5t^2 \pm 8qn^2$
$8t^2 \pm 6qn^2$	$10t^2 \pm 7qn^2$	$6t^2 \pm 8qn^2$
$11t^2 \pm 6qn^2$	$12t^2 \pm 7qn^2$	$10t^2 \pm 8qn^2$
$14t^2 \pm 6qn^2$	$13t^2 \pm 7qn^2$	$11t^2 \pm 8qn^2$
General Forms.	General Forms.	General Forms.
$(3p+2)t^2 \pm 6qn^2$ $3pt^2 + 6qn^2$	$(7p+3)t^2 \pm 7qn^2$ $(7p+5)t^2 \pm 7qn^2$ $(7p+6)t^2 \pm 7qn^2$	$(8p \pm 2)t^2 \pm 8qn^2$ $(8p \pm 3)t^2 \pm 8qn^2$
Modulus 9.	**Modulus 10.**	**Modulus 11.**
$2t^2 \pm 9qn^2$	$2t^2 \backsim 10qn^2$	$2t^2 \pm 11qn^2$
$3t^2 \backsim 9qn^2$	$3t^2 \backsim 10qn^2$	$6t^2 \pm 11qn^2$
$5t^2 \pm 9qn^2$	$7t^2 \backsim 10qn^2$	$7t^2 \pm 11qn^2$
$6t^2 \backsim 9qn^2$	$8t^2 \backsim 10qn^2$	$8t^2 \pm 11qn^2$
$8t^2 \pm 9qn^2$	$12t^2 \backsim 10qn^2$	$10t^2 \pm 11qn^2$
$11t^2 \pm 9qn^2$	$13t^2 \backsim 10qn^2$	$13t^2 \pm 11qn^2$
General Forms.	General Forms.	General Forms.
$(9p+2)t^2 \pm 9qn^2$ $(9p \pm 3)t^2 \backsim 9qn^2$ &c	$(5p \pm 2)t^2 \backsim 10qn^2$	$(11p+2)t^2 \pm 11qn^2$ $(11p+6)t^2 \pm 11qn^2$ &c

In this table it is only necessary to remark that q must always be taken prime to the modulus.

SQUARE *Root*, a number considered as the root of a second power or square number: or a number which multiplied by itself, produces the given number. See EXTRACTION *of Roots*, and also the article ROOT, where tables of squares and roots are inserted.



T. SQUARE, or *Tee* SQUARE, an instrument used in drawing, so called from its resemblance to the capital letter T. This instrument consists of two straight rulers AB and CD, fixed at right angles to each other. To which is sometimes added a third EF, moveable about the pin C, to set it to make any angle with CD.—It is very useful for drawing parallel and perpendicular lines, on the face of a smooth drawing-board.

SQUARED-*square*, SQUARED-*cube*, &c. See POWER.

SQUARING. See Quadrature.

Squaring *the Circle*, is the making or finding a square whose area shall be equal to that of a given circle. The best mathematicians have not yet been able to resolve this problem accurately, and perhaps never will. But they can easily come to any proposed degree of approximation whatever; for instance, so near as not to err so much in the area, as a grain of sand would cover, in a circle whose diameter is equal to that of the orbit of Saturn. The following proportion is near enough the truth for any real use, viz, as 1 is to ·88622692, so is the diameter of any circle, to the side of the square of an equal area. Therefore, if the diameter of the circle be called d, and the side of the equal square s;

$$\text{then is } s = \cdot 88622692d = \tfrac{39}{44}d \text{ nearly,}$$

$$\text{and } d = \frac{s}{\cdot 88622692} = \tfrac{44}{39}s \text{ nearly.}$$

See Circle, Diameter, and Quadrature.

STADIUM, an ancient Greek long measure, said to contain 125 geometrical paces, or 625 Roman feet; corresponding to our furlong. Eight stadia make a geometrical or Roman mile; and 20, according to Dacier, a French league: but according to others, 800 stadia make 41⅔ leagues.—Guilletiere observes, that the stadium was only 600 Athenian feet, which amount to 625 Roman, or 566 French, or 604 English feet: so that the stadium should have been only 113 geometrical paces. It must be observed however, that the stadium was different at different times and places.

Thus, according to the measures of Hipparchus, 769 stades make a degree on a great circle of the earth, or about 11⅓ to the English mile. By the result of Ptolemy's measures 716½ stades make a degree, or 10⅓ an English mile. According to Vernon, Stuart, and Chaudler, all of whom measured it, the Panathenæan stade was rather more than 600 Greek feet in length. Now the Greek foot is to the English, as 107·29 to 100; therefore the length of that stade was about 604 English feet, or 9⅓ nearly Panathæan stades were equal to an English mile.—For an interesting disquisition on the different kinds of stades, see the Quarterly Review, vol. 5, p. 278 &c.

Eratosthenes, in his measurement of the earth, makes the circumference of it equal to 90,000,000 stadia, or one degree equal to 250,000 stadia. Now if in this valuation we make use of the Egyptian stadium, 60 of which make 3024 toises, we shall have for the length of the degree about 35,000 toises, which is too little by the modern measurement, its true length being about 57050 toises.

The Olympic stadium, it is said, was about 94 feet 3 inches French measure; and supposing this to have been that employed by Eratosthenes, we should have for the length of a terrestrial degree 65625 toises, which is much too great.

It follows therefore, either that we are unacquainted with the true length of the stadium, at least that employed by Eratosthenes, or that this celebrated ancient astronomer was much deceived in his measurement: both of those are probable; and perhaps some of the other stadia, which are mentioned by different authors, are in a great measure the result only of their own imagination: thus, M. Picard, after supposing this measurement of Eratosthenes to be exact, thence deduces the value of the stadium, making it equal to 51 toises 10 inches.

STAFF, *Almucantar's*, *Augural*, *Back*, *Cross*, *Fore*, *Offset*, *&c.* See these several articles.

STANLEY (Thomas), f. r. s. a learned writer, son of Sir Thomas Stanley, of Hertfordshire, died in Westminster April 12, 1678. He studied at Pembroke Hall in the university of Cambridge, with great credit, where he took the degree of a. m. 1640, after which he went on his travels. On his return he entered of the Middle Temple, but did not follow the law. He was one of the early fellows of the Royal Society, being elected in July 1661, and was esteemed a very learned and worthy member. He edited some of the ancient classics, with notes; and published several ingenious poems of his own, as well as some translations. But the work on account of which he claims a place in this Dictionary, is his History of Philosophy and Lives of Philosophers, in folio. This was first published in 3 parts, in 1655, 1656, and 1660. And in 1662 came out his Chaldaic Philosophy also.

STAR, Stella, in Astronomy, a general name for all the heavenly bodies. The stars are distinguished into fixed and erratic or wandering.

Erratic or *Wandering* Stars, are those which are continually changing their places and distances, with regard to each other. These are what are properly called planets. Though to the same class may likewise be referred comets or blazing stars.

Fixed Stars, called also barely Stars, by way of eminence, are those which have usually been observed to keep the same distance, with regard to each other. The chief circumstances observable in the fixed stars, are their distance, magnitude, number, nature, and motion.

Distance of the Fixed Stars. The fixed stars are so extremely remote from us, that we have no distances in the planetary system to compare to them. Their immense distance appears from hence, that they have no sensible parallax; that is, that the diameter of the earth's annual orbit, which is nearly 190 millions of miles, bears no sensible proportion to their distance.

Mr. Huygens (Cosmotheor. lib. 4) attempts to determine the distance of the stars, by making the aperture of a telescope so small, that the sun through it appears no larger than Sirius; which he found to be only as 1 to 27664 of his diameter, when seen with the naked eye. So that, were the sun's distance 27,664 times as much as it is, it would then be seen of the same diameter with Sirius. And hence, supposing Sirius to be a sun of the same magnitude with our sun, the distance of Sirius will be found to be 27,664 times the distance of the sun, or 345 million times the earth's diameter.

Dr. David Gregory investigated the distance of Sirius by supposing it of the same magnitude with the sun, and of the same apparent diameter with Jupiter in opposition: as may be seen at large in his Astronomy, lib. 3, prop. 47.

Cassini (Mem. Acad. 1717), by comparing Jupiter and Sirius, when viewed through the same telescope, inferred, that the diameter of that planet was 10 times as great as that of the star; and the diameter of Jupiter being 50″, he concluded that the diameter of Sirius was about 5″; supposing then the real magnitude of Sirius to be equal to that of the sun, and the distance of the sun from us 12,000 diameters of the earth, and the apparent diameter of Sirius being to that of the sun as 1 to 384, the distance of Sirius becomes equal to 4,608,000 diameters of the earth.

These methods of Huygens, Gregory, and Cassini, are conjectural and precarious; both because the sun and Sirius are supposed of equal magnitude, and also because it is supposed the diameter of Sirius is determined with sufficient exactness.

Mr. Michell has proposed an inquiry into the probable parallax and magnitude of the fixed stars, from the quantity of light which they afford us, and the peculiar circumstances of their situation. With this view he supposes, that they are, on a medium, equal in magnitude and natural brightness to the sun; and then proceeds to inquire, what would be the parallax of the sun, if he were to be removed so far from us, as to make the quantity of the light, which we should then receive from him, no more than equal to that of the fixed stars. Accordingly, he assumes Saturn in opposition, as equal, or nearly equal in light to the brightest fixed star. As the mean distance of Saturn from the sun is equal to about 2082 of the sun's semidiameters, the density of the sun's light at Saturn will consequently be less than at his own surface, in the ratio of the square of 2082 or 4,334,724 to 1: if Saturn therefore reflected all the light that falls upon him, he would be less luminous in the same proportion. And besides, his apparent diameter, in the opposition, being but about the 105th part of that of the sun, the quantity of light which we receive from him must be again diminished in the ratio of the square of 105 or 11,025 to 1. Consequently, by multiplying these two numbers together, we shall have the whole of the light of the sun to that of Saturn, as the square nearly of 220,000 or 48,400,000,000 to 1. Hence, removing the sun to 220,000 times his present distance, he would still appear at least as bright as Saturn, and his whole parallax upon the diameter of the earth's orbit would be less than 2 seconds: and this must be assumed for the parallax of the brightest of the fixed stars, on the supposition that their light does not exceed that of Saturn.

By a like computation it may be found, that the distance, at which the sun would afford us as much light as we receive from Jupiter, is not less than 46,000 times his present distance, and his whole parallax in that case, on the diameter of the earth's orbit, would not be more than 9 seconds; the light of Jupiter and Saturn, as seen from the earth, being in the ratio of about 22 to 1, when they are both in opposition, and supposing them to reflect equally in proportion to the whole of the light that falls upon them. But if Jupiter and Saturn, instead of reflecting the whole of the light that falls upon them, should really reflect only a part of it, as a 4th, or a 6th, which may be the case, the above distances must be increased in the ratio of 2 or 2½ to 1, to make the sun's light no more than equal to theirs; and his parallax would be less in the same proportion. Supposing then that the fixed stars are of the same magnitude and brightness with the sun, it is no wonder that their parallax should hitherto have escaped observation; since in this case it could hardly amount to 2 seconds, and probably not more than one in Sirius himself, though he had been placed in the pole of the ecliptic; and in those that appear much less luminous, as γ Draconis, which is only of the 3d magnitude, it could hardly be expected to be sensible with such instruments as have hitherto been used. However, Mr. Michell suggests, that it is not impracticable to construct instruments capable of distinguishing even to the 20th part of a second, provided the air will admit of that degree of exactness. This ingenious writer apprehends that the quantity of light which we receive from Sirius, does not exceed the light we receive from the least fixed star of the 6th magnitude, in a greater ratio than that of 1000 to 1, nor less than that of 400 to 1; and the smaller stars of the 2d magnitude seem to be about a mean proportional between the other two. Hence the whole parallax of the least fixed stars of the 6th magnitude, supposing them of the same size and native brightness with the sun, should be from about 2‴ to 3‴, and their distance from about 8 to 12 million times that of the sun: and the parallax of the smaller stars of the 2d magnitude, on the same supposition, should be about 12‴, and their distance about 2 million times that of the sun.

This author further suggests, that, from the apparent situation of the stars in the heavens, it is highly probable that the stars are collected together in clusters in some places, where they form systems, while in others there are either few or none of them; whether this disposition be owing to their mutual gravitation, or to some other law or appointment of the Creator. Hence it may be inferred, that such double stars, &c, as appear to consist of two or more stars placed very near together, do really consist of stars placed near together, and under the influence of some general law: and he proceeds to inquire whether, if the stars be collected into systems, the sun does not likewise make one of some system, and which fixed stars those are that belong to the same system with him.

Those stars, he apprehends, which are found in clusters, and surrounded by many others at a small distance from them, belong probably to other systems, and not to ours. And those stars, which are surrounded with nebulæ, are probably only very large stars which, on account of their superior magnitude, are singly visible, while the others, which compose the remaining parts of the same system, are so small as to escape our sight. And those nebulæ in which we can discover either none or only a few stars, even with the assistance of the best telescopes, are probably systems that are still more distant than the rest. For other particulars of this inquiry, see Philos. Trans. vol. 57.

As the distance of the fixed stars is best determined by their parallax, various methods have been pursued, though hitherto without success, for investigating it; the result of the most accurate observations having given us little more than a distant approximation; from which however we may conclude, that the nearest of the fixed stars cannot be less than 40 thousand diameters of the whole annual orbit of the earth distant from us.

The method pointed out by Galileo, and attempted by Hooke, Flamsteed, Molyneux, and Bradley, of taking the distances of such stars from the zenith as pass very near it, has given us a more just idea of the immense distance of the stars, and furnished an approximation to their parallax, much nearer the truth, than any we had before.

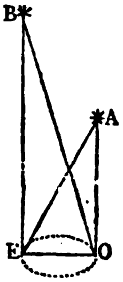
Dr. Bradley assures us (Philos. Trans. No. 406) that had the parallax amounted to a single second, or two at most, he should have perceived it in the great number of observations which he made, especially upon γ Draconis; and that it seemed to him very probable, that the annual parallax of this star does not amount to a single second.

and consequently that it is above 400 thousand times further from us than the sun.

But Dr. Herschel, to whose industry and ingenuity, in exploring the heavens, astronomy is so much indebted, remarks, that the instrument used on this occasion, being the same with the present zenith sectors, can hardly be allowed capable of showing an angle of one or even two seconds, with accuracy: and besides, the star on which the observations were made, is only a bright star of the 3d magnitude, or a small star of the 2d; and that therefore its parallax is probably much less than that of a star of the first magnitude. So that we are not warranted in inferring, that the parallax of the stars in general does not exceed 1″, whereas those of the first magnitude may have, notwithstanding the result of Dr. Bradley's observations, a parallax of several seconds.

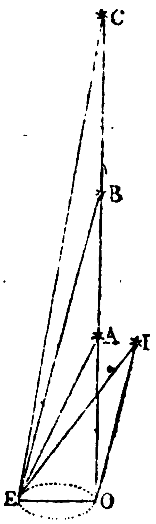
As to the method of zenith distances, it is liable to considerable errors, on account of refraction, the change of position of the earth's axis, arising from nutation, precession of the equinoxes, or other causes, and the aberration of light.

Dr. Herschel has proposed another method, by means of double stars, which is free from these errors, and of such a nature, that the annual parallax, even if it should not exceed the 10th part of a second, may still become visible, and be ascertained at least much nearer than heretofore. This method, which was first proposed in an imperfect manner by Galileo, and has been also mentioned by other authors, is capable of every improvement which the telescope and mechanism of micrometers can furnish. To give a general idea of it, let O and E be two opposite points of the annual orbit, taken in the same plane with two stars, A, B, of unequal magnitudes. Let the angle AOB be observed when the earth is at O, and AEB be observed when the earth is at E. From the difference of these angles, when there is any, the parallax of the stars may be computed, according to the theory subjoined. These two stars ought to be as near as possible to each other, and also to differ as much in magnitude as we can find them.



This theory of the annual parallax of double stars, with the method of computing from thence what is usually called the parallax of the fixed stars, or of single stars of the first magnitude, such as are nearest to us, supposes 1st, that the stars are all about the size of the sun; and 2dly, that the difference in their apparent magnitudes, is owing to their different distances, so that a star of the 2d, 3d, or 4th magnitude, is 2, 3, or 4 times as far off as one of the first. These principles, which Dr. Herschel premises as postulata, have so great a probability in their favour, that they will scarcely be objected to by those who are in the least acquainted with the doctrine of chances. See Mr. Michell's Inquiry, &c, already cited. And Philos. Trans. vol. 57, pa. 234 ---- 240. Also Dr. Halley, on the Number, Order, and Light of the fixed Stars, in the Philos. Trans. vol. 31.

Therefore, let EO be the whole diameter of the earth's annual orbit; and let A, B, C be three stars situated in the ecliptic, in such a manner, that they may appear all in one line OABC when the earth is at O. Now if OA, AB, BC be equal to each other, A will be a star of the first magnitude, B of the second, and C of the third. Let us next suppose the angle OAE, or parallax of the whole orbit of the earth, to be 1″ of a degree; then, because very small angles, having the same subtense EO, may be considered as in the inverse ratio of the lines OA, OB, OC, &c, we shall have EBO $= \frac{1}{2}''$, and ECO $= \frac{1}{3}''$, &c, also because EA $=$ AB nearly, the angle AEB $=$ ABE $= \frac{1}{2}''$; and because BC $= \frac{1}{2}$BO $= \frac{1}{2}$BE nearly, the angle BEC $= \frac{1}{2}$BCE $= \frac{1}{6}''$, and hence AEC $= \frac{1}{2} + \frac{1}{6} = \frac{2}{3}''$; whence it follows that, when the earth is at E, the stars A and B appear at $\frac{1}{2}''$ distant from each other, the stars A and C at $\frac{2}{3}''$ distant, and the stars B and C only $\frac{1}{6}''$ distant. In like manner may be deduced a general expression for the parallax that will become visible in the change of distance between the two stars, by the removal of the earth from one extreme of her orbit to the other. Let P denote the total parallax of a fixed star of the magnitude of the M order, and m the number of the order of a smaller star, p denoting the partial parallax to be observed by the change in the distance of a double star; then is $p = \frac{m-M}{m}P$, or $P = \frac{mp}{m-M}$, which gives P, when p is found by observation.



For Ex. Suppose a star of the 1st magnitude should have a small star of the 12th magnitude near it; then will the partial parallax we are to expect to see be $\frac{12-1}{12}P = \frac{11}{12}P$, or $\frac{11}{12}$ of the total parallax of the larger star; and if we should, by observation, find the partial parallax between two such stars to amount to 1″, then will the total parallax $P = \frac{12}{11}p = 1''\frac{1}{11}$. Again, if the stars be of the 3d and 24th magnitude, the total parallax will be $P = \frac{24}{24-3}p = \frac{24}{21}p = \frac{8}{7}p$; so that, if by observation p be found to be $\frac{1}{10}$ of a second, the whole parallax P will amount $\frac{8}{70} = 0{\cdot}11428''$.

Further, the stars being still in the ecliptic, suppose they should appear in one line, when the earth is in some other part of her orbit between E and O; then will the parallax be still expressed by the same algebraic formula, and one of the maxima will still lie at E, the other at O; but the whole effect will be divided into two parts, which will be in proportion to each other, as radius − sine to radius + sine, of the star's distance from the nearest conjunction or opposition.

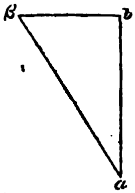
When the stars are any where out of the ecliptic, situated so as to appear in one line OABC perpendicular to EO, the maximum of parallax will still be expressed by $\frac{m-M}{m}P$; but there will arise another additional parallax in the conjunction and opposition, which will be to that which is found 90° before or after the sun, as the sine (s) of the latitude of the stars seen at O, is to radius (1); and the effect of this parallax will be divided into two parts; half of it lying on one side of the large star, the other half on the other side of it. And this latter parallax will also be compounded with the former, so that the distance of the stars in the conjunction and opposition will then be represented by the diagonal of a parallelogram, whose sides are the two semiparallaxes; a general expression for which will be $\frac{m-M}{2m}P\sqrt{1+s^2}$, or $\frac{1}{2}p\sqrt{1+s^2}$.

When the stars are in the pole of the ecliptic, s will be $= 1$, and the last formula becomes $\frac{1}{2}p\sqrt{2} = \cdot 7071p$.

Again, let the stars be at some distance, as 5″, from each other, and let them be both in the ecliptic. This case is resolvable into the first; for imagine the star A to be situated at I; then the angle AEI may be accounted equal to AOI; and as the foregoing formula, $p = \frac{m-M}{m}P$, gives us the angles AEB, AEC, we are to add AEI or 5″ to AEB, which will give IEB. In general, let the distance of the stars be d, and let the observed distance at E be D; then will $D = d + p$, and therefore the whole parallax of the annual orbit will be expressed by $\frac{D-d}{m-M}Dd = P$.

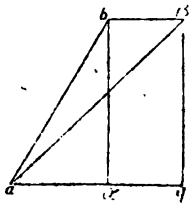
Suppose now the stars to differ only in latitude, one being in the ecliptic, the other at some distance as 5″ north, when seen at O. This case may also be resolved by the former; for imagine the stars B and C to be elevated at right angles above the plane of the figure, so that AOB, or AOC, may make an angle of 5″ at O; then instead of the lines OABC, EA, EB, EC, imagine them all to be planes at right angles to the figure; and it will appear that the parallax of the stars in longitude, must be the same as if the small star had been without latitude. And since the stars B, C, by the motion of the earth from O to E, will not change their latitude, we shall have the following construction for finding the distance of the stars AB and AC at E, and from thence the parallax P.

Let the triangle $ab\beta$ represent the situation of the stars; ab is the subtense of 5″, the angle under which they are supposed to be seen at O. The quantity $b\beta$ by the former theorem is found $= \frac{m-M}{m}P$, which is the partial parallax, that would have been seen by the earth's moving from O to E, if both stars had been in the ecliptic; but, on account of the difference in latitude, it will now be represented by $a\beta$, the hypothenuse of the triangle $ab\beta$: therefore in general, putting $ab = d$, $a\beta = D$, we have $\frac{m}{m-M}\sqrt{D^2 - d^2} = P$. Hence, D being found by observation, and the three d, m, M given, the total parallax is obtained.



When the stars differ in longitude as well as latitude, this case may be resolved in the following manner. Let the triangle $ab\beta$ represent the situation of the stars, $ab = d$ being their distance seen at O, $a\beta = D$ their distance seen at E. That the change $b\beta$, which is produced by the earth's motion, will be truly expressed by $\frac{m-M}{m}P$, may be proved as before, by supposing the star a to have been placed at α. Now let the angle of position $ba\alpha$ be taken by a micrometer, or by any other method sufficiently exact; then, by resolving the triangle $ab\alpha$, we obtain the longitudinal and latitudinal differences $a\alpha$ and $b\alpha$ of the two stars. Put $a\alpha = x$, $b\alpha = y$, and it will be $x + b\beta = aq$, whence

$$D = \sqrt{\left(\left(x + \frac{m-M}{m}P\right)^2 + y^2\right)}; \text{ and } P = \frac{\sqrt{(D^2-y^2)}-x}{m-M}m.$$



If neither of the stars should be in the ecliptic, nor have the same longitude or latitude, the last theorem will still serve to calculate the total parallax, whose maximum will lie in E. There will also arise another parallax, whose maximum will be in the conjunction and opposition, which will be divided, and lie on different sides of the large star; but as the whole parallax is extremely small, it is not necessary to investigate every particular case of this kind; for by reason of the division of the parallax, which renders observations taken at any other time, except where it is greatest, very unfavourable, the formula would be of little use.

Dr. Herschel closes his account of this theory, with a general observation on the time and place where the maxima of parallax will happen. Thus, when two unequal stars are both in the ecliptic, or, not being in the ecliptic, have equal latitudes, north or south, and the larger star has most longitude, the maximum of the apparent distance will be when the sun's longitude is 90° more than the star's, or when observed in the morning: and the minimum, when the longitude of the sun is 90° less than that of the star, or when observed in the evening. But when the small star has most longitude, the maximum and minimum, as well as the time of observation, will be the reverse of the former. And when the stars differ in latitude, this makes no alteration in the place of the maximum or minimum, nor in the time of observation; that is, it is immaterial which of the two stars has the greater latitude. Philos. Trans. vol. 72, art. 11.

The distance of the star γ Draconis appears, by Bradley's observations, already recited, to be at least 400,000 times that of the sun, and the distance of the nearest fixed star, not less than 40,000 diameters of the earth's annual orbit: that is, the distance from the earth, of the former at least - - 38,000,000,000,000 miles, and the latter not less than - 7,600,000,000,000 miles. As these distances are immensely great, it may both be amusing, and assist in giving a more familiar idea, to compare them with the velocity of some moving body, by which they may be measured.

The swiftest motion we know of, is that of light, which passes from the sun to the earth in about 8 minutes; and yet even this would be above 6 years traversing the first space, and near a year and a quarter in passing from the nearest fixed star to the earth. But a cannon ball, moving on a medium at the rate of about 20 miles in a minute, would be 3 million 8 hundred thousand years in passing from γ Draconis to the earth, and 760 thousand years in passing from the nearest fixed star. Sound, which moves at the rate of about 13 miles in a minute, would be 5 million 600 thousand years in traversing the former distance, and 1 million 128 thousand, in passing through the latter.

The celebrated Huygens pursued speculations of this kind so far, as to believe it not impossible, that there may be stars at such inconceivable distances, that their light has not yet reached the earth since their creation.

Dr. Halley has also advanced, what he says seems to be a metaphysical paradox (Philos. Trans. number 364), viz. that the number of fixed stars must be more than finite, and some of them at more than a finite distance from others: and Addison has justly observed, that this thought is far from being extravagant, when we consider that the universe is the work of infinite power, prompted by infinite goodness, and having an infinite space to exert itself in; so that our imagination can set no bounds to it.

Magnitude of the fixed Stars. The magnitudes of the stars appear to be very different from one another; which difference may probably arise, partly from a diversity in their real magnitude, but principally from their different distances. To the bare eye, the stars appear of some sensible magnitude, owing to the glare of light arising from the numberless reflections from the aërial particles &c about the eye; this makes us imagine the stars to be much larger than they would appear, if we saw them only by the few rays which come directly from them, so as to enter our eyes without being intermixed with others. Any person may be sensible of this, by looking at a star of the first magnitude through a long narrow tube; which, though it takes in as much of the sky as would hold a thousand such stars, scarce renders that one visible.

The stars, on account of their apparently various sizes, have been distributed into several classes, called magnitudes. The 1st class, or stars of the first magnitude, are those that appear largest, and may probably be nearest to us. Next to these, are those of the 2d magnitude; and so on to the 6th, which comprehends the smallest stars visible to the naked eye. All beyond these, that can be perceived by the help of telescopes, are called telescopic stars. Not that all the stars of each class appear exactly of the same magnitude; there being great difference in this respect; and those of the first magnitude appearing almost all different in lustre and size. There are also other stars, of intermediate magnitudes, which astronomers cannot refer to one class more than another, and therefore they place them between the two. Procyon, for instance, which Ptolemy makes of the first magnitude, and Tycho of the 2d, Flamsteed lays down as between the 1st and 2d. So that, instead of 6 magnitudes, we may say there are almost as many orders of stars, as there are stars; on account of the great variations observable in the magnitude, colour, and brightness of them.

There seems to be but little probability of discovering with certainty the real size of any of the fixed stars; we must therefore be content with an approximation, deduced from their parallax, if this should ever be found; and the quantity of light they afford us, compared with that of the sun. And to this purpose, Dr. Herschel informs us, that with a magnifying power of 6450, and by means of his new micrometer, he found the apparent diameter of α Lyræ to be 0″·355.

The stars are also distinguished, with regard to their situation, into asterisms, or constellations; which are only assemblages of several neighbouring stars, considered as constituting some determinate figure, as of an animal, &c, from which it is therefore denominated: a division as ancient as the book of Job, in which mention is made of Orion, the Pleiades, &c.

Besides the stars thus distinguished into magnitudes and constellations, there are others not reduced to either. Those not reduced into constellations, are called informes, or unformed stars; of which kind several, so left at large by the ancients, have since been formed into new constellations by the modern astronomers, and especially by Hevelius. Those not reduced to classes or magnitudes, are called nebulous stars; but such as only appear faintly in clusters, in form of little lucid spots, nebulæ, or clouds.

Ptolemy mentions five of such nebulæ, viz, one at the extremity of the right hand of Perseus, which appears through the telescope, thick set with stars; one in the middle of the crab, called Præsepe, or the Manger, in which Galileo counted above 40 stars; one unformed near the sting of the Scorpion; another in the eye of Sagittarius, in which two stars may be seen in a clear sky with the naked eye, and several more with the telescope; and the fifth in the head of Orion, in which Galileo counted 21 stars.

Flamsteed observed a cloudy star before the bow of Sagittarius, which consists of a great number of small stars; and the star *d* above the right shoulder of this constellation is encompassed with several more. Flamsteed and Cassini also discovered one between the great and little dog, which is very full of stars, that are visible only by the telescope.

But the most remarkable of all the cloudy stars, is that in the middle of Orion's sword, in which Huygens and Dr. Long observed 12 stars, 7 of which (3 of them, now known to be 4, being very close together) seem to shine through a cloud, very lucid near the middle, but faint and ill defined about the edges. But the greatest discoveries of nebulæ and clusters of stars, we owe to the powerful telescopes of Dr. Herschel, who has given accounts of some thousands of such nebulæ, in many of which the stars seem to be innumerable, like grains of sand on the sea shore, or, as Milton has so beautifully described the milky way, they seem powdered with stars. See Philos. Trans. 1784, 1785, 1786, 1789. See Galaxy, and Magellanic *Clouds*, and *lucid* Spots.

Cassini is of opinion, that the brightness of these proceeds from stars so minute, as not to be distinguished by the best glasses: and this opinion is fully confirmed by the observations of Dr. Herschel, whose powerful telescopes show those lucid specks to be composed entirely of masses of small stars, like heaps of sand.

There are also many stars which, though they appear single to the naked eye, are yet discovered by the telescope to be double, triple, &c. Of these, several have been observed by Cassini, Hooke, Long, Maskelyne, Hornsby, Pigott, Mayer, &c; but Dr. Herschel has been much the most successful in observations of this kind; and his success has been chiefly owing to the very extraordinary magnifying powers of the Newtonian 7 feet reflector which he has used, and the advantage of an excellent micrometer of his own construction. The powers which he has used, have been 146, 227, 278, 460, 754, 932, 1159, 1536, 2010, 3168, and even 6450. He has already formed a catalogue, containing 269 double stars, 227 of which, as far as he knows, have not been noticed by any other person. Among these there are also some stars that are treble, double-double, quadruple, double-treble, and multiple. His catalogue comprehends the names of the stars, and the number in Flamsteed's catalogue, or such a description of those that are not contained in it, as will be found sufficient to distinguish them; also the comparative size of the stars; their colours as they appeared to his view; their distances determined in several different ways; their angle of position with regard to the parallel of declination; and the dates when he first perceived the stars to be double, treble, &c. His observations appear to commence with the year 1776, but almost all of them were made in the years 1779, 1780, 1781.

Dr. Herschel has distributed the double stars contained in his catalogue, into 6 different classes. In the first he has placed all those which require a very superior telescope, with the utmost clearness of air, and every other favourable circumstance, to be seen at all, or well enough to judge of them; and there are 24 of these. To the 2d class belong all those double stars that are proper for estimations by the eye, and very delicate measures by the micrometer; the number being 38. The 3d class comprehends all those double stars, that are between $5''$ and $15''$ asunder; the number of them being 46. The 4th, 5th, and 6th classes contain double stars that are from $15''$ to $30''$, and from $30''$ to $1'$, and from $1'$ to $2'$ or more asunder; of which there are 44 in the 4th class, 51 in the 5th class, and 66 in the 6th class: the last of this class is α Tauri, number 87 of Flamsteed, whose apparent diameter, on the meridian measured with a power of 460 at a mean of two observations $1''\ 46'''$, and with a power of 932 at a mean of two observations $1''\ 12'''$. See the list at large, Philos. Trans. vol. 72, art. 12.

The stars are also distinguished, in each constellation, by numbers, or by the letters of the alphabet. This kind of distinction was introduced by John Bayer, in his Uranometria, 1654; where he denotes the stars, in each constellation, by the letters of the Greek alphabet, α, β, γ, δ, ε, &c, viz, the most remarkable star of each by α, the 2d by β, the 3d by γ, &c; and when there are more stars in a constellation than the characters in the Greek alphabet, he denotes the rest, in their order, by the Roman letters A, b, c, d, &c. But as the number of the stars, that have been observed and registered in catalogues, since Bayer's time, is greatly increased, as by Flamsteed and others, the additional ones have been marked by the ordinal numbers 1, 2, 3, 4, 5, &c.

The *Number of* STARS. The number of the stars appears to be immensely great, perhaps infinite; yet have astronomers long since ascertained the number of such as are visible to the eye, which are much fewer than at first sight could be imagined. See CATALOGUE *of the Stars*.—Of the 3000 contained in Flamsteed's catalogue, there are many that are only visible through a telescope; and a good eye scarce ever sees more than a thousand at the same time in the clearest heaven; the appearance of that immense number which are frequent in clear winter nights, arising from our sight's being deceived by their twinkling, and from our viewing them confusedly, and not reducing them to any order. But nevertheless we cannot but imagine that the stars are almost, if not altogether, infinite. See Halley, on the number, order, and light of the fixed stars, Philos. Trans. number 364.

Riccioli, in his New Almagest, affirms, that a man who shall say there are above 20 thousand times 20 thousand, would say nothing improbable. For a good telescope, directed indifferently to almost any point of the heavens, discovers multitudes that are lost to the naked eye; particularly in the milky way, which some take to be an assemblage of stars, too remote to be seen singly, but so closely disposed as to give a luminous appearance to that part of the heavens where they are. And this fact has been confirmed by Herschel's observations: though it is disputed by others, who contend that the milky way must be owing to some other cause.

In the single constellation of the Pleiades, instead of 6, 7, or 8 stars seen by the best eye; Dr. Hooke, with a telescope 12 feet long, told 78, and with larger glasses many more, of different magnitudes. And F. de Rheita affirms, that he has observed above 2000 stars in the single constellation of Orion. The same author found above 188 in the Pleiades. And Huygens, looking at the star in the middle of Orion's sword, instead of one, found that there were 12. Galileo found 80 in the space of the belt of Orion's sword, 21 in the nebulous star of his head; and above 500 in another part of him, within the compass of one or two degrees space, and more than 40 in the nebulous star Præsepe.

The Changes that have happened in the STARS are very considerable. The first change that is on record, was about 120 years before Christ; when Hipparchus, discovering a new star to appear, was first induced to make a catalogue of the stars, that posterity might perceive any future changes of the like nature.

In the year 1572, Cornelius Gemma and Tycho Brahé observed another new star in the constellation Cassiopeia, which was likewise the occasion of Tycho's making a new catalogue. At first its magnitude and brightness exceeded the largest of the stars, Sirius and Lyra; and even equalled the planet Venus when nearest the earth, and was seen in fair day-light. It continued 16 months; towards the latter end of which it began to dwindle, and at length, in March 1574, it totally disappeared, without any change of place in all that time.

Leovicius tells us of another star appearing in the same constellation, about the year 945, which resembled that of 1572; and he quotes another ancient observation, by which it appears that a new star was seen about the same place in 1264. Dr. Keil thinks these were all the same star; and indeed the periodical intervals, or distance of time between these appearances, were nearly equal, being from 318 to 319 years; and if so, its next appearance may be expected about 1890.

Fabricius, in 1596, discovered another new star, called the stella mira, or wonderful star, in the neck of the whale, which has since been found to appear and disappear periodically, 7 times in 6 years, continuing in its greatest lustre for 15 days together; and is never quite extinguished. Its course and motion are described by Bulliald, in a treatise printed at Paris in 1667. Dr. Herschel has lately, viz, in the years 1777, 1778, 1779, and 1780, made several observations on this star, an account of which may be seen in the Philos. Trans. vol. 70, art. 21.

In the year 1600, William Jansen discovered a changeable star in the neck of the Swan, which gradually decreased till it became so small as to be thought to disappear entirely, till the years 1657, 1658, and 1659, when it regained its former lustre and magnitude; but soon decayed again, and is now of the smallest size.

In the year 1604, a new star was seen by Kepler, and several of his friends, near the heel of the right foot of Serpentarius, which was particularly bright and sparkling; and it was observed to be every moment changing into some of the colours of the rainbow, except when it is near the horizon, at which time it was generally white. It surpassed Jupiter in magnitude, but was easily distinguished from him, by the steady light of the planet. It disappeared about the end of the year 1605, and has not been seen since that time.

Simon Marius discovered another in Andromeda's girdle, in 1612 and 1613; though Bulliald says it had been seen before, in the 15th century.

In July 1670, Hevelius discovered a second changeable star in the Swan, which was so diminished in October as to be scarce perceptible. In April following it regained its former lustre, but wholly disappeared in August. In March 1672 it was seen again, but appeared very small, and has not been visible since.

In 1686 a third changeable star was discovered by Kirchius in the Swan, viz, the star χ of that constellation, which returned periodically in about 405 days.

In 1672 Cassini saw a star in the neck of the Bull, which he thought was not visible in Tycho's time, nor when Bayer made his figures.

It is certain, from the old catalogues, that many of the ancient stars are not now visible. This has been particularly remarked with regard to the Pleiades.

M. Montanari, in his letter to the Royal Society in 1670, observes that there are now wanting in the heavens two stars of the 2d magnitude, in the stern of the ship Argo, and its yard, which had been seen till the year 1664. When they first disappeared is not known; but he assures us there was not the least glimpse of them in 1668. He adds, that he has observed many more changes in the fixed stars, even to the number of a hundred. And many other changes of the stars have been noticed by Cassini, Maraldi, and other observers. See Gregory's Astron. lib. 2, prop. 30.

But the greatest numbers of variable stars have been observed of late years, and the most accurate observations made on their periods, &c, by Herschel, Goodricke, Pigott, &c, in the late volumes of the Philos. Trans. particularly in the vol. for 1786, where the last of these gentlemen has given a catalogue of all that have been hitherto observed, with accounts of the observations that have been made upon them.

Various hypotheses have been devised to account for such changes and appearances in the stars. It is not probable they could be comets, as they had no parallax, even when largest and brightest. It has been supposed that the periodical stars have vast dark spots, or dark sides, and very slow rotations on their axes, by which means they must disappear when the darker side is turned towards us. And as for those which break out suddenly with such lustre, these may perhaps be suns whose fuel is almost spent, and again supplied by some of their comets falling upon them, and occasioning an uncommon blaze and splendor for some time; which it is conjectured may be one use of the cometary part of our system.

Maupertuis, in his Dissertation on the Figures of the Celestial Bodies (pa. 61—63), is of opinion that some stars, by their prodigious swift rotation on their axes, may not only assume the figures of oblate spheroids, but that by the great centrifugal force arising from such rotations, they may become of the figures of mill-stones, or be reduced to flat circular planes, so thin as to be quite invisible when their edges are turned towards us, as Saturn's ring is in such position. But when very excentric planets or comets go round any flat star in orbits much inclined to its equator, the attraction of the planets or comets in their perihelions must alter the inclination of the axis of that star; on which account it will appear more or less large and luminous, as its broad side is more or less turned towards us. And thus he imagines we may account for the apparent changes of magnitude and lustre of those stars, and also for their appearing and disappearing.

Hevelius apprehends (Cometograph. pa. 380), that the sun and stars are surrounded with atmospheres, and that by whirling round their axes with great rapidity, they throw off great quantities of matter into those atmospheres, and so cause great changes in them; and that thus it may come to pass that a star, which, when its atmosphere is clear, shines out with great lustre, may at another time, when it is full of clouds and thick vapours, appear greatly diminished in brightness and magnitude, or even become quite invisible.

Nature of the fixed STARS. The immense distance of the stars leaves us greatly at a loss about the nature of them. What we can gather for certain from their phenomena, is as follows: 1st, That the fixed stars are greater than our earth: because if that were not the case, they could not be visible at such an immense distance. 2nd, The fixed stars are farther distant from the earth than the farthest of the planets. For we frequently find the fixed stars hid behind the body of the planets: and besides, they have no parallax, which the planets have. 3rd, The fixed stars shine with their own light; for they are much farther from the sun than Saturn, and appear much smaller than Saturn; but since, notwithstanding this, they are found to shine much brighter than that planet, it is evident they cannot borrow their light from the same source as Saturn does, viz, the sun; but since we know of no other luminous body beside the sun, whence they might derive their light, it follows that they shine with their own native light.

Besides, it is known, that the more a telescope magnifies, the less is the aperture through which the star is seen; and consequently, the fewer rays it admits into the eye. Now since the stars appear less in a telescope which magnifies two hundred times, than they do to the naked eye, insomuch that they seem to be only indivisible points, it proves at once that the stars are at immense distances from us, and that they shine by their own proper light. If they shone by borrowed light, they would be as invisible without telescopes as the satellites of Jupiter are; for these satellites appear larger when viewed with a good telescope than the largest fixed stars. Hence, 1. We deduce, that the fixed stars are so many suns; for they have all the characters of suns. 2. That in all probability the stars are not smaller than our sun. 3. That it is highly probable each star is the centre of a system, and has planets or earths revolving round it, in the same manner as round our sun, i. e. it has opaque bodies illuminated, warmed, and cherished by its light and heat. As we have incomparably more light from the moon than from all the stars together, it is absurd to imagine that the stars were made for no other purpose than to cast a faint light upon the earth; especially since many more require the assistance of a good telescope to find them out, than are visible without that instrument. Our sun is surrounded by a system of planets and comets, all which would be invisible from the nearest fixed star; and from what we already know of the immense distance of the stars, it is easy to infer, that the sun, seen from such a distance, would appear no larger than a star of the first magnitude.

From all this it is highly probable, that each star is a sun to a system of worlds moving round it, though unseen by us; especially as the doctrine of a plurality of worlds is rational, and greatly manifests the power, the wisdom, and the goodness of the great Creator. How immense, then, does the universe appear! Indeed, it must either be infinite, or infinitely near it.

Kepler, it is true, denies that each star can have its system of planets as ours has; and takes them all to be fixed in the same surface or sphere; urging, that were one twice or thrice as remote as another, it would be twice or thrice as small, supposing their real magnitudes equal; whereas there is no difference in their apparent magnitudes, justly observed, at all. But to this it is opposed, that Huygens has not only shown, that fires and flames are visible at distances where other bodies, comprehended under equal angles, disappear; but it should likewise seem, that the optic theorem about the apparent diameters of objects, being reciprocally proportional to their distances from the eye, does only hold while the object has some sensible ratio to its distance.

For periodical stars, &c, see Changes, *&c, of Stars*, supra.

Motion of the Stars. The fixed stars have several kinds of apparent motion; one called the first, common, or diurnal motion, arising from the earth's rotation about its axis; and by which they seem to be carried along with the sphere or firmament, in which they appear fixed, round the earth, from east to west, in the space of 24 hours.

The other, called the second, or proper motion, is that by which they appear to go backwards from west to east, round the poles of the ecliptic, with an exceeding slow motion, as describing a degree of their circle only in the space of 71½ years, or 50⅓ seconds in a year. This apparent motion is owing to the recession of the equinoctial points, which is 50⅓ seconds of a degree in a year backward, or contrary to the order of the signs of the zodiac. In consequence of this second motion, the longitude of the stars will be always increasing. Thus, for example, the longitude of Cor Leonis was found at different periods, to be as follows: viz,

	Year.		Long.	
By Ptolemy, in	138	to be	2°	30′
By the Persians, in	1115		17	30
By Alphonsus, in	1364		20	40
By Prince of Hesse, in	1586		24	11
By Tycho, in	1601		24	17
By Flamsteed, in	1690		25	31½

Whence the proper motion of the stars, according to the order of the signs, in circles parallel to the ecliptic, is easily inferred.

It was Hipparchus who first suspected this motion, on comparing his own observations with those of Timocharis and Aristyllus. Ptolemy, who lived three centuries after Hipparchus, demonstrated the same by undeniable arguments. The increase of longitude in a century, as stated by different astronomers, is as follows:

By Tycho Brahé	1°	25′	0″
Copernicus	1	23	40⅕
Flamsteed and Riccioli	1	23	20
Bulliald	1	24	54
Hevelius	1	24	46⅚
Dr. Bradley, &c.	1	23	55

which is at the rate of 50⅓ seconds per year.

From these data, the increase in the longitude of a star for any given time, is easily found, and thence its longitude at any time: ex. gr. the longitude of Sirius, in Flamsteed's tables, for the year 1690, being 9° 49′ 1″, its longitude for the year 1800, is found by multiplying the interval of time, viz, 110 years, by 50⅓, the product 5537″, or 1° 32′ 17″, added to the given longitude 9° 49′ 1″ gives the longitude 11° 21′ 18″ for the year 1800.

The chief phenomena of the fixed stars, arising from their common and proper motion, besides their longitude, are their altitudes, right ascensions, declinations, occultations, culminations, risings, and settings.

Some have supposed that the latitudes of the stars are invariable. But this supposition is founded on two assumptions, which are both controverted among astronomers. The one of these is, that the orbit of the earth continues unalterably in the same plane, and consequently that the ecliptic is invariable; the contrary of which is now very generally allowed.

The other assumption is, that the stars are so fixed as to keep their places immoveably. Ptolemy, Tycho, and others, comparing their observations with those of the ancient astronomers, have adopted this opinion. But from the result of the comparison of the best modern observations, with such as were formerly made with any tolerable degree of exactness, there appears to have been a real change in the position of some of the fixed stars, with respect to each other; and several stars of the first magnitude have already been observed, and others suspected, to have a proper motion of their own.

Dr. Halley (Philos. Trans. No. 355), has observed, that the three following stars, the Bull's eye, Sirius, and Arcturus, are now found to be above half a degree more southerly than the ancients reckoned them: that this difference cannot arise from the errors of the transcribers, because the declinations of the stars, set down by Ptolemy, as observed by Timocharis, Hipparchus, and himself, show their latitudes given by him are such as those authors intended: and it is scarce to be believed that those three observers could be deceived in so plain a matter. To this he adds, that the bright star in the shoulder of Orion has, in Ptolemy, almost a whole degree more southerly latitude than at present: that an ancient observation, made at Athens in the year 509, as Bulliald supposes, of an appulse of the moon to the Bull's eye, shows that star to have had less latitude at that time than it now has: that as to Sirius, it appears by Tycho's observations, that he found him 4½′ more northerly than he is at this time. All these observations, compared together, seem to favour an opinion, that some of the stars have a proper motion of their own, which changes their places in the sphere of the heavens: this change of place, as Dr. Halley observes, may show itself in so long a time as 1800 years, though it be entirely imperceptible in the space of one single century; and it is likely to be soonest discovered in such stars as those just now mentioned; because they are all of the first magnitude, and may, therefore, probably be some of the nearest to our solar system. Arcturus, in particular, affords a strong proof of this: for if its present declination be compared with its place, as determined either by Tycho or Flamsteed, the difference will be found to be much greater than what can be suspected to arise from the uncertainty of their observations. See Arcturus, and Mr. Hornsby's inquiry into the quantity and direction of the proper motion of Arcturus, Phil. Trans. vol. 63, part 1, pa. 93, &c.

For an account of Dr. Bradley's observations, see the sequel of this article, also Aberration.

Dr. Herschel has also lately observed, that the distance of the two stars forming the double star γ Draconis, is

54″ 48‴, and their position 44° 19′ n. preceding. Whereas, from the right ascension and declination of these stars in Flamsteed's catalogue, their distance, in his time, appears to have been 1′ 11″·418, and their position 44° 23′ n. preceding. Hence he infers, that as the difference in the distance of these two stars is so considerable, we can hardly account for it, otherwise than by admitting a proper motion in one or the other of the stars, or in our solar system: most probably he says, neither of the three is at rest. He also suspects a proper motion in one of the double stars, in Cauda Lyncis Media, and in ο Ceti. Phil. Trans. vol. 72, part 1, pa. 117, 143, 150.

It is reasonable to expect, that other instances of the like kind must also occur among the great number of visible stars, because their relative positions may be altered by various means. For if our own solar system be conceived to change its place with respect to absolute space, this might, in process of time, occasion an apparent change in the angular distances of the fixed stars; and in such a case, the places of the nearest stars being more affected than of those that are very remote, their relative position might seem to alter, though the stars themselves were really immoveable; and vice versa, we may surmise from the observed motion of the stars, that our sun, with all its planets and comets, may have a motion towards some particular part of the heavens, on account of a greater quantity of matter collected in a number of stars and their surrounding planets there situated, which may perhaps occasion a gravitation of our whole solar system towards it. If this surmise should have any foundation, as Dr. Herschel observes, ubi supra, pa. 103, it will show itself in a series of some years; since from that motion there will arise another kind of hitherto unknown parallax (suggested by Mr. Michell, Philos. Trans. vol. 57, pa. 252), the investigation of which may account for some part of the motions already observed in some of the principal stars; and for the purpose of determining the direction and quantity of such a motion, accurate observations of the distance of stars, that are near enough to be measured with a micrometer, and a very high power of telescopes, may be of considerable use, as they will undoubtedly give us the relative places of those stars to a much greater degree of accuracy than they can be had by instruments or sectors, and thereby much sooner enable us to discover any apparent change in their situation, occasioned by this new kind of secular or systematical parallax, if we may so express the change arising from the motion of the whole solar system.

And, on the other hand, if our system be at rest, and any of the stars really in motion, this might likewise vary their apparent positions, and the more so, the nearer they are to us, or the swifter their motions are; or the more proper the direction of the motion is to be rendered perceptible by us. Since then the relative places of the stars may be changed from such a variety of causes, considering the amazing distance at which it is certain some of them are placed, it may require the observations of many ages to determine the laws of the apparent changes, even of a single star; much more difficult, therefore, must it be to settle the laws relating to all the most remarkable of them.

When the causes which affect the places of all the stars in general are known; such as the precession, aberration, and nutation, it may be of singular use to examine nicely the relative situations of particular stars, and especially of those of the greatest lustre, which, it may be presumed, lie nearest to us, and may therefore be subject to more sensible changes, either from their own motion, or from that of our system. And if, at the same time the brighter stars are compared with each other, we may likewise determine the relative positions of some of the smallest that appear near them, whose places can be ascertained with sufficient exactness, we may perhaps be able to judge to what cause the change is owing, if any be observable. The uncertainty that we are at present under, with respect to the degree of accuracy with which former astronomers could observe, makes us unable to determine several things relating to this subject; but the improvements, which have of late years been made in the methods of taking the places of the heavenly bodies, are so great, that a few years may hereafter be sufficient to settle some points, which cannot now be settled; by comparing even the earliest observations with those of the present age.

Dr. Hooke communicated several observations on the apparent motions of the fixed stars; and as this was a matter of great importance in astronomy, several of the learned were desirous of verifying and confirming his observations. An instrument was accordingly contrived by Mr. George Graham, and executed with surprising exactness.

With this instrument the star γ, in the constellation Draco, was frequently observed by Messrs. Molyneux, Bradley, and Graham, in the years 1725, 1726; and the observations were afterwards repeated by Dr. Bradley with an instrument contrived by the same ingenious person, Mr. Graham, and so exact, that it might be depended on to half a second. The result of these observations was, that the star did not always appear in the same place, but that its distance from the zenith varied, and that the difference of the apparent places amounted to 21 or 22 seconds. Similar observations were made on other stars, and a like apparent motion was found in them, proportional to the latitude of the star. This motion was by no means such as was to have been expected, as the effect of a parallax, and it was some time before any way could be found of accounting for this new phenomenon. At length Dr. Bradley resolved all its variety, in a satisfactory manner, by the motion of light and the motion of the earth compounded together. See ABERRATION and LIGHT, and Phil. Trans. No. 406, pa. 364.

That excellent astronomer had no sooner discovered the cause, and settled the laws of aberration of the fixed stars, than his attention was again excited by another new phenomenon, viz, an annual change of declination in some of the fixed stars, which appeared to be sensibly greater than a precession of the equinoctial points, of 50¼″ in a year, would have occasioned. This apparent change of declination was observed in the stars near the equinoctial colure; and there appearing at the same time an effect of a quite contrary nature, in some stars near the solstitial colure, which seemed to alter their declination less than a precession of 50″ required, Dr. Bradley was thereby convinced, that all the phenomena in the different stars could not be accounted for merely by supposing that he had assumed a wrong quantity for the precession of the equinoctial points. He had also, after many trials, sufficient reason to conclude, that these second unexpected deviations of the stars were not owing to any imperfection of his instruments. At length, from repeated observations he began to guess at the real cause of these phenomena.

It appeared from the Doctor's observations, during his residence at Wansted, from the year 1727 to 1732, that some of the stars near the solstitial colure had changed their declinations 9″ or 10″ less than a precession of 50″ would have produced; and, at the same time, that others near the equinoctial colure had altered theirs about the same quantity more than like a precession would have occasioned: the north pole of the equator seeming to have approached the stars, which come to the meridian with the sun about the vernal equinox, and the winter solstice; and to have receded from those which come to the meridian with the sun about the autumnal equinox and the summer solstice.

From the consideration of these circumstances, and the situation of the ascending node of the moon's orbit when he first began to make his observations, he suspected that the moon's action on the equatorial parts of the earth might produce these effects. For if the precession of the equinox be, according to Sir Isaac Newton's principles, caused by the actions of the sun and moon on those parts; the plane of the moon's orbit, being at one time, above 10 degrees more inclined to the plane of the equator, than at another, it was reasonable to conclude, that the part of the whole annual precession which arises from her action, would, in different years, be varied in its quantity; whereas the plane of the ecliptic, in which the sun appears, keeping always nearly the same inclination to the equator, that part of the precession which is owing to the sun's action, may be the same every year; and from hence it would follow, that though the mean annual precession, proceeding from the joint actions of the sun and moon, were 50″; yet the apparent annual precession might sometimes exceed, and sometimes fall short of that mean quantity, according to the various situations of the nodes of the moon's orbit.

In the year 1727, the moon's ascending node was near the beginning of Aries, and consequently her orbit was as much inclined to the equator as it can at any time be; and then the apparent annual precession was found, by the Doctor's first year's observations, to be greater than the mean; which proved, that the stars near the equinoctial colure, whose declinations are most of all affected by the precession, had changed theirs, above a tenth part more than a precession of 50″ would have caused. The succeeding year's observations proved the same thing; and, in three or four years' time, the difference became so considerable as to leave no room to suspect it was owing to any imperfection either of the instrument or observation.

But some of the stars, that were near the solstitial colure, having appeared to move, during the same time, in a manner contrary to what they ought to have done by an increase of the precession; and the deviations in them being as remarkable as in the others, it was evident that something more than a mere change in the quantity of the precession would be requisite to solve this part of the phenomenon. On comparing the observations of stars near the solstitial colure, that were almost opposite to each other in right ascension, they were found to be equally affected by this cause. For whilst γ Draconis appeared to have moved northward, the small star, which is the 35th Camelopardali Hevelii, in the British catalogue, seemed to have gone as much towards the south; which showed, that this apparent motion in both those stars might proceed from a nutation of the earth's axis; whereas the comparison of the Doctor's observations of the same stars formerly enabled him to draw a different conclusion, with respect to the cause of the annual alterations arising from the motion of light. For the apparent alteration in γ Draconis, from that cause, being as large again as in the other small star, proved, that that did not proceed from a nutation of the earth's axis; as, on the contrary, this might.

On making the like comparison between the observations of other stars, that lie nearly opposite in right ascension, whatever their situations were with respect to the cardinal points of the equator, it appeared, that their change of declination was nearly equal, but contrary; and such as a nutation or motion of the earth's axis would effect.

The moon's ascending node being now returned to the beginning of Capricorn in the year 1732, the stars near the equinoctial colure appeared about that time to change their declinations no more than a precession of 50″ required; while some of those near the solstitial colure altered theirs above 2″ in a year less than they ought. Soon after the annual change of declination of the former was perceived to be diminished, so as to become less than 50″ of precession would cause; and it continued to diminish till the year 1736, when the moon's ascending node was about the beginning of Libra, and her orbit had the least inclination to the equator. But by this time, some of the stars near the solstitial colure had altered their declinations 18″ less since the year 1727, than they ought to have done from a precession of 50″. For γ Draconis, which in those 9 years would have gone about 8″ more southerly, was observed, in 1736, to appear 10″ more northerly than it did in the year 1727.

As this appearance in γ Draconis indicated a diminution of the inclination of the earth's axis to the plane of the ecliptic, and as several astronomers had supposed that inclination to diminish regularly; if this phenomenon depended on such a cause, and amounted to 18″ in 9 years, the obliquity of the ecliptic would, at that rate, alter a whole minute in 30 years; which is much faster than any observations before made would allow. The Doctor had therefore reason to think, that some part of this motion at least, if not the whole, was owing to the moon's action on the equatorial parts of the earth, which he conceived might cause a libratory motion of the earth's axis. But as he was unable to judge, from only 9 years' observation, whether the axis would entirely recover the same position that it had in the year 1727, he found it necessary to continue his observations through a whole period of the moon's nodes; at the end of which he had the satisfaction to see, that the stars returned into the same positions again, as if there had been no alteration at all in the inclination of the earth's axis; which fully convinced him, that he had guessed rightly as to the cause of the phenomenon. This circumstance proves likewise, that if there be a gradual diminution of the obliquity of the ecliptic, it does not arise only from an alteration in the position of the earth's axis, but rather from some change in the plane of the ecliptic itself; because the stars, at the end of the period of the moon's nodes, appeared in the same places, with respect to the equator, as they ought to have done if the earth's axis had retained the same inclination to an invariable plane.

The Doctor having communicated these observations, and his opinion of their cause, to the late Mr. Machin, that excellent geometrician soon after sent him a table, containing the quantity of the annual precession in the various positions of the moon's nodes, as also the corresponding nutations of the earth's axis; which was computed on the supposition that the mean annual precession is 50″, and that the whole is governed by the pole of the moon's orbit only; and therefore Mr. Machin imagined, that the numbers in the table would be too large, as, in fact, they were found to be. But it appeared that the changes which Dr. Bradley had observed, both in the annual precession and nutation, kept the same law, as to increasing and decreasing, with the numbers of Mr. Machin's table. Those were calculated on the supposition, that the pole of the equator, during a period of the moon's nodes, moved round in the periphery of a little circle, whose centre was 23° 29′ distant from the pole of the ecliptic; having itself also an angular motion of 50″ in a year about the same pole. The north pole of the equator was conceived to be in that part of the small circle which is farthest from the north pole of the ecliptic, at the same time when the moon's ascending node is in the beginning of Aries; and in the opposite point of it, when the same node is in Libra.

If the diameter of the little circle, in which the pole of the equator moves, be supposed equal to 18″, which is the whole quantity of the nutation, as collected from Dr. Bradley's observations of the star γ Draconis, then all the phenomena of the several stars which he observed will be very nearly solved by this hypothesis. But for the particulars of his solution, and the application of his theory to the practice of astronomy, we must refer to the excellent author himself; our intention being only to give the history of the invention.

The corrections arising from the aberration of light, and from the nutation of the earth's axis, must not be neglected in astronomical observations; since such neglects might produce errors of near a minute in the polar distance of some stars. As to the allowance to be made for the aberration of light, Dr. Bradley assures us, that having again examined those of his own observations, which were most proper to determine the transverse axis of the ellipsis, which each star seems to describe, he found it to be nearest to 40″: and this is the number he makes use of in his computations relating to the nutation. Dr. Bradley says, in general, that experience has taught him, that the observations of such stars as lie nearest the zenith, generally agree best with one another, and are therefore fittest to prove the truth of any hypothesis. Phil. Trans. num. 485, vol. 45, pa. 1, &c. See our article Nutation.

M. Dalembert has published a treatise, entitled, Recherches sur la Precession des Equinoxes, et sur la Nutation de la Terre dans le Systeme Newtonien, 4to. Paris, 1749. The calculations of this learned gentleman agree in general with Dr. Bradley's observations. But M. Dalembert finds, that the pole of the equator describes an ellipsis in the heavens, the ratio of whose axes is that of 4 to 3; whereas, according to Dr. Bradley, the curve described is either a circle or an ellipsis, the ratio of whose axis is as 9 to 8.

The several stars in each constellation, as in Taurus, Bootes, Hercules, &c, see under the proper article of each constellation, Taurus, Bootes, Hercules, &c.— To learn to know the several fixed stars by the globe, see Globe.

Circumpolar Stars. See Circumpolar.

Morning Star. See Morning.

Place of a Star. See Place.

Pole Star. See Pole.

Twinkling of the Stars. See Twinkling.

Unformed Stars. See Informes.

Catalogues of the stars, with their situations in right-ascension and declination, may be seen in Mr. Vince's and most other books on Astronomy; also in Zach's and Wollaston's tables, and the French Connoissance des Tems, &c.

Star, in Electricity, denotes the appearance of the electric matter on a point into which it enters. Beccaria supposes that the Star is occasioned by the difficulty with which the electric fluid is extricated from the air, which is an electric substance. See Brush.

Star, in Fortification, denotes a small fort, having 5 or more points, or saliant and re-entering angles, flanking one another, and their faces 90 or 100 feet long.

Star, in Pyrothechny, a composition of combustible matters; which being borne, or thrown aloft into the air, exhibits the appearance of a real star.—Stars are chiefly used as appendages to rockets, a number of them being usually inclosed in a conical cap, or cover, at the head of the rocket, and carried up with it to its utmost height, where the stars, taking fire, are spread around, and exhibit an agreeable spectacle.

To make Stars.—Mix 3lbs of saltpetre, 11 ounces of sulphur, one of antimony, and 3 of gunpowder dust: or, 12 ounces of sulphur, 6 of saltpetre, 5½ of gunpowder dust, 4 of olibanum, one of mastic, camphor, sublimate of mercury, and half an ounce of antimony and orpiment. Moisten the mass with gumwater, and make it into little balls, of the size of a chesnut; which dry either in the sun, or in the oven. These being set on fire in the air, will represent stars.

Star-*Board* denotes the right hand side of a ship, when a person on board stands with the face looking forward towards the head or fore part of the ship. In contradistinction from Larboard, which denotes the left hand side of the ship in the same circumstances.—They say, Starboard the helm, or Helm a starboard, when the man at the helm should put the helm to the right hand side of the ship.

Falling **Star**, or *Shooting* **Star**, a luminous meteor darting rapidly through the air, and resembling a Star falling.—The explication of this phenomenon has puzzled all philosophers, till the modern discoveries in electricity have led to the most probable account of it. Signior Beccaria makes it pretty evident, that it is an electrical appearance, and recites the following fact in proof of it. About an hour after sunset, he and some friends that were with him, observed a falling star directing its course towards them, and apparently growing larger and larger, but it disappeared not far from them; when it left their faces, hands, and clothes, the earth, and all the neighbouring objects, suddenly illuminated with a diffused and lambent light, not attended with any noise at all. During their surprise at this appearance, a servant informed them that he had seen a light shine suddenly in the garden, and especially upon the streams which he was throwing to water it. All these appearances were evidently electrical; and Beccaria was confirmed in his conjecture, that electricity was the cause of them, by the quantity of electric matter which he had seen gradually advancing towards his kite, which had very much the appearance of a falling star. Sometimes also he saw a kind of glory round the kite, which followed it when it changed its place, but left some light, for a small space of time, in the place it had quitted. Priestley's Elect. vol. 1, pa. 434, 8vo. See Ignis *Fatuus*.

Star-*fort*, or *Redoubt*, in Fortification. See Star, Redoubt, and Fort.

STARLINGS, or Sterlings, or *Jettees*, a kind of case made about a pier built on stilts, &c, to secure it. See Stilts.

STATICS, a branch of mathematics which considers weight or gravity, and the motion of bodies resulting from it. Those who define mechanics, the science of motion, make statics a part of it; viz, that part which considers the motion of bodies arising from gravity. Others make them two distinct doctrines; restraining mechanics to the doctrine of motion and weight, as depending on, or connected with, the power of machines; and statics to the doctrine of motion, considered merely as arising from the weight of bodies, without any immediate respect to machines. In this way, statics should be the doctrine or theory of motion; and mechanics, the application of it to machines.

For the laws of statics, see Gravity, Descent, &c.

STATION, or Stationary, in Astronomy, the position or appearance of a planet in the same point of the zodiac, for several days. This happens from the observer being situated on the earth, which is far out of the centre of their orbits, by which they seem to proceed irregularly; being sometimes seen to go forwards, or from west to east, which is their natural direction; sometimes to go backwards, or from east to west, which is their retrogradation; and between these two states there must be an intermediate one, where the planet appears neither to go forwards nor backwards, but to stand still, and keep the same place in the heavens, which is called her Station, and the planet is then said to be Stationary.

Apollonius Pergæus has shown how to find the stationary point of a planet, according to the old theory of the planets, which supposes them to move in epicycles; which was followed by Ptolemy in his Almag. lib. 12, cap. 1, and others, till the time of Copernicus. Concerning this, see Regiomontanus in Epitome Almagesti, lib. 12, prop. 1; Copernicus's Revolutiones Cœlest. lib. 5, cap. 35 and 36; Kepler in Tabulis Rudolphinis, cap. 24; Riccioli's Almag. lib. 7, sect. 5, cap. 2: Herman in Miscellan. Berolinens. pa. 197. Dr. Halley, Mr. Facio, Mr. Demoivre, Dr. Keil, and others have treated on this subject. See also the articles Retrograde and Stationary in this Dictionary.

Station, in Practical Geometry &c, is a place pitched upon to make an observation, or take an angle, or such like, as in surveying, measuring heights-and-distances, levelling, &c. An accessible height is taken from one station; but an inaccessible height or distance is only to be taken by making two stations, from two places whose distance asunder is known. In constructing maps of counties, provinces, &c, stations are fixed upon certain eminences &c of the country, and angles taken from thence to the several towns, villages, &c.—In surveying, the instrument is to be adjusted by the needle, or otherwise, to answer the points of the horizon at every station; the distance from hence to the last station is to be measured, and an angle is to be taken to the next station; which process repeated includes the chief practice of surveying.—In levelling, the instrument is rectified, or placed level at each station, and observations made forwards and backwards.

There is a method of measuring distances at one station, in the Philos. Trans. numb. 7, by means of a telescope. I have heard of another, by Mr. Ramsden; and have seen other ingenious ways by Mr. Green, &c, consisting of a permanent scale of divisions, placed at any point whose distance is required; then the number of divisions seen through the telescope, gives the distance sought.

Station-*Line*, in Surveying, and *Line of Station*, in Perspective. See Line.

STATIONARY, in Astronomy, the state of a planet when, to an observer on the earth, it appears for some time to stand still, or remain immoveable in the same place in the heavens. For as the planets, to such an observer, have sometimes a progressive motion, and sometimes a retrograde one, there must be some point between the two where they must appear stationary. Now a planet will be seen stationary, when the line that joins the centres of the earth and planet is constantly directed to the same point in the heavens, which is when it keeps parallel to itself. For all right lines drawn from any point of the earth's orbit, parallel to one another, do all point to the same star; the distance of these lines being insensible, in comparison of that of the fixed stars.

The planet Herschel is seen stationary at the distance of 104° from the sun; Saturn at somewhat more than 90°; Jupiter at the distance of 52°; and Mars at a much greater distance; Venus at 47°, and Mercury at 28°.

Herschel is stationary 12 days, Saturn 8, Jupiter 4, Mars 2, Venus $1\frac{1}{2}$, and Mercury $\frac{1}{2}$ a day: though the several stations are not always equal; because the orbits of the planets are not circles which have the sun in their centre.

STEAM, the vapour arising from water, or any other liquid or moist body, when considerably heated. Subterranean steams often affect the surface of the earth in a remarkable manner, and promote or prevent vegetation more than any thing else. It has been imagined that steams may be the generative cause of both minerals and metals, and of all the peculiarities of springs. See Philos. Trans. vol. 5, pa. 1154.—Of the use of the air to elevate the steams of bodies, see pa. 2048 and 297 ib.—Concerning the warm and fertilizing temperature and steams of the earth, see Philos. Trans. vol. 10, pa. 307 and 357. See also Dr. Hamilton "On the Ascent of Vapours."

The steam raised from hot water is an elastic fluid, which, like air, has its elasticity proportional to its density when the heat is the same, or proportional to the heat when the density is the same. The steam raised with the ordinary heat of boiling water, is nearly 3000 times rarer than water, or about $3\frac{1}{2}$ times rarer than air, having its elasticity about equal to that of the common air of the atmosphere. And by great heat it has been found that the steam may be expanded into 14000 times the space of water, or may be made about 5 times stronger than the atmosphere. But from some accidents that have happened, it appears that steam, suddenly raised from water, or moist substances, by the immediate application of strong heat, is vastly stronger than the atmosphere, or even than gunpowder itself. We have an instance of this in what happened at a foundery of cannon at Moorfields, when upon the hot metal first running into the mould in the earth, some small quantity of water in the bottom of it was suddenly changed into steam, which by its explosion, blew the foundery to atoms. I remember another such accident at a foundery at Newcastle; the founder having purchased, among some old brass, a hollow brass ball that had been used for many years as a valve in a pump, withinside of which it seems some water had insinuated itself; and having put it into his fire to melt, when it had become very hot, it suddenly burst with a prodigious noise, and blew the adjacent parts of the furnace in pieces.

The observations on the different degrees of temperature acquired by water in boiling, under different pressures of the atmosphere, and the formation of the vapour from water under the receiver of an air-pump, when, with the common temperatures, the pressure is diminished to a certain degree, have taught us that the expansive force of vapour or steam is different in the different temperatures, and that in general it increases in a variable ratio as the temperature is raised.

But there was wanting, on this important subject, a series of exact and direct experiments, by means of which, having given the degree of temperature in boiling water, we might know the expansive force of the steam rising from it; and vice versa. There was wanting also an analytical theorem, expressing the relation between the temperature of boiling water, and the pressure with which the force of its steam is in equilibrium. This has now been accomplished by M. Betancourt, an ingenious Spanish philosopher, the particulars of which are described in a memoir communicated to the French Academy of Sciences in 1790, and ordered to be printed in their collection of the Works of Strangers.

The apparatus which M. Betancourt makes use of, is a copper vessel or boiler, with its cover firmly soldered on. The cover has three holes, which close up with screws: the first is to put the water in and out; through the second passes the stem of a thermometer, which has the whole of its scale or graduations above the vessel, and its ball within, where it is immersed either in the water or the steam according to the different circumstances; through the third hole passes a tube forming a communication between the cavity of the boiler and one branch of an inverted syphon, which, containing mercury, acts as a barometer for measuring the pressure of the elastic vapour within the boiler. There is a fourth hole, in the side of the vessel, into which is inserted a tube, with a turn-cock, making a communication with the receiver of an air-pump, for extracting the air from the boiler, and to prevent its return.

The apparatus being prepared in good order, and distilled water introduced into the boiler by the first hole, and then stopped, as well as the end of the inverted syphon or barometer, M. Betancourt surrounded the boiler with ice, to lower the temperature of the water to the freezing point, and then extracting all the air from the boiler by means of the air-pump, the difference between the columns of mercury in the two branches of the barometer is the measure of the spring of the vapour arising from the water in that temperature. Then, lighting the fire below the boiler, he raised gradually the temperature of the water from 0 to 110 degrees of Reaumur's thermometer; being the same as from 32 to 212 degrees of Fahrenheit's; and for each degree of elevation in the temperature, he observed the height of the column of mercury which measured the elasticity or pressure of the vapour.

The results of M. Betancourt's experiments are contained in a table of four columns, which are but little different, according to the different quantities of water in the vessel. It is here observable, that the increase in the expansive force of the vapour, is at first very slow; but gradually increasing faster and faster, till at last it becomes very rapid. Thus, the strength of the vapour, at 80 degrees, is only equal to 28 French inches of mercury; but at 110 degrees it is equal to no less than 98 inches, that is 3 times and a half more for the increase of only 30 degrees of heat.

To express analytically the relation between the degrees of temperature of the vapour, and its expansive force, this author employs a method devised by M. Prony; which consists in conceiving the heights of the columns of mercury, measuring the expansive force, to represent the ordinates of a curve, and the degrees of heat as the abscisses of the same; making the ordinates equal to the sum of several logarithmic ones, which contain two indeterminates, and determining these quantities so that the curve may agree with a sufficient number of observations taken throughout their whole extent. Then constructing the curve which results immediately from the experiments, and that given by the formula, these two curves are found to coincide almost perfectly together; the small differences being doubtless owing to the little irregularities in the experiments and in dividing the scale; so that the phenomena may be considered as truly represented by the formula.

M. Betancourt made also experiments with the vapour from spirit of wine, similar to those made with water; constructing the curve, and giving the formula proper to the same. From which is derived this remarkable result, that, for any one and the same degree of heat, the strength of the vapour of spirit of wine, is to that of water, always in the same constant ratio, viz, that of 7 to 3 very nearly; the strength of the former being always 2⅓ times the strength of the latter, with the same degree of heat in the liquid.

Of the Formula, or Equation to the Curve.

The equation to the curve of temperature and pressure, denoting the relation between the abscisses and ordinates, or between the temperature of the vapour and its strength, is, for water,

$$y = b^{a + cx} - b^{a + c'x} - b^{e + c''x} + b^{e' + c'''x}.$$

Where x denotes the abscisses of the curve, or the degrees of Reaumur's thermometer; and y the corresponding ordinates, or the heights of the column of mercury in Paris inches, representing the strength or elasticity of the vapour answering to the number x of degrees of the thermometer. Then, by comparing this formula with a proper number of the experiments, the values of the constant quantities come out as below:

$b = 10$

$a = 0{\cdot}068831$

$c = 0{\cdot}019438$

$c' = 0{\cdot}013490$

$e = -\ 4{\cdot}689760$

$c'' = 0{\cdot}058622$

$e' = -\ 3{\cdot}937600$

$c''' = 0{\cdot}049220$

Hence it is evident by inspection, that the terms of the equation are very easy to calculate. For, b being the radix or root of the common system of logarithms, and all the terms on the second side of the equation being the powers of b, these terms are consequently the tabular natural numbers having the variable exponents for their logarithms. Now as x rises only to the first power, and is multiplied by a constant number, and another constant number being added to the product, gives the variable exponent, or logarithm; to which then is immediately found the corresponding natural number in the table of logarithms.

In the above formula, the two last terms may be entirely omitted, as very small, as far as to the 90th degree of the thermometer; and even above that temperature those two terms make but a small part of the whole formula.

And for the spirit of wine the formula is

$$y = b^{a + cx} + b^{a' + c'x} - b^{e + c''x} + b^{e' + c''x} - A.$$

Where x and y, as before, denote the absciss and ordinate of the curve, or the temperature and expansive force of the vapour from the spirit of wine; also the values of the constant quantities are as below:

$b =$ 10·
$a = -$ 0·04853
$c =$ 0·02393
$a' = -$ 0·63414
$c' = -$ 0·096532
$e = -$ 2·509542
$c'' =$ 0·046473
$e' = -$ 1·790192
$c''' =$ 0·029448
$A =$ 1·12647

This formula is of the same nature as the former, having also the like ease and convenience of calculation; and perhaps more so; as the second term $b^{a' + c'x}$, having its exponent wholly negative, soon diminishes to no value, so as to be omitted from the 10th degree of temperature; also the difference between the last two terms $- b^{e + c''x} + b^{e' + c''x}$ may be omitted till the 70th degree, for the same reason. So that, to the 10th degree of temperature the theorem is only $y = b^{a + cx} + b^{a' + c'x} - A$; and from the 10th to the 70th degree it is barely $y = b^{a + cx} - A$; after which, for the last 15 or 20 degrees, for great accuracy, the last two terms may be taken in.

A compendium of the table of the experiments here follows, for the vapour of both water and spirit of wine, the temperature by Reaumur's thermometer, and the barometer in French inches.

Table of the Temperature and Strength of the Vapour of Water and Spirit of Wine, by Reaumur's Thermometer and French Inches.

Deg. of Reau. Ther.	Height of the Barometer for	
	Vapour of Water.	Vapour of Spirit of Wine.
1	0·0176	0·0043
2	0·0346	0·0208
3	0·0538	0·0478
4	0·0747	0·0837
5	0·1038	0·1279
6	0·1211	0·1794
7	0·1508	0·2377
8	0·1741	0·3024
9	0·2073	0·3733
10	0·2304	0·4502
11	0·2681	0·5130
12	0·3039	0·6058
13	0·3419	0·7040
14	0·3877	0·8077
15	0·4258	0·9172
16	0·4778	1·0330
17	0·5208	1·1553
18	0·5730	1·2846
19	0·6283	1·4212
20	0·6872	1·5655
21	0·7497	1·7180
22	0·8159	1·8791
23	0·8863	2·0494
24	0·9610	2·2293
25	1·0402	2·4194
26	1·1239	2·6202
27	1·2127	2·8325
28	1·3068	3·0568
29	1·4065	3·2937
30	1·5019	3·5441
31	1·6333	3·8037
32	1·7413	4·0883
33	1·8671	4·3837
34	1·9980	4·6958
35	2·1374	5·0256
36	2·2846	5·3741
37	2·4401	5·6423
38	2·6045	6·1315
39	2·7780	6·5426
40	2·9711	6·9770
41	3·1544	7·4360
42	3·3583	7·9211
43	3·5735	8·4336
44	3·8005	8·9751
45	4·0399	9·5476
46	4·2922	10·1516
47	4·5582	10·7906
48	4·8386	11·4606
49	5·1346	12·1800
50	5·4453	12·9340
51	5·7706	13·7300
52	6·1194	14·5720
53	6·4834	15·4610
54	6·8667	16·4000
55	7·2798	17·3930
56	7·6948	18·4420
57	8·1412	19·5081
58	8·6221	20·6286
59	9·1071	21·6071
60	9·6280	23·0544
61	10·1767	24·3451
62	10·7098	25·6107
63	11·3602	27·1444
64	11·9976	28·6483
65	12·6687	30·2262
66	13·3743	31·8795
67	14·1161	33·6114
68	14·8958	35·4258

Deg. of Reau. Ther.	Height of the Barometer for	
	Vapour of Water.	Vapour of Spirit of Wine.
69	15·7153	37·3232
70	16·577	39·3076
71	17·482	41·3807
72	18·433	43·5465
73	19·433	45·8042
74	20·485	48·1589
75	21·587	50·6096
76	22·746	53·1593
77	23·965	55·8095
78	25·260	58·3968
79	26·588	61·3057
80	28·006	64·3524
81	29·455	67·4095
82	30·980	70·4967
83	32·575	73·7647
84	34 251	77·0764
85	35·984	80·4708
86	37 800	83·9351
87	39·697	87·4625
88	41·642	91·1366
89	43·730	94·6580
90	45·870	98·2764
91	48·092	
92	50·408	
93	52·785	
94	55·253	
95	57·801	
96	60·423	
97	63·108	
98	65·877	
99	68·692	
100	71·552	
101	74·444	
102	77·359	
103	80·268	
104	83·259	
105	85·992	
106	88·735	
107	91·367	
108	93·815	
109	96·039	
110	98·356	

M. Betancourt deduces several useful and ingenious consequences and applications from this course of experiments. He shows, for instance, that the effect of steam engines must, in general, be greater in winter than in summer; owing to the different degrees of temperature in the water of injection. And from the very superior strength of the vapour of spirit of wine, over that of water, he argues that, by trying other fluids, some may be found, not very expensive, whose vapour may be so much stronger than that of water, with the same degree of heat, that it may be substituted instead of water in the boilers of steam-engines, to the great saving in the very heavy expence of fuel: nay, he even declares, that spirit of wine itself might thus be employed in a machine of a particular construction, which, with the same quantity of fuel, and without any increase of expence in other things, shall produce an effect greatly superior to what is obtained from the steam of water. He makes several other observations on the working and improvement of steam-engines.

Another use of these experiments, deduced by M. Betancourt, is, to measure the height of mountains, by means of a thermometer, immersed in boiling water, which he thinks may be done with a precision equal, if not superior, to that of the barometer. As soon as I had obtained exact results of my experiments, says he, and was convinced that the degree of heat received by water depends absolutely on the pressure upon its surface, I endeavoured to compare my observations with such as have been made on mountains of different heights, to know what is the degree of heat which water can receive when the barometer stands at a determinate height; but from so few observations having been made of this kind, and the different ways employed in graduating instruments, it is difficult to draw any certain consequences from them.

The first observation which M. Betancourt compared with his experiments, is one mentioned in the Memoirs of the Academy of Sciences, anno 1740, page 92. It is there said, that M. Monnier having made water boil upon the mountain of Canigou, where the barometer stood at 20·18 inches, the thermometer immersed in this water stood at a point answering to 71 degrees of Reaumur: whereas in M. Betancourt's table of experiments, at an equal pressure upon the surface of the water, the thermometer stood at 73·7 degrees. This difference he thinks is owing partly to the want of precision in the observation, and partly to the different method of graduating the thermometer, and the neglect of purging the barometer tube of air.

M. Betancourt next compared his experiments with some observations made by M. Deluc on the tops of several mountains; in which, after reducing the scales of this gentleman to the same measures as his own, he finds a very great degree of coincidence between them. The following table contains a specimen of these comparisons, the instances being taken at random from Deluc's treatise on the Modifications of the Atmosphere.

Degrees of Heat in Boiling Water upon the Tops of Mountains, observed by Deluc.				Heat of the Water in M. Betancourt's Experiments.
Places of Observation.	Heat of the air.	Height of the Bar.	Heat of the Water by Th.	
Beaucaire - -	14¼	28·248	80·37	80·29
Geneva - - - -	12½	27·056	79·33	79·33
Grange Town	16¼	24·510	77·11	77·42
Lans le Bourg		24·145	77·18	77·14
Grange le F.	15	24·089	76·76	77·09
Grenairon - -	10¼	20·427	73·26	73·89
Glaciere de B.	6½	19·677	72·56	73·24

Where it is remarkable, that the difference between the two is of no consequence in such matters.

Many other advantages might be deduced from the exact knowledge of the effect which the pressure of the atmosphere has upon the heat which water can receive; one of which, M. Betancourt observes, is of too great importance in physics not to be mentioned. As soon as the thermometer became known to philosophers, almost every one endeavoured to find out two fixed points to direct them in dividing the scale of the instrument; having found that those of the freezing and boiling of water were nearly constant in different places, they gave these the preference over all others: but having discovered that water is capable of receiving a greater or less quantity of heat, according to the pressure of the atmosphere upon its surface, they felt the necessity of fixing a certain constant value to that pressure, which it was almost generally agreed should be equal to a column of 28 French inches of mercury. This agreement however did not remove all the difficulties. For instance, if it were required to construct at Madrid a thermometer that might be comparable with another made at Paris, the thing would be found impossible by the means hitherto known, because the barometer never rises so high as 27 inches at Madrid; and it was not certainly known how much the scale of the thermometer ought to be increased to have the point of boiling water in a place where the barometer is at 28 inches. But by making use of the foregoing observations, the thing appears very easy, and it is to be hoped that by the general knowledge of them, thermometers may be brought to great perfection, the accurate use of which is of the greatest importance in physics.

Besides, without being confined to the height of the barometer in the open air, in a given place, we may regulate a thermometer according to any one assigned heat of water, by means of such an apparatus as M. Betancourt's. For, in order to graduate a thermometer, having a barometer ready divided; it is evident that by knowing, from the foregoing table of experiments, the degree of heat answering to any one expansive force, we can thence assign the degree of the thermometer corresponding to a certain height of the barometer. A determination admitting of great precision, especially in the higher temperatures, where the motion of the barometer is so considerable in respect to that of the barometer.

Steam *Engine*, an engine originally invented for raising water by means of the expansive force of the steam, or vapour, produced from water or other liquids in a state of ebullition. This has been often called the fire-engine, because of the fire used in boiling the liquid: but the latter term has of late been properly confined to machines for extinguishing fires. The steam-engine is justly esteemed one of the most curious, important, and serviceable, mechanical inventions, not only of modern, but of any times; particularly when it is considered with regard to some of its late improvements, which render it applicable to all kinds of mill-work, to plaining, sawing, boring, and rolling machines, and indeed to almost every purpose that requires a powerful first mover, the energy of which may be modified at the pleasure of the mechanist.

The steam-engine for raising water is commonly a forcing pump, having its rod fixed to one end of a lever, which is worked by the weight or pressure of the atmosphere upon a piston, at the other end, a temporary vacuum being made below it, by suddenly condensing the steam, that had been let into the cylinder in which this piston works, by a jet of cold water thrown into it. A partial vacuum being thus made, the weight of the atmosphere presses down the piston, and raises the other end of the straight lever with the water from the well &c. Then immediately a hole is uncovered in the bottom of the cylinder, by which a fresh supply of hot steam rushes in from a boiler of water below it, which proves a counterbalance for the atmosphere above the piston, on which the weight of the pump rods at the other end of the lever carries that end down, and raises the piston of the steam cylinder. Immediately the steam hole is shut, and the cock opened for injecting the cold water into the cylinder of steam, which condenses it to water again, and thus making another vacuum below the piston, the atmosphere above it presses it down, and raises the pump-rods with another lift of water; and so on continually. This is the common principle: but there are also other modes of applying the force of the steam, as we shall see in the following short history of this invention and its various improvements.

The earliest account to be met with of the invention of this engine, is in the marquis of Worcester's small book entitled a Century of Inventions (being a description of 100 notable discoveries), published in the year 1663, where he proposed the raising of great quantities of water by the force of steam, raised from water by means of fire; and he mentions an engine of that kind, of his own contrivance, which could raise a continual stream like a fountain 40 feet high, by means of two cocks which were alternately and successively turned by a man to admit the steam, and to re-fill the vessel with cold water, the fire being continually kept up.

Sir Samuel Morland also wrote a book on this engine; in which he not only showed the practicability of the plan, but went so far as to calculate the power of different cylinders. This book is now extant in manuscript, in the Harleian collection, in the British Museum, the description of which is found in the improved Harleian catalogue, vol. iii, num. 5771; and it is also pointed out in the preface to that volume, sect. 22. Morland dates his invention in 1682, being 17 years prior to Savery's patent mentioned below. It was presented to the French king in 1683, at which time experiments were actually shown at St. Germain's. As Morland held places under king Charles the 2nd, it is natural to conclude that he would not have gone over to France, to offer the invention to Louis 14th, had he not found it slighted at home. It seems to have remained obscure in both countries till 1699, when Savery, who probably knew more of Morland's invention than he owned, obtained a patent; and in the very same year, M. Amontons proposed something similar to the French Academy seemingly as his own.

This invention then not meeting with encouragement, probably owing to the confused state of public affairs at that time, it was neglected, and lay dormant several years, until one Captain Thomas Savery, having read the marquis of Worcester's books, several years afterwards, tried many experiments on the force and power of steam; and at last hit upon a method of applying it to raise water. He then bought up and destroyed all the marquis's books that could be got, and claimed the honour of the invention to himself, and obtained a patent for it, pretending that he had discovered this secret of nature by accident. He contrived an engine which, after many experiments, he brought to some degree of usefulness, so as to raise water in small quantities; but he could not succeed in raising it to any great height, or in large quantities, for the draining of mines; to effect which by his method, the steam was required to be so strong as would have burst all his vessels; so that he was obliged to limit himself to raising the water only to a small height, or in small quantities. The largest engines he erected, was for the York-buildings Company in London, for supplying the inhabitants in the Strand and that neighbourhood with water.

The principle of this machine was as follows: H (fig. 3, pl. 33) represents a copper boiler placed on a furnace; E is a strong iron vessel, communicating with the boiler by means of a pipe at top, and with the main pipe AB by means of a pipe I at bottom; AB is the main pipe immersed in the water at B; D and C are two fixed valves, both opening upwards, one being placed above, and the other below the pipe of communication I. Lastly, at G is a cock that serves occasionally to wet and cool the vessel E, by water from the main pipe, and F is a cock in the pipe of communication between the vessel E and the boiler.

The engine is set to work, by filling the copper in part with water, and also the upper part of the main pipe above the valve C, the fire in the furnace being lighted at the same time. When the water boils strongly, the cock F is opened, the steam rushes into the vessel E, and expels the air from thence through the valve C. The vessel E thus filled, and violently heated by the steam, is suddenly cooled by the water which falls upon it by turning the cock C; the cock F being at the same time shut, to prevent any fresh accession of steam from the boiler. Hence, the steam in E becoming condensed, it leaves the cavity within nearly a vacuum: and therefore the pressure of the atmosphere at B forces the water through the valve D till the vessel E is nearly filled. The condensing cock G is then shut, and the steam cock F again opened; hence the steam, rushing into E, expels the water through the valve C, as it before did the air. Thus E becomes again filled with hot steam, which is again cooled and condensed by the water from G, the supply of steam being cut off by shutting F, as in the former operation: the water consequently rushes through D, by the pressure of the atmosphere at B, and E is again filled. This water is forced up the main pipe through C, by opening F and shutting G, as before. And thus it is easy to conceive, that by the alternate operation of opening and shutting the cocks, water will be continually raised, as long as the boiler continues to supply the steam.

For the sake of perspicuity, the drawing is divested of the apparatus that serves to turn the two cocks at once, and of the contrivances for filling the copper to the proper quantity. But it may be found complete, with a full account of its uses and application, in Mr. Savery's book intituled the Miner's Friend, and in Dr. Gregory's Mechanics, vol. 2. The engines of this construction were usually made to work with two receivers or steam vessels, one to receive the steam, while the other was raising water by the condensation. This engine has been since improved, by admitting the end of the condensing pipe G into the vessel E, by which means the steam is more suddenly and effectually condensed than by water on the outside of the vessel.

The advantages of this engine are, that it may be erected in almost any situation, that it requires but little room, and is subject to very little friction in its parts.—Its disadvantages are, that great part of the steam is condensed and loses its force upon coming into contact with the water in the vessel E, and that the heat and elasticity of the steam must be increased in proportion to the height that the water is required to be raised to. On both these accounts a large fire is required, and the copper must be very strong, when the height is considerable, otherwise there is danger of its bursting.

While captain Savery was employed in perfecting his engine, Dr. Papin of Marburg was contriving one on the same principles, which he describes in a small book published in 1707, intitled Ars Nova ad Aquam Ignis adminiculo efficacissime elevandam. Captain Savery's engine however was much more complete than that proposed by Dr. Papin.

About the same time also Mons. Amontons of Paris was engaged in the same pursuit: but his method of applying the force of steam was different from those beforementioned; for he intended it to drive or turn a wheel, which he called a fire-mill, that was to work pumps for raising water; but he never brought it to perfection. Each of these three gentlemen claimed the originality of the invention; but it is most probable they all took the hint from the book published by the marquis of Worcester, as before-mentioned.

In this imperfect state it continued, without further improvements, till the year 1705, when Mr. Newcomen, an ironmonger, and Mr. John Ceudley, a glazier, both of Dartmouth, contrived another way to raise water by steam, bringing the engine to work with a beam and piston, and where the steam, even at the greatest depths of mines, is not required to be greater than the pressure of the atmosphere: and this structure of the engine is that which has since been chiefly used. These gentlemen obtained a patent for the sole use of this invention, for 14 years. The first proposal they made for draining of mines by this engine, was in the year 1711; but they were very coldly received by many persons in the south of England, who did not understand the nature of it. In 1712 they came to an agreement with the owners of a colliery at Griff in Warwickshire, where they erected an engine with a cylinder of 22 inches diameter. At first they were under great difficulties in many things; but by the assistance of some good workmen they got all the parts put together in such a manner, as to answer their intention tolerably well: and this was the first engine of the kind erected in England. There was at first one man to attend the steam-cock, and another to attend the injection-cock: but they afterwards contrived a method of opening and shutting them by some small machinery connected with the working beam. The next engine erected by these patentees, was at a colliery in the county of Durham, about the year 1718, where was concerned, as an agent, Mr. Henry Beighton, F. R. S. and conductor of the Ladies' Diary from the year 1714 to the 1744: this gentleman, not approving of the intricate manner of opening and shutting the cocks by strings and catches, as in the former engine, substituted the hanging-beam for that purpose as at present used, and likewise made improvements in the pipes, valves, and some other parts of the engine.

In a few years afterwards, these engines became better understood than they had been; and their advantages, especially in draining of mines, became more apparent: and from the great number of them erected, they received additional improvements from different persons, till they arrived at their present degree of perfection: as will appear in the sequel, after we have a little considered the general principles of this engine, which are as follow.

Principles of the Steam Engine.

The principles on which this engine acts, are truly philosophical; and when all the parts of the machine are proportioned to each other according to these principles, it never fails to answer the intention of the engineer.

1. It has been proved in pneumatics, that the pressure

of the atmosphere on a square inch at the earth's surface, is about 14¾lb avoirdupois at a medium, or 11½lb on a circular inch, that is on a circle of an inch diameter. And,

2. If a vacuum be made by any means in a cylinder, which has a moveable piston suspended at one end of a lever equally divided, the air will endeavour to rush in, and will press down the piston, with a force proportionate to the area of the surface, and will raise an equal weight at the other end of the lever.

3. Water may be rarefied near 14000 times by being expanded into steam, and violently heated: the particles of it are so strongly repellent, as to drive away air of the common density, only by a heat sufficient to keep the water in a boiling state, when the steam is almost 3000 times rarer than water, or 3½ times rarer than air, as appears by an experiment of Mr. Beighton's: by increasing the heat, the steam may be rendered much stronger; but this requires great strength in the vessels. This steam may be again condensed into its former state by a jet of cold water dispersed through it; so that 14000 cubic inches of steam admitted into a cylinder, may be reduced into the space of one cubic inch of water only, by which means a partial vacuum is obtained.

4. Though the pressure of the atmosphere be about 14¾ pounds upon every square inch, or 11½ pounds upon a circular inch; yet, on account of the friction of the several parts, the resistance from some air which is unavoidably admitted with the jet of cold water, and from some remainder of steam in the cylinder, the vacuum is very imperfect, and the piston does not descend with a force exceeding 8 or 9 pounds upon every square inch of its surface.

5. The gallon of water of 282 cubic inches weighs 10⅕ pounds avoirdupois, or a cubic foot 62½ pounds, or 1000 ounces. The piston being pressed by the atmosphere with a force proportional to its area in inches, multiplied by about 8 or 9 pounds, depresses that end of the lever, and raises a column of water in the pumps of equal weight at the other end, by means of the pump-rods suspended to it. When the steam is again admitted, the pump-rods sink by their superior weight, and the piston rises; and when that steam is condensed, the piston descends, and the pump-rods lift; and so on alternately as long as the piston works.

It has been observed above, that the piston does not descend with a force exceeding 8 or 9 pounds on every square inch of its surface; but by reason of accidental frictions, and alterations in the density of the air, it will be safest, in calculating the power of the cylinder, to allow something less than 8 pounds for the pressure of the atmosphere upon every square inch, viz, 7lb. 10 oz. = 7·64lb, or just 6lb. upon every circular inch; and it being allowed that the gallon of water, of 282 cubic inches, weighs 10⅕lb, from these premises the dimensions of the cylinder, pumps, &c, for any steam-engine, may be deduced as follows:—

Suppose

c = the cylinder's diameter in inches,
p = the pump's ditto,
f = the depth of the pit in fathoms,
g = gallons drawn by a stroke of 6 feet or a fathom,
h = the hogsheads drawn per hour,
s = the number of strokes per minute.

Then c^2 is the area of the cylinder in circular inches, therefore $6c^2$ is the power of the cylinder in pounds.

And $\frac{p^2 \times \cdot 7854 \times 72}{282}$ or $\frac{1}{5}p^2$ is $= g$ the gallons contained in one fathom or 72 inches of any pump; which multiplied by f fathoms, gives $\frac{1}{5}p^2f$ for the gallons contained in f fathoms of any pump whose diameter is p. Hence $\frac{1}{5}p^2f \times 10\frac{1}{5}$lb. gives $2p^2f$ nearly, for the weight in pounds of the column of water which is to be equal to the power of the cylinder, which was before found equal to $6c^2$. Hence then we have the 2d equation, viz, $6c^2 = 2p^2f$, or $3c^2 = p^2f$; the first equation being $\frac{1}{5}p^2 = g$, or $p^2 = 5g$. From which two equations, any particular circumstance may be determined.

Or if, instead of 6lb, for the pressure of the air on each circular inch of the cylinder, that force be supposed any number as a pounds; then will the power of the cylinder be ac^2, and the 2d equation becomes $ac^2 = 2p^2f = 10fg$, by substituting $5g$ instead of p^2.

And farther, $63h = 60gs$, or $21h = 20gs$.

From a comparison of these equations, the following theorems are derived, which will determine the size of the cylinder and pumps of any steam-engine capable of drawing a certain quantity of water from any assigned depth, with the pressure of the atmosphere on each circular inch of the cylinder's area.

These theorems are more particularly adapted to one pump in a pit. But it often happens in practice, that an engine has to draw several pumps of different diameters from different depths; and in this case, the square of the diameter of each pump must be multiplied by its depth, and double the sum of all the products will be the weight of water drawn at each stroke, which is to be used instead of $2p^2f$ for the power of the cylinder.

The following is a table, calculated from the foregoing theorems, of the powers of cylinders from 30 to 70 inches diameter; and the diameters and lengths of pumps which those cylinders are capable of working, from a 6 inch bore to that of 20 inches, together with the quantity of water drawn per stroke and per hour, allowing the engine to make 12 strokes of 6 feet per minute, and the pressure of the atmosphere at the rate of 7lb 10oz per square inch, or 6 lb per circular inch.

A TABLE *of* THEOREMS *for the readier computing the Powers of a* STEAM-ENGINE.

1 $a = \frac{2fp^2}{c^2} = \frac{10fg}{c^2} = \frac{21fh}{2c^2s}$

2 $c = \sqrt{\frac{2fp^2}{a}} = \sqrt{\frac{10fg}{a}} = \sqrt{\frac{21fh}{2as}}$

3 $f = \frac{ac^2}{2p^2} = \frac{ac^2}{10g} = \frac{2ac^2s}{21h}$

4 $g = \frac{p^2}{5} = \frac{ac^2}{10f} = \frac{21h}{20s}$

5 $h = \frac{4p^2s}{21} = \frac{20gs}{21} = \frac{2ac^2s}{21f}$

6 $p = \sqrt{5g} = \sqrt{\frac{ac^2}{2g}} = \sqrt{\frac{21h}{4s}}$

7 $s = \frac{21h}{4p^2} = \frac{21h}{20g} = \frac{21fh}{3ac^2}$

TABLE *of the Power and Effects of* STEAM-ENGINES, *allowing* 12 *Strokes, of* 6 *Feet long each, per Minute, and the Pressure of the Air* 7*lb* 10*oz per Square Inch, or* 6*lb per Circular Inch.*

	The Diameters of the Pumps in Inches.															Power of the cylinders and weight of water in pounds.
The Diameters of the Cylinders in Inches.	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	
30	75	55	42	33	27	22	19	16	14	12	10	·	·	·	·	5400
31	80	58	45	35	29	24	20	17	15	13	11	10	·	·	·	5766
32	83	61	47	37	30	25	21	18	16	13	12	10	·	·	·	6144
33	90	67	51	40	32	27	22	19	17	14	13	11	10	·	·	6534
34	94	70	53	42	34	28	23	20	18	15	14	12	10	·	·	6936
35	102	75	57	45	37	30	26	22	19	16	14	13	11	·	·	7350
36	·	79	[illegible]	48	39	32	27	23	20	17	15	14	12	10	·	7776
37	·	84	64	51	41	34	29	24	21	18	16	14	12	11	10	8214
38	·	88	68	53	43	35	30	26	22	19	17	15	13	12	10	8664
39	·	93	71	56	45	37	32	27	23	20	18	16	14	12	11	9126
40	·	98	75	59	48	39	34	28	24	21	19	17	15	13	12	9600
42	·	108	83	65	53	43	38	31	27	23	21	18	16	14	13	10584
44	·	·	90	71	58	48	41	34	30	26	23	20	18	16	14	11616
46	·	·	99	78	63	52	45	37	33	29	25	21	19	17	16	12696
48	·	·	·	85	69	57	49	41	35	31	27	24	21	19	17	13824
50	·	·	·	92	75	62	53	44	38	34	29	26	23	21	19	15000
52	·	·	·	100	81	67	57	48	41	36	31	28	25	22	20	16224
54	·	·	·	·	87	72	61	52	44	38	34	30	27	24	22	17496
56	·	·	·	·	94	78	66	56	48	42	37	32	29	26	23	18816
58	·	·	·	·	101	83	70	59	51	44	39	34	31	28	25	20184
60	·	·	·	·	·	89	75	63	55	48	42	37	33	30	27	21600
62	·	·	·	·	·	95	80	68	58	51	45	39	35	32	28	23064
64	·	·	·	·	·	·	85	72	62	54	48	42	38	34	30	24546
66	·	·	·	·	·	·	90	77	66	57	51	45	40	36	32	26676
68	·	·	·	·	·	·	96	82	70	61	54	48	42	38	34	27744
70	·	·	·	·	·	·	·	86	75	64	57	50	45	40	36	29400
Quan. drawn at one stroke in gallons.	7·2	10	13	16·2	20	24·2	28·8	33·8	39·2	45	51·2	57·8	64·8	72·2	80	
Quan. drawn in one hour in hogsheads.	82	114	148	184	228	276	328	385	447	513	583	659	738	823	912	
Diameter of pumps.	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	

Let us now describe the several parts of an engine, and exemplify the application of the foregoing principles, in the construction of one of the completest of the modern engines. See fig. 4. pl. 33.

A represents the fire-place under the boiler, for the boiling of the water, and the ash-hole below it.

B, the boiler, filled with water about 3 feet above the bottom, made of iron plates.

C, the steam pipe, through which the steam passes from the boiler into the receiver.

D, the receiver, a close iron vessel, in which is the regulator or steam-cock, which opens and shuts the hole of communication at each stroke.

E, the communication pipe between the receiver and the cylinder; it rises 5 or 6 inches up, in the inside of the cylinder bottom, to prevent the injected water from descending into the receiver.

F, the cylinder, of cast iron, about 10 feet long, bored smooth in the inside; it has a broad flanch in the middle on the outside, by which it is supported when hung in the cylinder-beams.

G, the piston, made to fit the cylinder exactly: it has a flanch rising 4 or 5 inches upon its upper surface, between which and the side of the cylinder a quantity of junk or oakum is stuffed, and kept down by weights, to prevent the entrance of air or water and the escaping of steam.

H, the chain and piston shank, by which it is connected to the working beam.

I I, the working beam or lever: it is made of two or more large logs of timber, bent together at each end, and kept at the distance of 8 or 9 inches from each other in the middle by the gudgeon, as represented in the Plate. The arch heads, II, at the ends, are for giving a perpendicular direction to the chains of the piston and pump-rods.

K, the pump-rod which works in the sucking pump. L, and draws the water from the bottom of the pit to the surface.

M, a cistern, into which the water drawn out of the pit is conducted by a trough, so as to keep it always full: and the superfluous wateris carried off by another trough.

N, the jack-head pump, which is a sucking-pump wrought by a small lever or working-beam, by means of a chain connected to the great beam or lever near the arch *g* at the inner end, and the pump-rod at the outer end. This pump commonly stands near the corner of the front of the house, and raises the column of water up to the cistern O, into which it is conducted by a trough.

O, the jack-head cistern for supplying the injection, which is always kept full by the pump N: it is fixed so high as to give the jet a sufficient velocity into the cylinder when the cock is opened. This cistern has a pipe on the opposite side for conveying away the superfluous water.

P P, the injection-pipe, of 3 or 4 inches diameter, which turns up in a curve at the lower end, and enters the cylinder bottom: it has a thin plate of iron upon the end *d*, with 3 or 4 adjutage holes in it, to prevent the jet of cold water of the jack-head cistern from flying up against the piston, and yet to condense the steam each stroke, when the injection-cock is open.

e, a valve upon the upper end of the injection-pipe within the cistern, which is shut when the engine is not working, to prevent any waste of the water.

f, a small pipe which branches off from the injection-pipe, and has a small cock to supply the piston with a little water to keep it air-tight.

Q, the working plug, suspended by a chain to the arch *g* of the working beam. It is usually a heavy piece of timber, with a slit vertically down its middle, and holes bored horizontally through it, to receive pins for the purpose of opening and shutting the injection and steam cocks, as it ascends and descends by the motion of the working beam.

h, the handle of the steam-cock or regulator, it is fixed to the regulator by a spindle which comes up through the top of the receiver. The regulator is a circular plate of brass or cast iron, which is moved horizontally by the handle *h*, and opens or shuts the communication at the lower end of the pipe E within the receiver. It is represented in the plate by a circular dotted line.

i i, the spanner, which is a long rod or plate of iron for communicating motion to the handle of the regulator: to which it is fixed by means of a slit in the latter, and some pins put through to fasten it.

k l, the vibrating lever, called the Y, having the weight *k* at one end and two legs at the other end. It is fixed to an horizontal axis, moveable about its centre-pins or pivots *mn*, by means of the two shanks *op* fixed to the same axis, which are alternately thrown backwards and forwards by means of two pins in the working plug; one pin on the outside depressing the shank *o*, throws the loaded end *k* of the Y from the cylinder into the position represented in the plate, and causes the leg *l* to strike against the end of the spanner; which forcing back the handle of the regulator or steam cock, opens the communication, and permits the steam to fly into the cylinder. The piston immediately rising by the admission of the steam, the working beam I I rises; which also raises the working-plug, and another pin which goes through the slit raises the shank *p*, which throws the end *k* of the Y towards the cylinder, and, striking the end of the spanner, forces it forward, and shuts the regulator steam-cock.

q r, the lever for opening and shutting the injection-cock, called the F. It has two toes from its centre, which take between them the key of the injection-cock. When the working-plug has ascended nearly to its greatest height, and shut the regulator, a pin catches the end *q* of the F and raises it up, which opens the injection-cock, admits a jet of cold water to fly into the cylinder, and condensing the steam, makes a vacuum; then the pressure of the atmosphere bringing down the piston in the cylinder, and also the plug-frame, another pin fixed in it catches the end of the lever in its descent, and, by pressing it down, shuts the injection-cock, at the same time the regulator is opened to admit steam, and so on alternately; when the regulator is shut the injection is open, and when the former is open the latter is shut.

R, the hot-well, a small cistern made of planks, which receives all the waste water from the cylinder.

S, the sink-pit to convey away the water which is injected into the cylinder at each stroke. Its upper end is even with the inside of the cylinder bottom; its lower end has a lid or cover moveable on a hinge which serves as a valve to let out the injected water, and shuts close each stroke of the engine, to prevent the water being forced up again when the vacuum is made.

T, the feeding-pipe, to supply the boiler with water from the hot-well. It has a cock to let in a large or small quantity of water as occasion requires, to make up for what is evaporated: it goes nearly down to the boiler bottom.

U, two gage-cocks, the one larger than the other, to try when a proper quantity of water is in the boiler: on opening the cocks, if one give steam and the other water, it is right; if they both give steam, there is too little water in the boiler; and if they both give water, there is too much.

W, a plate which is screwed on to a hole on the side of the boiler, to allow a passage into the boiler for the convenience of cleaning or repairing it.

X, the steam-clack or puppet valve, a brass valve on the top of a pipe opening into the boiler, to let off the steam when it is too strong. It is loaded with lead, at the rate of one pound to an inch square; and when the steam is nearly strong enough to keep it open, it will do for the working of the engine.

f, the snifting valve, by which the air is discharged from the cylinder each stroke, which was admitted with the injection, and would otherwise obstruct the due operation of the engine.

tt, the cylinder-beams; which are strong joists going through the house for supporting the cylinder.

v, the cylinder cap of lead, soldered on the top of the cylinder, to prevent the water upon the piston from flashing over when it rises too high.

w, the waste-pipe, which conducts the superfluous water from the top of the cylinder to the hot-well.

xx, iron bars, called the catch-pins, fixed horizontally through each arch-head, to prevent the beam descending too low in case the chain should break.

yy, two strong wooden springs, to weaken the blow given by the catch-pins when the stroke is too long.

zz, two friction-wheels, on which the gudgeon or centre of the great beam is hung; they are the third or fourth part of a circle, and move a little each way as the beam vibrates. Their use is to diminish the friction of the axis, which, in so heavy a lever, would otherwise be very great.

When this engine is to be set to work, the boiler must be filled about 3 or 4 feet deep with water, and a large fire made under it; and when the steam is found to be of a sufficient strength by the puppet-clack, then by thrusting back the spanner, which opens the regulator or steam-cock, the steam is admitted into the cylinder, which raises the piston to the top of the cylinder, and forces out all the air at the snifting valve; then by turning the key of the injection-cock, a jet of cold water is admitted into the cylinder, which condenses the steam and produces a vacuum; the atmosphere then pressing upon the piston, forces it down to the lower part of the cylinder, and makes a stroke by raising the column of water at the other end of the beam. After two or three strokes are made in this manner, by a man opening and shutting the cocks to try if they be right, the pins may then be put into the pin-holes in the working plug, and the engine left to turn the cocks of itself; which it will do with greater exactness than can be done by a man.

There are in some engines, methods of shutting and opening the cocks different from the one above described, but perhaps none better adapted to the purpose; and as the principles on which they all act are originally the same, any difference in the mechanical construction of the small machinery will have no influence of consequence on the total effect of the grand machine.

The furnace or fire-place should not have the bars so close as to prevent the free admission of fresh air to the fire, nor so open as to permit the coals to fall easily through them; for which purpose two inches or thereabouts is sufficient for the distance betwixt the bars. The size of the furnace depends on that of the boiler; but in every case the ash-hole ought to be capacious to admit the air, and the greater its height the better. If the flame is conducted in a flue or chimney round the outside of the boiler, or in a pipe round the inside of it, it ought to be gradually diminished from the entrance at the furnace to its egress at the chimney; and the section of the chimney at that place should not exceed the section of the flue or pipe, and should also be somewhat less at the chimney-top.

The boiler or vessel in which the water is rarefied by the force of fire, may be made of iron plates or cast iron, or such other materials as can withstand the effects of the fire, and the elastic force of the steam. It may be considered as consisting of two parts; the upper part which is exposed to the steam, and the under part which is exposed to the fire. The form of the latter should be such as to receive the full force of the fire in the most advantageous manner, so that a certain quantity of fuel may have the greatest possible effect in heating and evaporating the water; which is best done by making the sides cylindrical, and the bottom a little concave, and then conducting the flame by an iron flue or pipe round the inside of the boiler beneath the surface of the water, before it reach the chimney. For, by this means, after the fire in the furnace has heated the water by its effect on the bottom, the flame heats it again by the pipe being wholly included in the water, and having every part of its surface in contact with it; which is preferable to carrying it in a flue or chimney round the outside of the boiler, as a third or a half of the surface of the flame only could be in contact with the boiler, the other being spent upon the brick-work. This lower cylindric part may be less in its diameter than the upper part, and may contain from 4 to 6 feet perpendicular depth of water in it.

The upper part of the boiler is best made hemispherical, for resisting the elasticity of the steam; yet any other form may do, provided it be of sufficient strength for the purpose. The quick motion of the engine depends much on the capaciousness of the boiler-top; for if it be too small, it requires the steam to be heated to a great degree, to increase its elastic force so much as to work the engine. If the top is so capacious as to contain eight or ten times the quantity of steam used each stroke, it will require no more fire to preserve its elasticity than is sufficient to keep the water in a proper state of boiling; this, therefore, is the best size for a boiler top. If the diameter of the cylinder be c, and works a six-foot stroke, and the diameter of the boiler be supposed b, then

$$200c^2 = b^3, \text{ or } b = \sqrt[3]{200c^2}.$$

The effect of the injection in condensing the steam in the cylinder, depends on the height of the reservoir and the diameter of the adjutage. If the engine makes a 6 feet stroke, then the jackhead cistern should be 12 feet perpendicular above the bottom of the cylinder or the adjutage. The size of the adjutage may be from 1 to 2 inches in diameter; or if the cylinder be very large, it is proper to have three or four holes rather than one large one, in order that the jet may be dispersed the more effectually over the whole area of the cylinder. The injection pipe, or pipe of conduct, should be so large as to supply the injection freely with water; if the diameter of the injection pipe be called p, and the diameter of the adjutage, a, then $4a^2 = p^2$, and $a^2 = \frac{1}{4}p^2$, or $a = \frac{1}{2}p$.

For a further account of these engines, see Desaguliers's Exp. Philos. vol. 2, sect. 14, pa. 465, &c; or for an abstract, Martin's Phil. Brit. number 461, or Nicholson's Nat. Philos. pa. 83 &c. And for an account of the improvement made in the fire-engine by Mr. Payne, see Philos. Trans. number 461, or Martin's Philos. Brit. pa. 87 &c. See also Gregory's Mechanics, vol. 2, for a particular description of different steam-engines, containing the latest improvements that they have undergone.

Mr. Blakey communicated to the Royal Society, in 1752, remarks on the best proportions for steam-engine cylinders of a given content: and Mr. Smeaton describes an engine of this kind, invented by Mr. De Moura of Portugal, being an improvement of Savery's construction, to render it capable of working itself: for both which accounts, see Philos. Trans. vol. 47, art. 29 and 72.

We are informed in the new edit. of the Biograph. Brit. under the article Brindley, that in 1756 this gentleman, so well known for his concern in our inland navigations, undertook to erect a steam-engine near Newcastle-under-Line, on a new plan. The boiler of it was made with brick and stone, instead of iron plates, and the water was heated by iron flues of a peculiar construction; by which contrivances the consumption of fuel, necessary for working a steam-engine, was reduced one half. He introduced also into his engine wooden cylinders, made in the manner of cooper's ware, instead of iron ones; the former being both cheaper and more easily managed in the shafts: and he likewise substituted wood for iron in the chains which worked at the end of the beam. He had formed designs of introducing other improvements into the construction of this useful engine; but was discouraged by obstacles that were thrown in his way.

Mr. Blakey, some years ago, obtained a patent for his improvement of Savery's steam-engine, by which it is excellently adapted for raising water out of ponds, rivers, wells, &c, and for forcing it up to any height wanted for supplying houses, gardens, and other places; though it has not power sufficient to drain off the water from a deep mine. The principles of his construction are explained by Mr. Ferguson, in the Supplement to his Lectures, pa. 19; and a more particular description of it, accompanied with a drawing, is given by the patentee himself in the Gentleman's Magazine for 1769, pa. 392.

Mr. Blakey, it is said, is the first person who ever thought of making use of air as an intermediate body between steam and water; by which means the steam is always kept from touching the water, and consequently from being condensed by it: and on this new principle he has obtained a patent. The engine may be built at a trifling expence, in comparison of the common steam-engine now in use; it will seldom need repairs, and will not consume half so much fuel. And as it has no pumps with pistons, it is clear of all their friction; and the effect is equal to the whole strength or compressive force of the steam; which the effect of the common engine never is, on account of the great friction of the pistons in their pumps.

Ever since Mr. Newcomen's invention of the steam fire-engine, the great consumption of fuel with which it is attended, has been complained of as an immense drawback on the profits of our mines. It is a known fact, that every engine of considerable size consumes to the amount of three thousand pounds worth of coals in a year. Hence many of our engineers have endeavoured, in the construction of these engines, to save fuel. For this purpose, the fire-place has been diminished, the flame has been carried round from the bottom of the boiler in a spiral direction, and conveyed through the body of the water in a tube before its arrival at the chimney; some have used a double boiler, so that fire might act in every possible point of contact; and some have built a moor-stone boiler, heated by three tubes of flame passing through it. But the most important improvements which have been made in the steam-engine for more than forty years past, we owe to the skill of Mr. James Watt; of which we shall give some account: premising, that the internal structure of his new engines so much resembles that of the common ones, that those who are acquainted with them will not fail to understand the mechanism of his from the following description: He has contrived to observe a uniform heat in the cylinder of his engines, by suffering no cold water to touch it, and by protecting it from the air, or other cold bodies, by a surrounding case filled with steam, or with hot air or water, and by coating it over with substances that transmit heat slowly. He makes his vacuum to approach nearly to that of the barometer, by condensing the steam in a separate vessel, called the condenser, which may be cooled at pleasure without cooling the cylinder, either by an injection of cold water, or by surrounding the condenser with it, and generally by both. He extracts the injection water, and detached air, from the cylinder or condenser by pumps, which are wrought by the engine itself, or blows them out by the steam. As the entrance of air into the cylinder would stop the operation of the engines, and as it is hardly to be expected that such enormous pistons as those of steam-engines can move up and down, and yet be absolutely tight as in the common engines; a stream of water is kept always running upon the piston, which prevents the entry of the air: but this mode of securing the piston, though not hurtful in the common ones, would be highly prejudicial to the new engines. Their piston is therefore made more accurately; and the outer cylinder, having a lid, covers it, the steam is introduced above the piston; and when a vacuum is produced under it, acts upon it by its elasticity, as the atmosphere does upon common engines by its gravity. This way of working effectually excludes the air from the inner cylinder, and gives the advantage of adding to the power, by increasing the elasticity of the steam.

In Mr. Watt's engines, the cylinder, the great beams, the pumps, &c, stand in their usual positions. The cylinder is smaller than usual, in proportion to the load, and is very accurately bored. In the most complete engines, it is surrounded at a small distance, with another cylinder, furnished with a bottom and a lid. The interstice between the cylinders communicates with the boilers by a large pipe, open at both ends: so that it is always filled with steam, and thereby maintains the inner cylinder always of the same heat with the steam, and prevents any condensation within it, which would be more detrimental than an equal condensation in the outer one. The inner cylinder has a bottom and piston as usual: and as it does not reach up quite to the lid of the outer cylinder, the steam in the interstice has always free access to the upper side of the piston. The lid of the outer cylinder has a hole in its middle; and the piston rod, which is truly cylindrical, moves up and down through that hole, which is kept steam-tight by a collar of oakum screwed down upon it. At the bottom of the inner cylinder, there are two regulating valves, one of which admits the steam to pass from the interstice into the inner cylinder below the piston, or shuts it out at pleasure: the other opens or shuts the end of a pipe, which leads to the condenser. The condenser consists of one or more pumps furnished with clacks and buckets (nearly the same as in common pumps) which are wrought by chains fastened to the great working beam of the engine. The pipe, which comes from the cylinder, is joined to the bottom of these pumps, and the whole condenser stands immersed in a cistern of cold water supplied by the engine. The place of this cistern is either within the house or under the floor, between the cylinder and the lever wall; or without the house between that wall and the engine shaft, as conveniency may require. The condenser being exhausted of air by blowing, and both the cylinders being filled with steam, the regulating valve which admits the steam into the inner cylinder is shut, and the other regulator which communicates with the condenser is opened, and the steam rushes into the vacuum of the condenser with violence: but there it comes into contact with the cold sides of the pumps and pipes, and meets a jet of cold water, which was opened at the same time with the exhaustion regulator; these instantly deprive it of its heat, and reduce it to water; and the vacuum remaining perfect, more steam continues to rush in, and is condensed until the inner cylinder is exhausted. Then the steam which is above the piston, ceasing to be counteracted by that which was below it, acts upon the piston with its whole elasticity, and forces it to descend to the bottom of the cylinder, and so raises the buckets of the pumps which are hung to the other end of the beam. The exhaustion regulator is now shut, and the steam one opened again, which, by letting in the steam, allows the piston to be pulled up by the superior weight of the pump rods; and so the engine is ready for another stroke.

But the nature of Mr. Watt's improvement will be perhaps better understood from the following description of it as referred to a figure.—The cylinder or steam vessel A, of this engine (fig. 5, pl. 33), is shut at bottom and opened at top as usual; and is included in an outer cylinder or case BB, of wood or metal, covered with materials which transmit heat slowly. This case is at a small distance from the cylinder, and close at both ends. The cover C has a hole in it, through which the piston rod E slides; and near the bottom is another hole F, by which the steam from the boiler has always free entrance into this case or outer cylinder, and by the interstice GG between the two cylinders has access to the upper side of the piston HH. To the bottom of the inner cylinder A is joined a pipe I, with a cock or valve K, which is opened and shut when necessary, and forms a passage to another vessel L called a condenser, made of thin metal. This vessel is immersed in a cistern M full of cold water, and it is contrived so as to expose a very great surface externally to the water, and internally to the steam. It is also made air-tight, and has pumps N wrought by the engine, which keep it always exhausted of air and water.

Both the cylinders A and BB being filled with steam, the passage K is opened from the inner one to the condenser L, into which the steam violently rushes by its elasticity, because that vessel is exhausted; but as soon as it enters it, coming into contact with the cold matter of the condenser, it is reduced to water, and, the vacuum still remaining, the steam continues to rush in till the inner cylinder A below the piston is left empty. The steam which is above the piston, ceasing to be counteracted by that which is below it, acts upon the piston HH, and forces it to descend to the bottom of the cylinder, and so raises the bucket of the pump by means of the lever. The passage K between the inner cylinder and the condenser is then shut, and another passage O is opened, which permits the steam to pass from the outer cylinder, or from the boiler into the inner cylinder under the piston; and then the superior weight of the bucket and pump rods pulls down the outer end of the lever or great beam, and raises the piston, which is suspended to the inner end of the same beam.

The advantages that accrue from this construction are, first, that the cylinder being surrounded with the steam from the boiler, it is kept always uniformly as hot as the steam itself, and is therefore incapable of destroying any part of the steam, which should fill it, as the common engines do. Secondly, the condenser being kept always as cold as water can be procured, and colder than the point at which it boils in vacuo, the steam is perfectly condensed, and does not oppose the descent of the piston; which is therefore forced down by the full power of the steam from the boiler, which is somewhat greater than that of the atmosphere.

In the common steam-engines, when they are loaded to 7 pounds upon the inch, and are of a middle size, the quantity of steam which is condensed in restoring to the cylinder the heat which it had been deprived of by the former injection of cold water, is about one full of the cylinder, besides what it really required to fill that vessel; so that twice the full of the cylinder is employed to make it raise a column of water equal to about 7 pounds for each square inch of the piston: or, to take it more simply, a cubic foot of steam raises a cubic foot of water about 8 feet high, besides overcoming the friction of the engine, and the resistance of the water to motion.

In the improved engine, about one full and a fourth of the cylinder is required to fill it, because the steam is one-fourth more dense than in the common engine. This engine raises a load equal to 12 pounds and a half upon the square inch of the piston; and each cubic foot of steam of the density of the atmosphere, raises one cubic foot of water 22 feet high. The working of these engines is more regular and steady than the common ones, and from what has been said, their other advantages seem to be very considerable.

It is said, that the savings amount at least to two-thirds of the fuel, which is an important object, especially where coals are dear. The new engines will raise from twenty thousand to twenty-four thousand cubic feet of water, to the height of 24 feet by one hundred weight of good pit coal: and Mr. Watt has proposed to produce engines on the same principles, though somewhat differing in construction, which will require still much less fuel, and be more convenient for the purposes of mining, than any kind of engine yet used. * Mr. Watt has also contrived a kind of mill wheel, which turns round by the power of steam exerted within it.

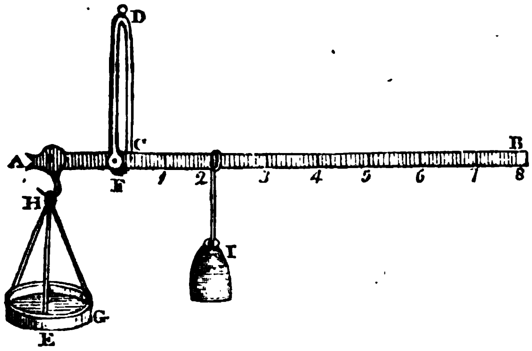
The improvements above recited were invented by Mr. James Watt, at Glasgow, in Scotland, in 1764: he obtained the king's letters patent for the sole use of his invention in 1768; but meeting with difficulties in the execution of a large machine, and being otherwise employed, he laid aside the undertaking till the year 1774, when, in conjunction with Mr. Boulton near Birmingham, he completed both a reciprocating and rotative or wheel engine. He then applied to parliament for a prolongation of the term of his patent, which was granted by an act passed in 1775. Since that time, Mr. Watt and Mr. Boulton have erected several engines in various parts of England. The terms they offer to the public are, to take in lieu of all profits, one-third part of the annual savings in fuel, which their engine makes when compared with a common engine of the same dimensions in the neighbourhood. The engines are built at the expence of those who use them, and Messrs. Boulton and Watt furnish such drawings, directions, and attendance, as may be necessary to enable a resident engineer to complete the machine. See the appendix to Pryce's Mineralogia, &c, 1778.

Mr. Hornblower and others have also made ingenious improvements on the steam-engine.

See another view of a steam engine at fig. 3, pl. 31.

Steam-*Boat*, *&c.* Besides the steam-engines employed for drawing the water out of deep-mines, (and without the discovery of such machines the country now would have been almost deprived of the use of coals,) steam has been gradually applied, as a power, to give motion to various other machines, and for other purposes where great and œconomical powers are required, with the best effects; such as saw-mills, pile-driving, deepening and cleansing of rivers and canals, the draft of numerous waggons on rail-ways, with many other useful and beneficial purposes; and lastly to the purposes of navigation, by impelling large vessels on rivers and canals, for the cheap and expeditious conveyance of passengers and goods of all kinds. Many vessels of this kind are now employed in this way, in this country, and in North America, and elsewhere. As early as the year 1801, such a vessel was tried on the Forth and Clyde inland navigation; but was laid aside, among other reasons, on account of the injury it threatened the banks of the canal by the agitation of the water. In America, the first steam-boat was launched at New-York on the 3d of October 1807, and began to ply on the river between that city and Albany, a distance of about 160 miles; and now numerous large vessels of that kind are employed on the navigation between New York and Canada, as well as on the Mississippi, and several other rivers in that country. The first attempt, on a large scale, to navigate by steam on the river Clyde, was made in the year 1812; and many other vessels, of great burden, are now daily employed there, conveying at each time several hundred tons of goods, or many hundred passengers, in a commodious, cheap, and expeditious manner. Vessels of the same kind are also successively establishing on many other rivers in the country; as on the Thames, the Tyne, &c, &c, and even coasting on the sea.—Accounts of these vessels and navigations have been given in several publications; particularly in the Monthly Magazine, in many places of the volumes 36 and 37, as also in the Philos. Magazine, vol. 45, in both cases accompanied with drawings and descriptions of the machinery.

STEELYARD, or **Stilyard**, in Mechanics, a kind of balance, called also, Statera Romana, or the Roman Balance, by means of which the weights of different bodies are discovered by using one single weight only.



The common steelyard consists of an iron beam AB, in which is assumed a point at pleasure, as C, on which is raised a perpendicular CD. On the shorter arm AC is hung a scale to receive the bodies weighed: the moveable weight I is shifted backward and forward on the beam, till it be a counterbalance to 1, 2, 3, 4, &c pounds placed in the scale; and the points are noted where the constant weight I weighs, as 1, 2, 3, 4, &c pounds. From this construction of the steelyard, the manner of using it is evident.

These instruments in the hands of designing men are easily converted to the purpose of deception; as, one cannot so readily know whether they be truly constructed or not, as we can with the common balance; on which account, and some other inconveniencies attending the use of them, they are not very generally employed in mercantile transactions.

These imperfections in the common steelyard, led C. Paul, inspector of weights at Geneva, to employ his thoughts on so far improving steelyards, that either in the delicate operations of the arts, or in those of the same kind which are often so necessary in the practice of physical sciences, those instruments might be substituted with advantage for common balances.

It would be contrary to our plan to enter at any length upon the description of the instrument which Mr. Paul has constructed for this purpose, being merely a mechanical contrivance; it is however very ingenious, a description of which may be seen in Gregory's Mechanics, vol. 2, pa. 405, and in the Philosophical Magazine, vol. 3, where there is also a representation of the instrument.

Chinese **Steelyard**. The Chinese carry this statera about them to weigh their gems, and other things of value. The beam or yard is a small rod of wood or ivory, about a foot in length: upon this are three rules of measure, made of a fine silver-studded work; they all begin from the end of the beam, whence the first is extended 8 inches, the second $6\frac{1}{2}$, the third $8\frac{1}{2}$. The first is the European measure, the other two seem to be Chinese measures. At the other end of the yard hangs a round scale, and at three several distances from this end are fastened so many slender strings, as different points of suspension. The first distance makes $1\frac{3}{5}$ or $\frac{8}{5}$ of an inch, the second $3\frac{1}{5}$ or double the first, and the third $4\frac{4}{5}$ or triple of the first. When they weigh any thing, they hold up the yard by some one of these strings, and hang a sealed weight, of about $1\frac{3}{4}$oz troy weight, upon the respective divisions of the rule, as the thing requires. Grew's Museum, pa. 369.

Spring **Steelyard**, is a kind of portable balance, serving to weigh any matter, from 1 to about 40 pounds. It is composed of a brass or iron tube, into which goes a rod, and about that is wound a spring of tempered steel in a spiral form. On this rod are the divisions of pounds and parts of pounds, which are made by successively hanging on, to a hook fastened to the other end, 1, 2, 3, 4, &c, pounds.

Now the spring being fastened by a screw to the bottom of the rod; the greater the weight is that is hung upon the hook, the more will the spring be contracted, and consequently a greater part of the rod will come out of the tube; the proportions or quantities of which greater weights are indicated by the figures appearing against the extremity of the tube.

Steelyard-*Swing*. In the Philos. Trans. (No. 462, sect. 5) is given an account of a steelyard-swing, proposed as a mechanical method for assisting children labouring under deformities, owing to the contraction of the muscles on one side of the body. The crooked person is suspended with cords under his arm, and these are placed at equal distances from the centre of the beam. It is supposed that the gravity of the body will affect the contracted side, so as to put the muscles upon the stretch; and hence by degrees the defect may be remedied.

STEEPLE, a building usually raised on the western end of a church to contain the bells.—Steeples are denominated from their form, either spires, or towers. The first are such as rise continually diminishing like a cone or other pyramid. The latter are mere parallelopipedons, or some other prism, and are covered at top with a platform.—In each kind there is usually a sort of windows, or loop-holes, to let out the sound, and so contrived as to throw it downward.

Masius, in his treatise on bells, treats also of steeples. The most remarkable in the world, it is said, is that at Pisa, which leans so much to one side, that you fear every moment it will fall; yet is in no danger. This odd disposition, he observes, is not owing to a shock of an earthquake, as is generally imagined; but was contrived so at first by the architect; as is evident from the ceilings, windows, doors, &c, which are all in the bevel.

STEERAGE, in a ship, that part next below the quarter-deck, before the bulk-head of the great cabin, where the steersman stands in most ships of war. In large ships of war it is used as a hall, through which it is necessary to pass to or from the great cabin. In merchant ships it is mostly the habitation of the lower officers and ship's crew.

STEERAGE, in Sea-language, is also used to express the effort of the helm: and hence

STEERAGE-*way* is that degree of progressive motion communicated to a ship, by which she becomes susceptible of the effect of the helm to govern her course.

STEERING, in Navigation, the art of directing the ship's way by the movements of the helm; or of applying its efforts to regulate her course when she advances. The perfection of steering consists in a vigilant attention to the motion of the ship's head, so as to check every deviation from the line of her course in the first instant of its motion; and in applying as little of the power of the helm as possible. By this means she will run more uniformly in a straight path, or decline less to the right and left; whereas, if a greater effort of the helm be employed, it will produce a greater declination from the course, and not only increase the difficulty of steering, but also make a crooked and irregular path through the water.

The helmsman, or steersman, should diligently watch the movements of the head by the land, clouds, moon, or stars; because, though the course is in general regulated by the compass, yet the vibrations of the needle are not so quickly perceived, as the sallies of the ship's head to the right or left, which, if not immediately restrained, will acquire additional velocity in every instant of their motion, and require a more powerful impulse of the helm to reduce them; the application of which will operate to turn her head as far on the contrary side of her course.

The phrases used in steering a ship, vary according to the relation of the wind to her course. Thus, when the wind is large or fair, the phrases used by the pilot or officer who superintends the steerage, are port, starboard, and steady: the first of which is intended to direct the ship's course further to the right; the second to the left; and the last is designed to keep her exactly in the line on which she advances, according to the intended course. The excess of the first and second movement is called hard-a-port, and hard-a-starboard; the former of which gives her the greatest possible inclination to the right, and the latter an equal tendency to the left.—If, on the contrary, the wind be scant or foul, the phrases are luff, thus, and no nearer: the first of which is the order to keep her close to the wind; the second, to retain her in her present situation; and the third, to keep her sails full.

STELLA. See STAR.

STENTOROPHONIC *Tube*, a speaking trumpet, or tube employed to speak to a person at a great distance. It has been so called from Stentor, a person mentioned in the 5th book of the Iliad, who, as Homer tells us, could call out louder than 50 men. With the celebrated stentorophonic horn of Alexander the Great, it is said, he could give orders to his army at the distance of 100 stadia, which is about 12 English miles.

The present speaking trumpet, it is said, was invented by Sir Samuel Moreland. But Derham, in his Physico-Theology, lib. 4, chap. 3, says, that Kircher found out this instrument 20 years before Moreland, and published it in his Mesurgia; and it is further said that Gaspar Schottus had seen one at the Jesuits' College at Rome. Also one Conyers, in the Philos. Trans. No. 141, gives a description of an instrument of this kind, different from those commonly made. Gravesande, in his Philosophy, disapproves of the usual figures of these instruments; he would have them to be parabolic conoids, with the focus of one of its parabolic sections at the mouth.—Concerning this instrument, see Sturmy's Collegium Curiosum, Pt. 2, Tentam. 8; also Philos. Trans. vol. 6, pa. 3056, vol. 12, pa. 1027.

STEREOGRAPHIC *Projection of the Sphere*, is that in which the eye is supposed to be placed in the surface of the sphere. Or it is the projection of the circles of the sphere on the plane of some one great circle, when the eye, or a luminous point, is placed in the pole of that circle.—For the fundamental principles and chief properties of this kind of projection, see PROJECTION.

STEREOGRAPHY, is the art of drawing the forms of solids upon a plane.

STEVIN, STEVINUS (SIMON), a Flemish mathematician of Bruges, who died in 1633. He was master of mathematics to prince Maurice of Nassau, and inspector of the dykes in Holland. It is said he was the inventor of the sailing chariots, sometimes made use of in Holland. He was a good practical mathematician and mechanist, and was author of several useful works: as, treatises on Arithmetic, Algebra, Geometry, Statics, Optics, Trigonometry, Geography, Astronomy, Fortification, and many others, in the Dutch language, which were translated into Latin, by Snellius, and printed in 2 volumes folio. There are also two editions in the French language, in folio, both printed at Leyden, the one in 1608, and the other in 1634, with curious notes and additions, by Albert Girard.—For a particular account of Stevin's inventions and improvements in Algebra, which were many and ingenious, see our article Algebra, vol. 1, pa. 89.

STEWART (the Rev. Dr. MATTHEW), late professor of mathematics in the university of Edinburgh, was the son of the reverend Mr. Dugald Stewart, minister of Rothsay in the Isle of Bute, and was born at that place in the year 1717. After having finished his course at the grammar school, being intended by his father for the church, he was sent to the university of Glasgow, and was entered there as a student in 1734. His academical studies were prosecuted with diligence and success; and he was so happy as to be particularly distinguished by the friendship of Dr. Hutcheson, and Dr. Simson the celebrated geometrician, under whom he made great progress in that science.

Mr. Stewart's views made it necessary for him to attend the lectures in the university of Edinburgh in 1741; and that his mathematical studies might suffer no interruption, he was introduced by Dr. Simson to Mr. Maclaurin, who was then teaching with so much success, both the geometry and the philosophy of Newton, and under whom Mr. Stewart made that proficiency which was to be expected from the abilities of such a pupil, directed by those of so great a master. But the modern analysis, even when thus powerfully recommended, was not able to withdraw his attention from the relish of the ancient geometry, which he had imbibed under Dr. Simson. He still kept up a regular correspondence with this gentleman, giving him an account of his progress, and of his discoveries in geometry, which were now both numerous and important, and receiving in return many curious communications with respect to the Loci Plani, and the Porisms of Euclid. Mr. Stewart pursued this latter subject in a different and new direction. In doing which, he was led to the discovery of those curious and interesting propositions, which were published, under the title of General Theorems, in 1746. They were given without the demonstrations; but did not fail to place their discoverer at once among the geometricians of the first rank. They are, for the most part, porisms, though Mr. Stewart, careful not to anticipate the discoveries of his friend, gave them only the name of theorems. They are among the most beautiful, as well as most general propositions, known in the whole compass of geometry, and are perhaps only equalled by the remarkable locus to the circle in the second book of Apollonius, or by the celebrated theorem of Mr. Cotes.

Such is the history of the invention of these propositions; and the occasion of the publication of them was as follows. Mr. Stewart, while engaged in them, had entered into the church, and become minister of Roseneath. It was in that retired and romantic situation, that he discovered the greater part of those theorems. In the summer of 1746, the mathematical chair in the university of Edinburgh became vacant, by the death of Mr. Maclaurin. The General Theorems had not yet appeared; Mr. Stewart was known only to his friends; and the eyes of the public were naturally turned on Mr. Stirling, who then resided at Leadhills, and who was well known in the mathematical world. He however declined appearing as a candidate for the vacant chair; and several others were named, among whom was Mr. Stewart. On this occasion he printed the General Theorems, which gave their author a decided superiority above all the other candidates. He was accordingly elected professor of mathematics in the university of Edinburgh, in September 1747.

The duties of this office gave a turn somewhat different to his mathematical pursuits, and led him to think of the most simple and elegant means of explaining those difficult propositions, which were hitherto only accessible to men deeply versed in the modern analysis. In doing this, he was pursuing the object which, of all others, he most ardently wished to attain, viz. the application of geometry to such problems as the algebraic calculus alone had been thought able to resolve. His solution of Kepler's problem was the first specimen of this kind which he published; and it was perhaps impossible to have produced one more to the credit of the method he followed, or of the abilities with which he applied it. Among the excellent solutions hitherto given of this famous problem, there were none of them at once direct in its method, and simple in its principles. Mr. Stewart was so happy as to attain both these objects. He founds his solution on a general property of curves, which, though very simple, had perhaps never been observed; and by a most ingenious application of that property, he shows how the approximation may be continued to any degree of accuracy, in a series of results which converge with great rapidity.

This solution appeared in the second volume of the Essays of the Philosophical Society of Edinburgh, for the year 1756. In the first volume of the same collection, there are some other propositions of Mr. Stewart's, which are an extension of a curious theorem in the 4th book of Pappus. They have a relation to the subject of Porisms, and one of them forms the 91st of Dr. Simson's Restoration.

It has been already mentioned, that Dr. Stewart had formed the plan of introducing into the higher parts of mixed mathematics, the strict and simple form of ancient demonstration. The prosecution of this plan produced the Tracts Physical and Mathematical, which were published in 1761. In the first of these, Dr. Stewart lays down the doctrine of centripetal forces, in a series of propositions, demonstrated (if we admit the quadrature of curves) with the utmost rigour, and requiring no previous knowledge of the mathematics, except the elements of plane Geometry, and of Conic Sections. The good order of these propositions, added to the clearness and simplicity of the demonstrations, renders this tract perhaps the best elementary treatise of Physical Astronomy that is any where to be found.

In the three remaining tracts, our author had it in view to determine, by the same rigorous method, the effect of those forces which disturb the motions of a secondary planet. From this he proposed to deduce, not only a theory of the moon, but a determination of the sun's distance from the earth. The former, it is well known, is the most difficult subject to which mathematics have been applied, and the resolution required and merited all the clearness and simplicity which our author possessed in so eminent a degree. It must be regretted therefore, that the decline of Dr. Stewart's health, which began soon after the publication of the tracts, did not permit him to pursue this investigation.

The other object of the tracts was, to determine the distance of the sun, from his effect in disturbing the motions of the moon; and Dr. S.'s inquiries into the lunar irregularities had furnished him with the means of accomplishing it.

The theory of the composition and resolution of forces enables us to determine what part of the solar force is employed in disturbing the motions of the moon; and therefore, could we measure the instantaneous effect of that force, or the number of feet by which it accelerates or retards the moon's motion in a second, we should be able to determine how many feet the whole force of the sun would make a body, at the distance of the moon, or of the earth, descend in a second of time, and consequently how much the earth is, in every instant, turned out of its rectilineal course. Thus the curvature of the earth's orbit, or, which is the same thing, the radius of that orbit, that is, the distance of the sun from the earth, would be determined. But the fact is, that the instantaneous effects of the sun's disturbing force are too minute to be measured; and that it is only the effect of that force, continued for an entire revolution, or some considerable portion of a revolution, which astronomers are able to observe.

There is yet a greater difficulty which embarrasses the solution of this problem. For as it is only by the difference of the forces exerted by the sun on the earth and on the moon, that the motions of the latter are disturbed, the farther off the sun is supposed to be, the less will be the force by which he disturbs the moon's motions; yet that force will not diminish beyond a fixed limit, and a certain disturbance would obtain, even if the distance of the sun were infinite. Now the sun is actually placed at so great a distance, that all the disturbances, which he produces on the lunar motions, are very near to this limit, and therefore a small mistake in estimating their quantity, or in reasoning about them, may give the distance of the sun infinite, or even impossible. But all this did not deter Dr. Stewart from undertaking the solution of the problem, with no other assistance than that which geometry could afford. Indeed the idea of such a problem had first occurred to Mr. Machin, who, in his book on the laws of the moon's motion, has just mentioned it, and given the result of a rude calculation (the method of which he does not explain), which assigns 8″ for the parallax of the sun. He made use of the motion of the nodes; but Dr. Stewart considered the motion of the apogee, or of the longer axis of the moon's orbit, as the irregularity best adapted to his purpose. It is well known that the orbit of the moon is not immoveable; but that, in consequence of the disturbing force of the sun, the longer axis of that orbit has an angular motion, by which it goes back about 3 degrees in every lunation, and completes an entire revolution in 9 years nearly. This motion, though very remarkable and easily determined, has the same fault, with respect to the present problem, that was ascribed to the other irregularities of the moon: for a very small part of it only depends on the parallax of the sun; and of this Dr. Stewart seems not to have been perfectly aware.

The propositions however which defined the relation between the sun's distance and the mean motion of the apogee, were published among the tracts, in 1761. The transit of Venus happened also in that year: and the astronomers returned, who had viewed that curious phenomenon, from the most distant stations; and no very satisfactory result was obtained from a comparison of their observations. Dr. Stewart then resolved to apply the principles he had already laid down; and, in 1763, he published his essay on the Sun's Distance, where the computation being actually made, the parallax of the sun was found to be no more than 6″·9, and consequently his distance almost 29875 semidiameters of the earth, or nearly 119 millions of miles.

A determination of the sun's distance, that so far exceeded all former estimations of it, was received with surprise, and the reasoning on which it was founded was likely to undergo a severe examination. But, even among astronomers, it was not every one who could judge in a matter of such difficult discussion. Accordingly, it was not till about 5 years after the publication of the sun's distance, that there appeared a pamphlet, under the title of Four Propositions, intended to point out certain errors in Dr. Stewart's investigation, which had given a result much greater than the truth. From his desire of simplifying, and of employing only the geometrical method of reasoning, he was reduced to the necessity of rejecting quantities, which were considerable enough to have a great effect on the last result. An error was thus introduced, which, had it not been for certain compensations, would have become immediately obvious, by giving the sun's distance near three times as great as that which has been mentioned.

The author of the pamphlet, referred to above, was the first who remarked the dangerous nature of these simplifications, and who attempted to estimate the error to which they had given rise. This author remarked what produced the compensation above mentioned, viz, the immense variation of the sun's distance, which corresponds to a very small variation of the motion of the moon's apogee. And it is but justice to acknowledge that, besides being just in the points already mentioned, they are very ingenious, and written with much modesty and good temper. The author, who at first concealed his name, but has now consented to its being made public, was Mr. Dawson, a surgeon at Sedberg in Yorkshire, and one of the most ingenious mathematicians and philosophers this country now possesses.

A second attack was soon after this made on the Sun's Distance, by Mr. Landen; but by no means with the same good temper which has been remarked in the former. He fancied to himself errors in Dr. Stewart's investigation, which have no existence; he exaggerated those that were real, and seemed to triumph in the discovery of them with unbecoming exultation. If there are any subjects on which men may be expected to reason dispassionately, they are certainly the properties of number and extension; and whatever pretexts moralists or divines may have for abusing one another, mathematicians can lay claim to no such indulgence. The asperity of Mr. Landen's animadversions ought not therefore to pass uncensured, though it be united with sound reasoning and accurate discussion. But Mr. Landen, in the zeal of correction, brings many other charges against Dr. Stewart, the greater part of which seem to have no good foundation. Such are his objections to the second part of the investigation, where Dr. Stewart finds the relation between the disturbing force of the sun, and the motion of the apses of the lunar orbit. For this part, instead of being liable to objection, is deserving of the greatest praise, since it resolves, by geometry alone, a problem which had eluded the efforts of some of the ablest mathematicians, even when they availed themselves of the utmost resources of the integral calculus. Sir Isaac Newton, though he assumed the disturbing force very near the truth, computed the motion of the apses from thence only at one half of what it really amounts to; so that, had he been required, like Dr. Stewart, to invert the problem, he would have committed an error, not merely of a few thousandth parts, as the latter is alleged to have done, but would have brought out a result double of the truth. (Princip. Math. lib. 3, prop. 3.) Machin and Callendrini, when commenting on this part of the Principia, found a like inconsistency between their theory and observation. Three other celebrated mathematicians, Clairaut, Dalembert, and Euler, severally experienced the same difficulties, and were led into an error of the same magnitude. It is true, that, on resuming their computations, they found that they had not carried their approximations to a sufficient length, which when they had at last accomplished, their results agreed exactly with observation. Mr. Walmsley and Dr. Stewart were, I think, the first mathematicians who, employing in the solution of this difficult problem, the one the algebraic calculus, and the other the geometrical method, were led immediately to the truth; a circumstance so much for the honour of both, that it ought not to be forgotten. It was the business of an impartial critic, while he examined our author's reasonings, to have remarked and to have weighed these considerations.

The Sun's Distance was the last work which Dr. Stewart published; and though he lived to see the animadversions made on it, that have been taken notice of above, he declined entering into any controversy. His disposition was far from polemical; and he knew the value of that quiet, which a literary man should rarely suffer his antagonists to interrupt. He used to say, that the decision of the point in question was now before the public; that if his investigation was right, it would never be overturned, and that if it was wrong, it ought not to be defended.

A few months before he published the Essay just mentioned, he gave to the world another work, entitled Propositiones More Veterum Demonstratæ. It consists of a series of geometrical theorems, mostly new; investigated, first by an analysis, and afterwards synthetically demonstrated by the inversion of the same analysis. This method made an important part in the analysis of the ancient geometricians; but few examples of it have been preserved in their writings, and those in the Propositiones Geometricæ are therefore the more valuable.

Dr. Stewart's constant use of the geometrical analysis had put him in possession of many valuable propositions, which did not enter into the plan of any of the works that have been enumerated. Of these, not a few have found a place in the writings of Dr. Simson, where they will for ever remain, to mark the friendship of these two mathematicians, and to evince the esteem which Dr. Simson entertained for the abilities of his pupil. Many of these are in the work upon the Porisms, and others in the Conic Sections, viz, marked with the letter x; also a theorem in the edition of Euclid's data.

Soon after the publication of the Sun's Distance, Dr. Stewart's health began to decline, and the duties of his office became burdensome to him. In the year 1772, he retired to the country, where he afterwards spent the greater part of his life, and never resumed his labours in the university. He was however so fortunate as to have a son to whom, though very young, he could commit the care of them with the greatest confidence. Mr. Dugald Stewart, having begun to give lectures for his father from the period above-mentioned, was elected joint professor with him in 1775, and gave an early specimen of those abilities, which have not been confined to a single science.

After mathematical studies (on account of the bad state of health into which Dr. Stewart was falling) had ceased to be his business, they continued to be his amusement. The analogy between the circle and hyperbola had been an early object of his admiration. The extensive views which that analogy is continually opening; the alternate appearance and disappearance of resemblance in the midst of so much dissimilitude, make it an object that astonishes the experienced, as well as the young geometrician. To the consideration of this analogy therefore the mind of Dr. Stewart very naturally returned, when disengaged from other speculations. His usual success still attended his investigations; and he has left among his papers some curious approximations to the areas, both of the circle and hyperbola. For some years toward the end of his life, his health scarcely allowed him to prosecute study even as an amusement. He died the 23d of January 1785, at 68 years of age.

The habits of study, in a man of original genius, are objects of curiosity, and deserve to be remembered. Concerning those of Dr. Stewart, his writings have made it unnecessary to remark, that from his youth he had been accustomed to the most intense and continued application. In consequence of this application, added to the natural vigour of his mind, he retained the memory of his discoveries in a manner that will hardly be believed. He seldom wrote down any of his investigations, till it became necessary to do so for the purpose of publication. When he discovered any proposition, he would set down the enunciation with great accuracy, and on the same piece of paper would construct very neatly the figure to which it referred. To these he trusted for recalling to his mind, at any future period, the demonstration, or the analysis, however complicated it might be. Experience had taught him that he might place this confidence in himself without any danger of diappointment; and for this singular power, he was probably more indebted to the activity of his invention, than to the mere tenaciousness of his memory.

Though Dr. Stewart was extremely studious, he read but few books, and thus verified the observation of Dalembert, that, of all the men of letters, mathematicians read least of the writings of one another. Our author's own investigations occupied him sufficiently; and indeed the world would have had reason to regret the misapplication of his talents, had he employed, in the mere acquisition of knowledge, that time which he could dedicate to works of invention.

It was Dr. Stewart's custom to spend the summer at a delightful retreat in Ayrshire, where, after the academical labours of the winter were ended, he found the leisure necessary for the prosecution of his researches. In his way thither, he often made a visit to Dr. Simson of Glasgow, with whom he had lived from his youth in the most cordial and uninterrupted friendship. It was pleasing to observe, in these two excellent mathematicians, the most perfect esteem and affection for each other, and the most entire absence of jealousy, though no two men ever trode more nearly in the same path. The similitude of their pursuits served only to endear them to each other, as it will ever do with men superior to envy. Their sentiments and views of the science they cultivated, were nearly the same; they were both profound geometricians; they equally admired the ancient mathematicians, and were equally versed in their methods of investigation; and they were both apprehensive that the beauty of their favourite science would be forgotten, for the less elegant methods of algebraic computation. This innovation they endeavoured to oppose; the one, by reviving those books of the ancient geometry which were lost; the other, by extending that geometry to the most difficult inquiries of the moderns. Dr. Stewart, in particular, had remarked the intricacies, in which many of the greatest of the modern mathematicians had involved themselves in the application of the calculus, which a little attention to the ancient geometry would certainly have enabled them to avoid. He had observed too the elegant synthetical demonstrations that, on many occasions, may be given of the most difficult propositions, investigated by the inverse method of fluxions. These circumstances had perhaps made a stronger impression than they ought, on a mind already filled with admiration of the ancient geometry, and produced too unfavourable an opinion of the modern analysis. But if it be confessed that Dr. Stewart rated, in any respect too high, the merit of the former of these sciences, this may well be excused in the man whom it had conducted to the discovery of the General Theorems, to the solution of Kepler's Problem, and to an accurate determination of the Sun's disturbing force. His great modesty made him ascribe to the method he used, that success which he owed to his own abilities.

The foregoing account of Dr. Stewart and his writings, is chiefly extracted from the learned history of them, by Mr. Playfair, in the 1st volume of the Edinburgh Philosophical Transactions, pa. 57, &c.

STIFEL, STIFELIUS (MICHAEL), a Protestant minister, and very skilful mathematician, was born at Eslingen, a town in Germany; and died at Jena in Thuringia, in the year 1567, at 58 years of age according to Vossius, but some others say 80. Stifel was one of the best mathematicians of his time. He published, in the German language, a treatise on Algebra, and another on the Calendar or Ecclesiastical computation. But his chief work, is the Arithmetica Integra, a complete and excellent treatise, in Latin, on arithmetic and algebra, printed in 4to at Norimberg 1544. In this work there are a number of ingenious inventions, both in common arithmetic and in algebra; of which, those relating to the latter are amply explained under the article Algebra in this dictionary, vol. 1.; to which may be added some particulars concerning the arithmetic, from the first volume of my Tracts, pa. 231, &c. In this original work are contained many curious things, some of which have mistakingly been ascribed to a much later date. He here treats pretty fully and ably, of progressional and figurate numbers, and in particular of the triangular table, for constructing both them and the coefficients of the terms of all powers of a binomial; which has been so often used since his time for these and other purposes, and which more than a century after was, by Pascal, otherwise called the Arithmetical Triangle, and who only mentioned some additional properties of the table. Stifel shows, that the horizontal lines of the table furnish the coefficients of the terms of the corresponding powers of a binomial; and teaches how to make use of them in the extraction of roots of all powers whatever. Cardan seems to ascribe the invention of that table to Stifelius; but I apprehend that is only to be understood of its application to the extraction of roots.

It is remarkable too, how our author, at pa. 35 &c of the same book, treats of the nature and use of logarithms; not under that name indeed, but under the idea of a series of arithmeticals, adapted to a series of geometricals. He there explains all their uses; such as, that the addition of them answers to the multiplication of their geometricals; subtraction to division; multiplication of exponents to involution; and dividing of exponents to evolution. He also exemplifies the use of them in cases of the Rule-of-three, and in finding mean proportionals between given terms, and such like, exactly as is done in logarithms. So that he seems to have been in the full possession of the idea of logarithms, and wanted only the necessity of troublesome calculations to induce him to make a table of such numbers.

Stifelius wrote also pretty largely on magic squares.

Stifel was a zealous, though weak disciple of Luther. He took it into his head to become a prophet, and he predicted that the end of the world would happen on a certain day in the year 1553, by which he terrified many people. When the proposed day arrived, he repaired early, with multitudes of his followers, to a particular place in the open air, spending the whole day in the most fervent prayers and praises, in vain looking for the coming of the Lord, and the universal conflagration of the elements, &c.

STILE. See STYLE.

STILYARD. See STEELYARD.

STOFLER (JOHN), a German mathematician, was born at Justingen in Suabia, in 1452, and died in 1531, at 79 years of age. He taught mathematics at Tubinga, where he acquired a great reputation, which however he in a great measure lost again, by intermeddling with the prediction of future events. He announced a great deluge, which he said would happen in the year 1524, a prediction with which he terrified all Germany, where many persons prepared vessels proper to escape with from the floods. But happily the prediction failing, it enraged the astrologer, though it served to convince him of the vanity of his prognostications.—He was author of several works in mathematics and astrology, full of foolish and chimerical ideas; such as,

1. Elucidatio Fabric. Ususque Astrolabii; fol. 1513.
2. Procli Sphæram Comment. fol. 154.
3. Cosmographicæ aliquot Descriptiones; 4to, 1537.

STONE (EDMUND), a respectable mathematician, who was author of several ingenious works. I know not the particular place or date of his birth, but it was probably in the shire of Argyle, and towards the conclusion of the 17th century. Nor have we any memoirs of his life, except what are contained in a letter from the Chevalier de Ramsay, author of the Travels of Cyrus, in a letter to father Castel, a Jesuit at Paris, and published in the Memoires de Trevoux, pa. 109, as follows: "True genius overcomes all the disadvantages of birth, fortune, and education; of which Mr. Stone is a rare example. Born a son of a gardener of the duke of Argyle, he arrived at 8 years of age before he learnt to read.—By chance a servant having taught young Stone the letters of the alphabet, there needed nothing more to discover and expand his genius. He applied himself to study, and he arrived at the knowledge of the most sublime geometry and analysis, without a master, without a conductor, without any other guide but pure genius.

"At 18 years of age he had made these considerable advances without being known, and without knowing himself the prodigies of his acquisitions. The duke of Argyle, who joined to his military talents a general knowledge of every science that adorns the mind of a man of his rank, walking one day in his garden, saw lying on the grass a Latin copy of Sir Isaac Newton's celebrated Principia. He called some one to him to take and carry it back to his library. Our young gardener told him that the book belonged to him. To you? replied the Duke. Do you understand geometry, Latin, Newton? I know a little of them, replied the young man with an air of simplicity arising from a profound ignorance of his own knowledge and talents. The Duke was surprised; and having a taste for the sciences, he entered into conversation with the young mathematician: he asked him several questions, and was astonished at the force, the accuracy, and the candour of his answers. But how, said the Duke, came you by the knowledge of all these things? Stone replied, A servant taught me, ten years since, to read: does one need to know any thing more than the 24 letters in order to learn every thing else that one wishes? The Duke's curiosity redoubled—he sat down upon a bank, and requested a detail of all his proceedings in becoming so learned.

" 'I first learned to read, said Stone: the masons were then at work upon your house: I went near them one day, and I saw that the architect used a rule, compasses, and that he made calculations. I inquired what might be the meaning of and use of these things; and I was informed that there was a science called Arithmetic; I purchased a book of arithmetic, and I learned it.—I was told there was another science called Geometry: I bought the books, and I learnt geometry. By reading I found that there were good books in these two sciences in Latin: I bought a dictionary, and I learned Latin. I understood also that there were good books of the same kind in French: I bought a dictionary, and I learned French. And this, my lord, is what I have done: it seems to me that we may learn every thing when we know the 24 letters of the alphabet.'

" This account charmed the Duke. He drew this wonderful genius out of his obscurity; and he provided him with an employment which left him plenty of time to apply himself to the sciences. He discovered in him also the same genius for music, for painting, for architecture, for all the sciences which depend on calculations and proportions.

" I have seen Mr. Stone. He is a man of great simplicity. He is at present sensible of his own knowledge; but he is not puffed up with it. He is possessed with a pure and disinterested love for the mathematics; though he is not solicitous to pass for a mathematician; vanity having no part in the great labour he sustains to excel in that science. He despises fortune also; and he has solicited me twenty times to request the duke to give him less employment, which may not be worth the half of that he now has, in order to be more retired, and less taken off from his favourite studies. He discovers sometimes, by methods of his own, truths which others have discovered before him; and he is charmed to find on these occasions that he is not a first inventor, and that others have made a greater progress than he thought. Far from being a plagiary, he attributes ingenious solutions, which he gives to certain problems, to the hints he has found in others, although the connexion is but very distant," &c.

Mr. Stone was author and translator of several useful works; viz, A New Mathematical Dictionary, in 1 vol. 8vo, first printed in 1726.

2. Fluxions, in 1 vol. 8vo, 1730. The Direct Method is a translation from the French, of Hospital's Analyse des Infiniments Petits; and the Inverse Method was supplied by Stone himself.

3. The Elements of Euclid, in 2 vols. 8vo, 1731. A neat and useful edition of this work, with an account of the life and writings of Euclid, and a defence of his elements against modern objectors.

4. Dr. Barrow's Geometrical Lectures, translated from the Latin, 1 vol. 8vo, 1735.

Besides other smaller works.

Stone was a fellow of the Royal Society, and had inserted in the Philos. Trans. (vol. 41, pa. 218) an " Account of two species of lines of the 3d order, not mentioned by Sir Isaac Newton, or Mr. Stirling."

STONES, *Meteoric*, certain semi-metallic masses which sometimes fall from the atmosphere. See AEROLITE

STRABO, a celebrated Greek geographer, philosopher, and historian, was born at Amasia, and was descended from a family settled at Gnossus in Crete. He was the disciple of Xenarchus, a Peripatetic philosopher, but at length attached himself to the Stoics. He contracted a strict friendship with Cornelius Gallus, governor of Egypt; and travelled into several countries, to observe the situation of places, and the customs of nations.

Strabo flourished under Augustus; and died under Tiberius about the year 25, at a very advanced age.—He composed several works; all of which are lost, except his Geography, in 17 books; which are justly esteemed very precious remains of antiquity. The first two books are employed in showing, that the study of geography is not only worthy of a philosopher, but even necessary to him; the 3d describes Spain; the 4th, Gaul and the Britannic isles; the 5th and 6th, Italy and the adjacent isles; the 7th, which is imperfect at the end, Germany, the countries of the Getæ and Illyrii, Taurica, Chersonesus, and Epirus; the 8th, 9th, and 10th, Greece with the neighbouring isles; the four following, Asia within Mount Taurus; the 15th and 16th, Asia without Taurus, India, Persia, Syria, Arabia; and the 17th, Egypt, Ethiopia, Carthage, and other parts of Africa.

Strabo's work was published with a Latin version by Xylander, and notes by Isaac Casaubon, at Paris 1620, in folio; but a better edition is that of Amsterdam in 1707, in 2 volumes folio, by the learned Theodore Janson of Almelooveen, with the entire notes of Xylander, Casaubon, Meursius, Cluver, Holsten, Salmasius, Bochart, Ez. Spanheim, Cellar, and others. To this edition is subjoined the Chrestomathiæ, or Epitome of Strabo; which, according to Mr. Dodswell, who has written a very elaborate and learned dissertation about it, was made by some unknown person, between the years of Christ 676 and 996. It has been found of some use, not only in helping to correct the original, but in supplying in some measure the defect in the 7th book. Mr. Dodswell's dissertation is prefixed to this edition. An edition has lately been published at Oxford.

STRAIT, or STRAIGHT, or STREIGHT, in Hydrography, is a narrow channel or arm of the sea, shut up between lands on either side, and usually affording a passage out of one great sea into another. As the Straits of Magellan, of Le Maire, of Gibraltar, &c.

STRAIT is also sometimes used, in Geography, for an isthmus, or neck of land between two seas, preventing their communication.

STRENGTH, *vis*, force, power. Some authors suppose the strength of animals, of the same kind, to depend on the quantity of blood; but most on the size of the bones, joints, and muscles; though we find by daily experience, that the animal spirits contribute greatly to strength at different times.

Emerson has most particularly treated of the strength of bodies depending on their dimensions and weight. In the general scholium after his propositions on this subject, he adds; If a certain beam of timber be able to support a given weight; another beam, of the same timber, similar to the former, may be taken so great, as to be able but just to bear its own weight: while any larger beam cannot support itself, but must break by its own weight; but any less beam will bear something more. For the strength being as the cube of the depth; and the stress, being as the length and quantity of matter, is as the 4th power of the depth; it is plain, therefore, that the stress increases in a greater ratio than the strength. Whence it follows, that a beam may be taken so large, that the stress may far exceed the strength: and that, of all similar beams, there is but one that will just support itself, and nothing more. And the like holds true in all machines, and in all animal bodies. And hence there is a certain limit, in regard to magnitude, not only in all machines and artificial structures, but also in natural ones, which neither art nor nature can go beyond; supposing them made of the same matter, and in the same proportion of parts.

Hence it is impossible that mechanic engines can be increased to any magnitude at pleasure. For when they arrive at a particular size, their several parts will break and fall asunder by their own weight. Neither can any buildings of vast magnitudes be made to stand, but must fall to pieces by their great weight, and go to ruin.

It is likewise impossible for nature to produce animals of any vast size at pleasure: except some sort of matter can be found, to make the bones of, which may be so much harder and stronger than any hitherto known: or else that the proportion of the parts be so much altered, and the bones and muscles made thicker in proportion; which will make the animal distorted, and of a monstrous figure, and not capable of performing any proper actions. And being made similar and of common matter, they will not be able to stand or move; but, being burthened with their own weight, must fall down. Thus, it is impossible that there can be any animal so large as to carry a castle upon his back; or any man so strong as to remove a mountain, or pull up a large oak by the roots: nature will not admit of these things; and it is impossible that there can be animals of any sort beyond a determinate size.

Fish may indeed be produced to a larger size than land animals; because their weight is supported by the water. But yet even these cannot be increased to immensity, because the internal parts will press upon one another by their weight, and destroy their fabric.

On the contrary, when the size of animals is diminished, their strength is not diminished in the same proportion as the weight. For which reason a small animal will carry far more than a weight equal to its own, while a great one cannot carry so much as its weight. And hence it is that small animals are more active, will run faster, jump farther, or perform any motion quicker, for their weight, than large animals: for the less the animal, the greater the proportion of the strength to the stress. And nature seems to know no bounds as to the smallness of animals, at least in regard to their weight.

Neither can any two unequal and similar machines resist any violence alike, or in the same proportion; but the greater will be more hurt than the less. And the same is true of animals; for large animals by falling break their bones, while lesser ones, falling higher, receive no damage. Thus a cat may fall two or three yards high, and be no worse, and an ant from the top of a tower.

It is likewise impossible in the nature of things, that there can be any trees of immense size; if there were any such, their limbs, boughs, and branches, must break off and fall down by their own weight. Thus it is impossible there can be an oak a quarter of a mile high; such a tree cannot grow or stand, but its branches will drop off by their own weight. And hence also smaller plants can better sustain themselves than large ones.

As to the due proportion of strength in several bodies, according to their particular positions, and the weights they are to bear; he further observes that, if a piece of timber is to be pierced with a mortise-hole, the beam will be stronger when it is taken out of the middle, than when taken out of either side. And in a beam supported at both ends, it is stronger when the hole is made in the upper side than when made in the under, provided a piece of wood is driven hard in to fill up the hole.

If a piece is to be spliced upon the end of a beam, to be supported at both ends; it will be the stronger when spliced on the under side of a beam: but if the piece is supported only at one end, to bear a weight on the other; it is stronger when spliced on the upper side.

When a small lever, &c, is nailed to a body, to move it or suspend it by; the strain is greater upon the nail nearest the hand, or point where the power is applied.

If a beam be supported at both ends; and the two ends reach over the props, and be fixed down immoveable; it will bear twice as much weight, as when the ends only lie loose or free upon the supporters.

When a slender cylinder is to be supported by two pieces; the distance of the pins ought to be nearly $\frac{3}{5}$ of the length of the cylinder, and the pins equidistant from its ends; and then the cylinder will endure the least bending or strain by its weight.

The strength of a beam or bar, to resist a fracture by a force acting laterally, is as a solid, made by a section of the beam in the place where the force is applied, into the distance of its centre of gravity, from the point or line where the breach will end.

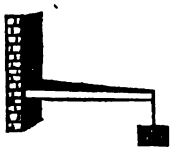
In square beams, the lateral strengths are as the cubes of the breadths or depths: and in cylindrical beams, the strengths are as the cubes of the diameters; the same is also true of all beams whose sections are similar figures, that is, the strengths are as the cubes of the corresponding dimensions.

In rectangular beams the lateral strengths are conjointly as the breadths and squares of the depths. Hence the lateral strength of a beam with its narrower face upwards, is to its strength with its broader face upwards, as the breadth of the broader face to the narrower one.

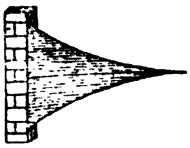
The lateral strengths of prismatic beams of the same materials, are as the areas of the sections and the distances of their centres of gravity, directly, and as their lengths and weights, inversely. This is true whether the beams be both supported at one end or at both; and in the latter case, a beam of any length is equal in strength to another of the same breadth and depth and of only half the length, when supported at one end.

The lateral strengths of two cylinders (of the same matter) of equal weight and length, one of which is hollow and the other solid, are to each other as the diameters of their ends.—The lateral strengths of tubes and solid cylinders of equal length and similar materials, are as the areas of their ends and their diameters conjointly.

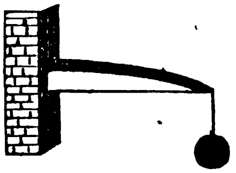
The strongest rectangular beam which can be cut out of a given cylinder, is that of which the squares of the breadth and depth, and the square of the cylinder's diameter, are respectively as the numbers 1, 2, and 3.—When a triangular beam is supported at both ends, its strength when the edge of the beam is uppermost, is to the strength when the other side is uppermost, as 2 to 1.



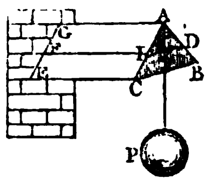
A beam fixed at one end, and bearing a weight at the other; if it be cut in the form of a wedge, and placed with its parallel sides parallel to the horizon; it will be equally strong every where; and no sooner break in one place than another.



When a beam has all its sides cut in form of a concave parabola, having the vertex at the end, and its absciss perpendicular to the axis of the solid, and the base a square, or a circle, or any regular polygon; such a beam fixed horizontally, at one end, is equally strong throughout for supporting its own weight.



If a beam be placed horizontally with one end fixed to a wall, and a weight be hung at the other, then if its breadth be the same throughout, it will be equally strong in all parts, when the vertical sides are in the form of a parabola.



Moreover, if AE be a beam in form of a triangular prism; and if AD = $\frac{1}{9}$AB, and AI = $\frac{1}{9}$AC, and the edge or small similar prism ADIF be cut away parallel to the base; the remaining beam DIBEF will bear a greater weight P, than the whole ABCEG, or the part will be stronger than the whole; which is a paradox in Mechanics.

Also when a wall faces the wind, and if the vertical section of it be a right-angled triangle; or if the fore part next the wind &c be perpendicular to the horizon, and the back part a sloping plane; such a wall will be equally strong in all its parts to resist the wind, if the parts of the wall cohere strongly together; but when it is built of loose materials, it is better to be convex on the back part in form of a parabola.

When a wall is to support a bank of earth or any fluid body, it ought to be built concave in form of a semicubical parabola, whose vertex is at the top of the wall, provided the parts of the wall adhere firmly together. But if the parts be loose, then a right line or sloping plane ought to be its figure. Such walls will be equally strong throughout.

All spires of churches in the form of cones or pyramids, are equally strong in all parts to resist the wind. But when the parts do not cohere together, then they ought to be parabolic conoids, to be equally strong throughout.

Likewise if there be a pillar erected in form of the logarithmic curve, the asymptote being the axis; it cannot be crushed to pieces in one part sooner than in another, by its own weight. And if such a pillar be turned upside down, and suspended by the thick end, it will not be more liable to separate in one part than another, by its own weight.

As to the strength of several sorts of wood, drawn from experiments, Mr. E. says, On a medium, a piece of good oak, an inch square, and a yard long, supported at both ends, will bear in the middle, for a very short time, about 330lb avoirdupois, but will break with more than that weight. But such a piece of wood should not, in practice, be trusted for any length of time, with more than a third or a fourth part of that weight. And the proportion of the strength of several sorts of wood, he found to be as follows:

Box, oak, plumtree, yew - - -	11
Ash, elm - - - - - - - -	8½
Thorn, walnut - - - - - - -	7½
Apple tree, elder, red fir, holly, plane	7
Beech, cherry, hazle - - - -	6⅔
Alder, asp, birch, white-fir, willow	6
Iron - - - - - - - - - - -	107
Brass - - - - - - - - - -	50
Bone - - - - - - - - - - -	22
Lead - - - - - - - - - - -	6½
Fine free stone - - - - - - -	1

As to the strength of bodies in direction of the fibres, he observes, A cylindric rod of good clean fir, of an inch circumference, drawn in length, will bear at extremity 400lb; and a spear of fir 2 inches diameter, will bear about 7 ton.—A rod of good iron, of an inch circumference, will bear near 3 ton weight. And a good hempen rope of an inch circumference, will bear 1000lb. at extremity.

All this supposes these bodies to be sound and good throughout; but none of them should be put to bear more than a third or a fourth part of that weight, especially for any length of time. From what has been said; if a spear of fir, or a rope, or a spear of iron, of *d* inches diameter, were to lift ¼ the extreme weight; then

The fir would bear 8⅘*dd* hundred weight.

The rope would bear 22*dd* hundred weight.

The iron would bear 6¼*dd* ton weight.

See on this subject Gregory's Mechanics, vol. 1, pa. 104, and following: as also Emerson on the same subject in his 4to edition. Also my Course of Mathematics, vol. 2.

As to animals; men may apply their strength several ways, in working a machine. A man of ordinary strength turning a roller by the handle, can act for a whole day against a resistance equal to 30lb. weight; and if he works 10 hours a day, he will raise a weight of 30lb. through 3½ feet in a second of time; or if the weight be greater, he will raise it so much less in proportion. But a man may act, for a small time, against a resistance of 50lb. or more.

If two men work at a windlass, or roller, they can more easily draw up 70lb, than one man can 30lb, provided the elbow of one of the handles be at right angles to that of the other. And with a fly, or heavy wheel, applied to it, a man may do ⅓ part more work; and for a little while he can act with a force, or overcome a continual resistance, of 80lb; and work a whole day when the resistance is but 40lb.—Men used to bear loads, such as porters, will carry, some 150lb, others 200 or 250lb, according to their strength.—A man can draw but about 70 or 80lb. horizontally; for he can but apply about half his weight.—If the weight of a man be 140lb, he can act with no greater a force in thrusting horizontally, at the height of his shoulders, than 27lb.

As to horses; a horse is, generally speaking, as strong as 5 men. A horse will carry 240 or 270lb. A horse draws to greatest advantage, when the line of direction is a little elevated above the horizon, and the power acts against his breast: and he can draw 200lb. for 8 hours a day, at 2½ miles an hour. If he draw 240lb, he can work but 6 hours, and not go quite so fast. And in both cases, if he carries some weight, he will draw the better for it. And this is the weight a horse is supposed to be able to draw over a pulley out of a well. But in a cart, a horse may draw 1000lb, or even double that weight, or a ton weight, or more.

As the most force a horse can exert, is when he draws a little above the horizontal position: so the worst way of applying the strength of a horse, is to make him carry or draw uphill: And three men in a steep hill, carrying each 100lb, will climb up faster than a horse with 300lb. Also, though a horse may draw in a round walk of 18 feet diameter; yet such a walk should not be less than 25 or 30 feet diameter. Emerson's Mechan. pa. 111 and 177.

For more on this subject, see Girard's treatise on the Resistance of Solids; Gregory's Mechanics, vol. 1; my Course of Mathematics, vol. 3, &c.

STRESS, in Mechanics, a familiar term used to denote the load, weight, or force, which a bar or beam has to sustain; being as the *distress* or oppression which it has to sustain and support; whether arising from its own weight, or from any adventitious load or force whatever. This is commonly treated of in conjunction with the strength, as in the preceding article; or as in the Mechanics of Emerson or of Gregory, or in my Course of Mathematics, &c.

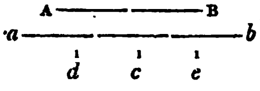
STRIKE, or STRYKE, a measure, containing 4 bushels, or half a quarter.

STRIKING-*wheel*, in a clock, the same as that by some called the pin-wheel, because of the pins which are placed on the round or rim, the number of which is the quotient of the pinion divided by the pinion of the detent-wheel. In sixteen-day clocks, the first or great wheel is usually the pin-wheel; but in such as go 8 days, the second wheel is the pin-wheel, or striking-wheel.

STRING, in Music. See CHORD.—If two strings or chords of a musical instrument only differ in length; their tones, or the number of vibrations they make in the same time, are in the inverse ratio of their lengths. If they differ only in thickness, their tones are in the inverse ratio of their diameters.

As to the tension of strings, to measure it regularly, they must be conceived stretched or drawn by weights; and then, cæteris paribus, the tones of two strings are in a direct ratio of the square roots of the weights that stretch them; that is, ex. gr. the tone of a string stretched by a weight 4, is an octave above the tone of a string stretched by the weight 1.

It is an observation of very old standing, that if a viol or lute-string be touched with the bow, or the hand, another string on the same instrument, or even on another, not far from it, if in unison with it, or in octave, or the like, will at the same time tremble of itself. But it is now found, that it is not the whole of that other string that thus trembles, but only the parts, severally, according as they are unisons to the whole, or the parts, of the string so struck. Thus, supposing AB to be an upper octave to *ab*, and therefore an unison to each half of it, stopped at *c*; if while *ab* is open, AB be struck, the two halves of this other, that is, *ac*, and *cb*, will both tremble; but the middle point will be at rest; as will be easily perceived, by wrapping a bit of paper lightly about the string *ab*, and moving it successively from one end of the string to the other. In like manner, if AB were an upper 12th to *ab*, and consequently an unison to its three parts *ad*, *de*, *eb*; then, *ab* being open, if AB be struck, the three parts of the other, *ad*, *de*, *eb* will severally tremble; but the points *d* and *e* remain at rest.



This, Dr. Wallis tells us, was first discovered by Mr. William Noble of Merton college; and after him by Mr. T. Pigot of Wadham college, without knowing that Mr. Noble had observed it before. To which may be added, that M. Sauveur, long afterwards, proposed it to the Royal Academy at Paris, as his own discovery, which in reality it might be; but upon his being informed, by some of the members then present, that Dr. Wallis had published it before, he immediately resigned all the honour of it. Philos. Trans.

STURM, or STURMIUS (JOHN CHRISTOPHER), a celebrated mathematician and philosopher, was born at Hippolstein, 1635, where also he died in 1703. He was first minister of a church in Germany during 5 years; and then became professor of mathematics and natural philosophy, at Altdorf in Germany. He exerted himself greatly in the cause of literature, and was very useful, by his lectures and otherwise, in explaining and diffusing the knowledge and discoveries made in that remarkable age, the 17th century; as manifest by all his writings.—He was author of several useful works, on the mathematics and philosophy, the most esteemed of which are, 1, his Mathesis Enucleata, in one vol. 8vo; 2. Mathesis Juvenilis, in 2 large volumes 8vo; 3. A large collection of letters to Dr. Henry More, of Cambridge, on the controversy concerning the weight and spring of the air; 4. Sturm published also a German translation of Archimedes. 5. But his most considerable work was, the Collegium Experimentale sive Curiosum, in quo primaria hujus seculi Inventa et Experimenta Physico-Mathematica, an. 1672, quibusdam naturæ demonstrativa &c, Norimb. an. 1676, in 4to.—In 1684 the author gave a second, and much larger part, of the like collection of discoveries, made till that era; with an appendix of further additions and explanations to the particulars in the first part. Editions of these two parts, and of the letters to Dr. More, were also printed in 1701 and 1715, together with several appendixes, the whole being usually bound in one large volume in 4to.

This is a very curious work, containing a multitude of interesting experiments, neatly illustrated by copper-plate figures printed upon almost every page, by the side of the letter-press. Of these, the 10th experiment is an improvement on father Lana's project for navigating a small vessel suspended in the atmosphere by several globes exhausted of air. But a more particular and satisfactory account of this work may be seen in the Philos. Trans. vol. 10, pa. 505, or in my Abridgment, vol. 2, pa. 265.

STYLE, in Chronology, a particular manner of counting time; as the old style, the new style. See CALENDAR.

Old STYLE, is the Julian manner of computing, as instituted by Julius Cæsar, in which the mean year consists of 365¼ days.

New STYLE, is the Gregorian manner of computation, instituted by pope Gregory the 13th, in the year 1582, and is used by most catholic countries, and many other states of Europe. The Gregorian, or new style, agrees with the true solar year, which contains only 365 days 5 hours 49 minutes. In the year of Christ 200, there was no difference of styles. In the year 1582, when the new style was first introduced, there was a difference of 10 days. At present there is 12 days difference. At the diet of Ratisbon, in the year 1700, it was decreed by the body of protestants of the empire, that 11 days should be retrenched from the old style, to accommodate it for the future to the new; and another day having been retrenched in the year 1800, it makes the difference of 12 days, as above stated. The same regulation has since passed into Sweden, Denmark, and into England, where it was established in the year 1752, when it was enacted, that in all dominions belonging to the crown of Great-Britain, the supputation, according to which the year of our Lord begins on the 25th day of March, shall not be used from and after the last day of December 1751; and that from thenceforth, the 1st day of January every year shall be reckoned to be the first day of the year: and that the natural day next immediately following the 2d day of September 1752, shall be accounted the 14th day of September, omitting the 11 intermediate nominal days of the common calendar. It is further enacted, that all kinds of writings, &c, shall bear date according to the new method of computation, and that all courts and meetings &c, feasts, fasts, &c, shall be held and observed accordingly. And for preserving the calendar in the same regular course for the future, it is enacted, that the several years of our Lord 1800, 1900, 2100, 2200, 2300, &c, except only every 400th year, of which the year 2000 shall be the first, shall be common years of 365 days, and that the years 2000, 2400, 2800, &c, and every other 400th year from the year 2000 inclusive, shall be leap years, consisting of 366 days. See Bissextile and Calendar.

The following table shows by what number of days the new style differs from the old, from 5900 years before the birth of Christ, to 5900 years after it. The days under the sign — (viz from 6000 years before to 200 years after Christ) are to be subtracted from the old style, to reduce it to the new; and the days under the sign + (viz from 200 to 5900 years after Christ) are to be added to the old style, to reduce it to the new.—All the years mentioned in the table are leap years in the old style; but those only that are marked with an L are leap years in the new.

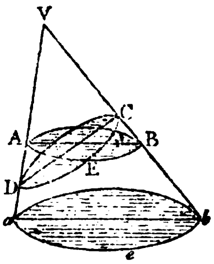
Years before Christ. New Style.	Days diff. —	Years after Christ. New Style.	Days diff. ∓
5900	46	L 0	−2
5800	45	100	−1
5700	44	200	0
L 5600	44	300	+1
5500	43	L 400	1
5400	42	500	2
5300	41	600	3
L 5200	41	700	4
5100	40	L 800	4
5000	39	900	5
4900	38	1000	6
L 4800	38	1100	7
4700	37	L 1200	7
4600	36	1300	8
4500	35	1400	9
L 4400	35	1500	10
4300	34	L 1600	10
4200	33	1700	11
4100	32	1800	12
L 4000	32	1900	13
3900	31	L 2000	13
3800	30	2100	14
3700	29	2200	15
L 3600	29	2300	16
3500	28	L 2400	16
3400	27	2500	17
3300	26	2600	18
L 3200	26	2700	19
3100	25	L 2800	19
3000	24	2900	20
2900	23	3000	21
L 2800	23	3100	22
2700	22	L 3200	22
2600	21	3300	23
2500	20	3400	24
L 2400	20	3500	25
2300	19	L 3600	25
2200	18	3700	26
2100	17	3800	27
L 2000	17	3900	28
1900	16	L 4000	28
1800	15	4100	29
1700	14	4200	30
L 1600	14	4300	31
1500	13	L 4400	31
1400	12	4500	32
1300	11	4600	33
L 1200	11	4700	34
1100	10	L 4800	34
1000	9	4900	35
900	8	5000	36
L 800	8	5100	37
700	7	L 5200	37
600	6	5300	38
500	5	5400	39
L 400	5	5500	40
300	4	L 5600	40
200	3	5700	41
100	2	5800	42
L 0	2	5900	43

The French nation, during the revolution in the year 1792, commenced another new style, or computation of time; according to which, the year commenced usually on our 22d of September. The year is divided into 12 months of 30 days each; and each month into 3 decades of 10 days each. For the names and computations of which, see the article Calendar.—They have lately however returned to the former general way of counting time.

Style, in Dialling, denotes the cock or gnomon, raised above the plane of the dial, to project a shadow.—The edge of the style, which by its shadow marks the hours on the face of the dial, is to be set according to the latitude, always parallel to the axis of the world.

STYLOBATA, or **Stylobaton**, in Architecture, the same with the pedestal of a column. It is sometimes taken for the trunk of the pedestal, between the cornice and the base, and is then called truncus. It is also otherwise named abacus.

SUBCONTRARY position, in Geometry, is when two



equiangular triangles, as VAB and VCD are so placed as to have one common angle V at the vertex, and yet their bases not parallel. Consequently the angles at the bases are equal, but on the contrary sides; viz, the $\angle A = \angle C$, and the $\angle B = \angle D$.

If the oblique cone VAB or vab, having the circular base AEB, or aeb, be so cut by a plane DEC, that the angle D be = the $\angle B$, or the $\angle C = \angle A$, then the cone is said to be cut, by this plane, in a subcontrary position to the base AEB, or aeb; and in this case the section DEC is always a circle, as well as the base AEB or aeb.

SUBDUCTION, in Arith. the same as Subtraction.

SUBDUPLE *Ratio*, is when any number or quantity is the half of another, or contained twice in it. Thus, 3 is said to be subduble of 6, as 3 is the half of 6, or is twice contained in it.

SUBDUPLICATE *Ratio*, of any two quantities, is the ratio of their square roots, being the opposite to duplicate ratio, which is the ratio of the squares. Thus, of the quantities, a and b, the subduplicate ratio is that of $\sqrt{a}$ to $\sqrt{b}$ or $a^{\frac{1}{2}}$ to $b^{\frac{1}{2}}$, as the duplicate ratio is that of a^2 to b^2.

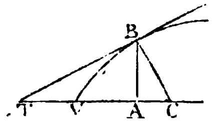
SUBLIME *Geometry*, the higher geometry, or that of curve lines. See GEOMETRY.

SUBLUNARY, is said of all things below the moon; as all things on the earth, or in its atmosphere, &c.

SUBMULTIPLE, the contrary of a multiple, being a number or quantity which is contained exactly a certain number of times in another of the same kind; or it is the same as an aliquot part of it. Thus, 3 is a submultiple of 21, or an aliquot part of it, because 21 is a multiple of 3.

SUBMULTIPLE *Ratio*, is the ratio of a submultiple or aliquot part, to its multiple; as the ratio of 3 to 21.

SUBNORMAL, in Geometry, is the subperpendicular AC, or line under the perpendicular to the curve BC, a term used in curve lines to denote the distance AC in the axis, between the ordinate AB, and the perpendicular BC to the curve or to the tangent. And the said perpendicular BC is the normal.—In all curves, the subnormal AC is a 3d proportional to the subtangent TA and the ordinate AB; and in the parabola, it is equal to half the parameter of the axis.



SUBSTITUTION, in Algebra, is the putting and using, in an equation, one quantity instead of another which is equal to it, but expressed after another manner. See REDUCTION of Equations..

SUBSTILE, or SUBSTYLE, in Dialling, a right line upon which the stile or gnomon of a dial is erected, being the common section of the face of the dial and a plane perpendicular to it passing through the stile.—The angle included between this line and the stile, is called the elevation or height of the stile.

In polar, horizontal, meridional, and northern dials, the substilar line is the meridional line, or line of 12 o'clock; or the intersection of the plane of the dial with that of the meridian.—In all declining dials, the substile makes an angle with the hour line of 12, and this angle is called the distance of the substile from the meridian.—In easterly and westerly dials, the substilar line is the line of 6 o'clock, or the intersection of the dial plane with the prime vertical.

SUBTANGENT *of a Curve*, is the line TA in the axis below the tangent TB, or limited between the tangent and ordinate to the point of contact. (See the last figure above.)—The tangent, subtangent, and ordinate, make a right-angled triangle.

In all parabolic and hyperbolic figures, the subtangent is equal to the absciss multiplied by the exponent of the power of the ordinate in the equation of the curve. Thus, in the common parabola, whose property or equation is $px = y^2$, the subtangent is equal to $2x$, double the absciss. And if $ax^2 = y^3$, or $px = y^{\frac{3}{2}}$, then the subtangent is $= \frac{3}{2}x$. Also if $a^m x = y^{m+n}$, or $px = y^{\frac{m+n}{n}}$, the subtan. is $\frac{m+n}{n}x$. See *Method of* TANGENTS.

SUBTENSE, in Geometry, of an arc, is the same as the chord of the arc; but of an angle, it is a line drawn across from the one leg of the angle to the other, or between the two extremes of the arc that measures the angle.

SUBTRACTION, or SUBSTRACTION, in Arithmetic, is the taking of one number or quantity from another, to find the remainder, or difference between them; and is usually made the second rule in arithmetic. The greater number or quantity is called the minuend, the less is the subtrahend, and the remainder is the difference. Also the sign of subtraction is −, or minus.

SUBTRACTION *of Whole Numbers*, is performed by setting the less number below the greater, as in addition, units under units, tens under tens, &c; and then, proceeding from the right hand towards the left, subtract or take each lower figure from that above it, and set down the several remainders or differences underneath; and these will compose the whole remainder or difference of the two given numbers. But when any one of the figures of the under number is greater than that of the upper, from which it is to be taken, you must add 10 (in your mind) to that upper figure, then take the under one from this sum, and set the difference underneath, carrying or adding 1 to the next under figure to be subtracted. Thus, for example, to subtract 2904821 from 37409732

Minuend	37409732
Subtrahend	2904821
Difference	34504911
Proof	37409732

To prove Subtraction: Add the remainder or difference to the less number, and the sum will be equal to the greater when the work is right.

SUBTRACTION *of Decimals*, is performed in the same manner as in whole numbers, by observing only to set the figures or places of the same kind under each other. Thus:

From	351·04	·479	27
Take	72·71	·0573	0·936
Rem. Diff.	278·33	·4217	26·064

To Subtract Vulgar Fractions. Reduce the two fractions to a common denominator, if they have different ones; then take the less numerator from the greater, and set the remainder over the common denominator, for the difference sought.—It is best to set the less fraction after the greater, with the sign (−) of subtraction between them, and the mark of equality (=) after them.

Thus, $\frac{7}{9} - \frac{2}{9} = \frac{5}{9}$.

And $\frac{3}{5} - \frac{4}{7} = \frac{21}{35} - \frac{20}{35} = \frac{1}{35}$.

Subtraction, *in Algebra*, is performed by changing the signs of all the terms of the subtrahend, to their contrary signs, viz, + into −, and − into +; and then uniting the terms with those of the minuend after the manner of addition of Algebra.

$$\begin{array}{ll}
\text{Ex. From} & +\,6a \\
\text{Take} & +\,2a \\
\text{Rem.} & 6a - 2a = 4a. \\
\text{From} & +\,6a \\
\text{Take} & -\,2a \\
\text{Rem.} & 6a + 2a = 8a. \\
\text{From} & -\,6a \\
\text{Take} & +\,2a \\
\text{Rem.} & -\,6a - 2a = -\,8a. \\
\text{From} & -\,6a \\
\text{Take} & -\,4a \\
\text{Rem.} & -\,6a + 4a = -\,2a. \\
\text{From} & 2a - 3x + 5z - 6 \\
\text{Take} & 6a + 4x + 5z + 4 \\
\text{Rem.} & -\,4a - 7x \quad 0 \; - 10
\end{array}$$

SUBTRIPLE, is when one quantity is the third part of another; as 2 is subtriple of 6. And **Subtriple** *Ratio*, is the ratio of 1 to 3.

SUBTRIPLICATE *Ratio*, is the ratio of the cube roots. So the subtriplicate ratio of a to b, is the ratio of $\sqrt[3]{a}$ to $\sqrt[3]{b}$, or of $a^{\frac{1}{3}}$ to $b^{\frac{1}{3}}$.

SUCCESSION *of Signs*, in Astronomy, is the order in which they are reckoned, or follow one another, and according to which the sun enters them; called also consequentia. As Aries, Taurus, Gemini, Cancer, &c. When a planet goes according to the order and succession of the signs, or in consequentia, it is said to be direct; but retrograde when they move the contrary way, or in antecedentia, as from Gemini to Taurus, then to Aries, &c.

SUCCULA, in Mechanics, a bare axis or cylinder with staves in it to move it round; but without any tympanum, or peritrochium.

SUCKER, in Mechanics, a name by which sometimes is called the piston or bucket, in a sucking pump; and sometimes the pump itself is so called.

SUCKING *Pump*, the common pump, working by two valves opening upwards. See **Pump**.

SUISETH (**Ricardi** or **Raymundi**), an early writer on Arithmetic. A long account of his book, called the Calculator, is given in Brucker's History of Philosophy.

SUM, the quantity produced by addition, or by adding two or more numbers or quantities together. So the sum of 6 and 4 is 10, and the sum of a and b is $a + b$.

SUMMER, the name of one of the seasons of the year, being one of the quarters when the year is divided into 4 parts, or one half when the year is divided only into two, summer and winter. In the former case, summer is the quarter during which, in northern climates, the sun is passing through the three signs Cancer, Leo, Virgo, or from the time of the greatest declination, till it comes to the equinoctial again, or have no declination; which is from about the 21st of June, to the 22d of September. In the latter case, summer contains the 6 warmer months, while the sun is on one side of the equinoctial; and winter the other 6 months, when the sun is on the other side of it.

Summer *Beam*, in carpentry, a large piece of timber which, being supported on two pillars or posts, serves as a lintel to a gate, door, or window, &c.

Summer *Solstice*, the time or point when the sun attains his greatest declination, and is nearest the zenith of the place. See **Solstice**.

SUN, **Sol**, ⊙, in Astronomy, the great luminary that is placed in the centre of our system, and about which all the planets revolve, in different periods, and at different distances. It is the great fountain of light and heat to all those bodies, warming and refreshing both their animal and vegetable inhabitants with the refulgence of his beams; without which, all nature would be involved in impenetrable darkness. The comets also revolve about the sun, but in excentric orbits, being sometimes very near him; and at others, at an incalculable distance from him.

The ancient astronomers conceived the earth to be the centre of the universe, having the sun, and all the other celestial bodies revolving about it; but this absurd doctrine was at last confuted and annihilated by Copernicus; though not without many angry disputes, and malignant persecutions, particularly by the church of Rome, because it seemed to contradict some parts of Scripture. Truth however at length prevailed; and gave to the sun his due place in the centre of our system.

It has since been discovered, that the sun has a motion on its own axis, in about 25½ days, as appears from the maculæ or spots on his disc. For, some of these spots have made their first appearance near the edge or margin of the sun, from thence they have seemed gradually to pass over the sun's face to the opposite edge, then disappear; and hence, after an absence of about 14 days, they have reappeared in their first place, and have taken the same course over again; finishing their entire circuit in 27 days $12^h\ 20^m$; which is hence inferred to be the period of the sun's rotation round his axis: and therefore the periodical time of the sun's revolution to a fixed star is $25^d\ 15^h\ 16^m$; because in $27^d\ 12^h\ 20^m$ of the month of May, when the observations were made, the earth describes an angle about the sun's centre of 26° 22′, and therefore as the angular motion $360° + 26°\ 22' : 360 :: 27^d\ 12^h\ 20^m : 25^d\ 15^h\ 16^m$. This motion of the spots is from west to east: whence we conclude the motion of the sun, to which the other is owing, to be from east to west. The more correct period of the sun's rotation is now stated at 25 days 10 hours.

Besides this motion round his axis, the sun, on account of the various attractions of the surrounding planets, is agitated by a small motion round the centre of gravity of the system.—Whether the sun and stars have any proper motion of their own in the immensity of space, however small, is not absolutely certain; though some very accurate observers have intimated conjectures of this kind, and have shown that such a general motion is not improbable. See **Stars**.

As for the apparent annual motion of the **Sun** *round the earth;* it is easily shown, by astronomers, that the real annual motion of the earth, about the sun, will cause such an appearance. A spectator in the sun would see the earth move from west to east, for the same reason as we see the sun move from east to west: and all the phenomena resulting from this annual motion in whichever of the bodies it be, will appear the same from either. And hence arises that apparent motion of the sun, by which he is seen to advance insensibly towards the eastern stars; in so much that, if any star, near the ecliptic, rise at any time with the sun; after a few days the sun will be got more to the east of the star, and the star will rise and set before him.

Nature, Properties, Figure, &c, of the SUN.

Those who have maintained that the substance of the sun is fire, argue in the following manner: The sun shines, and his rays, collected by concave mirrors, or convex lenses, will burn, consume, and melt the most solid bodies, or else convert them into ashes, or glass: therefore, as the force of the solar rays is diminished, by their divergency, in a duplicate ratio of the distances reciprocally taken; it is evident that their force and effect are the same, when collected by a burning lens, or mirror, as if we were at such distance from the sun, where they were equally dense. The sun's rays therefore, in the neighbourhood of the sun, produce the same effects, as might be expected from the most vehement fire: consequently the sun is of a fiery substance.

Hence it follows, that its surface is probably every where fluid; that being the condition of flame. Indeed, whether the whole body of the sun be fluid, as some think; or solid, as others; they do not presume to determine: but as there are no other marks, by which to distinguish fire from other bodies, but light, heat, a power of burning, consuming, melting, calcining, and vitrifying; they do not see what objection should be made to the hypothesis that the sun is a globe of fire, like our fires, invested with flame: and, supposing that the maculæ are formed out of the solar exhalations, they infer that the sun is not pure fire; but that there are heterogeneous parts mixed with it.

Philosophers have been much divided in opinion with respect to the nature of fire, light, and heat, and the causes that produce them: and they have given very different accounts of the agency of the sun, with which, whether we consider them as substances or qualities, they are intimately connected, and on which they seem primarily to depend. Some, among whom we may reckon Sir Isaac Newton, consider the rays of light as composed of small particles, which are emitted from shining bodies, and move with uniform velocities in uniform mediums, but with variable velocities in mediums of variable densities. These particles, say they, act upon the minute constituent parts of bodies, not by impact, but at some indefinitely small distance; they attract and are attracted; and in being reflected or refracted, they excite a vibratory motion in the component particles. This motion increases the distance between the particles, and thus occasions an augmentation of bulk, or an expansion in every dimension, which is the most certain characteristic of fire. This expansion, which is the beginning of a disunion of the parts, being increased by the increasing magnitude of the vibrations proceeding from the continued agency of light, it may easily be apprehended, that the particles will at length vibrate beyond their sphere of mutual attraction, and thus the texture of the body will be altered or destroyed; from solid it may become fluid, as in melted gold; or from being fluid, it may be dispersed in vapour, as in boiling water.

Others, as Boerhaave, represent fire as a substance sui generis, unalterable in its nature, and incapable of being produced or destroyed; naturally existing in equal quantities in all places, imperceptible to our senses, and only discoverable by its effects, when, by various causes, it is collected for a time into a less space than that which it would otherwise occupy. The matter of this fire is not in any wise supposed to be derived from the sun: the solar rays, whether direct or reflected, are of use only as they impel the particles of fire in parallel directions: that parallelism being destroyed, by intercepting the solar rays, the fire instantly assumes its natural state of uniform diffusion. According to this explication, which attributes heat to the matter of fire, when driven in parallel directions, a much greater degree must be given it when the quantity, so collected, is amassed into a focus; and yet the focus of the largest speculum does not heat the air or medium in which it is found, but only bodies of densities different from that medium.

M. Deluc (Lettres Physiques) is of opinion, that the solar rays are the principal cause of heat; but that they heat such bodies only as do not allow them a free passage. In this remark he agrees with Newton: but then he differs totally from him, as well as from Boerhaave, concerning the nature of the rays of the sun. He does not admit the emanation of any luminous corpuscles from the sun, or other self-shining substances, but supposes all space to be filled with an ether of great elasticity and small density, and that light consists in the vibrations of this ether, as sound consists in the vibrations of the air. "Upon Newton's supposition," says an excellent writer, "the cause by which the particles of light and the corpuscles constituting other bodies are mutually attracted and repelled, is uncertain. The reason of the uniform diffusion of fire, of its vibration, and repercussion, as stated in Boerhaave's opinion, is equally inexplicable. And in the last mentioned hypothesis, we may add to the other difficulties attending the supposition of an universal ether, the want of a first mover to make the sun vibrate."

Dr. Herschel has given, in the Philos. Trans. an ingenious paper on the physical construction of the sun. This plausible and ingenious theory is suggested by a variety of observations on the solar phenomena. The sun, he supposes, has an atmosphere resembling that of the earth; and this atmosphere consists of various elastic fluids, some of which exhibit a shining brilliancy, while others are merely transparent. Whenever the lucid fluid is removed, the body of the sun may be seen through those that are transparent. In like manner, an observer placed in the moon, will see the solid body of the earth only in those places where the transparent fluids of our atmosphere will permit him. In others, the opaque vapours will reflect the sun's light, without permitting his view to penetrate to the surface of our globe.

By changes in the atmosphere of Jupiter too, Dr. H. accounts for the phenomena of its belts: and on the same principle he illustrates the various appearances of spots observed in the sun. Such phenomena, he thinks, may be easily and satisfactorily explained, if it be allowed that the real solid body of the sun itself is seen on these occasions, though we seldom see more than its shining atmosphere. He apprehends that there are considerable inequalities in the surface of the sun; and that there may be elevations not less than 5 or 600 miles high. That a very high country, or chain of mountains, may oftener become visible, by the removal of the obstructing fluid, than the lower regions, on account of its not being so deeply covered by it. See *Solar* SPOTS.

All the phenomena of the spots, of the faculæ, and of the livid surface of the sun, concur to establish the existence of a solar atmosphere of very considerable extent, and to evince its composition of various elastic fluids, that are more or less lucid and transparent: but the lucid one is that which furnishes us with light. The generation of this lucid fluid, in the solar atmosphere, is a phenomenon similar to the generation of clouds in our atmosphere, which are produced by the decomposition of its constituent elastic fluids: but with this difference, that the continual and very extensive decompositions of the elastic fluids of the sun, are of a phosphoric nature, and attended with lucid appearances, by giving out light. To the objection that such decompositions, and the consequent emission of light, would exhaust the sun, Dr. H. replies that, in the decomposition of phosphoric fluids, every other ingredient besides light may return to the body of the sun. This waste, however, must be quite insensible, even in a very long period, when the extreme subtilty of light is considered: and besides, it may possibly be supplied by those telescopic comets, many of which are observed, which have no appearance of any solid nucleus, seeming to be mere collections of vapours condensed about a centre.

The sun, contemplated with the assistance of the doctor's theory, "appears to be nothing else than a very large, eminent, lucid planet, evidently the first, or indeed the only primary one of our system; all others being truly secondary to it. Its similarity to the other globes of the solar system, with regard to its solidity, its atmosphere, and its diversified surface; the rotation on its axis, and the fall of heavy bodies, lead us on to suppose that it is most probably also inhabited, like the rest of the planets, by beings whose organs are adapted to the peculiar circumstances of that vast globe."

Should it be objected that the heat of the sun renders it unfit for a habitable world, Dr. H. answers, that heat is produced by the sun's rays only when they act on a calorific medium, and that they are the cause of the production of heat, by uniting with the matter of fire, which is contained in the substances that are heated. Dr. H. suggests other considerations, intended to invalidate the objection. He then deduces from analogy a variety of arguments, in order to confirm the notion of the sun's being habitable; and infers that, if the sun be capable of accommodating inhabitants, the other stars, which are suns may be appropriated to the same use; and thus, says he, we see at once what an extensive field for animation opens itself to our view. Philos. Trans. Abridg. vol. 17, pa. 478. See also SPOTS.

Dr. Herschel has made many interesting experiments on the nature of the sun's rays, and has thus firmly established a fact which had long been disputed between philosophers, namely, the separate identity of light and heat: that they are both subject to the laws of reflection and refraction; that they are each of different refrangibility, are liable to be stopped in certain proportions when transmitted through diaphanous bodies; and that they are liable to be scattered on rough surfaces. These curious facts were discovered by the doctor in his optical experiments on coloured glass, in which he was led to examine the difference between the coloured rays of the sun with regard to their heating power. He thereby discovered that the most refracted rays of light, the violet, possess the lowest heating power; and the least refracted, the red, the greatest power, and the mean rays of the prismatic spectrum showed an intermediate power. Thus, in the red rays the thermometer, by the average of several experiments, rose 6⅞ degrees; in the green rays 3¼ degrees; and the violet 2 degrees; or in round numbers, the effect of the red rays was to that of the green as 9 to 4, and to that of the violet as 7 to 2.

Pursuing those experiments, the same philosopher found the range of dispersion of the rays of heat by the prism, to differ most essentially from that of light; for on applying thermometers of great sensibility and successively in a line, beginning at the violet rays, proceeding along the prismatic spectrum, he found not only the heat increased by advancing towards the red, or least refracted rays, but that the heat was greatest at a small distance beyond the extreme limits of the spectrum, that is, where no rays of light at all fell; and still continuing to advance the thermometer in the same line, the heat then gradually diminished, till it became too small to be noticed. This most curious and important discovery shows, therefore, both an entire separation of heat from light in the solar ray, and refrangibility of one from the other, which together go near to establish the separate identity of caloric and light, and cause precisely the same arguments used to demonstrate the materiality of light to apply to the materiality of heat.

These experiments of Dr. Herschel have been fully confirmed by sir H. Englefield, whose apparatus were somewhat different, and more accurate. The particulars of which are as follow. The coloured rays of the spectrum were successively and singly thrown on a lens (all the others being excluded by a screen), the thermometer with a blackened ball being placed in its focus, and allowed to remain there some time after it had ceased to rise, that the full effect might be secured. Thus circumstanced,

In the blue ray from	55°	to	56°
green	54	to	58
yellow	56	to	62
red	56	to	72
Quite out of visible light	61	to	79

These experiments were repeated several times, and in all with very closely corresponding results, and the most striking and novel phenomenon was manifest in all, namely, the rise of the thermometer, when passed beyond the extreme point of the luminous spectrum on the red side, and its fall when again carried back into the red light.

As to the Figure of the SUN; this, like the planets, is not perfectly globular, but spheroidical, being higher about the equator than at the poles. The reason of which is this: the sun has a motion about his own axis; and therefore the solar matter will have an endeavour to recede from the axis, and that with the greater force as their distances from it, or the circles they move in, are greater: but the equator is the greatest circle; and the rest, towards the poles, continually decrease; therefore the solar matter, though at first in a spherical form, will endeavour to recede from the centre of the equator further than from the centres of the parallels. Consequently, since the gravity, by which it is retained in its place, is supposed to be uniform throughout the whole sun, it will really recede from the centre more at the equator, than at any of the parallels; and hence the sun's diameter will be greater through the equator, than through the poles; that is, the sun's figure is not perfectly spherical, but spheroidical.

Several particulars of the Sun, related by Newton, in his Principia, are as follow: 1. That the density of the sun's heat, which is proportional to his light, is 7 times as great at Mercury as with us; and therefore our water there would be all carried off in vapour: for he found by experiments of the thermometer, that a heat but 7 times greater than that of the sun beams in summer, will serve to make water boil.

2. That the quantity of matter in the sun is to that in Jupiter, nearly as 1100 to 1; and that the distance of that planet from the sun, is in the same ratio to the sun's semidiameter.

3. That the matter in the sun is to that in Saturn, as 2360 to 1; and the distance of Saturn from the sun is in a ratio but little less than that of the sun's semidiameter. And hence, that the common centre of gravity of the sun and Jupiter is nearly in the superficies of the sun; of the sun and Saturn, a little within it.

4. And by the same mode of calculation it will be found, that the common centre of gravity of all the planets, cannot be more than the length of the solar diameter distant from the centre of the sun. This common centre of gravity he proves is at rest; and therefore though the sun, by reason of the various positions of the planets, may be moved every way, yet it cannot recede far from the common centre of gravity, and this, he thinks, ought to be accounted the centre of our world. Book 3, prop. 12.

5. By means of the solar spots it hath been discovered, that the sun revolves round his own axis, without moving considerably out of his place, in about 25 days, and that the axis of this motion is inclined to the ecliptic in an angle of 87° 30 nearly. The sun's apparent diameter being sensibly longer in December than in June, the sun must be proportionably nearer to the earth in winter than in summer; in the former of which seasons therefore will be the perihelion, in the latter the aphelion: and this is also confirmed by the earth's motion being quicker in December than in June, as it is by about $\frac{1}{15}$ part. For since the earth always describes equal areas in equal times, whenever it moves swifter, it must needs be nearer to the sun: and for this reason there are about 8 days more from the sun's vernal equinox to the autumnal, than from the autumnal to the vernal.

6. That the sun's diameter is equal to 100 diameters of the earth; and therefore the body of the sun must be 1,000,000 times greater than that of the earth.—Mr. Azout assures us, that he observed, by a very exact method, the sun's diameter to be no less than 31′ 45″ in his apogee, and not greater than 32′ 45″ in his perigee.

7. According to Newton, in his theory of the moon, the mean apparent diameter of the sun is 32′ 12″.—The sun's horizontal parallax is now fixed at $8''\frac{6}{10}$.

8. If you divide 360 degrees (the whole ecliptic) by the quantity of the solar year, it will give 59′ 8″ &c, which therefore is the medium quantity of the sun's daily motion; and if this 59′ 8″ be divided by 24, you have the sun's horary motion equal to 2′ 28″: and if this last be divided by 60, it will give his motion in a minute, &c. And in this way are the tables of the sun's mean motion constructed, as placed in books of astronomical tables and calculations.

SUNDAY, the first day of the week; thus called by our idolatrous ancestors, because set apart for the worship of the sun.—It is sometimes called the Lord's Day, because kept as a feast in memory of our Lord's resurrection on this day: and also Sabbath-day, because substituted under the new law instead of the sabbath in the old law.—It was Constantine the Great who first made a law for the observation of Sunday; and who, according to Eusebius, appointed that it should be regularly celebrated throughout the Roman empire.

SUNDAY *Letter*. See DOMINICAL *Letter*.

SUPERFICIAL, relating to Superficies.

SUPERFICIES, or SURFACE, in Geometry, the outside or exterior face of any body. This is considered as having the two dimensions of length and breadth only, but no thickness; and therefore it makes no part of the substance or solid content or matter of the body. The terms or bounds or extremities of a superficies, are lines; and superficies may be considered as generated by the motions of lines.—Superficies are either rectilinear, curvilinear, plane, concave, or convex.

Rectilinear SUPERFICIES, is bounded by right lines.

Curvilinear SUPERFICIES, is bounded by curve lines.

Plane SUPERFICIES, is that which has no inequality in it, nor risings, nor sinkings, but lies evenly and straight throughout, so that a right line may wholly coincide with it in all parts and directions.

Convex SUPERFICIES, is that which is curved and rises outwards.

Concave SUPERFICIES, is curved and sinks inward.

The measure or quantity of a surface, is called its area. And the finding of this measure or area, is sometimes called the quadrature of it, meaning the reducing it to an equal square, or to a certain number of smaller squares. For all plane figures, and the surfaces of all bodies, are measured by squares; as square inches or square feet, or square yards, &c; that is, squares whose sides are inches, or feet, or yards, &c. Our least superficial measure is the square inch, and other squares are taken from it according to the proportion in the following table of superficial or square measure.

144 square inches = 1 square foot
9 square feet = 1 square yard
30¼ square yards = 1 square pole
16 square poles = 1 square chain
10 square chains = 1 acre
640 acres = 1 square mile.

The superficial measure of all bodies and figures depends entirely on that of a rectangle; and this is found by drawing or multiplying the length by the breadth of it; as is proved from plane geometry only, in my Mensuration, pt. 2, sect. 1, prob. 1. From the area of the rectangle we obtain that of any oblique parallelogram, which, by geometry, is equal to a rectangle of equal base and altitude; thence a triangle, which is the half of such a parallelogram or rectangle; and hence, by composition, we obtain the superficies of all other figures whatever, as these may be considered as made up of triangles only.

Besides this way of deriving the superficies of all figures, which is the most simple and natural, as proceeding on common geometry alone, there are certain other methods; such as the methods of exhaustions, of fluxions, &c. See these articles in their places, as also QUADRATURES.

Line of SUPERFICIES, a line usually found on the sector, and Gunter's scale. The description and use of which, see under SECTOR, and GUNTER'S *Scale*.

SUPPLEMENT, of an arch, or angle, in Geometry or Trigonometry, is what it wants of a semicircle, or of 180 degrees; as the complement is what it wants of a quadrant, or of 90 degrees. So, the supplement of 50° is 130°; as the complement of it is 40°.

SURD, in Arithmetic and Algebra, denotes a number or quantity that is incommensurate to unity; or that is inexpressible in rational numbers by any known way of notation, otherwise than by its radical sign or index.—This is otherwise called an irrational or incommensurable number, as also an imperfect power.

The square roots of all numbers except 1, 4, 9, 16, 25, 36, &c, (which are the squares of the whole numbers 1, 2, 3, 4, 5, 6, &c,) are surds, or incommensurables; after the same manner, the cube roots of all numbers except the cubes of 1, 2, 3, 4, 5, 6, &c, are surds. And it is usual to denote such root by setting before it the proper mark of radicality, which is $\surd$, and placing above this radical sign the number that shows what kind of root is intended. Thus, $\sqrt[2]{2}$ or $\sqrt{2}$ signifies the square root of 2, and $\sqrt[3]{10}$ the cube root of 10; which roots, because it is impossible to express them in numbers exactly, are properly called surd roots.

Another way of notation, by which roots are expressed, is by fractional indices, without the radical sign: thus, as x^2, x^3, x^4, &c, denote the square, cube, 4th power, &c, of x; so $x^{\frac{1}{2}}$, $x^{\frac{1}{3}}$, $x^{\frac{1}{4}}$, &c, denote the square root, cube root, 4th root, &c, of the same quantity x.—The reason of which is evident; for since $\sqrt{x}$ is a geometrical mean proportional between 1 and x, so $\frac{1}{2}$ is an arithmetical mean between 0 and 1; and therefore, as 2 is the index of the square of x, $\frac{1}{2}$ will be the proper index of its square root, &c.

It may be observed that, for convenience, or the sake of brevity, quantities which are not naturally surds, are often expressed in the form of surd roots. Thus $\sqrt{4}$, $\sqrt{\frac{9}{4}}$, $\sqrt[3]{27}$, are the same as 2, $\frac{3}{2}$, 3.

Surds are either *simple* or *compound*.

Simple SURDS, are such as are expressed by one single term; as $\sqrt{2}$, or $\sqrt[3]{a}$, &c.

Compound SURDS, are such as consist of two or more simple surds connected together by the signs + or −; as $\sqrt{3} + \sqrt{2}$, or $\sqrt{3} - \sqrt{2}$, or $\sqrt[3]{(5 + \sqrt{2})}$: which last is called an universal root, and denotes the cubic root of the sum arising by adding 5 and the root of 2 together.

Of certain Operations by Surds.

1. Such surds as $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, &c, though they are themselves incommensurable with unity, according to the definition, are commensurable in power with it, because their powers are integers, which are multiples of unity. They may also be sometimes commensurable with one another; as $\sqrt{8}$ and $\sqrt{2}$, which are to each other as 2 to 1, as is found by dividing them by their greatest common measure, which is $\sqrt{2}$, for then those two become $\sqrt{4} = 2$, and 1 the ratio.

2. *To reduce Rational Quantities to the form of any proposed Surd Roots.*—Involve the rational quantity according to the index of the power of the surd, and then prefix before that power the proposed radical sign.

Thus $a = \sqrt{a^2} = \sqrt[3]{a^3} = \sqrt[4]{a^4} = \sqrt[n]{a^n}$, &c.
and $4 = \sqrt{16} = \sqrt[3]{64} = \sqrt[4]{256} = \sqrt[n]{4^n}$, &c.

And in this way may a simple surd fraction, whose radical sign refers to only one of its terms, be changed into another, which shall include both numerator and denominator. Thus,

$\frac{\sqrt{2}}{5}$ is reduced to $\sqrt{\frac{2}{25}}$, and $\frac{5}{\sqrt[3]{4}}$ to $\sqrt[3]{\frac{125}{4}}$: thus also the quantity a reduced to the form of $x^{\frac{1}{n}}$ or $\sqrt[n]{x}$, is $(a^n)^{\frac{1}{n}}$ or $\sqrt[n]{a^n}$. And thus may roots with rational coefficients be reduced so as to be wholly affected by the radical sign; as $a\sqrt[n]{x} = \sqrt[n]{a^n x}$.

3. *To reduce Simple Surds, having different radical signs (which are called heterogeneal Surds) to others that may have one common radical sign, or which are homogeneal: Or to reduce roots of different names to roots of the same name.*—Involve the powers reciprocally, each according to the index of the other, for new powers; and multiply their indices together, for the common index. Otherwise, as surds may be considered as powers with fractional exponents, reduce these fractional exponents to fractions having the same value and a common denominator.—Thus, by the 1st method,

$\sqrt[n]{a}$ and $\sqrt[m]{x}$ become $\sqrt[mn]{a^m}$ and $\sqrt[mn]{x^n}$;

and, by the 2d method,

$a^{\frac{1}{n}}$ and $x^{\frac{1}{m}}$ become $(a^m)^{\frac{1}{mn}}$ and $(x^n)^{\frac{1}{mn}}$.

Also $\sqrt{3}$ and $\sqrt[3]{2}$ are reduced to $\sqrt[6]{27}$ and $\sqrt[6]{4}$, which are equal to them, and have a common radical sign.

4. *To reduce Surds to their most simple expressions, or to the lowest terms possible.*—Divide the surd by the greatest power, of the same name with that of the root, which is contained in it, and which will measure or divide it without a remainder; then extract the root of that power, and place it before the quotient or surd so divided; this will produce a new surd of the same value with the former, but in more simple terms. Thus, $\sqrt{16a^2x}$, by dividing by $16a^2$, and prefixing its root $4a$, before the quotient $\sqrt{x}$, becomes $4a\sqrt{x}$; in like manner, $\sqrt{12} = \sqrt{(4 \times 3)} = 2\sqrt{3}$;

And $\sqrt[3]{ab^3x}$ reduces to $b\sqrt[3]{ax}$.
Also $\sqrt[3]{81} = \sqrt[3]{27 \times 3} = \sqrt[3]{3^3 \times 3} = 3\sqrt[3]{3}$.
And $\sqrt{288} = \sqrt{144 \times 2} = 12\sqrt{2}$.

5. *To Add and Subtract Surds.*—When they are reduced to their lowest terms, if they have the same irrational part, add or subtract their rational coefficients, and to the sum or difference subjoin the common irrational part.

Thus, $\sqrt{75} + \sqrt{48} = 5\sqrt{3} + 4\sqrt{3} = 9\sqrt{3}$;
and $\sqrt{150} - \sqrt{54} = 5\sqrt{6} - 3\sqrt{6} = 2\sqrt{6}$;
also $\sqrt{a^2x} + \sqrt{c^2x} = a\sqrt{x} + c\sqrt{x} = (a+c)\sqrt{x}$.

Or such surds may be added and subtracted, by first squaring them (by uniting the square of each part with double their product), and then extracting the root universal of the whole. Thus, for the first example above,

$\sqrt{75} + \sqrt{48} = \sqrt{(75 + 48 + 2\sqrt{75 \times 48})} =$
$\sqrt{(123 + 2\sqrt{3600})} = \sqrt{(123 + 120)} =$
$\sqrt{243} = 9\sqrt{3}$, the same as before.

If the quantities cannot be reduced to the same irrational part, they can only be connected by the signs + or −.

6. *To Multiply and Divide Surds.*—If the terms have the same radical, they will be multiplied and divided like powers, viz, by adding their indices for multiplication, and subtracting them for division. Thus,

$$\sqrt{a} \times \sqrt[3]{a} = a^{\frac{1}{2}} \times a^{\frac{1}{3}} = a^{\frac{3}{6}} \times a^{\frac{2}{6}} = a^{\frac{5}{6}} = \sqrt[6]{a^5};$$

$$\text{and } \sqrt{2} \times \sqrt[3]{2} = 2^{\frac{5}{6}} = \sqrt[6]{2^5} = \sqrt[6]{32};$$

$$\text{also } \sqrt{a} \div \sqrt[3]{a} = a^{\frac{1}{2}} \div a^{\frac{1}{3}} = a^{\frac{1}{6}} = \sqrt[6]{a};$$

$$\text{and } \sqrt{2} \div \sqrt[3]{2} = 2^{\frac{1}{6}} = \sqrt[6]{2}.$$

If the quantities be different, but under the same radical sign; multiply or divide the quantities, and place the radical sign to the product or quotient.

Thus, $\sqrt{2} \times \sqrt{5} = \sqrt{10}$;
and $\sqrt[3]{a^2} \times \sqrt[3]{c} = \sqrt[3]{a^2c}$;
also $\sqrt[3]{54} \div \sqrt[3]{2} = \sqrt[3]{27} = 3$.

But if the surds have not the same radical sign, reduce them to such as shall have the same radical sign, and proceed as before.

Thus, $\sqrt[m]{a} \times \sqrt[n]{b} = \sqrt[mn]{a^n} \times \sqrt[mn]{b^m} = \sqrt[mn]{a^nb^m}$;
and $\sqrt{2} \times \sqrt[3]{4} = \sqrt[6]{2^3} \times \sqrt[6]{4^2} = \sqrt[6]{8 \times 16} = \sqrt[6]{128}$.

If the surds have any rational coefficients, their product or quotient must be prefixed.

Thus, $5\sqrt{6} \times 2\sqrt{3} = 10\sqrt{18} = 30\sqrt{2}$;
and $8\sqrt{5} \div 2\sqrt{6} = 4\sqrt{\tfrac{5}{6}}$.

7. *Involution and Evolution of Surds.*—Surds are involved, or raised to any power, by multiplying their indices by the index of the power; and they are evolved or extracted, by dividing their indices by the index of the root.

Thus, the square of $\sqrt[3]{2}$ or of $2^{\frac{1}{3}}$, is $2^{\frac{2}{3}} = \sqrt[3]{4}$;
and the cube of $\sqrt{5}$ or of $5^{\frac{1}{2}}$, is $5^{\frac{3}{2}} = \sqrt{125}$;
also the square root of $\sqrt[3]{4}$ or $4^{\frac{1}{3}}$, is $4^{\frac{1}{6}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$.

Or thus: Involve or extract the quantity under the radical sign according to the power or root required, continuing the same radical sign.

So the square of $\sqrt[3]{2}$ is $\sqrt[3]{4}$;
and the square root of $\sqrt[3]{4}$, is $\sqrt[3]{2}$.

Unless the index of the power is the same as the name of the surd, or a multiple of it, for in that case the power of the surd becomes rational. Thus, the square of $\sqrt{3}$ is 3, and the cube of $\sqrt[3]{a^2}$ is a^2.

Simple surds are commensurable in power, and by being multiplied by themselves give, at length, rational quantities: but compound surds, multiplied by themselves, commonly give irrational products. Yet, in this case, when any compound surd is proposed, there is another compound surd, which, multiplied by it, gives a rational product.

Thus, $\sqrt{a} + \sqrt{b}$ multiplied by $\sqrt{a} - \sqrt{b}$ gives $a - b$;
and $\sqrt[3]{a} - \sqrt[3]{b}$ mult. by $\sqrt[3]{a^2} + \sqrt[3]{ab} + \sqrt[3]{b^2}$ gives $a - b$.
The finding of such a surd as multiplying the proposed surd gives a rational product, is made easy by three theorems, delivered by Maclaurin, in his Algebra, pa. 109 &c.

This operation is of use in reducing surd expressions to more simple forms. Thus, suppose a binomial surd divided by another, as $\sqrt{20} + \sqrt{12}$ by $\sqrt{5} - \sqrt{3}$, the quotient might be expressed by $\frac{\sqrt{20} + \sqrt{12}}{\sqrt{5} - \sqrt{3}} = \frac{2\sqrt{5} + 2\sqrt{3}}{\sqrt{5} - \sqrt{3}}$; but this will be expressed in a more simple form, by multiplying both numerator and denominator by such a surd as makes the product of the denominator become a rational quantity: thus, multiplying them by $\sqrt{5} + \sqrt{3}$, the fraction or quotient becomes

$$2 \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} - \sqrt{3}} \times \frac{\sqrt{5} + \sqrt{3}}{\sqrt{5} + \sqrt{3}} = 2 \times \frac{(\sqrt{5} + \sqrt{3})^2}{5 - 3 = 2} = \sqrt{(5 + \sqrt{3})^2} = 8 + 2\sqrt{15}.$$

To do this generally, see Maclaurin's Alg. p. 113.

When the square root of a surd is required, it may be found nearly, by extracting the root of a rational quantity that approximates to its value. Thus, to find the square root of $3 + 2\sqrt{2}$; first find the $\sqrt{2} = 1{\cdot}41421$; hence $3 + 2\sqrt{2} = 5{\cdot}82842$, the root of which is nearly $2{\cdot}41421 = 1 + \sqrt{2}$.

In like manner we may proceed with any other proposed root. And if the index of the root be very high, a table of logarithms may be used to advantage: thus, to extract the root $\sqrt[7]{(5 + \sqrt[13]{17})}$; take the logarithm of 17, divide it by 13, find the number answering to the quotient, add this number to 5, find the log. of the sum, and divide it by 7, and the number answering to this quotient will be nearly equal to $\sqrt[7]{(5 + \sqrt[13]{17})}$.

But it is sometimes requisite to express the roots of surds exactly by other surds. Thus, in the first example, the square root of $3 + 2\sqrt{2}$ is $1 + \sqrt{2}$, because $(1 + \sqrt{2})^2 = 1 + 2\sqrt{2} + 2 = 3 + \sqrt{2}$. For the method of performing this, the curious reader may consult Maclaurin's Algeb. pa. 115, where also rules for trinomials &c may be found. See also the article **Binomial** *Roots*, in this Dictionary.

For extracting the higher roots of a binomial, whose two members when squared are commensurable numbers, we have a rule in Newton's Arith. pa. 59, but without demonstration. This is supplied by Maclaurin, in his Alg. pa. 120: as also by Gravesande, in his Matheseos Univers. Elem. pa. 211.

It sometimes happens, in the solution of cubic equations, that binomials of this form $a \pm b\sqrt{-1}$ occur, the cube roots of which must be found; and to these Newton's rule cannot always be applied, because of the impossible or imaginary factor $\sqrt{-1}$; yet if the root be expressible in rational numbers, the rule will often yield to it in a short way, not merely tentative, the trials being confined to known limits. See Maclaurin's Alg. pa. 127. It may be further observed, that such roots, whether expressible in rational numbers or not, may be found by evolving the quantity $a + b\sqrt{-1}$ by Newton's binomial theorem, and summing up the alternate terms. Maclaurin, p. 130.

Those who are desirous of a general and elegant solution of the problem, to extract any root of an impossible binomial $a + b\sqrt{-1}$, or of a possible binomial $a + \sqrt{b}$, may have recourse to the appendix to Saunderson's Algebra, and to the Philos. Trans. No. 451. On the management of surds, see also the numerous authors upon Algebra.

SURFACE, in Geometry. See **Superficies**.

A *Mathematical* **Surface** is the mere exterior face of a body, but is not any part of it, being of no thickness, but only the bare figure or termination of the body.

A *Physical* **Surface** is considered as of some very small thickness.

SURSOLID, in Arithmetic, the 5th power of a number, considered as a root. The number 2, for instance, considered as a root, produces the powers thus:

$2 = 2$ the root or 1st power,
$2 \times 2 = 4$ the square or 2d power,
$2 \times 4 = 8$ the cube or 3d power,
$2 \times 8 = 16$ the biquadratic or 4th power,
$2 \times 16 = 32$ the sursolid or 5th power.

Sursolid *Problem*, is that which cannot be resolved but by curves of a higher kind than the conic sections.

SURVEYING, the art of measuring land; which comprises the three following parts; viz, taking the dimensions of any tract or piece of ground; the delineating or laying it down in a map or draught; and finding the superficial content or area of the same; besides the dividing and laying out of lands. The first of these is what is properly called Surveying; the second is called plotting, or protracting, or mapping; and the third casting up, or computing the contents.

The first again consists of two parts, the making of observations for the angles, and the taking of lineal measures for the distances. The former of these is performed by some of the following instruments; the theodolite, circumferentor, semicircle, plain-table, or compass, or even by the chain itself: the latter is performed by means either of the chain, or the perambulator. The description and manner of using each of these, see under its respective article.

It is useful in surveying, to take the angles which the bounding lines form with the magnetic needle, in order to check the angles of the figure, and to plot them conveniently afterwards. But, as the difference between the true and magnetic meridian perpetually varies in all places, and at all times; it is impossible to compare two surveys of the same place, taken at distant times, by magnetic instruments, without making due allowance for this variation. See observations on this subject, by Mr. Molineux, Philos. Trans. No. 230, pa. 625.

The second branch of surveying is performed by means of the protractor, and plotting scale. The description of which, see under their proper names.

If the lands in the survey are hilly, and not in any one plane, the measured lines cannot be truly laid down on paper, till they are reduced to one plane, which must be the horizontal one, because angles are taken in that plane. And in this case, when observing distant objects, for their elevation or depression, the following table shows the links or parts to be subtracted from each chain in the hypothenusal line, when the angle is the corresponding number of degrees.

A TABLE *of the links to be subtracted out of every chain in hypothenusal lines, of several degrees of altitude or depression, for reducing them to horizontal.*

		links			*links*
4°	3′	¼	19°	57′	6
5	44	½	21	34	7
7	1	¾	23	4	8
8	7	1	24	30	9
11	29	2	25	50	10
14	4	3	27	8	11
16	16	4	28	22	12
18	12	5	29	32	13

For example, if a station line measure 1250 links, or 12½ chains, on an ascent, or a descent, of 11°; here it is after the rate of almost two links per chain, and it will be exact enough to take only the 12 chains at that rate, which make 24 links in all, to be deducted from 1250, which leaves 1226 links, for the length to be laid down. Practical surveyors say, it is best to make this deduction at the end of every chain-length while measuring, by drawing the chain forward every time as much as the deduction is; viz, in the present instance, drawing the chain on 2 links at each chain-length.

The third branch of surveying, namely computing the contents, is performed by reducing the several inclosures and divisions into triangles, trapeziums, and parallelograms, but especially the two former; then finding the areas or contents of these several figures, and adding them together.

The Practice of Surveying.

1. Land is measured with a chain, called Gunter's chain, of 4 poles or 22 yards in length, which consists of 100 equal links, each link being $\frac{22}{100}$ of a yard, or $\frac{66}{100}$ of a foot, or 7·92 inches long, that is nearly 8 inches or $\frac{2}{3}$ of a foot.

An acre of land is equal to 10 square chains, that is, 10 chains in length and 1 chain in breadth.

Or it is 40 × 4 or 160 square poles.
Or it is 220 × 22 or 4840 square yards.
Or it is 1000 × 100 or 100000 square links.

These being all the same quantity.

Also, an acre is divided into 4 parts called roods, and a rood into 40 parts called perches, which are square poles, or the square of a pole of 5½ yards long, or the square of ¼ of a chain, or of 25 links, which is 625 square links. So that the divisions of land measure will be thus:

625 sq. links = 1 pole or perch
40 perches = 1 rood
4 roods = 1 acre.

The length of lines, measured with a chain, are set down in links as integers, every chain in length being 100 links; and not in chains and decimals. Therefore, after the content is found, it will be in square links; then cut off five figures on the right hand for decimals, and the rest will be acres. Those decimals are then multiplied by 4 for roods, and the decimals of these again by 40 for perches.

2. Among the various instruments for surveying, the plain-table is the easiest and most generally useful, especially in crooked difficult places, as in a town among houses, &c. Yet there are cases in which this cannot be conveniently used, as different surveys require different instruments, and the surveyor must judge which is the fittest instrument and method, and use it accordingly: nay, sometimes no instrument at all, but barely the chain itself, is the best method, particularly in regular open fields lying together; and even when using the plain-table, it is often of advantage to measure such large open parts with the chain only, and from those measures lay them down upon the table.

The perambulator is used for measuring roads, and other great distances on level ground, and by the sides of rivers. It has a wheel of 8¼ feet, or half a pole, in circumference, upon which the machine turns; and the distance measured is pointed out by an index, which is moved round by clock work.

Levels, with telescopic or other sights, are used to find the level between place and place, or how much one place is higher or lower than another.

An offset-staff is a very useful and necessary instrument, for measuring the offsets and other short distances. It is 10 links in length, being divided and marked at each of the 10 links.

Ten small arrows, or rods of iron or wood, are used to mark the end of every chain length, in measuring lines. And sometimes pickets, or staves with flags, are set up as marks or objects of direction.

Various scales are also used in protracting and measuring on the plan or paper; such as plane scales, line of chords, protractor, compasses, reducing scales, parallel and perpendicular rulers, &c. Of plane scales, there should be several sizes, as a chain in 1 inch, a chain in $\frac{3}{4}$ of an inch, a chain in $\frac{1}{2}$ of an inch, &c. And of these, the best for use are those that are laid on the very edges of the ivory scale, to prick off distances by, without compasses.

3. *The Field Book.*

In surveying with the plain-table, a field-book is not used, as every thing is drawn on the table immediately when it is measured. But in surveying with the theodolite, or any other instrument, some kind of a field-book must be used, to write down in it a register or account of all that is done and occurs relative to the survey in hand. And this book every one contrives and rules as he thinks fittest for himself.

But a few skilful surveyors now make use of a new method for the field-book, namely, beginning at the bottom of the page and writing upwards; by which they sketch a neat boundary on either hand, as they pass it; an example of which will be given below, with the plan of the ground to accompany it.

In smaller surveys and measurements, a good way of setting down the work, is, to draw, by the eye, on a piece of paper, a figure resembling that which is to be measured; and so write the dimensions, as they are found, against the corresponding parts of the figure. And this method may be practised to a considerable extent, even in the larger surveys.

For all the parts of surveying, and with all the instruments, see my large book on Mensuration; also my Course of Mathematics.

The New Method of Surveying.

Instead of the foregoing method, an ingenious friend (Mr. Abraham Crocker), after mentioning the new and improved method of keeping the field-book by writing from bottom to top of the pages, observes that, "In the former method of measuring a large estate, the accuracy of it depends on the correctness of the instruments used in taking the angles. To avoid the errors incident to such a multitude of angles, other methods have of late years been used by some few skilful surveyors; the most practical, expeditious, and correct, seems to be the following.

"As was advised in former methods, so in this, choose two or more eminences, as grand stations, and measure a principal base line from one station to the other, noting every hedge, brook, or other remarkable object as you pass by it; measuring also such short perpendicular lines to such bends of hedges as may be near at hand. From the extremities of this base line, or from any convenient parts of the same, go off with other lines to some remarkable object situated towards the sides of the estate, without regarding the angles they make with the base line or with one another; still remembering to note every hedge, brook or other object that you pass by. These lines, when laid down by intersections, will with the base line form a grand triangle upon the estate; several of which, if need be, being thus laid down, you may proceed to form other smaller triangles and trapezoids on the sides of the former: and so on, until you finish with the enclosures individually.

"To illustrate this excellent method, let us take AB (in the plan of an estate, fig. 1, pl. 34) for the principal base line. From B go off to the tree at C; noting down, in the field-book, every cross hedge, as you measure on; and from C measure back to the first station at A, noting down every thing as before directed.

"This grand triangle being completed, and laid down on the rough-plan paper, the parts, exterior as well as interior, are to be completed by smaller triangles and trapezoids.

"When the whole plan is laid down on paper, the contents of each field might be calculated by the methods laid down below in mensuration.

"In countries where the lands are enclosed with high hedges, and where many lanes pass through an estate, a theodolite may be used to advantage, in measuring the angles of such lands; by which means, a kind of skeleton of the estate may be obtained, and the lane-lines serve as the bases of such triangles and trapezoids as are necessary to fill up to the interior parts."

The method of measuring the other cross lines, off-sets and interior parts and enclosures, appears in the plan, fig. 1, last referred to.

16. Another ingenious correspondent (Mr. John Rodham of Richmond, Yorkshire) has also communicated the following example of the new method of surveying, accompanied by the field-book, and its corresponding plan. His account of the method is as follows.

The field-book is ruled into three columns. In the middle one are set down the distances on the chain line at which any mark, offset, or other observation is made; and in the right and left hand columns are entered, the offsets and observations made on the right and left hand respectively of the chain line.

It is of great advantage, both for brevity and perspicuity, to begin at the bottom of the leaf and write upwards; denoting the crossing of fences, by lines drawn across the middle column, or only a part of such a line on the right and left opposite the figures, to avoid confusion, and the corners of fields, and other remarkable turns in the fences where offsets are taken to, by lines joining in the manner the fences do, as will be best seen by comparing the book with the plan annexed, fig. 2, pl. 34.

The marks called *a*, *b*, *c*, &c, are best made in the fields, by making a small hole with a spade, and a chip or small bit of wood, with the particular letter upon it, may be put in, to prevent one mark being taken for another, on any return to it. But in general, the name of a mark is very easily had by referring in the book to the line it was made in. After the small alphabet is gone through, the capitals may be next, the print letters afterwards, and so on, which answer the purpose of so many different letters; or the marks may be numbered.

The letter in the left hand corner at the beginning of every line, is the mark or place measured from; and, that at the right hand corner at the end, is the mark measured to. But when it is not convenient to go exactly from a mark, the place measured from, is described such a distance from one mark towards another; and where a mark is not measured to, the exact place is ascertained by saying, turn to the right or left hand, such a distance to such a mark, it being always undertood that those distances are taken in the chain line.

The characters used, are ┌ for turn to the right hand, ┐ for turn to the left hand, and ∧ placed over an offset, to show that it is not taken at angles with the chain line, but in the line with some straight fence; being chiefly used when crossing their directions, and is a better way of obtaining their true places than by offsets at right angles.

When a line is measured whose position is determined, either by former work (as in the case of producing a given line or measuring from one known place or mark to another) or by itself (as in the third side of a triangle) it is called a fast line, and a double line across the book is drawn at the conclusion of it; but if its position is not determined (as in the second side of a triangle) it is called a loose line, and a single line is drawn across the book. When a line becomes determined in position, and is afterwards continued, a double line half through the book is drawn.

When a loose line is measured, it becomes absolutely necessary to measure some other line that will determine its position. Thus, the first line *ah*, being the base of a triangle, is always determined; but the position of the second side *hj*, does not become determined, till the third side *jb* is measured; then the triangle may be constructed, and the position of both is determined.

At the beginning of a line, to fix a loose line to the mark or place measured from, the sign of turning to the right or left hand must be added (as at *j* in the third line); otherwise a stranger, when laying down the work may as easily construct the triangle *hjb* on the wrong side of the line *ah*, as on the right one: but this error cannot be fallen into, if the sign above named be carefully observed.

In choosing a line to fix a loose one, care must be taken that it does not make a very acute or obtuse angle; as in the triangle *p*B*r*, by the angle at B being very obtuse, a small deviation from truth, even the breadth of a point at *p* or *r*, would make the error at B when constructed very considerable; but by constructing the triangle *p*B*q*, such a deviation is of no consequence.

Where the words *leave off* are written in the field-book, it is to signify that the taking of offsets is from thence discontinued; and of course something is wanting between that and the next offset.

The field-book above referred to, is engraved on plate 35, in four parts, representing so many pages, each of which is supposed to begin at the bottom, and end at top. And the map or plan belonging to it, in fig. 2, pl. 34.

Surveying of Harbours.

The method of surveying harbours, and of forming maps of them, as also of the adjacent coasts, sands, &c, depends on the same principles, and is chiefly conducted like that of common surveying. The operation is indeed more complicated and laborious; as it is necessary to erect a number of signals, and to mark a variety of objects along the coast, with different bearings from one another, and the several parts of the harbour; and likewise to measure a great number of angles at different stations, whether on the land or the water. For this purpose, the best instrument is Hadley's quadrant, as all these operations may be performed by it, not only with greater ease, but also with much more precision, than can be hoped for by any other means; as it is the only instrument in use, in which neither the exactness of the observations, nor the ease with which they may be made, are sensibly affected by the motion of a vessel: and hence a single observer, in a boat, may generally determine the situation of any place at pleasure, with a sufficient degree of exactness, by taking the angles subtended by several pairs of objects properly chosen upon shores at different places about him; but it will be still better to have two observers, or the same observer at different stations, to take the like angles to the several objects, and also to the stations. By this means, two angles and one side are given, in every triangle, from whence the situation of every part of them will be known. By such observations, when carefully made with good instruments, the situation of places may be easily determined to 20 or 30 feet, or less, upon every 3 or 4 miles. See Philos. Trans. vol. 55, pa. 70; also Mackenzie's Maritime Surveying.

Surveying *Cross*. See Cross.

Surveying *Quadrant*. See Quadrant.

Surveying *Scale*, the same with Reducing Scale.

Surveying *Wheel*. See Perambulator.

SURVIVORSHIP, the doctrine of Reversionary payments that depend upon certain contingencies, or contingent circumstances. Payments which are not to be made till some future period, are termed reversions, to distinguish them from payments that are to be made immediately.

Reversions are either certain or contingent. Of the former sort, are all sums or annuities, payable certainly or absolutely at the expiration of any terms, or on the extinction of any lives. And of the latter sort, are all such reversions as depend on any contingency; and particularly the survivorship of any lives beyond or after other lives. An account of the former may be found under the articles Assurance, Annuities, and Life-annuities. But the latter form the most intricate and difficult part of the doctrine of reversions and life-annuities; and the books in which this subject is treated most at large, and at the same time with the most precision, are Mr. Simpson's Select Exercises; Dr. Price's Reversionary Payments; Mr. Morgan's Annuities and Assurances on Lives and Survivorships; and Mr. Bayly's Annuities. The whole likewise of the 3d volume of Dodson's Mathematical Repository is on this subject; but his investigations are founded on Demoivre's false hypothesis, viz, of an equal decrement of life through all its stages, and which is explained under Life-annuities: but as this hypothesis does not agree near enough to fact and experience, the rules deduced from it cannot be sufficiently correct. For this reason, Dr. Price, and also Mr. Maseres, cursitor baron of the exchequer (in two volumes lately published, entitled the Principles of the Doctrine of Life Annuities), have discarded the valuations of lives founded upon it; and the former in particular, in order to obviate all occasion for using them, has substituted in their stead, a great variety of new tables of the probabilities and values of lives, at every age and in every situation; calculated, not upon any hypothesis, but in strict conformity to the best observations. These tables, added to other new tables of the same kind, in Mr. Baron Maseres's work just mentioned, form a complete set of tables, by which all questions relating to annuities on lives and survivorships, may be answered with as much correctness as the nature of the subject admits of.

Rules for calculating correctly, in most cases, the values of reversions depending on survivorships, may be found in the several treatises just mentioned. Mr. Morgan, in particular, has gone a good way towards exhausting this subject, as far as any questions can include in them any survivorships between two or three lives, either for terms, or the whole duration of the lives.

There is, however, one circumstance necessary to be attended to in calculating such values, to which no regard could be paid till lately. This circumstance is the shorter duration of the lives of males than of females; and the consequent advantage in favour of females in all cases of survivorship. In the 4th edition of Dr. Price's Treatise on Reversionary Payments, this fact is not only ascertained, but separate tables of the duration and values of lives are given for males and females.

SUSPENSION, in Mechanics, as in a balance, are those points in the axis or beam where the weights are applied, or from which they are suspended.

SUTTON's *Quadrant*. See QUADRANT.

SWAN, in Astronomy. See CYGNUS.

SWALLOW's-TAIL, in Fortification, is a single tenaille, which is narrower towards the place than towards the country.

SWANPAN, or *Chinese Abacus*, an instrument for performing arithmetical operations, described by Du Halde in his History of China. See ABACUS.

SWING-*Wheel*, in a royal pendulum, is that wheel which drives the pendulum. In a watch, or balance clock, it is called the *crown-wheel*.

SYDEREAL *Day*, or *Year*. See SIDEREAL.

SYMMETRY, the relation of parity, both in respect of length, breadth, and height, of the parts necessary to compose a beautiful whole. Symmetry arises from that proportion which the Greeks call analogy, which is the relation of conformity of all the parts of a building, and of the whole, to some certain measure; upon which depends the nature of symmetry.

According to Vitruvius, symmetry consists in the union and conformity of the several members of a work to their whole, and of the beauty of each of the separate parts to that of the entire work; regard being had to some certain measure: so the body, for instance, is framed with symmetry, by the due relation which the arm, elbow, hand, fingers, &c, have to each other, and to their whole.

SYMPHONY, is a consonance or concert of several sounds agreeable to the ear; whether they be vocal or instrumental, or both; called also harmony. The symphony of the ancients went no farther than to two or more voices or instruments set to unison; for they had no such thing as music in parts; as is very well proved by Perrault: at least, if ever they knew such a thing, it must have been lost very early.

It is to Guido Aretine, about the year 1022, that most writers agree in ascribing the invention of composition: it was he, they say, who first joined in one harmony several distinct melodies; and brought it even to the length of 4 parts, viz, bass, tenor, counter-tenor, and treble.

The term symphony is now applied to instrumental music, both that of pieces designed only for instruments, as sonatas and concertos, and that in which the instruments are accompanied with the voice, as in operas, &c. A piece is said to be in grand symphony, when, besides the bass and treble, it has also two other instrumental parts, viz, tenor and 5th of the violin.

SYNCHRONISM, the being or happening of several things together, at or in the same time. The happening or performing of several things in equal times, as the vibrations of pendulums, &c, is more properly called isochronism; though some authors confound the two.

SYNCOPATION, in Music, is a striking or breaking of the time; by which the distinctness of the several times or parts of the measure is interrupted.

SYNCOPATION, or SYNCOPE, is more particularly used for the connecting the last note of one measure or bar with the first of the following measure; so as to make only one note of both.

SYNCOPATION is also used when a note of one part ends on the middle of a note of the other part. This is otherwise called binding.

SYNODICAL *Month*, is the period or interval of time in which the moon passes from one conjunction with the sun to another. This period is also called a Lunation, since in this period the moon puts on all her phases, or appearances, as to increase and decrease.—Kepler found the quantity of the mean synodical month to be 29 days, 12 hrs. 44 min. 3 sec. 11 thirds.

SYNTHESIS, denotes a method of composition, as opposed to analysis. In the synthesis, or synthetic method, we pursue the truth by reasons drawn from principles before established, or assumed, and propositions formerly proved; thus proceeding by a regular chain till we come to the conclusion; and hence called also the direct method, and composition, in opposition to analysis or resolution. Such is the method in Euclid's Elements, and most demonstrations of the ancient mathematicians, which proceed from definitions and axioms, to prove theorems &c, and from those theorems proved, to demonstrate others. See ANALYSIS.

SYNTHETICAL *Method*, the method by synthesis, or composition, or the direct method. See SYNTHESIS.

SYPHON. See SIPHON.

SYRINGE, in Hydraulics, a small simple machine, serving first to imbibe or suck in a quantity of water, or other fluid, and then to expel the same with violence in a small jet. The syringe is a small single sucking pump, without a valve, the water ascending in it on the same principle. It consists, like the pump, of a small cylinder, with an embolus or sucker, moving up and down in it by means of a handle, and fitting it very close within. At the lower end is either a small hole, or a smaller tube fixed to it than the body of the instrument, through which the fluid or the water is drawn up, and squirted out again.

This ascent of the water the ancients, who supposed a plenum, attributed to nature's abhorrence of a vacuum; but the moderns, more reasonably, as well as more intelligibly, attribute it to the pressure of the atmosphere on the exterior surface of the fluid. For, by drawing up the embolus, the cavity of the cylinder would become a vacuum, or the air left there extremely rarefied; so that being no longer a counterbalance to the air incumbent on the surface of the fluid, this prevails, and forces the water through the little tube, or hole, up into the body of the syringe.

SYSTEM, in a general sense, denotes an assemblage or chain of principles and conclusions: or the whole of any doctrine, the several parts of which are bound together, and follow or depend on each other: as a system of astronomy, a system of planets, a system of philosophy, a system of motion, &c.

SYSTEM, in Astronomy, denotes an hypothesis or a supposition of a certain order and arrangement of the several parts of the universe; by which astronomers explain all the phenomena or appearances of the heavenly bodies, their motions, changes, &c. This is more peculiarly called the System of the world, and sometimes the Solar System.

System and hypothesis have much the same signification; unless perhaps hypothesis be a more particular system, and system a more general hypothesis. Some late authors indeed make another distinction: an hypothesis, say they, is a mere supposition or fiction, founded rather on imagination than reason; while a system is built on the firmest ground, and raised by the severest rules; it is founded on astrononomical observations, and physical causes, and confirmed by geometrical demonstrations.

The most celebrated systems of the world, are the Ptolemaic, the Copernican or Pythagorean, and the Tychonic: the economy of each of which is as follows.

Ptolemaic SYSTEM is so called from the celebrated astronomer Ptolemy. In this system, the earth is placed at rest, in the centre of the universe, while the heavens are considered as revolving about it, from east to west, and carrying along with them all the heavenly bodies, the stars and planets, in the space of 24 hours. The principal assertors of this system, are Aristotle, Hipparchus, Ptolemy, and many of the old philosophers; and indeed almost all astronomers, for a great number of ages, supported this system. But the late improvements in philosophy and reasoning, have utterly exploded this erroneous system from the place it so long held in the minds of men.

Copernican SYSTEM, is that system of the world which places the sun at rest, in the centre of the world, and the earth and planets revolving about him, in their several orbits. See this more particularly explained under the article COPERNICAN *System*.

Solar or *Planetary* SYSTEM, is usually confined to narrower bounds; the stars, by their immense distance, and the little relation they seem to bear to us, being accounted no part of it. It is highly probable that each fixed star is itself a sun, and the centre of a particular system, surrounded with a company of planets &c, which, in different periods, and at different distances, perform their courses round their respective sun, which enlightens, warms, and cherishes them. Hence we have a very magnificent idea of the world, and the immensity of it. Hence also arises a kind of system of systems.

The planetary system, described under the article COPERNICAN, is the most ancient in the world. It was first of all, as far as we know, introduced into Greece and Italy by Pythagoras; from whom it was called the Pythagorean System. It was followed by Philolaus, Plato, Archimedes, &c: but it was lost under the reign of the Peripatetic philosophy; till happily retrieved about the year 1500 by Copernicus.

Tychonic SYSTEM, was taught by Tycho, a Dane; who was born an. dom. 1546. It supposes that the earth is fixed in the centre of the universe or firmament of stars, and that all the stars and planets revolve round the earth in 24 hours; but it differs from the Ptolemaic system, as it not only allows a menstrual motion to the moon round the earth, and that of the satellites about Jupiter and Saturn, in their proper periods, but it makes the sun to be the centre of the orbits of the primary planets Mercury, Venus, Mars, Jupiter, &c, in which they are carried round the sun in their respective years, as the sun revolves round the earth in a solar year; and all these planets, together with the sun, are supposed to revolve round the earth in 24 hours. This hypothesis was so embarrassed and perplexed, that very few persons embraced it. It was afterwards altered by Longomontanus and others, who allowed the diurnal motion of the earth on its own axis, but denied its annual motion round the sun. This hypothesis, partly true and partly false, is called the semi-Tychonic system. See the figure and economy of these systems, in plates 36 and 37.

SYSTEM, in Music, denotes a compound interval; or an interval composed, or conceived to be composed of several less intervals. Such is the octave, &c.

SYSTYLE, in Architecture, the manner of placing columns, where the space between the two fusts consists of 2 diameters, or 4 modules.

SYZYGY, a term equally used for the conjunction and opposition of a planet with the sun.—On the phenomena and circumstances of the syzygies, a great part of the lunar theory depends. See MOON. For,

1. It is shown in the physical astronomy, that the force which diminishes the gravity of the moon in the syzygies, is double that which increases it in the quadratures; so that, in the syzygies, the gravity of the moon is diminished by a part which is to the whole gravity, as 1 to 89·36; for in the quadratures, the addition of gravity is to the whole gravity, as 1 to 178·73.

2. In the syzygies, the disturbing force is directly as the distance of the moon from the earth, and inversely as the cube of the distance of the earth from the sun. And at the syzygies, the gravity of the moon towards the earth receding from its centre, is more diminished than according to the inverse ratio of the square of the distance from that centre.—Hence, in the moon's motion from the syzygies to the quadratures, the gravity of the moon towards the earth is continually increased, and the moon is continually retarded in her motion; but in the moon's motion from the quadratures to the syzygies, her gravity is continually diminished, and the motion in her orbit is accelerated.

3. Further, in the syzygies, the moon's orbit, or circuit round the earth, is more convex than in the quadratures; for which reason she is less distant from the earth at the former than the latter.—Also, when the moon is in the syzygies, her apses go backward, or are retrograde.—Moreover, when the moon is in the syzygies, the nodes move in antecedentia fastest; then slower and slower, till they become at rest when the moon is in the quadratures.—Lastly, when the nodes are come to the syzygies, the inclination of the plane of the orbit is the least of all.

However, these several irregularities are not equal in each syzygy, being all somewhat greater in the conjunction than in the opposition.

T.

TABLE, in Architecture, a smooth, simple member or ornament, of various forms, but most commonly in that of a parallelogram.

TABLE, in Perspective, is sometimes used for the perspective plane, or the transparent plane on which the objects are formed in their respective appearance.

TABLE *of Pythagoras*, is the same as the MULTIPLICATION table; which see; as also PYTHAGORAS's table.

Tables, in Mathematics, are systems or series of numbers, calculated to be ready at hand for expediting calculations in the various branches of mathematics; as, tables of powers, or roots, of reciprocals, of products, &c.

Astronomical Tables, are computations of the motions, places, and other phenomena of the planets, both primary and secondary. The oldest astronomical tables extant are those of Ptolemy, found in his Almagest. These however are not now of much use, as they no longer agree with the motions of the heavens.

In 1252, Alphonso XI, king of Castile, undertook the correcting of them, chiefly by the assistance of Isaac Hazen, a learned Jew; and spent 400,000 crowns on the business. Thus arose the Alphonsine tables, to which that prince himself prefixed a preface. But the deficiency of these also was soon perceived by Purbach and Muller, or Regiomontanus; on which the latter, and after him Walther Warner, applied themselves to celestial observations, for further improving them; but death, or various difficulties, prevented the effect of these laudable designs.

Copernicus, in his books of the celestial revolutions, gives other tables, calculated by himself, partly from his own observations, and partly from the Alphonsine tables.

From Copernicus's observations and theorems, Erasmus Reinhold afterwards compiled the Prutenic tables, which have been printed several times, and in several places.

Tycho Brahé, even in his youth, became sensible of the deficiency of the Prutenic tables: which determined him to apply himself with so much vigour to celestial observations. From these he adjusted the motions of the sun and moon; and Longomontanus, from the same observations, constructed tables of the motions of the planets, which he added to the theories of the same, published in his Astronomia Danica; those being called the Danish tables. And Kepler also, from the same observations, published in 1627 his Rudolphine tables, which are much esteemed.

These were afterwards, viz, in 1650, changed into another form, by Maria Cunitia, whose astronomical tables, comprehending the effect of Kepler's physical hypothesis, are very easy, satisfying all the phenomena without any mention of logarithms, and with little or no trouble of calculation. So that the Rudolphine calculus is here greatly improved.

Nic. Mercator made a similar attempt in his Astronomical Institution, published in 1676. As did also J. Bap. Morini, whose abridgement of the Rudolphine tables was prefixed to a Latin version of Street's Astronomia Carolina, published in 1705. Lansberg indeed endeavoured to discredit the Rudolphine tables, and framed perpetual tables, as he calls them, of the heavenly motions. But his attempt was never much regarded by the astronomers; and our countryman Horrox warmly attacked him, in his defence of the Keplerian astronomy.

Since the Rudolphine tables, many others have been framed, and published: as the Philolaic tables of Bulliald; the Britannic Tables of Vincent Wing, calculated on Bulliald's hypothesis; the Britannic tables of John Newton; the French ones of the count Pagan; the Caroline tables of Street, all calculated on Ward's hypothesis; and the Novelmajestic tables of Riccioli. Among these, however, the Philolaic and Caroline tables are esteemed the best; insomuch that Mr. Whiston, by the advice of Mr. Flamsteed, thought fit to subjoin the Caroline tables to his astronomical lectures.

The Ludovician tables, published in 1702, by Lahire, were constructed wholly from his own observations, and without the assistance of any hypothesis; which, before the invention of the micrometer telescope and the pendulum clock, was held impossible.

Dr. Halley also long laboured to perfect another set of tables; which were printed in 1719, but not published till 1752.

M. Monnier, in 1746, published, in his Institutions Astronomiques, tables of the motions of the sun and moon, with the satellites; as also of refractions, and the places of the fixed stars. Lahire also published tables of the planets, and Lacaille tables of the sun: Gael Morris published tables of the sun and moon, and Mayer constructed tables of the moon, which were published by the board of longitude. Tables of the same have also been computed by Charles Mason, from the principles of the Newtonian philosophy, which are found to be very accurate, and are employed in computing the Nautical Ephemeris. Many other sets of astronomical tables have also been published by various persons and academies; and divers sets of them may be found in the modern books of astronomy, navigation, &c, of which those are esteemed among the best that are printed in Lalande's Astronomy; in Vince's Astronomy; also Delambre's, Burg's, and Burckhardt's tables, &c. For an account of several, and especially of those published annually under the direction of the commissioners of longitude, see Almanac, Ephemeris, and Longitude.

For Tables *of the Stars*, see Catalogue.

Tables of *Sines, Tangents,* and *Secants,* used in trigonometry, &c, are usually called Canons. See Sine.

Tables *of Logarithms, Rhumbs, &c,* used in trigonometry, navigation, &c, see Logarithm, and Rhumb.

Tables, *Loxodromic,* and *of Difference of Latitude and Departure,* are tables used in computing the way and reckoning of a ship on a voyage, and are published in most books of navigation.

TACQUET (Andrew), a Jesuit of Antwerp, who died in 1660. He was a most laborious and voluminous writer in mathematics. His works were collected, and printed at Antwerp in one large volume in folio, 1669. Tacquet was one of those learned Jesuits who chiefly cultivated the liberal sciences in the 16th and 17th centuries. Besides the collection of his works above-mentioned, he had before published very neat editions of the Elements of Geometry, and of Arithmetic, both in 8vo. In matters of astronomy, his fear of the church censures seems to have prevented him from more effectually defending the Copernican system of the world. A very particular and satisfactory account of the collection of his works may be seen in the Philos. Trans. vol. 3, pa. 868, or in my Abridgment, vol. 1, pa. 314.

TACTION, in Geometry, the same as tangency, or touching. See Tangent.

TALUS, or Talud, in Architecture, the inclination or slope of a work; as of the outside of a wall, when its thickness is diminished by degrees, as it rises in height, to make it the firmer.

Talus, in Fortification, means also the slope of a work, whether of earth or masonry.

The *Exterior Talus* of a work, is its slope on the side outwards or towards the country; which is always made as small as possible, to prevent the enemy's escalade, unless the earth be bad, for then it is necessary to allow a considerable talus for its parapet, and sometimes to support the earth with a slight wall, called a revetement.

The *Interior Talus* of a work, is its slope on the inside, towards the place. This is larger than the former, and it has, at the angles of the gorge, and sometimes in the middle of the curtains, ramps, or sloping roads for mounting upon the terreplain of the rampart.

Superior TALUS *of the Parapet*, is a slope on the top of the parapet, that allows of the soldiers defending the covert-way with small-shot, which they could not do if it were level.

TAMBOUR, in Architecture, a term applied to the Corinthian and Composite capitals, as bearing some resemblance to a tambour or drum.

TAMUZ, in Chronology, the 4th month of the Jewish ecclesiastical year, answering to part of our June and July. The 17th day of this month is observed by the Jews as a fast, in memory of the destruction of Jerusalem by Nebuchadnezzar, in the 11th year of Zedekiah, and the 588th before Christ.

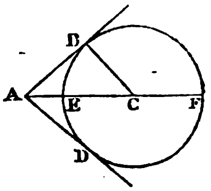
TANGENCIES (Problem of). This general problem in Geometry furnishes the subject of one of the 12 treatises described by Pappus in the preface to the 7th book of his Mathematical Collections. The general problem is this: Of points, right-lines, and circles, any three being given; to describe a circle that shall pass through the given points, and also touch the given lines.

This naturally divides into 10 distinct propositions; which, if the three things be thus denoted, viz, a point by the mark (·), a line by (1), and a circle by (0), may be stated very briefly according to the several data, in the following order: viz, (·· 1), (· 11), (110), (· 10), (100), (·· 0), (· 00), (000), (···), (111).

The original treatise on this subject, by Apollonius, having been lost, the restoration of it has lately been attempted by several persons; viz, by Vieta, under the title of Apollonius Gallus; and many of the deficiencies were supplied by Ghetaldus. Afterwards the tactions were restored by various other mathematicians, both geometrically and algebraically. A treatise on them by G. Camerar was published at Gotha and Amsterd. in 1795; but it contains only an edition of Vieta's treatise, with notes and additions, and a curious history of the problem. The history is interesting, from the accounts it contains of the labours of some foreign mathematicians on this problem, which are little known in this country. He gives the preface and lemmata of the tactions in Greek, with some various readings of several manuscripts of Pappus. Though Vieta's solutions are elegant, yet they are in several respects deficient: there is not a full distinction either of the cases, or of the necessary determinations: no analysis is given, and no attempt to restore the Apollonian solutions by the use of the lemmata in Pappus, which had been assumed in the work of Apollonius.

In the remaining papers of Dr. Rob. Simson, it seems, are found solutions of some of the cases of this problem. Also the treatises of Vieta and Ghetaldus have been translated into English, with the addition of a supplement, by the Rev. John Lawson, and a further addition of Fermat's Treatise on Spherical Tangencies. And Mr. Leslie has given, in his Geometry, as examples of the geometrical analysis, solutions to many of the cases of this problem. Also Mr. John Lawson published a neat edition, in English, of the two books on Tangencies, 1771, in 4to.

TANGENT, in Geometry, is a line that touches a curve, &c, that is, which meets it in a point without cutting it, though it be produced both ways; as the tangent AB of the circle BD. The point B, where the tangent touches the curve, is called the point of contact.



The direction of a curve at the point of contact, is the same as that of the tangent. It is demonstrated in Geometry,

1. That a tangent to a circle, as AB, is perpendicular to the radius BC drawn to the point of contact.

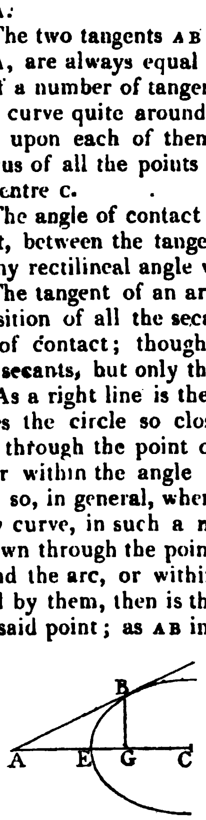
2. The tangent AB is a mean proportional between AF and AE, the whole secant and the external part of it; and the same for any other secant drawn from the same point A.

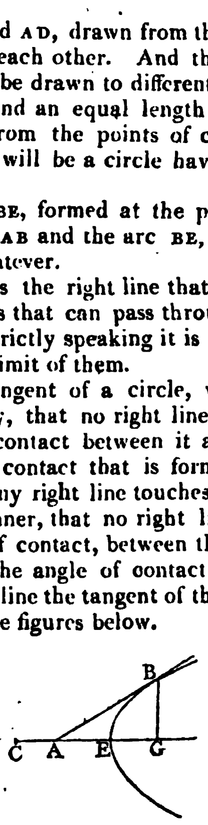
3. The two tangents AB and AD, drawn from the same point A, are always equal to each other. And therefore also, if a number of tangents be drawn to different points of the curve quite around, and an equal length BA be set off upon each of them from the points of contact, the locus of all the points A will be a circle having the same centre C.

4. The angle of contact ABE, formed at the point of contact, between the tangent AB and the arc BE, is less than any rectilineal angle whatever.

5. The tangent of an arc is the right line that limits the position of all the secants that can pass through the point of contact; though strictly speaking it is not one of the secants, but only the limit of them.

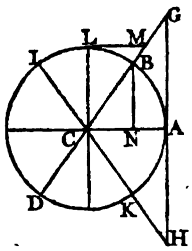
6. As a right line is the tangent of a circle, when it touches the circle so closely, that no right line can be drawn through the point of contact between it and the arc, or within the angle of contact that is formed by them; so, in general, when any right line touches an arc of any curve, in such a manner, that no right line can be drawn through the point of contact, between the right line and the arc, or within the angle of contact that is formed by them, then is that line the tangent of the curve at the said point; as AB in the figures below.





7. In all the conic sections; if C be the centre of the figure, and BG an ordinate drawn from the point of contact and perpendicular to the axis; then is CG : CE : : CE : CA, or the semiaxis CE is a mean proportional between CG and CA.

TANGENT, in Trigonometry. A TANGENT *of an arc*, is a right line touching one extremity of the arc, and limited by a secant or line drawn through the centre and the other extremity of the arc.



So, AG is the tangent of the arc AB, or of the arc ABD; and AH is the tangent of the arc AI, or of the arc AIDK.

The same are also the tangents of the angles that are subtended or measured by the arcs.

Hence, 1. The tangents in the 1st and 3d quadrants are positive, in the 2d and 4th negative, or drawn the contrary way. But of 0 or 180° the semicircle, the tangent is 0 or nothing; while those of 90° or a quadrant, and 270° or 3 quadrants, are both infinite; the former infinitely positive, and the latter infinitely negative. That is,

Between 0 and 90°, or between 180° and 270°, the tangents are positive.

Between 90° and 180°, or between 270° and 360°, the tangents are negative.

2. The tangent of an arc and the tangent of its supplement, are equal, but of contrary affections, the one being positive, and the other negative;

as of a and $180° - a$, where a is any arc.

$\left.\begin{array}{l}\text{Also } 180° + a \\ \text{and } \qquad\quad a\end{array}\right\}$ have the same tangent, and are of the same affection.

$\left.\begin{array}{l}\text{Or } \quad 180° + a \\ \text{and } \; 180° - a\end{array}\right\}$ have the same tangent, but of different affections.

3. The tangent of an arc is a 4th proportional to the cosine, the sine, and the radius; that is, CN : NB : : CA : AG. Hence, a canon of sines being made or given, the canon of tangents is easily constructed from them.

Co-TANGENT, contracted from complement-tangent, is the tangent of the complement of the arc or angle, or of what it wants of a quadrant or 90°. So LM is the cotangent of the arc AB, being the tangent of its complement BL.

The tangent is reciprocally as the cotangent; or the tangent and cotangent are reciprocally proportional with the radius. That is tang. is as $\frac{1}{\text{cotan.}}$, or tang. : radius : : radius : cotan. And the rectangle of the tangent and cotangent is equal to the square of the radius; that is, tan. × cot. = radius ².

If a denote any arc, and t its tangent, radius 1: then is $a = t - \frac{1}{3}t^3 + \frac{1}{5}t^5 - \frac{1}{7}t^7 +$ &c. Whence, since tan. 45° = 1, we have arc $45° = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} -$ &c. And conversely $t = a + \frac{1}{3}a^3 + \frac{2}{15}a^5 + \frac{17}{315}a^7 +$ &c.

Further, $t = \frac{\sin a}{\cos a} = \frac{1}{\cot a} = \sqrt{(\frac{1}{\cos^2 a} - 1)} = \frac{\sin a}{\sqrt{(1-\sin^2 a)}} = \frac{\sqrt{(1-\cos^2 a)}}{\cos a} = \frac{2\tan\frac{1}{2}a}{1-\tan^2\frac{1}{2}a} = \frac{2}{\cot\frac{1}{2}a - \tan\frac{1}{2}a} = \cot a - 2\cot 2a = \frac{1-\cos 2a}{\sin 2a} = \frac{\sin 2a}{1+\cos 2a} = \sqrt{\frac{1-\cos 2a}{1+\cos 2a}} = \frac{\tan(45+\frac{1}{2}a) - \tan(45-\frac{1}{2}a)}{2}$.

$$\text{Also, } \tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \quad \tan b}.$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \cdot \tan b}.$$

$$\tan a + \tan b = \frac{\sin(a+b)}{\cos a \cdot \cos b}.$$

$$\cot a + \cot b = \frac{\sin(a+b)}{\sin a \cdot \sin b}.$$

See the treatises on trigonometry by Emerson, Cagnoli, Mauduit, and several others.

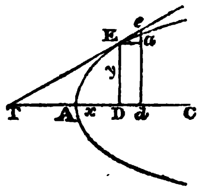
Artificial TANGENTS, or *Logarithmic* TANGENTS, are the logarithms of the tangents of arcs; so called, in contradistinction from the natural tangents, or the tangents expressed by the natural numbers.

Line of TANGENTS, is a line usually placed on the sector, and Gunter's scale; the description and uses of which see under the article SECTOR.

Sub-TANGENT, a line lying beneath the tangent, being the part of the axis intercepted by the tangent and the ordinate to the point of contact: as the line AG in the 2d and 3d figures above.

Method of TANGENTS, is a method of determining the quantity of the tangent and subtangent of any algebraic curve; the equation of the curve being given. This method is one of the great results of the doctrine of fluxions. It is of great use in geometry; because that in determining the tangents of curves, we determine at the same time the quadrature of the curvilinear spaces: on which account it deserves to be here particularly treated on.

To Draw the Tangent, or to find the Subtangent, of a curve.



If AE be any curve, and E any point in it, to which it is required to draw a tangent TE. Draw the ordinate DE: then if we can determine the subtangent TD, by joining the points T and E, the line TE will be the tangent sought.

Let *dae* be another ordinate indefinitely near to DE, meeting the curve, or tangent produced, in *e*: and let E*a* be parallel to the axis AD. Then is the elementary triangle E*ae* similar to the triangle TDE;

and therefore - ea : aE : : ED : DT;
but - - - - ea : aE : : flux. ED : flux. AD;
therefore - - flux. ED : flux. AD : : DE : DT;
that is - - - $\dot{y} : \dot{x} : : y : \frac{y\dot{x}}{\dot{y}} =$ DT,

which is therefore the value of the subtangent sought; where x is the absciss AD, and y the ordinate DE.

Hence comes this general rule · By the given equation of the curve, find the value either of $\dot{x}$ or $\dot{y}$, or of $\frac{\dot{x}}{\dot{y}}$, which substitute for it in the expression DT $= \frac{y\dot{x}}{\dot{y}}$, and, when reduced to its simplest terms, it will be the value of the subtangent sought. This we may illustrate in the following examples.

Ex. 1. The equation defining a circle is $2ax - xx = y^2$, where a is the radius; and the fluxion of this is $2a\dot{x} - 2x\dot{x} = 2y\dot{y}$; hence $\frac{\dot{x}}{\dot{y}} = \frac{y}{a-x}$; this multiplied by y, gives $\frac{y\dot{x}}{\dot{y}} = \frac{y^2}{a-x} = \frac{\text{DE}^2}{\text{CD}} =$ the subtangent TD, or CD : DE : : DE : TD, which is a property of the circle also known from common geometry.

Ex. 2. The equation defining the common parabola is $ax = y^2$, a being the parameter, and x and y the absciss and ordinate in all cases. The fluxion of this is $a\dot{x} = 2y\dot{y}$; hence $\frac{\dot{x}}{\dot{y}} = \frac{2y}{a}$; consequently $\frac{y\dot{x}}{\dot{y}} = \frac{2y^2}{y} = \frac{2ax}{a} = 2x =$ TD; that is, the subtangent TD is double the absciss AD, or TA is = AD, which is a well-known property of the parabola.

Ex. 3. The equation defining an ellipsis is $c^2(2ax - x^2) = a^2y^2$, where a and c are the semiaxes. The fluxion of it is $c^2(2a\dot{x} - 2x\dot{x}) = 2a^2y\dot{y}$; hence $\frac{y\dot{x}}{\dot{y}} = \frac{a^2y^2}{c^2(a-x)} = \frac{c^2(2ax-x^2)}{c^2(a-x)} = \frac{2a-x}{a-x}x =$ TD the subtangent; or by adding CD which is $= a - x$, it becomes

$\text{CT} = \frac{2ax - x^2}{a - x} + a - x = \frac{a^2}{a - x} = \frac{\text{CA}^2}{\text{CD}}$, or CD : CA : : CA : CT, a well-known property of the ellipse.

Ex. 4. The equation defining the hyperbola is $c^2(2ax + x^2) = a^2y^2$, which is similar to that for the ellipse, having only $+ x^2$ for $- x^2$; hence the conclusion is exactly similar also, viz,

$\frac{2a + x}{a + x}x$ or $\frac{2ax + xx}{a + x} = \text{TD}$, which taken from CD or $a + x$, gives $\text{CT} = \frac{\text{CA}^2}{\text{CD}}$, or CD : CA : : CA : CT.

And so on, for the tangents to other curves.

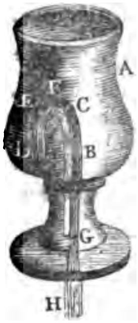
The Inverse Method of TANGENTS. This is the reverse of the foregoing, and consists in finding the nature of the curve that has a given subtangent. The method of solution is to put the given subtangent equal to the general expression $\frac{y\dot{x}}{\dot{y}}$, which serves for all kinds of curves; then the equation reduced, and the fluents taken, will give the fluential equation of the curve sought.

Ex. 1. To find the curve line whose subtangent is $= \frac{2y^2}{a}$. Here $\frac{2y^2}{a} = \frac{y\dot{x}}{\dot{y}}$; hence $2y\dot{y} = a\dot{x}$, and the fluents of this give $y^2 = ax$, the equation to a parabola, which therefore is the curve sought.

Ex. 2. To find the curve whose subtangent is $= \frac{yy}{2a - x}$, or a third proportional to $2a - x$ and y. Here $\frac{yy}{2a - x} = \frac{y\dot{x}}{\dot{y}}$; hence $y\dot{y} = 2a\dot{x} - x\dot{x}$, the fluents of which give $y^2 = ax - x^2$, the equation to a circle, which therefore is the curve sought.

Or, in a more general sense, this is the same thing as finding the fluents of such forms as involve several variable quantities. See INVERSE, &c. Also the Fluxional Treatises by Maclaurin, Simpson, Emerson, Dealtry, Bossu, Lacroix, Lagrange, &c.

TANTALUS's *Cup*, in Hydraulics, is a cup, as A, with a hole in the bottom, and the longer leg of a syphon BCED cemented into the hole; so that the end D of the shorter leg DE may always touch the bottom of the cup within. Then, if water be poured into this cup, it will rise in the shorter leg by its upward pressure, extruding the air before it through the longer leg, and when the cup is filled above the bend of the syphon at E, the pressure of the water in the cup will force it over the bend; from whence it will descend in the longer leg EB, and through the bottom at G, till the cup be quite emptied. The legs of this syphon are almost close together, and it is sometimes concealed by a small hollow statue, or figure of a man placed over it; the bend E being within the neck of the figure as high as the chin. So that poor thirsty Tantalus stands up to the chin in water, according to the fable, imagining it will rise a little higher, as more water is poured in, and he may drink; but instead of that, when the water comes up to his chin, it immediately begins to descend, and therefore, as he cannot stoop to follow it, he is left in the same distressful state of thirst as before. Ferguson's Lect. pa. 72, 4to.



TARRANTIUS (LUCIUS), surnamed *Firmanus*, because he was a native of Firmum, a town in Italy, flourished at the same time with Cicero, and was one of his friends. He was a mathematical philosopher, and therefore was thought to have great skill in judicial astrology. He was particularly famous by two horoscopes which he drew, the one the horoscope of Romulus, and the other of Rome. Plutarch says, "Varro, who was the most learned of the Romans in history, had a particular friend named Tarrantius, who, out of curiosity, applied himself to draw horoscopes, by means of astronomical tables, and was esteemed the most eminent in his time." Historians controvert some particular circumstances of his calculations; but all agree in conferring on him the honorary title *Prince of astrologers.*

TARTAGLIA, or TARTALEA (NICHOLAS), a noted mathematician, was born at Brescia in Italy, in 1479, of a very poor family; and was in that town when the French plundered it on their return from Naples. On this occasion he received many wounds, several of them on the head, which affected his speech, causing him to stutter. It is not known how he learned to read; but, to acquire writing, he was obliged to steal from a teacher a set of the letters of the alphabet. Hence, it is easy to imagine what difficulties he must have surmounted in acquiring his knowledge. Yet we find he was a considerable master or preceptor in mathematics in the year 1521, when the first of his collection of questions and answers was written, which he afterwards published in the year 1546, under the title of Quesiti et Inventioni diverse, at Venice, where he then resided as a public lecturer on mathematics, having removed to this place about the year 1534. This work consists of 9 chapters, containing solutions to a number of questions on all the different branches of mathematics and philosophy then in vogue. The last or 9th of these contains the questions in algebra, among which are those celebrated letters and communications between Tartalea and Cardan, by which our author put the latter in possession of the rules for cubic equations, which he first discovered in the year 1530.

But the first work of Tartalea's that was published, was his Nova Scientia inventa, in 4to, at Venice in 1537. This is a treatise on the theory and practice of gunnery, and the first of the kind, he being the first writer on the flight and path of balls and shells. This work was translated into English, by Lucar, and printed at London in 1588, in folio, with many notes and additions by the translator. Tartalea published at Venice, in folio, 1543, the whole books of Euclid, accompanied with many curious notes and commentaries. But the last and chief work of Tartalea, was his Trattato di Numeri et Misure, in folio, 1556 and 1560. This is a universal treatise on arithmetic, algebra, geometry, mensuration, &c. It contains also many other curious particulars of the disputes between our author and Cardan, which ended only with the death of Tartalea, in 1557, before the last part of this work was published in the year 1558. One of the ingenious inventions of Tartalea was the method of finding the area of a triangle from having the three sides given.

For many other circumstances concerning Tartalea and his writings, see the article ALGEBRA, vol. 1.

TAURUS, *the Bull*, in Astronomy, one of the 12 signs in the zodiac, and the second in order. The Greeks fabled that this was the bull which carried Europa safely across the seas to Crete; and that Jupiter, in reward for so signal a service, placed the creature, whose form he had assumed on that occasion, among the stars, and that this is the constellation formed of it. But it is probable that the Egyptians, or Babylonians, or whoever invented the constellations of the zodiac, placed this figure in that part of it which the sun entered about the time of the bringing forth of calves; like as they placed the ram in the first part of spring, as the lambs appear before them, and the two kids (for that was the original figure of the sign Gemini), afterwards, to denote the time of the goats bringing forth their young.

In the constellation Taurus there are some remarkable stars that have names; as Aldebaran in the south or right eye of the bull, the cluster called the Pleiades in the neck, and the cluster called Hyades in the face. The stars in the constellation Taurus, in Ptolemy's catalogue are 44, in Tycho's catalogue 43, in Hevelius's catalogue 51, and in the Britannic catalogue 141.

TAYLOR (Dr. Brook), a very able mathematician and secretary of the Royal Society, was born at Edmonton, in Middlesex, 1685. In 1701 he entered St. John's College, Cambridge; and in 1708 wrote his tract on the centre of oscillation. In 1712 he was elected into the Royal Society, of which he was chosen secretary two years after. Dr. Taylor had many excellent papers, on philosophy and mathematics, inserted in the Philos. Trans. from vol. 27 to vol. 32, inclusively; besides which, he published some other excellent works, viz, Methodus Incrementorum, in 4to, 1715, containing many excellent tracts, particularly a curious and general theorem, on the manner of expressing a variable quantity by all the orders of its differentials or fluxions; also the problem of the vibrations of a tense cord, of which he gave the first solution. The same year also came out his Principles of Linear Perspective, first establishing the true practice of that art, on principles which have been ever since followed by all other authors. Dr. Taylor was a profound and elegant mathematician of the old school of Newton, Jones, Cotes, &c, and one of the chief writers in the disputes with the Bernoullis and other eminent writers on the continent. Dr. Brook died at an early age, 46, in the year 1731.

Taylor's *Theorem*, in the higher mathematics, is a very elegant and fertile formula, given by Dr. Brook Taylor, in cor. 2, pr. 7, pa. 23, of his Method of Increments. The purport of it is as follows:—If x and z be any two variable quantities, having any given relation; then, while x by flowing uniformly, is increased by $\dot{x}$, z will be increased by $\dot{z} + \frac{1}{1.2}\ddot{z} + \frac{1}{1.2.3}\dddot{z}$ + &c; in which the values of $\dot{z}$, $\ddot{z}$, &c, are to be determined from the given equation.

The demonstration of this theorem has been given by several eminent writers; as Frisi; Lacroix, pa. 25, Calcul. Differential.; Francœur, Mathematiques Pures, tom. 2, pa. 243, &c; and by Dr. Brinkley, vol. 7, Transactions of the Irish Acad. with applications of the use of the theorem, in finding fluxions *per saltum*, and in approximating to the roots of equations, &c.

TEBET, or **Thevet**, the 4th month of the civil year of the Hebrews, and the 10th of their ecclesiastical year. It answered to part of our December and January, and had only 29 days.

TEETH, of various kinds of machines, as of mill wheels, &c. These are often called cogs by the workmen; and by working in the pinions, rounds, or trundles, the wheels are made to turn one another. Mr. Emerson (in his Mechanics, prop. 25), treats of the theory of teeth, and shows that they ought to have the figure of epicycloids, for properly working in one another. Camus (in his Cours de Mathematique, tom. 2, pa. 349, &c, edit. 1767) treats more fully on the same subject; and demonstrates that the teeth of the two wheels should have the figures of epicycloids, but that the generating circles of these epicycloids should have their diameters only the half of what Mr. Emerson makes them.

Mr. Emerson observes, that the teeth ought not to act upon one another before they arrive at the line which joins their centres. And though the inner or under sides of the teeth may be of any form; yet it is better to make them both sides alike, which will serve to make the wheels turn backwards. Also a part may be cut away on the back of every tooth, to make way for those of the other wheel. And the more teeth that work together, the better; at least one tooth should always begin before the other has done working. The teeth ought to be disposed in such manner as not to disturb or hinder one another, before they begin to work; and there should be a convenient length, depth, and thickness given to them, as well for strength, as that they may more easily disengage themselves.

TELEGRAPH, a machine brought into use by the French nation, in the year 1793, contrived to communicate words or signals from one person to another at a great distance, in a very small space of time.

The telegraph, though it has been generally known and used by the moderns only for a few years, is by no means an entirely modern invention; some kind of signals for distant communication having probably existed in all ages, and in all nations. There is reason to believe that among the Greeks something of this kind was in use; as the burning of Troy was certainly known in Greece very soon after it happened, and before any person had returned from thence.—The Chinese, when they send couriers on the great canal, or when any great man travels there, make signals by fire from one day's journey to another, to have every thing prepared. And most of the barbarous nations used formerly, and often do still, to give the alarm of war, or the approach of an enemy, by fires lighted on the hills or rising grounds.

The object proposed is, to obtain an intelligible figurative language, which may be distinguished at a distance, and by which the obvious delay in the dispatch of orders or information by messenger may be avoided. On first reflection we find the practical modes of such distant communication must be confined to sound and vision. Each of which is in a great degree subject to the state of the atmosphere: as, independent of the wind's direction, it is known that the air is sometimes so far deprived of its elasticity, or whatever other quality the conveyance of sound depends on, that the heaviest ordnance is scarce heard farther than the shot flies; it is also well known, that in thick hazy weather the largest objects become totally obscured at a short distance. No instrument therefore designed for the purpose can be perfect. We can only endeavour to diminish these defects as much as may be.

Polybius names the different instruments used by the ancients, for communicating information, *πυρσειαι*, *pyrsiæ*, because the signals were always made by means of fire or lights. At first they communicated information of events, in an imperfect manner, merely by torches. A new method was invented by Cleoxenus, or as others say by Democlitus, which was much improved by Polybius, as he himself informs us; and which he describes as follows: Take the letters of the (Greek) alphabet, and divide them into 5 parts, each of which will consist of 5 letters, except the last division, which will have only 4. Let these be fixed on a board in five columns. The man who is to give the signals is then to begin by holding up 2 torches, which he is to keep aloft till the other person has also shown two; which is only to ascertain that both sides are ready; these two torches being then withdrawn. Both parties are provided with boards, on which the letters are disposed as before described. The person then, who gives the signal, is to hold up torches on the left, to point out to the other party from which column he shall take the letters, as they are pointed out to him. If it is to be from the first column, he holds up one torch; if from the second, 2; and so on for the others. He is then to hold up torches on the right, to denote the particular letter of the column that is to be taken, according to their place in the column. The man who gives the signals has an instrument consisting of two tubes, so placed as that, by looking through one of them, he can see only the right side, and through the other only the left, of him who is to answer. The board is set up near this instrument; and the station on the right and left surrounded with a wall, 10 feet broad, and about the height of a man, that the torches raised above it may be clearly seen, and to conceal them when taken down. Thus, then, it is easy to conceive how the letters of a short sentence, one after another, are communicated from station to station, as far as required.—And this is the pyrsia or télegraph recommended by Polybius.

It seems the Romans had a method in their walled cities, either by a hollow formed in the masonry, or by tubes affixed to it, so to confine and augment sound, as to convey information to any part they wished; and in lofty houses it is now sometimes the custom to have a pipe, by way of speaking trumpet, to give orders from the upper apartments to the lower: by this mode of confining sound its volume may be carried to a very great distance; but beyond a certain extent the sound, losing articulation, would only convey alarm, not give directions.

Every city among the ancients had its watch-towers; and the castra stativa of the Romans, had always some spot, elevated either by nature or art, from whence signals were given to the troops cantoned or foraging in the neighbourhood. But I believe they had not arrived to greater refinement than that on seeing a certain signal they were immediately to repair to their appointed stations.

A beacon or bonfire made of the first inflammable materials that offered, as the most obvious, is perhaps the most ancient mode of general alarm; and by being previously concerted, the number or point where the fires appeared might have its particular intelligence affixed. The same observations may be referred to the throwing up of rockets, whose number or point from whence thrown may have its affixed signification.

Flags or ensigns with their various devices are of the earliest invention, especially at sea; where, from the first idea, which most probably was that of a vane to show the direction of the wind, they have been long adopted as the distinguishing mark of nations, and are now so neatly combined by the ingenuity of a great naval commander, that by his system every requisite order and question is received and answered by the most distant ships of a fleet.

To the adopting this or a similar mode in land service, the following are objections: That in the latter case, the variety of matter necessary to be conveyed, is so infinitely greater, that the combinations would become too complicated. And if the person for whom the information is intended should be in the direction of the wind, the flag would then present a straight line only, and at a little distance be scarce visible. The Romans were so well aware of this inconvenience of flags, that many of their standards were solid, and the name manipulus denotes the rudest of their modes, which was a truss of hay fixed on a pole.

But it does not seem that the moderns had thought of such a thing as a telegraph till the year 1663, when the marquis of Worcester, in his Century of Inventions, affirmed that he had discovered "a method by which, at a window, as far as eye can discover black from white, a man may hold discourse with his correspondent, without noise made or notice taken; being according to occasion given, or means afforded, *ex re nata*, and no need of provision beforehand; though much better if foreseen, and course taken by mutual consent of parties." This could be done only by means of a telegraph, which, in the next sentence, is declared to have been rendered so perfect, that by means of it the correspondence could be carried on "by night as well as by day, though as dark as pitch is black."

Dr. Hooke, whose genius as a mechanical inventor was perhaps never surpassed, delivered a "Discourse to the Royal Society, May 21st, 1684, showing a way how to communicate one's mind at great distances." In this discourse he asserts the possibility of conveying intelligence from one place to another, at the distance of 30, 40, 100, 120, &c, miles, "in as short a time as a man can write what he would have sent." He takes to his aid the then recent invention of the telescope, and explains the method by which characters exposed at one station, may be rendered visible at the others. He directs, "first, for the stations; if they be far distant, it will be necessary that they should be high, and lie exposed to the sky, that there be no higher hill or part of the earth beyond them, that may hinder the distinctness of the characters, that are to appear dark, the sky beyond them appearing white: by which means also the thick and vaporous air near the ground will be passed over and avoided." "Next, the height of the stations is advantageous, upon the account of the refractions or inflections of the air." "Next, in choosing of these stations, care must be taken, as near as may be, that there be no hill that interposes between them, that is almost high enough to touch the visual ray; because in such cases, the refraction of the air of that hill will be very apt to disturb the clear appearance of the object." "The next thing to be considered is, what telescopes will be necessary for such stations." "One of these telescopes must be fixed at each extreme station, and two of them in each intermediate; so that a man for each glass sitting and looking through them, may plainly discover what is done in the next adjoining station, and with his pen write down on paper the characters there exposed in their due order; so that there ought to be two persons at each extreme station, and three at each intermediate; so that, at the same time, intelligence may be conveyed forwards and backwards. Next, there must be certain times agreed on, when the correspondents are to expect; or else there must be set at the top of the pole, in the morning, the hour appointed by either of the correspondents, for acting that day; if the hour be appointed, pendulum clocks may adjust the moment of expectation and observing." "Next, there must be a convenient apparatus of characters, whereby to communicate any thing with great ease, distinctness, and secrecy. And there must be either day characters, or night characters." The day characters "may all be made of three slit deals:" the night characters "may be made with links, or other lights, disposed in a certain order." The Doctor invented 24 simple characters, each formed of right lines, for the letters of the alphabet; and several single characters, made up of semicircles, for whole sentences. He recommended that three very long masts or poles should be placed vertically, and joined at top by one strong horizontal beam; that a large screen should be placed at one of the upper corners of this frame, behind which all the deal-board characters should hang, and by the help of proper chords should quickly be drawn forwards to be exposed, and then drawn back again behind the screen. "By these means," says the Doctor, "all things may be made so convenient, that the same character may be seen at Paris within a minute after it has been exposed at London, and the like in proportion for greater distances; and that the characters may be exposed so quick after one another, that a composer shall not much exceed the exposer in swiftness." Among the cases of this contrivance, the inventor mentions these: "The first is for cities or towns besieged; and the second for ships upon the sea; in both which cases it may be practised with great certainty, security, and expedition."

The whole of Dr. Hooke's paper was published in Derham's collection of his Experiments and Observations; from which it appears that he had brought the telegraph to a state of far greater maturity and perfection than M. Amonton's, who attempted the same thing about the year 1702; and indeed to a state but little inferior to several which have been proposed during the last 20 years.

It was not however till the French revolution that the telegraph was applied to useful purposes. Whether M. Chappe, who is said to have invented the telegraph, first used by the French about the end of 1793, knew any thing of Hooke's or Amonton's invention or not, it is impossible to say; but his telegraph was constructed on principles nearly similar, the description of which here follows:

The following account of this curious instrument is copied from Barrere's report in the sitting of the French Convention of August 15, 1794.—"The new-invented telegraphic language of signals is an artful contrivance to transmit thoughts, in a peculiar way, from one distance to another, by means of machines, which are placed at different distances, of from 12 to 15 miles from one another, so that the expression reaches a very distant place in the space of a few minutes. Last year an experiment of this invention was tried in the presence of several Commissioners of the Convention. From the favourable report which the latter made of the efficacy of the contrivance, the Committee of Public Welfare tried every effort to establish, by this means, a correspondence between Paris and the frontier places, beginning with Lisle. Almost a whole twelvemonth has been spent in collecting the necessary instruments for the machines, and to teach the people employed how to use them. At present, the telegraphic language of signals is prepared in such a manner, that a correspondence may be conducted with Lisle upon every subject, and that every thing, nay even proper names, may be expressed; an answer may be received, and the correspondence thus be renewed several times a day. The machines are the invention of Citizen Chappe, and were constructed under his own eye; he also directs their establishment at Paris. They have the advantage of resisting the changes in the atmosphere, and the inclemencies of the seasons. The only thing which can interrupt their effect is, if the weather is so very bad and turbid that the objects and signals cannot be distinguished. By this invention, remoteness and distance almost disappear; and all the communications of correspondence are effected with the rapidity of the twinkling of an eye. The operations of government can be very much facilitated by this contrivance, and the unity of the republic can be the more consolidated by the speedy communication with all its parts. The greatest advantage which can be derived from this correspondence is, that, if one chooses, its object shall only be known to certain individuals, or to one individual alone, or to the extremities of any distance; so that the Committee of Public Welfare may now correspond with the representative of the people at Lisle without any other persons getting acquainted with the object of the correspondence. Hence it follows that, were Lisle even besieged, we should know every thing at Paris that might happen in that place, and could send thither the decrees of the Convention without the enemy's being able to discover or to prevent it."—The figure of the French machine, as given in some English prints, is represented in fig. 3, pl. 34.

Such is the account given of the French invention. Various improved contrivances have been since made in England, and a pamphlet has lately been published, giving an account of some of them, by the Rev. J. Gamble, under the title of Observations and Telegraphic Experiments.

As to the French machine, it is evident that to every angular change of the greater beam or of the lesser end arms, a different letter or figure may be annexed. But where the whole difference consists in the variation of the angle of the greater or lesser pieces, much error may be expected, from the inaccuracy either of the operator or the observer: besides other inconveniences arising from the great magnitude of the machinery.

Another idea is perfectly numerical; which is to raise and depress a flag or curtain a certain number of times for each letter, according to a previously concerted system: as, suppose one elevation to mean A, two to mean B, and so on through the alphabet. But in this case, the least inaccuracy in giving or noting the number changes the letter; and besides, the last letters of the alphabet would be a tedious operation.

Another method that has been proposed, is an ingenious combination of the magnetical experiment of Comus, and the telescopic micrometer. But as this is only an imperfect idea of Mr. Garnet's very ingenious machine, described in the latter part of this article, no further notice need be taken of it here.

Mr. Gamble proposes one on a new idea of his own. The principle of it is simply that of a Venetian blind, or rather what are called the lever boards of a brewhouse, which, when horizontal, present so small a surface to the distant observer, as to be lost to his view, but are capable of being in an instant converted into a screen of a magnitude adapted to the required distance of vision.—Let AB and CD (fig. 4, pl. 34), two upright posts fixed in the ground, and joined by the braces BD and EF, be considered as the frame work for 9 lever boards working upon centres in EB and DF, and opening in three divisions by iron rods connected with each three of the lever boards.

Let *abcd* and *efgh* be two lesser frames fixed to the great one, having also three lever boards in each, and moving by iron rods, in the same manner as the others. If all these rods be brought so near the ground as to be in the management of the operator, he will then have five, of what may be called, keys to play on. Now as each of the handles *iklmn* commands three lever boards, by raising any one of them, and fixing it in its place by a catch or hook, it will give a different appearance to the machine; and by the proper variation of these five movements, there will be more than 25 of what may be called mutations, in each of which the machine exhibits a different appearance, and to which any letter or figure may be annexed at pleasure.

Should it be required to give intelligence in more than one direction, the whole machine may be easily made to turn to different points on a strong centre, after the manner of a single-post windmill.—To use this machine by night, another frame must be connected with the back part of the telegraph, for raising five lamps, of different colours, behind the openings of the lever boards; these lamps by night answer for the openings by day. M. Gamble gives also particular directions for placing and using the machine, and for writing down the several figures or movements.

Mr. John Garnet's most simple and ingenious contrivance, is as follows. This is merely a bar or plank turning upon a centre, like the sail of a windmill, and being moved into any position, the distant observer turns the tube of a telescope into the same position, by bringing a fixed wire within it to coincide with or parallel to the bar, which is a thing extremely easy to do. The centre of motion of the bar has a small circle about it, with letters and figures around the circumference, and an index moving round with the bar, pointing to any letter or mark that the operator wishes to set the bar to, or to communicate to the observer. The eye end of the telescope without has a like index and circle, with the corresponding letters or other marks. The consequence is obvious: the telescope being turned about till its wire cover or become parallel to the bar, the index of the former necessarily points out the same letter or mark in its circle, as that of the latter; and the communication of sentiment is immediate and perfect. The use of this machine is so easy, that I have seen it put into the hands of two common labouring men, who had never seen it before, and they have immediately held a quick and distant conversation together.

The more particular description and figure of this machine, is as follows. ABDE (fig. 5, pl. 34), is the telegraph, on whose centre of gravity C, about which it revolves, is a fixed pin, which goes through a hole or socket in the firm upright post G, and on the opposite side of which is fixed an index CI. Concentric to C, on the same post, is fixed a wooden or brass circle, of 6 or 8 inches diameter, divided into 48 equal parts, 24 of which represent the letters of the alphabet, and between the letters, are numbers. So that the index, by means of the arm AB, may be moved to any letter or number. The length of the arm should be 2½ or 3 feet for every mile of distance. Two revolving lamps of different colours suspended occasionally at A and B, the ends of the arm, would serve equally at night.

Let *ss* (fig. 6, pl. 34) represent the transverse section of the outward tube of a telescope to its axis, and *xx* the like section of the sliding or adjusting tube, on which is fixed an index II. On the part of the outward tube next to the observer, there is fixed a circle of letters and numbers, similarly divided and situated to the circle in figure 5; then the index II, by means of the sliding or adjusting tube, may be turned to any letter or number.—Now there being a hair, or fine silver wire *fg*, fixed in the focus of the eye-glass, in the same direction as the index II; so that when the arm AB (fig. 5) of the telegraph is viewed at a distance through the telescope, the hair may be turned, by means of the sliding tube, to the same direction of the arm AB; then the index II (fig. 6) will point to the same letter or number on its own circle, as the index I (fig. 5) points to on the telegraphic circle.—If, instead of using the letters and numbers to form words at length, they be employed as signals, three motions of the arm will give above a hundred thousand different signals.

Two ingenious telegraphs have also been invented by Captain Pasley, of the Royal Engineers; descriptions of which are given in the Philosophical Magazine, Nos. 115 and 116.

It seems there are now in use in England, four grand lines of telegraphs, communicating with London: viz, to Portsmouth, to Plymouth, to Deal, and to Yarmouth. "There are 12 stations between London and Portsmouth, and 31 between London and Plymouth, of which 8 are part of the Portsmouth line, till they separate in the New Forest. The other chains extend from London to Yarmouth, formed by 19 stations, and from London to Deal, formed by 10 stations, making in the whole 64 separate telegraphs. Their distances average about 8 miles, yet some of them 12 or 14 miles; the distances being often increased by the want of commanding heights: in the Yarmouth line particularly they make a considerable detour northward.

After about 20 years' experience, they calculate on about 200 days in a year, on which signals can be transmitted throughout the day; about 60 others on which they pass only part of the day, or at particular stations; and about 100 days on which few of the stations can see the others. The powers of the stations in this respect are exceedingly various. Dead flats are found to be universally unfavourable. On the contrary, stations between hill and hill, looking across a valley, or series of valleys, are mostly clear; and water surfaces are found to produce fewer obscure days than land in any situation. The stillness of the morning and evening are found to be the most favourable times for observations. The least favourable period of the day is an hour or two before and after noon, particularly on dead levels, where the play of the sun's rays on the rising exhalations renders distant vision very obscure.

The transmission of a message from London to Portsmouth usually occupies about 15 minutes; but, by an experiment tried for the purpose, a single signal has been transmitted to Plymouth and back again in 3 minutes, which by the telegraph route was at least 500 miles. In this instance however notice had been given to make ready, and every captain was at his post to receive and return the signals. The speed was at the rate of 170 miles in a minute, or 3 miles per second, or 3 seconds at each station; a facility truly wonderful! The number of signals produced by the English telegraph is 63—by which they represent the ten digits, and the letters of the alphabet, with many generic words, and all the numbers expressed by the combination of the digits 63 ways. The signals are sufficiently various to express any 3 or 4 words in twice as many changes of the shutters.

The telescopes used are Dollond's achromatics; though a simple Galilean might serve equally well, or better. The field of this telescope is quite large enough; and, having but two lenses, one of which is a thin concave, it gives the object with more brightness. It may seem strange too, that, to ease the operator, it was never contrived to exhibit the fixed spectrum on the principle of a portable camera, so that, without wearying the eye, the motion of the distant telegraph might have been exhibited on a plain surface, and seen with both eyes like as on the leaf of a book." Mo. Mag. vol. 39, pa. 202.

TELESCOPE, an optical instrument which serves for discovering and viewing distant objects, either directly by glasses, or by reflection, by means of specula, or mirrors. Accordingly,

Telescopes are either refracting or reflecting; the former consisting of different lenses, through which the objects are seen by rays refracted through them to the eye; and the latter of specula, from which the rays are reflected and passed to the eye. The lens or glass turned towards the object, is called the object-glass; and that next the eye, the eye-glass; and when the telescope consists of more than two lenses, all but that next the object are called eye-glasses. The latter consisting of different metallic speculums, finely polished and figured, so as to magnify the objects by reflection.

The invention of the telescope is one of the noblest and most useful these ages have to boast of: by means of it, the wonders of the heavens are discovered to us, and astronomy is brought to a degree of perfection which former ages could have no idea of. The discovery indeed was owing rather to chance than design; so that it is the good fortune of the discoverer, rather than his skill or ability, we are indebted to: on this account it concerns us the less to know, who it was that first hit upon this admirable invention. Be that as it may, it is certain it must have been casual, since the theory it depends upon was not then known.

John Baptista Porta, a Neapolitan, according to Wolfius, first made a telescope, which he infers from this passage in the Magia Naturalis of that author, printed in 1560: "If you do but know how to join the two (viz, the concave and convex glasses) rightly together, you will see both remote and near objects, much larger than they otherwise appear, and withal very distinct. In this we have been of good help to many of our friends, who either saw remote things dimly, or near ones confusedly; and have made them see every thing perfectly." But it is certain, that Porta did not understand his own invention, and therefore neither troubled himself to bring it to greater perfection, nor ever applied it to celestial observation. Besides, the account given by Porta of his concave and convex lenses, is so dark and indistinct, that Kepler, who examined it by desire of the emperor Rudolph, declared to that prince, that it was perfectly unintelligible.

Thirty years afterwards, or in 1590, a telescope 16 inches long was made, and presented to prince Maurice of Nassau, by a spectacle maker of Middleburg: but authors are divided about his name. Sirturus, in a treatise on the telescope, printed in 1618, will have it to be John Lippersheim: and Peter Borelli, in a volume expressly on the inventor of the telescope, published in 1655, states that it was Zacharias Jansen, or, as Wolfius writes it, Hansell. Now the invention of Lippersheim is fixed by some in the year 1609, and by others in 1605: Fontana, in his Novæ Observationes Cælestium et Terrestrium Rerum, printed at Naples in 1646, claims the invention in the year 1608. But Borelli's account of the discovery of telescopes is so circumstantial, and so well authenticated, as to render it very probable that Jansen was the original inventor.

In 1620, James Metius of Alcmaer, brother of Adrian Metius who was professor of mathematics at Franeker, came with Drebel to Middleburg, and there bought telescopes of Jansen's children, who had made them public; and yet this Adr. Metius has given his brother the honour of the invention, in which he is mistakenly followed by Descartes.

But none of these artificers made telescopes of above a foot and a half: Simon Marius in Germany, and Galileo in Italy, it is said, first made long ones fit for celestial observations; though, from the recently discovered astronomical papers of the celebrated Harriot, author of the Algebra, it appears that he must have employed telescopes in viewing the solar maculæ, which he did quite as early as they were observed by Galileo. Whether Harriot made his own telescopes, or whether he had them from Holland, does not appear; it seems however that Galileo's were made by himself; for Le Rossi relates, that Galileo, being then at Venice, was told of a kind of optic glass made in Holland, which brought objects nearer: upon which, setting himself to think how it should be, he ground two pieces of glass into form as well as he could, and fitted them to the two ends of an organ-pipe; and with these he showed at once all the wonders of the invention to the Venetians, on the top of the tower of St. Mark. The same author adds, that from this time Galileo devoted himself wholly to the improving and perfecting the telescope; and that he hence almost deserved all the honour usually done him, of being reputed the inventor of the instrument, and of its being from him called Galileo's tube. Galileo himself, in his Nuncius Sidereus, published in 1610, acknowledges that he first heard of the instrument from a German; and that, being merely informed of its effects, first by common report, and a few days after by letter from a French gentleman, James Badovere, at Paris, he himself discovered the construction by considering the nature of refraction. He adds, in his Sagiatore, that he was at Venice when he heard of the effects of prince Maurice's instrument, but nothing of its construction; that the first night after his return to Padua, he solved the problem, and made his instrument the next day, and soon after presented it to the Doge of Venice, who, in honour of his grand invention, gave him the ducal letters, which settled him for life in his lectureship, at Padua, and doubled his salary, which then became treble of what any of his predecessors had enjoyed before. And thus Galileo may be considered as an inventor of the telescope, though not the first inventor.

F. Mabillon indeed relates, in his travels through Italy, that in a monastery of his own order, he saw a manuscript copy of the works of Commestor, written by one Conradus, who lived in the 13th century; in the 3d page of which was seen a portrait of Ptolemy, viewing the stars through a tube of 4 joints or draws: but Mabillon does not say that the tube had glasses in it. Indeed it is more than probable, that such tubes were then used for no other purpose but to defend and direct the sight, or to render it more distinct, by singling out the particular object looked at, and shutting out all the foreign rays reflected from others, whose proximity might have rendered the image less precise. And this conjecture is verified by experience; for we have often observed that without a tube, by only looking through the hand, or even the fingers, or a pin-hole in a paper, the objects appear more clear and distinct than otherwise. Be this as it may, it is certain that the optical principles, on which telescopes are founded, are contained in Euclid, and were well known to the ancient geometricians; and it has been for want of attention to them, that the world was so long without that admirable invention; as doubtless there are many others lying hid in the same principles, only waiting for reflection or accident to bring them forth.

To the foregoing abstract of the history of the invention of the telescope, it may be proper to add some particulars relating to the claims of our own celebrated countryman, friar Bacon, who died in 1294. Mr. W. Molyneux, in his Dioptrica Nova, pa. 256, declares his opinion, that Bacon did perfectly well understand all kinds of optic glasses, and knew likewise the method of combining them, so as to compose some such instrument as our telescope: and his son, Samuel Molyneux, asserts more positively, that the invention of telescopes, in its first original, was certainly put in practice by an Englishman, friar Bacon; though its first application to astronomical purposes may probably be ascribed to Galileo. The passages to which Mr. Molyneux refers, in support of Bacon's claims, occur in his Opus Majus, pa. 348 and 357 of Jebb's edit. 1773. The first is as follows: Si vero non sint corpora plana, per quæ visus videt, sed sphæria, tunc est magna diversitas; nam vel concavitas corporis est versus oculum vel convexitas: whence it is inferred, that he knew what a concave and a convex glass was. The second is comprised in a whole chapter, where he says, De visione fracta majora sunt; nam de facili patet per canones supra dictos, quod maxima possunt apparere minima, et e contra, et longe distantia videbuntur propinquissime, et e converso. Nam possumus sic figurare perspicua, et taliter ea ordinare respectu nostri visus et rerum, quod frangentur radii, et flectentur quorsumcunque voluerimus, ut sub quocunque angulo voluerimus, videbimus rem prope vel longe, &c. Sic etiam faceremus solem et lunam et stellas descendere secundum apparentiam hic inferius, &c: that is, Greater things than these may be performed by refracted vision; for it is easy to understand by the canons above mentioned, that the greatest objects may appear exceeding small, and the contrary; also that the most remote objects may appear just at hand, and the converse; for we can give such figures to transparent bodies, and dispose them in such order with respect to the eye and the objects, that the rays shall be refracted and bent towards any place we please; so that we shall see the object near at hand or at a distance, under any angle we please, &c. So that thus the sun, moon, and stars may be made to descend hither in appearance, &c. Mr. Molyneux has also cited another passage out of Bacon's Epistle ad Parisiensem, of the Secrets of Art and Nature, cap. 5, to this purpose, Possunt etiam sic figurari perspicua, ut longissime posita appareant propinqua, et è contrario; ita quod ex incredibili distantia legeremus literas minutissimas, et numeraremus res quantumquo parvas, et stellas faceremus apparere quo vellemus: that is, Glasses, or diaphanous bodies may be so formed, that the most remote objects may appear just at hand, and the contrary; so that we may read the smallest letters at an incredible distance, and may number things though never so small, and may make the stars also appear as near as we please.

Moreover, Doctor Jebb, in the dedication of his edition of the Opus Majus, produces a passage from a manuscript, to show that Bacon actually applied telescopes to astronomical purposes: Sed longe magis quam hæc, says he, oporteret homines haberi, qui bene, immo optime, scirent perspectivam et instrumenta ejus—quia instrumenta astronomia non vadunt nisi per visionem secundum leges istius scientiæ.

From these passages, it is not unreasonable to conclude, that Bacon had actually combined glasses so as to have produced the effects which he mentions, though he did not complete the construction of telescopes. Dr. Smith, however, to whose judgment particular deference is due, is of opinion that the celebrated friar wrote hypothetically, without having made any actual trial of the things he mentions: to which purpose he observes, that this author does not assert one single trial or observation upon the sun or moon, or any thing else, though he mentions them both: on the other hand, he imagines some effects of telescopes that cannot possibly be performed by them. He adds, that persons unexperienced in looking through telescopes expect, in viewing any object, as for instance the face of a man, at the distance of one hundred yards, through a telescope that magnifies one hundred times, that it will appear much larger than when they are close to it: this he is satisfied was Bacon's notion of the matter; and hence he concludes that he had never looked through a telescope.

It is remarkable that there is a passage in Thomas Digge's Stratioticos, pa. 359, where he affirms that his father, Leonard Digges, among other curious practices, had a method of discovering, by perspective glasses set at due angles, all objects pretty far distant that the sun shone upon, which lay in the country round about; and that this was by the help of a manuscript book of Roger Bacon of Oxford, who he conceived was the only man besides his father (since Archimedes) who knew it. This is the more remarkable, because the Stratioticos was first printed in 1579, more than 30 years before Metius or Galileo made their discovery of those glasses; and therefore it has hence been thought that Roger Bacon was the first inventor of telescopes, and Leonard Digges the next reviver of them. But from what Thomas Digges says of this matter, it would seem that the instrument of Bacon, and of his father, was something of the nature of a camera obscura, or, if it were a telescope, that it was of the reflecting kind; though the term perspective glass seems to favour a contrary opinion.

There is also another passage to the same effect in the preface to the Pantometria of Leonard Digges, but published by his son Thomas Digges, some time before the Stratioticos, and a second time in the year 1591. The passage runs thus: "My father by his continuall painfull practises, assisted with demonstrations mathematical, was able, and sundrie times hath by Proportional Glasses duely situate in convenient angles, not only discovered things farre off, read letters, numbered peeces of money with the very coyne and superscription thereof, cast by some of his freends of purpose upon downes in open fields, but also seven myles off declared what hath beene doone at that instant in private places: He hath also sundrie times by the sunne beames fixed (should be fired) powder, and dischargde ordinance halfe a mile and more distante," &c.

But to whomsoever we ascribe the honour of first inventing the telescope, the rationale of this admirable instrument, depending on the refraction of light in passing through mediums of different forms, was first explained by the celebrated Kepler, who also pointed out methods of constructing others, of superior powers, and more commodious application, than that first used: though something of the same kind, it is said, was also done by Maurolycus, whose treatise De Lumine et Umbra was published in 1575.

The Principal Effects of TELESCOPES, depend upon this plain maxim, viz, that objects appear larger in proportion to the angles which they subtend at the eye; and the effect is the same, whether the pencils of rays, by which objects are made visible to us, come directly from the objects themselves, or from any place nearer to the eye, where they may have been united, so as to form an image of the object; because they issue again from those points in certain directions, in the same manner as they did from the corresponding points in the objects themselves. In fact therefore, all that is effected by a telescope, is first to make such an image of a distant object, by means of a lens or mirror, and then to give the eye some assistance for viewing that image as near as possible; so that the angle, which it shall subtend at the eye, may be very large, compared with the angle which the object itself would subtend in the same situation. This is done by means of an eye-glass, which so refracts the pencils of rays, as that they may afterwards be brought to their several foci, by the natural humours of the eye. But if the eye had been so formed as to be able to see the image, with sufficient distinctness, at the same distance, without an eye-glass, it would appear to him as much magnified, as it does to another person who makes use of a glass for that purpose, though he would not in all cases have so large a field of view.

Though no image be actually formed by the foci of the pencil without the eye, yet if, by the help of an eye-glass, the pencils of rays shall enter the pupil, just as they would have done from any place without the eye, the visual angle will be the same as if an image had been actually formed in that place. Priestley's History of Light &c. pa. 69, &c.

As to the Grinding of Telescopic Glasses, the first persons who distinguished themselves in that way, were two Italians, Eustachio Divini at Rome, and Campani at Bologna, whose fame was much superior to that of Divini, or that of any other person of his time; though Divini himself pretended, that in all the trials that were made with their glasses, his of a great focal distance performed better than those of Campani, and that his rival was not willing to try them fairly, viz, with equal eye-glasses. It is however generally supposed, that Campani really excelled Divini, both in the goodness and focal length of his object-glasses.

It was with Campani's Telescopes that Cassini discovered the nearest satellites of Saturn. They were made at the express desire of Lewis XIV, and were of 86, 100, and 136, Paris feet focal length.

Campani's laboratory was purchased, after his death, by pope Benedict XIV, who made a present of it to the academy at Bologna called the Institute; and by the account which Fougeroux has given, we learn that (except a machine which Campani constructed, to work the basons on which he ground his glasses) the goodness of his lenses depended on the clearness of his glass, his Venetian tripoli, the paper with which he polished his glasses, and his great skill and address as a workman. It does not appear that he made many lenses of a very great focal distance. Accordingly Dr. Hooke, who probably speaks with the partiality of an Englishman, says that some glasses, made by Divini and Campani, of 36 and 50 feet focal distance, did not excel Telescopes of 12 or 15 feet made in England. He adds, that sir Paul Neile made telescopes of 36 feet, pretty good; and one of 50, but not of proportionable goodness.

Afterwards, Mr. Reive first, and then Mr. Cox, who were the most celebrated in England, as grinders of optic glasses, made some good telescopes of 50 and 60 feet focal distance; and Mr. Cox made one of 100, but how good Dr. Hooke could not assert. Borelli also in Italy made object-glasses of a great focal length, one of which he presented to the Royal Society. But, with respect to the focal length of telescopes, these and all others were far exceeded by those of Auzout, who made one object-glass of 600 feet focus; but he was never able to manage it, so as to make it useful. And Hartsoeker, it is said, made some of a still greater focal length. Philos. Trans. Abr. vol. i, pa. 666. Hooke's Exper. by Derham, pa. 261. Priestley, pa. 211. See GRINDING.

Telescopes are of several kinds, distinguished by the number and form of their lenses, or glasses, and denominated from their particular uses &c: such are the Terrestrial or land Telescope, the Celestial or astronomical Telescope; to which may be added, the Galilean or Dutch Telescope, the Reflecting Telescope, the Refracting Telescope, the Aërial Telescope, Achromatic Telescope, &c.

Galileo's, or the Dutch telescope, is one consisting of a convex object-glass, and a concave eye-glass.

This is the most ancient form of any, being the only kind made by the inventors, Galileo, &c, or known, before Huygens. The first telescope, constructed by Galileo, magnified only 3 times; but he soon made another, which magnified 18 times: and afterwards, with great trouble and expence, he constructed one that magnified 33 times; with which he discovered the satellites of Jupiter, and the spots of the sun. The construction, properties, &c, of it, are as follow:

Construction of Galileo's, or the *Dutch* TELESCOPE.—In a tube prepared for the purpose, at one end is fitted a convex object lens, either a plain convex, or convex on both sides, but a segment of a very large sphere: at the other end is fitted an eye-glass, concave on both sides, and the segment of a less sphere, so disposed as to be at the distance of the virtual focus before the image of the convex lens.

Let AB (fig. 10, pl. 28) be a distant object, from every point of which pencils of rays issue, and falling on the convex glass DE, tend to their foci at FSG. But a concave lens HI (the focus of which is at FG) being interposed, the converging rays of each pencil are made parallel when they reach the pupil; so that by the refractive humours of the eye, they can easily be brought to a focus on the retina at PRQ. Also the pencils themselves diverging, as if they came from X, MXO is the angle under which the image will appear, which is much larger than the angle under which the object itself would have appeared. Such then is the telescope that was at first discovered and used by philosophers: the great inconvenience of which is, that the field of view, which depends, not on the breadth of the eye-glass, as in the astronomical telescope, but on the breadth of the pupil of the eye, is exceedingly small: for since the pencils of the rays enter the eye very much diverging from one another, but few of them can be intercepted by the pupil; and this inconvenience increases with the magnifying power of the telescope, so that philosophers may now well wonder at the patience and address with which Galileo and others, with such an instrument, made the discoveries they did. And yet no other telescope was thought of for many years after the discovery. Descartes, who wrote 30 years after, mentions no other as actually constructed, though Kepler had suggested some. Hence,

1. In an instrument thus framed, all people, except myopes, or short-sighted persons, must see objects distinctly in an erect situation, and increased in the ratio of the distance of the virtual focus of the eye-glass, to the distance of the focus of the object-glass.

2. But for myopes to see objects distinctly through such an instrument, the eye-glass must be set nearer the object-glass, so that the rays of each pencil may not emerge parallel, but may fall diverging upon the eye; in which case the apparent magnitude will be altered a little, though scarce sensibly.

3. Since the focus of a plano-convex object lens, and the virtual focus of a plano-concave eye-lens, are at the distance of the diameter; and the focus of an object-glass convex on both sides, and the virtual focus of an eye-glass concave on both sides, are at the distance of a semi-diameter; if the object-glass be plano-convex, and the eye-glass plano-concave, the telescope will increase the diameter of the object, in the ratio of the diameter of the concavity to that of the convexity: if the object-glass be convex on both sides, and the eye-glass concave on both sides, it will magnify in the ratio of the semidiameter of the concavity to that of the convexity: if the object-glass be plano-convex, and the eye-glass concave on both sides, the semidiameter of the object will be increased in the ratio of the diameter of the convexity to the semidiameter of the concavity: and lastly, if the object-glass be convex on both sides, and the eye-glass plano-concave, the increase will be in the ratio of the diameter of the concavity to the semidiameter of the convexity.

4. Since the ratio of the semidiameters is the same as that of the diameters, telescopes magnify the object in the same manner, whether the object-glass be plano-convex, and the eye-glass plano-concave; or whether the one be convex on both sides, and the other concave on both.

5. Since the semidiameter of the concavity has a less ratio to the diameter of the convexity than its diameter has, a telescope magnifies more if the object-glass be plano-convex, than if it be convex on both sides. The case is the same if the eye-glass be concave on both sides, and not plano-concave.

6. The greater the diameter of the object-glass, and the less that of the eye-glass, the less ratio has the diameter of the object, viewed with the naked eye, to its semidiameter when viewed with a telescope, and consequently the more is the object magnified by it.

7. Since a telescope exhibits so much a less part of the object, as it increases its diameter more, for this reason, mathematicians were determined to look out for another telescope, after having clearly found the imperfection of the first, which was discovered by chance. Nor were their endeavours vain, as appears from the astronomical telescope described below.

If the semidiameter of the eye-glass have too small a ratio to that of the object-glass, an object through the telescope will not appear sufficiently clear, because the great divergency of the rays will occasion the several pencils representing the several points of the object on the retina, to consist of too few rays.

It is also found that equal object-lenses will not bear the same eye-lenses, if they be differently transparent, or if there be a difference in their polish; a less transparent object-glass, or one less accurately ground, requiring a more spherical eye-glass than another more transparent, &c.

Hevelius recommends an object-glass convex on both sides, whose diameter is 4 feet; and an eye-glass concave on both sides, whose diameter is 4½ tenths of a foot. An object-glass, equally convex on both sides, whose diameter is 5 feet, he observes, will require an eye-glass of 5½ tenths; and adds, that the same eye-glass will also serve an object-glass of 8 or 10 feet.

Hence, as the distance between the object-glass and eye-glass is the difference between the distance of the virtual focus of the eye-glass, and the distance of the focus of the object-glass; the length of the telescope is had by subtracting that from this. That is, the length of the telescope is the difference between the diameters of the object-glass and eye-glass, if the former be plano-convex, and the latter plano-concave; or the difference between the semidiameters of the object-glass and eye-glass, if the former be convex on both sides, and the latter concave on both; or the difference between the semidiameter of the object-glass and the diameter of the eye-glass, if the former be convex on both sides, and the latter plano-concave; or lastly the difference between the diameter of the object-glass and the semidiameter of the eye-glass, if the former be plano-convex, and the latter concave on both sides. Thus, for instance, if the diameter of an object-glass, convex on both sides, be 4 feet; and that of an eye-glass, concave on both sides, be 4½ tenths of a foot; then the length of the telescope will be 1 foot and 7¾ tenths.

Astronomical TELESCOPE; this is one that consists of an object-glass, and an eye-glass, both convex. It is so called from being wholly used in astronomical observations.

It was Kepler who first suggested the idea of this telescope: having explained the rationale, and pointed out the advantages of it in his Catoptrics, in 1611. But the first person who actually made an instrument of this construction, was father Scheiner, who has given a description of it in his Rosa Ursina, published in 1630. To this purpose he says, if you insert two similar convex lenses in a tube, and place your eye at a convenient distance, you will see all terrestrial objects, inverted indeed, but magnified and very distinct, with a considerable extent of view. He afterwards subjoined an account of a telescope of a different construction, with two convex eye-glasses, which again reverses the images, and makes them appear in their natural position. Father Reita however soon after proposed a better construction, using three eye-glasses instead of two.

Construction of the Astronomical TELESCOPE. The tube being prepared, an object-glass, either plano-convex, or convex on both sides, but a segment of a large sphere, is fitted in at one end; and an eye-glass, convex on both sides, which is the segment of a small sphere, is fitted to the other end; at the common distance of the foci.

Thus the rays of each pencil issuing from every point of the object ABC, (fig. 3, pl. 36) passing through the object-glass DEF, become converging, and meet in their foci at IHG, where an image of the object will be formed. If then another convex lens KM, of a shorter focal length, be so placed, as that its focus shall be in IHG, the rays of each pencil, after passing through it, will become nearly parallel, so as to meet upon the retina, and form an enlarged image of the object at RST. If the process of the rays be traced, it will presently be perceived that this image must be inverted. For the pencil that issues from A, has its focus in G, and again in R, on the same side with A. But as there is always one inversion in simple vision, this want of inversion produces just the reverse of the natural appearance. The field of view in this telescope will be large, because all the pencils that can be received on the surface of the lens KM, being converging after passing through it, are thrown into the pupil of the eye, placed in the common intersection of the pencils at P.

Theory of the Astronomical TELESCOPE.—An eye placed near the focus of the eye-glass, of such a telescope, will see objects distinctly, but inverted, and magnified in the ratio of the distance of the focus of the eye-glass to the distance of the focus of the object-glass.

If the sphere of concavity in the eye-glass of the Galilean telescope, be equal to the sphere of convexity in the eye-glass of another telescope, their magnifying power will be the same. The concave glass however being placed between the object-glass and its focus, the Galilean telescope will be shorter than the other, by twice the focal length of the eye-glass. Consequently, if the length of the telescopes be the same, the Galilean will have the greater magnifying power. Vision is also more distinct in these telescopes, owing in part perhaps to there being no intermediate image between the eye and the object. Besides, the eye-glass being very thin in the centre, the rays will be less liable to be distorted by irregularities in the substance of the glass. Whatever be the cause, we can sometimes see Jupiter's satellites very clearly in a Galilean telescope, of 20 inches or 2 feet long, when one of 4 or 5 feet, of the common sort, will hardly make them visible.

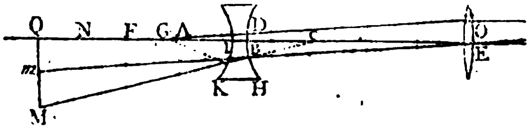
As the astronomical telescope exhibits objects inverted, it serves commodiously enough for observing the stars, as it is not material whether they be seen erect or inverted; but for terrestrial objects it is much less proper, as the inverting often prevents them from being known. But if a plane well-polished metal speculum, of an oval figure, and about an inch long, and inclined to the axis in an angle of 45°, be placed behind the eye-glass; then the eye, conveniently placed, will see the image, hence reflected, in the same magnitude as before, but in an erect situation; and therefore, by the addition of such a speculum, the astronomical telescope is thus rendered fit to observe terrestrial objects.

Since the focus of the glass, convex on both sides, is distant from the glass itself a semidiameter, and that of a plano-convex glass, a diameter; if the object-glass be convex on both sides, the telescope will magnify the semidiameter of the object, in the ratio of the diameter of the eye-glass to the diameter of the object-glass; but if the object-glass be a plano-convex, in the ratio of the semidiameter of the eye-glass to the diameter of the object-glass. And therefore a telescope magnifies more if the object-glass be a plano-convex, than if convex on both sides. And for the same reason, a telescope magnifies more when the eye-glass is convex on both sides, than when it is plano-convex.

A telescope magnifies the more, as the object-glass is a segment of a great sphere, and the eye-glass of a less one. And yet the eye-glass must not be too small in respect of the object-glass; for if it be, it will not refract rays enough to the eye from each point of the object; nor will it separate sufficiently those that come from different points; by which means the vision will be rendered obscure and confused.—Dechales observes, that an object-lens of 2½ feet will require an eye-glass of 1½ tenth of a foot; and an object-glass of 8 or 10 feet, an eye-glass of 4 tenths; in which he is confirmed by Eustachio Divini.

To shorten the Astronomical TELESCOPE; that is, to construct a telescope so, as that, though shorter than the common one, it shall magnify as much.

Having provided a drawing tube, fit in it an object-lens EO which is a segment of a moderate sphere; let the first eye-glass BD be concave on both sides, and so placed in the tube, as that the focus of the object-glass A may be behind it, but nearer to the centre of the concavity G: then will the image be thrown in Q, so as that GA : GI : : AB : QI. Lastly, fit in another object-glass, convex on both sides, and a segment of a smaller sphere, so as that its focus may be in Q.



This telescope will magnify the diameter of the object more than if the object-glass were to represent its image at the same distance EQ; and consequently a shorter telescope, constructed this way, is equivalent to a longer in the common way. See Wolfius Elem. Math. vol. 3, pa. 245.

Sir Isaac Newton furnishes us with another method of constructing the telescope, in his catoptrical or reflecting telescope, the construction of which is given below. See *Achromatic* TELESCOPE.

Aërial TELESCOPE, a kind of astronomical telescope, the lenses of which are used without a tube. In strictness however, the aërial telescope is rather a particular manner of mounting and managing long telescopes for celestial observation in the night-time, by which the trouble of long unwieldy tubes is saved, than a particular kind of telescope; and the contrivance was one of Huygens's. This invention was successfully practised by the inventor himself and others, particularly with us by Mr. Pound and Dr. Bradley, with an object-glass of 123 feet focal distance, and an apparatus belonging to it, made and presented by Huygens to the Royal Society, and described in his Astroscopia Compendiaria Tubi Optici Molimine Liberata, printed at the Hague in 1684.

The principal parts of this telescope may be comprehended from a view of fig. 4, pl. 36, where AB is a long pole, or a mast, or a high tree, &c, in a groove of which slides a piece that carries a small tube LK in which is fixed an object-glass; which tube is connected by a fine line, with another small tube oq, which contains the eye-glass, &c.

Lahire contrived a little machine for managing the object-glass, which is described Mem. de l'Acad. 1715. See Smith's Optics, book 3, chap. 10.

Hartsoeker, who made telescopes of a very considerable focal length, contrived a method of using them without a tube, by fixing them to the top of a tree, a high wall, or the roof of a house. Miscel. Berol. vol. 1, pa. 261.

Huygens's great telescope, with which Saturn's true face, and one of his satellites were first discovered, consists of an object-glass of 12 feet, and an eye-glass of a little more than 3 inches; though he frequently used a telescope of 23 feet long, with two eye-glasses joined together, each $1\frac{1}{2}$ inch diameter; so that the two were equal to one of 3 inches.

The same author observes, that an object-glass of 30 feet requires an eye-glass of $3\frac{3}{10}$ inches; and has given a table of proportions for constructing astronomical telescopes, an abridgement of which is as follows:

Distance of Focus of Object-Glasses.	Diameter of Aperture.	Distance of Focus of Eye-Glass.	Power or Magnitude of Diameter.
Feet.	Inches and Decim.	Inches and Decim.	
1	0·55	0·61	20
2	0·77	0·85	28
3	0·95	1·05	34
4	1·09	1·20	40
5	1·23	1·35	44
6	1·34	1·47	49
7	1·45	1·60	53
8	1·55	1·71	56
9	1·64	1·80	60
10	1·73	1·90	63
15	2·12	2·33	77
20	2·45	2·70	89
25	2·74	3·01	100
30	3·00	3·30	109
40	3·46	3·81	120
50	3·87	4·26	141
60	4·24	4·66	154
70	4·58	5·04	166
80	4·90	5·39	178
90	5·20	5·72	189
100	5·49	6·03	200
120	6·00	6·60	218
140	6·48	7·12	235
160	6·93	7·62	252
180	7·35	8·09	267
200	7·75	8·53	281
220	8·12	8·93	295
240	8·48	9·33	308
260	8·83	9·71	321
280	9·16	10·08	333
300	9·49	10·44	345
400	10·95	12·05	400
500	12·25	13·47	445
600	13·42	14·76	488

Dr. Smith (Rem. pa. 78) observes, that the magnifying powers of this table are not so great as Huygens himself intended, or as the best object-glasses now made will admit of. For the author, in his Astroscopia Compendiaria, mentions an object-glass of 34 feet focal distance, which, in astronomical observations, bore an eye-glass of $2\frac{1}{2}$ inches focal distance, and consequently magnified 163 times. According to this standard, a telescope of 35 feet ought to magnify 166 times, and of 1 foot 28 times; whereas the table allows but 118 times to the former, and but 20 to the latter. Now $\frac{166}{118}$ or $\frac{28}{20} = 1{\cdot}4$; by which if we multiply the numbers in the given column of magnifying powers, we shall gain a new column, showing how much those object-glasses ought to magnify if wrought up to the perfection of this standard.

The new apertures and eye-glasses must also be taken in the same proportions to each other, as the old ones have in the table; or the eye-glasses may be found by dividing the length of each telescope by its magnifying power. And thus a new table may be easily made for this or any other more perfect standard when offered.

The rule for computing this table depends on the following theorem, viz, that in refracting telescopes of different lengths, a given object will appear equally bright and equally distinct, when their linear apertures and the focal distances of their eye-glasses are severally in a subduplicate ratio of their lengths, or focal distances of their object-glasses; and then also the breadth of their apertures will be in the subduplicate ratio of their lengths.

The rule is this: Multiply the number of feet in the focal distance of any proposed object-glass by 3000, and the square-root of the product will give the breadth of its aperture in centesms, or 100th parts of an inch; that is, $\sqrt{3000F}$ is the breadth of the aperture in centesms of an inch, where F is the focal distance of the object-glass in feet. Also, the same breadth of the aperture increased by the 10th part of itself, gives the focal distance of the eye-glass in centesms of an inch. And the magnifying powers are as the breadths of the apertures.

If, in different telescopes, the ratio between the object-glass and eye-glass be the same, the object will be magnified the same in both. Hence some may conclude the making of large telescopes a needless trouble. But it must be remembered, that an eye-glass may be in a less ratio to a greater object-glass than to a smaller: thus, for example, in Huygens's telescope of 25 feet, the eye-glass is 3 inches: now, keeping this proportion in a telescope of 50 feet, the eye-glass should be 6 inches; but the table shows that $4\frac{1}{4}$ are sufficient. Hence, from the same table it appears, that a telescope of 50 feet magnifies in the ratio of 1 to 141; whereas that of 25 feet only magnifies in the ratio of 1 to 100.

Since the distance of the lens is equal to the aggregate of the distances of the foci of the object and eye-glasses; and since the focus of a glass convex on each side is a semidiameter's distance from the lens, and that of a plano-convex at a diameter's distance from the same; the length of a telescope is equal to the aggregate of the semidiameters of the lenses, if the object-glass be convex on both sides; and to the sum of the semidiameter of the eye-glass and the whole diameter of the object-glass, if the object-glass be a plano-convex.

But as the diameter of the eye-glass is very small in respect of that of the object-glass, the length of the telescope is usually estimated from the distance of the object-glass; i.e. from its semidiameter if it be convex on both sides, or its whole diameter if plano-convex. Thus, a telescope is said to be 12 feet, if the semidiameter of the object-glass, convex on both sides, be 12 feet, &c.

Since myopes see near objects best; for them, the eye-glass is to be removed nearer to the object-glass, that the rays refracted through it may be the more diverging.

To take in the larger field at one view, some make use of two eye-glasses, the foremost of which is a segment of a larger sphere than that behind; to this it must be added, that if two lenses be joined immediately together, so as the one may touch the other, the focus is removed to double the distance which that of one of them would be at.

Land TELESCOPE, or *Day* TELESCOPE, is one adapted for viewing objects in the day-time, on or about the earth. This contains more than two lenses, usually it has a convex object-glass, and three convex eye-glasses; exhibiting objects erect, yet different from that of Galileo.

In this telescope, after the rays have passed the first eye-glass HI (fig. 3, pl. 38), as in the former construction, instead of being there received by the eye, they pass on to another equally convex lens, situated at twice its focal distance from the other, so that the rays of each pencil, being parallel in that whole interval, those pencils cross one another in the common focus, and the rays constituting them are transmitted parallel to the second eye-glass LM; after which, the rays of each pencil converge to other foci at NO, where a second image of the object is formed, but inverted with respect to the former image in EF. This image then being viewed by a third eye-glass QR, is painted upon the retina at XYZ, exactly as before, only in a contrary position.

Father Reita was the author of this construction; which is effected by fitting in at one end of a tube an object-glass, which is either convex on both sides, or plano-convex, and a segment of a large sphere; to this add three eye-glasses, all convex on both sides, and segments of equal spheres; disposing them in such a manner as that the distance between any two may be the aggregate of the distances of their foci. Then will an eye applied to the last lens, at the distance of its focus, see objects very distinctly, erect, and magnified in the ratio of the distance of the focus of one eye-glass, to the distance of the focus of the object-glass.

Hence, 1. An astronomical telescope is easily converted into a land telescope, by using three eye-glasses for one; and the land telescope, on the contrary, into an astronomical one, by taking away two eye-glasses, the faculty of magnifying still remaining the same.

2. Since the distance of the eye-glasses is very small, the length of the telescope is much the same as if you only used one.

3. The length of the telescope is found by adding five times the semidiameter of the eye-glasses, to the diameter of the object-glass when this is a plano-convex, or to its semidiameter when convex on both sides.

Huygens first observed, both in the astronomical and land telescope, that it contributes considerably to the perfection of the instrument, to have a ring of wood or metal, with an aperture, a little less than the breadth of the eye-glass, fixed in the place where the image is found to radiate upon the lens next the eye; by means of which, the colours, which are apt to disturb the clearness and distinctness of the object, are prevented, and the whole compass taken in at one view, perfectly defined.

Some make land telescopes of three lenses, which yet represent objects erect, and magnified as much as the former. But such telescopes are subject to very great inconveniences, both as the objects in them are tinged with false colours, and as they are distorted about the margin.

Others again use five lenses, and even more; but as some parts of the rays are intercepted in passing every lens, objects are thus exhibited very dimly.

Telescopes of this kind, longer than 20 feet, will be of hardly any use in observing terrestrial objects, on account of the continual motion of the particles of the atmosphere, which these powerful telescopes render visible, and give a tremulous motion to the objects themselves.

The great length of dioptric telescopes, adapted to any important astronomical purpose, rendered them extremely inconvenient for use; as it was necessary to increase their length in no less a proportion than the duplicate of the increase of their magnifying power: so that, in order to magnify twice as much as before, with the same light and distinctness, the telescope required to be lengthened 4 times; and to magnify thrice as much, 9 times the length, and so on. This unwieldiness of refracting telescopes, possessing any considerable magnifying power, was one cause, why the attention of astronomers, &c, was directed to the discovery and construction of reflecting telescopes. And indeed a refracting telescope, even of 1000 feet focus, supposing it possible to make use of such an instrument, could not be made to magnify with distinctness more than 1000 times; whereas a reflecting telescope, of 9 or 10 feet, will magnify 12 hundred times. The perfection of refracting telescopes, it is well known, is very much limited by the aberration of the rays of light from the geometrical focus: and this arises from two different causes, viz, from the different degrees of refrangibility of light, and from the sphericity, which is not of a proper curvature for collecting the rays in a single point. Till the time of Newton, no optician had imagined that the object-glasses of telescopes were subject to any other error besides that which arose from their spherical figure, and therefore all their efforts were directed to the construction of them, with other kinds of curvature: but that author had no sooner demonstrated the different refrangibility of the rays of light, than he discovered in this circumstance a new and a much greater cause of error in telescopes. Thus, since the pencils of each kind of light have their foci in different places, some nearer and some farther from the lens, it is evident that the whole beam cannot be brought into any one point, but that it will be drawn the nearest to a point in the middle place between the focus of the most and least refrangible rays; so that the focus will be a circular space of a considerable diameter. Newton shows that this space is about the 55th part of the aperture of the telescope, and that the focus of the most refrangible rays is nearer to the object-glass than the focus of the least refrangible ones, by about the 27½ part of the distance between the object-glass, and the focus of the mean refrangible rays. But he says, that if the rays flow from a lucid point, as far from the lens on one side as their foci are on the other, the focus of the most refrangible rays will be nearer to the lens than that of the least refrangible, by more than the 14th part of the whole distance. Hence, he concludes, that if all the rays of light were equally refrangible, the error in telescopes, arising from the sphericity of the glass, would be many hundred times less than it now is; because the error arising from the spherical form of the glass, is to that arising from the different refrangibility of the rays of light, as 1 to 5449. See ABERRATION.

Upon the whole he observes, that it is a wonder that telescopes represent objects so distinctly as they do. The reason of which is, that the dispersed rays are not scattered uniformly over all the circular space above-mentioned, but are infinitely more dense in the centre than in any other part of the circle; and that in the way from the centre to the circumference they grow continually rarer and rarer, till at the circumference they become infinitely rare: for which reason, these dispersed rays are not copious enough to be visible, except about the centre of the circle. He also mentions another argument, to prove, that the different refrangibility of the rays of light is the true cause of the imperfection of telescopes. For the dispersions of the rays arising from the spherical figures of object-glasses, are as the cubes of their apertures; and therefore, to cause telescopes of different lengths to magnify with equal distinctness, the apertures of the object-glasses, and the charges or magnifying powers ought to be as the cubes of the square roots of their lengths, which does not answer to experience. But the errors of the rays, arising from the different refrangibility, are as the apertures of the object-glasses; and thence, to make telescopes of different lengths to magnify with equal distinctness, their apertures and charges ought to be as the square roots of their lengths; and this answers to experience.

Were it not for this different refrangibility of the rays, telescopes might be brought to a sufficient degree of perfection, by composing the object-glass of two glasses with water between them. For by this means, the refractions on the concave sides of the glasses will very much correct the errors of the refractions on the convex sides, so far as they arise from their spherical figure: but on account of the different refrangibility of different kinds of rays, Newton did not see any other means of improving telescopes by refraction only, except by increasing their length. Newton's Optics, pa. 73, 83, 89, 3d edition.

This important desideratum in the construction of dioptric telescopes, has been since discovered by the ingenious Mr. Dollond; an account of which is given below.

Achromatic **TELESCOPE**, is a name given to the refracting telescope, invented by Mr. John Dollond, and so contrived as to remedy the aberration arising from colours, or the different refrangibility of the rays of light. See ACHROMATIC.

The principles of Mr. Dollond's discovery and construction, have been already explained under the articles ABERRATION, and ACHROMATIC. The improvement made by Mr. Dollond in his telescopes, by making two object-glasses of crown-glass, and one of flint, which was tried with success when concave eye-glasses were used, was completed by his son Peter Dollond; who, conceiving that the same method might be practised with success with convex eye-glasses, found, after a few trials, that it might be done. Accordingly he finished an object-glass of 5 feet focal length, with an aperture of 3¾ inches, composed of two convex lenses of crown-glass, and one concave of white flint glass. But apprehending afterward that the apertures might be admitted still larger, he completed one of 3½ feet focal length, with the same aperture of 3¾ inches. Philos. Trans. vol. 55, pa. 56.

But besides the obligation we are under to Mr. Dollond, for correcting the aberration of the rays of light in the focus of object-glasses, arising from their different refrangibility, he made another considerable improvement in telescopes, viz, by correcting, in a great measure, both this kind of aberration, and also that which arises from the spherical form of lenses, by an expedient of a very different nature, viz, increasing the number of eye-glasses. If any person, says he, would have the visual angle of a telescope to contain 20 degrees, the extreme pencils of the field must be bent or refracted in an angle of 10 degrees; which, if it be performed by one eye-glass, will cause an aberration from the figure, in proportion to the cube of that angle: but if two glasses be so proportioned and situated, as that the refraction may be equally divided between them, they will each of them produce a refraction equal to half the required angle; and therefore, the aberration being in this case proportional to double the cube of half the angle, will be but a 4th part of that which is in proportion to the cube of the whole angle; because twice the cube of 1 is but ¼ of the cube of 2: so that the aberration from the figure, where two eye-glasses are rightly proportioned, is but a 4th part of what it must unavoidably be, where the whole is performed by a single eye-glass. By the same way of reasoning, when the refraction is divided among three glasses, the aberration will be found to be but the 9th part of what would be produced from a single glass; because 3 times the cube of 1 is but the 9th part of the cube of 3. Whence it appears, that by increasing the number of eye-glasses, the indistinctness, near the borders of the field of a telescope, may be very much diminished, though not entirely taken away.

The method of correcting the errors arising from the different refrangibility of light, is of a different consideration from the former: for, whereas the errors from the figure can only be diminished in a certain proportion to the number of glasses, in this they may be entirely corrected, by the addition of only one glass; as we find in the astronomical telescope, that two eye-glasses, rightly proportioned, will cause the edges of objects to appear free from colours quite to the borders of the field. Also, in the day telescope, where no more than two eye glasses are absolutely necessary for erecting the object, we find, by the addition of a third rightly situated, that the colours, which would otherwise confuse the image, are entirely removed: but this must be understood with some limitation; for though the different colours, which the extreme pencils must necessarily be divided into by the edges of the eye-glasses, may in this manner be brought to the eye in a direction parallel to each other, so as, by its humours, to be converged to a point in the retina, yet if the glasses exceed a certain length, the colours may be spread too wide to be capable of being admitted through the pupil or aperture of the eye; which is the reason that, in long telescopes, constructed in the common way, with three eye-glasses, the field is always very much contracted.

These considerations first set Mr. Dollond on contriving how to enlarge the field, by increasing the number of eye-glasses, without lessening the distinctness or brightness of the image: and though others had laboured at the same work before, yet observing that the five-glass telescopes, sold in the shops, would admit of further improvement, he endeavoured to construct one with the same number of glasses in a better manner; which so far answered his expectations, as to be allowed by the best judges to be a considerable improvement on the former. Encouraged by this success, he resolved to try if he could not make some further enlargement of the field, by the addition of another glass, and by placing and proportioning the glasses in such a manner, as to correct the aberrations as much as possible, without any detriment to the distinctness: and at last he obtained as large a field as is convenient or necessary, and that even in the longest telescopes that can be made. These telescopes, with 6 glasses, having been well received both at home and abroad, the author has settled the date of the invention in a letter addressed to Mr. Short, and read at the Royal Society, March 1, 1753. Philos. Trans. vol. 48, art. 14.

Of the achromatic telescopes, invented by Mr. Dollond, there are several different sizes, from one foot to 8 feet in length, made and sold by his sons P. and J. Dollond. In the 17-inch improved achromatic telescope, the object-glass is composed of three glasses, viz, two convex of crown-glass, and one concave of white flint glass: the focal distance of this combined object-glass is about 17 inches, and the diameter of the aperture 2 inches. There are 4 eye-glasses contained in the tube, to be used for land objects; the magnifying power with these is near 50 times; and they are adjusted to different sights, and to different distances of the object, by turning a finger screw at the end of the outer tube. There is another tube, containing two eye-glasses that magnify about 70 times, for astronomical purposes. The telescope may be directed to any object by turning two screws in the stand on which it is fixed, the one giving a vertical motion, and the other a horizontal one. The stand may be inclosed in the inside of the brass tube.

The object-glass of the 2½ and 3½ feet telescopes is composed of two glasses, one convex of crown glass, and the other concave of white flint glass; and the diameters of their apertures are 2 inches and 2¼ inches. Each of them is furnished with two tubes; one for land objects, containing four eye-glasses, and another with two eye-glasses for astronomical uses. They are adjusted by buttons on the outside of the wooden tube; and the vertical and horizontal motions are given by joints in the stands. The magnifying power of the least of these telescopes, with the eye-glass for land objects, is near 50 times, and with those for astronomical purposes, 80 times; and that of the greatest for land objects is near 70 times, but for astronomical observations 80 and 130 times; for this has two tubes, either of which may be used as occasion requires. This telescope is also moved by a screw and rackwork, and the screw is turned by means of a Hook's joint.

These opticians also construct an achromatic pocket perspective glass, or Galilean telescope; so contrived, that all the different parts are put together and contained in one piece 4½ inches long. This small telescope is furnished with 4 concave eye-glasses, the magnifying powers of which are 6, 12, 18, and 28 times. With the greatest power of this telescope, the satellites of Jupiter and the ring of Saturn may be easily seen. They have also contrived an achromatic telescope, the sliding tubes of which are made of very thin brass, which pass through springs or tubes; the outside tube being either of mahogany or brass. These telescopes, which from their convenience for gentlemen in the army are called military telescopes, have 4 convex eye-glasses, whose surfaces and focal lengths are so proportioned, as to render the field of view very large. They are of 4 different lengths and sizes, usually called one foot, 2, 3, and 4 feet: the first is 14 inches when in use, and 5 inches when shut up, having the aperture of the object-glass $1\frac{1}{10}$ inch, and magnifying 22 times: the second 28 inches for use, 9 inches shut up, the aperture $1\frac{6}{10}$ inch, and magnifying 35 times; the third 40 inches, and 10 inches shut, with the aperture 2 inches, and magnifying 45 times; and the fourth 52 inches, and 14 inches shut, with the aperture 2¼ inches, and magnifying 55 times.

Euler, who, in a memoir of the Academy of Berlin for the year 1757, pa. 323, had calculated the effects of all possible combinations of lenses in telescopes and microscopes, published another long memoir on the subject of these telescopes, showing with precision of what advantages they are naturally capable. See Miscel. Taurin. vol. 3, part 2, pa. 92.

Mr. Caleb Smith, having paid much attention to the subject of shortening and improving telescopes, thought he had found it possible to rectify the errors which arise from the different degrees of refrangibility, on the principle that the sines of refraction of rays differently refrangible, are to one another in a given proportion, when their sines of incidence are equal; and the method he proposed for this purpose, was to make the specula of glass, instead of metal, the two surfaces having different degrees of concavity. But it does not appear that this scheme was ever carried into practice. See Philos. Trans. No. 456, pa. 326.

The ingenious Mr. Ramsden has lately described a new construction of eye-glasses for such telescopes as may be applied to mathematical instruments. The construction which he proposes, is that of two plano-convex lenses, both of them placed between the eye and the observed image formed by the object-glass of the instrument, and thus correcting not only the aberration arising from the spherical figure of the lenses, but also that arising from the different refrangibility of light. For a more particular account of this construction, its principle, and its effects, see Philos. Trans. vol. 73, art. 5.

A construction, similar at least in its principle to that above, is ascribed, in the Synopsis Optica Honorati Fabri, to Eustachio Divini, who placed two equal narrow plano-convex lenses, instead of one eye lens, to his telescopes, which touched at their vertices; the focus of the object-glass coinciding with the centre of the plano-convex lens next it. And this, it is said, was done at once both to make the rays that come parallel from the object fall parallel upon the eye, to exclude the colours of the rainbow from it, to augment the angle of sight, the field of view, the brightness of the object, &c. This was also known to Huygens, who sometimes made use of the same construction, and gives the theory of it in his Dioptrics. See Hugenii Opera Varia, vol. 4, ed. 1728.

Telescope, *Reflecting*, or *Catoptric*, or *Catadioptric*, is a telescope which, instead of lenses, consists chiefly of mirrors, and exhibits remote objects by reflection instead of refraction.

A brief account of the history of the invention of this important and useful telescope, is as follows. The ingenious Mr. James Gregory, of Aberdeen, has been commonly considered as the first inventor of this telescope.—But it seems the first thought of a reflector had been suggested by Mersenne, about 20 years before the date of Gregory's invention: a hint to this purpose occurs in the 7th proposition of his Catoptrics, which was printed in 1651: and it appears from the 3d and 29th letters of Descartes, in vol. 2 of his Letters, which it is said were written in 1639, though they were not published till the year 1666, that Mersenne proposed a telescope with specula to Descartes in that correspondence; though indeed in a manner so very unsatisfactory, that Descartes, who had given particular attention to the improvement of the telescope, was so far from approving the proposal, that he endeavoured to convince Mersenne of its fallacy. This point has been largely discussed by Le Roi in the Encyclopedia, art. Telescope, and by Montucla in his Hist. des Mathem. tom. 2, pa. 643.

Whether Gregory had seen Mersenne's treatise on optics and catoptrics, and whether he availed himself of the hint there suggested, or not, perhaps cannot now be determined. He was led however to the invention by seeking to correct two imperfections in the common telescope: the first of these was its too great length, which made it troublesome to manage; and the second was the incorrectness of the image. It had been already demonstrated, that a pencil of rays could not be collected in a single point by a spherical lens; and also, that the image transmitted by such a lens would be in some degree incurvated. These inconveniencies he thought might be obviated by substituting for the object-glass a metallic speculum, of a parabolical figure, to receive the image, and to reflect it towards a small speculum of the same metal; this again was to return the image to an eye-glass placed behind the great speculum, which was, for that purpose, to be perforated in its centre. This construction he published in 1663, in his Optica Promota. But as Gregory, according to his own account, possessed no mechanical skill, and could not find a workman capable of realizing his invention, after some fruitless trials, he was obliged to give up the thoughts of bringing telescopes of this kind into use.

Sir Isaac Newton however interposed, to save this excellent invention from perishing, and to bring it forward to maturity. Having applied himself to the improvement of the telescope, and imagining that Gregory's specula were neither very necessary, nor likely to be executed, he began with prosecuting the views of Descartes, who aimed at making a more perfect image of an object, by grinding lenses, not to the figure of a sphere, but to that formed from one of the conic sections. But, in the year 1666, having discovered the different refrangibility of the rays of light, and finding that the errors of telescopes, arising from that cause alone, were much more considerable than such as were occasioned by the spherical figure of lenses, he was constrained to turn his thoughts to reflectors. The plague however interrupted his progress in this business; so that it was towards the end of 1668, or in the beginning of 1669, when, despairing of perfecting telescopes by means of refracted light, and recurring to the construction of reflectors, he set about making his own specula, and early in the year 1672 completed two small reflecting telescopes. In these he ground the large speculum into a spherical concave, being unable to accomplish the parabolic form proposed by Gregory; but though he then despaired of performing that work by geometrical rules, yet (as he writes in a letter that accompanied one of these instruments, which he presented to the Royal Society) he doubted not but that the thing might in some measure be accomplished by mechanical devices. With a perseverance equal to his ingenuity, he, in a great measure, overcame another difficulty, which was to find a metallic substance that would be of a proper hardness, have the fewest pores, and receive the smoothest polish: this difficulty he deemed almost insurmountable, when he considered that every irregularity in a reflecting surface would make the rays of light deviate 5 or 6 times more out of their due course, than the like irregularities in a refracting surface. After repeated trials, he at last found a composition that answered in some degree, leaving it to those who should come after him to find a better. These difficulties have accordingly been since obviated by other artists, particularly by Dr. Mudge, the rev. Mr. Edwards, and Dr. Herschel, &c. Newton having succeeded so far, he communicated to the Royal Society a full and satisfactory account of the construction and performance of his telescope. The Society, by their secretary Mr. Oldenburg, transmitted an account of the discovery to Mr. Huygens, celebrated as a distinguished improver of the refractor; who not only replied to the Society in terms expressing his high approbation of the invention, but drew up a favourable account of the new telescope, which he caused to be published in the Journal des Sçavans of the year 1672, and by this mode of communication it was soon known over Europe. See Huygenii Opera Varia, tom. 4.

Notwithstanding the excellence and utility of this contrivance, and the honourable manner in which it was announced to the world, it seems to have been greatly neglected for nearly half a century. Indeed when Newton had published an account of his telescopes in the Philos. Trans. M. Cassegrain, a Frenchman, in the Journal des Sçavans of 1672, claimed the honour of a similar invention; and said, that, before he heard of Newton's improvement, he had hit upon a better construction, by using a small convex mirror instead of the reflecting prism. This telescope, which was the Gregorian one disguised, the large mirror being perforated, and which it is said was never executed by the author, is much shorter than the Newtonian; and the convex mirror, by dispersing the rays, serves greatly to increase the image made by the large concave mirror.

Newton made many objections to Cassegrain's construction, but several of them equally affect that of Gregory, which has been found to answer remarkably well in the hands of good artists.

Dr. Smith took the pains to make many calculations of the magnifying power, both of Newton's and Cassegrain's telescopes, in order to their further improvement, which may be seen in his Optics, Rem. p. 97.

Mr. Short, it is also said, made several telescopes on the plan of Cassegrain.

Dr. Hooke constructed a reflecting telescope (mentioned by Dr. Birch in his Hist. of the Royal Soc. vol. 3, pa. 122) in which the great mirror was perforated, so that the spectator looked directly towards the object, and it was produced before the Royal Society in 1674. On this occasion it was said that this construction was first proposed by Mersenne, and afterwards repeated by Gregory, but that it had never been actually executed before it was done by Hooke. A description of this instrument may be seen in Hooke's Experiments, by Derham, pa. 269.

The Society also made an unsuccessful attempt, by employing an artificer to imitate the Newtonian construction; however, about half a century after the invention of Newton, a reflecting telescope was produced to the world, of the Newtonian construction, which the venerable author, ere yet he had finished his very distinguished course, had the satisfaction to find executed in such a manner, as left no room to fear that the invention would longer continue in obscurity. This effectual service to science was accomplished by Mr. John Hadley, who, in the year 1723, presented to the Royal Society a telescope, which he had constructed on Newton's plan. The two telescopes which Newton had made, were but 6 inches long, were held in the hand for viewing objects, and in power were compared to a 6-feet refractor: but the radius of the sphere, to which the principal speculum of Hadley's was ground, was 10 feet 5¼ inches, and consequently its focal length was 62⅝ inches. In the Philos. Trans. Abr. vol. 6, pa. 646, 664, may be seen a drawing and description of this telescope, and also of a very ingenious but complex apparatus, by which it was managed. One of these telescopes, in which the focal length of the large mirror was not quite 5¼ feet, was compared with the celebrated Huygenian telescope, which had the focal length of its object-glass 123 feet; and it was found that the former would bear such a charge, as to make it magnify the object as many times as the latter with its due charge; and that it represented objects as distinctly, though not altogether so clear and bright. With this reflecting telescope might be seen whatever had been hitherto discovered by that of Huygens, particularly the transits of Jupiter's satellites, and their shades over the disk of Jupiter, the black list in Saturn's ring, and the edge of the shade of Saturn cast upon his ring. Five satellites of Saturn were also observed with this telescope, and it afforded other observations on Jupiter and Saturn, which confirmed the good opinion which had been conceived of it by Pound and Bradley.

Mr. Hadley, after finishing two telescopes of the Newtonian construction, applied himself to make them in the Gregorian form, in which the large mirror is perforated. This scheme he completed in the year 1726.

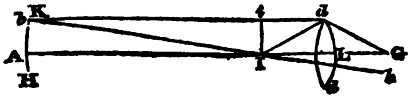
Dr. Smith prefers the Newtonian construction to that of Gregory; but if long experience be admitted as a final judge in such matters, the superiority must be adjudged to the latter; as it is now, and has been for many years past, the only instrument in request.

Mr. Hadley spared no pains, after having completed his construction, to instruct Mr. Molyneux and Dr. Bradley; and when these gentlemen had made a good proficiency in the art, being desirous that these telescopes should become more public, they liberally communicated to some of the chief instrument-makers of London, the knowledge they had acquired from him: and thus, as it is reasonable to imagine, reflectors were completed by other and better methods than even those in which they had been instructed. Mr. James Short in particular signalized himself as early as the year 1734, by performances of this kind. He at first made his specula of glass; but finding that the light reflected from the best glass specula was much less than the light reflected from metallic ones, and that glass was very liable to change its form by its own weight, he applied himself to improve metallic specula; and, by giving particular attention to their curvature, he was able to give them greater apertures than other workmen could do; and by a more accurate adjustment of the specula, &c, he greatly improved the whole instrument. By some which he made, in which the larger mirror was 15 inches focal distance, he and some other persons were able to read in the Philos. Trans. at the distance of 500 feet; and they several times saw the five satellites of Saturn together, which greatly surprised Mr. Maclaurin, who gave this account of it, till he found that Cassini had sometimes seen them all with a 17 feet refractor. Short's telescopes were all of the Gregorian construction. It is supposed that he discovered a method of giving the parabolic figure to his great speculum; a degree of perfection which Gregory and Newton despaired of attaining, and which Hadley it seems had never attempted in either of his telescopes. However, the secret of working that configuration, whatever it was, it seems died with that ingenious artist; though lately in some degree discovered by Dr. Mudge and others.

On the history of reflecting telescopes, see Dr. David Gregory's Elem. of Catopt. and Dioptr. Appendix by Desaguliers: Smith's Optics, book 3, c. 2, Rem. on art. 489: and Sir John Pringle's excellent Discourse on the Invention &c of the Reflecting Telescope.

Construction of the Reflecting Telescope of the Newtonian form.—Let ABCD (fig. 1, pl. 38) be a large tube, open at AD, and closed at BC, and its length at least equal to the distance of the focus from the metallic spherical concave speculum GH placed at the end BC. The rays EG, FH, &c, proceeding from a remote object PR, intersect one another somewhere before they enter the tube, so that EG and eg are those that come from the lower part of the object, and *fh*FH from its upper part: these rays, after falling on the speculum GH, will be reflected so as to converge and meet in *mn*, where they will form a perfect image of the object. But as this image cannot be seen by the spectator, they are intercepted by a small plane metallic speculum KK, intersecting the axis at an angle of 45°, by which the rays tending to *mn*, will be reflected towards a hole LL in the side of the tube, and the image of the object will be thus formed in *qs*; which image will be less distinct, because some of the rays which would otherwise fall on the concave speculum GH, are intercepted by the plane speculum: it will nevertheless appear pretty distinct, because the aperture AD of the tube, and the speculum GH, are large. In the lateral hole LL is fixed a convex lens, whose focus is at *sq*; and therefore this lens will refract the rays that proceed from any point of the image, so as at their emergence they will appear parallel, and those that proceed from the extreme points *sq*, will converge after refraction, and form an angle at *o*, where the eye is placed; which will see the image *sq*, as if it were an object, through the lens LL: consequently the object will appear enlarged, inverted, bright, and distinct. In LL may be placed lenses of different convexities, which, by being moved nearer to the image and farther from it, will represent the object more or less magnified, if the surface of the speculum GH be of a figure truly spherical. If, instead of one lens LL, three lenses be disposed in the same manner with the three eye-glasses of the refracting telescope, the object will appear erect, but less distinct than when it is observed with one lens. On account of the position of the eye in this telescope, it is extremely difficult to direct the instrument towards any object: Huygens therefore first thought of adding to it a small refracting telescope, having its axis parallel to that of the reflector: this is called a finder or director. The Newtonian telescope is also furnished with a suitable apparatus for the commodious use of it.

To determine the magnifying power of this telescope, it is to be considered that the plane speculum KK is of no use in this respect: let us then suppose that one ray proceeding from the object coincides with the axis GLIA of



the lens and speculum : let *bb* be another ray proceeding from the lower extremity of the object, and passing through the focus I of the speculum KH; this will be reflected in the direction *bid*, parallel to the axis GLA, and falling on the lens *d*L*d*, will be refracted to G, so that GL will be equal to LI, and *d*G = *d*I. To the naked eye the object would appear under the angle I*bi* = *b*IA; but by means of the telescope it appears under the angle *d*GL = *d*IL = I*di*; and the angle I*di* is to the angle I*bi* as *b*I to I*d*; consequently the apparent magnitude by the telescope, is to that with the naked eye, as the distance of the focus of the speculum from the speculum, to the distance of the focus of the lens from the lens.

Construction of the Gregorian Reflecting Telescope.—Let TYYT (fig. 2, pl. 38) be a brass tube, in which L*ld*D is a metallic concave speculum, perforated in the middle at X; and EF a less concave mirror, so fixed by the arm or strong wire RT, which is moveable by means of a long screw on the outside of the tube, as to be moved nearer to, or farther from the larger speculum L*ld*D; its axis being kept in the same line with that of the great one. Let AB represent a very remote object, from each part of which issue pencils of rays, as *cd*, CD, from A the upper extremity of the object, and IL, *il*, from the lower part B; the rays IL, CD, from the extremities, crossing each other before they enter the tube. These rays, falling upon the larger mirror LD, are reflected from it into the focus KH, where they form an inverted image of the object AB, as in the Newtonian telescope. From this image the rays, issuing as from an object, fall upon the small mirror EF, the centre of which is at *e*, so that after reflection they would meet in their foci at QQ, and there form an erect image. But since an eye at that place could see but a small part of an object, in order to bring rays from more distant parts of it into the pupil, they are intercepted by the plano-convex lens MN, by which means a smaller erect image is formed at PV, which is viewed through the meniscus SS, by an eye at O. This meniscus both makes the rays of each pencil parallel, and magnifies the image PV. At the place of this image all the foreign rays are intercepted by the perforated partition ZZ. For the same reason the hole near the eye O is very narrow. When nearer objects are viewed by this telescope, the small speculum EF is removed to a greater distance from the larger LD, so that the second image may be always formed in PV: and this distance is to be adjusted (by means of the screw on the outside of the great tube) according to the form of the eye of the spectator. It is also necessary that the axis of the telescope should pass through the middle of the speculum EF, and its centre, the centre of the speculum LL, and the middle of the hole X, the centres of the lenses MN, SS, and the hole near O. As the hole X in the speculum LL can reflect none of the rays issuing from the object, that part of the image which corresponds to the middle of the object, must appear to the observer more dark and confused than the extreme parts of it. Besides, the speculum EF will also intercept many rays proceeding from the object; and therefore, unless the aperture TT be large, the object must appear in some degree obscure.

The magnifying power of this telescope is estimated in the following manner. Let LD be the larger mirror (fig. 4, pl. 38), having its focus at G, and aperture in A; and FF the small mirror with the focus of parallel rays in I, and the axis of both the specula and lenses MN, SS, be in the right line DIGAOK. Let *bb* be a ray of light coming from the lower extremity of a very distant visible object, passing through the focus G, and falling upon the point *b* of the speculum LD; which, after being reflected from *b* to F in a direction parallel to the axis of the mirror DAK, is reflected by the speculum F so as to pass through the focus I in the direction FIN to N, at the extremity of the lens MN, by which it would have been refracted to K; but by the interposition of another lens SS is brought to O, so that the eye in O sees half the object under the angle TOS. The angle G*b*F, or AG*b*, under which the object is viewed by the naked eye, is to SOT under which it is viewed by the telescope, in the ratio of G*b*F to IF*i* = *n*IN, of *n*IN to NK*n*, and of NK*n* to SOT.

But G*b*F : IF*i* : : DI : GA,
and *n*IN : *n*KN : : *n*K : *n*I,
and *n*KN : SOT : : TO : TK;
theref. G*b*F : SOT : : DI × *n*K × TO : GA × *n*I × TK.

Musschenbroek's Introd. vol. 2, p. 819.

In reflecting telescopes of different lengths, a given object will appear equally bright and equally distinct, when their linear apertures, and also their linear breadths, are as the 4th roots of the cubes of their lengths; and consequently when the focal distances of their eye-glasses are also as the 4th roots of their lengths. See the demonstration of this proposition in Smith's Optics, art. 361.

Hence he has deduced a rule, by which he has computed the following table for telescopes of different lengths, taking, for a standard, the middle eye-glass and aperture of Hadley's Reflecting telescope, described in Philos. Trans. No. 376 and 378: the focal distances and linear apertures being given in 1000th parts of an inch.

Table for Telescopes of different Lengths.

Length of the Telescope, or Focal Distance of the Concave.	Focal Distance of the Eye-Glass.	Linear amplifying or magnifying Power.	Linear Aperture of the Concave Metal.
Feet.	Inches.	......	Inches.
½	0·167	36	0·864
1	0·199	60	1·440
2	0·236	102	2·448
3	0·261	138	3·312
4	0·281	171	4·104
5	0·297	202	4·843
6	0·311	232	5·568
7	0·323	260	6·240
8	0·334	287	6·888
9	0·344	314	7·536
10	0·353	340	8·160
11	0·362	365	8·760
12	0·367	390	9·360
13	0·377	414	9·936
14	0·384	437	10·488
15	0·391	460	11·040
16	0·397	483	11·592
17	0·403	506	12·143

Mr. Hadley's telescope, above-mentioned, magnified 228 or 230 times; but we are informed that an object-metal of 3¼ feet focal distance was wrought by Mr. Hauksbee to so great a perfection, as to magnify 226 times, and therefore it was scarcely inferior to Hadley's of 5¼ feet. If Hauksbee's telescope be taken for a new standard, it follows that a speculum of one foot focal distance ought to magnify 93 times, whereas the above table allows it but 60. Now $\frac{93}{60} = 1·55$, and the given column of magnifying powers multiplied by this number, gives a new column, showing how much the object-metals ought to magnify if wrought up to the perfection of Hauksbee's. And thus a new table may be easily made for this or any other more perfect standard, taking also the new eye-glasses and apertures in the same ratio to one another as the old ones have in this table. Smith's Optics, Rem. pa. 79.

The magnifying power of any telescope may be easily found by experiment, viz, by looking with one eye through the telescope upon an object of known dimensions, and at a given distance, and throwing the image upon another object seen with the naked eye. Dr. Smith has given a particular account of the process, Rem. pa. 79.

But the easiest method of all, is to measure the diameter of the aperture of the object-glass, and that of the little image of it, which is formed at the place of the eye. For the proportion between these gives the ratio of the magnifying power, provided no part of the original pencil be intercepted by the bad construction of the telescope. For in all cases the magnifying power of telescopes, or microscopes, is measured by the proportion of the diameter of the original pencil, to that of the pencil which enters the eye. Priestley's Hist. of Light, pa. 747.

The most considerable, and indeed truly astonishing magnifying powers, that have ever been used, are those of Dr. Herschel's reflecting telescopes. Some account of these, and of the discoveries made by them, has been already introduced under the article Star; for his method of ascertaining them, see Philos. Trans. vol. 72, pa. 173 &c. See also several of the other late volumes of the Philos. Trans. Likewise vol. 17, pa. 593, of my Abridg. of the Philos. Trans. for a description of Herschel's 40-foot reflecting telescope, with an engraved representation of all its machinery; see also plate xv of this Dictionary.

Dr. Herschel observes, that though opticians have proved, that two eye-glasses will give a more correct image than one, he has always (from experience) persisted in refusing the assistance of a second glass, which is sure to introduce errors greater than those he would correct. "Let us resign," says he, "the double eye-glass to those who view objects merely for entertainment, and who must have an exorbitant field of view. To a philosopher, this is an unpardonable indulgence. I have tried both the single and double eye-glass of equal powers, and always found that the single eye-glass had much the superiority in point of light and distinctness. With the double eye-glass I could not see the belts in Saturn, which I very plainly saw with the single one. I would however except all those cases where a large field is absolutely necessary, and where power joined to distinctness is not the sole object of our view." Philos. Trans. vol. 72, pa. 95.

Meridian TELESCOPE, is one that is fixed at right angles to an axis, and turned about it in the plane of the meridian; and is otherwise called a Transit Instrument.—The common use of this is to correct the motion of a clock or watch, by daily observing the exact time when the sun or a star comes to the meridian. It serves also for a variety of other uses. The transverse axis is placed horizontal by a spirit level. For the farther description and method of fixing this instrument by means of its levels &c, see Smith's Optics, pa. 321. See also TRANSIT *Instrument*.

TELESCOPICAL *Stars*, are such as are not visible to the naked eye, being only discernible by means of a telescope.—All stars less than those of the 6th magnitude, are telescopic to an ordinary eye.

TEMPERAMENT, in Music, is defined by Rousseau to be an operation which, by means of a slight alteration in the intervals, causes the difference between two contiguous sounds to disappear, makes each of these sounds seem identical with the other, which, without offending the ear, may still preserve their respective intervals or distances one from the other. By this operation the scale is rendered more simple, and the number of sounds which would otherwise be necessary retrenched. Had not the scale been thus modified, instead of 12 sounds alone which are contained in the octave, more than 60 would be indispensably required to form what is properly called Modulation in every tone.

It is proved by computation, that on the organ, the harpsichord, and every other instrument with keys, there is not, and there scarcely can be, any chords properly in tune, save the octave alone. The cause is this, that though 3 thirds major, or 4 thirds minor, ought to form a just octave, those are found to surpass, and these not to reach it.

TEMPERATURE, the degree or quantity of heat in any substance or place; as, in the atmosphere, in a climate, in the earth, in the ocean, &c. In all these cases, the heat is greater in the lower latitudes, than in the higher: being greatest at the equator, and gradually less all the way to each pole, where it is least.

TEMPERATURE *of the Atmosphere*, is greatest at the bottom, next the earth's surface, where it is warmed by the contact of the earth, and by the reflection of the sun's heat from it. From hence, gradually in ascending up in the atmosphere, the heat is always the less, till, in the upper regions, there is perpetual cold or frost, and that more or less, at equal elevations, in all latitudes. In so much that, at a certain elevation above the sea, peculiar to each latitude, the mountains exhibit perpetual frost or snow, if not higher than where vapours ascend in the atmosphere; which appearance of ice or snow terminates, however cold, at the highest point of the ascent of vapours. This latter point may be termed the upper altitude termination, as the former is the lower. And the heights of these two terms, for the different latitudes, have been observed as they are here exhibited in the following table; the latitude for every 5° being placed in the first column, and the altitude, in feet, of the lower and upper terms, in the 2d and 3d columns.

Latitude.	Alt. lower Termin.	Alt. upper Termin.	Latitude.	Alt. lower Termin.	Alt. upper Termin.
0°	15573	28000	45°	7658	13730
5	15457	27784	50	6260	11253
10	15067	27084	55	4912	8830
15	14498	26061	60	3684	6546
20	13719	24661	65	2516	4676
25	13020	23423	70	1557	2803
30	11592	20838	75	748	1346
35	10664	19169	80	120	207
40	9016	16207			

By dividing each number in the 2d column, by its corresponding number in the 3d, the quotients generally come out ·556, or nearly $\frac{5}{9}$, excepting some very few irregular numbers, which must have been errors in the observations. I find also that the numbers in both these columns are very nearly proportional to the squares of the cosines of the latitudes, excepting a few of the numbers belonging to the very high latitudes; and indeed that those in the 3d column ought to be expressed by this formula $28000c^2$, where c denotes the cosine of the latitude, to radius 1. Hence $\frac{5}{9}$ of $28000c^2$, or $\frac{140000}{9}c^2$ will give the proper numbers for the 2d column. And hence the irregular numbers, in both the columns, may be corrected.

Temperature *of the Climate*, is that of the air which we breathe, at the earth's surface, or the bottom of the atmosphere. This temperature is higher as the place is nearer the equator, and as the time or the season is nearer the warmest part of the year, near the summer equinox.

At London, by a mean of the observations, for each month, made at the Royal Society, from the year 1772 to 1780, it appears that the mean annual temperature there, is 51°·9, or in whole numbers 52°; and the monthly temperature is as follows:

January	35°·9	July	65°·3
February	42 ·3	August	65 ·8
March	46 ·4	September	59 ·6
April	49 ·9	October	52 ·8
May	56 ·6	November	44 ·4
June	63 ·2	December	41 ·0

The greatest usual cold is 20°, and happens in January; the greatest usual heat is 81°, and happens generally in July.

At Petersburg, lat. 59° 56′, the mean annual temperature is 38°·8. The greatest cold observed was that at which mercury freezes, that is, − 39°, or 39 below 0; but the greatest mean degree of cold for several years was − 25°; and the greatest summer heat, on a mean, is 79°, though once it amounted to 94°.

With respect to different latitudes, from theory it would seem that the heat must vary with some function of the square of the sine or of the cosine of the latitude. Accordingly, the rule given by Tobias Mayer of Gottingen, for the mean annual temperature, is $84 - 53s^2$, where s is the sine of the latitude; or which may be otherwise expressed by $31 + 53c^2$, where c denotes the cosine of the latitude, to the radius 1. And by this rule is computed the following table.

Lat.	Temp.	Lat.	Temp.	Lat.	Temp.
0°	84·0	35°	66·6	70°	37·2
5	83·6	40	62·0	75	34.5
10	82·3	45	57·5	80	32 6
15	80·4	50	52·9	85	31·4
20	77·8	55	48·4	90	31·0
25	74·5	60	44·3		
30	70·7	65	40·4		

Temperature *of the Earth*, is various at different depths below the surface, to a certain depth or limit, where it is stationary, being at about 80 or 90 feet deep. It is found by observation, that the same degree of heat occurs in all subterraneous places at the same depth, varying a little at different depths, but is never less than 36° of Fahrenheit's thermometer. At 80 or 90 feet, and sometimes much less, the temperature varies very little, and generally approaches to the mean annual heat. Thus, the temperature of springs is nearly the same as the mean annual heat, and varies very little in different seasons. The temperature of the cave at the observatory of Paris, of about 90 feet deep, is about 53¼ degrees: varying only about half a degree in very cold years. The internal heat of the earth in our climate is always above 40°, and therefore the snow generally begins to melt first at the bottom. Mr. Boyle kept a thermometer for a year, in a cave 80 feet deep, and found the liquid remain stationary all the time. Dr. Withering made a similar experiment on a well 84 feet deep, at Edgbaston, near Birmingham, the temperature of which was found to be 49° in every month of the year 1798. A remarkable circumstance however is observable in experiments made on pits or wells of a moderate depth. Mr. Gough kept a monthly account of the temperature of a well, for the years 1795 and 1798, of only 20 feet deep, and he found the annual variation was under 4°. And it is remarkable that the temperature of the earth, at the depth of 20 feet from the surface, is at the highest in October, when a thermometer in the atmosphere makes the monthly mean coincide with that of the year: on the contrary, the subterranean temperature does not arrive at a minimum before the end of March, 2 or 3 months later than the coldest weather above ground.

Temperature *of the Sea*, like that of the land, is also different at different depths, but at great depths is found to be nearly constant. In winter, when the surface of water is much cooled by contact with the colder air, the deeper and warmer water at the bottom, being specifically lighter, rises and tempers the top; and as the colder water constantly descends during the winter, in the following summer the surface is generally warmer than at any depths; whereas in winter it is colder. As the water in the high latitudes is, by cold, rendered heavier than that in lower warm latitudes; hence occurs a continual current from the poles to the equator, which sometimes carries down large masses of ice, which cool the air to a great extent. The temperatures of land and water differ more in winter than in summer.

The following table exhibits the results of several observations on the temperature of the air, and of the sea at different depths, in several latitudes, and at different seasons of the year.

Latitude.		Time.		Heat of the Air.	Depth of the Sea.	Heat of the Sea.	Heat of the Surface.
					Feet.		
57°	0′n	Jan.	8	46·0	6	40	37
-	-		10	43·6	50	43·6	43·6
55	40 n		20	47	110	51·5	40
39	30 n		28	53	110	59	59
2	55 n	Feb.	25	81	58	81	81
2	50 n		26	83	110	81	84·5
67	0 n	June	20	48·5	4680	26	
78	0 n		30	40·5	708	31	
69	0 n	Aug.	31	59·5	4038	32	
0	0	Sept.	5	75·5	510	66	74
24	0 s		26	72·5	480	70	70
34	4 s	Oct.	11	60·5	600	57	59

TENACITY, in Natural Philosophy, is that quality of bodies by which they sustain a considerable pressure or force without breaking; and is the opposite quality to fragility or brittleneness. Mem. Acad. Berlin, 1745, pa. 47.

TENAILLE, in Fortification, a kind of outwork, consisting of two parallel sides, with a front, having a re-entering angle. In fact, that angle, and the faces which compose it, are the tenaille. The tenaille is of two kinds, simple and double.

Simple or *Single* TENAILLE, is a large outwork, consisting of two faces or sides, including a re-entering angle.

Double, or *Flanked* TENAILLE, is a large outwork, consisting of two simple tenailles, or three saliant and two re-entering angles.

The great defects of tenailles are, that they take up too much room, and on that account are advantageous to the enemy; that the re-entering angle is not defended; the height of the parapet preventing the seeing down into it, so that the enemy can lodge there under cover; and the sides are not sufficiently flanked. For these reasons, tenailles are now mostly excluded out of fortification by the best engineers, and never made but where time does not serve to form a hornwork.

TENAILLE *of the Place*, is the front of the place, comprehended between the points of two neighbouring bastions; including the curtain, the two flanks raised on the curtain, and the two sides of the bastions which face each other. So that the tenaille, in this sense, is the same with what is otherwise called the Face of a fortress.

TENAILLE *of the Ditch*, is a low work raised before the curtain, in the middle of the foss or ditch; the parapet of which is only 2 or 3 feet higher than the level ground of the ravelin.

The use of tenailles in general, is to defend the bottom of the ditch by a grazing fire, and likewise the level ground of the ravelin, which cannot be so conveniently defended from any other place. The first sort do not defend the ditch so well as the others, because they are too oblique a defence; but as they are not subject to be enfiladed, Vauban has generally preferred them in the fortifying of places. Those of the second kind defend the ditch much better than the first, and add a low flank to those of the bastions; but as these flanks are liable to be enfiladed, they have not been much used This defect however might be remedied, by making them so as to be covered by the extremities of the parapets of the opposite ravelins, or by some other work. And the same thing may be said of the third sort as of the second.

The *Ram's-horn* is a curved tenaille, raised in the foss before the flanks, and presenting its convexity to the covered way. This work seems preferable to either of the other tenailles, both on account of its simplicity, and the defence for which it is constructed.

TENAILLONS, in Fortification, are works constructed on each side of the ravelin, much like the lunettes. They differ, as one of the faces of a tenaillon is in the direction of the ravelin, whereas that of the lunette is perpendicular to it.

TENOR, in Music, the first mean or middle part, or that which is the ordinary pitch, or tenor, of the voice, when not either raised to the treble, or lowered to the bass.

TENSION, the state of a thing tight, or stretched. Thus, animals sustain and move themselves by the tension of their muscles and nerves. A chord, or string, gives an acuter or deeper sound, as it is in a greater or less degree of tension, that is, more or less stretched or tightened.

The *Tension* of a cord in *Mechanics*, is the force which acts at one end thereof when the other is fixed, or it is equivalent to that force. Thus, in the case of an equilibrium of forces applied to a physical point; if we consider that point as fixed, the tension of each cord is precisely the force applied at each cord to move the point; but if there be not an equilibrium, as will happen, for example, when two unequal powers act at its extremities; the tension is in this case the least of the two forces; for the tension will evidently be the same, as if one of the extremities were fixed, and the least of the two forces acted solely at the other end.

TERM, in Geometry, is the extreme of any magnitude, or that which bounds and limits its extent. So the terms of a line, are points; of a superficies, lines; of a solid, superficies.

TERMS, of an equation, or of any quantity, in Algebra, are the several names or members of which it is composed, separated from each other by the signs + or −. So, the quantity $ax + 2bc - 3ax^2$, consists of the three terms ax and $2bc$ and $3ax^2$.

In an equation, the terms are the parts which contain the several powers of the same unknown letter or quantity: for if the same unknown quantity be found in several members in the same degree or power, they shall pass but for one term, which is called a compound one, in distinction from a simple or single term. Thus, in the equation $x^3 + \overline{a - 3b} \,.\, x^2 - acx = b^3$, the four terms are x^3 and $(a - 3b)x^2$ and acx and b^3; of which the second term $(a - 3b)x^2$ is compound, and the other three are simple terms.

TERMS, of a Product, or of a Fraction, or of a Ratio, or of a Proportion, &c, are the several quantities employed in forming or composing them. Thus, the terms

of the product ab, are a and b;
of the fraction $\frac{5}{8}$, are 5 and 8;
of the ratio 6 to 7; are 6 and 7;
of the proportion $a : b :: 5 : 9$, are a, b, 5, 9.

TERMS are also used for the several times or seasons of the year in which the public colleges or universities, or courts of law, are open, or sit. Such are the Oxford and Cambridge terms; also the terms for the courts of King's-Bench, Common Pleas, and the Exchequer, which are the high courts of common law. But the high court of Parliament, the Chancery, and inferior courts, do not observe the terms.—The rest of the year, out of term-time, is called *vacation*.

There are four law terms in the year; viz,

Hilary-Term, which, at London, begins the 23d day of January, and ends the 12th of February.

Easter-Term, which begins the 3d Wednesday after Easter-day, and ends on the Monday next after Ascension-day.

Trinity-Term, which begins the Friday next after Trinity-Sunday, and ends the 4th Wednesday after Trinity-Sunday.

Michaelmas-Term, which begins the 6th of November, and ends the 28th of November.

All these terms have also their returns, the days of which are expressed in the following table or synopsis.

Table of the Law Terms, and their Returns.

Term	Begin.	1st Return	2d Return	3d Return	4th Return	5th Return	End
Hilary	January 23	January 20	January 27	February 3	February 9	- - - -	February 12
Easter	3 Wed. af. Eas.	2 Wks. af. East.	3 Wks. af. East.	4 Wks. af. East.	5 Wks. af. East.	Ascension day	Mon. af. Ascens.
Trinity	Fri. af. Trin. S.	Trinity Mond.	1 Wk. af. Trin.	2 Wks. af. Trin.	3 Wks. af. Trin.	- - - -	4th We. af. Tri. S.
Mich.	November 6	November 3	November 12	November 18	November 25	- - - -	November 28

When the beginning or ending of any of these terms happens on a Sunday, it is held on the Monday following.

Oxford **Terms.** These are four; which begin and end as below:

Terms	Begin.	End
Lent Term	January 14	Sat. bef. Palm-Sun.
Easter Term	Wed. af. Low-Sun.	Sat. bef. Whitsun.
Trinity Term	Wed. af. Whitsun.	Sat. after the Act
Michaelmas T.	October 10	December 17.

The act is 1st Monday after the 6th of July.—When the day of the beginning or ending happens on a Sunday, the terms begin or end the day after.

Cambridge-**Terms.** These are three, as below:

Terms	Begin.	End
Lent Term	January 13	Frid. bef. Palm-Sun.
Easter Term	Wed. aft. Low-Sun.	Frid. aft. Commence.
Michaelmas	October 10	December 16.

The commencement is the 1st Tuesday in July.—There is no difference on account of the beginning or ending being Sunday.

Scottish **Terms.** In Scotland, Candlemas term begins January 23d, and ends February the 12th. Whitsuntide-term begins May 25th, and ends June 15th. Lammas-term begins July the 20th, and ends August the 8th. Martinmas-term begins November the 3d, and ends November the 29th.

Irish **Terms.** In Ireland the terms are the same as at London, except Michaelmas-term which begins October the 13th, and adjourns to November the 3d, and thence to the 6th.

TERMINATOR, in Astronomy, a name sometimes given to the circle of illumination, from its property of terminating the boundaries of light and darkness.

TERRA, in Geography. See **Earth.**

Terra-*firma*, in Geography, is sometimes used for a continent, in contradistinction to islands. Thus, Asia, the Indies, and South America, are usually distinguished into terra-firmas and islands.

TERRAQUEOUS, in Geography, an epithet given to our globe or earth, considered as consisting of land and water, which together constitute one mass.

TERRE-**plein**, or **Terre-plain**, in Fortification, the top, platform, or horizontal surface of the rampart, upon which the cannon are placed, and where the defenders perform their office. It is so called, because it lies level, having only a little slope outwardly to counteract the recoil of the cannon. Its breadth is from 24 to 30 feet; being terminated by the parapet on the outer side, and inwardly by the inner talus.

TERRELLA, or little earth, is a magnet turned of a spherical figure, and placed so as that its poles, equator, &c, do exactly correspond with those of the world. It was so first called by Gilbert, as being a just representation of the great magnetic globe we inhabit. Such a terrella, it was supposed, if nicely poised, and hung in a meridian like a globe, would be turned round like the earth in 24 hours by the magnetic particles pervading it; but experience has shown that this is a mistake.

TERRESTRIAL, something relating to the earth. As terrestrial globe, terrestrial line, &c.

TERTIAN; denotes an old measure, containing 84 gallons, so called because it is the 3d part of a tun.

TERTIATE, in Gunnery. To tertiate a great gun, is to examine the thickness of the metal at the muzzle, by which to judge of the strength of the piece, and whether it be sufficiently fortified or not.

TETRACHORD, in Music, called by the moderns a fourth, is a concord or interval of 4 tones.—The tetrachord of the ancients, was a rank of 4 strings, accounting the tetrachord for one tone, as it is often taken in music.

TETRADIAPASON, or *Quadruple Diapason*, is a musical chord, otherwise called a quadruple eighth, or a nine-and-twentieth.

TETRAEDRON, or **Tetrahedron**, in Geometry, is one of the five Platonic or regular bodies or solids, comprehended under 4 equilateral and equal triangles. Or it is a triangular pyramid of 4 equal and equilateral faces. It is demonstrated in geometry, that the side of a tetraedron is to the diameter of its circumscribing sphere, as $\sqrt{2}$ to $\sqrt{3}$; consequently they are incommensurable.

If a denote the linear edge or side of a tetraedron, b its whole superficies, c its solidity, r the radius of its inscribed sphere, and R the radius of its circumscribing sphere; then the general relation among all these is expressed by the following equations, viz,

$$\begin{aligned}
a &= 2r\sqrt{6} &&= \tfrac{2}{3}R\sqrt{6} &&= \sqrt{\tfrac{1}{3}b\sqrt{3}} &&= \sqrt[3]{6c\sqrt{2}}. \\
b &= 24r^2\sqrt{3} &&= \tfrac{8}{3}R^2\sqrt{3} &&= a^2\sqrt{3} &&= 6\sqrt[3]{c^2}\sqrt{3}. \\
c &= 8r^3\sqrt{3} &&= \tfrac{8}{27}R^3\sqrt{3} &&= \tfrac{1}{12}a^3\sqrt{2} &&= \tfrac{1}{36}b\sqrt{2b\sqrt{3}}. \\
R &= 3r &&= \tfrac{1}{4}a\sqrt{6} &&= \tfrac{1}{4}\sqrt{2b\sqrt{3}} &&= \tfrac{3}{2}\sqrt[3]{\tfrac{1}{3}c\sqrt{3}}. \\
r &= \tfrac{1}{3}R &&= \tfrac{1}{12}a\sqrt{6} &&= \tfrac{1}{12}\sqrt{2b\sqrt{3}} &&= \tfrac{1}{2}\sqrt[3]{\tfrac{1}{3}c\sqrt{3}}.
\end{aligned}$$

See my Mensuration, pa. 186 &c, 4th edit. See also the articles **Regular**, and **Bodies.**

TETRAGON, in Geometry, a quadrangle, or a figure having 4 angles. Such as a square, a parallelogram, a rhombus, and a trapezium. It sometimes also means peculiarly a square.

TETRAGONIAS, a meteor, whose head is of a quadrangular figure, and its tail or train is long, thick, and uniform. It does not differ much from the meteor called *Trabs* or beam.

TETRAGONISM, a term which some authors use to express the quadrature of the circle, because the quadrature is the finding a square equal to it.

TETRASPASTON, in Mechanics, a machine in which are 4 pulleys.

TETRASTYLE, in the ancient Architecture, a building, and particularly a temple, with 4 columns in front.

THALES, a celebrated Greek philosopher, and the first of the seven wise men of Greece, was born at Miletum, about 640 years before Christ. After acquiring the usual learning of his own country, he travelled into Egypt and several parts of Asia, to learn astronomy, geometry, mystical divinity, natural knowledge or philosophy, &c. In Egypt he met for some time great favour from the king, Amasis; but he lost it again, by the freedom of his remarks on the conduct of kings, which it is said occasioned his return to his own country, where he communicated the knowledge he had acquired to many disciples, among the principal of whom were Anaximander, Anaximenes, and Pythagoras, and was the author of the Ionian sect of philosophers. He always however lived very retired, and refused the proffered favours of many great men. He was often visited by Solon; and it is said he took great pleasure in the conversation of Thrasybulus, whose excellent wit made him forget that he was tyrant of Miletum.

Laertius, and several other writers, agree that Thales was the father of the Greek philosophy; being the first that made any researches into natural knowledge and mathematics. His doctrine was, that water was the principle of which all the bodies in the universe are composed; that the world was the work of God; and that God sees the most secret thoughts in the heart of man. He observed that, in order to live well, we ought to abstain from what we find fault with in others: that bodily felicity consists in health, and that of the mind in knowledge: that the most ancient of beings is God, because he is uncreated: that nothing is more beautiful than the world, because it is the work of God; nothing more extensive than space, quicker than spirit, stronger than necessity, wiser than time. He used also to observe, that we ought never to say that to any one which may be turned to our prejudice; and that we should live with our friends as with persons that may become our enemies.

In geometry, it has been said, he was a considerable inventor, as well as an improver; particularly in that part concerning triangles. And all the writers agree, that he was the first, even in Egypt, who took the height of the pyramids by the shadow.

His knowledge and improvements in astronomy were very considerable. He divided the celestial sphere into five circles or zones, the arctic and antarctic circles, the two tropical circles, and the equator. He observed the apparent diameter of the sun, which he made equal to half a degree; and formed the constellation of the Little Bear. He also observed the nature and course of eclipses, and calculated them exactly; one in particular, memorably recorded by Herodotus, as it happened on a day of battle between the Medes and Lydians; which, Laertius says, he had foretold to the Ionians. And the same author informs us, that he divided the year into 365 days. Plutarch not only confirms his general knowledge of eclipses, but that his doctrine was, that an eclipse of the sun is occasioned by the intervention of the moon, and that an eclipse of the moon is caused by the intervention of the earth.

His morals were as just, as his mathematics well grounded, and his judgment in civil affairs equal to either. He was very averse to tyranny, and esteemed monarchy little better in any shape.—Diogenes Laertius relates, that, walking to contemplate the stars, he fell into a ditch; on which a good old woman, that attended him, exclaimed, "How canst thou know what is doing in the heavens, when thou seest not what is at thy feet?"—He went to visit Crœsus, who was marching a powerful army into Cappadocia, and enabled him to pass the river Halys without making a bridge. Thales died soon after, at above 90 years of age, it is said, at the Olympic games, where, oppressed with heat, thirst, and a load of years, he, in public view, sunk into the arms of his friends.

Concerning his writings, it remains doubtful whether he left any behind him; at least none have come down to us. Augustine mentions some books of Natural Philosophy; Simplicius, some written on Nautic Astrology; Laertius, two treatises on the Tropics and Equinoxes; and Suidas, a treatise on Meteors, written in verse.

THAMMUZ, in Chronology, the 10th month of the year of the Jews, containing 29 days, and answering to our June.

THEMIS, in Astronomy, a name given by some to the 3d satellite of Jupiter.

THEODOLITE, an instrument much used in surveying, for taking angles, distances, altitudes, &c. This instrument is variously made; different persons having their several ways of contriving it, each attempting to make it more simple and portable, more accurate and expeditious, than others. It usually consists of a brass circle, about a foot diameter, cut in form of fig. 5, pl. 36; having its limb divided into 360 degrees, and each degree subdivided either diagonally, or otherwise, into minutes. Underneath, at *cc*, are fixed two little pillars *bb* (fig. 6), which support an axis, bearing a telescope, for viewing remote objects.

On the centre of the circle moves the index c, which is a circular plate, having a compass in the middle, the meridian line of which answers to the fiducial line *aa*; at *bb* are fixed two pillars to support an axis, bearing a telescope like the former, whose line of collimation answers to the fiducial line *aa*. At each end of either telescope is, or may be, fixed a plain sight, for the viewing of nearer objects.

The ends of the index *aa* are cut circularly, to fit the divisions of the limb B; and when that limb is diagonally divided, the fiducial line at one end of the index shows the degrees and minutes on the limb. It is also furnished with cross spirit levels, for setting the plane of the circle truly horizontal; and a vertical arch, divided into degrees, for taking angles of elevation and depression. The whole instrument is mounted with a ball and socket, upon a three-legged staff.

Many theodolites however have no telescopes, but only four plain sights, two of them fastened on the limb, and two on the ends of the index. Two different ones, mounted on their stand, are represented in fig. 4 and 5, plate 21.

The use of the theodolite is abundantly shown in that of the semicircle, which is only half a theodolite. And the index and compass of the theodolite serve also for a circumferentor, and are used as such. The ingenious Mr. Ramsden made a most excellent theodolite, for the use of the military survey now carrying on in England.

THEODOSIUS, a celebrated mathematician, who flourished in the times of Cicero and Pompey; but the time and place of his death are unknown. This Theodosius, the Tripolite, as mentioned by Suidas, is probably the same with Theodosius the philosopher of Bithynia, who Strabo says excelled in the mathematical sciences, as also his sons; for the same person might have travelled from the one of those places to the other, and spent part of his life in each of them; like as Hipparchus was called by Strabo the Bithynian; but by Ptolemy and others the Rhodian.

Theodosius chiefly cultivated that part of geometry which relates to the doctrine of the sphere, concerning which he published three books. The first of these contains 22 propositions; the second 23; and the third 14; all demonstrated in the pure geometrical manner of the ancients. Ptolemy made great use of these propositions, as well as all succeeding writers. These books were translated by the Arabians, out of the original Greek, into their own language. From the Arabic, the work was again translated into Latin, and printed at Venice. But the Arabic version being very defective, a more complete edition was published in Greek and Latin, at Paris 1558, by John Pena, Regius Professor of Astronomy. And Vitello acquired reputation by translating Theodosius into Latin. This author's works were also commented on and illustrated by Clavius, Heleganius, and Gaurinus, and lastly by Dechales, in his Cursus Mathematicus. Theodosius's Spherics was also translated, and published, by our countryman the learned Dr. Barrow, in the year 1675, illustrated and demonstrated in a new and concise method. By this author's account, Theodosius appears not only to be a great master in this more difficult part of geometry, but the first considerable author of antiquity who has written on that subject. Another edition was published at Oxford 1707 in 8vo. by Jos. Hunt.

Theodosius wrote also concerning the Celestial Houses; also of Days and Nights; copies of which, in Greek, are in the king's library at Paris; of which there was a Latin edition, published by Peter Dasypody, in the year 1572.

THEON, of Alexandria, a celebrated Greek philosopher and mathematician, who flourished in the 4th century, about the year 380, in the time of Theodosius the Great; but the time and manner of his death are unknown. His genius and disposition for the study of philosophy were very early improved by a close application to study; so that he acquired such a proficiency in the sciences, as to render his name venerable in history; and to procure him the honour of being president of the famous Alexandrian school. One of his pupils was the admirable Hypatia, his daughter, who succeeded him in the presidency of the school; a trust, which, like himself, she discharged with the greatest honour and usefulness. [See her life in its place in the first volume of this Dictionary.]

The study of nature led Theon to many just conceptions concerning God, and to many useful reflections in the science of moral philosophy; hence, it is said, he wrote with great accuracy on divine providence. And he seems to have made it his standing rule, to judge the truth of certain principles, or sentiments, from their natural or necessary tendency. Thus, he says, that a full persuasion, that the Deity sees every thing we do, is the strongest incentive to virtue; for he insists, that the most profligate have power to refrain their hands, and hold their tongues, when they think they are observed, or overheard, by some person whom they fear or respect. With how much more reason then, says he, should the apprehension and belief, that God sees all things, restrain men from sin, and constantly excite them to their duty? He also represents this belief, concerning the Deity, as productive of the greatest pleasure imaginable, especially to the virtuous, who might depend with greater confidence on the favour and protection of Providence. For this reason, he recommends nothing so much as meditation on the presence of God: and he recommended it to the civil magistrate, as a restraint on such as were profane and wicked, to have the following inscription written, in large characters, at the corner of every street; GOD SEES THEE, O SINNER.

Theon wrote notes and commentaries on some of the ancient mathematicians. He composed also a book, entitled Progymnasmata, a rhetorical work, written with great judgment and elegance; in which he criticised on the writings of some illustrious orators and historians; pointing out, with great propriety and judgment, their beauties and imperfections; and laying down proper rules for propriety of style. He recommends conciseness of expression, and perspicuity, as the principal ornaments. This book was printed at Basle, in the year 1541; but the best edition is that of Leyden, in 1626, in 8vo.

THEOPHRASTUS, a celebrated Greek philosopher, was the son of Melanthus, and was born at Eretus in Bœotia. He was at first the disciple of Lucippus, then of Plato, and lastly of Aristotle; whom he succeeded in his school, about the 322d year before the Christian era, and taught philosophy at Athens with great applause. He said of an orator without judgment, "that he was a horse without a bridle." He used also to say, "There is nothing so valuable as time, and those who lavish it are the most inexcusable of all prodigals."—He died at about 100 years of age.

Theophrastus wrote many works, the principal of which are the following.—1. An excellent moral treatise entitled, Characters, which, he says in the preface, he composed at 99 years of age. Isaac Casaubon has written learned commentaries on this small treatise. It has been translated from the Greek into French, by Bruyere; and it has also been translated into English.—2. A curious treatise on Plants.—3. A treatise on fossils or stones; of which Dr. Hill has given a good edition, with an English translation, and learned notes, in 8vo.

THEOREM, a proposition which terminates in theory, and which considers the properties of things already made or done. Or, a theorem is a speculative proposition, deduced from several definitions compared together. Thus, if a triangle be compared with a parallelogram standing on the same base, and of the same altitude, and partly from their immediate definitions, and partly from other of their properties already determined, it is inferred that the parallelogram is double the triangle; that proposition is a theorem.

Theorem stands contradistinguished from problem, which denotes something to be done or constructed, as a theorem proposes something to be proved or demonstrated.

There are two things to be chiefly regarded in every theorem, viz, the proposition, and the demonstration. In the first is expressed what agrees to some certain thing, under certain conditions, and what does not. In the latter, the reasons are laid down by which the understanding comes to conceive that it does or does not agree to it. Theorems are of various kinds: as,

Universal THEOREM, is that which extends to any quantity without restriction, universally. As this, that the rectangle or product of the sum and difference of any two quantities, is equal to the difference of their squares.

Particular THEOREM, is that which extends only to a particular quantity. As this, in an equilateral rectilinear triangle, each angle is equal to 60 degrees.

Negative THEOREM, is that which expresses the impossibility of any assertion. As, that the sum of two biquadrate numbers cannot make a square number.

Local THEOREM, is that which relates to a surface. As, that triangles of the same base and altitude are equal.

Plane THEOREM, is that which relates to a surface that is either rectilinear or bounded by the circumference of a circle. As, that all angles in the same segment of a circle are equal.

Solid THEOREM, is that which considers a space terminated by a solid line; that is, by any of the three conic sections. As this, that if a right line cut two asymptotic parabolas, its two parts terminated by them shall be equal.

Reciprocal THEOREM, is one whose converse is true. As, that if a triangle have two sides equal, it has also two angles equal: the converse of which is likewise true, viz, that if the triangle have two angles equal, it has also two sides equal.

THEORY, a doctrine which terminates in the sole speculation or consideration of its object, without any view to the practice or application of it. To be learned in an art, &c, the theory is sufficient; to be a master of it, both the theory and practice are requisite.—Machines often promise very well in theory, but fail in the practice.—We say, theory of the moon, theory of the rainbow, of the microscope, of the camera obscura, &c.

THEORIES *of the Planets*, &c, are systems or hypotheses, according to which the astronomers explain the reasons of the phenomena or appearances of them.

THERMOMETER, an instrument for measuring the temperature of the air, &c, as to heat and cold.

The invention of the thermometer is attributed to several persons by different authors, viz, to Sanctorio, Galileo, father Paul, and to Drebbel. Thus, the invention is ascribed to Cornelius Drebbel of Alcmar, about the beginning of the 17th century, by his countrymen Boerhaave (Chem. 1, pa. 152, 156), and Musschenbroek (Introd. ad Phil. Nat. vol. 2, pa. 625).—Fulgenzio, in his Life of Father Paul, gives him the honour of the first discovery.—Vincenzio Viviani (Vit. de l'Galil. pa. 67; also Oper. di Galil. pref. pa. 47) speaks of Galileo as the inventor of thermometers.—But Sanctorino (Com. in Galen. Art. Med. pa. 736, 842, Com. in Avicen. Can. Fen. 1, pa. 22, 78, 219) expressly assumes to himself this invention: and Borelli (De Mot. Animal. 2, prop. 175) and Malpighi (Oper. Posth. pa. 30) ascribe it to him without reserve. Upon which Dr. Martine remarks, that these Florentine academicians are not to be suspected of partiality in favour of one of the Patavinian school.

But whoever was the first inventor of this instrument, it was at first very rude and imperfect; and as the various degrees of heat were indicated by the different contraction or expansion of air, it was afterwards found to be an uncertain and sometimes a deceiving measure of heat, because the bulk of the air was affected, not only by the difference of heat, but also by the variable weight of the atmosphere.—There are various kinds of thermometers, the construction, defects, theory, &c, of which, are as follow.

The Air THERMOMETER.—This instrument depends on the rarefaction of the air. It consists of a glass tube BE (fig. 1, pl. 39) connected at one end with a large glass ball A, and at the other end immersed in an open vessel, or terminating in a ball DE, with a narrow orifice at D; which vessel, or ball, contains any coloured liquor that will not easily freeze. Aquafortis tinged of a fine blue colour with solution of vitriol or copper, or spirit of wine tinged with cochineal, will answer this purpose. But the ball A must be first moderately warmed, so that a part of the air contained in it may be expelled through the orifice D; and then the liquor pressed by the weight of the atmosphere, will enter the ball DE, and rise, for example, to the middle of the tube at C, at a mean temperature of the weather; and in this state the liquor by its weight, and the air included in the ball and tube ABC, by its elasticity, will counterbalance the weight of the atmosphere. As the surrounding air becomes warmer, the air in the ball and the upper part of the tube, expanding by heat, will drive the liquor into the lower ball, and consequently its surface will descend; on the contrary, as the ambient air becomes colder, that in the ball is condensed, and the liquor, pressed by the weight of the atmosphere, will ascend: so that the liquor in the tube will ascend or descend more or less, according to the state of the air contiguous to instrument. To the tube is affixed a scale of the same length, divided upwards and downwards, from the middle C, into 100 equal parts, by means of which may be observed the ascent and descent of the liquor in the tube, and consequently the variations also in the temperature of the atmosphere.

A similar thermometer may be constructed by putting a small quantity of mercury, not exceeding the bulk of a pea, into the tube BC (fig. 4, pl. 39), bent into wreaths, that taking up the less height, it may be the more manageable, and less liable to harm; divide this tube into any number of equal parts to serve for a scale. Here the approaches of the mercury towards the ball A will show the increase of the degree of heat. The reason of which is the same as in the former.

The defect of both these instruments consists in this, that they are liable to be acted on by a double cause: for, not only a decrease of heat, but also an increase of weight of the atmosphere, will make the liquor rise in the one, and the mercury in the other; and, on the contrary, either an increase of heat, or decrease of the weight of the atmosphere, will cause them to descend.

For these, and other reasons, thermometers of this kind have long been disused. However, M. Amontons, in 1702, with a view of perfecting the aërial thermometer, contrived his Universal Thermometer. Finding that the changes produced by heat and cold in the bulk of the air were subject to invincible irregularities, he substituted for these the variations produced by heat in the elastic force of this fluid. This thermometer consisted of a long tube of glass (fig. 3, pl. 39) open at one end, and recurved at the other end, which terminated in a ball. A certain quantity of air was compressed into this ball by the weight of a column of mercury, and also by the weight of the atmosphere. The effect of heat on this included air was to make it sustain a greater or less weight; and this effect was measured by the variation of the column of mercury in the tube, corrected by that of the barometer, with respect to the changes of the weight of the external air. This instrument, though much more perfect than the former, is nevertheless subject to very considerable defects and inconveniences. Its length of 4 feet renders it unfit for a variety of experiments, and its construction is difficult and complex: it is extremely inconvenient for carriage, as a very small inclination of the tube would suffer the included air to escape: also the friction of the mercury in the tube, and the compressibility of the air, contribute to render the indications of this instrument extremely uncertain. Besides, the dilatation of the air is not so regularly proportional to its heat, nor is its dilatation by a given heat nearly so uniform as he supposed. This depends much on its moisture; for dry air does not expand near so much by a given heat, as air stored with watery particles. For these, and other reasons, enumerated by Deluc (Recherches sur les Mod. de l'Atmo. tom. 1, pa. 278 &c), this instrument was imitated by very few, and never came into general use.

Of the Florentine THERMOMETER.—The academists del Cimento, about the middle of the 17th century, considering the inconveniencies of the air thermometers above described, attempted another, that should measure heat and cold by the rarefaction and condensation of spirit of wine; though much less than those of air, and consequently the alterations in the degree of heat likely to be much less sensible.

The spirit of wine coloured, was included in a very fine and cylindrical glass tube (fig. 2, pl. 39), exhausted of its air, having a hollow ball at one end A, and hermetically sealed at the other end D. The ball and tube are filled with rectified spirit of wine to a convenient height, as to C, when the weather is of a mean temperature, which may be done by inverting the tube into a vessel of stagnant coloured spirit, under a receiver of the air-pump, or in any other way. When the thermometer is properly filled, the end D is heated red hot by a lamp, and then hermetically sealed, leaving the included air of about ⅓ of its natural density, to prevent the air which is in the spirit from dividing it in its expansion. To the tube is applied a scale, divided from the middle, into 100 equal parts, upwards and downwards.

Now spirit of wine rarefying and condensing very considerably; as the heat of the ambient atmosphere increases, the spirit will dilate, and so ascend in the tube; and as the heat decreases, the spirit will descend; and the degree or quantity of the motion will be shown by the attached scale.

These thermometers could not be subject to any inconvenience by an evaporation of the liquor, or a variable gravity of the incumbent atmosphere. Instruments of this kind were first introduced into England by Mr. Boyle, and they soon came into general use among philosophers in other countries. They are however subject to considerable inconveniences, from the weight of the liquor itself, and from the elasticity of the air above it in the tube, both which prevent the freedom of its ascent; besides, the rarefactions are not exactly proportional to the surrounding heat. Moreover spirit of wine is incapable of bearing very great heat or very great cold: it boils sooner than any other liquor; and therefore the degrees of heat of boiling fluids cannot be determined by this thermometer. And though it retains its fluidity in pretty severe cold, yet it seems not to condense very regularly in them: and at Torneao, near the polar circle, the winter cold was so severe, as Maupertuis informs us, that the spirits were frozen in all their thermometers. So that the degrees of heat and cold, which spirit of wine is capable of indicating, is much too limited to be of very great or general use.

Another great defect of these, and other thermometers, is, that their degrees cannot be compared with each other. It is true they mark the variations of heat and cold; but each marks for itself, and after its own manner; because they do not proceed from any point of temperature that is common to all of them.

From these and various other imperfections in these thermometers, it happens, that the comparisons of them become so precarious and defective: and yet the most curious and interesting use of them, is what ought to arise from such comparison. It is by this we should know the heat or cold of another season, of another year, another climate, &c; and what is the greatest degree of heat or cold that men and other animals can subsist in.

Reaumur contrived a new thermometer, (fig. 3, pl. 39) in which the inconveniences of the former are proposed to be remedied. He took a large ball and tube, the content or dimensions of which are known in every part; he graduated the tube, so that the space from one division to another might contain a 1000th part of the liquor, which liquor would contain 1000 parts when it stood at the freezing point: then putting the ball of his thermometer and part of the tube into boiling water, he observed whether it rose 80 divisions: if it exceeded these, he changed his liquor, and by adding water lowered it, till upon trial it should just rise 80 divisions; or if the liquor, being too low, fell short of 80 divisions, he raised it by adding rectified spirit to it. The liquor thus prepared suited his purpose, and served for making a thermometer of any size, whose scale would agree with his standard. Such liquor, or spirits, being about the strength of common brandy, may easily be had any where, or made of a proper degree of density by raising or lowering it.

The abbé Nollet made many excellent thermometers upon Reaumur's principle. Dr. Martine however expresses his apprehensions that thermometers of this kind cannot admit of such accuracy as might be wished. The balls or bulbs, being large, as 3 or 4 inches in diameter, are neither heated nor cooled soon enough to show the variations of heat. Small bulbs and small tubes, he says, are much more convenient, and may be constructed with sufficient accuracy. Though it must be allowed that Reaumur, by his excellent scale, and by depriving the spirit of its air, and expelling the air by means of heat from the ball and tube of his thermometer, has brought it to as much perfection as may be; yet it is liable to some of the inconveniences of spirit thermometers, and is much inferior to mercurial ones. These two kinds do not agree together in indicating the same degrees of intense cold; for when the mercury has stood at 22° below 0, the spirit indicated only 18°, and when the mercury stood at 28° or 37° below 0, the spirit rested at 25° or 29°. See the description of Reaumur's thermometer at large in Mem. de l'Acad. des Scienc. an. 1730, pa. 645, Hist. pa. 15. Ib. an. 1731, pa. 354, Hist. pa. 7.

Mercurial THERMOMETER.—It is a most important circumstance in the construction of thermometers, to procure a fluid that measures equal variations of heat by corresponding equal variations in its own bulk: and the fluid which possesses this essential requisite in the most perfect degree, is mercury: the variations in its bulk approaching nearer to a proportion with the corresponding variations of its heat, than any other fluid. Besides, it is the most easy to purge of its air; and is also the most proper for measuring very considerable variations of heat and cold, as it will bear more cold before freezing, and more heat before boiling, than any other fluid. Mercury is also more sensible than any other fluid, air excepted, or conforms more speedily to the several variations of heat. Moreover, as mercury is an homogeneous fluid, it will in every thermometer exhibit the same dilatation or condensation by the same variations of heat.

Dr. Halley, though apprised only of some of the remarkable properties of mercury above recited, seems to have been the first who suggested the application of this fluid to the construction of thermometers. Philos. Trans. vol. 3, pa. 505.

Boerhaave (Chem. 1, pa. 720) says, these mercurial thermometers were first contrived by Olaus Roemer; but the claims of Fahrenheit of Amsterdam, who gave an account of his invention to the Royal Society in 1724, (Philos. Trans. No. 381,) have been generally allowed. And though Prius and others, in England, Holland, France, and other countries, have made this instrument as well as Fahrenheit, yet most of the mercurial thermometers are graduated according to his scale, and are called Fahrenheit's thermometers.

The cone or cylinder, which these thermometers are often made with, instead of the ball, is made of glass of a moderate thickness, lest, when the exhausted tube is hermetically sealed, its internal capacity should be diminished by the weight of the ambient atmosphere. When the mercury is thoroughly purged of its air and moisture by boiling, the thermometer is filled with a sufficient quantity of it; and before the tube is hermetically sealed, the air is wholly expelled from it by heating the mercury, so that it may be rarefied and ascend to the top of the tube. To the side of the tube is annexed a scale (fig. 7, pl. 39), which Fahrenheit divided into 600 parts, beginning with that of the severe cold which he had observed in Iceland in 1709, or that produced by surrounding the bulb of the thermometer with a mixture of snow or beaten ice and sal ammoniac or sea salt. This he apprehended to be the greatest degree of cold, and accordingly he marked this, as the beginning of his scale, with 0; the point at which mercury begins to boil, he conceived to show the greatest degree of heat, and this he made the limit of his scale. The distance between these two points he divided into 600 equal parts or degrees; and by trials he found at the freezing point, when water just begins to freeze, or snow or ice just begins to thaw, that the mercury stood at 32 of these divisions, therefore called the degree of the freezing point; and when the tube was immersed in boiling water, the mercury rose to 212, which therefore is the boiling point, and is just 180 degrees above the former or freezing point. But the present method of making the scale of these thermometers, which is the kind in most common use, is first to immerge the bulb of the thermometer in ice or snow just beginning to thaw, and mark the place where the mercury stands with 32; then immerge it in boiling water, and again mark the place where the mercury stands in the tube, which mark with the No. 212, exceeding the former by 180; dividing therefore the intermediate space into 180 equal parts, will give the scale of the thermometer, and which may afterwards be continued upwards and downwards at pleasure.

Other thermometers of a similar construction have been accommodated to common use, having but a portion of the above scale. They have been made of a small size and portable form, and adapted with appendages to particular purposes; and the tube with its annexed scale has often been enclosed in another thicker glass tube, also hermetically sealed, to preserve the thermometer from injury. And all these are called Fahrenheit's thermometers.

In 1733, M. Delisle of Petersburg constructed a mercurial thermometer (see fig. 3, pl. 34), on the principles of Reaumur's spirit thermometer. In his thermometer, the whole bulk of quicksilver, when immerged in boiling water, is conceived to be divided into 100,000 parts; and from this one fixed point the various degrees of heat, either above or below it, are marked in these parts on the tube or scale, by the various expansion or contraction of the quicksilver in all imaginable varieties of heat.—Dr. Martine apprehends it would have been better if Delisle had made the integer 100,000 parts, or fixed point, at freezing water, and from thence computed the dilatations or condensations of the quicksilver in those parts; as all the common observations of the weather, &c, would have been expressed by numbers increasing as the heat increased, instead of decreasing, or counting the contrary way. However, in practice it will not be very easy to determine exactly all the divisions from the alteration of the bulk of the contained fluid. And besides, as glass itself is dilated by heat, though in a less proportion than quicksilver, it is only the excess of the dilatation of the contained fluid above that of the glass that is observed; and therefore if different kinds of glass be diffetently affected by a given degree of heat, this will make a seeming difference in the dilatations of the quicksilver in the thermometers constructed in the Newtonian method, either by Reaumur's rules or Delisle's. Accordingly it has been found, that the quicksilver in Delisle's thermometers has stood at different degrees of the scale when immerged in thawing snow: having stood in some at 154°, while in others it has been at 156 or even 158°.

Metallic THERMOMETER.—This is a name given to a machine composed of two metals, which, while it indicates the variations of heat, serves to correct the errors hence resulting in the going of pendulum clocks and watches. Instruments of this kind have been contrived by Graham, Le Roy, Ellicot, Harrison, and other eminent artificers. See the Philos. Trans. vol. 44, pa. 689, and vol. 45, pa. 129, and vol. 51, pa. 823, where the particular descriptions &c may be seen.

M. Deluc has likewise described two thermometers of metal, which he uses for correcting the effects of heat upon a barometer, and an hygrometer of his construction connected with them. See Philos. Trans. vol. 68, pa. 437.

Oil THERMOMETERS.—To this class belongs Newton's thermometer, constructed in 1701, with linseed oil, instead of spirit of wine. This fluid has the advantage of being sufficiently homogeneous, and capable of 15 times greater rarefaction than that of spirit of wine. It has not been observed to freeze even in very great colds; and it sustains a great heat, about 4 times that of water, before it boils. With these advantages it was made use of by Sir I. Newton, who discovered by it the comparative degree of heat for boiling water, melting wax, boiling spirit of wine, and melting tin; beyond which it does not appear that this thermometer was applied. The method he used for adjusting the scale of this oil thermometer, was as follows: supposing the bulb, when immerged in thawing snow, to contain 10,000 parts, he found the oil expanded by the heat of the human body so as to take up a 39th more space, or 10256 such parts; and by the heat of water boiling strongly 10725; and by the heat of melting tin 11516. So that, reckoning the freezing point as a common limit between heat and cold, he began his scale there, marking it 0, and the heat of the human body he made 12°; and consequently, the degrees of heat being proportional to the degrees of rarefaction, or 256 : 725 : : 12 : 34, this number 34 will express the heat of boiling water; and, by the same rule, 72 that of melting tin. Philos. Trans. No. 270.

There is an insuperable inconvenience attending all thermometers made with oil, or any other viscid fluid, viz, that such liquor adheres too much to the sides of the tube, and so inevitably disturbs the regularity and uniformity of the thermometer.

Of the fixed points of THERMOMETERS.—Various methods have been proposed by different authors, for finding a fixed point or degree of heat, from which to reckon the other degrees, and adjust the scale; so that different observations and instruments might be compared together. Mr. Boyle was very sensible of the inconveniencies arising from the want of a universal scale and mode of graduation; and he proposed either the freezing of the essential oil of aniseeds, or of distilled water, as a term to begin the numbers at, and from thence to graduate them according to the proportional dilatations or contractions of the included spirits.

Dr. Halley (Philos. Trans. vol. 3.) seems to have been fully apprised of the bad effects of the indefinite method of constructing thermometers, and wished to have them adjusted to some determined points. What he seems to prefer, for this purpose, is the degree of temperature found in subterranean places, where the heat in summer or cold in winter appears to have no influence. But this degree of temperature, Dr. Martine shows, is a term for the universal construction of thermometers, both inconvenient and precarious, as it cannot be easily ascertained, and as the difference of soils and depths may occasion a considerable variation. Another term of heat, which he thought might be of use in a general graduation of thermometers, is that of boiling spirit of wine that has been highly rectified.

The first trace that occurs of the method of actually applying fixed points or terms to the thermometer, and of graduating it, so that the unequal divisions of it might correspond to equal degrees of heat, is the project of Renaldini, professor at Padua, in 1694: it is thus described in the Acta Erud. Lips. "Take a slender tube, about 4 palms long, with a ball fastened to the same; pour into it spirit of wine, enough just to fill the ball, when surrounded with ice, and not a drop over: in this state seal the orifice of the tube hermetically, and provide 12 vessels, each capable of containing a pound of water, and somewhat more; and into the first pour 11 ounces of cold water, into the second 10 ounces, into the third 9, &c; this done, immerge the thermometer in the first vessel, and pour into it one ounce of hot water, observing how high the spirit rises in the tube, and noting the point with unity; then remove the thermometer into the second vessel, into which are to be poured 2 ounces of hot water, and note the place the spirit rises to with 2: by thus proceeding till the whole pound of water is spent, the instrument will be found divided into 12 parts, denoting so many terms or degrees of heat; so that at 2 the heat is double to that at 1, at 3 triple, &c."

But this method, though plausible, Wolfius shows, is deceitful, and built on false suppositions; for it takes for granted, that we have one degree of heat, by adding one ounce of hot water to 11 of cold; two degrees by adding 2 ounces to 10, &c: it supposes also, that a single degree of heat acts on the spirit of wine, in the ball, with a single force; a double with a double force, &c: lastly it supposes, that if the effect be produced in the thermometer by the heat of the ambient air, which is here produced by the hot water, the air has the same degree of heat with the water.

Soon after this project of Renaldini, viz, in 1701, Newton constructed his oil thermometer, and placed the base or lowest fixed point of his scale at the temperature of thawing snow, and 12 at that of the human body, &c, as above explained.—Deluc observes, that the 2d term of this scale should have been at a greater distance from the first, and that the heat of boiling water would have answered the purpose better than that of the human body.

In 1702, Amontons contrived his universal thermometer, the scale of which was graduated in the following manner. He chose for the first term, the weight that counterbalanced the air included in his thermometer, when it was heated by boiling water: and in this state he so adjusted the quantity of mercury contained in it, till the sum of its height in the tube, and of its height in the barometer at the moment of observation, was equal to 73 inches. Fixing this number at the point to which the mercury in the tube rose by plunging it in boiling water, it is evident that if the barometer at this time was at 28 inches, the height of the column of mercury in the thermometer, above the level of that in the ball, was 45 inches; but if the height of the barometer was less by a certain quantity, the column of the thermometer ought to be greater by the same quantity, and reciprocally. He formed his scale on the supposition, that the weight of the atmosphere was always equal to that of a column of mercury of 28 inches, and he divided it into inches from the point 73 downward, marking the divisions with 72, 71, 70, &c, and subdividing the inches into lines. But as the weight of the atmosphere is variable, the barometer must be observed at the same time with the thermometer, that the number indicated by this last instrument may be properly corrected, by adding or subtracting the quantity which the mercury is below or above 28 inches in the barometer. In this scale, then, the freezing point is at 51½ inches, corresponding to 32 degrees of Fahrenheit, and the heat of boiling water at 73 inches, answering to 212 of Fahrenheit's; and thus they may be easily compared together.

The fixed points of Fahrenheit's thermometer, as has been already observed, are the congelation produced by sal ammoniac and the heat of boiling water. The interval between these points is divided into 212 equal parts; the former of these points being marked 0, and the other 212.

Reaumur in his thermometer, the construction of which he published in 1730, begins his scale at an artificial congelation of water in warm weather, which, as he uses large bulbs for his glasses, gives the freezing point much higher than it should be, and at boiling water he marks 80 degrees, which point Dr. Martine thinks is more vague and uncertain than his freezing point. In order to determine the correspondence of his scale with that of Fahrenheit, it is to be considered that his boiling water heat, is really only the boiling heat of weakened spirit of wine, coinciding nearly, as Dr. Martine apprehends, with Fahrenheit's 180 degrees. And as his $10\frac{1}{4}$ degrees is the constant heat of the cave of the observatory at Paris, or Fahrenheit's 53°, he thence finds his freezing point, instead of answering just to 32°, to be somewhat above 34°.

In Celsius's thermometer (exhibited in plate 39, fig. 8), which is mercurial, the two fixed terms are the degree at which ice begins to thaw, and that which answers to the heat of boiling water. The interval between these two limits is divided into a hundred equal parts, and the zero of the scale, which is the inferior limit, corresponds to 32° of Fahrenheit; so that 9 degrees of Fahrenheit's scale are equivalent to 5 degrees of Celsius's. This thermometer is now generally called the Centigrade thermometer.

Delisle's thermometer, an account of which he presented to the Petersburg Academy in 1733, has only one fixed point, which is the heat of boiling water, and, contrary to the common order, the several degrees are marked from this point downward, according to the condensations of the contained quicksilver; and consequently by numbers increasing as the heat decreases. The freezing point of Delisle's scale, Dr. Martine makes near to his 150°, corresponding to Fahrenheit's 32°, by means of which they may be compared; but Ducrest says, that this point ought to be marked at least at 154°.

Ducrest, in his spirit thermometer, constructed in 1740, made use of two fixed points; the first, or 0, indicated the temperature of the earth, and was marked on his scale in the cave of the Paris Observatory; and the other was the heat of boiling water, which that spirit in his thermometer was made to endure, by leaving the upper part of the tube full of air. He divided the interval between these points into 100 equal parts; calling the divisions upward, degrees of heat, and those below 0, degrees of cold.—It is said that he has since regulated his thermometer by the degree of cold indicated by melting ice, which he found to be $10\frac{2}{5}$.

The Florentine thermometers were of two kinds. In one sort the freezing point, determined by the degree at which the spirit stood in the ordinary cold of ice or snow (probably in a thawing state) and coinciding with 32° of Fahrenheit, fell at 20°; and in the other kind at $13\frac{1}{2}$. And the natural heat of the viscera of cows and deer, &c, raised the spirit in the latter, or less sort, to about 40°, coinciding with their summer heat, and nearly with 102° in Fahrenheit's; and in their other or long thermometer, the spirit, when exposed to the great midsummer heat in their country, rose to the point at which they marked 80°.

In the thermometer of the Paris Observatory, made of spirit of wine by Lahire, the fluid always stands at 48° in the cave of the observatory, corresponding to 53 degrees in Fahrenheit's; and his 28° corresponded with 51 inches 6 lines in Amontons' thermometer, and consequently with the freezing point, or 32° of Fahrenheit's.

In Poleni's thermometer, made after the manner of Amontons', but with less mercury, 47 inches corresponded, according to Dr. Martine, with 51 in that of Amontons, and 53 with $59\frac{1}{2}$.

In the standard thermometer of the Royal Society of London, according to which thermometers were for a long time constructed in England, Dr. Martine found that $34\frac{1}{2}$ degrees answered to 64° in Fahrenheit, and 0 to 89.

In the thermometers graduated for adjusting the degrees of heat proper for exotic plants, &c, in stoves and greenhouses, the middle temperature of the air is marked at 0, and the degrees of heat and cold are numbered both above and below. Many of these are made on no regular and fixed principles. But in that formerly much used, called Fowler's regulator, the spirit fell, in melting snow, to about 34° below 0; and Dr. Martine found that his 16° above 0, nearly coincided with 64°. of Fahrenheit.

Dr. Hales (Statical Essays, vol. 1, pa. 58), in his thermometer, made with spirit of wine, and used in experiments on vegetation, began his scale with the lowest degree of freezing, or 32° of Fahrenheit, and carried it up to 100°, which he marked where the spirit stood when the ball was heated in hot water, upon which some wax floating first began to coagulate, and this point Dr. Martine found to correspond with 142° of Fahrenheit. But by experience it is found that Hales's 100 falls considerably above 142.

In the Edinburgh thermometer, made with spirit of wine, and used in the meteorological observations published in the Medical Essays, the scale is divided into inches and tenths. In melting snow the spirit stood at $8\frac{2}{10}$, and the heat of the human skin raised it to $22\frac{2}{10}$. Dr. Martine found that the heat of the person who graduated it, was 97 of Fahrenheit.

The new French thermometer, called the Centigrade Thermometer, contains 100 degrees between the freezing and boiling points of water; and those degrees further divided decimally by a vernier, &c.

From the abstract of the history of the construction of thermometers, it appears that freezing and boiling water have furnished the distinguishing points that have been marked upon almost all thermometers. The inferior fixed point is that of freezing, which some have determined by the freezing of water, and others by the melting of ice, plunging the ball of the thermometer into the water and ice, while melting, which is the best way. The superior fixed point of almost all thermometers, is the heat of boiling water. But this point cannot be considered as fixed and certain, unless the heat be produced by the same degree of boiling, and under the same weight of the atmosphere; for it is found that the higher the barometer, or the heavier the atmosphere, the greater is the heat when the water boils. It is now agreed therefore that the operation of plunging the ball of the thermometer in the boiling water, or suspending it in the steam of the same in an enclosed vessel, should be performed when the water boils violently, and when the barometer stands at 30 English inches, in a temperature of 55° of the atmosphere, marking the height of the thermometer then for the degree of 212 of Fahrenheit; the point of melting ice being 32 of the same; thus having 180 degrees between those two fixed points, so determined. This was Mr. Bird's method, who it is apprehended first attended to the state of the barometer, in the making of thermometers. But these instruments may be made equally true under any pressure of the atmosphere, by making a proper allowance for the difference in the height of the barometer from 30 inches. M. Deluc, in his Recherches sur les Mod. de l'Atmosphere, from a series of experiments, has given an equation for the allowance on account of this difference, in Paris measure, which has been verified by sir George Shuckburgh, Philos. Trans. 1775 and 1778; also Dr. Horsley, Dr. Maskelyne, and sir George Shuckburgh have adapted the equation and rules, to English measures, and have reduced the allowances into tables for the use of the artist. Dr. Horsley's rule, deduced from Deluc's, is this: $\frac{99}{1000000}$ log. z − 92·804 = *h*, where *h* denotes the height of a thermometer plunged in boiling water, above the point of melting ice, in degrees of Bird's Fahrenheit, and *z* the height of the barometer in 10ths of an inch. From this rule he has computed the following table, for finding the heights, to which a good Bird's Fahrenheit will rise, when immersed in boiling water, in all states of the barometer, from 27 to 31 English inches; which will serve, among other uses, to direct instrument-makers in making a true allowance for the effect of the variation of the barometer, if they should be obliged to finish a thermometer at a time when the barometer is above or below 30 inches; though it is best to fix the boiling point when the barometer is at that height.

Equation of the Boiling Point.

Barometer.	Equation.	Difference.
31·0	+ 1·57	0·78
30·5	+ 0·79	0·79
30·0	0·00	0·80
29·5	− 0·80	0·82
29·0	− 1·62	0·83
28·5	− 2·45	0·85
28·0	− 3·31	0·86
27·5	− 4·16	0·88
27·0	− 5·04	

The numbers in the first column of this table express heights of the quicksilver in the barometer in English inches and decimal parts: the 2d column shows the equation to be applied, according to the sign prefixed, to 212° of Bird's Fahrenheit, to find the true boiling point for every such state of the barometer. The boiling point for all intermediate states of the barometer may be had with sufficient accuracy by taking proportional parts, by means of the 3d column of differences of the equations. See Philos. Trans. vol. 64, art. 30; also Dr. Maskelyne's paper, vol. 64, art. 20.

Sir Geo. Shuckburgh (Philos. Trans. vol. 69, pa. 362) has also given several tables and rules relating to the boiling point, both from his own observations and Deluc's, from which is extracted the following table, for the use of artists in constructing the thermometer.

Height of the Barom.	Corr. of the Boiling Point.	Differences.	Correct. accord. to Deluc.	Differences.
26·0	− 7·09	0·91	− 6·83	0·90
26·5	− 6·18	0·91	− 5·93	0·89
27·0	− 5·27	0·90	− 5·04	0·88
27·5	− 4·37	0·89	− 4·16	0·87
28·0	− 3·48	0·89	− 3·31	0·86
28 5	− 2·59	0·87	− 2·45	0·83
29·0	− 1·72	0·87	− 1·62	0·82
29·5	− 0·85	0·85	− 0·80	0·80
30·0	0·00	0·85	0·00	0·79
30·5	+ 0·85	0·84	+ 0·79	0·78
31·0	+ 1·60		+ 1·57	

The Royal Society too, fully sensible of the importance of adjusting the fixed points of thermometers, appointed a committee of seven gentlemen to consider of the best method for this purpose; and their report may be seen in the Philos. Trans. vol. 67, art. 37.

They observe, that though the boiling point be placed so much higher on some of the thermometers now made, than on others, yet this does not produce any considerable error in the observations of the weather, at least in this climate; for an error of 1½ degree in the position of the boiling point, will make an error only of half a degree in the position of 92°, and of not more than a quarter of a degree in the point of 62°. It is only in nice experiments, or in trying the heat of hot liquors, that this error in the boiling point can be of much signification.

In adjusting the freezing, as well as the boiling point, the quicksilver in the tube ought to be kept of the same heat as that in the ball. When the freezing point is placed at a considerale distance from the ball, the pounded ice should be piled up very near to it; if it be not so piled, then the observed point, to be very accurate, should be corrected, according to the annexed table.

Heat of the Air.	Correction.
42°	·00087
52	·00174
62	·00261
72	·00348
82	·00435

The correction in this table is expressed in 1000th parts of the distance between the freezing point and the surface of the ice: ex. gr. if the freezing point stand 6 inches above the surface of the ice, and the heat of the room be 62, then the point of 32 should be placed 6 × ·00261, or ·01566 of an inch below the observed point.

The committee further observe, that in examining the heat of liquors, care should be taken that the quicksilver in the tube of the thermometer be heated to the same degree as that in the ball; or if this cannot be done conveniently, the observed heat should be corrected on that account; for the manner of doing which, and a table calculated for that purpose, see Philos. Trans. vol. 67, art. 37.

It was for some time thought, especially from the experiments at Petersburg, that quicksilver suffered a cold of several hundred degrees below 0 before it congealed and became fixed and malleable; but later experiments have shown that this persuasion was merely owing to a deception in the experiments, and later ones have made it appear that its point of congelation is no lower than − 40°, or rather − 39°, of Fahrenheit's scale. But that it will bear however to be cooled a few degrees below that point, to which it leaps up again on beginning to congeal; and that its rapid descent in a thermometer, through many hundred degrees, when it has once passed the above-mentioned limit, proceeds merely from its great contraction in the act of freezing. See Philos. Trans. vol. 73, art. *20, 20, 21.

Miscellaneous Observations.

It is absolutely necessary that those who would derive any advantage from these instruments, should agree in using the same liquor, and in determining, according to the same method, the two fundamental points. If they agree in these fixed points, it is of no great importance whether they divide the interval between them into a greater or a less number of equal parts. The scale of Fahrenheit, in which the fundamental interval between 212°, the point of boiling water, and 32° that of melting ice, is divided into 180 parts, should be retained in the northern countries, where Fahrenheit's thermometer is used: and the scale in which the fundamental interval is divided into 80 parts, will serve for those countries where Reaumur's thermometer is adopted. But no inconvenience is to be apprehended from varying the scale for particular uses, provided care be taken to signify into what number of parts the fundamental interval is divided, and the point where 0 is placed.

With regard to the choice of tubes, it is best to have them exactly cylindrical through their whole length. The capillary tubes are preferable to others, because they require smaller bulbs, and they are also more sensible, and less brittle. The most convenient size for common experiments has the internal diameter about the 40th or 50th of an inch, about 9 inches long, and made of thin glass, that the rise and fall of the mercury may be better seen.

For the whole process of filling, marking, and graduating, see Deluc's Recherches &c, tom. 1, pa. 393, &c.

To change the Degrees of one Thermometer to another.

The most usual thermometers employed in Europe, are, Fahrenheit's, Reaumur's, and the new French or Centigrade. Now the range or space on the tube, between the points of freezing and boiling water, in the first is divided into 180 degrees, in the second 80, and in the last 100, which three numbers are in the proportion of the three 9, 4, 5; by means of which small proportional numbers, therefore, any number of degrees of one of these scales, is easily changed into the corresponding number of either of the others; viz, by saying, as the proportional number of the latter, is to that of the former, so is the proposed degrees of the former, to those on the latter: observing, however, that when Fahrenheit's is one of the thermometers compared, which begins with the freezing point at 32, where the other two begin with 0, then the degrees of Fahrenheit must be diminished or increased by 32, as the case may require.

Also the same may be done by the following simple theorems; in which R denotes the degrees in Reaumur's scale, F those of Fahrenheit's, and C those of the Centigrade thermometer.

$$1 \ldots\ldots F = \tfrac{9}{4} R + 32 = \tfrac{9}{5} C + 32.$$
$$2 \ldots\ldots R = \tfrac{4}{9}(F - 32) = \tfrac{4}{5} C.$$
$$3 \ldots\ldots C = \tfrac{5}{9}(F - 32) = \tfrac{5}{4} R.$$

Experiments with THERMOMETERS.

The following is a table of some observations made with Fahrenheit's thermometer, the barometer standing at 29 inches, or little higher.

At 600°	Mercury boils.
546	Oil of vitriol boils.
242	Spirit of nitre boils.
240	Lixivium of tartar boils.
213	Cow's milk boils.
212	Water boils.
206	Human urine boils.
190	Brandy boils.
175	Alcohol boils.
156	Serum of blood and white of eggs harden.
146	Kills animals in a few minutes.
108	to 99, Hens hatch eggs.
107 103	Heat of skin in ducks, geese, hens, pigeons, partridges, and swallows.
106	Heat of skin in a common ague and fever.
103 100	Heat of skin in dogs, cats, sheep, oxen, swine, and most other quadrupeds.
99	to 92, Heat of the human skin in health.
97	Heat of a swarm of bees.
96	A perch died in 3 minutes in water so warm.
80	Heat of air in the shade, in very hot weather.
74	Butter begins to melt.
64	Heat of air in the shade, in warm weather.
55	Mean temperature of air in England.
43	Oil of olives begins to stiffen and grow opake.
32	Water just freezing, or snow and ice just melting.
30	Milk freezes.
28	Urine and common vinegar freeze.
25	Blood out of the body freezes.
20	Burgundy, Claret, and Madeira freeze.
5	Greatest cold in Pennsylvania in 1731-2, lat. 40°.
4	Greatest cold at Utrecht in 1728-9.
0	A mixture of snow and salt, which can freeze oil of tartar per deliquium, but not brandy.
−39	Mercury freezes.

Martine's Essays, pa. 284, &c.

On the general subject of thermometers also see Martine's Essays, Medical and Philosophical. Desaguliers's Exp. Phil. vol. 2, pa. 289. Musschenbroeck's Int. ad Phil. Nat. vol. 2, pa. 625, ed. 1762. Deluc's Recherches sur les Modif. &c, tom. 1, part. 2, ch. 2. Nollet's Leçons de Physique, tom. 4, pa. 375.

THERMOMETERS *for particular uses.*—In 1757, lord Cavendish presented to the Royal Society an account of a curious construction of thermometers, of two different forms; one contrived to show the greatest degree of heat, and the other the greatest cold, that may happen at any time in a person's absence. Philos. Trans. vol. 50, pa. 300.

Since the publication of Mr. Canton's discovery of the compressibility of spirits of wine and other fluids, there are two corrections necessary to be made in the result given by lord Cavendish's thermometer. For in estimating, for instance, the temperature of the sea at any depth, the thermometer will appear to have been colder than it really was: and besides, the expansion of spirits of wine by any given number of degrees of Fahrenheit's thermometer, is greater in the higher degrees than in the lower. For the method of making these two corrections by Mr. Cavendish, see Phipps's Voyage to the North Pole, pa. 145.

Instruments of this kind, for determining the degree of heat or cold in the absence of the observer, have been invented and described by others. Van Swinden (Diss. sur la Comparaison du Therm. pa. 253 &c) describes one, which he says was the first of the kind, made on a plan communicated by Bernoulli to Leibnitz. Mr. Kraft, he also informs us, made one nearly like it. Mr. Six has, in 1782, proposed another construction of a thermometer of the same kind, described in the Philos. Trans. vol. 72, pa. 72 &c.

M. Deluc has described the best method of constructing a thermometer, fit for determining the temperature of the air, in the measuring of heights by the barometer. He has also shown how to divide the scale of a thermometer, so as to adapt it for astronomical purposes in the observation of refractions. See Recherches &c, tom. 2, pa. 35 and 265.

Mr. Cavallo, in 1781, proposed the construction of a

Thermometrical Barometer, which, by means of boiling water, might indicate the various gravity of the atmosphere, or the height of the barometer. This thermometer, he observes, with its apparatus, might be packed up into a small portable box, and serve for determining the heights of mountains &c, with greater facility, than with the common portable barometer. The instrument, in its present state, consists of a cylindrical tin vessel, about 2 inches in diameter, and 5 inches high, in which vessel the water is contained, which may be made to boil by the flame of a large wax-candle. The thermometer is fastened to the tin vessel in such a manner, as that its bulb may be about an inch above the bottom. The scale of this thermometer, which is of brass, exhibits on one side of the glass tube a few degrees of Fahrenheit's scale, viz, from 200° to 216°. On the other side of the tube are marked the various barometrical heights, at which the boiling water shows those particular degrees of heat which are set down in sir Geo. Shuckburgh's table. With this instrument the barometrical height is shown within one-10th of an inch. The degrees of this thermometer are rather longer than one 9th of an inch, and therefore may be divided into many parts, especially by a Nonius. But a considerable imperfection arises from the smallness of the tin vessel, which does not admit a sufficient quantity of water; but when the quantity of water shall be sufficiently large, as for instance 10 or 12 ounces, and is kept boiling in a proper vessel, its degree of heat under the same pressure of the atmosphere is very settled; whereas when a thermometer is kept in a small quantity of boiling water, the mercury in its stem does not stand very steady, sometimes rising or falling so much as half a degree. Mr. Cavallo proposes a further improvement of this instrument, in the Philos. Trans. vol. 71, pa. 524.

The ingenious Mr. Wedgwood, so well known for his various improvements in the different sorts of pottery ware, has contrived to make a thermometer for measuring the higher degrees of heat, by means of a distinguishing property of argillaceous bodies, viz, the diminution of their bulk by fire. This diminution commences in a low red heat, and proceeds regularly, as the heat increases, till the clay becomes vitrified. The total contraction of some good clays which he has examined in the strongest of his own fires, is considerably more than one-fourth part in every dimension. By measuring the contraction of such substances then, Mr. Wedgwood contrived to measure the most intense heats of ovens, furnaces, &c. For the curious particulars of which, see Philos. Trans. vol. 72, pa. 305 &c.

In 1790 a paper was presented to the Royal Society of Edinburgh, describing two thermometers, newly invented, by Dr. John Rutherford, of Middle Balilish; the one for registering the highest, and the other for registering the lowest degree of heat, to which the thermometer has risen or fallen during the absence of the observer. An account of them may be found in the third volume of the Transactions of the Society.

A new self-registering thermometer has more lately been invented by Mr. Keith of Ravelstone, which we consider as the most ingenious, simple, and perfect, of any that has hitherto appeared. Its simplicity is so great that it requires only a very short description to make it intelligible.

AB (fig. 5, pl. 39) is a thin glass tube about 14 inches long and $\frac{3}{4}$ths of an inch caliber, close or hermetically sealed at the top. To the lower end, which is open, there is joined the crooked glass-tube BE, 7 inches long, and $\frac{4}{10}$ths of an inch caliber, and open at top. The tube AB is filled with the strongest spirit of wine, and the tube BE with mercury. This is properly a spirit of wine thermometer, and the mercury is used merely to support a piece of ivory or glass, to which is affixed a wire for raising one index and depressing another, according as the mercury rises or falls. F is a small conical piece of ivory or glass, of such a weight as to float on the surface of the mercury. To the float is joined a wire called the float-wire, which reaches upwards to H, where it terminates in a knee bent at right angles. The float-wire, by means of an eye at *a*, moves easily along the small harpsichord wire GK. L, L are two indexes made of thin black oiled silk, which slide upwards or downwards with a force not more than two grains. The one placed above the knee, points out the greatest rise, and the one placed below it points out the greatest fall of the thermometer.

When the instrument is to be prepared for an observation, both indexes are to be brought close to the knee H. It is evident that when the mercury rises, the float and float-wire, which can be moved with the smallest force, will be pushed upwards till the mercury becomes stationary. As the knee of the float-wire moves upwards, it will carry along with it the upper index L. When the mercury again subsides, it leaves the index at the highest point to which it was raised, for it will not descend by its own weight. As the mercury falls, the float-wire does the same; it therefore brings along with it the lower index L, and continues to depress it till it again become stationary, or ascend in the tube; in which case it leaves the lower index behind it as it had formerly left the upper. The scale to which the indexes point is placed parallel to the slender harpsichord wire. It may be seen more distinctly in fig. 6. That the scale and indexes may not be injured by the wind and rain, a cylindrical glass cover, close at top, and made so as exactly to fit the part FG, is placed over it.

Mr. Leslie, the author of the very ingenious Treatise on Heat, has invented a differential thermometer for the measurement of minute variations of temperature. It consists of two tubes, each terminating in a small bulb of the same dimensions, joined by the blow-pipe, and bent in the form of a U, a small portion of dark coloured liquor having previously been introuduced into one of the bulbs. After many trials, the fluid best adapted to the purpose was found to be a solution of carmine in concentrated sulphuric acid. By managing the included air with the heat of the hand, this red liquor is made to stand at the required point of the opposite tube. This is the zero of a scale fastened to that tube, and divided into equal parts above and below that point. The instrument is then fixed on a stand. It is manifest, that when the liquor is at rest, or points at zero, the column is pressed in opposite directions by two portions of air equal in elasticity, and containing equal quantities of caloric. Whatever heat then may be applied to the whole instrument, provided both bulbs receive it in the same degree, the liquor must remain at rest. But if the one ball receive the slightest excess of temperature, the air which it contains will be proportionally expanded, and will push the liquor against the air in the other full with a force varying as the difference between the temperatures of these two portions of air: thus the equilibrium will be destroyed, and the fluid will rise in the opposite tube. The degrees of the scale through which it passes will mark the successive augmentations in the temperature of the ball which is exposed to the greatest heat. So that this instrument is a balance of extreme delicacy for comparing the temperatures of its two scales.

When thermometers are contrived to measure very great degrees of heat by the expansions they produce in substances, or, on the contrary, the expansions corresponding to different temperatures, they are denominated *Pyrometers.* See the descriptions of the principal of these under their proper article.—On the subject of thermometers, see also my Philosophical Recreations, &c. vol. 4, pa. 43, &c.

THERMOSCOPE, the same as Thermometer.

THIR, in Chronology, the name of the 5th month of the Ethiopians, which corresponds, according to Ludolf, to the month of January.

THIRD, in Music, a concord resulting from a mixture of two sounds containing an interval of 2 degrees: being called a third, because containing 3 terms, or sounds, between the extremes.

There is a greater and a less third. The former takes its form from the sesquiquarta ratio, 4 to 5. The logarithm or measure of the octave $\frac{2}{1}$ being 1·00000, the measure of the greater third $\frac{5}{4}$ will be 0·32193.—The greater third is by practitioners often taken for the third part of an octave; which is an error, since three greater thirds fall short of the octave by a diesis; for $\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{128}{125} = \frac{2}{1}$.

The lesser third takes its form from the sesquiquinta ratio 5 to 6; the measure or logarithm of this lesser third $\frac{6}{5}$, being 0·26303, that of the octave $\frac{2}{1}$ being 1·00000.—Both these thirds are of great use in melody; making as it were the foundation and life of harmony.

Third-*Point*, or *Tierce-point*, in Architecture, the point of section in the vertex of an equilateral triangle.—Arches or vaults of the third point, are those consisting of two arches of a circle, meeting in an angle at top.

THREE-*legged-Staff*, an instrument consisting of three wooden legs, made with joints, so as to shut up together, and to take off in the middle for the better carriage. It has usually a ball and socket on the top; and its use is to support and adjust instruments for astronomy, surveying, &c.

THUNDER, a noise in the lower region of the air, excited by a sudden explosion of electrical clouds; which are therefore called thunder-clouds.—The phenomenon of thunder is variously accounted for. Seneca, Rohault, and some other authors, both ancient and modern, account for thunder, by supposing two clouds impending over each other, the upper and rarer of which, becoming condensed by a fresh accession of air raised by warmth from the lower parts of the atmosphere, or driven upon it by the wind, immediately falls forcibly down upon the lower and denser cloud; by which fall, the air interposed between the two being compressed, that next the extremities of the two clouds is forced out, and leaves room for the extremity of the upper cloud to close tight upon the under; thus a great quantity of the air is enclosed, which at length escaping through some winding irregular vent or passage, occasions the noise called thunder.

But this lame device could only reach at most to the case of thunder heard without lightning; and therefore recourse has been had to other modes of solution. Thus, it has been said that thunder is not occasioned by the falling of clouds, but by the kindling of sulphurous exhalations, in the same manner as the noise of the aurum fulminans. "There are sulphurous exhalations," says Newton, "always ascending in to the air when the earth is dry; there they ferment with the nitrous acids, and, sometimes taking fire, generate thunder, lightning, &c."

The effects of thunder are so like those of fired gunpowder, that Dr. Wallis thinks we need not scruple to ascribe them to the same cause; and the principal ingredients in gunpowder, we know, are nitre and sulphur; charcoal only serving to keep the parts separate, for their better kindling. Hence, if we conceive in the air a convenient mixture of nitrous and sulphurous particles; and those, by any cause, to be set on fire, such explosion may well follow, and with such noise and light as attend the firing of gunpowder; and being once kindled, it will run from place to place, different ways, as the exhalations happen to lead it; much as is found in a train of gunpowder.

But a third, and most probable opinion is, that thunder is the report or noise produced by an electrical explosion in the clouds. Ever since the year 1752, in which the identity of the matter of lightning and of the electrical fluid has been ascertained, philosophers have generally agreed in considering thunder as a concussion produced in the air by an explosion of electricity. For the illustration and proof of this theory, see Electricity, and Lightning.

It may here be observed, that Mr. Henry Eeles, in a letter written in 1751, and read before the Royal Society in 1752, considers the electrical fire as the cause of thunder, and accounts for it on this hypothesis; and he tells us, that he did not know of any other person's having made the same conjecture. Philos. Trans. vol. 47, p. 524 &c.—That rattling in the noise of thunder, which makes it seem as if it passed through arches, or were variously broken, is probably owing to the sound being excited among clouds hanging over one another, and the agitated air passing irregularly between them.—The explosion, if high in the air, and remote from us, will do no mischief; but when near, it may destroy trees, animals, &c.

This proximity, or small distance, may be estimated nearly by the interval of time between seeing the flash of lightning and hearing the report of the thunder, estimating the distance after the rate of 1142 feet per second of time, or $3\frac{2}{3}$ seconds to the mile. Dr. Wallis observes, that commonly the difference between the two is about 7 seconds, which, at the rate above mentioned, gives the distance almost 2 miles. But sometimes it comes in a second or two, which argues the explosion very near us, and even among us. And in such cases, the doctor assures us, he has sometimes foretold the mischiefs that followed.

The noise of thunder, and the flame of lightning, are easily made by art. If a mixture of oil or spirit of vitriol be made with water, and some filings of steel added to it, there will immediately arise a thick smoke, or vapour, out of the mouth of the vessel; and if a lighted candle be applied to this, it will take fire, and the flame will immediately descend into the vessel, which will be burst to pieces with a noise like that of a cannon. This is so far analogous to thunder and lightning, that a great explosion and fire are occasioned by it; but in this they differ, that this matter when once fired is destroyed, and can give no more explosions; whereas, in the heavens, one clap of thunder usually follows another, and there is a continued succession of them for a long time. Mr. Homberg explained this by the lightness of the air above us, in comparison of that near, which therefore would not suffer all the matter so kindled to be dissipated at once, but keeps it for several returns.

THUNDERBOLT. When lightning acts with extraordinary violence, and breaks or shatters any thing, it is called a thunderbolt, which the vulgar, to fit it for such effects, suppose to be a hard body, and even a stone.—But that we need not have recourse to a hard solid body to account for the effects commonly attributed to the thunderbolt, will be evident to any one, who considers those of the pulvis fulminans, and of gunpowder; but more especially the astonishing powers of electricity, when only collected and employed by human art, and much more when directed and exercised in the course of nature.—When we consider the known effects of electrical explosions, and those produced by lightning, we shall be at no loss to account for the extraordinary operations vulgarly ascribed to thunderbolts. As stones and bricks struck by lightning are often found in a vitrified state, we may reasonably suppose, with Beccaria, that some stones in the earth, having been struck in this manner, gave occasion to the vulgar opinion of the thunderbolt.

THUNDER-*Clouds*, in Physiology, are those clouds which are in a state fit for producing lightning and thunder.—From Beccaria's exact and circumstantial account of the external appearances of thunder-clouds, the following particulars are extracted. The first appearance of a thunder storm, which usually happens when there is little or no wind, is one dense cloud, or more, increasing very fast in size, and rising into the higher regions of the air. The lower surface is black and nearly level; but the upper finely arched, and well defined. Many of these clouds often seem piled upon one another, all arched in the same manner; but they are contiually uniting, swelling, and extending their arches.

At the time of the rising of this cloud, the atmosphere is commonly full of a great many separate clouds, that are motionless, and of whimsical shapes. All these, on the appearance of the thunder-cloud, draw towards it, and become more uniform in their shapes as they approach; till, coming very near the thunder-cloud, their limbs mutually stretch towards each other, and they immediately coalesce into one uniform mass. These he calls adscititious clouds, from their coming in, to enlarge the size of the thunder-cloud. But sometimes the thunder-cloud will swell, and increase very fast, without the conjunction of any adscititious clouds; the vapours in the atmosphere forming themselves into clouds wherever it passes. Some of the adscititious clouds appear like white fringes, at the skirts of the thunder-cloud, or under the body of it, but they keep continually getting darker and darker, as they approach to unite with it.

When the thunder-cloud is grown to a great size, its lower surface is often ragged, particular parts being detached towards the earth, but still connected with the rest. Sometimes the lower surface swells into various large protuberances bending uniformly downward; and sometimes one whole side of the cloud will have an inclination to the earth, and the extremity of it nearly touch the ground. When the eye is under the thunder-cloud, after it is grown larger and well formed, it is seen to sink lower, and to darken prodigiously; at the same time that a number of small adscititious clouds (the origin of which can never be perceived) are seen in a rapid motion, driving about in very uncertain directions under it. While these clouds are agitated with the most rapid motions, the rain commonly falls in the greatest plenty, and if the agitation be exceedingly great, it commonly hails.

While the thunder-cloud is swelling, and extending its branches over a large tract of country, the lightning is seen to dart from one part of it to another, and often to illuminate its whole mass. When the cloud has acquired a sufficient extent, the lightning strikes between the cloud and the earth, in two opposite places, the path of the lightning lying through the whole body of the cloud and its branches. The longer this lightning continues, the less dense does the cloud become, and the less dark its appearances; till at length it breaks in different places, and shows a clear sky.

These thunder-clouds were sometimes in a positive as well as a negative state of electricity. The electricity continued longer of the same kind, in proportion as the thunder-cloud was simple, and uniform in its direction; but when the lightning changed its place, there commonly happened a change in the electricity of the apparatus over which the clouds passed. It would change suddenly after a very violent flash of lightning, but the change would be gradual when the lightning was moderate, and the progress of the thunder-cloud slow. Beccar. Lettere dell' Elettricismo pa. 107; or Priestley's Hist. Elec. vol. 1, pa. 397. See also LIGHTNING.

THUNDER-*House*, in Electricity, is an instrument invented by Dr. James Lind, for illustrating the manner in which buildings receive damage from lightning, and to evince the utility of metallic conductors in preserving them from it.

A (fig. 1, pl. 40), is a board about ¾ of an inch thick, and shaped like the gable end of a house. This board is fixed perpendicularly on the bottom board B, upon which the perpendicular glass pillar CD is also fixed in a hole about 8 inches distant from the basis of the board A. A square hole ILMK, about a quarter of an inch deep, and nearly one inch wide, is made in the board A, and is filled with a square piece of wood, nearly of the same dimensions. It is nearly of the same dimensions, because it must go so easily into the hole, that it may drop off, by the least shaking of the instrument. A wire LK is fastened diagonally to this square piece of wood. Another wire IH of the same thickness, having a brass ball H, screwed on its pointed extremity, is fastened upon the board A: so also is the wire MN, which is shaped in a ring at O. From the upper extremity of the glass pillar CD, a crooked wire proceeds, having a spring socket F, through which a double knobbed wire slips perpendicularly, the lower knob G of which falls just above the knob H. The glass pillar DC must not be made very fast into the bottom board; but it must be fixed so that it may be pretty easily moved round its own axis, by which means the brass ball G may be brought nearer to or farther from the ball H, without touching the part EFG. Now when the square piece of wood LMIK (which may represent the shutter of a window or the like) is fixed into the hole so that the wire LK stands in the dotted representation IM, then the metallic communication from H to O is complete, and the instrument represents a house furnished with a proper metallic conductor; but if the square piece of wood LMIK be fixed so that the wire LK stands in the direction LK, as represented in the figure, then the metallic conductor HO, from the top of the house to its bottom, is interrupted at IM, in which case the house is not properly secured.

Fix the piece of wood LMIK, so that its wire may be as represented in the figure, in which case the metallic conductor HO is discontinued. Let the ball H be fixed at about half an inch perpendicular distance from the ball H; then, by turning the glass pillar DC, remove the former ball from the latter; by a wire or chain connect the wire EF with the wire Q of the jar P; and let another wire or chain, fastened to the hook O, touch the outside coating of the jar. Connect the wire Q with the prime conductor, and charge the jar; then, by turning the glass pillar DC, let the ball G come gradually near the ball H, and when they are arrived sufficiently near together, you will observe, that the jar explodes, and the piece of wood LMIK is pushed out of the hole to a considerable distance from the thunder-house.

Now the ball G, in this experiment, represents an electrified cloud, which, when it is arrived sufficiently near the top of the house A, the electricity strikes it; and as this house is not secured with a proper conductor, the explosion breaks part of it, i. e. knocks off the piece of wood IM.

Repeat the experiment with only this variation, viz, that this piece of wood IM be situated so that the wire LK may stand in the situation IM; in which case the conductor HO is not discontinued; and you will observe that the explosion will have no effect on the piece of wood LM; this remaining in the hole unmoved; which shows the usefulness of the metallic conductor.

Farther, unscrew the brass ball H from the wire HI, so that this may remain pointed, and with this difference only in the apparatus repeat both the above experiments, and you will find that the piece of wood IM is in neither case moved from its place, nor will any explosion be heard; which not only demonstrates the preference of conductors with pointed terminations to those with blunted ones, but also shows that a house, furnished with sharp terminations, though not furnished with a regular conductor, is almost sufficiently guarded against the effects of lightning.

Mr. Henley, having connected a jar containing 509 square inches of coated surface with his prime conductor, observed that if it was so charged as to raise the index of his electrometer to 60°, by bringing the ball on the wire of the thunder-house, to the distance of half an inch from that connected with the prime conductor, the jar would be discharged, and the piece in the thunder-house thrown out to a considerable distance. Using a pointed wire for a conductor to the thunder-house, instead of the knob, the charge being the same as before, the jar was discharged silently, though suddenly; and the piece was not thrown out of the thunder-house. In another experiment, having made a double circuit to the thunder-house, the first by the knob, the second by a sharp-pointed wire, at an inch and a quarter distance from each other, but of exactly the same height (as in fig. 2) the charge being the same; though the knob was brought first under that connected with the prime conductor, which was raised half an inch above it, and followed by the point, yet no explosion could fall upon the knob; the point drew off the whole charge silently, and the piece in the thunder-house remained unmoved.—Philos. Trans. vol. 64, pa. 136. See POINTS in Electricity.

THURSDAY, the 5th day of the Christians' week, but the 6th of the Jews.' The name is from Thor, one of the Saxon gods.

THUS, in Sea-Language, a word used by the pilot in directing the helmsman or steersman to keep the ship in her present situation when sailing with a scant wind, so that she may not approach too near the direction of the wind, which would shiver her sails, nor fall to leeward, and run farther out of her course.

TIDES, two periodical motions of the waters of the sea; called also the flux and reflux, or the flow and ebb.

The tides are found to follow periodically the course of the sun and moon, both as to time and quantity. And hence it has been suspected, in all ages, that the tides were somehow produced by the influence of these luminaries. Thus, several of the ancients, and among others, Pliny, Ptolemy, and Macrobius, were acquainted with the influence of the sun and moon on the tides; and Pliny says expressly, that the cause of the ebb and flow is in the sun, which attracts the waters of the ocean; and adds, that the waters rise in proportion to the proximity of the moon to the earth. It is indeed now well known, from the discoveries of Newton, that the tides are caused by the gravitation of the earth towards the sun and moon. Indeed the sagacious Kepler, long ago, conjectured this to be the cause of the tides: "If," says he, "the earth ceased to attract its waters towards itself, all the water in the ocean would rise and flow into the moon; the sphere of the moon's attraction extends to our earth, and draws up the water." Thus thought Kepler, in his Introd. ad Theor. Mart. This surmise, for it was then no more, is now abundantly verified in the theory laid down by Newton, and by Halley, as well as other eminent mathematicians, from his principles.

As to the Phenomena of the TIDES: 1. The sea is observed to flow, for about 6 hours, from south towards north; the sea gradually swelling; so that, entering the mouths of rivers, it drives back the river-waters towards their heads, or springs. After a continual flux of 6 hours, the sea appears to rest for about a quarter of an hour; after which it begins to ebb, or retire back again, from north to south, for 6 hours more; in which time, the waters sinking, the rivers resume their natural course. Then, after another seeming pause of a quarter of an hour, the sea again begins to flow, as before; and so on alternately.

2. Hence, the sea ebbs and flows twice a day, but falling every day gradually later and later, by about 48 minutes, the period of a flux and reflux being on an average about 12 hours 24 minutes, and the double of each 24 hours and 48 minutes; which is the period of a lunar day, or the time between the moon's passing a meridian and coming to it again. So that the sea flows as often as the moon passes the meridian, both the arch above the horizon, and that below it; and ebbs as often as she passes the horizon, both on the eastern and western side.

Other phenomena of the tides are as below; and the reasons of them will be noticed in the theory of the tides that follows.

3. The elevation towards the moon a little exceeds the opposite one. And the quantity of the ascent of the water is diminished from the equator towards the poles.

4. From the sun, every natural day, the sea is twice elevated, and twice depressed, the same as for the moon. But the solar times are much less than the lunar ones, on account of the immense distance of the sun; yet they are both subject to the same laws.

5. The tides which depend on the actions of the sun and moon, are not distinguished, but compounded, and so

forming as to sense one united tide, increasing and decreasing, and thus making neap and spring tides: for, by the action of the sun, the lunar tide is only changed; which change varies every day, by reason of the inequality between the natural and lunar day.

6. In the syzygies the elevations from the action of both luminaries concur, and the sea is more elevated. But the sea ascends less in the quadratures; for where the water is elevated by the action of the moon, it is depressed by the action of the sun; and vice versa. Therefore, while the moon passes from the syzygy to the quadrature, the daily elevations are continually diminished: on the contrary, they are increased while the moon moves from the quadrature to the syzygy. At a new moon also, *cæteris paribus*, the elevations are greater; and those that follow one another the same day, are more different than at full moon.

7. The greatest elevations and depressions are not observed till the 2d or 3d day after the new or full moon. And if we consider the luminaries receding from the plane of the equator, we shall perceive that the agitation is diminished, and becomes less, according as the declination of the luminaries becomes greater.

8. In the syzygies, and near the equinoxes, the tides are observed to be the greatest, both luminaries being in or near the equator.

9. The actions of the sun and moon are greater, the nearer those bodies are to the earth; and the less, as they are farther off. Also the greatest tides happen near the equinoxes, or rather when the sun is a little to the south of the equator, that is, a little before the vernal, and after the autumnal equinox. But yet this does not happen regularly every year, because some variation may arise from the situation of the moon's orbit, and the distance of the syzygy from the equinox.

10. All these phenomena obtain, as described, in the open sea, where the ocean is extended enough to be subject to these motions. But the particular situations of places, as to shores, capes, straits, &c, disturb these general rules. Yet it is plain, from the most common and universal observations, that the tides follow the laws above laid down.

11. The mean force of the moon to move the sea, is to that of the sun, nearly as 4½ to 1. And therefore, if the action of the sun alone produce a tide of 2 feet, which it has been stated to do, that of the moon will be 9 feet; from which it follows, that the spring tides will be 11 feet, and the neap tides 7 feet high. But as to such elevations as far exceed these, they happen from the motion of the waters against some obstacles, and from the sea violently entering into straits or gulphs where the force is not broken till the water rises higher.

Theory of the TIDES.

1. If the earth were entirely fluid, and quiescent, it is evident that its particles, by their mutual gravity towards each other, would form the whole mass into the figure of an exact sphere. Then suppose some power to act on all the particles of this sphere with an equal force, and in parallel directions; by such a power the whole mass will be moved together, but its figure will suffer no alteration by it, being still the same perfect sphere, whose centre will have the same motion as each particle.

On this supposition, if the motion of the earth, round the common centre of gravity of the earth and moon, were destroyed, and the earth left to the influence of its gravitation towards the moon, as the acting power above mentioned; then the earth would fall or move straight towards the moon, but still retaining its true spherical figure.

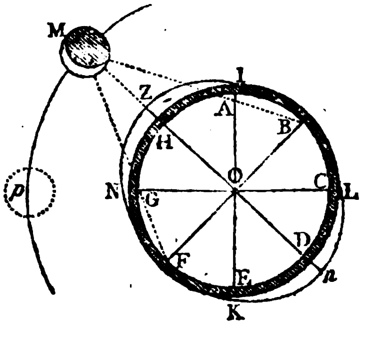
But the fact is, that the effects of the moon's action, as well as the action itself, on different parts of the earth, are not equal: those parts, by the general law of gravity, being most attracted that are nearest the moon, and those being least attracted that are farthest from her, while the parts that are at a middle distance are attracted by a mean degree of force: besides, all the parts are not acted on in parallel lines, but in lines directed towards the centre of the moon: on both which accounts the spherical figure of the fluid earth must suffer some change from the action of the moon. So that, in falling, as above, the nearer parts, being most attracted, would fall quickest; the farther parts, being least attracted, would fall slowest; and the fluid mass would be lengthened out, and take a kind of spheroidical form.

Hence it appears, and what must be carefully observed, that it is not the action of the moon itself, but the inequalities in that action, that cause any variation from the spherical figure; and that, if this action were the same in all the particles as in the central parts, and operating in the same direction, no such change would ensue.

Let us now admit the parts of the earth to gravitate towards its centre: then, as this gravitation far exceeds the action of the moon, and much more exceeds the differences of her actions on different parts of the earth, the effect that results from the inequalities of these actions of the moon, will be only a small diminution of the gravity of those parts of the earth which it endeavoured in the former supposition to separate from its centre; that is, those parts of the earth which are nearest to the moon, and those that are farthest from her, will have their gravity toward the earth somewhat abated, to say nothing of the lateral parts. So that supposing the earth fluid, the columns from the centre to the nearest, and to the farthest parts, must rise, till by their greater height they be able to balance the other columns, whose gravity is less altered by the inequalities of the moon's action. And thus the figure of the earth must still be an oblong spheroid.

Considering now the earth, instead of falling toward the moon by its gravity, as projected in any direction, so as to move round the centre of gravity of the earth and moon: it is evident that in this case, the several parts of the fluid earth will still preserve their relative positions; and the figure of the earth will remain the same as if it fell freely toward the moon; that is, the earth will still assume a spheroidal form, having its longest diameter directed toward the moon.

From the above reasoning it appears, that the parts of the earth directly under the moon, as at H, and also the opposite parts at D, will have the flood or high water at the same time; while the parts, at B and F, at 90° distance, or where the moon appears in the horizon, will have the ebbs or lowest wa-



ters at that time. Hence, as the earth turns round its axis from the moon to the same body again in 24 hours 48 minutes, this oval of water must shift with it; and thus there will be two tides of flood and two of ebb in that time.

But it is farther evident that, by the motion of the earth about her axis, the most elevated part of the water is carried beyond the moon in the direction of the rotation. So that the water continues to rise after it has passed directly under the moon, though the immediate action of the moon there begins to decrease, and comes not to its greatest elevation till it has got about half a quadrant farther. It continues also to descend after it has passed at 90° distance from the point below the moon, to a like distance of about half a quadrant. The greatest elevation therefore is not in the line drawn through the centres of the earth and moon, nor the lowest points where the moon appears in the horizon, but all these about half a quadrant removed eastward from these points, in the direction of the motion of rotation. Thus in open seas, where the water flows freely, the moon M is generally past the north and south meridian, as at *p*, when the high water is at *z* and at *n*: the reason of which is plain, because the moon acts with the same force after she has passed the meridian, and thus adds to the libratory or waving motion, which the water acquired when she was in the meridian; and therefore the time of high water is not precisely at the time of her coming to the meridian, but some time after, &c.

Besides, the tides answer not always to the same distance of the moon, from the meridian, at the same places; but are variously affected by the action of the sun, which brings them on sooner when the moon is in her first and third quarters, and keeps them back later when she is in her 2d and 4th; because, in the former case the tide raised by the sun alone would be earlier than the tide raised by the moon, and in the latter case later.

2. We have hitherto adverted only to the action of the moon in producing tides; but it is manifest that, for the same reasons, the inequality of the sun's action on different parts of the earth, would produce a like effect, and a like variation from the exact spherical figure of a fluid earth. So that in reality there are two tides every natural day from the action of the sun, as there are in the lunar day from that of the moon, subject to the same laws; and the lunar tide, as we have observed, is somewhat changed by the action of the sun, and the change varies every day on account of the inequality between the natural and the lunar day. Indeed the effect of the sun in producing tides, because of his immense distance, must be considerably less than that of the moon, though the gravity toward the sun be much greater: for it is not the action of the sun or moon itself, but the inequalities in that action, that have any effect: the sun's distance is so great, that the diameter of the earth is but as a point in comparison with it, and therefore the difference between the sun's actions on the nearest and farthest parts, becomes vastly less than it would be if the sun were as near as the moon. However the immense bulk of the sun makes the effect still sensible, even at so great a distance; and therefore, though the action of the moon has the greatest share in producing the tides, the action of the sun adds sensibly to it when they conspire together, as in the full and change of the moon, when they are nearly in the same line with the centre of the earth, and therefore unite their forces; consequently, in the syzygies, or at new and full moon, the tides are the greatest, being what are called the Spring-Tides. But the action of the sun diminishes the effect of the moon's action in the quarters, because the one raises the water in that case where the other depresses it; therefore the tides are the least in the quadratures, and are called Neap-Tides.

Newton has calculated the effects of the sun and moon respectively on the tides, from their attractive powers. The former he finds to be to the force of gravity, as 1 to 12868200, and to the centrifugal force at the equator as 1 to 44527. The elevation of the waters by this force is considered by Newton as an effect similar to the elevation of the equatorial parts above the polar parts of the earth, arising from the centrifugal force at the equator; and as it is 44527 times less, he finds it to be 24½ inches, or 2 feet and ½ an inch.

To find the force of the moon on the water, Newton compares the spring-tides at the mouth of the river Avon, below Bristol, with the neap-tides there, and finds the proportion as 9 to 5; whence, after several necessary corrections, he concludes that the force of the moon to that of the sun, in raising the waters of the ocean, is as 4·4815 to 1; so that the force of the moon is able of itself to produce an elevation of 9 feet 1¾ inch, and the sun and moon together may produce an elevation of about 11 feet 2 inches, when at their mean distances from the earth, or an elevation of about 12¾ feet, when the moon is nearest the earth. The height to which the water is found to rise, on coasts of the open and deep ocean, is agreeable enough to this computation.

Dr. Horsley estimates the force of the moon to that of the sun, as 5·0469 to 1, in his edit. of Newton's Princip. See the Princip. lib. 3, sect. 3, pr. 36 and 37; also Maclaurin's Dissert. de Causa Physica Fluxus et Refluxus Maris apud Phil. Nat. Princ. Math. Comment. le Seur & Jacquier, tom. 3, pa. 272. And other calculators make the proportion still more different.

3. It must be observed, that the spring-tides do not happen precisely at new and full moon, nor the neap-tides at the quarters, but a day or two after; because, as in other cases, so in this, the effect is not greatest or least when the immediate influence of the cause is greatest or least. As, for example, the greatest heat is not on the day of the solstice, when the immediate action of the sun is greatest, but some time after; so likewise, if the actions of the sun and moon should suddenly cease, yet the tides would continue to have their course for some time; and like also as the waves of the sea continue after a storm.

4. The different distances of the moon from the earth produce a sensible variation in the tides. When the moon approaches toward the earth, her action on every part increases, and the differences of that action, on which the tides depend, also increase; and as the moon approaches, her action on the nearest parts increases more quickly than that on the remote parts, so that the tides increase in a higher proportion as the moon's distances decrease. In fact, it is shown by Newton, that the tides increase in proportion as the cubes of the distances decrease; so that the moon at half her distance would produce a tide 8 times greater.

The moon describes an oval about the earth, and at her nearest distance produces a tide sensibly greater than at her greatest distance from the earth: and hence it is that two great spring-tides never succeed each other immediately; for if the moon be at her least distance from the earth at the change, she must be at her greatest distance at the full, having made half a revolution in the intervening time, and therefore the spring-tide then will be much less than that at the last change was; and for the same reason, if a great spring-tide happen at the time of full moon, the tide at the ensuing change will be less.

5. The spring-tides are highest, and the neap-tides lowest, about the time of the equinoxes, or the latter end of March and September; and, on the contrary, the spring-tides are the lowest, and the neap-tides the highest, at the solstices, or about the latter end of June and December: so that the difference between the spring and neap tides, is much more considerable about the equinoctial than the solstitial seasons of the year. To illustrate and evince the truth of this observation, let us consider the effect of the luminaries on the tides, when in and out of the plane of the equator. Now it is manifest, that if either the sun or moon were in the pole, they could not have any effect on the tides; for their action would raise all the water at the equator, or at any parallel, quite around, to a uniform height; and therefore any place of the earth, in describing its parallel to the equator, would not meet, in its course, with any part of the water more elevated than another; so that there could be no tide in any place, that is, no alteration in the height of the waters.

On the other hand, the effect of the sun or moon is greatest when in the equinoctial; for then the axis of the spheroidal figure, arising from their action, moves in the greatest circle, and the water is put into the greatest agitation; and hence it is that the spring-tides produced when the sun and moon are both in the equinoctial, are the greatest of any, and the neap-tides the least of any about that time. And when the luminary is any where between the equinoctial and the pole, the tides are the smaller.

6. The highest spring-tides are after the autumnal and before the vernal equinox; the reason of which is, because the sun is nearer the earth in winter than in summer.

7. Since the greatest of the two tides happening in every diurnal revolution of the moon, is that in which the moon is nearest the zenith or nadir; for this reason, while the sun is in the northern signs, the greater of the two diurnal tides in our climates, is that arising from the moon above the horizon; when the sun is in the southern signs, the greatest is that arising from the moon below the horizon. Thus it is found by observation that the evening tides in the summer exceed the morning tides, and in winter the morning tides exceed the evening tides. The difference is found at Bristol to amount to 15 inches, and at Plymouth to 12. It would be still greater, but that the fluid always retains an impressed motion for some time; so that the preceding tides affect always those that follow them. Upon the whole, while the moon has a north declination, the greatest tides in the northern hemisphere are when she is above the horizon, and the reverse while her declination is south.

8. Such would the tides regularly be, if the earth were all over covered with the sea very deep, so that the water might freely follow the influence of the sun and moon; but, by reason of the shoalness of some places, and the narrowness of the straits in others, through which the tides are propagated, there arises a great diversity in the effect according to the various circumstances of the places. Thus, a very slow and imperceptible motion of the whole body of water, where it is very deep, as 2 miles for instance, will suffice to raise its surface 10 or 12 feet during the period of a tide: whereas, if the same quantity of water were to be conveyed through a channel of 40 fathoms deep, it would require a very rapid stream to effect it in so large inlets as are the English channel, and the German ocean; whence the tide is found to set strongest in those places where the sea grows narrowest, the same quantity of water being in that case to pass through a smaller passage. This is particularly observable in the straits between Portland and Cape la Hogue in Normandy, where the tide runs like a sluice: and would be yet more so between Dover and Calais, if the tide coming round the island did not check it.

This force, when once impressed, continues to carry the water above the ordinary height in the ocean, especially where the water meets a direct obstacle, as it does in St. Maloes; and where it enters into a long channel which, running far into the land, becomes very strait at its extremity, as it does into the Severn sea at Chepstow and Bristol.

This shoalness of the sea, and the intercurrent continents, are the reasons that in the open ocean the tides rise but to very small heights in proportion to what they do in wide-mouthed rivers, opening in the direction of the stream of the tide; and that high water is not soon after the moon's appulse to the meridian, but some hours after it, as it is observed on all the western coast of Europe and Africa, from Ireland to the Cape of Good Hope; in all which a south-west moon makes high water; and the same it is said is the case on the western side of America. So that tides happen to different places at all distances of the moon from the meridian, and consequently at all hours of the day.

To allow the tides their full motion, the ocean in which they are produced, ought to be extended from east to west 90 degrees at least; because that is the distance between the places where the water is most raised and depressed by the moon. Hence it appears that it is only in the great oceans that such tides can be produced, and why in the larger Pacific ocean they exceed those in the Atlantic ocean. Hence also it is obvious, why the tides are not so great in the torrid zone, between Africa and America, where the ocean is narrower, as in the temperate zones on either side; and hence we may also understand why the tides are so small in islands that are very far distant from the shores. It is farther manifest that, in the Atlantic ocean, the water cannot rise on one shore but by descending on the other; so that at the intermediate islands it must continue at a mean height between its elevations on those two shores. But when tides pass over shoals, and through straits into bays of the sea, their motion becomes more various, and their height depends on many circumstances.

To be more particular. The tide that is produced on the western coasts of Europe, in the Atlantic, corresponds to the situation of the moon already described. Thus it is high water on the western coasts of Ireland, Portugal and Spain, about the 3d hour after the moon has passed the meridian: from thence it flows into the adjacent channels, as it finds the easiest passage. One current from it, for instance, runs up by the south of England, and another comes in by the north of Scotland; they take a considerable time to move through this extent, making always high water sooner in the places to which they first come; and it begins to fall at these places while the currents are still going on to others that are farther distant in their course. As they return, they are not able to raise the tide, because the water runs faster off than it returns, till, by a new tide propagated from the open ocean, the return of the current is stopped, and the water begins to rise again. The tide propagated by the moon in the German ocean, when she is 3 hours past the meridian, takes 12 hours to come from thence to London bridge: so that when it is high water there, a new tide is already come to its height in the ocean; and in some intermediate place it must be low water at the same time. Consequently when the moon has north declination, and we should expect the tide at London to be the greatest when the moon is above the horizon, we find it is least; and the contrary when she has south declination.

At several places it is high water 3 hours before the moon comes to the meridian; but that tide, which the moon drives as it were before her, is only the tide opposite to that which was raised by her when she was 9 hours past the opposite meridian.

It would be endless to recount all the particular solutions, which are easy consequences from this doctrine: as, why the lakes and seas, such as the Caspian sea and the Mediterranean sea, the Black sea and the Baltic, have little or no sensible tides: for lakes are usually so small, that when the moon is vertical she attracts every part of them alike, so that no part of the water can be raised higher than another: and having no communication with the ocean, it can neither increase nor diminish their water, to make it rise and fall; and seas that communicate by such narrow inlets, and are of so immense an extent, cannot speedily receive and empty water enough to raise or sink their surface in any sensible degree.

In general; when the time of high water at any place is mentioned, it is to be understood on the days of new and full moons.—Among pilots, it is customary to reckon the time of flood, or high water, by the point of the compass the moon bears on, at that time, allowing ¾ of an hour for each point. Thus, on the full and change days, in places where it is flood at noon, the tide is said to flow north and south, or at 12 o'clock: in other places, on the same days, where the moon bears 1, 2, 3, 4, or more points to the east or west of the meridian when it is high water, the tide is said to flow on such point; thus, if the moon bears SE, at flood, it is said to flow SE and NW, or 3 hours before the meridian, that is, at 9 o'clock; if it bears SW, it flows SW and NE, or at 3 hours after the meridian; and in like manner for the other points of the moon's bearing.

The times of high water in any place fall about the same hours after a period of about 15 days, or between one spring tide and another; but during that period, the times of high water fall each day later by about 48 minutes.

On the subject of this article, see Newton Princ. Math. lib. 3, prop. 24, and De System. Mundi sect. 38, &c. Apud Opera edit. Horsley, tom. 3, pa. 52 &c, pa. 203 &c. Maclaurin's Account of Newton's Discoveries, book 4, ch. 7. Ferguson's Astron. ch. 17. Robertson's Navig. book 6, sect. 7, 8, 9. Lalande's Astron. vol. 4. See also the article Physical Astronomy in this dictionary.

TIDE *Dial*, an instrument contrived by Mr. Ferguson, for exhibiting and determining the state of the tides: for the construction and use of which, see his Astron. pa. 297.

TIDE *Tables*, are tables commonly exhibiting the times of high water at sundry places, as they fall on the days of the full and change of the moon, and sometimes the height of them also. These are common in most books on navigation, particularly Robertson's, and the tables requisite to be used with the Nautical Almanac. See one at HIGH-*water*.

TIERCE, or TEIRCE, a liquid measure, as of wine, oil, &c, containing 42 gallons, or the 3d part of a pipe; whence its name.

TIME, a succession of phenomena in the universe; or a mode of duration, marked by certain periods and measures; chiefly indeed by the motion and revolution of the luminaries, and particularly of the sun.

The idea of time in general, Locke observes, we acquire by considering any part of infinite duration, as set out by periodical measures: the idea of any particular time, or length of duration, as a day, an hour, &c, we acquire first by observing certain appearances at regular and seemingly equidistant periods. Now, by being able to repeat these lengths or measures of time as often as we will, we can imagine duration, where nothing really endures or exists; and thus we imagine to-morrow, or next year, &c.

Some of the later school-philosophers define time to be the duration of a thing whose existence is neither without beginning nor end: by this, time is distinguished from eternity.

Aristotle and the Peripatetics define it, numerus motus secundum prius et posterius, or a multitude of transient parts of motion, succeeding each other, in a continual flux, in the relation of priority and posteriority. Hence it should follow that time is motion itself, or at least the duration of motion, considered as having several parts, some of which are continually succeeding to others. But on this principle, time or temporal duration would not agree to bodies at rest, which yet nobody will deny to exist in time, or to endure for a time.

To avoid this inconvenience, the Epicureans and Corpuscularians made time to be a kind of flux different from motion, consisting of infinite parts, continually and immediately succeeding each other, and this from eternity to eternity. But others directly explode this notion, as establishing an eternal being, independent of God. For how should there be a flux before any thing existed to flow? and what should that flux be, a substance, or an accident? According to the philosophic poet,

> "Time of itself is nothing, but from thought
> Receives its rise; by labouring fancy wrought
> From things consider'd, while we think on some
> As present, some as past, or yet to come.
> No thought can think on time, that's still confest,
> But thinks on things in motion or at rest."

And so on. Vide Lucretius, book i.

Time may be distinguished, like place, into absolute and relative.

Absolute TIME, is time considered in itself, and without any relation to bodies, or their motions.

Relative or *Apparent* TIME, is the sensible measure of any duration by means of motion.

Some authors divide time into astronomical and civil.

Astronomical TIME, is that which is taken purely from the motion of the heavenly bodies, without any other regard.

Civil Time, is the former time accommodated to civil uses, and formed or distinguished into years, months, days, &c.

Apparent Time, is that deduced from the motion of the heavenly bodies, as of the sun: which is unequal. And

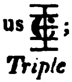
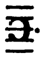
Equal, Mean, or *True* Time, is that which is shown by a good clock, which it is supposed never varies in its rate of going.

Equation of Time, is the difference between true and apparent time.

Time, in Music, is an affection of sound, by which it is said to be long or short, with regard to its continuance in the same tone or degree of tune.

Musical time is distinguished into common or duple time, and triple time.

Double, Duple, or *Common Time*, is when the notes are in a duple duration of each other, viz, a semibreve equal to 2 minims, a minim to 2 crotchets, a crotchet to 2 quavers, &c.

Common or double time is of two kinds. The first when every bar or measure is equal to a semibreve, or its value in any combination of notes of a less quantity. The second is where every bar is equal to a minim, or its value in less notes. The movements of this kind of measure are various, but there are three common distinctions; the first slow, denoted at the beginning of the line by the mark c; the 2d brisk, marked thus ; and the 3d very brisk, thus marked .

Triple Time is when the durations of the notes are triple of each other, that is, when the semibreve is equal to 3 minims, the minim to 3 crotchets, &c; and it is marked T.

.Time-*keepers*, in a general sense, denote instruments adapted for measuring time. See Chronometer. In a more peculiar and definite sense, time-keeper is a term first applied by Mr. John Harrison to his watches, constructed and used for determining the longitude at sea, and for which he received, at different times, the parliamentary reward of 20 thousand pounds. And several other artists have since received also considerable sums for their improvements of time-keepers; as Arnold, Mudge, &c. See Longitude. This appellation is now become common among artists, to distinguish such watches as are made with extraordinary care and accuracy for nautical or astronomical observations.

The principles of Mr. Harrison's time-keeper, as they were communicated by himself, to the commissioners appointed to receive and publish the same in the year 1765, are as below: " In this time-keeper there is the greatest care taken to avoid friction, as much as can be, by the wheel moving on small pivots, and in ruby-holes, and high numbers in the wheels and pinions.

" The part which measures time goes but the 8th part of a minute without winding up; so that part is very simple, as this winding-up is performed at the wheel next to the balance-wheel; by which means there is always an equal force acting at that wheel, and all the rest of the work has no more to do in the measuring of time than the person that winds up once a day.

" There is a spring in the inside of the fusee, which I will call a secondary main spring. This spring is always kept stretched to a certain tension by the main spring; and during the time of winding-up the time-keeper, at which time the main-spring is not suffered to act, this secondary spring supplies its place.

" In common watches in general, the wheels have about one-third the dominion over the balance, that the balance-spring has; that is, if the power which the balance-spring has over the balance be called three, that from the wheel is one: but in this my time-keeper, the wheels have only about one 18th part of the power over the balance that the balance spring has; and it must be allowed, the less the wheels are connected with the balance, the better. The wheels in a common watch having this great dominion over the balance, they can, when the watch is wound up, and the balance at rest, set the watch a-going; but when my time-keeper's balance is at rest, and the spring is wound up, the force of the wheels can no more put it in motion, than the wheels of a common regulator can, when the weight is wound-up, set the pendulum a vibrating; nor will the force from the wheels move the balance when at rest, to a greater angle in proportion to the vibration that it is to fetch, than the force of the wheels of a common regulator can move the pendulum from the perpendicular when it is at rest.

" My time-keeper's balance is more than three times the weight of a large sized common watch balance, and three times its diameter; and a common watch balance goes through about 6 inches of space in a second, but mine goes through about 24 inches in that time: so that had my instrument only these advantages over a common watch, a good performance might be expected from it. But my time-keeper is not affected by the different degrees of heat and cold, nor agitation of the ship; and the force from the wheels is applied to the balance in such a manner, together with the shape of the balance-spring, and (if I may be allowed the term) an artificial cycloid, which acts at this spring; so that from these contrivances, let the balance vibrate more or less, all its vibrations are performed in the same time; and therefore if it go at all, it must go true. So that it is plain from this, that such a time-keeper goes entirely from principle, and not from chance."—Those who may desire to see a minute account of the construction of Mr. Harrison's time-keeper, may consult the publication by order of the commissioners of longitude.

We shall here subjoin a short view of the improvements in Mr. Harrison's watch, from the account presented to the board of longitude by Mr. Ludlam, one of the gentlemen to whom, by order of the commissioners, Mr. Harrison discovered and explained the principle on which his time-keeper is constructed. The defects in common watches which Mr. Harrison proposes to remedy, are chiefly these: 1. That the main spring acts not constantly with the same force on the wheels, and through them on the balance: 2. That the balance, either urged with an unequal force, or meeting with a different resistance from the air, or the oil, or the friction, vibrates through a greater or less arch: 3 That these unequal vibrations are not performed in equal times: and, 4. That the force of the balance-spring is altered by a change of heat.

To remedy the first defect, Mr. Harrison has contrived that his watch shall be moved by a very tender spring, which never unrolls itself more than one-eighth part of a turn, and acts upon the balance through one wheel only. But such a spring cannot keep the watch in motion a long time. He has therefore, joined another, whose office is to wind up the first spring 8 times in every minute; and which is itself wound up but once a day. To remedy the second defect, he uses a much stronger balance spring than in a common watch. For if the force of this spring upon the balance remains the same, while the force of the other varies, the errors arising from that variation will be the less, as the fixed force is the greater. But a stronger spring will require either a heavier or a larger balance. A heavier balance would have a greater friction. Mr. Harrison therefore increases the diameter of it. In a common watch it is under an inch, but in Mr. Harrison's 2 inches 2 tenths. However, the methods already described only lessening the errors, and not removing them, Mr. Harrison uses two ways to make the times of the vibrations equal, though the arches may be unequal: one is, to place a pin, so that the balance-spring pressing against it, has its force increased, but increased less when the variations are larger: the other, to give the pallets such a shape, that the wheels press them with less advantage, when the vibrations are larger. To remedy the last defect, Mr. Harrison uses a bar compounded of two thin plates of brass and steel, about 2 inches in length, riveted in several places together, fastened at one end and having two pins at the other, between which the balance spring passes. If this bar be straight in temperate weather (brass changing its length by heat more than steel) the brass side becomes convex when it is heated, and the steel side when it is cold: and thus the pins lay hold of a different part of the spring in different degrees of heat, and lengthen or shorten it as the regulator does in a common watch.

The principles on which Mr. Arnold's time-keeper is constructed are these: The balance is unconnected with the wheel work, except at the time it receives the impulse to make it continue its motion, which is only while it vibrates 10° out of 380° which is the whole vibration; and during this small interval it has little or no friction, but what is on the pivots, which work in ruby holes on diamonds. It has but one pallet, which is a plane surface formed out of a ruby, and has no oil on it. Watches of this construction, says Mr. Lyons, go while they are wound up; they keep the same rate of going in every position, and are not affected by the different forces of the spring; and the compensation for heat and cold is absolutely adjustable. Phipps's Voyage to the North Pole, pa. 230. See Longitude.

TISRI, or Tizri, in Chronology, the first Hebrew month of the civil year, and the 7th of the ecclesiastical or sacred year. It answered to part of our September and October.

TOD *of wool*, is mentioned in the statute 12 Carol. II. c. 32, as a weight containing 2 stone, or 28 pounds.

TOISE, a French measure, containing 6 of their feet, similar to our fathom.—The length of the French toise, is to the English fathom, as 1065 to 1000, or as 213 to 200.

TON, is 20 hundred weight, or 2240 lbs.

TONDIN, or Tandino, in Architecture. See Tore.

TONE, or Tune, in Music, a property of sound, by which it comes under the relation of grave and acute; or the degree of elevation any sound has, from the degree of swiftness of the vibrations of the parts of the sonorous body.—For the cause, measure, degree, difference, &c, of tones, see Tune.

The word Tone is taken in four different senses among the ancients. 1, For any sound. 2, For a certain interval; as when it is said the difference between the diapente and diatessaron is a tone. 3, For a certain locus or compass of the voice; in which sense they used the Dorian, Phrygian, Lydian tones. 4. For tension; as when they speak of an acute, a grave, or a middle tone. Wallis's Append. Ptolem. Harm. pa. 172.

Tone is more particularly used, in music, for a certain degree or interval of tune, by which a sound may be either raised or lowered from one extreme of a concord to the other, so as still to produce true melody. In tempered scales of music, the tones are made equal, but in a true and accurate practice of singing they are not so. Pepusch, in Philos. Trans. No. 481. Besides the concords, or harmonical intervals, musicians admit three less kinds of intervals, which are the measures and component parts of the greater, and are called degrees. Of these degrees, two are called tones, and the third a semitone. Their ratios in number are 8 to 9, called a greater tone; 9 to 10 called a lesser tone; and 15 to 16, a semitone.

The tones arise out of the simple concords, and are equal to their differences. Thus the greater tone, 8 : 9, is the difference of a 5th and a 4th; the less tone 9 : 10, the difference of a less 3d and a 4th, or of a 5th and a greater 6th; and the semitone 15 : 16, the difference of a greater 3d and a 4th.

Of these tones and semitones every concord is compounded, and consequently every one is resolvable into a certain number of them. Thus, the less 3d consists of one greater tone and one semitone: the greater 3d, of one greater tone and one less tone: the 4th, of one greater tone, one less tone, and one semitone: and the 5th, of two greater tones, one less tone, and one semitone.

TONNAGE, of a ship, is the weight or loading it is supposed to bear. The rule commonly used for computing it, is to multiply the length of the keel by the breadth of the beam, and that again by half the same breadth of the beam; the last product divided by 94, gives the number of tons burthen. Thus, if the length of keel be 100 feet, and the breadth of beam 30; then $\frac{100 \times 30 \times 15}{94} = 478$, is the tonnage.

TONSTALL (Cuthbert), a learned English divine and mathematician, was born in the year 1476. He entered a student at the university of Oxford about the year 1491; but afterwards, being driven from thence by the plague, he went to Cambridge, and shortly after to the university of Padua in Italy, which was then in a flourishing state of literature, where his genius and learning acquired him great respect from every one, particularly for his knowledge in mathematics, philosophy, and jurisprudence.

On his return home, he met with great favours from the government, obtaining several church preferments, and the office of secretary to the cabinet of the king, Henry the 8th. This prince, having also employed him on several foreign embassies, was so well satisfied with his conduct, that he first gave him the bishopric of London in 1522, and afterwards that of Durham in 1530.

Tonstall approved at first of the dissolution of the marriage of his benefactor with Catharine of Spain, and even wrote a book in favour of that dissolution; but he afterwards condemned that work, and experienced a great reverse of fortune. He was ejected from the see of Durham for his religion in the time of Edward the 6th, to which however he was restored again by queen Mary in the beginning of her reign, but was again expelled in 1559 when queen Elizabeth was settled in her throne; and he died in a prison a few months after, in the 84th year of his age.

Tonstall was doubtless one of the most learned men of his time. "He was," says Wood, "a very good Grecian and Ebritian, an eloquent rhetorician, a skilful mathematician, a noted civilian and canonist, and a profound divine. But that which maketh for his greatest commendation, is, that Erasmus was his friend, and he a fast friend to Erasmus, in an epistle to whom from Sir Thomas More, I find this character of Tonstall, that, "As there was no man more adorned with knowledge and good literature, no man more severe and of greater integrity for his life and manners; so there was no man a more sweet and pleasant companion, with whom a man would rather choose to converse."

His writings that were published, were chiefly, 1. In Laudem Matrimonii, Lond. 1518, 4to.—But that for which he is chiefly entitled to a place in this work, was his book on arithmetic, viz,—2. De Arte Supputandi, Lond. 1522, 4to, dedicated to Sir Thomas More. This was afterwards several times printed abroad.—3. A Sermon on Palm Sunday before king Henry the 8th, &c. Lond. 1539 and 1633, 4to.—4. De Veritate Corporis & Sanguinis Domini in Eucharistia. Lutet. 1554, 4to.—5. Compendium in decem Libros Ethicorum Aristotelis. Par. 1554, in 8vo.—6. Contra impios Blasphematores Dei prædestinationis opera. Antw. 1555, 4to.—7. Godly and devout Prayers in English and Latin. 1558, in 8vo.

TOPOGRAPHY, is a description or draught of some particular place, or small tract of land; as that of a city, or town, manor or tenement, field, garden, house, castle, or the like; such as surveyors set out in their plots, or make draughts of, for the information and satisfaction of the proprietors. Topography differs from chorography, as a particular from a more general.

TORELLI (**Joseph**), a respectable Italian mathematician, was born at Verona in November 1721, and died in Sept. 1781. His father was a merchant, in good circumstances, who died soon after his son's birth; so that the care of Torelli's education devolved on his mother; by whom his infant mind was most attentively cultivated, and to whose care might be ascribed many of those amiable qualities, which distinguished his more advanced age. Having laid a good foundation by private instruction at Verona, he prosecuted his studies at the university of Padua, with great assiduity, in the various branches of literature and science.

Having spent four years at Padua, where he conciliated the general esteem of the learned, and where he obtained a doctor's degree, he returned to his own country. Being in easy circumstances, he declined engaging in any profession, but devoted his whole time and attention to general study, both of languages and mathematics. He became accordingly an excellent proficient in several of the ancient and modern tongues. The Greek and Hebrew he well understood; he wrote Latin with ease and correctness; and his acquaintance with the French, Spanish, and English, enabled him to peruse the best writers with pleasure and improvement. To his knowledge of the languages he added a very extensive acquaintance with the arts and sciences; so that he was no less distinguished as a mathematician and philosopher, than as a critical scholar.

Torelli was author of a great number and variety of compositions, which sufficiently evince his distinguished abilities and application. But that from which he has obtained his chief celebrity, is an edition of the collected works of Archimedes, printed at Oxford, 1792, folio, in Greek and Latin. The preparing of this work had been indeed the labour of most part of his life. Having been completely prepared for publication, and even the diagrams cut which were to accompany the demonstrations, the manuscript was disposed of after his death to the curators of the Clarendon press, by whose order it was printed under the immediate care of Dr. Robertson.

It appears that there have been few persons, in any country, or in any period of time, who were better qualified for preparing a correct edition of Archimedes. As a Greek scholar, he was capable of correcting the mistakes, supplying the defects, and illustrating the obscure passages, that occurred in treatises originally written in the Greek tongue: his knowledge of Latin, and a facility, acquired by habit, of writing in this language, rendered him a fit person to translate the Greek into pure and correct Latin; and his comprehensive acquaintance with mathematics and philosophy qualified him for conducting the whole work with judgment and accuracy.

TORNADO, a violent gust of wind arising suddenly from the shore, and afterwards veering round all points of the compass like a hurricane. It is very frequent on the coast of Guinea.

TORRENT, in Hydrography, a temporary stream of water, falling suddenly from mountains, &c, where there have been great rains, or an extraordinary thaw of snow; sometimes making great ravages in the plains.

TORRICELLI (**Evangeliste**), an illustrious mathematician and philosopher of Italy, was born at Faenza in 1608, and trained up in Greek and Latin literature by an uncle, who was a monk. Natural inclination led him to cultivate mathematical knowledge, which he pursued some time without a master; but at about 20 years of age, he went to Rome, where he continued the pursuit of it under father Benedict Castelli. Castelli had been a scholar of the great Galileo, and had been appointed by the pope professor of mathematics at Rome. Torricelli made such progress under this master, that having read Galileo's Dialogues, he composed a Treatise concerning motion on his principles. Castelli, surprised at the performance, carried it and read it to Galileo, who heard it with great pleasure, and conceived a high esteem and friendship for the author. On this, Castelli proposed to Galileo, that Torricelli should come and live with him; recommending him as the most proper person he could have, since he was the most capable of comprehending those sublime speculations, which his own great age, infirmities, and want of sight, prevented him from giving to the world. Galileo accepted the proposal, and Torricelli the employment, as things of all others the most advantageous to both. Galileo was at Florence, at which place Torricelli arrived in 1641, and began to take down what Galileo dictated, to regulate his papers, and to act in every respect according to his directions. But he did not long enjoy the advantages of this situation, as Galileo died at the end of only three months.

Torricelli was then about returning to Rome; but the Grand Duke engaged him to continue at Florence, making him his own mathematician for the present, and promising him the professor's chair as soon as it should be vacant. Here he applied himself intensely to the study of mathematics, physics, and astronomy, making many improvements and some discoveries. Among others, he greatly improved the art of making microscopes and telescopes; and it is generally acknowledged that he first found out the method of ascertaining the weight of the atmosphere by a proportionate column of quicksilver, the barometer being called from him the Torricellian tube, and Torricellian experiment. In short, great things were expected from him, and great things would probably have been further performed by him, if he had lived: but he died, after a few days illness, in 1647, when he had just completed the 39th year of his age.

Torricelli published at Florence in 1644, a volume of ingenious pieces, entitled, Opera Geometrica, in 4to. There was also published at the same place, in 1715, Lezzioni Accademiche, consisting of 96 pages in 4to. These are discourses that had been pronounced by him on different occasions. The first of them was to the academy of La Crusca, by way of thanks for admitting him into their body. The rest are upon subjects of mathematics and physics. Prefixed to the whole is a long life of Torricelli by Thomas Buonaventuri, a Florentine gentleman.

TORRICELLIAN, a term very frequent among physical writers, used in the phrases, Torricellian tube, or Torricellian experiment, on account of the inventor Torricelli, a disciple of the great Galileo.

Torricellian *Tube*, is the barometer tube, being a glass tube, open at one end, and hermetically sealed at the other, about 3 feet long, and $\frac{1}{5}$ of an inch in diameter.

Torricellian *Experiment*, or the filling the barometer tube, is performed by filling the Torricellian tube with mercury, then stopping the open orifice with the finger, inverting the tube, and plunging that orifice into a vessel of stagnant mercury. This done, the finger is removed, and the tube sustained perpendicular to the surface of the mercury in the vessel.

The consequence is, that part of the mercury falls out of the tube into the vessel, and there remains only enough in the tube to fill about 30 inches of its capacity, above the surface of the stagnant mercury in the vessel; these being sustained in the tube by the pressure of the atmosphere on the surface of the stagnant mercury; and according as the atmosphere is more or less heavy, or as the winds, blowing upward or downward, heave up or depress the air, and so increase or diminish its weight and spring, more or less mercury is sustained, from 28 to 31 inches.—The Torricellian experiment constitutes what is now called the barometer.

Torricellian *Vacuum*, is the vacuum produced by filling a tube with mercury, and when inverted allowing it to descend to such a height as is counterbalanced by the pressure of the atmosphere, as in the Torricellian experiment and barometer, the vacuum being that part of the tube above the surface of the mercury.

TORRID *Zone*, is that round the middle of the earth, extending to 23½ degrees on both sides of the equator.

TORUS, or **Tore**, in Architecture, is a large round moulding in the bases of the columns.

TOUCAN, or *American Goose*, is one of the modern constellations of the southern hemisphere, consisting of 9 small stars.

TRACTION, or *Drawing*, is the act of a moving power, by which the moveable is brought nearer to the mover, called also attraction.

Traction, *Angle of*, in Mechanics, is the angle which the direction of the power makes with any given plane.

TRACTRIX, or **Tractix**, in Geometry, a curve line called also Catenaria; which see.

TRAJECTORY, a term often employed to denote the path of any body moving either in a void, or in a medium that resists its motion; or even for any curve passing through a given number of points. Thus Newton, Princip. lib. 1, prob. 22, proposes to describe a trajectory that shall pass through five given points.

Trajectory *of a Comet*, is its path or orbit, or the line it describes in its motion. This path, Hevelius, in his Cometographia, will have to be very nearly a right line; but Dr. Halley concludes it to be, as it really is, a very excentric ellipsis; though its place may often be well computed on the supposition of its being a parabola. —Newton, in prop. 41 of his 3d book, shows how to determine the trajectory of a comet from three observations; and in his last prop. how to correct a trajectory graphically described.

TRAMMELS, in Mechanics, an instrument used by artificers for drawing ovals upon boards, &c. One part of it consists of a cross with two grooves at right angles; the other is a beam carrying two pins which slide in those grooves, and also the describing pencil. All the engines for turning ovals are constructed on the same principles with the trammels: the only difference is, that in the trammels the board is at rest, and the pencil moves upon it: in the turning engine, the tool, which supplies the place of the pencil, is at rest, and the board moves against it. See a demonstration of the chief properties of these instruments by Mr. Ludlam, in the Philos. Trans. vol. 70, pa. 378 &c.

TRANSACTIONS, *Philosophical*, are a collection of the principal papers and matters read before certain philosophical societies, as the Royal Society of London, and the Royal Society of Edinburgh. These transactions contain the several discoveries and histories of nature and art, either made by the members of those societies, or communicated by them from their correspondents, with the various experiments, observations, &c, made by them, or transmitted to them, &c.

The Philos. Trans. of the Royal Society of London were begun in 1665, by Mr. Oldenburg, the then secretary of that Society, and were continued by him till the year 1677. They were then discontinued on his death, till January 1678, when Dr. Grew resumed the publication of them, and continued it for the months of December 1678, and January and February 1679, after which they were intermitted till January 1683. During this last interval their want was in some measure supplied by Dr. Hooke's Philosophical Collections. They were also interrupted for 3 years, from December 1687 to January 1691, besides other smaller interruptions, amounting to near a year and a half more, before October 1695, since which time the transactions have been carried on regularly to the present day, with various degrees of credit and merit.

Till the year 1752 these transactions were published in numbers quarterly, and the printing of them was always the single act of the respective secretaries till that time; but then the Society thought fit that a committee should be appointed to consider the papers read before them, and to select out of them such as they should judge most proper for publication in the future transactions. For this purpose, the members of the council, for the time being, constitute a standing committee: they meet on the first Thursday of every month, and no less than

7 of the members of the committee (of which number the president, or in his absence a vice-president, is always to be one) are allowed to be a quorum, capable of acting in relation to such papers; and the question with regard to the publication of any paper, is always decided by the majority of votes taken by ballot.

They are published annually in two parts, at the expence of the Society; and each fellow, or member, is entitled to receive one copy gratis of every part published after his admission into the Society. For many years past, the collection, in two parts, has made one volume in each year; and in the year 1793 the number of the volumes was 83, being 10 less than the number of the year in the century. They were formerly much respected for the great number of excellent papers and discoveries contained in them; but of late years there has been a great falling off, and the volumes have been sometimes considered as of very inferior merit, as well as quantity.

There was also an useful abridgment of those volumes of the transactions that were published before the year 1752, when the Society began to publish the transactions on their own account. Those to the end of the year 1700 were abridged, in 3 volumes, by Mr. John Lowthorp: those from the year 1700 to 1720 were abridged, in 2 volumes, by Mr. Henry Jones: and those from 1719 to 1733 were abridged, in 2 volumes, by Mr. John Eames and Mr. John Martyn; Mr. Martyn also continued the abridgment of those from 1732 to 1744 in 2 volumes, and of those from 1744 to 1750 in 2 volumes; making in all 11 volumes. But lately a complete Abridgment, in 18 large 4to volumes, of the whole, from the beginning, to the end of the year 1800, has been published by Dr. Charles Hutton, Dr. George Shaw, and Dr. Richard Pearson. In this abridgment all the papers are given in their original order, and a copious index is added, by which is shown the place of any article, either in the original or in the abridgment.

The Royal Society of Edinburgh instituted in 1783, have also published several volumes of their Philosophical Transactions; which are deservedly held in the highest respect for the importance of their contents.

The Society of Arts, &c, have also published a number of volumes of transactions, abounding with mechanical inventions and discoveries. There are also transactions of the American Society, of the Manchester Philosophical Society, of the Connecticut Society, &c. The Irish Academy, and most of the foreign philosophical societies, give to their transactions the title of Memoirs.

TRANSCENDENTAL *Quantities*, among Geometricians, are indeterminate ones; or such as cannot be expressed or fixed to any constant equation: such is a transcendental curve, or the like. M. Leibnitz has a dissertation in the Acta Erud. Lips. in which he endeavours to show the origin of such quantities; viz, why some problems are neither plain, solid, nor sursolid, nor of any certain degree, but transcend all algebraic equations. He also shows how it may be demonstrated without calculus, that an algebraic quadratrix for the circle or hyperbola is impossible: for if such a quadratrix could be found, it would follow, that by means of it any angle, ratio, or logarithm, might be divided in a given proportion of one right line to another, and this by one universal construction: and consequently the problem of the section of an angle, or the invention of any number of mean proportionals, would be of a certain finite degree. Whereas the different degrees of algebraic equations, and therefore the problem understood in general of any number of parts of an angle, or mean proportionals, is of an indefinite degree, and transcends all algebraical equations.

Others define transcendental equations, to be such fluxional equations as do not admit of fluents in common finite algebraical equations, but as expressed by means of some curve, or by logarithms, or by infinite series; thus the expression $\dot{y} = \frac{\dot{x}}{\surd(aa - xx)}$ is a transcendental equation, because the fluents cannot both be expressed in finite terms. And the equation which expresses the relation between an arc of a circle and its sine, is a transcendental equation; for Newton has demonstrated that this relation cannot be expressed by any finite algebraic equation, and therefore it can only be by an infinite or a transcendental equation.

It is also usual to rank exponential equations among transcendental ones; because such equations, though expressed in finite terms, have variable exponents, which cannot be expunged but by putting the equation into fluxions, or logarithms, &c. Thus, the exponential equation

$$y = a^x, \text{ gives } x \times \log. a = \log. y, \text{ or } \dot{x} \times \log. a = \frac{\dot{y}}{y}.$$

TRANSCENDENTAL *Curve*, in the Higher Geometry, is such a one as cannot be defined by an algebraic equation; or of which, when it is expressed by an equation, one of the terms is a variable quantity, or a curve line. And when such curve line is a geometrical one, or one of the first degree or kind, then the transcendental curve is said to be of the second degree or kind, &c.

These curves are the same with what Descartes, and others after him, call mechanical curves, and which they would have excluded out of geometry; contrary however to the opinion of Newton and Leibnitz; for as much as, in the construction of geometrical problems, one curve is not to be preferred to another as it is defined by a more simple equation, but as it is more easily described than that other: besides, some of these transcendental, or mechanical curves, are found of greater use than almost all the algebraical ones.

M. Leibnitz, in the Acta Erudit. Lips. has given a kind of transcendental equations, by which these transcendental curves are actually defined, and which are of an indefinite degree, or are not always the same in every point of the curve. Now whereas algebraists use to assume some general letters or numbers for the quantities sought, in these transcendental problems Leibnitz assumes general or indefinite equations for the lines sought; thus, for example, putting x and y for the absciss and ordinate, the equation he uses for a line required, is $a + bx + cy + exy + fxx + gyy$ &c $= 0$: by the help of which indefinite equation, he seeks for the tangent; and comparing that which results with the given property of tangents, he finds the value of the assumed letters a, b, c, &c, and thus defines the equation of the line sought.

If the comparison abovementioned do not succeed, he pronounces the line sought not to be an algebraical, but a transcendental one. This supposed; he proceeds to find the species of transcendency: for some transcendentals depend on the general division or section of a ratio, or upon logarithms, others on circular arcs, &c. Here then, besides the symbols x and y, he assumes a third, as v, to denote the transcendental quantity; and of these three he forms a general equation of the line sought, from which he finds the tangent according to the differential method, which succeeds even in transcendental quantities. This found, he compares it with the given properties of the tangents, and so discovers not only the values of a, b, c, &c, but also the particular nature of the transcendental quantity.

Transcendental problems are very well managed by the method of fluxions. Thus, for the relation of a circular arc and right line, let a denote the arc, and x the versed sine, to the radius 1, then is $a =$ fluent of $\frac{\dot{x}}{\sqrt{(2x-xx)}}$; and if the ordinate of a cycloid be y, then is $y = \sqrt{(2x-xx)} +$ fluent of $\frac{\dot{x}}{\sqrt{(2x-xx)}}$.

Thus is the analytical calculus extended to those lines which have hitherto been excluded, for no other cause but that they were thought incapable of it.

TRANSFORMATION, in Geometry, is the changing or reducing of a figure, or of a body, into another of the same area, or the same solidity, but of a different form. As, to transform or reduce a triangle to a square, or a pyramid to a parallelopipedon.

TRANSFORMATION *of Equations*, in Algebra, is the changing equations into others of a different form, but of equal value. This operation is often necessary, to prepare equations for a more easy solution, some of the principal cases of which are as follow.—1. The signs of the roots of an equation are changed, viz, the positive roots into negative, and the negative roots into positive ones, by only changing the signs of the 2d, 4th, and all the other even terms of the equation. Thus, the roots of the equation $x^4 - x^3 - 19x^2 + 49x - 30 = 0$, are $+1, +2, +3, -5$; whereas the roots of the same equation having only the signs of the 2d and 4th terms changed, viz, of $x^4 + x^3 - 19x^2 - 49x - 30 = 0$, are $-1, -2, -3, +5$.

2. To transform an equation into another that shall have its roots greater or less than the roots of the proposed equation by some given difference, proceed as follows: Let the proposed equation be the cubic $x^3 - ax^2 + bx - c = 0$; and let it be required to transform it into another, whose roots shall be less than the roots of this equation by some given difference d; if the root y of the new equation must be the less, take it $y = x - d$, and hence $x = y + d$; then instead of x and its powers substitute $y + d$ and its powers, and there will arise this new equation

$$(\text{A})\quad \left.\begin{array}{rrrr} y^3 + 3dy^2 & + 3d^2y & + d^3 \\ - ay^2 & - 2ady & - ad^2 \\ & + by & + bd \\ & & - c \end{array}\right\} = 0,$$

whose roots are less than the roots of the former equation by the difference d. If the roots of the new equation had been required to be greater than those of the original one, we must then have substituted $y = x + d$, or $x = y - d$, &c.

3. To take away the 2d or any other particular term out of an equation; or to transform an equation, so as the new equation may want its 2d, or 3d, or 4th, &c term of the given equation $x^3 - ax^2 + bx - c = 0$, which is transformed into the equation (A) in the last article. Now to make any term of this equation (A) vanish, is only to make the coefficient of that term $= 0$, which will form an equation that will give the value of the assumed quantity d, so as to produce the desired effect, viz, to make that term vanish. So, to take away the 2d term, make $3d - a = 0$, which makes the assumed quantity $d = \frac{1}{3}a$. To take away the 3d term, we must put the sum of the coefficients of that term $= 0$, that is $3d^2 - 2ad + b = 0$, or $3d^2 - 2ad = -b$; then by resolving this quadratic equation, there is found the assumed quantity $d = \frac{1}{3}a \pm \frac{1}{3}\sqrt{(a^2 - 3b)}$, by the substitution of which for d, the 3d term will be taken away out of the equation.

In like manner, to take away the 4th term, we must make the sum of its coefficients $d^3 - ad^2 + bd - c = 0$; and so on for any other term whatever. And in the same manner we must also proceed when the proposed equation is not a cubic, but of any height whatever, as

$$x^n - ax^{n-1} + bx^{n-2} - cx^{n-3}\ \&\text{c} = 0;$$

this is first, by substituting $y + d$ for x, to be transformed to this new equation

$$\left.\begin{array}{rrr} y^n + ndy^{n-1} & + \frac{1}{2}n(n-1)d^2y^{n-2} & \&\text{c} \\ - ay^{n-1} & - a(n-1)dy^{n-2} & \&\text{c} \\ & + by^{n-2} & \&\text{c} \end{array}\right\} = 0;$$

then, to take away the 2d term, we must make $nd - a = 0$, or $d = \frac{a}{n}$; to take away the 3d term, we must make $\frac{1}{2}n(n-1)d^2 - a(n-1)d + b = 0$, or $d^2 - \frac{2a}{n}d = -\frac{2b}{n(n-1)}$; and so on.

Whence it appears that, to take away the 2d term of an equation, we must resolve a simple equation; for the 3d term, a quadratic equation; for the 4th term, a cubic equation, and so on.

4. To multiply or divide the roots of an equation by any quantity; or to transform a given equation to another, that shall have its roots equal to any multiple or submultiple of those of the proposed equation. This is done by substituting, for x and its powers, $\frac{y}{m}$ or py, and their powers, viz, $\frac{y}{m}$ for x, to multiply the roots by m; and py for x, to divide the roots by p. Thus, to multiply the roots by m, substituting $\frac{y}{m}$ for x in the proposed equation

$x^n - ax^{n-1} + bx^{n-2}\ \&\text{c} = 0$, and it becomes

$\frac{y^n}{m^n} - \frac{ay^{n-1}}{m^{n-1}} + \frac{by^{n-2}}{m^{n-2}}\ \&\text{c} = 0$;

or multiply all by m^n, then is

$y^n - amy^{n-1} + bm^2y^{n-2} - cm^3y^{n-3}\ \&\text{c} = 0$,

an equation that has its roots equal to m times the roots of the proposed equation.

In like manner, substituting py for x, in the proposed equation, &c, it becomes

$$y^n - \frac{ay^{n-1}}{p} + \frac{by^{n-2}}{p^2} - \frac{cy^{n-3}}{p^3}\ \&\text{c} = 0,$$

an equation that has its roots equal to those of the proposed equation divided by p.

Whence it appears, that to multiply the roots of an equation by any quantity m, we must multiply its terms, beginning at the 2d term, respectively by the terms of the geometrical series, m, m^2, m^3, m^4, &c. And to divide the roots of an equation by any quantity p, that we must divide its terms, beginning at the 2d, by the corresponding terms of this series p, p^2, p^3, p^4, &c.

5. And sometimes, by these transformations, equations are cleared of fractions, or even of surds. Thus the equation

$x^3 - ax^2\sqrt{p} + bx - c\sqrt{p} = 0$, by putting $y = x\sqrt{p}$, or multiplying the terms, from the 2d, by the geometricals $\sqrt{p}$, p, $p\sqrt{p}$, is transformed to

$$y^3 - apy^2 + bpy - cp^2 = 0.$$

6. An equation, as $x^3 - ax^2 + bx - c = 0$, may be

transformed into another, whose roots shall be the reciprocals of the roots of the given equation, by substituting $\frac{1}{y}$ for x; by which it becomes

$$\frac{1}{y^3} - \frac{a}{y^2} + \frac{b}{y} - c = 0,$$ or, multiplying all by y^3, the same becomes $cy^3 - by^2 + ay - 1 = 0$.

On this subject, see Newton's Alg. on the Transmutation of Equations; Maclaurin's Algeb. pt. 2, chap. 3 and 4. Saunderson's Algebra, vol. 2, pa. 687, &c.

TRANSIT, in Astronomy, denotes the passage of any planet, just before or over another planet or star; or the passing of a star or planet over the meridian, or before an astronomical instrument. Venus and Mercury, in their transits over the sun, appear like dark specks.

The transits of Venus and Mercury over the sun's disc are very interesting phenomena, not merely on account of their rare and singular appearance, but also because of their use in determining the sun's parallax, and thence the real dimensions of the earth's orbit. Hence the times when these transits are to be seen have been very carefully computed. Dr. Halley computed the times of a number of these visible transits, for the 17th and 18th centuries, which were published in the Philos. Trans. No. 193, or my Abridg. vol. 3, pa. 448; and several others have been since computed. The following are the times when there were or will be transits of Mercury, from the year 1753 to 1894 inclusive.

1753	May 5	1832	May 5
1756	Nov. 6	1835	Nov. 7
1769	Nov. 9	1845	May 8
1776	Nov. 2	1848	Nov. 9
1782	Nov. 12	1861	Nov. 11
1786	May 5	1868	Nov. 4
1789	Nov. 5	1878	May 5
1799	May 7	1881	Nov. 7
1802	Nov. 8	1891	May 9
1815	Nov. 11	1894	Nov. 10
1822	Nov. 4		

It appears from this table, that the transits of Mercury always occur either in May or in November; but most frequently in the latter month; depending on the position of the elliptic projection of Mercury's orbit on the plane of the ecliptic. This ellipse is now so placed, that it presents to us its perihelion, during the winter, and its aphelion during the summer; and as it is very excentric, Mercury is much nearer the sun in the month of November than in May. Now if it be considered that the luminous cone formed by the visual rays, drawn from the earth to the sun, is contracted in the vicinity of the earth, while it is enlarged near the sun, the disc of which serves for its base; Mercury ought therefore to cut it more readily when it is near the sun, than when it is remote from it; and consequently the transits of Mercury ought to occur most frequently in the winter part of the year.

From the observations of the transit of Nov. 8, 1802, it was inferred that the node of the planet's orbit was in $1^s\ 15° 58' 56''$.

The transits of Venus across the sun's disc happen much less frequently than those of Mercury, because Venus is more distant from the sun. The following are all that occur between 1631 and 2110.

1631	Dec. 6	1874	Dec. 8
1639	Dec. 4	1882	Dec. 6
1761	June 5	2004	June 7
1769	June 3	2109	Dec. 10

Now the chief use of these conjunctions is, accurately to determine the sun's distance from the earth, or his parallax, which astronomers have in vain attempted to find by various other methods; for the minuteness of the requisite angles easily eludes the nicest instruments. But in observing the ingress of Venus into the sun, and her egress from the same, the interval between the moments of the internal contacts, observed to a second of time, that is, to $\frac{1}{15}$ of a second, or $4'''$ of an arch, may be obtained by the assistance of a moderate telescope, and a pendulum clock that goes uniformly for 6 or 8 hours. Now from two such observations, rightly made in proper places, the distance of the sun, within a 500th part, may be certainly concluded, &c.—The only observations that have been made, were those of 1639, 1761, and 1769; whence the sun's parallax has been inferred to be $8''\cdot6$. See PARALLAX and VENUS.

TRANSIT *Instrument*, in Astronomy, is a telescope fixed at right angles to a horizontal axis; this axis being so supported that the line of collimation may move in the plane of the meridian.

The axis, to the middle of which the telescope is fixed, should gradually taper towards its ends, and terminate in cylinders well turned and smoothed; and a proper weight or balance is put on the tube, so that it may stand at any elevation when the axis rests on the supporters. Two upright posts of wood or stone, firmly fixed at a proper distance, are to sustain the supporters to this instrument; these supporters are two thick brass plates, having well smoothed angular notches in their upper ends to receive the cylindrical arms of the axis; each of the notched plates is contrived to be moveable by a screw, which slides them upon the surfaces of two other plates immoveably fixed to the two upright posts; one plate moving in a vertical direction, and the other horizontally, they adjust the telescope to the planes of the horizon and meridian; to the plane of the horizon, by a spirit level hung in a position parallel to the axis, and to the plane of the meridian in the following manner. Observe the times by the clock when a circumpolar star, seen through this instrument, transits both above and below the pole; then if the times of describing the eastern and western parts of its circuit be equal, the telescope is then in the plane of the meridian; otherwise the notched plates must be gently moved till the time of the star's revolution is bisected by both the upper and lower transits, taking care at the same time that the axis keeps its horizontal position.

When the telescope is thus adjusted, a mark must be set up, or made, at a considerable distance (the greater the better) in the horizontal direction of the intersection of the cross wires, and in a place where it can be illuminated in the night-time by a lanthorn near it, which mark, being on a fixed object, will serve at all times afterwards to examine the position of the telescope, by first adjusting the tranverse axis by the level.

To adjust a clock by the sun's transit over the meridian, note the times by the clock, when the preceding and following edges of the sun's limb touch the cross wires: the difference between the middle time and 12 hours, shows how much the mean, or clock time, is faster and slower than the apparent or solar time, for that day; to which the equation of time being applied, it will show the time of mean noon for that day, by which the clock may be adjusted.

TRANSMISSION, in Optics, &c, denotes the property of a transparent or translucent body, by which it admits the rays of light to pass through its substance; in which sense, the word stands opposed to reflection.—For the cause of transmission, or the reason why some bodies transmit the rays, and others reflect them, see TRANSPARENCY and OPACITY.—The rays of light, Newton observes, are subject to fits of easy transmission and reflection. See LIGHT, and REFLECTION.

TRANSMUTATION, or TRANSFORMATION, in Geometry, denotes the reduction or change of one figure or body into another of the same area or solidity; as a triangle into a square, a pyramid into a cube, &c.

TRANSMUTATION, in the Higher Geometry, has been used for the converting of a figure into another of the same kind and order, whose respective parts rise to the same dimensions in an equation, and admit the same tangents, &c.—If a rectilineal figure be to be transmuted into another, it is sufficient that the intersections of the lines which compose it be transferred, and lines drawn through the same in the new figure. But if the figure to be transmuted be curvilinear the points, tangents, and other right lines, by means of which the curve line is to be defined, must be transferred.

TRANSOM, among Builders, the piece that is framed across a double light window.

TRANSOM, among Mathematicians, denotes the vane of a cross-staff; being a wooden member fixed across it, with a square upon which it slides, &c.

TRANSPARENCY, or TRANSLUCENCY, in Physics, a quality in certain bodies, by which they give passage to the rays of light. The transparency of natural bodies, as glass, water, air, &c, is ascribed by some, to the great number and size of the pores or interstices between the particles of those bodies. But this account is very defective; for the most solid and opaque body in nature, that we know of, contains a great deal more of pores than it does matter; surely a great deal more than is necessary for the passage of so very fine and subtle a body as light.

Aristotle, Descartes, &c, make transparency to consist in straightness or rectilineal direction of the pores; by means of which, say they, the rays can pass freely through, without striking against the solid parts, and so being reflected back again. But this account, Newton shows, is imperfect; the quantity of pores in all bodies being sufficient to transmit all the rays that fall upon them, however those pores be situated with respect to each other.

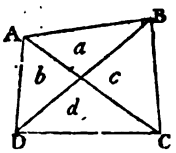
The reason then why all bodies are not transparent, is not to be ascribed to their want of rectilineal pores; but either to the unequal density of the parts, or to the pores being filled with some foreign matters, or to their being quite empty, by means of which the rays, in passing through, undergoing a great variety of reflections and refractions, are perpetually diverted different ways, till at length falling on some of the solid parts of the body, they are extinguished and absorbed.

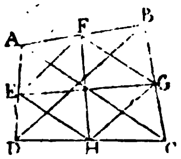
Thus cork, paper, wood, &c, are opake; while glass, diamonds, &c, are transparent; and the reason is, that in the neighbourhood of parts equal in density with respect to each other, as these latter bodies, the attraction being equal on every side, no reflection or refraction ensues: but the rays which entered the first surface of the body proceed quite through it without interruption, those few only excepted that chance to meet with the solid parts: but in the neighbourhood of parts that differ much in density, such as the parts of wood and paper are, both in respect of themselves and of the air, or the empty space in their pores; as the attraction is very unequal, the reflections and refractions must be very great; and therefore the rays will not be able to make their way through such bodies, but will be variously deflected, and at length quite stopped. See OPACITY.

TRANSPOSITION, in Algebra, is the bringing any term of an equation over to the other side of it. Thus if $a + x = c$, and we make $x = c - a$, then a is said to be transposed. This operation is to be performed in order to bring all the known terms to one side of the equation, and all those that are unknown to the other side of it; and every term thus transposed must always have its sign changed, from + to −, or from − to +; which in fact is no more than subtracting or adding such term on both sides of the equation. See REDUCTION of Equations.

TRANSVERSE-*Axis*, or *Diameter*, in the Conic Sections, is the first or principal diameter, or axis. See AXIS, DIAMETER, and LATUS TRANSVERSUM. In an ellipse, the transverse is the longest of all the diameters; but the shortest of all in the hyperbola; and in the parabola the diameters are all equal, or at least in a ratio of equality.

TRAPEZIUM, in Geometry, a plane figure of four straight sides, of which the opposites are not parallel.—When this figure has two of its sides parallel to each other, it is sometimes called a trapezoid.—The chief properties of the trapezium are as follow: 1. Any three sides of a trapezium taken together, are greater than the 4th side.—2. The two diagonals of any trapezium divide it into four proportional triangles, a, b, c, d; that is, the triangle $a : b :: c : d$.—3. The sum of all the four inward angles, A, B, C, D, taken together, is equal to 4 right angles, or 360°.





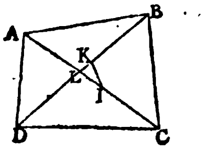
4. In a trapezium ABCD, if all the sides be bisected, in the points E, F, G, H, the figure EFGH formed by joining the points of bisection will be a parallelogram, having its opposite sides parallel to the corresponding diagonals of the trapezium, and the area of the said inscribed parallelogram is just equal to half the area of the trapezium.—5. The sum of the squares of the diagonals of the trapezium, is equal to double the sum of the squares of the diagonals of the parallelogram, or of the two lines drawn to bisect the opposite sides of the trapezium. That is,

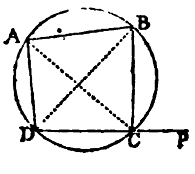
$$AC^2 + BD^2 = 2EG^2 + 2FH^2.$$

6. In any trapezium, the sum of the squares of all the four sides, is equal to the sum of the squares of the two diagonals together with 4 times the square of the line KI joining their middle points. That is,

$$AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2 + 4IK^2.$$

7. In any trapezium, the sum of the two diagonals, is less than the sum of any four lines that can be drawn, to the four angles, from any point within the figure, besides the intersection of the diagonals.—8. The area of any trapezium, is equal to half the rectangle or product under





either diagonal and the sum of the two perpendiculars drawn upon it from the two opposite angles.—9. The area of any trapezium may also be found thus: Multiply the two diagonals together, then that product, multiplied by the sine of their angle of intersection, to the radius 1, will be the area. That is, $AC \times BD \times \sin. \angle L =$ area. —10. The same area will be otherwise found thus: Square each side of a trapezium, add the squares of each pair of opposite sides together, subtract the less sum from the greater, multiply the remainder by the tangent of the angle of intersection of the diagonals (to radius 1), and ¼ of the product will be the area: that is,

$$[(AB^2 + DC^2) - (AD^2 + BC^2)] \times \tfrac{1}{4} \text{tang.} \angle L = \text{area.}$$

11. The area of a trapezoid, or one that has two sides parallel, is equal to the rectangle or product under the sum of the two parallel sides and the perpendicular distance between them.—12. If a trapezium be inscribed in a circle; the sum of any two opposite angles is equal to two right angles; and if the sum of two opposite angles be equal to two right angles, the sum of the other two will also be equal to two right angles, and a circle may be described about it; and farther, if one side, as DC, be produced, the external angle will be equal to the interior opposite angle. That is, (last fig. above)

$$\angle A + \angle C = \angle B + \angle D = 2 \text{ right angles,}$$
$$\text{and } \angle A = \angle BCP.$$

13. If a trapezium be inscribed in a circle; the rectangle of the two diagonals, is equal to the sum of the two rectangles contained under the opposite sides. That is, $AC \times BD = AB \times DC + AD \times BC$.

14. If a trapezium be inscribed in a circle; its area may be found thus: Multiply any two adjacent sides together, and the other two sides together; then add these two products together, and multiply the sum by the sine of the angle included by either of the pairs of sides that are multiplied together, and half this last product will be the area. That is, the area is equal either

to $(AB \times AD + CB \times CD) \times \tfrac{1}{2} \sin. \angle A$ or $\angle C$,
or $(AB \times BC + AD \times DC) \times \tfrac{1}{2} \sin. \angle B$ or $\angle D$.

15. Or, when the trapezium can be inscribed in a circle, the area may be otherwise found thus: Add all the four sides together, and take half the sum; then from this half sum subtract each side severally; multiply the four remainders continually together, and the square root of the last product will be the area.

16. Lastly, the area of the trapezium inscribed in a circle may be otherwise found thus:

$$\begin{aligned} \text{Put } m &= AB \times BC + AD \times DC, \\ n &= BA \times AD + BC \times CD, \\ p &= AB \times DC + AD \times BC, \\ r &= \text{radius of the circumscribing circle,} \end{aligned}$$

then $\sqrt{mnp} \div 4r =$ the area of the trapezium.

TRAPEZOID, sometimes denotes a trapezium that has two of its sides parallel to each other; and sometimes an irregular solid figure, having four sides not parallel to each other. See Trapezium.

TRAVERSE, in Gunnery, is the turning a piece of ordnance about, as upon a centre, to make it point in any particular direction.

Traverse, in Fortification, is a trench with a little parapet, sometimes two, one on each side, to serve as a cover from the enemy that might come in flank.

Traverse, in a wet foss, is a kind of gallery, made by throwing saucissons, joists, fascines, stones, earth, &c, into the foss, opposite the place where the miner is to be put, in order to fill up the ditch, and make a passage over it.

Traverse also denotes a wall of earth, or stone, raised across a work, to stop the shot from rolling along it.

Traverse is also used for any retrenchment, or line fortified with fascines, barrels or bags of earth, or gabions.

Traverse, in Navigation, is the variation of a ship's course, occasioned by the shifting of the winds, or currents, &c; or a traverse is a compound course, consisting of several different courses and distances.

Traverse *Sailing*, is the method of working, or calculating traverses, or compound courses, so as to bring them into one, &c. Traverse sailing is used when a ship, having sailed from one port towards another, whose course and distance from the former is known, is by reason of contrary winds, or other accidents, forced to shift and sail upon several courses, which are to be reduced into one course, in order to determine, after so many turnings and windings, the true course and distance made good, or the true point the ship is arrived at; and so to know what is the true distance, and the new course to be steered, to arrive at the intended port.

To Construct a Traverse. Assume a convenient point or centre, to begin at, to represent the place sailed from. From that point as a centre, with the chord of 60°, describe a circle, which quarter with two perpendicular lines intersecting in the centre, one to represent the meridian, or north-and-south line, and the other the east-and-west line. From the intersections of these lines with the circle, set off upon the circumference, the arcs or degrees, taken from the chords, for the several courses that have been sailed upon, marking the points they reach to, in the circumference, with the figures for the order or number of the courses, 1, 2, 3, 4, &c; and from the centre draw lines to these several points in the circumference, or conceive them to be drawn. On the first of these lines lay off the first distance sailed; from the extremity of this distance draw a line parallel to the second radius, or line drawn in the circle, upon which lay off the 2d distance; through the end of this 2d distance draw a line parallel to the 3d radius, for the direction of the 3d course, and on it lay off the 3d distance; and so on, through all the courses and distances. This done, draw a line from the centre to the end of the last distance, which will be the whole distance made good, and it will cut the circle in a point showing the course made good. Lastly, draw a line from the end of the last distance to the point representing the port bound to, and it will show the distance and course yet to be sailed, to gain that port.

To work a Traverse, or to compute it by the Traverse Table, of Difference of Latitude and Departure.

Make a little tablet with 6 columns; the 1st for the courses, the 2d for the distances, the 3d for the northing, the 4th for the southing, the 5th for the easting, and the 6th for the westing; first entering the several courses and distances, in so many lines, in the 1st and 2d columns. Then, from the traverse table, take out the quantity of the northings or southings, and eastings or westings, answering to the several given courses and distances, entering them on their corresponding lines, and in the proper columns of easting, westing, northing and southing. This done, add up into one sum the numbers in each of these last four columns, which will give four sums showing the whole quantity of easting, westing, northing, and southing made good; then take the difference between the whole easting and westing, and also between the northing and southing, so shall these show the spaces made good in these two directions, viz, east or west, and north or south; which being compared with the given difference of latitude and departure, will show those yet to be obtained in sailing to the desired port, and thence the course and distance to it.

Example. A ship from the latitude 28° 32′ north, bound to a port distant 100 miles, and bearing NE by N, has run the following courses and distances, viz, 1st, NW by N dist. 20 miles; 2d, SW 40 miles; 3d, NE by E 60 miles; 4th, SE 55 miles; 5th, W by S 41 miles; 6th, ENE 66 miles. Required her present latitude, with the direct course and distance made good, and those for the port bound to.

The numbers being taken out of the traverse table, and entered opposite the several courses and distances, the tablet will be as here follows:

Courses.	Dist.	N.	S.	E.	W.
NW by N	20	16·6	.	.	11·1
SW	40	.	28·3	.	28·3
NE b E	60	33·3	.	49·9	.
SE	55	.	38·9	38·9	.
W b S	41	.	8·0	.	40·2
ENE	66	25·3	.	61·0	.
		75·2	75·2	149·8	79·6
		75·2		79·6	
		0		70·2	Dep.

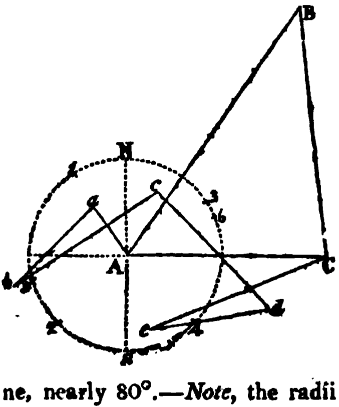
where the sums of the northings and southings, being both alike, 75·2, shows that the ship is come to the same parallel of latitude she set out from. And the difference between the sums of the eastings and westings, shows that the ship is 70·2 miles more to the eastward, that being the greater. Consequently the course made good is due east; and the distance is 70·2 miles.

But, by the traverse table, the northing and easting to the proposed course NE by N, and distance 100, are thus,

viz, northing 83·1 and easting 55·6
diff. from made good 0 and easting 70·2
give - - northing 83·1 and westing 14·6

yet to be made good to arrive at the intended port; and therefore, by finding these in the traverse table, answering to them are the intended course and distance, viz, distance 85, and course N 10° W.

The geometrical construction, according to the method before described, gives the figure annexed: where A is the port sailed from, B is the port bound to, C is the place come to, by sailing the several courses and distances A*a*, *ab*, *bc*, *cd*, *de*, and *e*C; then CB is the distance to be sailed to arrive at the port B, and its course, or direction with the meridian, is nearly 10°, or the angle ACB, made with the east-and-west line, nearly 80°.—*Note*, the radii from the centre to the several points in the circumference, are omitted, to prevent a confusion in the figure.



TRAVERSE-*Board*, in a ship, a small round board, hanging up in the steerage, and pierced full of holes in lines showing the points of the compass: upon which, by moving a small peg from hole to hole, the steersman keeps an account how many glasses, that is half hours, the ship steers upon any point.

TRAVERSE-*Table*, in Navigation, is the same with a table of difference of latitude and departure; being the difference of latitude and departure ready calculated to every point, half point, quarter point, degree, &c, of the quadrant; and for every distance, up to 50 or 100 or 120, &c. Though it may serve for any greater distance whatever, by adding two or more together; or by taking their halves, thirds, fourths, &c, and then doubling, tripling, quadrupling, &c, the difference of latitude and departure found to those parts of the distance.

This table is one of the most necessary and useful things a navigator has occasion for; for by it he can readily reduce all his courses and distances, run in the space of 24 hours, into one course and distance; whence he finds the latitude he is in, and the departure from the meridian.

One of the best tables of this kind is in Robertson's Navigation, at the end of book 7, vol. 1. The distances are there carried to 120, for the sake of more easy subdivisions; and it is divided into two parts; the first containing the courses for every quarter point of the compass, and the 2d adapted to every 15′, or quarter of a degree, in the quadrant. See TRAVERSE *Sailing*.

A specimen of such a traverse table is the following, otherwise called a table of difference of latitude and departure. The distances are placed at top and bottom of the columns, from 1 to 10; but may be extended to any quantity by multiplying the parts, and taking out at several times. The courses, or angles of a right-angled triangle, are in a column, on both sides, each in two parts, the one containing the even points and quarter points, and the other whole degrees, as [illegible] as to 45°, or half the quadrant, on the left-hand side, and the other half quadrant, from 45° to 90°, returned upwards from bottom to top on the right-hand side. The corresponding difference of latitude and departure are in two columns below or above the distances, viz, below them when the course or angle is within 45°, or found on the left-hand side; but above them when between 45 and 90°, or found on the right-hand side.

The same table serves also to work all cases of right-angled triangles, for any other purposes. For example, Suppose a given course be 15°, and distance 35 miles, to find the corresponding difference of latitude and the departure: Or, in a right-angled triangle, given the hypothenuse 35, and one angle 15°, to find the two legs.—Here, the distance 3 in the table must be accounted 30, moving the decimal point proportionally or one place in the other numbers; and those numbers taken out at twice, viz, once from the columns under 3 for the 30, and the other from the columns under the distance 5. Thus, on the line of 15°, and under the

	Dist.		*Lat.*		*Dep.*
	30	are	28·978	and	7·765
	5	are	4·830	and	1·294
therefore for	35	are	33·808	and	9·059

So that the other two legs of the triangle are 33·808 and 9·059. If the course had been 75°, or the complement of the former, which is only the other angle of the same triangle, and which is found on the same line of the table, but on the right-hand side of it; then the numbers in the columns will be the same as before, and will give the same sums for the two legs of the triangle, only with the contrary names, as to latitude and departure, which change places.

A TABLE *of the Difference of Latitude and Departure, for Degrees and Quarter Points.*

Course.		Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.		Course.	
Pts.	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D.	Pts.
	1	0·9998	0·0175	1·9997	0·0349	2·9995	0·0524	3·9994	0·0698	4·9992	0·0873	89	
	2	0·9994	0·0349	1·9988	0·0698	2·9982	0·1047	3·9976	0·1396	4·9970	0·1745	88	
0 ¼		0·9988	0·0491	1·9976	0·0981	2·9964	0·1472	3·9952	0·1963	4·9940	0·2453		7 ¾
	3	0·9986	0·0523	1·9973	0·1047	2·9959	0·1570	3·9945	0·2093	4·9931	0·2617	87	
	4	0·9976	0·0698	1·9951	0·1395	2·9927	0·2093	3·9903	0·2790	4·9878	0·3488	86	
	5	0·9962	0·0872	1·9924	0·1743	2·9886	0·2615	3·9848	0·3486	4·9810	0·4358	85	
0 ½		0·9952	0·0980	1·9904	0·1960	2·9856	0·2940	3·9807	0·3921	4·9759	0·4901		7 ½
	6	0·9945	0·1045	1·9890	0·2091	2·9836	0·3136	3·9781	0·4181	4·9726	0·5226	84	
	7	0·9925	0·1219	1·9851	0·2437	2·9776	0·3656	3·9702	0·4875	4·9627	0·6093	83	
	8	0·9903	0·1392	1·9805	0·2783	2·9708	0·4175	3·9611	0·5567	4·9513	0·6959	82	
0 ¾		0·9892	0·1467	1·9784	0·2935	2·9675	0·4402	3·9567	0·5869	4·9459	0·7337		7 ¼
	9	0·9877	0·1564	1·9754	0·3129	2·9631	0·4693	3·9508	0·6257	4·9384	0·7822	81	
	10	0·9848	0·1736	1·9696	0·3473	2·9544	0·5209	3·9392	0·6946	4·9240	0·8682	80	
	11	0·9816	0·1908	1·9633	0·3816	2·9449	0·5724	3·9265	0·7632	4·9081	0·9540	79	
1		0·9808	0·1951	1·9616	0·3902	2.9424	0·5853	3·9231	0·7804	4·9039	0·9754		7
	12	0·9781	0·2079	1·9563	0·4158	2·9344	0·6237	3·9126	0·8316	4·8907	1·0396	78	
	13	0·9744	0·2250	1·9487	0·4499	2·9231	0·6749	3·8975	0·8998	4·8718	1·1248	77	
	14	0·9703	0·2419	1·9406	0·4838	2·9108	0·7258	3·8812	0·9677	4·8515	1·2096	76	
1 ¼		0·9700	0·2430	1·9401	0·4860	2·9101	0·7289	3·8801	0·9719	4·8502	1·2149		6 ¾
	15	0·9659	0·2588	1·9319	0·5176	2·8978	0·7765	3·8637	1·0353	4·8296	1·2941	75	
	16	0·9613	0·2756	1·9225	0·5513	2·8838	0·8269	3·8450	1·1025	4·8063	1·3782	74	
1 ½		0·9569	0·2903	1·9139	0·5806	2·8708	0·8709	3·8278	1·1611	4·7847	1·4514		6 ½
	17	0·9563	0·2924	1·9126	0·5847	2·8689	0·8771	3·8252	1·1695	4·7815	1·4619	73	
	18	0·9511	0·3090	1·9021	0·6180	2·8532	0·9271	3·8042	1·2361	4·7553	1·5451	72	
	19	0·9455	0·3256	1·8910	0·6511	2·8366	0·9767	3·7821	1·3023	4·7276	1·6278	71	
1 ¾		0·9415	0·3369	1·8831	0·6738	2·8246	1·0107	3·7662	1·3476	4·7077	1·6844		6 ¼
	20	0·9397	0·3420	1·8794	0·6840	2·8191	1·0261	3·7588	1·3681	4·6985	1·7101	70	
	21	0·9336	0·3584	1·8672	0·7167	2·8007	1·0751	3·7343	1·4335	4·6679	1·7918	69	
	22	0·9272	0·3746	1·8544	0·7492	2·7816	1·1238	3·7087	1·4984	4·6359	1·8730	68	
2		0·9239	0·3827	1·8478	0·7654	2·7716	1·1480	3·6955	1·5307	4·6194	1·9134		6
	23	0·9205	0·3907	1·8410	0·7815	2·7615	1·1722	3·6820	1·5629	4·6025	1·9537	67	
	24	0·9135	0·4067	1·8270	0·8135	2·7406	1·2202	3·6542	1·6269	4·5677	2·0337	66	
	25	0·9063	0·4226	1·8126	0·8452	2·7189	1·2679	3·6252	1·6905	4·5315	2·1131	65	
2 ¼		0·9040	0·4276	1·8080	0·8551	2·7120	1·2827	3·6160	1·7102	4·5199	2·1378		5 ¾
	26	0·8988	0·4384	1·7976	0·8767	2·6964	1·3151	3·5952	1·7535	4·4940	2·1919	64	
	27	0·8910	0·4540	1·7820	0·9080	2·6730	1·3620	3·5640	1·8160	4·4550	2·2699	63	
	28	0·8829	0·4695	1·7659	0·9389	2·6488	1·4084	3·5318	1·8779	4·4147	2·3474	62	
2 ½		0·8819	0·4714	1·7638	0·9428	2·6458	1·4142	3·5277	1·8850	4·4096	2·3570		5 ½
	29	0·8746	0·4848	1·7492	0·9696	2·6239	1·4544	3·4985	1·9392	4·3731	2·4240	61	
	30	0·8660	0·5000	1·7320	1·0000	2·5981	1·5000	3·4641	2·0000	4·3301	2·5000	60	
2 ¾		0·8577	0·5141	1·7155	1·0282	2·5732	1·5423	3·4309	2·0564	4·2886	2·5705		5 ¼
	31	0·8572	0·5150	1·7143	1·0301	2·5715	1·5451	3·4287	2·0602	4·2858	2·5752	59	
	32	0·8480	0·5299	1·6961	1·0598	2·5441	1·5896	3·3922	2·1197	4·2402	2·6496	58	
	33	0·8387	0·5446	1·6773	1·0893	2·5160	1·6339	3·3547	2·1786	4·1934	2·7232	57	
3		0·8315	0·5556	1·6629	1·1111	2·4944	1·6667	3·3259	2·2223	4·1573	2·7778		5
	34	0·8290	0·5592	1·6581	1·1184	2·4871	1·6776	3·3162	2·2368	4·1452	2·7960	56	
	35	0·8192	0·5736	1·6383	1·1472	2·4575	1·7207	3·2766	2·2943	4·0958	2·8679	55	
	36	0·8090	0·5878	1·6180	1·1756	2·4271	1·7634	3·2361	2·3511	4·0451	2·9389	54	
3 ¼		0·8032	0·5957	1·6064	1·1914	2·4096	1·7871	3·2128	2·3828	4·0160	2·9785		4 ¾
	37	0·7986	0·6018	1·5973	1·2036	2·3959	1·8054	3·1945	2·4073	3·9932	3·0091	53	
	38	0·7880	0·6157	1·5760	1·2313	2·3640	1·8470	3·1520	2·4626	3·9401	3·0783	52	
	39	0·7771	0·6293	1·5543	1·2586	2·3314	1·8880	3·1086	2·5173	3·8857	3·1466	51	
3 ½		0·7730	0·6344	1·5460	1·2688	2·3190	1·9032	3·0920	2·5376	3·8650	3·1720		4 ½
	40	0·7660	0·6428	1·5321	1·2856	2·2981	1·9284	3·0642	2·5712	3·8302	3·2139	50	
	41	0·7547	0·6561	1·5094	1·3121	2·2641	1·9682	3·0188	2·6242	3·7736	3·2803	49	
	42	0·7431	0·6691	1·4803	1·3383	2·2294	2·0074	2·9726	2·6765	3·7157	3·3457	48	
3 ¾		0·7410	0·6716	1·4819	1·3431	2·2229	2·0147	2·9638	2·6862	3·7048	3·3578		4 ¼
	43	0·7314	0·6820	1·4628	1·3640	2·1941	2·0460	2·9254	2·7280	3·6568	3·4100	47	
	44	0·7193	0·6947	1·4387	1·3894	2·1580	2·0840	2·8774	2·7786	3·5967	3·4733	46	
4	45	0·7071	0·7071	1·4142	1·4142	2·1213	2·1213	2·8284	2·8284	3·5355	3·5355	45	4
Pts.	Deg.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Deg.	Pts.
		Dist. 1.		Dist. 2.		Dist. 3.		Dist. 4.		Dist. 5.			

A Table of the Difference of Latitude and Departure, for Degrees and Quarter Points.

Course.		Dist. 6.		Dist. 7.		Dist. 8.		Dist. 9.		Dist. 10.		Course.	
Pts.	D.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D	Pts.
	1	5·9991	0·1047	6·9989	0·1222	7·9988	0·1396	8·9986	0·1571	9·9985	0·1745	89	
	2	5·9963	0·2094	6·9957	0·2443	7·9951	0·2792	8·9945	0·3141	9·9939	0·3490	88	
0 ¼		5·9928	0·2944	6·9916	0·3435	7·9904	0·3925	8·9892	0·4416	9·9880	0·4907		7 ¾
	3	5·9918	0·3140	6·9904	0·3664	7·9890	0·4187	8·9877	0·4710	9·9863	0·5234	87	
	4	5·9854	0·4185	6·9829	0·4883	7·9805	0·5580	8·9781	0·6278	9·9756	0·6976	86	
	5	5·9772	0·5229	6·9734	0·6101	7·9696	0·6972	8·9658	0·7844	9·9619	0·8716	85	
0 ½		5·9711	0·5881	6·9663	0·6861	7·9615	0·7841	8·9567	0·8822	9·9518	0·9802		7 ½
	6	5·9671	0·6272	6·9617	0·7317	7·9562	0·8362	8·9507	0·9408	9·9452	1·0453	84	
	7	5·9553	0·7312	6·9478	0·8531	7·9404	0·9750	8·9329	1·0968	9·9255	1·2187	83	
	8	5·9416	0·8350	6·9319	0·9742	7·9221	1·1134	8·9124	1·2526	9·9027	1·3917	82	
0 ¾		5·9351	0·8804	6·9242	1·0271	7·9134	1·1738	8·9026	1·3206	9·8918	1·4674		7 ¼
	9	5·9261	0·9386	6·9138	1·0950	7·9015	1·2515	8·8892	1·4079	9·8769	1·5643	81	
	10	5·9088	1·0419	6·8937	1·2155	7·8785	1·3892	8·8633	1·5628	9·8481	1·7365	80	
	11	5·8898	1·1449	6·8714	1·3357	7·8530	1·5265	8·8346	1·7173	9·8165	1·9081	79	
1		5·8847	1·1705	6·8655	1·3656	7·8463	1·5607	8·8271	1·7558	9·8079	1·9509		7
	12	5·8689	1·2475	6·8470	1·4554	7·8252	1·6633	8·8033	1·8712	9·7815	2·0791	78	
	13	5·8462	1·3497	6·8206	1·5746	7·7950	1·7996	8·7693	2·0246	9·7437	2·2495	77	
	14	5·8218	1·4515	6·7921	1·6935	7·7624	1·9354	8·7327	2·1773	9·7030	2·4192	76	
1 ¼		5·8202	1·4579	6·7902	1·7009	7·7602	1·9438	8·7303	2·1868	9·7003	2·4298		6 ¾
	15	5·7956	1·5529	6·7615	1·8117	7·7274	2·0706	8·6933	2·3204	9·6593	2·5882	75	
	16	5·7676	1·6538	6·7288	1·9295	7·6901	2·2051	8·6513	2·4807	9·6120	2·7562	74	
1 ½		5·7416	1·7417	6·6986	2·0320	7·6555	2·3223	8·6125	2·6126	9·5694	2·9028		6 ½
	17	5·7378	1·7542	6·6941	2·0466	7·6504	2·3390	8·6067	2·6313	9·5630	2·9237	73	
	18	5·7063	1·8541	6·6574	2·1631	7·6084	2·4721	8·5595	2·7812	9·5106	3·0902	72	
	19	5·6731	1·9534	6·6186	2·2790	7·5642	2·6045	8·5097	2·9301	9·4552	3·2557	71	
1 ¾		5·6493	2·0213	6·5908	2·3582	7·5324	2·6951	8·4739	3·0320	9·4154	3·3689		6 ¼
	20	5·6382	2·0521	6·5779	2·3941	7·5175	2·7362	8·4572	3·0782	9·3969	3·4202	70	
	21	5·6015	2·1502	6·5351	2·5086	7·4686	2·8669	8·4022	3·2253	9·3358	8·5837	69	
	22	5·5631	2·2476	6·4903	2·6222	7·4175	2·9969	8·3447	3·3715	9·2718	3·7461	68	
2		5·5433	2·2961	6·4672	2·6788	7·3910	3·0615	8·3149	3·4441	9·2388	3·8268		6
	23	5·5230	2·3444	6·4435	2·7351	7·3640	3·1258	8·2845	3·5166	9·2050	3·9075	67	
	24	5·4813	2·4404	6·3948	2·8472	7·3084	3·2539	8·2219	3·6606	9·1355	4·0674	66	
	25	5·4378	2·5357	6·3442	2·9583	7·2505	3·3809	8·1568	3·8036	9·0631	4·2262	65	
2 ¼		5·4239	2·5653	6·3279	2·9929	7·2319	3·4204	8·1359	3·8480	9·0399	4·2756		5 ¾
	26	5·3928	2·6302	6·2916	3·0686	7·1904	3·5070	8·0891	3·9453	8·9879	4·3837	64	
	27	5·3460	2·7239	6·2370	3·1779	7·1280	3·6319	8·0191	4·0859	8·9101	4·5399	63	
	28	5·2977	2·8168	6·1806	3·2863	7·0636	3·7558	7·9465	4·2252	8·8295	4·6947	62	
2 ½		5·2915	2·8284	6·1734	3·2998	7·0554	3·7712	7·9373	4·2426	8·8192	4·7140		5 ½
	29	5·2477	2·9089	6·1223	3·3937	6·9970	3·8785	7·8716	4·3633	8·7462	4·8481	61	
	30	5·1961	3·0000	6·0622	3·5000	6·9282	4·0000	7·7942	4·5000	8·6603	5·0000	60	
2 ¾		5·1464	3·0846	6·0041	3·5987	6·8618	4·1128	7·7196	4·6269	8·5773	5·1410		5 ¼
	31	5·1430	3·0902	6·0002	3·6052	6·8573	4·1203	7·7145	4·6353	8·5717	5·1504	59	
	32	5·0883	3·1795	5·9363	3·7094	6·7843	4·2394	7·6324	4·7093	8·4805	5·2992	58	
	33	5·0320	3·2678	5·8707	3·8125	6·7094	4·3571	7·5480	4·9018	8·3867	5·4464	57	
3		4·9888	3·3334	5·8203	3·8890	6·6518	4·4446	7·4832	5·0001	8·3147	5·5557		5
	34	4·9742	3·3552	5·8033	3·9144	6·6323	4·4735	7·4613	5·0327	8·2904	5·5919	56	
	35	4·9149	3·4415	5·7341	4·0150	6·5532	4·5880	7·3724	5·1622	8·1915	5·7358	55	
	36	4·8541	3·5267	5·6631	4·1145	6·4721	4·7023	7·2812	5·2901	8·0902	5·8779	54	
3 ¼		4·8192	3·5742	5·6224	4·1699	6·4257	4·7656	7·2289	5·3613	8·0321	5·9570		4 ¾
	37	4·7918	3·6109	5·5904	4·2127	6·3891	4·8145	7·1877	5·4163	7·9864	6·0182	53	
	38	4·7281	3·6940	5·5161	4·3096	6·3041	4·9253	7·0921	5·5409	7·8801	6·1560	52	
	39	4·6629	3·7759	5·4400	4·4052	6·2172	5·0346	6·9943	5·6039	7·7715	6·2932	51	
3 ½		4·6381	3·8064	5·4111	4·4408	6·1841	5·0751	6·9571	5·7095	7·7301	6·3439		4 ½
	40	4·5963	3·8567	5·3623	4·4995	6·1284	5·1423	6·8944	5·7851	7·6604	6·4279	50	
	41	4·5283	3·9363	5·2830	4·5924	6·0377	5·2485	6·7924	5·9045	7·5471	6·5606	49	
	42	4·4589	4·0148	5·2020	4·6839	5·9452	5·3530	6·6883	6·0222	7·4314	6·6913	48	
3 ¾		4·4457	4·0294	5·1867	4·7009	5·9276	5·3725	6·6680	6·0440	7·4095	6·7156		4 ¼
	43	4·3881	4·0920	5·1195	4·7740	5·8508	5·4560	6·5822	6·1380	7·3135	6·8200	47	
	44	4·3160	4·1679	5·0354	4·8626	5·7547	5·5573	6·4741	6·2519	7·1934	6·9466	46	
4	45	4·2426	4·2426	4·9497	4·9497	5·6509	5·6569	6·3640	6·3640	7·0711	7·0711	45	4
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TREBLE, in Music, the highest or acutest of the four parts in symphony, or that which is heard the clearest and shrillest in a concert. In the like sense we say, a treble violin, treble hautboy, &c. In vocal music, the treble is usually committed to boys and girls; their proper part being the treble. The treble is divided into first or highest treble, and second or bass treble. The half treble is the same with the counter-tenor.

TRENCHES, in Fortification, are ditches which the besiegers cut to approach more securely to the place attacked; whence they are called lines of approach. Their breadth is 8 or 10 feet, and depth 6 or 7.—They say, mount the trenches, that is, go upon duty in them. To relieve the trenches, is to relieve such as have been upon duty there. The enemy is said to have cleared the trenches, when he has driven away or killed the soldiers who guarded them.

Tail of the TRENCH, is the place where it was begun. And the *Head* is the place where it ends.

Opening of the TRENCHES, is when the besiegers first begin to work upon them, or to make them; which is usually done in the night.

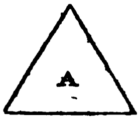
TREPIDATION, in the ancient astronomy, denotes what was called a libration of the 8th sphere; or a motion which the Ptolemaic system attributed to the firmament, to account for certain almost insensible changes and motions observed in the axis of the world; by means of which the latitudes of the fixed stars come to be gradually changed, and the ecliptic appears to approach reciprocally, first towards one pole and then towards the other.—This motion is also called the motion of the first libration.

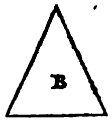
TRET, in Commerce, is an allowance made for the waste, or the dust, that may be mixed with any commodity; which is always 4 pounds on every 104 pounds weight. See TARE.

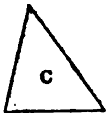
TRIANGLE, in Geometry, a figure bounded or contained by three lines or sides, and which consequently has three angles, whence the figure takes its name.

Triangles are either plane or spherical or curvilinear. Plane when the three sides of the triangle are right lines; but spherical when some or all of them are arcs of great circles on the sphere.

Plane triangles take several denominations, both from the relation of their angles, and of their sides, as below. And 1st with regard to the sides.





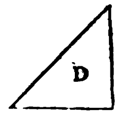


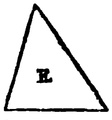
An *Equilateral Triangle*, is that which has all its three sides equal to one another; as A.

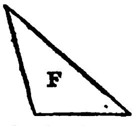
An *Isosceles* or *Equicrural Triangle*, is that which has two sides equal; as B.

A *Scalene Triangle* has all its sides unequal; as C.

Again, with respect to the angles.







A *Rectangular* or *Right-angled Triangle*, is that which has one right angle; as D.

An *Oblique Triangle* is that which has no right angle, but all oblique ones; as E or F.

An *Acutangular* or *Oxygone Triangle*, is that which has three acute angles; as E.

An *Obtusangular* or *Amblygone Triangle*, is that which has an obtuse angle; as F.

A *Curvilinear* or *Curvilineal Triangle*, is one that has all its three sides curve lines.

A *Mixtilinear Triangle*, is one that has its sides some of them curves, and some right lines.

A *Spherical Triangle* is one that has its sides, or at least some of them, arcs of great circles of the sphere.

Similar Triangles are such as have the angles in the one equal to the angles in the other, each to each.

The *Base* of a triangle, is any side on which a perpendicular is drawn from the opposite angle, called the vertex: and the two sides about the perpendicular, or the vertex, are called the legs.

The chief properties of plane triangles, are as follow, viz, In any plane triangle,

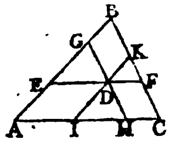
1. The greatest side is opposite to the greatest angle, and the least side to the least angle, &c. Also, if two sides be equal, their opposite angles are equal; and if the triangle be equilateral, or have all its sides equal, it will also be equiangular, or have all its angles equal to one another.—2. Any side of a triangle is less than the sum, but greater than the difference, of the other two sides.—3. The sum of all the three angles, taken together, is equal to two right angles.—4. If one side of a triangle be produced, the external angle, made by it and the adjacent side, is equal to the sum of the two opposite internal angles.—5. A line drawn parallel to one side of a triangle, cuts the other two sides proportionally, the corresponding segments being proportional, each to each, and to the whole sides; and the triangle cut off is similar to the whole triangle.

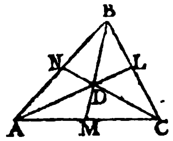
If a perpendicular be let fall from any angle of a triangle, as a vertical angle, upon the opposite side as a base; then, 6. The rectangle of the sum and difference of the sides, is equal to twice the rectangle of the base and the distance of the perpendicular from the middle of the base.—Or, which is the same thing in other words, 7. The difference of the squares of the sides, is equal to the difference of the squares of the segments of the base. Or, as the base is to the sum of the sides, so is the difference of the sides, to the difference of the segments of the base.—8. The rectangle of the legs or sides, is equal to the rectangle of the perpendicular and the diameter of the circumscribing circle.

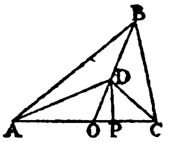
If a line be drawn bisecting any angle, to the base or opposite side; then, 9. The segments of the base, made by the line bisecting the opposite angle, are proportional to the sides adjacent to them.—10. The square of the line bisecting the angle, is equal to the difference between the rectangle of the sides and the rectangle of the segments of the base.

If a line be drawn from any angle to the middle of the opposite side, or bisecting the base; then, 11. The sum of the squares of the sides, is equal to twice the sum of the squares of half the base and the line bisecting the base.—12. The angle made by the perpendicular from any angle and the line drawn from the same angle to the middle of the base, is equal to half the difference of the angles at the base.—13. If through any point D, within a triangle ABC, three lines EF, GH, IK, be drawn parallel to the three sides of the triangle; the continual products or solids made by the alternate segments of these lines will be equal; viz,

$$DE \times DK \times DH = DG \times DF \times DI.$$







14. If three lines AL, BM, CN, be drawn from the three angles through any point D within a triangle, to the opposite sides; the solid products of the alternate segments of the sides are equal; viz,

$$AN \times BL \times CM = AM \times CL \times BN.$$

15. Three lines drawn from the three angles of a triangle to bisect the opposite sides, or to the middle of the opposite sides, do all intersect one another in the same point D, and that point is the centre of gravity of the triangle, and the distance AD of that point from any angle as D, is equal to double the distance DL from the opposite side; or one segment of any of these lines is double the other segment: moreover the sum of the squares of the three bisecting lines, is $\frac{3}{4}$ of the sum of the squares of the three sides of the triangle.—16. Three perpendiculars bisecting the three sides of a triangle, all intersect in one point, and that point is the centre of the circumscribing circle.—17. Three lines bisecting the three angles of a triangle, all intersect in one point, and that point is the centre of the inscribed circle.—18. Three perpendiculars drawn from the three angles of a triangle, upon the opposite sides, all intersect in one point.—19. If the three angles of a triangle be bisected by the lines AD, BD, CD (3d fig. above), and any one as BD be continued to the opposite side at O, and DP be drawn perp. to that side; then is $\angle ADO = \angle CDP$, or $\angle ADP = \angle CDO$.

20. Any triangle may have a circle circumscribed about it, or touching all its angles, and a circle inscribed within it, or touching all its sides.—21. The square of the side of an equilateral triangle, is equal to 3 times the square of the radius of its circumscribing circle.—22. If the three angles of one triangle be equal to the three angles of another triangle, each to each; then those two triangles are similar, and their like sides are proportional to one another, and the areas of the two triangles are to each other as the squares of their like sides.—23. If two triangles have any three parts of the one (except the three angles), equal to three corresponding parts of the other, each to each; those two triangles are not only similar, but also identical, or having all their six corresponding parts equal, and their areas also equal.—24. Triangles standing on the same base, and between the same parallels, are equal; and triangles on equal bases, and having equal altitudes, are equal.—25. Triangles on equal bases, are to one another as their altitudes: and triangles of equal altitudes, are to one another as their bases; also equal triangles have their bases and altitudes reciprocally proportional.—26. Any triangle is equal to half its circumscribing parallelogram, or half the parallelogram on the same or an equal base, and of the same or equal altitude.—27. Therefore the area of any triangle is found, by multiplying the base by the altitude, and taking half the product.—28. The area is also found thus: Multiply any two sides together, and multiply the product by the sine of their included angle, to radius 1, and divide by 2.—29. The area is also otherwise found thus, when the three sides are given: Add the three sides together, and take half their sum; then from this half sum subtract each side severally, and multiply the three remainders and the half sum continually together; then the square root of the last product will be the area of the triangle.—30. In a right-angled triangle, if a perpendicular be let fall from the right angle upon the hypothenuse, it will divide it into two other triangles similar to each other, and to the whole triangle.—31. In a right-angled triangle, the square of the hypothenuse is equal to the sum of the squares of the two sides; and, in general, any figure described on the hypothenuse, is equal to the sum of two similar figures described on the two sides.—32. In an isosceles triangle, if a line be drawn from the vertex to any point in the base; the square of that line together with the rectangle of the segments of the base, is equal to the square of one of the equal sides.—33. If one angle of a triangle be equal to 120°; the square of the base will be equal to the squares of both the sides, together with the rectangle of those sides; and if those sides be equal to each other, then the square of the base will be equal to three times the square of one side, or equal to 12 times the square of the perpendicular from the angle upon the base.—34. In the same triangle, viz, having one angle equal to 120°; the difference of the cubes of the sides, about that angle, is equal to a solid contained by the difference of the sides and the square of the base; and the sum of the cubes of the sides, is equal to a solid contained by the sum of the sides and the difference between the square of the base and twice the rectangle of the sides.

There are many other properties of triangles to be found in the geometrical writings; indeed Gregory St. Vincent has written a folio volume upon triangles; there are also several in his quadrature of the circle. See also other properties under the article TRIGONOMETRY, and under RIGHT-*Angled Triangle*.

Solution of TRIANGLES. See TRIGONOMETRY.

TRIANGLE, in Astronomy, one of the 48 ancient constellations, situated in the northern hemisphere. There is also the southern triangle in the southern hemisphere, which is a modern constellation. The stars in the northern triangle are, in Ptolemy's catalogue 4, in Tycho's 4, in Hevelius's 12, and in the British catalogue 16. The stars in the southern triangle are, in Sharp's catalogue, 5.

Arithmetical TRIANGLE, a kind of numeral triangle, or triangle of numbers, being a table of certain numbers disposed in form of a triangle. It was so called by Pascal; but he was not the inventor of this table, as some writers have imagined, its properties having been treated of by other authors, some centuries before him, as is shown in my Mathematical Tracts, vol. 1, tract 12.

The form of the triangle is as follows:

1					
1	1				
1	2	1			
1	3	3	1		
1	4	6	4	1	
1	5	10	10	5	1
1	6	15	20	&c	
1	7	21	&c		
1	8	&c			
1	9				

And it is constructed by adding always the last two numbers of the next two preceding columns together, to give the next succeeding column of numbers.

The first vertical column consists of units; the 2d a series of the natural numbers 1, 2, 3, 4, 5, &c; the 3d a series of triangular numbers 1, 3, 6, 10, &c; the 4th a series of pyramidal numbers, &c. The oblique diagonal rows, descending from left to right, are also the same as the vertical columns. And the numbers taken on the horizontal lines are the co-efficients of the different powers of a binomial. Many other properties and uses of these numbers have been delivered by various authors, as may be seen in the Introduction to my Mathematical Tables, pa. 7, 8, 75, 76, 77, 89, 2d edition.

After these, Pascal wrote a treatise on the Arithmetical Triangle, which is contained in the 5th volume of his works, published at Paris and the Hague in 1779, in 5 volumes, 8vo. In this publication is also a description, taken from the 1st volume of the French Encyclopedic, art. Arithmetique Machine, of that admirable machine invented by Pascal at the age of 19, furnishing an easy and expeditious method of making all kinds of arithmetical calculations without any other assistance than the eye and the hand.

TRIANGULAR, relating to a triangle; as

TRIANGULAR *Canon*, tables relating to trigonometry; as of sines, tangents, secants, &c.

TRIANGULAR *Compasses*, are such as have three legs or feet, by which any triangle, or three points, may be taken off at once. These are very useful in the construction of maps, globes, &c.

TRIANGULAR *Numbers*, are a kind of polygonal numbers; being the sums of arithmetical progressions, which have 1 for the common difference of their terms.
Thus, from these arithmeticals - 1 2 3 4 5 6,
are formed the triang. numb. - 1 3 6 10 15 21,
or the 3d column of the arithmetical triangle above mentioned.

Because the sum of n terms of such arithmetical progression is expressed by $\frac{n^2+n}{2}$; we shall evidently have the same formula to express generally the triangular numbers; or the triangle, which answers to any side represented by n.

Thus, if $n = 6$, the sixth triangular number taken in order will be $\frac{36+6}{2} = 21$. And if $n = 15$, the triangle is $\frac{225+15}{2} = 120$.

The sum of any number n of the terms of the triangular numbers, 1, 3, 6, 10, &c; is =

$$\frac{n^3}{6} + \frac{n^2}{2} + \frac{n}{3}, \text{ or } \frac{n}{1} \times \frac{n+1}{2} \times \frac{n+2}{3},$$

which is also equal to the number of shot in a triangular pile of balls, the number of rows, or the number in each side of the base, being n.

The sum of the reciprocals of the triangular series, infinitely continued, is equal to 2; viz,

$$1 + \tfrac{1}{3} + \tfrac{1}{6} + \tfrac{1}{10} + \tfrac{1}{15} \ \&c = 2.$$

For the rationale and management of these numbers, see Malcolm's Arith. book 5, ch. 2; and Simpson's Algeb. sec. 15.

TRIANGULAR *Quadrant*, is a sector furnished with a loose piece, by which it forms an equilateral triangle. Upon it is graduated and marked the calendar, with the sun's place, and other useful lines; and by the help of a string and a plummet, with the divisions graduated on the loose piece, it may be made to serve for a quadrant.

TRIBOMETER, in Mechanics, a term applied by Musschenbroek to an instrument invented by him for measuring the friction of metals. It consists of an axis formed of hard steel, passing through a cylindrical piece of wood: the ends of the axis, which are highly polished, are made to rest on the polished semicircular cheeks of various metals, and the degree of friction is estimated by means of a weight suspended by a fine silken string or ribband over the wooden cylinder. For a farther description and the figure of this instrument, with the results of various experiments performed with it, see Musschenb. Introd. ad Phil. Nat. vol. 1, p. 151.

TRIDENT, is a particular kind of parabola, used by Descartes in constructing equations of 6 dimensions. See the article *Cartesian* PARABOLA.

TRIGLYPH, in Architecture, is a member of the Doric frize, placed directly over each column, and at equal distances in the intercolumnation, having two entire glyphs or channels engraven in it, meeting in an angle, and separated by three legs from the two demi-channels of the sides.

TRIGON, a figure of three angles, or a triangle.

TRIGON, in Astronomy, denotes an aspect of two planets when they are 120 degrees distant from each other; called also a trine, being the 3d part of 360 degrees.

TRIGON, in Dialling, an instrument of a triangular form.

TRIGON, in Music, denoted a musical instrument, used among the ancients. It was a kind of triangular lyre, or harp, invented by Ibycus; and was used at feasts, being played on by women, who struck it either with a quill, or beat it with small rods of different lengths and weights, to occasion a diversity in the sounds.

TRIGONAL *Numbers*. See TRIANGULAR *Numbers*.

TRIGONOMETER, *Armillary*. See ARMILLARY *Trigonometer*.

TRIGONOMETRY, the art of measuring the sides and angles of triangles, either plane or spherical; whence it is accordingly called either plane trigonometry, or spherical trigonometry.

Every triangle has 6 parts, viz, 3 sides, and 3 angles; and it is necessary that three of these parts be given, to find the other three. In spherical trigonometry, the three parts that are given, may be of any kind, either all sides, or all angles, or part the one and part the other. But in plane trigonometry, it is necessary that one of the three parts at least be a side, since from three angles can only be found the proportions of the sides, but not the real quantities of them.

Trigonometry is an art of the greatest use in the mathematical sciences, especially in astronomy, navigation, surveying, dialling, geography, &c, &c. By the aid of it, we can determine the magnitude of the earth, the planets and stars, their distances, motions, eclipses, and almost all other useful arts and sciences. Accordingly we find this art has been cultivated from the earliest ages of mathematical knowledge.

Trigonometry, or the resolution of triangles, is founded on the mutual proportions which subsist between the sides and angles of triangles; which proportions are known by finding the relations between the radius of a circle and certain other lines drawn in and about the same, called chords, sines, tangents, and secants. The ancients Menelaus, Hipparchus, Ptolemy, &c, performed their trigonometry, by means of the chords. As to the sines, and the common theorems relating to them, they were introduced into trigonometry by the Moors or Arabians, from whom this art passed into Europe, with several other branches of science. The Europeans have introduced, since the 15th century, the tangents and secants, with the theorems relating to them. See the history and improvements at large, in the Introduction to my Mathematical Tables.

The proportion of the sines, tangents, &c, to their radius, is sometimes expressed in common or natural numbers, which constitute what are called the tables of natural sines, tangents, and secants. Sometimes it is expressed in logarithms, being the logarithms of the said natural sines, tangents, &c; and these constitute the table of artificial sines, &c. Lastly, sometimes the proportion is not expressed in numbers; but the several sines, tangents, &c, are actually laid down upon lines of scales; whence the line of sines, of tangents, &c. See SCALE.

In trigonometry, as angles are measured by arcs of a circle described about the angular point, so the whole circumference of the circle is divided into a great number of parts, as 360 degrees, and each degree into 60 minutes, and each minute into 60 seconds, &c; then any angle is said to consist of so many degrees, minutes, and seconds, as are contained in the arc that measures the angle, or that is intercepted between the legs or sides of the angle.

Now the sine, tangent, and secant, &c, of every degree and minute, &c, of a quadrant, are calculated to the radius 1, and ranged in tables for use; as also the logarithms of the same; forming the triangular canon. And these numbers, so arranged in tables, form every species of right-angled triangles, so that no such triangle can be proposed, but one similar to it may be there found, by comparison with which, the proposed one may be computed by analogy or proportion.

As to the scales of chords, sines, tangents, &c, usually placed on instruments, the method of constructing them is exhibited in the scheme annexed to the article SCALE; which, having the names added to each, needs no farther explanation.

There are usually three methods of resolving triangles, or the cases of trigonometry; viz, geometrical construction, arithmetical computation, and instrumental operation. In the 1st method, the triangle is constructed by drawing and laying down the several parts of their magnitudes given, viz, the sides from a scale of equal parts, and the angles from a scale of chords, or other instrument; then the unknown parts are measured by the same scales, and so they become known.

In the 2d method, having stated the terms of the proportion according to rule, which terms consist partly of the numbers of the given sides, and partly of the sines, &c, of angles taken from the tables, the proportion is then resolved like all other proportions, in which a 4th term is to be found from three given terms, by multiplying the 2d and 3d together, and dividing the product by the first. Or, in working with the logarithms, adding the log. of the 2d and 3d terms together, and from the sum subtracting the log. of the 1st term, then the number answering to the remainder is the 4th term sought.

To work a case instrumentally, as suppose by the log. lines on one side of the two-foot scales: Extend the compasses from the 1st term to the 2d, or 3d, which happens to be of the same kind with it; then that extent will reach from the other term to the 4th. In this operation, for the sides of triangles, is used the line of numbers (marked Num.); and for the angles, the line of sines or tangents (marked sin. and tan.) according as the proportion respects sines or tangents.

In every case of triangles, as has been hinted before, there must be given three parts, one at least of which must be a side. And then the different circumstances, as to the three parts that may be given, admit of three cases or varieties only; viz,

1st. When two of the three parts given, are a side and its opposite angle.—2d, When there are given two sides and their contained angle.—3d, And thirdly, when the three sides are given.

To each of these cases there is a particular rule, or proportion, adapted, for resolving it by.

1st. *The Rule for the 1st Case*, or that in which, of the three parts that are given, an angle and its opposite side are two of them, is this, viz, That the sides are proportional to the sines of their opposite angles,

That is,

As one side given :
To the sine of its opposite angle : :
So is another side given :
To the sine of its opposite angle.

Or, As the sine of an angle given :
To its opposite side : :
So is the sine of another angle given :
To its opposite side.

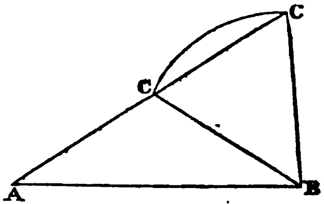
So that, to find an angle, we must begin the proportion with a given side that is opposite to a given angle; and to find a side, we must begin with an angle opposite to a given side.

Note. An angle found by this rule is ambiguous, or uncertain whether it be acute or obtuse, unless it be a right angle, or unless its magnitude be such as to prevent the ambiguity; because the sine answers to two angles, which are supplements to each other; and accordingly the geometrical construction forms two triangles with the same parts that are given, as in the example below; and when there is no restriction or limitation included in the question, either of them may be taken. The degrees in the table, answering to the sine is the acute angle; but if an angle be obtuse, subtract those degrees from 180°, and the remainder will be the obtuse angle. When a given angle is obtuse or a right one, there can be no ambiguity; for then neither of the other angles can be obtuse, and the geometrical construction will form but one triangle.

Ex. Suppose in the plane triangle ABC, there be given

$AB = 345$ yards
$BC = 232$ yards
$\angle A = 37° 20'$

to find the other side and the angles.



1. *Geometrically, by Construction.*

Draw an indefinite line, upon which set off $AB = 345$, from some convenient scale of equal parts.—Make the angle $A = 37° 20'$.—With a radius of 232, taken from the same scale of equal parts, and centre B, cross AC in the two points C, C. Lastly, join BC, BC, and the figure is constructed, which gives two triangles, and showing that the case is ambiguous.

Then the sides AC measured by the scale of equal parts, and the angles B and C measured by the line of chords, or other instrument, will be found to be nearly as follow; viz,

AC 174	∠B 27°	∠C 115½°
or 374½	or 78¼°	or 64½.

2. *Arithmetically, by Tables of Logs.*

First, to find the angles at C.

As side BC 232	- -	log. 2·3654880
To sin. op. ∠A 37° 20′	- -	9·7827958
So side AB 345	- -	2·5378191
To sin. op. ∠C 115° 36′ or 64° 24′	-	9·9551269

add ∠A	37	20	37	20
the sum	152	56	101	44
taken from	180	00	180	00
leaves ∠B	27	04 or	78	16.

Then, to find the side AC.

As sin ∠A 37° 20′	-	log. 9·7827958
To op. side BC 232	- -	2·3654880
So sin ∠B { 27 04	-	9·6580371
{ 78 16	-	9·9908291
To op. side AC 174·07	- -	2·2407293
or, 374·56	- -	2·5735213.

3. *Instrumentally, by Gunter's Lines.*

In the first proportion.—Extend the compasses from 232 to 345 upon the line of numbers; then that extent will reach, in the sines, from 37⅓° to 64½°, the angle C.

In the second proportion.—Extend the compasses from 37⅓° to 27° or 78¼°, on the sines; then that extent will reach, on the line of numbers, from 232 to 174 or 374½; the two values of the side AC.

2*d Case*, when there are given two sides and their contained angle, to find the rest, the rule is this:

As the sum of the two given sides :
Is to the difference of those sides : :
So is the tang. of half the sum of the two opposite angles, or cotangent of half the given angle :
To tang. of half the diff. of those angles.

Then the half diff. added to the half sum, gives the greater of the two unknown angles; and subtracted, leaves the less of the same two angles.

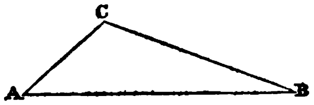
Hence, the angles being now all known, the remaining 3d side will be found by the former case.

Note. When the triangle is isosceles, the angles at the base are each equal to half the supplement of the given angle, or that at the vertex; whence the third side may be found directly by the former case.

Ex. Suppose, in the triangle ABC, there be given

AC = 154·33
BC = 309·86
∠C = 98° 3′

to find the other side and the angles.



1. *Geometrically.*—Draw two indefinite lines making the angle C = 98° 3′: upon these lines set off CA = 154⅓, and CB = 310: Join the points A and B, and the figure is constructed. Then, by measurement, as before, we find the ∠A = 57¼; ∠B 24¾; and side AB = 365.

2. *By Logarithms.*

As CB + CA = 464·19	-	log. 2·6666958
To CB − CA = 155·53	-	2·1918142
So tan. ½A + ½B = 40° 58½′	- -	9·9387803
To tan ½A − ½B = 16 13½	- -	9·4638987
sum gives ∠A 57 12		
diff. gives ∠B 24 45		
Then, As sin. ∠B = 24° 45′	-	log. 9·6218612
To side AC = 154·33	- -	2·1884504
So sin ∠C = 98° 3′, or 81° 57′	-	9·9956993
To side AB = 365	- - -	2·5622885

3. *Instrumentally.*—Extend the compasses from 464 to 155½ upon the line of numbers; then that extent will reach, upon the line of tangents, from 41° to 16¼°. Then, in the 2d proportion, extend the compasses from 24¾° to 82° on the sines; and that extent will reach, upon the numbers, from 154⅓ to 365, which is the third side.

3d *Case*, is when the three sides are given, to find the three angles; and the method of resolving this case is, to let a perpendicular fall from the greatest angle, upon the opposite side or base, dividing it into two segments, and the whole triangle into two right-angled triangles: then it will be,

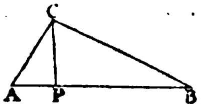
As the base, or sum of the two segments :
Is to the sum of the other two sides : :
So is the difference of those sides :
To the difference of the segments of the base.

Then half this difference of the two segments added to the half sum, or half the base, gives the greater segment, and subtracted, gives the less. Hence, in each of the two right-angled triangles, there are given the hypothenuse, and the base, besides the right angle, to find the other angles by the 1st case.

Ex. In the triangle ABC, suppose there are given the three sides, to find the three angles, viz,

AB = 365
AC = 154·33
BC = 309·86

to find the angles.



1. *Geometrically.*—Draw the base AB = 365: with the radius 154⅓ and centre A describe an arc; and with the radius 310 and centre B describe another arc, cutting the former in C; then join AC, and BC, and the triangle is constructed. And by measuring the angles, they are found, viz,

∠A = 57¼°; ∠B = 24¾°; ∠C = 98° nearly.

2. *Arithmetically.*—Having let fall the perpendicular CP, dividing the base into the two segments AP, PB, and the given triangle ABC into the two right-angled triangles ACP, BCP. Then

As AB = 365	-	log. 2·5622929
To CB + CA = 464·19	-	2·6666958
So CB − CA = 155·53	-	2·1918142
To BP − PA = 197·80		2·2962171
its half = 98·90		
½AB = 182·50		
sum BP = 281·40		
diff. AP = 83·60.		

Then, in the triangle APC, right-angled at P,

As AC = 154·33	-	log. 2·1884504
To sin. ∠P = 90°	- -	10·0000000
So AP = 83·6	- -	1·9222063
To sin. ∠ACP = 32° 48′	-	9·7337559
its comp. ∠A = 57 12.		

And in the triangle BCP, right-angled at P,

As BC = 309·86	-	log. 2.4911655
To sin. ∠P = 90°	-	10·0000000
So BP = 281·4	- -	2·4493241
To sin. ∠BCP = 65° 15′	-	9·9581586
its comp. ∠B = 24 45		
Also to ∠ACP = 32 48		
add ∠BCP = 65 15		
makes ∠ACB = 98 3		

3. *Instrumentally.*—In the 1st proportion, Extend the compasses from 365 to 464 on the line of numbers, and that extent will reach, on the same line, from $155\frac{1}{2}$ to 197·8 nearly.—In the 2d proportion, Extend the compasses from $154\frac{1}{3}$ to 83·6 on the line of numbers, and that extent will reach, on the sines, from 90° to $32\frac{3}{4}$° nearly.—In the 3d proportion, Extend the compasses from 310 to $281\frac{1}{4}$ on the line of numbers; then that extent will reach, on the sines, from 90° to $65\frac{1}{4}$°.

Another method of resolving this case, and that at one operation, is as follows:

1. Add together the logarithm of half the sum of the three given sides and the logarithm of the difference between this half sum and the side opposite the angle sought, and find the complement of their sum. 2. Then, to this complement, increased by 10 in the index, add the logarithms of the differences between the said half sum and each of the other two sides, and the result, divided by 2, will give the tangent of half the required angle.

Thus, resuming the same example, to find the angle A, the work will be as under:

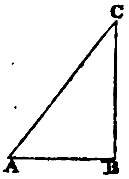
AB = 365·00
AC = 154·33
BC = 309·86 side op. required ∠A.

Sum	829·19	
½ sum	414·595 -	log. 2·6176241
dif.	104·73 - -	2·0200711
sum	- -	4·6376952
log. complem. of do.		5·3623048
add	- - -	10·0000000
Differences between the ½ sum of sides and AB, AC.	49·595	1·6954379
	260·265	2·4154158
sum of these	-	19·4731585
tan. ½∠A = 28° 36′	½ sum	9·7365792
2		
∠A = 57 12 the same as before.		

The foregoing three cases include all the varieties of plane triangles that can happen, both of right and oblique-angled triangles. But beside these, there are some other theorems that are useful upon many occasions, or suited to some particular forms of triangles, which are often more expeditious in use than the foregoing general ones; one of which, for right-angled triangles, as the case for which it serves so often occurs, may be here inserted, and is as follows:

Case 4. When, in a right-angled triangle, there are given the angles and one leg, to find the other leg, or the hypothenuse. Then it will be,

As radius :
To given leg AB : :
So tang. adjacent ∠A :
To the opp. leg BC, and : :
So sec. of same ∠A :
To hypot. AC.



Ex. In the triangle ABC, right-angled at B,

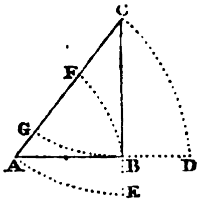
Given the leg AB = 162
and the ∠A = 53° 7′ 48″
conseq. ∠C = 36 52 12
} to find BC and AC.

1. *Geometrically.*—Draw the leg AB = 162: Erect the indefinite perpendicular BC: Make the angle A = $53\frac{1}{8}$°, and the side AC will cut BC in C, and form the triangle ABC. Then, by measuring, there will be found AC = 270, and BC = 216.

2. *Arithmetically.*

As radius	= 10	-	log. 10·0000000
To AB	= 162	- -	2·2095150
So tan. ∠A	= 53° 7′ 48″		10·1249372
To BC	= 216	- -	2·3344522
So sec. ∠A	= 53° 7′ 48″	-	10·2218477
To AC	= 270	- -	2.4313627

3. *Instrumentally.*—Extend the compasses from 45° at the end of the tangents (the radius) to the tangent of $50\frac{1}{8}$°; then that extent will reach, on the line of numbers, from 162 to 216, for BC. Again, extend the compasses from 36° 52′ to 90 on the sines; then that extent will reach, on the line of numbers, from 162 to 270 for AC.



Note, Another method, by making every side radius, is often added by the authors on trigonometry, which is thus: The given right-angled triangle being ABC, make first the hypothenuse AC radius, that is, with the extent of AC as a radius, and each of the centres A and C, describe arcs CD and AE; then it is evident that each leg will represent the sine of its opposite angle, viz, the leg BC the sine of the arc CD or of the angle A, and the leg AB the sine of the arc AE or of the angle C. Again, making either leg radius, the other leg will represent the tangent of its opposite angle, and the hypothenuse the secant of the same angle; thus, with radius AB and centre A describing the arc BF, BC represents the tangent of that arc, or of the angle A, and the hypothenuse AC the secant of the same; or with the radius BC and centre C describing the arc BG, the other leg AB is the tangent of that arc BG, or of the angle C, and the hypothenuse CA the secant of the same.

And then the general rule for all these cases is this, viz, that the sides bear to each other the same proportions as the parts or things which they represent. And this is called making every side radius.

For Plane Trigonometry considered analytically, see my Course of Mathematics, vol. 3, chap. 3.

Spherical TRIGONOMETRY, is the resolution and calculation of the sides and angles of spherical triangles, which are made by three intersecting arcs of great circles on a sphere. Here, any three of the six parts being given, even the three angles, the rest can be found; and the sides are measured or estimated by degrees, minutes, and seconds, as well as the angles.

Spherical Trigonometry is divided into right-angled and oblique-angled, or the resolution of right and oblique-angled spherical triangles. When the spherical triangle has a right angle, it is called a right-angled triangle, as well as in plane triangles; and when a triangle has one of its sides equal to a quadrant of a circle, it is called a quadrantal triangle.

For the resolution of spherical triangles, there are various theorems and proportions, which are similar to those in plane trigonometry, by substituting the sines of sides instead of the sides themselves, when the proportion respects sines; or tangents of the sides for the sides, when the proportion respects tangents, &c; some of the principal of which theorems are as follow:

Theor. 1. In any spherical triangle, the sines of the sides are proportional to the sines of their opposite angles.

Theor. 2. In any right-angled triangle,
As radius :
To sine of one side : :
So tang. of the adjacent angle :
To tang. of the opposite side.

Theor. 3. If a perpendicular be let fall from any angle, upon the base or opposite side of a spherical triangle; it will be,
As the sine of the sum of the two sides :
To the sine of their difference : :
So cotan. ½ sum angles at the vertex :
To tang. of half their difference.

Theor. 4.
As tang. half sum of the sides :
To tang. half their difference : :
So tang. ½ sum ∠s at the base :
To tang. half their difference.

Theor. 5.
As cotan. ½ sum of ∠s at the base :
To tang. half their difference : :
So tang. ½ sum of ∠s at the vertex :
To tang half their difference.

Theor. 6.
As tang. ½ sum segments of base :
To tang. half sum of the sides : :
So tang. half difference of the sides :
To tang. ½ diff. segments of base.

Theor. 7.
As sin. sum of ∠s at the base :
To sine of their difference : :
So tang. ½ sum segments of base :
To tang. of half their difference.

Theor. 8.
As sin. sum of segments of base :
To sine of their difference : :
So sin. sum of angles at the vertex :
To sine of their difference.

Theor. 9.
As sine of the base :
To sine of the vertical angle : :
So sin. of diff. segments of the base
To sin. diff. ∠s at vertex, when the perp. falls within : :
Or so sin. sum segments of base :
To sin. sum vertical ∠s, where the perp. falls without.

Theor. 10.
As cosin. half sum of the two sides :
To cosine of half their difference : :
So cotang. of half the included angle :
To tang. half sum of opposite angles.

Theor. 11.
As sin. of half sum of two sides :
To sine of half their difference : :
So cotang. half the included angle :
To tang. ½ diff. of the oppos. angles.

Theor. 12.
As cosin. half sum of two angles :
To cosine of half their difference : :
So tang. of half the included sides :
To tang. ½ sum of the opposite sides.

Theor. 13.
As sin. half sum of two angles :
To sine of half their difference : :
So tang. half the included side :
To tang. ½ diff. of the opposite sides.

Theor. 14. In a right-angled spherical triangle,
As sin. sum of hypot. and one side :
To sin. of their difference : :
So radius squared :
To square of tang. ½ contained angle.

Theor. 15. In any spherical triangle,
The product of the sines of two sides and of the cosine of the included angle, added to the product of the cosines of those sides, is equal to the cosine of the third side; the radius being 1.

Theor. 16. In any spherical triangle,
The product of the sines of two angles and of the cosine of the included side, minus the product of the cosines of those angles, is equal to the cosine of the third angle; the radius being 1.

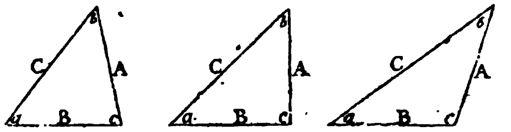
By some or other of these theorems may all the cases of spherical triangles be resolved, both right-angled and oblique: viz, the cases of right-angled triangles by the 1st and 2d theorems, and the oblique triangles by some of the other theorems.

In treatises on trigonometry are to be found many other theorems, as well as synopses or tables of all the cases, with the theorem that is peculiar or proper to each. See the Introduction to my Mathematical Tables, pa. 156 &c; or Robertson's Navigation, vol. 1, pa. 162. See also Napier's Catholic or Universal Rule in this Dictionary.

To the foregoing theorems may be added the following synopsis of rules for resolving all the cases of plane and spherical triangles, under the title of

Trigonometrical Rules.

1. In a right-lined triangle, whose sides are A, B, C, and their opposite angles a, b, c; having given any three of these, of which one is a side; to find the rest.



Put s for the sine, s′ the cosine, t the tangent, and t′ the cotangent of an arch or angle, to the radius r; also L for a logarithm, and L′ its arithmetical complement. Then

Case 1. When three sides A, B, C, are given.

Put $P = \frac{1}{2}(A + B + C)$ or semiperimeter.

Then $s.\ \frac{1}{2}c = r\sqrt{\frac{(P - A) \times (P - B)}{A \times B}}$.

And $s'.\ \frac{1}{2}c = r\sqrt{\frac{P \times (P - C)}{A \times B}}$.

$L.\,s.\ \frac{1}{2}c = \frac{1}{2}\left(L.\,(P - A) + L.\,(P - B) + L'A + L'B\right)$,
$L.'s.\ \frac{1}{2}c = \frac{1}{2}\left(L.\,P + L.\,(P - C) + L'A + L'B\right)$.

Note, When A = B, then

$s.\ \frac{1}{2}c = \frac{C}{A} \times \frac{r}{2}$. And $s'\ \frac{1}{2}c = r\sqrt{\frac{A^2 - \frac{1}{4}C^2}{A^2}}$.

Case 2. Given two sides A, B, and their included angle c.

Put $s = 90^\circ - \frac{1}{2}c$, and $t.\ d = \frac{A - B}{A + B} \times t.\ s$; then $a = s + d$; and $b = s - d$. And $C = \sqrt{\left(\frac{4AB + s^2\,\frac{1}{2}c}{rr} + (A - B)^2\right)}$.

Or in logarithms, putting $L.\,Q = 2\,L.\,(A - B)$, and

$\text{L. R} = \text{L. }2\text{A} + \text{L. }2\text{B} + 2\text{L. s. }\tfrac{1}{2}c - 20$, we shall have $\text{L. C} = \tfrac{1}{2}\text{ L. }(\text{Q} + \text{R})$.

If the angle c be right, or $= 90°$; then

$$\text{t. }a = \frac{\text{A}}{\text{B}}r;\ \text{t. }b = \frac{\text{B}}{\text{A}}r;$$

$$\text{C} = \frac{r}{\text{s. }a}\text{A},\ \text{or} = \frac{r}{\text{s. }b}\text{B},\ \text{or} = \sqrt{(\text{A}^2 + \text{B}^2)}.$$

If $\text{A} = \text{B}$; we shall have $a = b = 90° - \tfrac{1}{2}c$, and $\text{C} = \frac{\text{s. }\frac{1}{2}c}{r} \times 2\text{A}$.

Case 3. When a side and its opposite angle are among the terms given; then

$$\frac{\text{A}}{\text{s. }a} = \frac{\text{B}}{\text{s. }b} = \frac{\text{C}}{\text{s. }c};$$ from which equations any term wanted may be found.

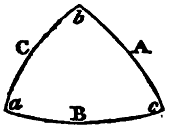
When an angle, as a, is $90°$, and A and C are given; then

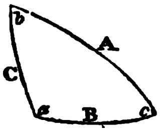
$$\text{B} = \sqrt{(\text{A}^2 - \text{C}^2)} = \sqrt{(\text{A} + \text{C})} \times (\text{A} - \text{C}).$$

And $\text{L. B} = \tfrac{1}{2}(\text{L. }(\text{A} + \text{C}) + \text{L. }(\text{A} - \text{C}))$.

Note, When two sides A, B, and an angle a opposite to one of them, are given; if A be less than B, then b, c, C have each two values; otherwise, only one value.

II. In a spherical triangle, whose three sides are A, B, C, and their opposite angles a, b, c; any three of these six terms being given, to find the rest.





Case 1. Given the three sides A, B, C.

Calling 2P the perim. or $\text{P} = \tfrac{1}{2}(\text{A} + \text{B} + \text{C})$.

$$\text{Then s.}\tfrac{1}{2}c = r\sqrt{\frac{\text{s.}(\text{P} - \text{A}) \times \text{s.}(\text{P} - \text{B})}{\text{s.A} \times \text{s.B}}}$$

$$\text{And s}'\tfrac{1}{2}c = r\sqrt{\frac{\text{s.P} \times \text{s.}(\text{P} - \text{C})}{\text{s.A} \times \text{s.B}}}.$$

$$\text{L.s.}\tfrac{1}{2}c = \tfrac{1}{2}(\text{L.s.}(\text{P} - \text{A}) + \text{L.s.}(\text{P} - \text{C}) + \text{L}'\text{s.A} + \text{L}'\text{s.B})$$
$$\text{L.s}'\tfrac{1}{2}c = \tfrac{1}{2}(\text{L.s.P} + \text{L.s.}(\text{P} - \text{C}) + \text{L}'\text{s.A} + \text{L}'\text{s.B}).$$

And the same for the other angles.

Case 2. Given the three angles.

Put $2p = a + b + c$. Then

$$\text{s.}\tfrac{1}{2}\text{C} = r\sqrt{\frac{\text{s}'p \times \text{s}'(p - c)}{\text{s.}a \times \text{s.}b}}.\ \text{And}$$

$$\text{s}'\tfrac{1}{2}\text{C} = r\sqrt{\frac{\text{s}'(p - a) \times \text{s}'(p - b)}{\text{s.}a \times \text{s.}b}}.$$

$$\text{L.s.}\tfrac{1}{2}\text{C} = \tfrac{1}{2}(\text{L.s}'p + \text{L.s}'(p - c) + \text{L}'\text{s.}a + \text{L}'\text{s.B}).$$
$$\text{L.s}'\tfrac{1}{2}\text{C} = \tfrac{1}{2}(\text{L.s}'(p - a) + \text{L.s}'(p - b) + \text{L}'\text{s.}a + \text{L.s.}'b)$$

And the same for the other sides.

Note. The sign $>$ signifies greater than, and $<$ less; also $\backsim$ the difference.

Case 3. Given A, B, and included angle c.

To find an angle a opposite the side A,
let $r : \text{s}'c :: \text{t. A} : \text{t. M}$, like or unlike A,
as c is $>$ or $< 90°$; also $\text{N} = \text{B} \backsim \text{M}$:
then $\text{s. N} : \text{s. M} :: \text{t. }c : \text{t. }a$, like or unlike
c as M is $>$ or $<$ B.

Or let $\text{s}'\tfrac{1}{2}(\text{A} + \text{B}) : \text{s}'\tfrac{1}{2}(\text{A} \backsim \text{B}) :: \text{t}'\tfrac{1}{2}c : \text{t. M}$,
which is $>$ or $< 90°$ as $\text{A} + \text{B}$ is $>$ or $< 180°$;
and $\text{s.}\tfrac{1}{2}(\text{A} + \text{B}) : \text{s.}(\text{A} \backsim \text{B}) :: \text{t}'\tfrac{1}{2}c : \text{t. N}$, $> 90°$.
then $a = \text{M} + \text{N}$; and $b = \text{M} - \text{N}$.

Again let $r : \text{s}'c :: \text{t. A} : \text{t. M}$, like or unlike A as c is $>$ or $< 90°$; and $\text{N} = \text{B} \backsim \text{M}$.

Then $\text{s}'\text{M} : \text{s}'\text{N} :: \text{s}'\text{A}, \text{s}'\text{C}$, like or unlike N as c is $>$ or $< 90°$. Or,

$$\text{s.}\tfrac{1}{2}\text{C} = \sqrt{\frac{\text{s.A} \times \text{s.B} \times \text{s}'^2\tfrac{1}{2}c}{rr} + \text{s}^2\tfrac{1}{2}(\text{A} \backsim \text{B})}.$$

In logarithms, put $\text{L. Q} = 2\text{L.s.}\tfrac{1}{2}(\text{A} \backsim \text{B})$; and $\text{L. R} = \text{L. s. A} + \text{L. s. B} + 2\text{L. s. }\tfrac{1}{2}c - 20$; then $\text{L.s.}\tfrac{1}{2}\text{C} = \tfrac{1}{2}\text{L.}(\text{Q} + \text{R})$.

Case 4. Given a, b, and included side C.

First, let $r : \text{s}'\text{C} :: \text{t. }a : \text{t}'m$, like or unlike a as C is $>$ or $< 90°$; also $n = b \backsim m$.

Then $\text{s}'n : \text{s}'m :: \text{t. C} : \text{t. A}$, like or unlike n as a is $>$ or $< 90°$.

Or, let $\text{s}'(\tfrac{1}{2}a + b) : \text{s}'\tfrac{1}{2}(a \backsim b) :: \text{t.}\tfrac{1}{2}\text{C} : \text{t. M}$, $>$ or $< 90°$ as $a + b$ is $>$ or $< 180°$;
and $\text{s}'\tfrac{1}{2}(a + b) : \text{s}\tfrac{1}{2}(a \backsim b) :: \text{t.}\tfrac{1}{2}\text{C} : \text{t N}$, $> 90°$;
then $\text{A} = \text{M} \pm \text{N}$; and $\text{B} = \text{M} \mp \text{N}$.

Again, let $r : \text{s}'\text{C} :: \text{t. }a : \text{t}'m$, like or unlike a as C is $>$ or $< 90°$;
and $n = b \backsim m$:
then $\text{s.}m. : \text{s.}n. :: \text{s}'a : \text{s}'c$, like or unlike a as m is $>$ or $<$ b.

Case 5. Given A, B, and an opposite angle a.

1st. $\text{s. A} : \text{s. }a. :: \text{s. B} : \text{s. }b$, $>$ or $< 90°$.

2nd. Let $r : \text{s}'\text{B} :: \text{t. }a : \text{t}'m$, like or unlike B as a is $>$ or $< 90°$;
and $\text{t. A} : \text{t. B} :: \text{s}'m : \text{s}'n$, like or unlike A as a is $>$ or $< 90°$:
then $c = m \pm n$, two values also.

3dly. Let $r : \text{s}'a :: \text{t. B} : \text{t. M}$, like or unlike B as a is $>$ or $< 90°$;
and $\text{s}'\text{B} : \text{s}'\text{A} :: \text{s}'\text{M} : \text{s}'\text{N}$, like or unlike A as a is $>$ or $< 90°$:
then $\text{C} = \text{M} \pm \text{N}$, two values also.

But if A be equal to B, or to its supplement, or between B and its supplement; then is b like to B: also c is $= m \mp n$, and $\text{C} = \text{M} \pm \text{N}$, as B is like or unlike a.

Case 6. Given a, b, and an opposite side A.

1st. $\text{s. }a. \cdot \text{s. A} :: \text{s. }b : \text{s. B}$, $>$ or $< 90°$.

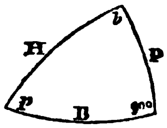
2nd. Let $r : \text{s}'b :: \text{t. A} :$ to t. M, like or unlike b as A is $>$ or $< 90°$;
and $\text{t. }a : \text{t. }b. :: \text{s. M} : \text{s. N}$, $>$ or $< 90°$:
then $\text{C} = \text{M} \pm \text{N}$, as a is like or unlike b.

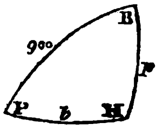
3dly. Let $r : \text{s}'\text{A} :: \text{t. }b : \text{t}'m$, like or unlike b as A $>$ or $< 90°$;
and $\text{s}'b : \text{s}'a :: \text{s. }m. : \text{s. }n$, $>$ or $< 90°$:
then $c = m \pm n$, as a is like or unlike b.

But if A be equal to B, or to its supplement, or between B and its supplement; then B is unlike b, and only the less values of N, n, are possible.

Note, When two sides A, B, and their opposite angles a, b, are known; the third side C, and its opposite angle c, are readily found thus:

$$\text{s.}\tfrac{1}{2}(a \backsim b) : \text{s.}\tfrac{1}{2}(a + b) :: \text{t.}\tfrac{1}{2}(\text{A} \backsim \text{B}) : \text{t.}\tfrac{1}{2}\text{C}.$$
$$\text{s.}\tfrac{1}{2}(\text{A} \backsim \text{B}) : \text{s.}\tfrac{1}{2}(\text{A} + \text{B}) :: \text{t.}\tfrac{1}{2}(a \backsim b) : \text{t.}\tfrac{1}{2}c.$$





III. In a right-angled spherical triangle, where H is the hypothenuse, or side opposite the right angle, B, P the other two sides, and b, p their opposite angles; any two of these five terms being given, to find the rest; the cases, with their solutions, are as in the following table.

The same table will also serve for the quadrantal triangle, or that which has one side $= 90°$, H being the angle opposite that side, B, P the other two angles, and *b*, *p* their opposite sides: observing, instead of H, to take its supplement: and mutually change the terms *like* and *unlike* for each other where H is concerned.

Case	Given	Req^d.	SOLUTIONS.
1	H	*b*	s. H. : *r* : : s.B : s.*b*, and is like B
	B	*p*	*r* : t′H : : t.B : s′*p* }, > or < 90° as H is like or unlike B
		P	s′B : *r* : : s′H : s′P
2	H	B	*r* : s H : : s.*b* : s.B, like *b*
	b	P	*r* : s′*b* : : t.H : t.P }, > or < 90° as H is like or unlike *b*
		p	*r* : s′H : : t.*b* : t′*p*
3	B	H	s.*b* : *r* : : s.B : s.H }, each > or < 90°; both values true
	b	P	*r* : t.B : : t′*b* : s.P
		P	s′B : *r* : : s′*b* : s.*p*
4	B	H	*r* : t′B : : s′*p* : t′H, > or < 90° as B is like or unlike *p*
	p	*b*	*r* : s′B : : s.*p* : s′*b*, like B
		P	*r* : s.B : : t.*p* : t.P, like *p*
5	B	H	*r* : s′B : : s′P : s′H, > or < 90° as B is like or unlike P
	P	*b*	*r* : s.P : : t′B : t′*b*, like B
		p	*r* : s.B : : t′P : t′*p*, like P
6	*b*	H	*r* ; t′*b* : : t′*p* : s′H, > or < 90° as *b* is like or unlike *p*
	p	B	s.*p* : *r* .: : s′*b* : s′B like *b*
		P	s.*b* : *r* : : s′*p* : s′P like *p*

The following propositions and remarks, concerning spherical triangles, (selected and communicated by the reverend Nevil Maskelyne, D. D. astronomer royal, F. R. S.) will also render the calculations of them perspicuous, and free from ambiguity.

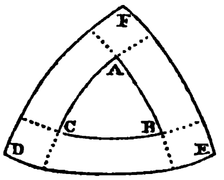
" 1. A spherical triangle is equilateral, isoscelar, or scalene, according as it has its three angles all equal, or two of them equal, or all three unequal; and *vice versa*.

2. The greatest side is always opposite the greatest angle, and the smallest side opposite the smallest angle.

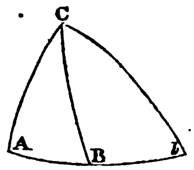
3. Any two sides taken together, are greater than the third.

4. If the three angles are all acute, or all right, or all obtuse; the three sides will be, accordingly, all less than 90°, or equal to 90°, or greater than 90°; and *vice versa*.

5. If from the three angles A, B, C, of a triangle ABC, as poles, there be described, upon the surface of the sphere, three arches of a great circle DE, DF, FE, forming by their intersections a new spherical triangle DEF; each side of the new triangle will be the supplement of the angle at its pole; and each angle of the same triangle, will be the supplement of the side opposite to it in the triangle ABC.



6. In any triangle ABC, or A*bc*, right-angled in A, 1st, The angles at the hypothenuse are always of the same kind as their opposite sides; 2dly, The hypothenuse is less or greater than a quadrant, according as the sides including the right-angle are of the same or different kinds; that is to say, according as these same sides are either both acute or both obtuse, or as one is acute and the other obtuse. And, *vice versa*, 1st, The sides including the right angle, are always of the same kind as their opposite angles: 2dly, The sides including the right angle will be of the same or different kinds, according as the hypothenuse is less or more than 90°; but one at least of them will be of 90°, if the hypothenuse is so."



Analytical TRIGONOMETRY. See my Course of Mathematics, last vol.

TRILATERAL, three-sided, a term applied to all figures of three sides, or triangles.

TRILLION, in Arithmetic, the number of a million of billions, or a million of million of millions.

TRIMMERS, in Architecture, pieces of timber framed at right angles to the joists, against the ways for chimneys to support the hearths, and the well-holes for stairs.

TRINE *Dimension*, or *threefold dimension*, includes length, breadth, and thickness. The trine dimension is peculiar to bodies or solids.

TRINITY *Sunday*, is the next after Whitsunday; so called, because on that day was anciently held a festival (as it still continues to be in the Romish church) in honor of the Holy Trinity.—The observance of this festival was first enjoined by the 6th canon of the council of Arles, in 1260; and John the 22d, who distinguished himself so much by his opinion concerning the beatific vision, it is said, fixed the office for this festival in 1334.

TRINODA, or TRINODIA *Terræ*, in some ancient writers, denotes the quantity of 3 perches of land.

TRINOMIAL, in Algebra, is a quantity, or a root, consisting of three parts or terms, connected together by the signs + or − : as $a + b - c$, or $x + y + z$.

TRIO, in Music, a part of a concert in which three persons sing; or rather a musical composition consisting of 3 parts.—Trios are the finest kind of musical composition, and please most in concerts.

TRIONES, in Astronomy, a kind of constellation, or assemblage of 7 stars in the Ursa Major, popularly called Charles's Wain.—From the Septem Triones the north pole takes the denomination Septentrio.

TRIPARTITION, is a division by 3, or the taking of the 3d part of any number or quantity.

TRIPLE, threefold. See RATIO and SUBTRIPLE.

TRIPLE, in Music, is one of the species of measure or time, and is taken from hence, that the whole, or half measure, is divisible into 3 equal parts, and is beaten accordingly.

TRIPLICATE *Ratio*, is the ratio which cubes, or any similar solids, bear to each other; and is the cube of the simple ratio, or this twice multiplied by itself. Thus 1 to 8 is the triplicate ratio of 1 to 2, and 1 to 27 triplicate of 1 to 3.

TRIS-Diapason, or *Triple Diapason Chord*, in Music, is what is otherwise called a triple eighth.

TRISECTION, the dividing a thing into three equal parts. The term is chiefly used in geometry, for the division of an angle into three equal parts. The trisection of an angle geometrically, is one of those great problems whose solution has been so much sought for by mathematicians, for 2000 years past; being, in this respect, on a footing with the famous one concerning the quadrature of the circle, and the duplicature of the cube.

The ancients trisected an angle by means of the conic sections, and the book of inclinations; and Pappus enumerates several ways of doing it, in the 4th book of his Mathematical Collections, prop. 31, 32, 33, 34, 35, &c. He further observes, that the problem of trisecting an angle, is a solid problem, or a problem of the 3d degree, being expressed by the resolution of a cubic equation, in which way it has been resolved by Vieta, and others of the moderns. See his angular sections, with those of other authors, and the trisection in particular by cubic equations, as in Guisne's Application of Algebra to Geometry, in l'Hospital's Conic Sections, and in Emerson's Trigonometry, book 1, sec. 4. The cubic equation by which the problem of trisection is resolved, is as follows: Let $2c$ denote the chord of a given arc, or angle, and x the chord of the 3d part of the same, to the radius 1; then is $x^3 - 3x = - 2c$, by the resolution of which cubic equation is found the value of x, or the chord of the 3d part of the given arc or angle, whose chord is c; and the resolution of this equation, by Cardan's rule, gives the chord

$$x = \sqrt[3]{(-c + \sqrt{(c^2 - 1)})} + \frac{1}{\sqrt[3]{(-c + \sqrt{(c^2 - 1)})}}.$$

$$\text{or } x = \sqrt[3]{(-c + \sqrt{(c^2 - 1)})} + \sqrt[3]{(-c - \sqrt{(c^2 - 1)})}.$$

TRISPAST, or Trispaston, in Mechanics, a machine with 3 pulleys, or an assemblage of 3 pulleys, for raising great weights; being a lower species of the polyspaston.

TRITE, in Music, the 3d musical chord in the system of the ancients.

TRITONE, in Music, a false concord, consisting of three tones, or a greater third, and a greater tone. Its ratio or proportion in numbers, is that of 45 to 32.

TROCHILE, in Architecture, is that hollow ring, or cavity, which runs round a column next to the tore.

TROCHLEA, in Mechanics, one of the mechanic powers, more usually called the pulley.

TROCHOID, in the Higher Geometry, a curve described by a point in any part of the radius of a wheel, during its rotatory and progressive motions. This is the same curve as what is more usually called the cycloid, where the construction and properties of it are shown.

TRONE *Weight*, the most ancient of the different weights used in Scotland.

Trone *Pound*, in Scotland, contains 20 Scotch ounces. Or because it is usual to allow one to the score, the trone pound is commonly 21 ounces.

Trone *Stone*, in Scotland, according to Sir John Skene, contains 19½ pounds.

TROPHY, in Architecture, an ornament which represents the trunk of a tree, charged or encompassed all around with arms or military weapons, both offensive and defensive.

TROPICAL, something relating to the tropics. As, Tropical-*Winds*. See Wind, and Trade-*Winds*.

Tropical *Year*, the space of time during which the sun passes round from a tropic, till his return to it again. See Year.

TROPICS, in Astronomy, two fixed circles of the sphere, drawn parallel to the equator, through the solstitial points, or at such distance from the equator, as is equal to the sun's greatest recess or declination, or to the obliquity of the ecliptic.

That on the north side of the equator, passes through the first point of Cancer, and is therefore called the Tropic of Cancer. And the other on the south side, passing through the first point of Capricorn, is called the Tropic of Capricorn.

To determine the distance between the two tropics, and thence the sun's greatest declination, or the obliquity of the ecliptic; observe the sun's meridian altitude, both in the summer and winter solstice, and subtract the latter from the former, so shall the remainder be the distance between the two tropics; and the half of this the quantity of the greatest declination, or the obliquity of the ecliptic; the medium of which is now 23° 27′ 46″ nearly.

Tropics, in Geography, are two lesser circles of the globe, drawn parallel to the equator through the beginnings of Cancer and Capricorn, being in the planes of the celestial tropics, and consequently at 23° 28′ distance nearly, either way from the equator.

TROY-*Weight*, anciently called trone-weight, is supposed to be taken from a weight of the same name in France, and that from the name of the town of Troyes there. The original of all weights used in England, was a corn or grain of wheat gathered out of the middle of the ear; and, when well dried, 32 of them were to make one pennyweight, 20 pennyweights 1 ounce, and 12 ounces 1 pound troy. Vide statutes of 51 Hen. III; 31 Ed. I. and 12 Hen. VII. But afterward it was thought sufficient to divide the said pennyweight into 24 equal parts, called grains, being the least weight now in common use; so that the divisions of troy weight now are these:

24 grains = 1 pennyweight *dwt.*
20 pennyweights = 1 ounce *oz.*
12 ounces = 1 pound *lb.*

By troy-weight are weighed jewels, gold, silver, and all liquors.

TRUCKS, among Gunners, are the small wooden wheels fixed on the axletrees of gun carriages, especially those for ship service, to move them about by.

TRUE *Conjunction*. See *True* Conjunction.

True *Place of a Planet or Star*, is a point in the heavens shown by a right line drawn from the centre of the earth, through the centre of the star or planet.

TRUMPET, *Listening or Hearing*, is an instrument invented by Joseph Landini, to assist the hearing of persons dull of that faculty, or to assist us to hear persons who speak at a great distance. Instruments of this kind are formed of tubes, with a wide mouth, and terminating in a small canal, which is applied to the ear. The form of these instruments evidently shows how they conduce to assist the hearing; for the greater quantity of the weak and languid pulses of the air being received and collected by the large end of the tube, are reflected to the small end, where they are collected and condensed; thence entering the ear in this condensed state, they strike the tympanum with a greater force than they could naturally have done from the ear alone. Hence it appears, that a speaking trumpet may be applied to the purpose of a hearing trumpet, by turning the wide end towards the sound, and the narrow end to the ear.

Speaking TRUMPET, is a tube of a considerable length, from 6 to 15 feet, used for speaking with to make the voice be heard to a greater distance. This tube, which is made of tin, is straight throughout its length, but opening to a large aperture outwards, and the other end terminating in a proper shape and size to receive both the lips in the act of speaking, the speaker pushing his voice or the sound outwards, by which means it may be heard at the distance of a mile or more.

The invention of this trumpet is held to be modern, and has been ascribed to Sir Samuel Moreland, who called it the tuba stentorophonica; and in a work of the same name, published at London in 1671, that author gave an account of it, and of several experiments made with it. With one of these instruments, of 5½ feet long, 21 inches diameter at the greater end, and 2 inches at the smaller, tried at Deal Castle, the speaker was heard to the distance of 3 miles, the wind blowing from the shore.

But it seems that Kircher has a better title to the invention; for it is certain that he had such an instrument before ever Moreland thought of his. That author, in his Phonurgia Nova, published in 1673, says, that the tromba, published last year in England, he invented 24 years before, and published in his Mesurgia. He adds, that Jac. Albanus Ghibbisius and Fr. Eschinardus ascribe it to him; and that G. Schottus testifies of him, that he had such an instrument in his chamber in the Roman college, with which he could call to, and receive answers from the porter.

But, considering how famous the tube or horn of Alexander the Great was, it is rather strange that the moderns should pretend to the invention. With his stentorophonic horn or tube he used to speak to his army, and make himself be distinctly heard, it is said, 100 stadia or furlongs. A figure of this tube is preserved in the Vatican; and it is nearly the same as that now in use. See STENTOROPHONIC.

The principle of this instrument is obvious; for as sound is stronger in proportion to the density of the air, it follows that the voice, in passing through a tube or trumpet, must be greatly augmented by the constant reflection and agitation of the air through the length of the tube, by which it is condensed, and its action on the external air greatly increased at its exit from the tube. It has been found, that a man speaking through a tube of 4 feet long, may be understood at the distance of 500 geometrical paces; with a tube 16⅔ feet, at the distance of 1800 paces; and with a tube 24 feet long, at more than 2500 paces.

Though some advantage in heightening the sound, both in speaking and hearing, be derived from the shape of the tube, and the width of the outer end, yet the effect depends chiefly on its length. As to the form of it, some have asserted that the best figure is that which is formed by the revolution of a parabola about its axis; the mouth-piece being placed in the focus of the parabola, and consequently the sonorous rays reflected parallel to the axis of the tube. But Mr. Martin observes, that this parallel reflection is by no means essential to increasing the sound: on the contrary, it prevents the infinite number of reflections and reciprocations of sound, in which, according to Newton, its augmentation chiefly consists; the augmentation of the impetus of the pulses of air being proportional to the number of repercussions from the sides of the tube, and therefore to its length, and to such a figure as is most productive of them. Hence he infers, that the parabolic trumpet is the most unfit of any for this purpose; and he endeavours to show, that the logarithmic or logistic curve gives the best form, viz, by a revolution about its axis. Martin's Philos. Brit. vol. 2, pa. 248, 3d edit.

But Cassegrain is of opinion that an hyperbola, having the axis of the tube for an asymptote, is the best figure for this instrument. Musschenb. Intr. ad Phil. Nat. tom. 2, pa. 926, 4to. For other constructions of speaking-trumpets, by Mr. Conyers, see Philos. Trans. No. 141, for 1678.

TRUNCATED *Pyramid* or *Cone*, is the frustum of one, being the part remaining at the bottom, after the top is cut off by a plane parallel to the base. See FRUSTUM.

TRUNNIONS, of a piece of ordnance, are those knobs or short cylinders of metal on the sides, by which it rests on the cheeks of the carriage.

TRUNNION-*Ring*, is the ring about a cannon, next before the trunnions.

TSCHIRNHAUSEN (ERNFROY WALTER), an ingenious mathematician, lord of Killingswald and of Stolzenberg in Lusatia, where he was born in 1651. After having served as a volunteer in the army of Holland in 1672, he travelled into most parts of Europe, as England, Germany, Italy, France, &c. He went to Paris for the third time in 1682; where he communicated to the Academy of Sciences, the discovery of the curves, called from him, Tschirnhausen's Caustics; and the Academy in consequence elected the inventor one of its foreign members. On returning to Italy, he was desirous of perfecting the science of optics; for which purpose he established two glass-works, whence resulted many new improvements in dioptrics and physics, particularly the noted burning-glass which he presented to the regent. It was to him too that Saxony owed its porcelane manufactory. Content with the enjoyment of literary fame, Tschirnhausen refused all other honours that were offered him. Learning was his sole delight. He searched out men of talents, and gave them encouragement. He was often at the expence of printing the useful works of other men, for the benefit of the public; and died, beloved and regretted, the 11th of September 1708.

Tschirnhausen wrote, De Medicina Mentis et Corporis, printed at Amsterdam in 1687. And the following memoirs were printed in the volumes of the Academy of Sciences:—1. Observations on Burning Glasses of 3 or 4 feet diameter; vol. 1699.—2. Observations on the Glass of a Telescope, convex on both sides, of 32 feet focal distance; 1700.—3. On the Radii of Curvature, with the finding of Tangents, Quadratures, and Rectifications of many curves; 1701.—4. On the Tangents of Mechanical Curves; 1702.—5. On a Method of Quadratures; 1702.

TUBE, a pipe, conduit, or canal; being a hollow cylinder, either of metal, wood, glass, or other matter, for the conveyance of air, or water, &c. The term is chiefly applied to those used in physics, astronomy, anatomy, &c. On other ordinary occasions, we more usually say pipe.

In the memoirs of the French Academy of Sciences, Varignon has given a treatise on the proportions for the diameters of tubes, to give any particular quantities of water. The result of his paper gives these two analogies, viz, that the diminutions of the velocity of water, occasioned by its friction against the sides of tubes, are as the diameters; the tubes being supposed equally long: and the quantities of water issuing out at the tubes, are as the square roots of their diameters, deducting out of them the quantity that each is diminished.

Tube, in Astronomy, is sometimes used for telescope; but more properly for that part of it into which the lenses are fitted, and by which they are directed and used.

TUESDAY, the 3d day of the week, so called from Tuesco, one of the Saxon Gods, similar to Mars; for which reason the astronomical mark for this day of the week, is ♂.

TUMBREL, is a kind of carriage with two wheels, used either in husbandry for dung, or in artillery to carry the tools of the pioneers, &c, and sometimes likewise the money of an army.

TUN, is a measure for liquids, as wine, oil, &c. The English ton contains 2 pipes, or 4 hogsheads, or 252 gallons.

TUNE, or Tone, in Music, is that property of sounds by which they come under the relation of acute and grave. If two or more sounds be compared together in this relation, they are either equal or unequal in the degree of tune: such as are equal, are called unisons. The unequal constitute what are called intervals, which are the differences of tone between sounds.

Sonorous bodies are found to differ in tone: 1st, According to the different kinds of matter; thus the sound of a piece of gold, is much graver than that of a piece of silver of the same shape and dimensions. 2d, According to the different quantities of the same matter in bodies of the same figure; as a solid sphere of brass of 1 foot diameter, sounds acuter than a sphere of brass of 2 feet diameter.

But the measures of tone are only to be sought in the relations of the motions that are the cause of sound, which are most discernible in the vibration of chords. Now, in general, we find that in two chords, all things being equal, excepting the tension, the thickness, or the length, the tones are different; which difference can only be in the velocity of their vibratory motions, by which they perform a different number of vibrations in the same time; as it is known that all the small vibrations of the same chord are performed in equal times. Now the frequenter or quicker those vibrations are, the more acute is the tone; and the slower and fewer they are in the same time, by so much the more grave is the tone. So that any given note of a tune is made by one certain measure of velocity of vibrations, that is, such a certain number of vibrations of a chord or string, in such a certain part of time, constitutes a determinate tone.

This theory is strongly supported by the best and latest writers on music, Holder, Malcolm, Smith, &c, both from reason and experience. Dr. Wallis, who owns it very reasonable, adds, that it is evident the degrees of acuteness are reciprocally as the lengths of the chords; though, he says, he will not positively affirm that the degrees of acuteness answer the number of vibrations, as their only true cause: but his diffidence arises from hence, that he doubts whether the thing has been sufficiently confirmed by experiment.

TUNNAGE. See Tonnage.

TURN, is used for a circular motion; in which sense it agrees with revolution.

Turn, in Clock or Watch-work, particularly denotes the revolution of a wheel or pinion. In calculation, the number of turns which the pinion has, is denoted in common arithmetic thus, 5) 60 (12, where the pinion 5, playing in a wheel of 60, moves round 12 times in one turn of the wheel. Now by knowing the number of turns which any pinion makes in one turn of the wheel it works in, is easily found how many turns a wheel or pinion has at a greater distance; as the contrat-wheel, crown-wheel, &c, by multiplying together the quotients, and the number produced is the number of turns, as in the example here annexed: the first of these three numbers has 11 turns, the next 9, and the last 8: by multiplying 11 by 9, it produces 99; that is, in one turn of the wheel 55, there are 99 turns of the second pinion 5, or the wheel 45, which runs concentrical or on the same arbor with the second pinion 5: and again multiplying 99 by the last quotient 8, it produces 792, which is the number of turns the third pinion 5 has. See Clock-*work*, and Pinion.

5) 55 (11
5) 45 (9
5) 40 (8

TURNING *to windward*, in Sea-Language, denotes that operation in sailing when a ship endeavours to make a progress against the direction of the wind, by a compound course, inclined to the place of her destination.—This method of navigation is otherwise called plying to windward.

TUSCAN *Order*, in Architecture, is the first, the simplest, and the strongest or most massive of any. Its column has 7 diameters in height; and its capital, base, and entablement, have no ornaments, and but few mouldings.

TWELFTH-*Day*, the festival of the Epiphany, or the manifestation of Christ to the Gentiles, so called, as being the twelfth day, exclusive, from the nativity or Christmas-day; of course it falls always on the 6th day of January.

TWILIGHT, in Astronomy, is that faint light which is perceived before sun-rising, and after sun-setting. The twilight is occasioned by the earth's atmosphere refracting the rays of the sun, and reflecting them among its particles.

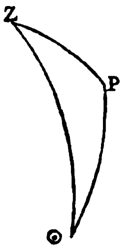
The depression of the sun below the horizon, at the beginning of the morning, and end of the evening twilight, has been variously stated, at different seasons, and by different observers: by Alhazen it was observed to be 19°; by Tycho 17°; by Rothman 24°; by Stevinus 18°; by Cassini 15°; by Riccioli, at the time of the equinox in the morning 16°, in the evening 20¼°; in the summer solstice in the morning 21° 25′, and in the winter 17° 15′. Whence it appears that the cause of the twilight is variable; but, on a medium, about 18° of the sun's depression will serve tolerably well for our latitude, for the beginning and end of twilight, and according to which Dr. Long, (in his Astronomy, vol. 1, pa. 258) gives the following table, of the duration of twilight, in different latitudes, and for several different declinations of the sun.

Latitude.	0	10	20	30	40	45	50	52½	55	60	65	70	75	80	85	90
☉ Enters	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m	h m
♋	1 18	1 21	1 28	1 41	2 8	2 39	w n	w n	w n	w n	w n	c d	c d	c d	c d	c d
♊ ♌	1 16	1 19	1 25	1 36	1 58	2 19	3 3	w n	w n	w n	w n	c d	c d	c d	c d	c d
♉ ♍	1 13	1 15	1 20	1 28	1 43	1 55	2 12	2 25	2 41	3 55	w n	w n	w n	c d	c d	c d
♈ ♎	1 12	1 13	1 17	1 24	1 35	1 44	1 55	2 2	2 10	2 33	3 8	4 18	w n	w n	w n	w n
♓ ♏	1 13	1 14	1 18	1 24	1 35	1 43	1 54	2 0	2 8	2 27	2 56	8 41	5 2	17 32	w n	w n
♒, ♐	1 16	1 17	1 21	1 28	1 40	1 49	2 1	2 8	2 18	2 43	3 26	11 38	11 14	10 32	8 38	c n
♑	1 18	1 19	1 23	1 30	1 43	1 53	2 6	2 15	2 26	2 57	4 4	10 24	9 30	7 46	c n	c n

Where *c d* signify that it is then continual day, *c n* continual night, and *w n* that the twilight lasts the whole night.

Prob.—To find the Beginning or End of Twilight.

In this problem, there are given the sides of an oblique spherical triangle, to find an angle; viz, given the side ZP the co-latitude of the place; P☉ the co-declination, or polar distance; and Z☉ the zenith distance, which is always equal to 108°, viz, 90° from the zenith to the horizon, and 18° more for the sun's distance below the horizon. For example, suppose the place London in latitude 51° 32′, and the time the 1st of May, when the sun's declination is 15° 12′ north. Here then ZP = 38° 28′ the complement of 51° 32′ and P☉ = 74° 48′, the complement of 15° 12′. Then the calculation is as follows.



P☉ = 74° 48′
PZ = 38 28
P☉ − PZ = 36 20 = D
Z☉ = 108 00
Z☉ + D = 144 20 | 72° 10′ = ½(Z☉ + D)
Z☉ − D = 71 40 | 35 50 = ½(Z☉ − D)

Then,

Co-ar. sin. polar dist.	= 74° 48′	-	0·01547
Co-ar. sin. colat. -	= 38 28	-	0·20617
Sine ½(Z☉ + D) -	= 72 10	-	9·97861
Sine ½(Z☉ − D) -	= 35 50	-	9·76747
Sum of these four logs.	-	-	19.96772
Half sum gives	74° 28½′	-	9·98386

Which doubled gives 148 57 for the angle ZPO.

This 148° 57′ reduced to time, at the rate of 15° per hour, gives $9^h\ 55^m\ 48^s$, either before or after noon; that is, the twilight begins at $2^h\ 4^m\ 12^s$ in the morning, and ends at $9^h\ 55^m\ 48^s$ in the evening on the given day at London.

To find the time of shortest twilight at any given place, say, as radius to the sine of the latitude, so is the tangent of T to the sine of the sun's declination at the time required.—The declination of the sun and the latitude of the place must be of contrary kinds.—Hence, at about 51 or 52 degrees north latitude, the twilight will be shortest at about the 2d or 3d of March, and the 11th or 12th of October.

TWINKLING *of the Stars*, denotes that tremulous motion which is observed in the light proceeding from the fixed stars.—This twinkling in the stars has been variously accounted for. Alhazen, a Moorish philosopher of the 12th century, considers refraction as the cause of this phenomenon. Vitello, in his Optics, (composed before the year 1270) pa. 449, ascribes the twinkling of the stars to the motion of the air, in which the light is refracted; and he observes, in confirmation of this hypothesis, that they twinkle still more when they are viewed in water put into motion.

Dr. Hooke (Microgr. pa. 231, &c) ascribes this phenomenon to the inconstant and unequal refraction of the rays of light, occasioned by the tremulous motion of the air and interspersed vapours, in consequence of variable degrees of heat and cold in the air, producing corresponding variations in its density, and also of the action of the wind, which must cause the successive rays to fall upon the eye in different directions, and consequently on different parts of the retina at different times, and also to hit and miss the pupil alternately; and this also is the reason, he says, why the limbs of the sun, moon, and planets appear to wave or dance.

These tremors of the air are manifest to the eye by the undulating motion of shadows cast from high towers; and by looking at objects through the smoke of a chimney, or through steams of hot water, or at objects situated beyond hot sands, especially if the air be moved transversely over them. But when stars are seen through telescopes that have large apertures, they twinkle but little, and sometimes not at all. For, as Newton has observed, (Opt. pa. 98) the rays of light which pass through different parts of the aperture, tremble each of them apart, and by means of their various and contrary tremors, fall at one and the same time upon different points in the bottom of the eye, and their quivering motions are too quick and confused to be separately perceived. And all these illuminated points constitute one broad lucid point, composed of those many trembling points confusedly and insensibly mixed with one another by very short and swift tremors, and so cause the star to appear broader than it is, and without any motion of the whole.

Dr. Jurin, in his Essay on Distinct and Indistinct Vision, has recourse to Newton's hypothesis of fits of easy refraction and reflection for explaining the twinkling of the stars: thus, he says, if the middle part of the image of a star be changed from light to dark, and the adjacent ring at the same time be changed from dark to light, as must happen from the least motion of the eye towards or from the star, this will occasion such an appearance as twinkling.

Mr. Michell (Philos. Trans. vol. 57, pa. 262) supposes that the arrival of fewer or more rays at one time, especially from the smaller or more remote fixed stars, may make such an unequal impression on the eye, as may at least have some share in producing this effect: since it may be supposed that even a single particle of light is sufficient to make a sensible impression on the organs of sight; so that very few particles arriving at the eye in a second of time, perhaps not more than three or four may be sufficient to make an object constantly visible. See Light.

Hence, he says, it is not improbable that the number of the particles of light which enter the eye in a second of time, even from Sirius himself, may not exceed 3 or 4 thousand, and from stars of the 2d magnitude they may probably not exceed 100. Now the apparent increase and diminution of the light, which we observe in the twinkling of the stars, seem to be repeated at intervals not very unequal, perhaps about 4 or 5 times in a second. He therefore thought it reasonable to suppose, that the inequalities which will naturally arise from the chance of the rays coming sometimes a little denser, and sometimes a little rarer, in so small a number of them, as must fall upon the eye in the 4th or 5th part of a second, may be sufficient to account for this appearance.

Since these observations were published however, Mr. Michell (as we are informed by Dr. Priestley in his Hist. of Light, pa. 495) has entertained some suspicion, that the unequal density of light does not contribute to this effect in so great a degree as he had imagined; especially as he has observed that even Venus does sometimes twinkle. This he once observed her to do remarkably when she was about 6 degrees high, though Jupiter, which was then about 16 degrees high, and was sensibly less luminous, did not twinkle at all. If, notwithstanding the great number of rays which doubtless come to the eye from such a surface as this planet presents, its appearance be liable to be affected in this manner, it must be owing to such undulations in the atmosphere, as will probably render the effect of every other cause altogether insensible.

Musschenbroek suspects (Introd. ad. Phil. Nat. vol. 2, sect. 1741, pa. 707) that the twinkling of the stars arises from some affection of the eye, as well as the state of the atmosphere. For, says he, in Holland, when the weather is frosty, and the sky very clear, the stars twinkle most manifestly to the naked eye, though not in telescopes; and since he does not suppose there is any great exhalation, or dancing of the vapour, at that time, he questions whether the vivacity of the light, affecting the eye, may not be concerned in the phenomenon.

But this philosopher might have satisfied himself with respect to this hypothesis, by looking at the stars near the zenith, when the light traverses but a small part of the atmosphere, and therefore might be expected to affect the eye most sensibly. For he would have found that they do not twinkle near so much as they do near the horizon, when much more of their light is intercepted by the atmosphere.

Some astronomers have lately endeavoured to explain the twinkling of the fixed stars, by the extreme minuteness of their apparent diameter; so that they suppose the sight of them is intercepted by every mote that floats in the air. To this purpose Dr. Long observes (Astron. vol. 1, pa. 170), that our air near the earth is so full of various kinds of particles, which are in continual motion, that some one or other of them is perpetually passing between us and any star we look at, which makes us every moment alternately see it and lose sight of it: and this twinkling of the stars, he says, is greatest in those that are nearest the horizon, because they are viewed through a great quantity of thick air, where the intercepting particles are most numerous; whereas stars that are near the zenith do not twinkle so much, because we do not look at them through so much thick air, and therefore the intercepting particles, being fewer, come less frequently before them. With respect to the planets, it is observed that, because they are much nearer to us than the stars, they have a sensible apparent magnitude, so that they are not covered by the small particles floating in the atmosphere, and therefore do not twinkle, but shine with a steady light.

The fallacy of this hypothesis appears from the observation of Mr. Michell, that no object can hide a star from us that is not large enough to exceed the apparent diameter of the star, by the diameter of the pupil of the eye; so that if a star were even a mathematical point, or of no diameter, the interposing object must still be equal in size to the pupil of the eye; and indeed it must be large enough to hide the star from both eyes at the same time.

The principal cause therefore of the twinkling of the stars, is now acknowledged to be the unequal refraction of light, in consequence of inequalities and undulations in the atmosphere.

Besides a variation in the quantity of light, it may here be added, that a momentary change of colour has likewise been observed in some of the fixed stars. Mr. Melville (Edinb. Essays, vol. 2, pa. 81) asserts, that when one looks stedfastly at Sirius, or any bright star, not much elevated above the horizon, its colour appears not to be constantly white, but as tinctured, at every twinkling, with red and blue. Mr. Melville could not entirely satisfy himself as to the cause of this phenomenon; observing that the separation of the colours by the refractive power of the atmosphere, is probably too small to be perceived. Mr. Michell's hypothesis above-mentioned, though not adequate to the explication of the twinkling of the stars, may pretty well account for this circumstance. For the red and blue rays being much fewer than those of the intermediate colours, and therefore much more liable to inequalities from the common effect of chance, a small excess or defect in either of them will make a very sensible difference in the colour of the stars.

TYCHONIC *System*, or *Hypothesis*, is an order or arrangement of the heavenly bodies, of an intermediate nature between the Copernican and Ptolemaic; and is so called from its inventor Tycho Brahé. See System.

TYMPAN, or Tympanum, in Architecture, is the area of a pediment, being that part which is on a level with the naked of the frize. Or it is the space included between the three cornices of a triangular pediment, or the two cornices of a circular one.

Tympan is also used for that part of a pedestal called the trunk or dye.

Tympan, among joiners, is also applied to the pannels of doors.

Tympan *of an Arch*, is a triangular space or table in the corners of sides of an arch, usually hollowed and enriched, sometimes with branches of laurel, olive-tree, or oak; or with trophies, &c; sometimes with flying figures, as fame, &c; or sitting figures, as those representing the cardinal virtues.

Tympan, in Mechanics, is a kind of wheel placed round an axis, or cylindrical beam, on the top of which are two levers, or fixed staves, for more easily turning the axis about, in order to raise a weight. The tympanum is much the same with the peritrochium; but that the cylinder of the axis of the peritrochium is much shorter and less than the cylinder of the tympanum.

TYMPANUM of a machine, is also used for a hollow wheel, in which people or animals walk, to turn it; such as that of some cranes, calenders, &c.

TYR, in the Ethiopian Calendar, the name of the 5th month of the Ethiopian year. It commences on the 25th of December of the Julian year.

TYSHAS, among the Ethiopians, the name of the 4th month of their year, commencing the 27th of November in the Julian year.

U AND V

V Is a numeral letter, in the Roman numeration, denoting 5 or five. And with a dash over the top thus V̄, it denoted 5000.

VACUUM, in Physics, a space empty or devoid of all matter.—Whether there be any such thing in nature as an absolute vacuum; or whether the universe be completely full, and there be an absolute plenum; is a question that has been agitated by the philosophers of all ages.

The ancients, in their controversies, distinguished two kinds; a Vacuum coacervatum, and a Vacuum interspersum, or disseminatum.

VACUUM *Coacervatum*, is conceived as a considerably large space destitute of matter; such, for instance, as there would be, should God annihilate all the air, and other bodies, within the walls of a chamber.—The existence of such a vacuum is maintained by the Pythagoreans, Epicureans, and the Atomists or Corpuscularians; most of whom assert, that such a vacuum actually exists without the limits of the sensible world. But the modern Corpuscularians, who hold a vacuum coacervatum, deny that appellation; as conceiving that such a vacuum must be infinite, eternal, and uncreated.

According then to the later philosophers, there is no vacuum coacervatum without the bounds of the sensible world; nor would there be any other vacuum, provided God should annihilate divers contiguous bodies, than what amounts to a mere privation, or nothing; the dimensions of such a space, which the ancients held to be real, being by these held to be mere negations; that is, in such a place there is so much length, breadth, and depth wanting, as a body must have to fill it. To suppose then, that when all the matter in a chamber is annihilated, there should yet be real dimensions, is to suppose, say they, corporeal dimensions without body; which is absurd.

The Cartesians however deny any vacuum coacervatum at all, and assert that if God should immediately annihilate all the matter, for example in a chamber, and prevent the ingress of any other matter, the consequence would be, that the walls would become contiguous, and include no space at all. They add, that if there be no matter in a chamber, the walls cannot be conceived otherwise than as contiguous; those things being said to be contiguous, between which there is not any thing intermediate: but if there be no body between, there is, say they, no extension between; extension and body being the same thing: and if there be no extension between, then the walls are contiguous, and where is the vacuum?——But this reasoning, or rather quibbling, is founded on the mistake, that body and extension are the same thing.

VACUUM Disseminatum, or Interspersum, is that supposed to be naturally interspersed in and among bodies, in the interstices between different bodies, and in the pores of the same body.—It is this kind of vacuum which is chiefly contested among the modern philosophers; the Corpuscularians strenuously asserting it; and the Peripatetics and Cartesians as tenaciously denying it. See CARTESIAN and LEIBNITZIAN.

The great argument urged by the Peripatetics against a vacuum interspersum, is, that there are divers bodies frequently seen to move contrary to their own nature and inclination; and that for no other apparent reason, but to avoid a vacuum: whence they conclude, that nature abhors a vacuum; and give us a new class of motions ascribed to the fuga vacui or nature's flying a vacuum. Such, they say, is the rise of water in a syringe, on the drawing up of the piston; and such is the ascent of water in pumps, and the swelling of the flesh in a cupping glass, &c.—But since the weight, elasticity, &c, of the air have been ascertained by sure experiments, those motions and effects are universally, and justly, ascribed to the gravity and pressure of the atmosphere.

The Cartesians deny, not only the actual existence, but even the possibility of a vacuum; and that on this principle, that extension being the essence of matter, or body, wherever extension is, there is matter; but mere space, or vacuity, is supposed to be extended; therefore it is material. Whoever asserts an empty space, say they, conceives dimensions in that space, i. e. he conceives an extended substance in it; and therefore he denies a vacuum, at the same time that he admits it.—But Descartes, if we may believe some accounts, rejected a vacuum from a complaisance to the taste which prevailed in his time, against his own first sentiments; and among his familiar friends he used to call his system his philosophical romance.

On the other hand, the corpuscular authors prove, not only the possibility, but the actual existence, of a vacuum, from divers considerations; particularly from that of motion in general; and that of the planets, comets, &c, in particular; as also from the fall of bodies; from the vibration of pendulums; from rarefaction and condensation; from the different specific gravities of bodies; and also from the divisibility of matter into parts.

1. First, there could be no linear or progressive motion without a vacuum; for if all space were full of matter, no body could be moved out of its place, for want of another place unoccupied, to move into. And this argument was stated even by Lucretius.

2. The motions of the planets and comets also confirm a vacuum. Thus, Newton argues, "that there is no such

fluid medium as æther," (to fill up the porous parts of all sensible bodies, and so make a plenum,) "seems probable; because the planets and comets proceed with so regular and lasting a motion, through the celestial spaces; for hence it appears that those celestial spaces are void of all sensible resistance, and consequently of all sensible matter. Consequently if the celestial regions were as dense as water, or as quicksilver, they would resist almost as much as water or quicksilver; but if they were perfectly dense, without any interspersed vacuity, though the matter were ever so fluid and subtle, they would resist more than quicksilver does: a perfectly solid globe, in such a medium, would lose above half its motion, in moving 3 lengths of its diameter; and a globe not perfectly solid, such as the bodies of the planets and comets are, would be stopped still sooner. Therefore, that the motion of the planets and comets may be regular, and lasting, it is necessary that the celestial spaces be void of all matter; except perhaps some few and much rarefied effluvia of the planets and comets, and the passing rays of light."

3. The same great author also deduces a vacuum from the consideration of the weights of bodies; thus: "All bodies about the earth gravitate towards it; and the weights of all bodies, equally distant from the earth's centre, are as the quantities of matter in those bodies. If the æther therefore, or any other subtile matter, were altogether destitute of gravity, or did gravitate less than in proportion to the quantity of its matter; because (as Aristotle, Descartes, and others, argue) it differs from other bodies only in the form of matter; the same body might, by the change of its form, gradually be converted into a body of the same constitution with those which gravitate most in proportion to the quantity of matter: and, on the other hand, the heaviest bodies might gradually lose their gravity, by gradually changing their form; and so the weights would depend upon the forms of bodies, and might be changed with them; which is contrary to all experiment."

4. The descent of bodies also proves, that all space is not equally full; for the same author proceeds, "If all spaces were equally full, the specific gravity of that fluid with which the region of the air would, in that case, be filled, would not be less than the specific gravity of quicksilver or gold, or any other the most dense body; and therefore neither gold, nor any other body, could descend in it. For bodies do not descend in a fluid, unless that fluid be specifically lighter than the body. But by the air-pump we can exhaust a vessel, till even a bit of down shall fall with a velocity equal to that of gold in the open air; and therefore the medium through which this feather falls, must be much rarer than that through which the gold falls in the other case. The quantity of matter therefore in a given space may be diminished by rarefaction: and why may it not be diminished ad infinitum? Add, that we conceive the solid particles of all bodies to be of the same density; and that they are only rarefiable by means of their pores; and hence a vacuum evidently follows."

5. "That there is a vacuum, is evident too from the vibrations of pendulums: for since those bodies, in places out of which the air is exhausted, meet with no resistance to retard their motion, or shorten their vibrations; it is obvious that there is no sensible matter in those spaces, or in the occult pores of those bodies."

6. That there are interspersed vacuities, appears from matter's being actually divided into parts, and from the figures of those parts; for, on supposition of an absolute plenum, we do not conceive how any part of matter could be actually divided from that next adjoining, any more than it is possible to divide actually the parts of absolute space from one another: for by the actual division of the parts of a continuum from each other, we conceive nothing else understood, but the placing of those parts at a distance from one another, which in the continuum were at no distance asunder: but such divisions between the parts of matter must imply vacuities between them.

7. As for the figures of the parts of bodies, on the supposition of a plenum, they must either be all rectilinear, or all concavo-convex; otherwise they would not adequately fill space; which we do not find to be true in fact.

8. The denying a vacuum supposes what it is impossible for any one to prove to be true, viz, that the material world has no limits.

However, we are told by some, that it is impossible to conceive a vacuum. But this surely must proceed from their having imbibed Descartes's doctrine, that the essence of body is constituted by extension; as it would be contradictory to suppose space without extension. To suppose that there are fluids penetrating all bodies and replenishing space, which neither resist nor act on bodies, merely in order to avoid admitting a vacuum, is feigning two kinds of matter without any necessity or foundation; or is tacitly giving up the question.

Since then the essence of matter does not consist in extension, but in solidity, or impenetrability, the universe may be said to consist of solid bodies moving in a vacuum: nor need we at all fear, lest the phenomena of nature, most of which are plausibly accounted for from a plenum, should become inexplicable when the plenitude is set aside. The principal ones, such as the tides; the suspension of the mercury in the barometer; the motion of the heavenly bodies, and of light, &c, are more easily and satisfactorily accounted for from other principles.

Vacuum *Boileanum*, is used to express that approach to a real vacuum, which we arrive at by means of the air-pump. Thus, any thing put in a receiver so exhausted, is said to be put in vacuo: and thus most of the experiments with the air-pump are said to be performed in vacuo, or in vacuo Boileano.

Some of the principal phenomena observed of bodies in vacuo, are; that the heaviest and lightest bodies, as gold and a feather, fall with equal velocity:—that fruits, as grapes, cherries, peaches, apples, &c, kept for any time in vacuo, retain their nature, freshness, colour, &c, and those withered in the open air recover their plumpness in vacuo:—all light and fire become immediately extinct in vacuo:—little or no sound is heard from a bell rung in vacuo:—a bladder half full of air, will distend the bladder, and lift up 40 pound weight in vacuo:—most animals soon expire in vacuo.

By experiments made in 1704, Dr. Derham found that animals which have two ventricles, and no foramen ovale, as birds, dogs, cats, mice, &c, die in less than half a minute; counting from the first exsuction: a mole died in one minute; a bat lived 7 or 8. Insects, as wasps, bees, grasshoppers, &c, seemed dead in two minutes; but after being left in vacuo 24 hours, they came to life again in the open air: snails continued 24 hours in vacuo, without appearing much affected.—Seeds planted in vacuo do not grow: Small beer dies, and loses all its taste, in vacuo: and air rushing through mercury into a vacuum, throws the mercury in a kind of shower upon the receiver, and produces a great light in a dark room.

The air-pump can never produce a perfect vacuum: as is evident from its structure, and the manner of its working: in effect, every exsuction only takes away a part of the air; so that there is still some left after any finite number of exsuctions. For, the air-pump has no longer any effect but while the spring of the air remaining in the receiver is able to lift up the valves; and when the rarefaction is come to that degree, you can arrive no nearer to a vacuum; unless perhaps the air valves can be opened mechanically, independent of the spring of the air, as it is said they are in some newly improved air-pumps.

Torricellian VACUUM, is that made in the barometer tube, between the upper end and the top of the mercury. This is probably never a perfect and entire vacuum; as all fluids are found to yield or to rise in elastic vapours, on the removal of the pressure of the atmosphere. See TORRICELLIAN, and BAROMETER.

VALVE, in Hydraulics, Pneumatics, &c, is a kind of lid or cover to a tube or vessel, contrived to open one way; but which, the more forcibly it is pressed the other way, the closer it shuts the aperture: so that it either admits the entrance of a fluid into the tube, or vessel, and prevents its return; or permits it to escape, and prevents its re-entrance.

Valves are of great use in the air-pump, and other wind machines: in which they are usually made of pieces of bladder. In hydraulic engines, as the emboli of pumps, they are mostly of strong leather, of a round figure, and fitted to shut the apertures of the barrels or pipes. Sometimes they are made of two round pieces of leather enclosed between two others of brass; having divers perforations, which are covered with another piece of brass, moveable upwards and downwards, on a kind of axis, which goes through the middle of them all. Sometimes they are made of brass, covered over with leather, and furnished with a fine spring, which gives way upon a force applied against it; but on the ceasing of that, returns the valve over the aperture. See PUMP. See also Desaguliers' Exper. Philos. vol. 2, pa. 156, and pa. 180.

VANE, in a ship, &c, a thin slip of some kind of matter, placed on high in the open air, turning easily round on an axis or spindle, and veered about by the wind, to show its direction or course.

VANES, in Mathematical or Philosophical Instruments, are sights made to slide and move upon cross-staves, fore-staves, quadrants, &c.

VAPOUR, in Meteorology, a watery exhalation raised up either by the heat of the sun, or any other heat, as fire, &c. Vapour is considered as a thin vesicle of water, or other humid matter, filled or inflated with air; which, being rarefied to a certain degree by the action of heat, ascends to some height in the atmosphere, where it is suspended, till it returns in form of rain, snow, or the like. An assemblage of a number of particles or vesicles of vapour, constitutes what is called a cloud.

Some use the term vapour indifferently, for all fumes emitted, either from moist bodies, as fluids of any kind; or from dry bodies, as sulphur, &c. But Newton, and other authors, better distinguish between humid and dry fumes, calling the latter exhalations.

For the manner in which vapours are raised, and again precipitated, see CLOUD, DEW, RAIN, BAROMETER, and particularly EVAPORATION.

It may here be added, with respect to the principles of solution adopted to account for evaporation, and largely illustrated under that article, that Dr. Halley, about the beginning of the 18th century, seems to have been acquainted with the solvent power of air on water; for he says, that supposing the earth to be covered with water, and the sun to move diurnally round it, the air would of itself imbibe a certain quantity of aqueous vapours, and retain them like salts dissolved in water; and that the air warmed by the sun would sustain a greater proportion of vapours, as warm water will hold more dissolved salts; which would be discharged in dews, similar to the precipitation of salts on the cooling of liquors. Philos. Trans. vol. 3.

Mr. Eeles, in 1755, endeavoured to account for the ascent of vapour and exhalation, and their suspension in the atmosphere, by means of the electric fire. The sun, he acknowledges, is the great agent in detaching vapour and exhalations from their masses, whether he acts immediately by himself, or by his rendering the electric fire more active in its vibrations: but their subsequent ascent he attributes entirely to their being rendered specifically lighter than the lower air, by their conjunction with electrical fire: each particle of vapour, with the electrical fluid that surrounds it, occupying a greater space than the same weight of air. Mr. Eeles also endeavours to show, that the ascent and descent of vapour, attended by this fire, are the cause of all the winds, and that they furnish a satisfactory solution of the general phenomena of the weather and barometer. Philos. Trans. vol. 49, pa. 124.

Dr. Darwin, in 1757, published remarks on the theory of Mr. Eeles, with a view of confuting it; and attempting to account for the ascent of vapours, by considering the power of expansion which the constituent parts of some bodies acquire by heat, and also that some bodies have a greater affinity to heat, or acquire it sooner, and retain it longer, than others. On these principles, he thinks, it is easily understood how water, whose parts appear from the æolipile to be capable of immeasurable expansion, should by heat alone become specifically lighter than the common atmosphere. A small degree of heat is sufficient to detach or raise the vapour of water from the mass to which it belongs; and the rays of the sun communicate heat only to those bodies by which they are refracted, reflected, or obstructed; whence, by their impulse, a motion or vibration is caused in the parts of such bodies. Hence he infers, that the sphericles of vapour will, by refracting the solar rays, acquire a constant heat, though the surrounding atmosphere remain cold. If it be asked, how clouds are supported in the absence of the sun? it must be remembered, that large masses of vapour must for a considerable time retain much of the heat they have acquired in the day; at the same time reflecting how small a quantity of heat was necessary to raise them, and that doubtless even a less will be sufficient to support them; as from the diminished pressure of the atmosphere at a given height, a less power may be able to continue them in their present state of rarefaction; and lastly, that clouds of particular shapes will be sustained or elevated by the motion they acquire from winds. Philos. Trans. vol. 50, pa. 246.

The quantity of vapour raised from the sea by the warmth of the sun, must be far greater than is commonly imagined. Dr. Halley has attempted to estimate it. For the result of his calculations, see EVAPORATION.

For the Effect of Vapour in the Formation of Springs, &c, see SPRING, and RIVER.

VARENIUS (BERNARD), a learned Dutch geographer and physician, of the 17th century, who was author of the best mathematical treatise on Geography, intitled, Geographia Universalis, in qua affectiones generalis Telluris explicantur. This excellent work has been translated into all languages, and was honoured by an edition, with improvements, by Sir Isaac Newton, for the use of his academical students at Cambridge.

VARIABLE, in Geometry and Analytics, is a term applied by mathematicians, to such quantities as are considered in a variable or changeable state, either increasing or decreasing. Thus, the abscisses and ordinates of an ellipsis, or other curve line, are variable quantities; because these vary or change their magnitude together, the one at the same time with the other. But some quantities may be variable by themselves alone, or while those connected with them are constant: as the abscisses of a parallelogram, whose ordinates may be considered as all equal, and therefore constant. Also the diameter of a circle, and the parameter of a conic section, are constant, while their abscisses are variable.

Variable quantities are usually denoted by the last letters of the alphabet, z, y, x, &c: while the constant ones are denoted by the leading letters, a, b, c, &c. Some authors, instead of variable and constant quantities, use the terms fluent and stable quantities. The indefinitely small quantity by which a variable quantity is continually increased or decreased, in very small portions of time, is called the differential, or increment or decrement of that quantity. And the rate of its increase or decrease at any point, is called its fluxion; while the variable quantity itself is called the fluent. And the calculation of these, is the subject of the new Methodus Differentialis, or Doctrine of Fluxions.

VARIABLE MOTION, in Mechanics, is that motion of a body when subject to the continual action of a force which changes, or is different at every instant. We have instances of variable motion, in the unbending of springs: though the velocity continues to be augmented, yet the degrees by which the augmentation proceeds are diminishing. It is the same with regard to the degrees by which the motion of a ship arrives at uniformity: the action of the wind on the sails diminishes in proportion as the vessel acquires greater velocity, because the action of the wind varies as the difference between its velocity and that of the sail on which it acts.

For an illustration of the different natures of constant and variable accelerating motions, see the art. *Acceleration.*

VARIATION, *of Quantities*, in Algebra. See CHANGES, and COMBINATION.

Calculus of VARIATIONS, is that by which, having given an expression or function containing two or more variable quantities, whose relation is expressed by a determinate law, we find what that function becomes when the law itself is supposed to experience any variation indefinitely small, occasioned by the variation of one or of several of the terms which express that law. The origin of this calculus is imputed to the circumstance of certain problems concerning the maxima and minima of quantities having been proposed by John Bernoulli, to the mathematicians of Europe. Such a problem was that in which it was required to find, of all curves passing through two fixed points, and situated in the same vertical plane, that along which a body would descend from the highest to the lowest point in the least time possible.

The first geometricians, remarking that nothing was obtained by putting the differential of the time $\frac{dz}{\sqrt{x}} = o$, found that they could obtain a solution by making the time a minimum for two successive elements of the curve: thus, if x, x', x'', were three vertical abscissas, and y, y', y'', the corresponding ordinates, the time would be expressed by

$$\sqrt{\frac{(x'-x)^2+(y'-y)^2}{x}} + \sqrt{\frac{(x''-x')^2+(y''-y')^2}{x'}},$$

the differential of which being taken, and put $= o$, gave a resulting equation $\frac{dy}{\sqrt{x} \times \sqrt{(dx^2+dy^2)}} = b$, a constant quantity; and hence proved the curve to be a cycloid. —Euler, with far greater analytical knowledge than John Bernoulli, next treated these problems in a general manner, in his tract entitled, "Methodus inveniendas lineas curvas maximi minimive proprietate gaudentes; sive solutio problematis isoperimetricalatissimo sensu accepti." M. Lagrange afterwards gave greater generality to this calculus, by making variable not only y, dy, d^2y, &c, but also x.

The explanation of M. Lacroix affords as clear an idea of the calculus of variations as any that we are acquainted with.

"Suppose," says he, "the variable quantities at first connected together by an equation, or by any other dependence, to change by reason of the form of the equation, or of the relation that results from the dependence established between them ceasing to be the same; this circumstance cannot be expressed in a more general manner, than by regarding the increments of x and y as absolutely independent of each other; since, in effect, this hypothesis not designating any particular relation between x and y, comprehends all. It follows thence, that the calculus of variations can only be employed for expressions, to which the differential calculus has already been applied; and it differs from the last only by the independence which it supposes between the variable quantities, which before were considered as connected by constant relations. The following example will illustrate this notion. The expression $\frac{ydx}{dy}$, which belongs to the subtangent of a curve, represents a determinate function of x, when y is considered as a function whose composition in terms of x is known: and if this last changes, the first changes also. There will be perhaps some difficulty in conceiving how we can submit to calculation the variability of a function which is only the abstract dependence in which several quantities are with regard to each other: but this difficulty is removed, by considering that the connection between the quantities y and x changes if the first be made to vary independently of the second. Thus, in the example before us, if we suppose x to remain the same, and y and $\frac{dy}{dx}$ to change, the relation between x and y must necessarily have changed also, since these quantities are the immediate consequences of that relation: $\frac{dy}{dx}$, in the form $\frac{ydx}{dy}$, may alone be made to vary, since it depends only on one value of y: but, if an expression affected by the sign $\int$ (denoting the integral of that expression) be considered, y and $\frac{dy}{dx}$ must be made to vary at the same time; for it follows from the theory for the formation of integrals, that the value of a like function depends on the consecutive values of y, which are deduced from those of $\frac{dy}{dx}$.

" It is evident that, to take under this point of view the differential of any expression whatever, it is sufficient to make y, dy, d^2y, &c, vary without altering x; but in treating this latter quantity as variable as the first, we arrive at results more general and symmetrical than what are otherwise obtained, and which lead to very interesting remarks on the nature of the differential forms. For these reasons, we shall adopt in this chapter the method of making x, dy, d^2y vary. That the symbols of this new species of differentiation, in which x and y are considered as independent, may not be confounded with the symbols of the first, in which one of the variable quantities is regarded as a function of the other, we shall employ, after the manner of Lagrange, the characteristic δ; and we shall suppose, with him, that when y changes only by virtue of the change of x, which becomes $x + dx$, its differential is dy: but that when the relation of y and x varies, these two quantities become respectively $x + \delta x$, $y + \delta y$; and we note by the name of variations, the increments δx and δy.

" Hence it follows that, as $du = \frac{du}{dx}dx + \frac{du}{dy}dy$, ($u$ being a function of x and y) so $\delta u = \frac{\delta u}{\delta x}\delta x + \frac{\delta u}{\delta y}\delta y$.

" In applying this to the example $\frac{ydx}{dy}$ we must regard $\frac{dx}{dy}$ as a function of x and y; whence it results that $\delta\frac{ydx}{dy} = \frac{dx\delta y}{dy} + y\delta(\frac{dx}{dy})$, and $\delta(\frac{dx}{dy}) = \frac{dy\delta dx - dx\delta dy}{dy^2} = \frac{dyd\delta x - dxd\delta y}{dy^2}$, for $\delta dx = d\delta x$, $\delta dy = d\delta y$ — "

M. Lacroix then proves $\delta dx = d\delta x$, &c. After the methods for finding the variations of any function whatever, is given the application of the calculus to the problems of maxima and minima.

VARIATION, in Astronomy.—The Variation of the Moon, called by Bulliald, the Reflection of her Light, is the third inequality observed in the moon's motion; by which, when out of the quadratures, her true place differs from her place twice equated. See PLACE, EQUATION, &c.—Newton makes the moon's variation to arise partly from the form of her orbit, which is an ellipsis; and partly from the inequality of the spaces, which the moon describes in equal times, by a radius drawn to the earth.

To find the Greatest Variation. Observe the moon's longitude in the octants; and to the time of observation compute the moon's place twice equated; then the difference between the computed and observed place, is the greatest variation.

Tycho makes the greatest variation 40′ 30″; and Kepler makes it 51′ 49″.—But Newton makes the greatest variation, at a mean distance between the sun and the earth, to be 35′ 10″: at the other distances, the greatest variation is in a ratio compounded of the duplicate ratio of the times of the moon's synodical revolution directly, and the triplicate ratio of the distance of the sun from the earth inversely. And therefore in the sun's apogee, the greatest variation is 33′ 14″, and in his perigee 37′ 11″; provided that the eccentricity of the sun be to the transverse semidiameter of the orbis magnus, as $16\frac{11}{16}$ to 1000. Or, taking the mean motions of the moon from the sun, as they are stated in Dr. Halley's tables, then the greatest variation at the mean distance of the earth from the sun will be 35′ 7″, in the apogee of the sun 33′ 27″, and in his perigee 36′ 51″. Philos. Nat. Princ. pr. 29, lib. 3.

VARIATION *of Curvature*, is the rate at which is varied the curvature of any curve, except that of the circle, which is constant.

VARIATION, in Geography, Navigation, &c, a term applied to the deviation of the magnetic needle, or compass, from the true north point, either towards the east or west; called also the declination. Or the variation of the compass is properly defined, the angle which a magnetic needle freely suspended makes with the meridian line on an horizontal plane; or an arch of the horizon, comprehended between the true and the magnetic meridians. In the sea-language, the variation is usually called north-easting, or north-westing.

All magnetic bodies are found to range themselves, in some sort, according to the meridian; but they seldom agree precisely with it: in one place they decline, from the north toward the east, in another toward the west; and that too differently at different times.

The variation of the compass could not long remain a secret, after the invention of the compass itself: accordingly Ferdinand, the son of Columbus, in his life written in Spanish, and printed in Italian at Venice in 1571, asserts, that his father observed it on the 14th of September 1492: though others seem to attribute the discovery of it to Sebastian Cabot, a Venetian, employed in the service of our king Henry VII, about the year 1500.—It now appears however, that this variation or declination of the needle was known even some centuries earlier, though it does not appear that the use of the needle itself in navigation was then known. For it seems there is in the library of the university of Leyden, a small manuscript tract on the magnet, in Latin, written by one Peter Adsiger, bearing date the 8th of August 1269; in which the declination of the needle is particularly mentioned. Mr. Cavallo has printed the chief part of this letter in the Supplement to his Treatise on Magnetism, with a translation; and it is to be wished he had printed the whole of so curious a paper. The curiosity of this letter, says Mr. Cavallo, consists in its containing almost all that is at present known on the subject, at least the most remarkable parts of it, mixed however with a good deal of absurdity. The laws of magnetic attraction, and of the communication of that power to iron, the directive property of the natural magnet, as well as of the iron that has been touched by it, and even the declination of the magnetic needle, are clearly and unequivocally mentioned in it.

As this variation differs in different places, Gonsales d'Oviedi found there was none at the Azores; whence some geographers thought fit in their maps to make the first meridian pass through one of these islands; it not being then known that the variation altered in time. See

MAGNET; also Gilbert de Magnete, Lond. 1600, pa. 4 and 5; or Purchas's Pilgrims, Lond. 1625, book 2, sect. 1.

Various are the hypotheses that have been framed to account for this extraordinary phenomenon: we shall only notice some of the later, and more probable: just premising, that Robert Norman, the inventor of the dipping-needle, disputes against Cortes's notion, that the variation was caused by a point in the heavens; contending that it should be sought for in the earth, and proposes how to discover its place.

The first is that of Gilbert (De Magnete, lib. 4, pa. 151 &c), which is followed by Cabeus, &c. This notion is, that it is the earth, or land, that draws the needle out of its meridian direction: hence they argue, that the needle varied more or less, as it was more or less distant from any great continent; and consequently that if it were placed in the middle of an ocean, equally distant from equal tracts of land on each side, eastward and westward, it would not decline either to the one or the other, but point exactly north and south. Thus, say they, in the Azores islands, which are equally distant from Africa on the east, and America on the west, there is no variation: but as you sail from thence towards Africa, the needle begins to decline toward the east, and that still more and more till you reach the shore. Proceed still farther eastward, the declination gradually diminishes again, by reason of the land left behind on the west, which continues to draw the needle. The same also obtains till you arrive at a place where the tracts of land on each side are equal; and there again the variation will be nothing. But the misfortune is, the law does not hold universally; for multitudes of observations of the variation, in different parts, made and collected by Dr. Halley, overturn the whole theory.

Others therefore have recourse to the frame and compages of the earth, considered as interspersed with rocks and shelves, which being generally found to run towards the polar regions, the needle comes to have a general tendency that way; but it seldom happens that their direction is exactly in the meridian, and the needle has consequently, for the most part, some variation.

Others maintain that divers parts of the earth have different degrees of the magnetic virtue, as some are more intermixed with heterogeneous matters, which prevent the free action or effect of it, than others are.

Others again ascribe all to magnetic rocks and iron mines, which, affording more of the magnetic matter than other parts, attract or draw the needle more.

Lastly, others imagine that earthquakes, or high tides, have disturbed and dislocated several considerable parts of the earth, and so changed the magnetic axis of the globe, which was originally the same with the axis of the earth itself.

But none of these theories can be the true one; for still that great phenomenon, the variation of the variation, i. e. the continual change of the declination, in one and the same place, is not accountable for, on any of these foundations, nor is it even consistent with them.

Doctor Hooke communicated to the Royal Society, in 1674, a theory of the variation; the substance of which is, that the magnet has its peculiar pole, distant 10 degrees from the pole of the earth, about which it moves, so as to make a revolution in 370 years: whence the variation, he says, has altered of late about 10 or 11 minutes every year, and will probably so continue to do for some time, when it will begin to proceed slower and slower, till at length it become stationary and retrograde, and so return back again. Birch's Hist. of the Royal Society, vol. 3, pa. 131.

Dr. Halley has given a new system, the result of numerous observations, and even of a number of voyages made at the public expence on this account. The light which this author has thrown upon this obscure part of natural history, is very great, and of important consequence in navigation, &c. In this system he has reduced the several variations in divers places to a precise rule, or order, which before appeared quite precarious and arbitrary. His theory will therefore deserve a more ample detail. The observations it is built upon, as laid down in the Philos. Trans. No. 148, or Abr. vol. 2, pa. 624, are as follow:

Observed Variations of the Needle in divers places, and at divers times.

Places observed at.	Longitude from London.		Latitude.		Year of Observation.	Variation observed.	
	°	′	°	′		°	′
London - -	0	0	51	31 n	1580	11	15 e
					1622	6	0 e
					1634	4	5 e
					1672	2	30 w
					1683	4	30 w
Paris - -	2	25 e	48	51 n	1640	3	0 e
					1666	0	0
					1681	2	30 w
Uraniburg -	13	0 e	55	54 n	1672	2	35 w
Copenhagen -	12	53 e	55	41 n	1649	1	53 e
					1672	3	45 w
Dantzick - -	19	0 e	54	23 n	1679	7	0 w
Montpelier -	4	0 e	43	37 n	1674	1	10 w
Brest - -	4	25 w	48	23 n	1680	1	45 w
Rome - -	13	0 e	41	50 n	1681	5	0 w
Bayonne - -	1	20 w	43	30 n	1680	1	20 w
Hudson's Bay -	79	40 w	51	0 n	1668	19	15 w
In Hudson's Straits	57	0 w	61	0 n	1668	29	30 w
Baffin's Bay, Sir T. Smith's Sound -	80	0 w	78	0 n	1616	57	0 w
At Sea - -	57	0 w	38	40 n	1682	7	30 w
At Sea - -	31	30 w	43	50 n	1682	5	30 w
At Sea - -	42	0 w	21	0 n	1678	0	40 e
Cape St. Augustine	35	30 w	28	0 s	1670	5	30 e
Off the mouth of River Plate	53	0 w	39	30 s	1670	20	30 e
Cape Frio - -	41	10 w	22	40 s	1670	12	10 e
Entrance of Magellan's Straits -	68	0 w	52	30 s	1670	17	0 e
West entrance of ditto -	75	0 w	53	0 s	1670	14	10 e
Baldivia - -	73	0 w	40	0 s	1670	8	10 e
Cape Aguillas -	16	30 e	34	50 s	1622	2	0 w
					1675	8	0 w
At Sea - -	1	0 e	34	30 s	1675	0	0
At Sea - -	20	0 w	34	0 s	1675	10	30 e
At Sea - -	32	0 w	24	0 s	1675	10	30 e
St. Helena -	6	30 w	16	0 s	1677	0	40 e
Isle Ascension -	14	30 w	7	50 s	1678	1	0 e
Johanna - -	44	0 e	12	15 s	1675	19	30 w
Mombasa -	40	0 e	4	0 s	1675	16	0 w
Zocatra - -	56	0 e	12	30 n	1674	17	0 w
Aden, Mouth of Red Sea -	47	30 e	13	0 n	1674	15	0 w

Places observed at.	Longitude from London.		Latitude.		Year of Observation.	Variation observed.	
	°	′	°	′		°	′
Diego Roiz -	61	0 e	20	0 s	1676	20	30 w
At Sea - -	64	30 e	0	0	1676	15	30 w
At Sea - -	55	0 e	27	0 s	1676	24	0 w
Bombay - -	72	30 e	19	0 n	1676	12	0 w
Cape Comorin -	76	0 e	8	15 n	1680	8	48 w
Ballasore - -	87	0 e	21	30 n	1680	8	10 w
Fort St. George .	80	0 e	13	15 n	1680	8	10 w
West Point of Java	104	0 e	6	40 s	1676	3	10 w
At Sea - -	58	0 e	39	0 s	1677	27	30 w
I. St. Paul -	72	0 e	38	0 s	1677	23	30 w
At Van Diemen's	142	0 e	42	25 s	1642	0	0
At New Zealand	170	0 e	40	50 s	1642	9	0 e
Three-kings Isle in ditto	169	30 e	34	35 s	1642	8	40 e
I. Rotterdam in the South Sea	184	0 e	20	15 s	1642	6	20 e
Coast of New Guinea -	149	0 e	4	30 s	1643	8	45 e
West Point of ditto	126	0 e	0	26 s	1643	5	30 e

On these observed variations Dr. Halley makes several remarks, as to the variation in different parts of the world at the time of his writing, eastward and westward, and the situation and direction of the lines or places of no variation: from the whole he deduces the following theory.

Dr. Halley's Theory of the Variation of the Magnetic Needle.—That the whole globe of the earth is one great magnet, having four magnetical poles, or points of attraction; near each pole of the equator two; and that in those parts of the world which lie nearly adjacent to any one of these magnetic poles, the needle is governed by it; the nearest pole being always predominant over the more remote.

The pole which at present is nearest to us, he conjectures to lie in or near the meridian of the Land's-end of England, and not above 7° from the north pole: by this pole, the variations in all Europe and Tartary, and the North Sea, are chiefly governed; though still with some regard to the other northern pole, whose situation is in the meridian passing about the middle of California, and about 15° from the north pole of the world, to which the needle has chiefly respect in all North America, and in the two oceans on either side of it, from the Azores westward to Japan, and farther.

The two southern magnetic poles, he imagines, are rather more distant from the south pole of the world; the one being about 16° from it, on a meridian 20° to the westward of the Magellanic Streights, or 95° west from London: this pole commands the needle in all South America, in the Pacific Ocean, and the greatest part of the Ethiopic Ocean. The other magnetic pole seems to have the greatest power, and the largest dominion of all, as it is the most remote from the pole of the world, being little less than 20° distant from it, in the meridian which passes through New Holland, and the island Celebes, about 120° east from London: this pole is predominant in the south part of Africa, in Arabia, and the Red Sea, in Persia, India, and its islands, and all over the Indian sea, from the Cape of Good Hope eastward, to the middle of the Great South Sea that divides Asia from America.

Such, he observes, seems to be the present disposition of the magnetic virtue throughout the whole globe of the earth. It is then shown how this hypothesis accounts for all the variations that have been observed of late, and how it answers to the several remarks drawn from the table. It is there inferred that from the whole it appears, that the direction of the needle, in the temperate and frigid zones, depends chiefly on the counterpoise of the forces of two magnetic poles of the same nature: as also why, under the same meridian, the variation should be in one place 29½ degrees west, and in another 20½ degrees east.

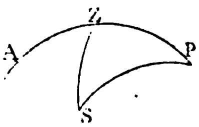
In the torrid zone, and particularly about the equator, respect must be had to all the four poles, and their positions must be well considered, otherwise it will not be easy to determine what the variation should be, the nearest pole being always strongest; yet so however as to be sometimes counterbalanced by the united forces of two more remote ones. Thus, in sailing from St. Helena, by the isle of Ascension, to the equator, on the north-west course, the variation is very little easterly, and unalterably the same in that whole track; because the South-American pole (which is much the nearest in the aforesaid places), requiring a great easterly variation, is counterpoised by the contrary attraction of the North-American and the Asiatic south poles; each of which singly is, in these parts, weaker than the American south pole; and on the north-west course the distance from this latter is very little varied; and as you recede from the Asiatic south-pole, the balance is still preserved by an access toward the North-American pole. In this case no notice is taken of the European north pole; its meridian being a little removed from the meridians of these places, and of itself requiring the same variations which are here found.

After the same manner may the variations in other places about the equator be accounted for, upon Dr. Halley's hypothesis.—But still this will do nothing as to accounting for the continual variation or change of the declination, in the same place.

To observe the Variation of the Needle.—Draw a meridian line, as directed under MERIDIAN; then a stile being erected in the middle of it, place a needle upon it, and draw the right line which it hangs over. Thus will the quantity of the variation appear.

Or thus: As the former method of finding the variation cannot be applied at sea, others have been devised, the principal of which are as follow. Suspend a thread and plummet over the compass, till the shadow pass through the centre of the card; observe the rhumb, or point of the compass which the shadow touches when it is the shortest. For the shadow is then a meridian line; and consequently the variation is determined.

Or thus: Observe the point of the compass on which the sun, or some star, rises and sets; bisect the arch intercepted between the rising and setting points, and the line of bisection will be the meridian line; consequently the variation is had as before. The same may also be obtained from two equal altitudes of the same star, observed either by day or night. Or thus: Observe the rhumb upon which the sun or star rises and sets; and from the latitude of the place find the eastern or western amplitude: for the difference between the amplitude, and the distance of the rhumb observed, from the eastern rhumb of the card, is the variation sought.



Or thus: Observe the altitude of the sun, or some star s, whose declination is known; and note the rhumb in the compass to which it then corresponds. Then in the triangle ZPS, are known three sides, viz, PZ the co-latitude, PS the co-declination, and ZS the co-altitude; the angle PZS is thence found by spherical trigonometry; the supplement to which, viz AZS, is the azimuth from the south. Then the difference between the azimuth and the observed distance of the rhumb from the south, is the variation sought. See *Azimuth* COMPASS.

The use of the variation is to correct the courses a ship has steered by the compass, which must always be done before they are worked, or calculated.

VARIATION *of the Variation*, is a gradual and continual change in the variation, observed in any place, by which the quantity of the variation is found to be different at different times.

This variation, according to Henry Bond (in his Longitude Found, Lond. 1670, pa. 6), "was first found to decrease by Mr. John Mair; 2dly, by Mr. Edmund Gunter: 3dly, by Mr. Henry Gellibrand; 4thly, by myself (Henry Bond) in 1640; and lastly, by Dr. Robert Hooke, and others, in 1665;" which they found out by comparing together observations made at the same place, at different times. The discovery was soon known abroad; for Kircher, in his treatise entitled Magnes, first printed at Rome in 1641, says that our countryman Mr. John Greaves had informed him of it, and then he gives a letter of Mersenne's, containing a distinct account of it.

This continual change in the variation, is gradual and universal, as appears by numerous observations. Thus, the variation was, at Paris,

in 1550 - 8° 0′E
in 1640 - 3 0E
in 1660 - 0 0
in 1681 - 2 2W
in 1759 - 18 10W
in 1760 - 18 20W
in 1794 - 21 54W
in 1798 - 22 15W
in 1799 - 22 0W
in 1800 - 22 12W
in 1801 - 22 1W
in 1802 - 21 45W
in 1803 - 21 59W
in 1804 - 22 15W
in 1810 - 22 16W

the variation towards the conclusion appearing obviously to vacillate about a limit. M. de la Lande (Exposition du Calcul Astronomique) observes, that the variation has changed, at Paris, 26° 20′ in the space of 150 years, allowing that in 1610 the variation was 8° E: and since 1740 the needle, which was always used by Maraldi, is more than 3° advanced toward the west, beyond what it was at that period; which is a change after the rate nearly of 9¼′ per year.

At Cape d'Agulhas, in 1600, it had no variation; (whence the Portuguese gave it that name);

in 1622 it was 2°W
in 1675 - 8 W
in 1692 - 11 W

which is a change of nearly 8′ per year.

At St. Helena, the variation, in 1600 was 8° 0′E
in 1623 - 6 0E
in 1677 - 0 40E
in 1692 - 1 0W

which is a change of nearly 5½′ per year.

At Cape Comorin, the variation, in 1620 was 14° 20′W
in 1680 - 8 44W
in 1688 - 7 30W

which is a change of nearly 6¼′ per year.

At London, the variation, in 1580 was 11° 15′E
in 1622 - 6 0E
in 1634 - 4 5E
in 1657 - 0 0
in 1672 - 2 30W
in 1692 - 6 0W
in 1723 - 14 17W
in 1747 - 17 40W
in 1780 - 22 41W
in 1812 - 24 16W

which till 1780, is a change after the rate of 10′ per year. But in the 32 years, from 1780 to 1812, the change was only 1° 35′, being only at the rate of 3′ a year. See Philos. Trans. No. 148 and No. 383, and also vol. 45, pa. 280, and vol. 66, pa. 393. On the subject of the variation, see also Norman's New Attractive 1614; Burrows's Discovery of the Variation 1581; Bond's Longitude Found 1676; &c.

Mr. Thomas Harding, in the Transactions of the Royal Irish Academy, vol. 4, has given observations on the variation of the magnetic needle, at Dublin, which are rather extraordinary. He says the change in the variation at that place is uniform; that from the year 1657, in which the variation was nothing (the same as at London in that year), it has been going on at the medium rate of 12′ 20″ annually, and was in May 1791, 27° 23′ west; exceeding that at London now by 3 or 4 degrees. He brings proof of his assertion of the uniformity of the variation, from different authentic records, and states the operations by which it is calculated. He concludes with recommending accuracy in marking the existing variation when maps are made, as not only conducing to the exact definition of boundaries, but as laying the best foundation for a discovery of the longitude by sea or land.

Theory of the Variation of the Variation.—According to Dr. Halley's theory, this change in the variation of the compass, is supposed owing to the difference of velocity in the motions of the internal and external parts of the globe. From the observations that have been cited, it seems to follow, that all the magnetical poles have a motion westward, but yet not exactly about the axis of the earth, for then the variations would continue the same in the same parallel of latitude, contrary to experience.

From the disagreement of such a supposition with experiments therefore, the learned author of the theory invented the following hypothesis: The external parts of the globe he considers as the shell, and the internal as a nucleus, or inner globe; and between the two he conceives a fluid medium. That inner earth having the same common centre and axis of diurnal rotation, may revolve with our earth every 24 hours: only the outer sphere having its turbinating motion somewhat swifter or slower than the internal ball; and a very minute difference in length of time, by many repetitions, becoming sensible; the internal parts will gradually recede from the external, and they will appear to move, either eastward or westward, by the difference of their motions.

Now, supposing such an internal sphere, having such a motion, the two great difficulties in the former hypothesis are easily solved; for if this exterior shell of earth be a magnet, having its pole at a distance from the poles of diurnal rotation; and if the internal nucleus be likewise a magnet, having its poles in two other places, distant also from the axis; and these latter, by a slow gradual motion, change their place in respect of the external, a reasonable account may then be given of the four magnetical poles before mentioned, and also of the changes of the needle's variation.

The author thinks that two of these poles are fixed, and the other two moveable; viz, that the fixed poles are the poles of the external cortex or shell of the earth; and the other the poles of the magnetical nucleus, included and moveable within the former. From the observations he infers, that the motion is westward, and consequently that the nucleus has not precisely attained the same velocity with the exterior parts in their diurnal rotation; but so very nearly equals it, that in 365 revolutions the difference is scarcely sensible.

That there is any difference of this kind, arises from hence, that the impulse by which the diurnal motion was impressed on the earth, was given to the external parts, and from thence in time communicated to the internal; but so as not yet perfectly to equal the velocity of the first motion impressed on the superficial parts of the globe, and still preserved by them.

As to the precise period, observations are wanting to determine it, though the author thinks we may reasonably conjecture that the American pole has moved westward 46° in 90 years, and that its whole period is performed in about 700 years.

Mr. Whiston, in his New Laws of Magnetism, raises several objections against this theory. See MAGNETISM.

M. Euler too, the son of the celebrated mathematician of that name, has controverted and censured Dr. Halley's theory. He thinks, that two magnetic poles, placed on the surface of the earth, will sufficiently account for the variation: and he then endeavours to show, how we may determine the declination of the needle, at any time, and on every part of the globe, from this hypothesis.

Euler first examines the case in which the two magnetic poles are diametrically opposite; 2d, he places them in the two opposite meridians, but at unequal distances from the poles of the world; 3d, he places them in the same meridians; finally, he considers them situated in two different meridians. These four cases may become equally important; because if it is determined that there are only two magnetic poles, and that these poles change their situations, it may some time hereafter be discovered that they pass through all the different positions.

Since the needle of the compass ought always to be in the plane which passes through the place of observation and two magnetic poles, the problem is reduced to the discovery of the angle contained between this plane and the plane of the meridian. M. Euler, after having examined the different cases, finds that they also express the earth's magnetism, represented in the chart published by Messrs. Mountaine and Dodson in 1744, particularly throughout Europe and North America, if the following principles are established; viz, Between the arctic pole and the magnetic pole 14° 53';—between the antarctic pole and the other magnetic pole 29° 23';—the angle at the north pole, formed by the meridians passing through the two magnetic poles 53° 18';—The longitude of the meridian which passes over the northern magnetic pole 250'.

As the observations which have been collected, with regard to the variation, are for the most part loose and inaccurate, it is impossible to represent them all with precision; and the great variations observed in the Indian ocean seem to require, says Euler, that the first three quantities should be 14,35, and 65 degrees.—See Cavallo's Treatise on Magnetism, pa. 117.

In the memoir of Messrs. Biot and Humboldt, "On the variations of the terrestrial magnetism in different latitudes," the position of the magnetic equator is determined from direct observations. The inclination of the plane of this circle to the astronomical equator, is stated to be 10° 58' 56", its occidental node on that equator being at 120° 2' 5" longitude west from Paris, the other node at 59° 57' 55" east of Paris. The points where the axis of the magnetic equator pierces the earth's surface, are, the northern point at 79° 1' 4' north lat. and 30° 2' 5" west long. from Paris; the southern point is situated in the same latitude south, and 149° 57' 55" east long. from Paris.

Variation of the Needle by Heat and Cold, otherwise called the *Diurnal* or *Daily Variation.*—There is a small variation of the variation of the magnetic needle, amounting only to a few minutes of a degree in the same place, at different hours of the same day, which is only discoverable by nice observations. Mr. George Graham made several observations of this kind in the years 1722 and 1723, professing himself altogether ignorant of the cause of the phenomena he observed. Philos. Trans. No. 383.

About the year 1750, Mr. Wargentin, secretary of the Swedish Academy of Sciences, took notice both of the regular diurnal variation of the needle, and also of its being disturbed at the time of the aurora borealis, as recorded in the Philos. Trans. vol. 47, pa. 126.

About the year 1756, Mr. Canton commenced a series of observations, amounting to near 4000, with an excellent variation-compass, of about 9 inches diameter. The number of days on which these observations were made, was 603, and the diurnal variation on 574 of them was regular, so as that the absolute variation of the needle westward was increasing from about 8 or 9 o'clock in the morning, till about 1 or 2 in the afternoon, when the needle became stationary for some time; after that, the absolute variation westward was decreasing, and the needle came back again to its former situation, or nearly so, in the night, or by the next morning. The diurnal variation is irregular when the needle moves slowly eastward in the latter part of the morning, or westward in the latter part of the afternoon; also when it moves much either way after night, or suddenly both ways in a short time. These irregularities seldom happen more than once or twice in a month, and are always accompanied, as far as Mr. Canton observed, with an aurora borealis.

Mr. Canton lays down and evinces, by experiment, the following principle, viz, that the attractive power of the magnet (whether natural or artificial) will decrease while the magnet is heating, and increase while it is cooling. He then proceeds to account for both the regular and irregular variation. It is evident, he says, that the magnetic parts of the earth in the north, on the east side and on the west side of the magnetic meridian, equally attract the north end of the needle. If then the eastern magnetic parts be heated faster by the sun in the morning, than the western parts, the needle will move westward, and the absolute variation will increase: when the attracting parts of the earth on each side of the magnetic meridian have their heat increasing equally, the needle will be stationary, and the absolute variation will then be greatest: but when the western magnetic parts are either heating faster or cooling slower than the eastern, the needle will move eastward, or the absolute variation will decrease; and when the eastern and western magnetic parts are cooling equally fast, the needle will again be stationary, and the absolute variation will then be a minimum.

By this theory, the diurnal variation in the summer ought to exceed that in winter; and accordingly it is found by observation, that the diurnal variation in the months of June and July is almost double of that in December and January.

The irregular diurnal variation must arise from some other cause than that of heat communicated by the sun; and here Mr. Canton has recourse to subterranean heat, which is generated without any regularity as to time, and which will, when it happens in the north, affect the attractive power of the magnetic parts of the earth on the north end of the needle. That the air nearest the earth will be most warmed by the heat of it, is obvious; and this has been often noticed in the morning, before day, by means of thermometers at different distances from the ground. Philos. Trans. vol. 48, pa. 526.

Mr. Canton has annexed to his paper on this subject, a complete year's observations; from which it appears, that the diurnal variation increases from January to June, and decreases from June to December. Philos. Trans. an. 1759, pa. 398. Abridg. vol. 11, pa. 421.

It has also been observed, that different needles, especially if touched with different loadstones, will differ a few minutes in their variation. See Poleni Epist. Phil. Trans. No. 421.

Dr. Lorimer (in the Supp. to Cavallo's Magnetism) adduces some ingenious observations on this subject. It must be allowed, says he, according to the observations of several ingenious gentlemen, that the collective magnetism of this earth arises from the magnetism of all the ferruginous bodies contained in it, and that the magnetic poles should therefore be considered as the centres of the powers of those magnetic substances. These poles must therefore change their places according as the magnetism of such substances is affected; and if with Mr. Canton we allow, that the general cause of the diurnal variation arises from the sun's heat in the forenoon and afternoon of the same day, it will naturally occur, that the same cause, being continued, may be sufficient to produce the general variation of the magnetic needle for any number of years. For we must consider, that ever since any attentive observations have been made on this subject, the natural direction of the magnetic needle in Europe has been constantly moving from east to west, and that in other parts of the world it has continued its motion with equal constancy.

As we must therefore admit, says Dr. Lorimer, that the heat in the different seasons depends chiefly on the sun, and that the months of July and August are commonly the hottest, while January and February are the coldest months of the year; and that the temperature of the other months falls into the respective intermediate degrees; so we must consider the influence of heat upon magnetism to operate in the like manner, viz, that for a short time it scarcely manifests itself; yet in the course of a century, the constancy and regularity of it becomes sufficiently apparent. It would therefore be idle to suppose, that such an influence could be derived from an uncertain or fortuitous cause. But if it be allowed to depend on the constancy of the sun's motion, and this appears to be a cause sufficient to explain the phenomena, we should (agreeably to Newton's first law of philosophizing) look no farther.

As we therefore consider, says he, the magnetic powers of the earth to be concentrated in the magnetic poles, and that there is a diurnal variation of the magnetic needle, these poles must perform a small diurnal revolution proportional to such variation, and return again to the same point nearly. Suppose then that the sun in his diurnal revolution passes along the northern tropic, or along any parallel of latitude between it and the equator; when he comes to that meridian in which the magnetic pole is situated, he will be much nearer to it, than in any other; and in the opposite meridian he will of course be the farthest from it. As the influence of the sun's heat will therefore act most powerfully at the least, and less forcibly at the greatest distance, the magnetic pole will consequently describe a figure something of the elliptical kind; and as it is well known that the greatest heat of the day is some time after the sun has passed the meridian, the longest axis of this elliptical figure will lie north-easterly in the northern, and south-easterly in the southern hemisphere. Again, as the influence of the sun's heat will not from those quarters have so much power, the magnetic poles cannot be moved back to the very same point from which they set out; but to one which will be a little more northerly and easterly, or more southerly and easterly, according to the hemispheres in which they are situated. The figures therefore which they describe, may more properly be termed elliptoidal spirals.

In this manner the variation of the magnetic needle in the northern hemisphere may be accounted for. But with respect to the southern hemisphere we must recollect, that though the lines of declination in the northern hemisphere have constantly moved from west to east, yet in the southern hemisphere, it is equally certain that they have moved from east to west, ever since any observations have been made on the subject. Hence then the lines of magnetic declination, or Halleyan curves, as they are now commonly called, appear to have a contrary motion in the southern hemisphere, to what they have in the northern; though both the magnetic poles of the earth move in the same direction, that is from west to east.

In the northern hemisphere there was a line of no variation, which had east variation on its eastern side, and west variation on its western side. This line evidently moved from west to east during the two last centuries; the lines of east variation moving before it, while the lines of west variation followed it with a proportional pace. These lines first passed the Azores or Western Islands, then the meridian of London, and after a certain number of years still later, they passed the meridian of Paris. But in the southern hemisphere there was another line of no variation, which had east variation on its western side, and west variation on its eastern; the lines of east variation moving before it, while those of the west variation followed it. This line of no variation first passed the Cape des Aiguilles, and then the Cape of Good Hope; the lines of 5°, 10°, 15°, and 20° west variation following it, the same as was the case in the northern hemisphere, but in the contrary direction.

We may just farther mention the idea of Dr. Gowin Knight, which was, that this earth had originally received its magnetism, or rather that its magnetical powers had been brought into action, by a shock, which entered near the southern tropic, and passed out at the northern one. His meaning appears to have been, that this was the course of the magnetic fluid, and that the magnetic poles were at first diametrically opposite to each other. Though, according to Mr. Canton's doctrine, they would not have long continued so; for from the intense heat of the sun in the torrid zone, according to the principles already explained, the north pole must have soon retired to the north-eastward, and the south pole to the south-eastward. It is also curious to observe, that on account of the southern hemisphere being colder upon the whole than the northern hemisphere, the magnetic poles would have moved with unequal pace: that is, the north magnetic pole would have moved farther in any given time to the north-east, than the south magnetic pole could have moved to the south-east. And, according to the opinions of the most ingenious authors on this subject, it is generally allowed, that at this time the north magnetic pole is considerably nearer to the north pole of the earth, than the south magnetic pole is to the south pole of the earth.

It may farther be added, that several ingenious sea officers are of opinion, that in the western parts of the English Channel the variation of the magnetic needle has already begun to decrease; having in no part of it ever amounted to 25°. There are however other persons who assert that the variation is still increasing in the Channel, and as far westward as the 15th degree of longitude and 51° of latitude, at which place they say that it amounts to about 30°.

Of the Variation Chart. Doctor Halley having collected a multitude of observations made on the variation of the needle in many parts of the world, was hence enabled to draw, on a Mercator's chart, certain lines, showing the variation of the compass in all those places over which they passed, in the year 1700, when he published the first chart of this kind, called the Variation Chart.

From the construction of this chart it appears, that the longitude of any of those places may be found by it, when the latitude and the variation in that place are known. Thus, having found the variation of the compass, draw a a parallel of latitude on the chart through the latitude found by observation; and the point where it cuts the curved line, whose variation is the same with that observed, will be the ship's place. A similar project of thus finding the longitude, from the known latitude and inclination or dip of the needle, was before proposed by Henry Bond, in his treatise entitled, The Longitude Found, printed in 1676.

This method however is attended with two considerable inconveniences: 1st, That wherever the variation lines run east and west, or nearly so, this way of finding the longitude becomes imperfect, as their intersection with the parallel of latitude must be very indefinite: and among all the trading parts of the world, this imperfection is at present found chiefly on the western coasts of Europe, between the latitudes of 45° and 53°; and on the eastern shores of North America, with some parts of the Western Ocean and Hudson's Bay, lying between the said shores: but for the other parts of the world, a variation chart may be attended with considerable benefit. However, the variation curves, when they run east and west, may sometimes be applied to good purpose in correcting the latitude, when meridian observations cannot be had, as it often happens on the northern coasts of America, in the Western Ocean, and about Newfoundland; for if the variation can be obtained exactly, then the east and west curve, answering to the variation in the chart, will show the latitude.

2dly, As the deviation of the magnetical meridian, from the true one, is subject to continual alteration, therefore a chart to which the variation lines are fitted for any year, must in time become useless, unless new lines, showing the state of the variation at that time, be drawn on the chart: but as the change in the variation is very slow, therefore new variation charts published every 7 or 8 years, will answer the purpose tolerably well. And thus it has happened that Halley's variation chart has become useless, for want of encouragement to renew it from time to time.

However, in the year 1744, Mr. William Mountaine and Mr. James Dodson published a new variation chart, adapted for that year, which was well received; and several instances of its great utility having been communicated to them, they fitted the variation lines anew for the year 1756, and in the following year published the 3d variation chart, and also presented to the Royal Society a curious paper concerning the variation of the magnetic needle, with a set of tables annexed, containing the result of upwards of 50 thousand observations, in six periodical reviews, from the year 1700 to 1756 inclusive, and adapted to every 5 degrees of latitude and longitude in the more frequented oceans; which paper and tables were printed in the Transactions for the year 1757.

From these tables of observations, such extraordinary and whimsical irregularities occur in the variation, that we cannot think it wholly under the direction of one general and uniform law; but rather conclude, with Dr. Knight, in the 87th prop. of his Treatise upon Attraction and Repulsion, that it is influenced by various and different magnetic attractions, perhaps occasioned by the heterogeous compositions in the great magnet, the earth.

Many other observations on the variation of the magnetic needle, are to be found in several volumes of the Philos. Trans. See particularly vol. 48; pa. 875; vol. 50, pa. 329; vol. 56, pa. 220; and vol. 61, pa. 422.

VARIATION *Compass.* See COMPASS.

VARIATION *of Curvature*, in Geometry, is used for that inequality or change which takes place in the curvature of all curves except the circle, by which their curvature is more or less in different parts of them. And this variation constitutes the quality of the curvature of any line.

Newton makes the index of the inequality, or variation of curvature, to be the ratio of the fluxion of the radius of curvature to the fluxion of the curve itself: and Maclaurin, to avoid the perplexity that different notions, connected with the same terms, occasion to learners, has adopted the same definition: but he suggests, that this ratio gives rather the variation of the ray of curvature, and that it might have been proper to have measured the variation of curvature rather by the ratio of the fluxion of the curvature itself to the fluxion of the curve; so that, the curvature being inversely as the radius of curvature, and consequently its fluxion as the fluxion of the radius itself directly, and the square of the radius inversely, its variation would have been directly as the measure of it according to Newton's definition, and inversely as the square of the radius of curvature.

According to this notion, it would have been measured by the angle of contact contained by the curve and circle of curvature, in the same manner as the curvature itself is measured by the angle of contact contained by the curve and tangent. The reason of this remark may appear from this example: The variation of curvature, according to Newton's explication, is uniform in the logarithmic spiral, the fluxion of the radius of curvature in this figure being always in the same ratio to the fluxion of the curve; and yet, while the spiral is produced, though its curvature decreases, it never vanishes; which must appear a strange paradox to those who do not attend to the import of Newton's definition. Newton's Method of Fluxions and Inf. Series, pa. 76. Maclaurin's Flux. art. 386. Philos. Trans. No. 468, pa. 342.

The variation of curvature at any point of a conic section, is always as the tangent of the angle contained by the diameter that passes through the point of contact, and the perpendicular to the curve at the same point, or to the angle formed by the diameter of the section and of the circle of curvature. Hence the variation of curvature vanishes at the extremities of either axis, and is greatest when the acute angle, contained by the diameter passing through the point of contact and the tangent, is least.

When the conic section is a parabola, the variation is as the tangent of the angle, contained by the right line drawn from the point of contact to the focus, and the perpendicular to the curve. See CURVATURE.

From Newton's definition may be derived practical rules for the variation of curvature, as follows:

1. Find the radius of curvature, or rather its fluxion; then divide this fluxion by the fluxion of the curve, and the quotient will give the variation of curvature; exterminating the fluxions when necessary, by the equation of the curve, or perhaps by expressing their ratio by help of the tangent, or ordinate, or subnormal, &c.

2. Since $\frac{\dot{z}^3}{-\dot{x}\ddot{y}}$, or $\frac{\dot{z}^3}{-\ddot{y}}$ (putting $\dot{x} = 1$) denotes the radius of curvature of any curve z, whose absciss is x, and ordinate y; if the fluxion of this be divided by $\dot{z}$, and $\dot{z}$ and $\ddot{z}$ be exterminated, the general value of the variation will come out $\frac{-3\dot{y}\ddot{y}^2 + \dddot{y}(1 + \dot{y}^2)}{\ddot{y}^2}$; then substituting the values of $\dot{y}$, $\ddot{y}$, $\dddot{y}$ (found from the equation of the curve) into this quantity, it will give the variation sought.

Ex. Let the curve be the parabola, whose equation is $ax = y^2$. Here then $2y\dot{y} = a\dot{x} = a$, and $\dot{y} = \frac{a}{2y}$; hence $\ddot{y} = \frac{-a\dot{y}}{2yy} = \frac{-aa}{4y^3}$, and $\dddot{y} = \frac{3aa\dot{y}}{2y^4} = \frac{3a^3}{8y^5}$. Therefore $\frac{-3\dot{y}\ddot{y}^2 + \dddot{y}(1 + \dot{y}^2)}{\ddot{y}^2} = -3\dot{y} + \dddot{y} \times \frac{1 + \dot{y}^2}{\ddot{y}^2} = \frac{-3a}{2y} + \frac{3a^3}{8y^5} \times (1 + \frac{aa}{4yy}) \times \frac{16y^6}{a^4} = \frac{6y}{a}$, the variation sought. Emerson's Flux. pa. 228.

VARIGNON (PETER), a celebrated French mathematician and priest, was born at Caen in 1654, and died 1722, at 68 years of age. He was the son of an architect in middling circumstances, but had a college education, being intended for the church. An accident threw a copy of Euclid's Elements in his way, which gave him a strong turn to that kind of learning. The study of geometry led him to the works of Descartes on the same science, and there he was struck with that new light which has, from thence, spread over the world.

At this time the abbé St. Pierre, who studied philosophy in the same college, became acquainted with him. A taste in common for rational subjects, whether physics or metaphysics, and continual disputations, formed the bonds of their friendship. They were mutually serviceable to each other in their studies. The abbé, to enjoy Varignon's company with greater ease, lodged him with himself; thus, growing still more sensible of his merit, he resolved to give him a fortune, that he might fully pursue his genius, and improve his talents; and, out of only 18 hundred livres a year, which he had himself, he conferred 300 of them on Varignon.

The abbé, persuaded that he could not do better than go to Paris to study philosophy, settled there in 1686, with M. Varignon, in the suburbs of St. Jacques. There each studied in his own way; the abbé applying himself to the study of men, manners, and the principles of government; whilst Varignon was wholly occupied with the mathematics. In these solitary suburbs he formed a connection with many other learned men; as Duhamel, Duverney, Delahire, &c. Duverney often asked his assistance in those parts of anatomy connected with mechanics: they examined together the positions of the muscles, and their directions; hence Varignon learned a good deal of anatomy from Duverney, which he repaid by the application of mathematical reasoning to that subject.

At length, in 1687, Varignon made himself known to the public by a treatise on New Mechanics, dedicated to the Academy of Sciences. His thoughts on this subject were, in effect, quite new. He discovered truths, and laid open their sources. In this work, he demonstrated the necessity of an equilibrium, in such cases where it happens, though the cause of it is not exactly known. This discovery Varignon made by the theory of compound motions, and is what this essay chiefly treats upon.

This new treatise on mechanics was greatly admired by the mathematicians, and procured the author two considerable places, the one of geometrician in the Academy of Sciences, the other of professor of mathematics in the College of Mazarine, to which he was the first person raised.

Varignon catched eagerly at the science of infinitesimals as soon as it appeared in the world, and became one of its most early cultivators. When that sublime and beautiful method was attacked in the Academy itself (for it could not escape the fate of all innovations), he became one of its most zealous defenders, and in its favour he put a violence upon his natural character, which abhorred all contention. He sometimes lamented, that this dispute had interrupted him in his enquiries into the integral calculation so far, that it would be difficult for him to resume his disquisition where he had left it off. He sacrificed infinitesimals to the interest of infinitesimals, and gave up the pleasure and glory of making a farther progress in them when called upon by duty to undertake their defence.

All the printed volumes of the Academy bear witness to his application and industry. His works are never detached pieces, but complete theories of the laws of motion, central forces, and the resistance of mediums to moving bodies. In these he makes such use of his rules, that nothing escapes him that has any connection with the subject he treats.

Geometrical certainty is by no means incompatible with obscurity and confusion, and those are sometimes so great, that it is surprising a mathematician should not lose his way in so dark and perplexing a labyrinth. The works of M. Varignon never occasion this disagreeable surprise, he makes it his chief care to place every thing in the clearest light; he does not, like some great geniuses, consult his ease by declining to take the trouble of being methodical, a trouble much greater than that of composition itself; he does not endeavour to acquire a reputation for profoundness, by leaving a great deal to be guessed by the reader.

Though Varignon's constitution did not seem easy to be impaired, assiduity and constant application brought upon him a severe disease in 1705. Great abilities are generally dangerous to the possessors. He was six months in danger, and three years in a languid state, which proceeded from his spirits being almost entirely exhausted. He said that sometimes, when delirious with a fever, he thought himself in the midst of a forest, where all the leaves of the trees were covered with algebraical calculations. Condemned by his physicians, his friends, and himself, to lay aside all study, he could not, when alone in his chamber, avoid taking up a book of mathematics, which he hid as soon as he heard any person coming. He again resumed the attitude and behaviour of a sick man, and seldom had occasion to counterfeit. Our author recovered from his disease; but the remembrance of what he had suffered did not make him more prudent for the future.

With regard to his character, Fontenelle observes, that it was at this time that a writing of his appeared, in which he censured Dr. Wallis for having advanced that there are certain spaces more than infinite, which that great geometrician ascribes to hyperbolas. He maintained, on the contrary, that they were finite. The criticism was softened with all the politeness and respect imaginable; but a criticism it was, though he had written it only for himself. He let M. Carré see it, when he was in a state that rendered him indifferent about things of that kind; and that gentleman, influenced only by the interest of the sciences, caused it to be printed in the memoirs of the Academy of Sciences, unknown to the author, who thus made an attack against his inclination.

Notwithstanding his great desire of peace, in the latter part of his life he was involved in a dispute. An Italian monk, well versed in mathematics, attacked him on the subject of tangents and the angle of contact in curves, such as they are conceived in the arithmetic of infinites; he answered by the last memoir he ever gave to the Academy, and the only one which turned on a dispute.

In the last two years of his life he was attacked with an asthmatic complaint. This disorder increased daily, and all remedies were ineffectual. He did not however cease from any of his customary business; so that, after having finished his lecture at the College of Mazarine, on the 22d of December 1722, he died suddenly the following night.

His character, says Fontenelle, was as simple as his superior understanding could require. He was not apt to be jealous of the fame of others: indeed he was at the head of the French mathematicians, and one of the best in Europe. It must be owned however, that when a new idea was offered to him, he was too hasty to object. The fire of his genius, the various insights into every subject, made too impetuous an opposition to those that were offered; so that it was not easy to obtain from him a favourable attention.

His works that were published separately, were,

1. Projet d'une Nouvelle Mechanique; 4to, Paris 1687.
2. Des Nouvelles Conjectures sur la Pesanteur.
3. Nouvelle Mechanique ou Statique, 2 tom. 4to, 1725.
4. Un Traité du Mouvement & de la Mesure des Eaux Courantes &c. 1725, in 4to.
5. Eclaircissement sur l'Analyse des Infiniment Petits, in 4to.
6. Des Cahiers de Mathématiques, ou Elémens de Mathématiques; in 1731.
7. Une Demonstration de la Possibilité de la Présence réelle du Corps de Jesus Christ dans l'Eucharistie; in a collection entitled, Pièces Fugitives sur l'Eucharistie, published in 1730; an extraordinary thing for a mathematician to undertake to demonstrate; which he does, as might be expected, not mathematically, but sophistically.

As to his memoirs in the volumes of the Academy of Sciences, they are far too numerous to be here particularized; they extend through almost all the volumes, down to his death in 1722.

VASA *Concordiæ*, in Hydraulics, are two vessels, so constructed, as that one of them, though full of wine, will not run a drop, unless the other, being full of water, run also. Their structure and apparatus may be seen in Wolfius, Element. Mathes. tom. 3, Hydraul.

VAUBAN (SEBASTIAN LE PRESTRE), a very great French engineer, was born in 1633. He displayed great abilities and skill in many sieges, and his services were rewarded with the first military honours. He was made governor of Lisle, commissary-general of the fortifications of France, and afterwards governor of the maritime parts of Flanders, and a marshal of France. He died in 1707, having brought fortification to a degree of perfection unknown before. His writings on these subjects are still in very high esteem.

VAULT, in Architecture, an arched roof, so contrived, as that the several stones of which it consists, by their disposition into the form of a curve, mutually sustain each other; as the arches of bridges, &c.

Vaults are to be preferred, on many occasions, to soffits, or flat ceilings, as they give a greater rise and elevation, and are also more firm and durable.

The ancients, Salmasius observes, had only three kinds of vaults: the first the fornix, made cradle-wise; the 2d, the testudo, tortoise-wise, or oven-wise; the 3d, the concha, made shell-wise. But the moderns subdivide these three kinds into a great many more, to which they give different names, according to their figures and use: some are circular, others elliptical, &c.

Again, the sweeps of some are larger, and others less portions of a sphere: all above hemispheres are called high, or surmounted vaults; all that are less than hemispheres, are low, or surbased vaults, &c.—In some the height is greater than the diameter; in others it is less: there are others again quite flat, only made with haunses; others oven-like, and others growing wider as they lengthen, like a trumpet.

Vaults are either single, double, cross, diagonal, horizontal, ascending, descending, angular, oblique, pendent, &c, &c. There are also Gothic vaults, with pendentives, &c.

Master VAULTS, are those which cover the principal parts of buildings; in contradistinction from the less, or subordinate vaults, which only cover some small part; as a passage, a gate, &c.

Double VAULT, is such a one as, being built over another, to make the exterior decoration range with the interior, leaves a space between the convexity of the one and the concavity of the other; as in the dome of St. Paul's at London, and that of St. Peter's at Rome.

VAULTS *with Compartiments*, are such whose sweep, or inner face, is enriched with pannels of sculpture, separated by platbands. These compartiments, which are of different figures, according to the vaults, and are usually gilt on a white ground, are made with stucco, on brick vaults; as in the church of St. Peter's at Rome; and with plaster, on timber vaults.

Theory of VAULTS—In a semicircular vault, or arch, being a hollow cylinder cut by a plane through its axis, standing on two imposts, and all the stones that compose it being cut and placed in such a manner, as that their joints, or beds, being prolonged, do all meet in the centre of the vault; it is evident that all the stones must be cut wedge-wise, or wider at top and above, than below; by virtue of which, they sustain each other, and mutually oppose the effort of their weight, which determines them to fall.

The stone in the middle of the vault, which is perpendicular to the horizon, and is called the key of the vault, is sustained on each side by the two contiguous stones, as by two inclined planes. The second stone, which is on the right or left of the key-stone, is sustained by a third; which, by virtue of the figure of the vault, is necessarily more inclined to the second, than the second is to the first; and consequently the second, in the effort it makes to fall, employs a less part of its weight than the first. For the same reason, all the stones, reckoning from the keystone, employ still a less and less part of their weight, to the last; which, resting on the horizontal plane, employs no part of its weight, or makes no effort to fall, as being entirely supported by the impost.

Now a great point to be aimed at in vaults, is, that all the several stones make an equal effort to fall: to effect this, it is evident that as each stone, reckoning from the key to the impost, employs a still less and less part of its whole weight; the first only employing, for example, one-half; the 2d, one-third; the 3d, one-fourth; &c; there is no other way to make these different parts equal, but by a proportionable augmentation of the whole; that is, the second stone must be heavier than the first, the third heavier than the second, and so on to the last, which should be vastly heavier.

Lahire demonstrates what that proportion is, in which the weights of the stones of a semicircular arch must be increased, to be in equilibrio, or to tend with equal forces to fall; which gives the firmest disposition that a vault can have. Before him, the architects had no certain rule to conduct themselves by; but did all at random. Reckoning the degrees of the quadrant of the circle, from the keystone to the impost, the length or weight of each stone must be so much greater, as it is farther from the key. Lahire's rule is, to augment the weight of each stone above that of the keystone, as much as the tangent of the arch to the stone exceeds the tangent of the arch of half the key. Now the tangent of the last stone becomes infinite, and consequently the weight should be so too; but as infinity has no place in practice, the rule amounts to this, that the last stone be loaded as much as possible, and the others in proportion, that they may the better resist the effort which the vault makes to separate them; which is called the shoot or drift of the vault.

M. Parent, and other authors, have since determined the curve, or figure, which the extrados or outside of a vault, whose intrados or inside is spherical, ought to have, that all the stones may be in equilibrio.

The above rule of Lahire's has since been found not accurate. See ARCH, and BRIDGE. See also my Treatise on the Principles of Bridges in my Tracts, and Emerson's Construction of Arches; also M. Berard's Statique des Vontes.

Key of a VAULT. See KEY, and VOUSSOIR.

Reins or *Fillings up of a* VAULT, are the sides which sustain it.

Pendentive of a VAULT. See PENDENTIVE.

Impost of a VAULT, is the stone upon which is laid the first voussoir, or arch-stone of the vault.

VEADAR, in Chronology, the 13th month of the Jewish ecclesiastical year, answering commonly to our March; this month is intercalated, to prevent the beginning of Nisan from being removed to the end of February.

VECTIS, in Mechanics, one of the simple mechanical powers, more usually called the Lever.

VECTOR, or *Radius Vector*, in Astronomy, is a line conceived to be drawn from any planet moving round a centre, or the focus of an ellipse, to that centre, or focus. It is so called, because it is that line by which the planet seems to be carried round its centre; and with which it describes areas proportional to the times.

VELOCITY, *Celerity*, or *Swiftness*, in Mechanics, is that affection of motion, by which a moving body passes over a certain space in a certain time. This is always proportional to the space moved over in a given time, when the velocity is uniform, or always the same during that time.

Velocity is either uniform or variable. Uniform, or equal velocity, is that with which a body passes over equal spaces in equal times. And it is variable, or unequal, when the spaces passed over in equal times are unequal; in which case it is either accelerated or retarded velocity; and this acceleration, or retardation, may also be equal or unequal, i. e. uniform or variable, &c. See ACCELERATION, and MOTION.

Velocity is also either absolute or relative. Absolute velocity is that we have hitherto been considering, in which the velocity of a body is considered simply in itself, or as passing over a certain space in a certain time. But relative or respective velocity, is that with which bodies approach to, or recede from one another, whether they both move, or one of them be at rest. Thus, if one body move with the absolute velocity of 2 feet per second, and another with that of 6 feet per second; then if they move directly towards each other, the relative velocity with which they approach is that of 8 feet per second; but if they move both the same way, so that the latter overtake the former, then the relative velocity with which that overtakes it, is only that of 4 feet per second, or only half of the former; and consequently it will require double the time of the former before they come in contact together.

Velocity *in a Right Line.*—When a body moves with a uniform velocity, the spaces passed over by it, in different times, are proportional to the times; also the spaces described by two different uniform velocities, in the same time, are proportional to the velocities; and consequently, when both times and velocities are unequal, the spaces described are in the compound ratio of the times and velocities. That is, $S \propto TV$, and $s \propto tv$; or $S : s :: TV : tv$. Hence also, $V : v :: \frac{S}{T} : \frac{s}{t}$, or the velocity is as the space directly and the time reciprocally.

But in uniformly accelerated motions; the last degree of velocity uniformly gained by a body in beginning from rest, is proportional to the time; and the space described from the beginning of the motion, is as the product of the time and velocity, or as the square of the velocity, or as the square of the time. That is, in uniformly accelerated motions, $v \propto t$, and $s \propto tv$ or $\propto v^2$ or $\propto t^2$. And, in fluxions, $\dot{s} = v\dot{t}$.

Velocity *of Bodies moving in Curves.*—According to Galileo's system of the descent of heavy bodies, which is now universally admitted among philosophers, the velocities of a body falling vertically are, at each moment of its fall, as the square roots of the heights from whence it has fallen; reckoning from the beginning of the descent. Hence he inferred, that if a body descend along an inclined plane, the velocities acquired at different times, will be in the same ratio: for since its velocity is all owing to its fall, and it only falls as much as there is perpendicular height in the inclined plane, the velocity should be still measured by that height, the same as if the descent were vertical.

The same principle led him also to conclude, that if a body fall through several contiguous inclined planes, making any angles with each other, much like a stick when broken, the velocity would still be regulated after the same manner, by the vertical heights of the different planes taken together, considering the last velocity as the same that the body would acquire by descending through the same perpendicular height.

This conclusion, it seems, continued to be acquiesced in, till the year 1672, when it was demonstrated to be false, by James Gregory, in a small piece of his, intitled Tentamina quædam Geometrica de Motu Penduli et Projectorum. This piece has been very little known, because it was only added to the end of an obscure and pseudonymous piece of his, then published, to expose the errors and vanity of Mr. Sinclar, professor of natural philosophy at Glasgow. This little jeu d'esprit of Gregory is entitled, "The great and new Art of Weighing Vanity: or a discovery of the Ignorance and Arrogance of the great and new Artist, in his Pseudo-Philosophical writings: by M. Patrick Mathers, Arch-Bedal to the University of St. Andrews." In the Tentamina, Gregory shows what the real velocity is, which a body acquires by descending down two contiguous inclined planes, forming an obtuse angle, and that it is different from the velocity a body acquires by descending perpendicularly through the same height; also that the velocity on quitting the first plane, is to that with which it enters the second, and in this latter direction, as radius to the cosine of the angle of inclination between the two planes.

This conclusion, however, Gregory observes, does not apply to the motions of descent down any curve lines, because the contiguous parts of curve lines do not form any angle between them, and consequently no part of the velocity is lost by passing from one part of the curve to the other; hence he infers, that the velocities acquired in descending down a continued curve line, are the same as by falling perpendicularly through the same height. This principle is then applied, by the author, to the motion of pendulums and projectiles.

Varignon too, in the year 1693, followed in the same track, showing that the velocity lost in passing from one right lined direction to another, becomes indefinitely small in the course of a curve line; and that therefore the doctrine of Galileo holds good for the descent of bodies down a curve line, viz, that the velocity at any point of the curve, is equal to that which would be acquired by falling through the same perpendicular altitude.

The nature of every curve is abundantly determined by the ratio of the ordinates to the corresponding abscisses; and the essence of curves in general may be conceived as consisting in this ratio, which may be varied in a thousand different ways. But this same ratio will be also that of two simple velocities, by whose joint effect a body may describe the curve in question; and consequently the essence of all curves, in general, is the same thing as the concourse or combination of all the forces which, taken two by two, may move the same body. Thus we have a most simple and general equation of all possible curves, and of all possible velocities. By means of this equation, as soon as the two simple velocities of a body are known, the curve resulting from them is immediately determined.

It may be observed, in particular, according to this equation, that an uniform velocity, combined with a velocity that always varies as the square roots of the heights, will produce the particular curve of a parabola, independent of the angle made by the directions of the two forces that give the velocities; and consequently a cannon ball, projected either horizontally or obliquely to the horizon, must always describe a parabola, were it not for the resistance of the air.

Circular **Velocity.** See **Circular.**

Initial **Velocity,** in Gunnery, denotes the velocity with which military projectiles issue from the mouth of the piece by which they are discharged. This, it is now known, is much more considerable than was formerly apprehended. For the method of estimating it, and the result of a variety of experiments, by Mr. Robins, and myself, &c, see the articles **Gun, Gunnery, Projectile, Resistance,** and my Tracts, vols. 2 and 3.

Mr. Robins had hinted in his New Principles of Gunnery, at another method of measuring the initial velocities of military projectiles, viz, from the arc of vibration of the gun itself, in the act of expulsion, when it is suspended by an axis like a pendulum. And Mr. Thompson, in his experiments (Philos. Trans. vol. 71, pa. 229), has pursued the same idea at considerable length, in a number of experiments, from which he deduces a rule for computing the velocity, which is somewhat different from that of Mr. Robins, but which agrees very well with his own experiments.

This rule however being drawn only from the experiments with a musket barrel, and with a small charge of powder, and besides being different from that in the theory as proposed by Robins; it was suspected that it would not obtain when applied to cannon, or other large pieces of ordnance, of different and various lengths, and to larger charges of powder. For this reason, a great number of experiments, as related in my Tracts, were instituted with cannon of various lengths, and charged with many different quantities of powder; and the initial velocities of the shot were computed both from the vibration of a ballistic pendulum, and from the vibration of the gun itself; but the consequence was, that these two hardly ever agreed together, and in many cases they differed by almost 400 feet per second in the velocity. A brief abstract for a comparison between these two methods, is contained in the following tablet, viz,

Comparison of the Velocities by the Gun and Pendulum.

Gun. No.	2 Ounces.			4 Ounces.			8 Ounces.			16 Ounces.		
	Velocity by		Diff.	Velocity by		Diff.	Velocity by		Diff.	Velocity by		Diff.
	Gun.	Pend.		Gun.	Pend.		Gun.	Pend.		Gun.	Pend.	
1	830	780	50	1135	1100	35	1445	1430	15	1345	1377	−32
2	863	835	28	1203	1180	23	1521	1580	−59	1485	1656	−171
3	919	920	−1	1294	1300	−6	1631	1790	−159	1680	1998	−318
4	929	970	−41	1317	1370	−53	1669	1940	−271	1730	2106	−376

In this table, the first column shows the number of the gun, as they were of different lengths; viz, the length of number 1 was 30⅓ inches, number 2 was 40⅓ inches, number 3 was 60 inches, and number 4 was 83 inches, nearly. After the first column, the rest of the table is divided into four spaces, for the four charges, 2, 4, 8, 16 ounces of powder: and each of these is divided into three columns, in the first of the three is the velocity of the ball as determined from the vibration of the gun; in the second is the velocity as determined from the vibration of the pendulum; and in the third is the difference between the two, being so many feet per second, which is marked with the negative sign, or −, when the former velocity is too little, otherwise it is positive.

From the comparison contained in this table, it appears, in general, that the velocities, determined by the two different methods, do not agree together; and that therefore the method of determining the velocity of the ball from the recoil of the gun, is not generally true, though Mr. Robins and Mr. Thompson had suspected it to be so: and consequently that the effect of the inflamed powder on the recoil of the gun, is not exactly the same when it is fired without a ball, as when it is fired with one. It also appears, that this difference is no ways regular, neither in the different guns with the same charge of powder, nor in the same gun with different charges: that with very small charges, the velocity by the gun is greater than that by the pendulum; but that the latter always gains upon the former, as the charge is increased, and soon becomes equal to it; afterwards it proceeds to exceed it more and more: that the particular charge, at which the two velocities become equal, is different in the different guns; and that this charge is less, or the equality sooner takes place, as the gun is longer. And all this, whether we use the actual velocity with which the ball strikes the pendulum, or the same increased by the velocity lost by the resistance of the air, in its flight from the gun to the pendulum.

Virtual VELOCITY. See VIRTUAL *Velocity*.

VENA CONTRACTA, a term employed by Sir Isaac Newton to denote that section of a stream of fluid issuing from an orifice in the side or bottom of a vessel, at the distance of its diameter from the orifice. When a fluid issues from a vessel, the velocity through the orifice does not arise from a continual acceleration of descending particles by the force of gravity, as in the case of a body falling freely, but it is communicated by the whole pressure of the surrounding fluid; in consequence of which, the water rushing towards the orifice in all directions causes a contraction in the stream; and at a distance from the orifice equal to its diameter. Sir Isaac Newton measured the diameter of the section of the stream (which section he called by the above name), and found it to be to the diameter of the orifice as 21 to 25: hence, the area of the orifice: the area of the vena contracta (they being supposed to be similar) : : $25^2 : 21^2$, which is very nearly as $\sqrt{2} : 1$; and as the velocity is inversely as the area of the section, the velocity at the vena contracta: the velocity at the orifice : : $\sqrt{2} : 1$. Also from the quantity of water, running out in a given time, and the area of the vena contracta, Sir Isaac also determined that the velocity at the vena contracta is that which a body acquires in falling down the altitude of the fluid above the orifice: hence the velocity at the orifice (being less than that at the vena contracta in the ratio of $\sqrt{2} : 1$) is that which a body would acquire in falling down half the altitude. See the art. WATER, *Motion of*.

VENTILATOR, a machine by which the noxious air of any close place, as an hospital, gaol, ship, chamber, &c, may be discharged and changed for fresh air.—The noxious qualities of bad air have been long known; and Dr. Hales and others have taken great pains to point out the mischiefs arising from foul air, and to prevent or remedy them. That philosopher proposed an easy and effectual one, by the use of his ventilators; the account of which was read before the Royal Society in May 1741; and a farther account of it may be seen in his Description of Ventilators, printed at London in 8vo, 1743; and still farther in part 2, pa. 32, printed in 1758; where the uses and applications of them are pointed out for ships, and prisons, &c. For what is said of the foul air of ships may be applied to that of gaols, mines, workhouses, hospitals, barracks, &c. In mines, ventilators may guard against the suffocations, and other terrible accidents arising from damps. The air of gaols has often proved infectious;

and we had a fatal proof of this, by the accident that happened some years since at the Old Bailey sessions. After that, ventilators were used in the prison, which were worked by a small windmill, placed on the top of Newgate; and the prison became more healthy.

Dr. Hales farther suggests, that ventilators might be of use in making salt; for which purpose there should be a stream of water to work them; or they might be worked by a windmill, and the brine should be in long narrow canals, covered with boards of canvas, about a foot above the surface of the brine, to confine the stream of air, so as to make it act on the surface of the brine, and carry off the water in vapours. Thus it might be reduced to a dry salt, with a saving of fuel, in winter and summer, or in rainy weather, or any state of the air whatever. Ventilators, he apprehends, might also serve for drying linen hung in low, long, narrow galleries, especially in damp or rainy weather, and also in drying woollen cloths, after they are fulled or dyed; and in this case, the ventilators might be worked by the fulling water-mill. Ventilators might also be a useful appendage to malt and hop kilns; and the same author is farther of opinion, that a ventilation of warm dry air from the adjoining stove, with a cautious hand, might be of service to trees and plants in greenhouses; where it is well known that air full of the rancid vapours which perspire from the plants, is very unfavourable to them, as well as the vapours from human bodies are to men: for fresh air is as necessary to the healthy state of vegetables, as it is to that of animals.—Ventilators are also of excellent use for drying corn, hops, and malt.—Gunpowder may be thoroughly dried, by blowing air up through it by means of ventilators; which is of great advantage to the strength of it. These ventilators, even the smaller ones, will also serve to purify most easily and effectually, the bad air of a ship's well, before a person is sent down into it, by blowing air through a trunk, reaching nearly to its bottom. And in a similar manner may stinking water, and ill-tasted milk, &c, be sweetened, viz, by passing a current of air through them, from bottom to top, which will carry the offensive particles along with it.

For these and other uses to which they might be applied, as well as for a particular account of the construction and disposition of ventilators in ships, hospitals, prisons, &c, and the benefits attending them, see Hales's Treatise on Ventilators, part 2 passim; and the Philos. Trans. vol. 49, pa. 332.

The method of drawing off air from ships by means of fire-pipes, which some have preferred to ventilators, was published by sir Robert Moray in the Philos. Trans. for 1665. These are metal pipes, about 2½ inches diameter, one of which reaches from the fire-place to the well of the ship, and other three branches go to other parts of the ship; the stove hole and ash hole being closed up, the fire is supplied with air through these pipes. The defects of these, compared with ventilators, are particularly examined by Dr. Hales, ubi supra, pa. 113.

In the latter part of the year 1741, M. Triewald, military architect to the king of Sweden, informed the secretary to the Royal Society, that he had in the preceding spring invented a machine for the use of ships of war, to draw out the foul air from under their decks, which exhausted 36172 cubic feet of air in an hour, or at the rate of 21732 tuns in 24 hours. In 1742 he sent one of these to France, which was approved of by the Academy of Sciences at Paris, and the navy of France was ordered to be furnished with the like ventilators.

Mr. Erasmus King proposed to have ventilators worked by the fire engines, in mines. And Mr. Fitzgerald has suggested an improved method of doing this, which he has also illustrated by figures. See Philos. Trans. vol. 50, pa. 727.

There are various ways of ventilation, or changing the air of rooms. Mr. Tidd contrived to admit fresh air into a room, by taking out the middle upper sash pane of glass, and fixing in its place a frame box, with a round hole in its middle, about 6 or 7 inches diameter; in which holes are fixed, behind each other, a set of sails of very thin broad copper-plates, which spread over and cover the circular hole, so as to make the air which enters the room, and turning round these sails, to spread round in thin sheets sideways; and so not to incommode persons, by blowing directly upon them, as it would do if it were not hindered by the sails.

This method however is very unseemly and disagreeable in good rooms; and therefore, instead of it, the late ingenious Mr. John Whitehurst substituted another; which was, to open a small square or rectanglar hole in the party wall of the room, in the upper part near the cieling, at a corner or part distant from the fire; and before it he placed a thin piece of metal or pasteboard, &c, attached to the wall in its lower part just below the hole, but declining from it upwards, so as to give the air, that enters by the hole, a direction upwards against the cieling, along which it sweeps and disperses itself through the room, without blowing in a current against any person. This method is very useful to cure smoky chimneys, by thus admitting conveniently fresh air. A picture placed before the hole prevents the sight of it from disfiguring the room. This, and many other methods of ventilating, he meant to have published, and was occupied on, when death put an end to his useful labours. These have since been published, viz in 1794, 4to, by Dr. Willan.

VENUS, in Astronomy, one of the inferior planets, but the brightest, and to appearance the largest, of all the planets; and is designed by the mark ♀, supposed to be a rude representation of a female figure, with her trailing robe. Venus is easily distinguished from all the other planets, by her whiteness and brightness, in which she exceeds all the rest, even Jupiter himself, and which is so considerable, that in a dusky place she causes an object to project a sensible shadow, and she is often visible in the day-time. Her place in the system is the second from the sun, viz, between Mercury and the earth, and in magnitude is about equal to the earth, or rather a little larger according to Dr. Herschel's observations.

As Venus moves round the sun, in a circle beneath that of the earth, she is never seen in opposition to him, nor indeed very far from him; but seems to move backward and forward, passing him from side to side, to the distance of about 47 or 48 degrees, both ways, which is her greatest elongation. When she appears west of the sun, which is from her inferior conjunction to her superior, she rises before him, or is a morning star, and is called Phosphorus, or Lucifer, or the Morning Star; and when she is eastwards from the sun, which is from her superior conjunction to her inferior, she sets after him, or is an evening star, and is then called Hesperus, or Vesper, or the Evening Star: being each of those in its turn for 290 days.

The real diameter of Venus is nearly equal to that of the earth, being about 7900 miles; her apparent mean diameter seen from the sun, or her horizontal parallax, 30″; but as seen from the earth 18″·79 according to Dr. Herschel, or 16″·7 by M. Lalande; her distance from the sun 68 million of miles; her eccentricity $\frac{1}{138}$ths of the same, or 490,000 miles; the inclination of her orbit to the plane of the ecliptic 3° 23′; the points of their intersection or nodes are 14° of ♊ and ♐; the place of her aphelion ♒ 10° 18′; her axis inclined to her orbit 75° 0′; her periodical course round the sun 224 days 17 hours; the diurnal rotation about her axis very uncertain, being according to Cassini only 23 hours, but according to the observations of Bianchini it is in 24 days 8 hours; though Dr. Herschel thinks it cannot be so much; and by M. Schroeter 23h. 21min. See also **Planets**.

Venus, when viewed through a telescope, is rarely seen to shine with a full face, but has phases and changes just like those of the moon, being increasing, decreasing, horned, gibbous, &c: her illuminated part being constantly turned toward the sun, or directed toward the east when she is a morning star, and toward the west when an evening star. These different phases of Venus were first discovered by Galileo; who thus fulfilled the prediction of Copernicus: for when this excellent astronomer revived the ancient Pythagorean system, asserting that the earth and planets move round the sun, it was objected that in such a case the phases of Venus should resemble those of the moon; to which Copernicus replied, that some time or other that resemblance would be found out. Galileo sent an account of the first discovery of these phases in a letter, written from Florence in 1611, to William de Medici, the duke of Tuscany's ambassador at Prague; desiring him to communicate it to Kepler. The letter is extant in the preface to Kepler's Dioptrics, and a translation of it in Smith's Optics, pa. 416. Having recited the observations he had made, he adds, "We have hence the most certain, sensible decision and demonstration of two grand questions, which to this day have been doubtful and disputed among the greatest masters of reason in the world. One is, that the planets in their own nature are opake bodies, attributing to Mercury what we have seen in Venus: and the other is, that Venus necessarily moves round the sun; as also Mercury and the other planets; a thing well believed indeed by Pythagoras, Copernicus, Kepler, and myself, but never yet proved, as now it is, by ocular inspection on Venus."

Cassini and Campani, in the years 1665 and 1666, both discovered spots in the face of Venus: from the appearances of which the former ascertained her motion about her axis; concluding that this revolution was performed in less than a day; or at least that the bright spot which he observed, finished its period either by revolution or libration in about 23 hours. And Lahire, in 1790, through a telescope of 16 feet, observed spots also in Venus; which he found to be larger than those in the moon.

The next observations of the same kind that occur, are those of signior Bianchini at Rome, in 1726, 1727, 1728, who, with Campani's glasses, discovered several dark spots in the disk of Venus, of which he gave an account and a representation in his book entitled Hesperi et Phosphori Nova Phenomena, published at Rome in 1728. From several successive observations Bianchini concludes, that a rotation of Venus about her axis was not completed in 23 hours, as Cassini imagined, but in 24⅓ days; that the north pole of this rotation faced the 20th degree of Aquarius, and was elevated 15 above the plane of the ecliptic, and that the axis kept parallel to itself, during the planet's revolution about the sun. Cassini the son, though he admits the accuracy of Bianchini's observations, disputes the conclusion drawn from them, and finally observes, that if we suppose the period of the rotation of Venus to be 23h. 20min. it agrees equally well with the observations both of his father and Bianchini; but if she revolve in 24d. 8h. then his father's observations must be rejected as of no consequence.

In the Philos. Trans. 1792, are published the results of a course of observations on the planet Venus, begun in the year 1780, by M. Schroeter, of Lilienthal, Bremen. From these observations, the author infers, that Venus has an atmosphere in some respects similar to that of our earth, but far exceeding that of the moon in density, or power to weaken the rays of the sun: that the diurnal period of this planet is probably much longer than that of other planets: that the moon also has an atmosphere, though less dense and high than that of Venus: and that the mountains of this planet are 5 or 6 times as high as those on the earth.

Dr. Herschel too, between the years 1777 and 1793, has made a long series of observations on this planet, accounts of which are given in the Philos. Trans. for 1793. The results of these observations are: that the planet revolves about its axis, but the time of it is uncertain: that the position of its axis is also very uncertain: that the planet's atmosphere is very considerable: that the planet has probably hills and inequalities on its surface, but he has not been able to see much of them, owing perhaps to the great density of its atmosphere: as to the mountains of Venus, no eye, he says, which is not considerably better than his, or assisted by much better instruments, will ever get a sight of them: and that the apparent diameter of Venus, at the mean distance from the earth, is 18″·79; whence it may be inferred, that this planet is somewhat larger than the earth, instead of being less, as former astronomers have imagined.

Sometimes Venus is seen in the disk of the sun, in form of a dark round spot. These appearances, called transits, happen but seldom, viz, when the earth is about her nodes at the time of her inferior conjunction. One of these transits was seen in England in 1639 by Mr. Horrox and Mr. Crabtree; and two in the last century, viz, the one June 6, 1761, and the other in June 1769. There will not happen another of them till the year 1874. See **Parallax**. Except such transits as these, Venus exhibits the same appearances to us regularly every 8 years; her conjunctions, elongations, and times of rising and setting, being very nearly the same, on the same days, as before.

In 1672 and 1686, Cassini, with a telescope of 34 feet, thought he saw a satellite move round this planet, at the distance of about $\frac{3}{5}$ of Venus's diameter. It had the same phases as Venus, but without any well-defined form; and its diameter scarce exceeded $\frac{1}{4}$ of the diameter of Venus. Dr. Gregory (Astron. lib. 6, prop. 3) thinks it more than probable that this was a satellite; and supposes that the reason why it is not more frequently seen, is the unfitness of its surface to reflect the rays of the sun's light; as is the case of the spots in the moon; for if the whole disk of the moon were composed of such, he thinks she could not be seen so far as to Venus.

Mr. Short, in 1740, with a reflecting telescope of 16¼ inches focus, perceived a small star near Venus: with another telescope of the same focus, magnifying 50 or 60 times, and fitted with a micrometer, he found its distance from Venus about 10′; and with a magnifying power of 240, he observed the star assume the same phases with Venus; its diameter seemed to be about $\frac{1}{3}$, or somewhat less, of the diameter of Venus; its light not so bright and vivid, but exceeding sharp and well defined. He viewed it for the space of an hour; but never had the good fortune to see it after the first morning. Philos. Trans. No. 459, pa. 646.

M. Montaign, of Limoges in France, preparing for observing the transit of 1761, discovered in the preceding month of May a small star, about the distance of 20′ from Venus, the diameter of it being about ¼ of that of the planet. Others have also thought they saw a like appearance. And indeed it must be acknowledged, that Venus may have a satellite, though it be difficult for us to see it. Its enlightened side can never be fully turned towards us, but when Venus is beyond the sun; in which case Venus herself appears little larger than an ordinary star, and therefore her satellite may be too small to be perceived at such a distance. When she is between us and the sun, her moon has its dark side turned towards us; and when Venus is at her greatest elongation, there is but half the enlightened side of the moon turned toward us, and even then it may be too far distant to be seen by us. But it was presumed, that the two transits of 1761, and 1769, would afford opportunity for determining this point; and yet we do not find, though many observers directed their attention to this object, that any satellite was then seen in the sun's disk; unless we except two persons, viz, an anonymous writer in the London Chronicle of May 18, who says that he saw the satellite of Venus on the sun the day of the transit, at St. Neot's in Huntingdonshire; that it moved in a track parallel to that of Venus, but nearer the ecliptic; that Venus quitted the sun's disk at 31 minutes after 8, and the satellite at 6 minutes after 9; and M. Montaign at Limoges, whose account of his observations is in the Memoirs of the Academy of Paris, whence the following certificate is extracted:—Certificate. "We having examined, by order of the Academy, the remarks of M. Baudouin on a new observation of the satellite of Venus, made at Limoges the 11th of May by M. Montaign. This fourth observation, of great importance for the theory of the satellite, has shown that its revolution must be longer than appeared by the first three observations. M. Baudouin believes it may be fixed at 12 days; as to its distance, it appears to him to be 50 semidiameters of Venus; whence he infers that the mass of Venus is equal to that of the earth. This mass of Venus is a very essential element to astronomy, as it enters into many computations, and produces different phenomena: &c.

Signed L'Abbé De La Caille,
De La Lande."

VERBERATION, in Physics, a term used to express the cause of sound, which arises from a verberation of the air, when struck, in divers manners, by the several parts of the sonorous body first put into a vibratory motion.

VERNAL, something belonging to the spring season: as vernal signs, vernal equinox, &c.

VERNIER, is a scale, or a division, well adapted for the graduation of mathematical instruments, so called from its inventor Peter Vernier, a gentleman of Franche Comté, who published the discovery in a small tract, entitled La Construction, l'Usage, et les Proprietez du Quadrant Nouveau de Mathematique &c, printed at Brussels in 1631. This was an improvement on the method of division proposed by Jacobus Curtius, printed by Tycho in Clavius's Astrolabe, in 1593. Vernier's method of division, or dividing plate, has been very commonly, though erroneously, called by the name of Nonius; the method of Nonius being very different from that of Vernier, and much less convenient.

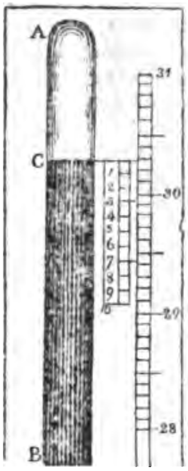
When the relative unit of any line is so divided into many small equal parts, those parts may be too numerous to be introduced, or if introduced, they may be too close to one another to be readily counted or estimated; for which reason there have been various methods contrived for estimating the aliquot parts of the small divisions, into which the relative unit of a line may be commodiously divided; and among those methods, Vernier's has been most justly preferred to all others. For a curious history of this, and other inventions of a similar nature, see Robins's Math. Tracts, vol. 2, pa. 265, &c.

This improved method of subdividing scaler divisions, was first published by Peter Vernier, a person of distinction in Franche Comte, in a very small tract, entitled, The Construction, the Use, and the Properties of a New Mathematical Quadrant, &c. In his dedication, having shown its preference to what has been done in the affair by Nunez and Clavius, he adds, Mine having all these advantages over the others, it is not without reason that I call it new and of my own invention.

In the preface also he claims it as his own invention, and says, by it a quadrant of 3 inches is rendered capable of determining minutes. In his book he shows how to apply it to instruments of different dimensions. His contrivance is a moveable arch divided into equal parts, one *less* in number than the divisions of the portion of the limb corresponding to it.

Vernier's scale then, is a small moveable arch, or scale, sliding along the limb of a quadrant, or any other graduated scale, and divided into equal parts, that are one less in number, than the divisions of the portion of the limb corresponding to it. So, if we want to subdivide the graduations on any scale into (for ex.) 10 equal parts; we must make the vernier equal in length to 11 of those graduations of the scale, but dividing the same length of the vernier itself only into 10 equal parts; for then it is evident that each division on the vernier will be $\frac{1}{10}$th part longer than the gradations on the instrument, or that the division of the former is equal to $\frac{1}{10}$ of the degree on the latter, as that gains 1 in 10 upon this.

Thus let AB be a part of the upper end of a barometer tube, the quicksilver standing at the point C; from 28 to 31 is a part of the scale of inches, viz, from 28 inches to 31 inches, divided into 10ths of inches; and the middle piece, from 1 to 10, is the vernier, that slides up and down in a groove, and having 10 of its divisions equal to 11-10ths of the inches, for the purpose of subdividing every 10th of the inch into 10 parts, or, the inches into centesms or 100th parts. In practice, the method of counting is by observing (when the vernier is set with its index at top, pointing exactly against the upper surface of the mercury in the tube) which division of the vernier it is that exactly, or nearest, coincides with a division in the scale of 10ths of inches, for that will show the number of 100ths, over the 10ths of inches next below the index at top. So, in the annexed figure, the top of the vernier is between 2 and 3 tenths above the 30 inches of the barometer; and because the 8th division of the vernier is seen to coincide with a division of the scale, this shows that it is 8 centesms more: so that the height of the quicksilver altogether, is 30·28, that is, 30 inches, and 28 hundredths, or 2 tenths and 8 hundredths.



If the scale were not inches and 10ths, but degrees of a quadrant, &c, then the 8 would be $\frac{8}{10}$ths of a degree, or 48′; or if every division on the scale be 10 minutes, then the vernier will subdivide it into single minutes, and the 8 will then be 8 minutes. And so for any other case.

By altering the number of divisions, either in the degrees or in the vernier, or in both, an angle can be observed to many different degrees of accuracy. Thus, if a degree on a quadrant be divided into 12 parts, each being 5 minutes, and the length of the vernier be 21 such parts, or 1¾°, and divided into 20 parts, then

$$\frac{1}{12} \times \frac{1}{20} = \frac{1^\circ}{240} = \frac{1'}{4} = 15'',$$

is the smallest division the vernier will measure to; Or, if the length of the vernier be $2\frac{7}{12}$°, and divided into 30 parts, then

$$\frac{1}{12} \times \frac{1}{30} = \frac{1^\circ}{360} = \frac{1'}{6} = 10'',$$

is the smallest part in this case: Also

$$\frac{1}{12} \times \frac{1}{50} = \frac{1^\circ}{600} = \frac{1'}{10} = 6'',$$

is the smallest part when the vernier extends 4¼°. See Robertson's Navigation, book 5, pa. 279.

But though the vernier was thus originally divided into one *less* than the correspondent length on the scale, yet a practice has gradually come into use, of dividing it into one part *more* than those of the scale; so that it is now more usual to practise this latter way, than the former.

M. Delambre, in his Abrégé d'Astronomie, 1813, pa. 48, says, The general formula is, to take $n-1$ parts of the limb, and to divide them into n parts on the vernier, or the formula is $\frac{n-1}{n}$; then the vernier gives the $\frac{1}{n}$ part of the division of the limb. This, says he, I call the *Direct Vernier*, because the numeration on it proceeds in the same way as on the limb. There is another, which I call the *Retrograde*, because the numeration on it reads the contrary way: this is rather less common; and its formula is $\frac{n+1}{n}$; but the principle is the same, as well as the use.

For the method of applying the vernier to a quadrant, see *Hadley's* QUADRANT. And for the application of this instrument to a telescope, and the principles of its construction, see Smith's Optics, book 3, sect. 861.

VERSED *Sine*, of an arch, is the part of the diameter intercepted between the sine and the commencement of the arc; and it is equal to the difference between the radius and the cosine. See *Versed* SINE. And for *coversed sine*, see COVERSED *Sine*.

VERTEX *of an Angle*, is the angular point, or the point where the legs or sides of the angle meet.

VERTEX *of a Figure*, is the uppermost point, or the vertex of the angle opposite to the base.

VERTEX *of a Curve*, is the extremity of the axis or diameter, or it is the point where the diameter meets the curve; which is also the vertex of the diameter.

VERTEX *of a Glass*, in Optics, the same as its pole.

VERTEX is also used, in Astronomy, for the point of the heavens vertically or perpendicularly over our heads, also called the zenith.

VERTEX, *Path of the*. See PATH.

VERTICAL, something relating to the vertex or highest point. As,

VERTICAL *Point*, in Astronomy, is the same with vertex, or zenith.—Hence a star is said to be vertical, when it happens to be in that point which is perpendicularly over any place.

VERTICAL *Circle*, is a great circle of the sphere, passing through the zenith and nadir of a place.—The vertical circles are also called azimuths. The meridian of any place is a vertical circle, viz, that particular one which passes through the north or south point of the horizon.—All the vertical circles intersect one another in the zenith and nadir.

The use of the vertical circles is to estimate or measure the height of the stars &c, and their distances from the zenith, which is reckoned on these circles; and to find their eastern and western amplitude, by observing how many degrees the vertical, in which the star rises or sets, is distant from the meridian.

Prime **VERTICAL**, is that verticle circle, or azimuth, which passes through the poles of the meridian; or which is perpendicular to the meridian, and passes through the equinoctial points.

Prime **VERTICALS**, in Dialling. See PRIME *Verticals*.

VERTICAL *of the Sun*, is the vertical which passes through the centre of the sun at any moment of time.—Its use is, in dialling, to find the declination of the plane on which the dial is to be drawn, which is done by observing how many degrees that vertical is distant from the meridian, after marking the point or line of the shadow on the plane at any times.

VERTICAL *Dial*. See *Vertical* DIAL.

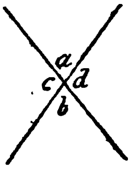
VERTICAL *Line*, in Dialling, is a line in any plane perpendicular to the horizon.—This is best found and drawn on an erect and reclining plane, by steadily holding up a string and plummet, and then marking two points of the shadow of the thread on the plane, a good distance from each other; and drawing a line through these marks.

VERTICAL *Line*, in Conics, is a line drawn on the vertical plane, and through the vertex of the cone.

VERTICAL *Line*, in Perspective. See *Vertical* LINE.

VERTICAL *Plane*, in Conics, is a plane passing through the vertex of a cone, and parallel to any conic section.

Vertical *Plane*, in Perspective. See Plane and Perspective.



Vertical *Angles*, or *Opposite Angles*, in Geometry, are such as have their legs or sides continuations of each other, and which consequently have the same vertex or angular point. So the angles *a* and *b* are vertical angles; as also the angles *c* and *d*. Vertical angles are equal to each other.

VERTICITY, is that property of the magnet or loadstone, or of a needle, &c touched with it, by which it turns or directs itself to some peculiar point, as to its pole.—The attraction of the magnet was known long before its verticity.

VERU, a comet, according to some writers resembling a spit, being nearly the same as the lonchites, only its head is rounder, and its train longer and sharper pointed.

VESPER, in Astronomy, called also Hesperus, and the Evening Star, is the planet Venus, when she is eastward of the sun, and consequently sets after him, and shines as an evening star.

VESPERTINE, in Astronomy, is when a planet is descending to the west after sun-set, or shines as an evening star.

VESTA, one of the small planetary bodies revolving between the planets Mars and Jupiter. It was discovered by M. Olbers the 29th of March, 1807, and is the nearest to Mars of the 4 small planets. See my Recreations, vol. 3, pa. 144.

VESTIBULE, in Architecture, a kind of entrance into a large building; being an open place before the hall, or at the bottom of the staircase.

VIA Lactea, in Astronomy, the milky way, or Galaxy. See Galaxy.

Via Solis, or *Sun's Way*, is used among astronomers, for the ecliptic line, or path in which the sun seems always to move.

VIBRATION, in Mechanics, a regular reciprocal motion of a body, as, for example, a pendulum, which being freely suspended, vibrates or swings from side to side of the vertical line. Mechanical authors, instead of vibration, often use the term oscillation, especially when speaking of a body that thus swings by means of its own gravity or weight.

The vibrations of the same pendulum are all isochronal; that is, they are performed in an equal time, at least in the same latitude; for in lower latitudes they are found to be slower than in higher ones. See Pendulum. In our latitude, a pendulum 39⅛ inches long, vibrates seconds, making 60 vibrations in a minute.

The vibrations of a longer pendulum take up a longer time than those of a shorter one, and that in the subduplicate ratio of the lengths, or the ratio of the square roots of the lengths. Thus, if one pendulum be 40 inches in length, and another only 10 inches, the former will be double the time of the latter in making a vibration; for $\sqrt{40} : \sqrt{10} :: \sqrt{4} : \sqrt{1}$, that is as 2 to 1. And because the number of vibrations, made in any given time, is reciprocally as the duration of one vibration, therefore the number of such vibrations is in the reciprocal subduplicate ratio of the lengths of the pendulums.

M. Mouton, a priest of Lyons, wrote a treatise, expressly to show, that by means of the number of vibrations of a given pendulum, in a certain time, may be established an universal measure throughout the whole world; and may fix the several measures that are in use among us, in such a manner, as that they might be recovered again, if at any time they should chance to be lost, as is the case of most of the ancient measures, which we now only know by conjecture.

The Vibrations *of a Stretched Chord*, or *String*, arise from its elasticity; which power being in this case similar to gravity, as acting uniformly, the vibrations of a chord follow the same laws as those of pendulums. Consequently the vibrations of the same chord equally stretched, though they be of unequal lengths, are isochronal, or are performed in equal times; and the squares of the times of vibration are to one another inversely as their tensions, or powers by which they are stretched.

The vibrations of a spring too are proportional to the powers by which it is bent. These follow the same laws as those of the chord and pendulum; and consequently are isochronal; which is the foundation of spring watches.

Vibrations are also used in Physics, &c, and for several other regular alternate motions. Sensation, for instance, is supposed to be performed by means of the vibratory motion of the contents of the nerves, begun by external objects, and propagated to the brain. This doctrine has been particularly illustrated by Dr. Hartley, who has extended it farther than any other writer, in establishing a new theory of our mental operations. The same ingenious author also applies the doctrine of vibrations to the explanation of muscular motion, which he thinks is performed in the same general manner as sensation and the perception of ideas. See his Observations on Man, vol. 1.

The several kinds and rays of light, Newton conceives to make vibrations of divers magnitudes; which, according to those magnitudes, excite sensations of several colours; much after the same manner as vibrations of air, according to their several magnitudes, excite sensations of several sounds. See the article Colour.

Heat, according to the same author, is only an accident of light, occasioned by the rays putting a fine, subtile, ethereal medium, which pervades all bodies, into a vibratory motion, which gives us that sensation. See Heat. From the vibrations or pulses of the same medium, he accounts for the alternate fits of easy reflection and ready transmission of the rays.—In the Philosophical Transactions it is observed, that the butterfly, into which the silkworm is transformed, makes 130 vibrations or motions of its wings, in one coition.

VIETA (Francis), a very celebrated French mathematician, was born in 1540 at Fontenai, or Fontenai-le-Comté, in Lower Poitou, a province of France. He was master of requests at Paris, where he died in 1603, being the 63d year of his age. Among other branches of learning in which he excelled, he was one of the most respectable mathematicians of the 16th century, or indeed of any age. His writings abound with marks of great originality, and the finest genius, as well as intense application. Indeed such was the vigour of his perseverance, that he has sometimes remained in his study for three days together, without eating or sleeping. His inventions and improvements in all parts of the mathematics were very considerable. He was in a manner the inventor and introducer of specious algebra, in which letters are used instead of numbers, as well as of many beautiful theorems in that science full explanation of which may be found under

the article Algebra, and still more in my Tracts, vol. 2. He made also considerable improvements in geometry and trigonometry. His angular sections are a very ingenious and masterly performance: by these he was enabled to resolve the problem of Adrian Roman, proposed to all mathematicians, amounting to an equation of the 45th degree. Romanus was so struck with his sagacity, that he immediately quitted his residence of Wirtsbourg in Franconia, and came to France to visit him, and solicit his friendship. His Apollonius Gallus, being a restoration of Apollonius's tract on Tangencies, and many other geometrical pieces to be found in his works, give proofs of the finest taste and genius for true geometrical speculations.—He gave some masterly tracts on trigonometry, both plane and spherical, which may be found in the collection of his works, published at Leyden in 1646, by Schooten, besides another large and separate volume in folio, published in the author's life-time at Paris in 1579, containing extensive trigonometrical tables, with the construction and use of the same, which are particularly described in the introduction to my Logarithms, pa. 4 &c. To this complete treatise on trigonometry, plane and spherical, are subjoined several miscellaneous problems and observations, such as, the quadrature of the circle, the duplication of the cube, &c. Computations are here given of the ratio of the diameter of a circle to the circumference, and of the length of the sine of 1 minute, both to a great many places of figures; by which he found that the sine of 1 minute is between 2908881959
and 2908882056;
also the diameter of a circle being 1000 &c, that the perimeter of the inscribed and circumscribed polygon of 393216 sides, will be as follows, viz, the
perimeter of the inscribed polygon 31415926535
perim. of the circumscribed polygon 31415926537
and that therefore the circumference of the circle lies between these two numbers.

Vieta having observed that there were many faults in the Gregorian Calendar, as it then existed, he composed a new form of it, to which he added perpetual canons, and an explication of it, with remarks and objections against Clavius, whom he accused of having deformed the true Lelian reformation, by not rightly understanding it.

Besides those, it seems a work greatly esteemed, and the loss of which cannot be sufficiently deplored, was his Harmonicon Cœleste, which, being communicated to father Mersenne, was, by some perfidious acquaintance of that honest-minded person, surreptitiously taken from him, and irrecoverably lost, or suppressed, to the great detriment of the literary world. There were also, it is said, other works of an astronomical kind, that have been buried in the ruins of time.

Vieta was also a profound decipherer, an accomplishment that proved very useful to his country. As the different parts of the Spanish monarchy lay very distant from one another, when they had occasion to communicate any secret designs, they wrote them in ciphers and unknown characters, during the disorders of the league: the cipher was composed of more than 500 different characters, which yielded their hidden contents to the penetrating genius of Vieta alone. His skill so disconcerted the Spanish councils for two years, that they published it at Rome, and other parts of Europe, that the French king had only discovered their ciphers by means of magic.

VINCULUM, in Algebra, a mark or character, either drawn over, or including, or some other way accompanying, a factor, divisor, dividend, &c, when it is compounded of several letters, quantities, or terms, to connect them together as one quantity, and show that they are to be multiplied, or divided, &c, together.

Vieta, I think, first used the bar or line over the quantities, for a vinculum, thus $\overline{a + b}$; and Albert Girard the parenthesis thus $(a + b)$; the former way being now chiefly used by the English, and the latter by most other Europeans. Thus $\overline{a + b} \times c$, or $(a + b) \times c$, denotes the product of c and the sum $a + b$ considered as one quantity. Also $\sqrt{a + b}$, or $\sqrt{}(a + b)$, denotes the square root of the sum $a + b$. Sometimes the mark : is set before a compound factor, as a vinculum, especially when it is very long, or an infinite series; thus $3a \times : 1 - 2x + 3x^2 - 4x^3 + 5x^5$ &c.

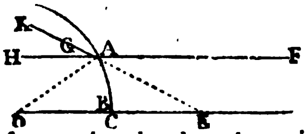
VINDEMIATRIX, or Vindemiator, a fixed star of the third magnitude, in the northern wing of the constellation Virgo.

VIRGO, in Astronomy, one of the signs or constellations of the zodiac, which the sun enters about the 21st or 22d of August; being one of the 48 old constellations, and is mentioned by the astronomers of all ages and nations, whose works have reached us. Anciently the figure was that of a girl, almost naked, with an ear of corn in her hand, evidently to denote the time of harvest among the people who invented this sign, whoever they were. But the Greeks much altered the figure, with clothes, wings, &c, and variously explained the origin of it by their own fables: thus, they tell us that the virgin, now exalted into the skies, was, while on earth, that Justitia, the daughter of Astræus and Ancora, who lived in the golden age, and taught mankind their duty; but who, when their crimes increased, was obliged to leave the earth, and take her place in the heavens. Again, Hesiod gives the celestial maid another origin, and says she was the daughter of Jupiter and Themis. There are also others who depart from both these accounts, and make her to have been Erigone, the daughter of Icarius; while others make her Parthene, the daughter of Apollo, who placed her there; and others, from the ear of corn, make it a representation of Ceres; and others, from the obscurity of her head, of Fortune.

The ancients, as they gave each of the 12 months of the year to the care of some one of the 12 principal deities, so they also threw into the protection of each of these one of the 12 signs of the zodiac. Hence Virgo, from the ear of corn in her hand, naturally fell to the lot of Ceres, and we accordingly find it called Signum Cereris.

The stars in the constellation Virgo, in Ptolemy's catalogue, are 32; in Tycho's 33; in Hevelius's 50; and in the Britannic 110.

VIRTUAL *Focus*, in Optics, is a point in the axis of a glass, where the continuation of a refracted ray meets it. Thus, let D be the centre, and DBE the axis of the glass AB; upon which falls the ray FA. Now this ray will not proceed straight forward, as AH, after passing the glass, but will take the course as AK, being deflected from the perpendicular AD. If then the refracted ray KA be produced, by



AE, to the axis at E, this point E Mr. Molineux calls the Virtual focus, or point of divergence.

VIRTUAL *Velocity*, of a point urged by any force, denotes the element of the space which it would describe in the direction of the power, when the system is supposed to have suffered an indefinitely small derangement.

The principle of virtual velocities, in mechanics, is much used by the foreign mathematicians, and is thus enunciated: If any system whatever, of bodies or points, be urged on powers in equilibrio, and there be given to this system any small motion, by virtue of which every point describes an indefinitely small space; then the sum of the products of each power multiplied by the space, which the point where it is applied would describe, according to the direction of the same power, will be always equal to zero or nothing; regarding as positive the small spaces described in the direction of the powers; and as negative, those described in the opposite sense.—This principle is due to Galileo. For its history and demonstration, Lagrange, Mecanique, pa. 8. See also Fournier's demonstration in 5° Caheir du Journal de l'Ecole Polytechnique.

VIS, a Latin word, signifying force or power; adopted by writers on physics, to express divers kinds of natural powers or faculties.

The term vis is either active or passive: the vis activa is the power of producing motion; the vis passiva is that of receiving or losing it. The vis activa is again subdivided into vis viva and vis mortua.

VIS *Absoluta*, or *Absolute Force*, is that kind of centripetal force which is measured by the motion that would be generated by it in a given body, at a given distance, and depends on the efficacy of the cause producing it.

VIS *Acceleratrix*, or *Accelerating Force*, is that centripetal force which produces an accelerated motion, and is proportional to the velocity which it generates in a given time; or it is as the motive or absolute force directly, and as the quantity of matter moved inversely.

VIS *Impressa* is defined by Newton to be the action exercised on any body to change its state, either of rest or uniform motion in a right line.—This force consists altogether in the action; and has no place in the body after the action is ceased: for the body perseveres in every new state by the vis inertiæ alone. This vis impressa may arise from various causes; as from percussion, pression, and centripetal force.

VIS *Inertiæ*. See INERTIA.

The vis inertiæ, the same great author elsewhere observes, is a passive principle, by which bodies persist in their motion or rest, and receive motion, in proportion to the force impressing it, and resist as much as they are resisted. See RESISTANCE.

VIS *Insita*, or *Innate Force* of matter, is a power of resisting, by which every body, as much as in it lies, endeavours to persevere in its present state, whether of rest or of motion uniformly forward in a right line. This force is always proportional to the quantity of matter in the body, and differs in nothing from the vis inertiæ, but in our manner of conceiving it.

VIS *Centripeta*. See CENTRIPETAL *Force*.

VIS *Motrix*, or *Moving Force* of a centripetal body, is the tendency of the whole body towards the centre, resulting from the tendency of all the parts, and is proportional to the motion which it generates in a given time; so that the vis motrix is to the vis acceleratrix, as the motion is to the celerity: and as the quantity of motion in a body is estimated by the product of the velocity into the quantity of matter, so the vis motrix arises from the vis acceleratrix multiplied by the quantity of matter. The followers of Leibnitz use the term vis motrix for the force of a body in motion, in the same sense as the Newtonians use the term vis inertiæ; this latter they allow to be inherent in a body at rest; but the former, or vis motrix, a force inherent in the same body only while in motion, which actually carries it from place to place, by acting upon it always with the same intensity in every physical part of the line which it describes.

VIS *Mortua*, and VIS *Viva*, in Mechanics, are terms used by Leibnitz and his followers for force, which they distinguish into two kinds, vis mortua, and vis viva; understanding by the former any kind of pressure, or an endeavour to move, not sufficient to produce actual motion, unless its action on a body be continued for some time; and by the latter, that force or power of acting which resides in a body in motion.

VISIBLE, something that is an object of vision or sight, or the property of a thing seen.—The Cartesians say that light alone is the proper object of vision. But according to Newton, colour alone is the proper object of sight; colour being that property of light by which the light itself is visible, and by which the images of opake bodies are painted on the retina.

As to the Situation and Place of Visible Objects:—Philosophers in general had formerly taken for granted, that the place to which the eye refers any visible object, seen by reflection or refraction, is that in which the visual ray meets a perpendicular from the object upon the reflecting or the refracting plane. That this is the case with respect to plane mirrors is universally acknowledged; and some experiments with mirrors of other forms seem to favour the same conclusion, and thus afford reason for extending the analogy to all cases of vision. If a right line be held perpendicularly over a convex or concave mirror, its image seems to make one line with it. The same is the case with a perpendicular right line held within water; for the part which is within the water appears to be a continuation of that which is without. But Dr. Barrow called in question this method of judging of the place of an object, and so opened a new field of inquiry and debate in this branch of science. This, with other optical investigations, he published in his Optical Lectures, first printed in 1674. According to him, we refer every point of an object to the place from which the pencils of light issue, or from which they would have issued, if no reflecting or refracting substance intervened. Pursuing this principle, Dr. Barrow proceeded to investigate the place in which the rays issuing from each of the points of an object, and that reach the eye after one reflection or refraction, meet; and he found that when the refracting surface was plane, and the refraction was made from a denser medium into a rarer, those rays would always meet in a place between the eye and a perpendicular to the point of incidence. If a convex mirror be used, the case will be the same; but if the mirror be plane, the rays will meet in the perpendicular, and beyond it if it be concave. The same author also determined, according to these principles, what form the image of a right line will take when it is presented in different manners to a spherical mirror, or when it is seen through a refracting medium.

Dr. Barrow however notices an objection against the maxim above-mentioned, concerning the supposed place of visible objects, and candidly owns that he was not able to give a satisfactory solution of it. The objection is this: Let an object be placed beyond the focus of a convex lens, and if the eye be close to the lens, it will appear confused, but very near to its true place. If the eye be a little withdrawn, the confusion will increase, and the object will seem to come nearer; and when the eye is very near the focus, the confusion will be very great, and the object will appear to be close to the eye. But in this experiment the eye receives no rays but those that are converging; and the point from which they issue is so far from being nearer than the object, that it is beyond it; notwithstanding which, the object is conceived to be much closer than it is, though no very distinct idea can be formed of its precise distance.

The first person who took much notice of Dr. Barrow's hypothesis, and the difficulty attending it, was Dr. Berkeley, who (in his Essay on a New Theory of Vision, pa. 30) observes, that the circle formed on the retina, by the rays which do not come to a focus, produce the same confusion in the eye, whether they cross one another before they reach the retina, or tend to it afterwards; and therefore that the judgment concerning distance will be the same in both cases, without any regard to the place from which the rays originally issued; so that in this case, by receding from the lens, as the confusion increases, which always accompanies the nearness of an object, the mind will judge that the object comes nearer. See *Apparent* Distance.

M. Bouguer (in his Traité d'Optique, pa. 104) adopts Barrow's general maxim, in supposing that we refer objects to the place from which the pencils of rays seemingly converge at their entrance into the pupil. But when rays issue from below the surface of a vessel of water, or any other refracting medium, he finds that there are always two different places of this seeming convergence: one of them of the rays that issue from it in the same vertical circle, and therefore fall with different degrees of obliquity on the surface of the refracting medium; and another, of those that fall upon the surface with the same degree of obliquity, entering the eye laterally with respect to one another. He says, sometimes one of these images is attended to by the mind, and sometimes the other; and different images may be observed by different persons. And he adds, that an object plunged in water affords an example of this duplicity of images.

G. W. Krafft has ably supported Barrow's opinion, that the place of any point seen by reflection from the surface of any medium, is that in which rays issuing from it, infinitely near to one another, would meet; and considering the case of a distant object viewed in a concave mirror, by an eye very near it, when the image, according to Euclid and other writers, would be between the eye and the object, and Barrow's rule cannot be applied, he says that in this case the speculum may be considered as a plane, the effect being the same, only that the image is more obscure. Com. Petrepol. vol. 12, pa. 252, 256. See Priestley's Hist. of Light &c, pa. 89, 688.

From the principle above illustrated, several remarkable phenomena of vision may be accounted for: as—That if the distance between two visible objects be an angle that is insensible, the distant bodies will appear as if contiguous: whence, a continuous body being the result of several contiguous ones, if the distances between several visibles subtend insensible angles, they will appear one continuous body; which gives a pretty illustration of the notion of a continuum.—Hence also parallel lines, and long vistas, consisting of parallel rows of trees, seem to converge more and more the farther they are extended from the eye; and the roofs and floors of long extended alleys seem, the former to descend, and the latter to ascend, and approach each other; because the apparent magnitudes of their perpendicular intervals are perpetually diminishing, while at the same time we mistake their distance.

As to the different Distances of Visible Objects.—The mind perceives the distance of visible objects, 1st, From the different configurations of the eye, and the manner in which the rays strike the eye, and in which the image is impressed upon it. For the eye disposes itself differently, according to the different distances it is to see; viz, for remote objects the pupil is dilated, and the crystalline brought nearer the retina, and the whole eye is made more globous; on the contrary, for near objects, the pupil is contracted, the crystalline thrust forwards, and the eye lengthened. The mode of performing this, however, has greatly divided the opinions of philosophers. See Priestley's Hist. of Light &c, pa. 638—652, where the several opinions of Descartes, Kepler, Lahire, Leroi, Porterfield, Jurin, Musschenbroek, &c, are stated and examined.

Again, the distance of visible objects is judged of by the angle the object makes; from the distinct or confused representation of the objects; and from the briskness or feebleness, or the rarity or density of the rays.

To this it is owing, 1st, That objects which appear obscure or confused, are judged to be more remote; a principle which the painters make use of to cause some of their figures to appear farther distant than others on the same plane. 2d, Hence also, rooms whose walls are whitened, appear the smaller; that fields covered with snow, or white flowers, seem less than when clothed with grass; that mountains covered with snow, in the night time, appear the nearer, and that opaque bodies appear the more remote in the twilight.

The Magnitude of Visible Objects.—The quantity or magnitude of visible objects, is known chiefly by the angle contained between two rays drawn from the two extremes of the object to the centre of the eye. An object appears so large as is the angle it subtends; or bodies seen under a greater angle appear greater; and those under a less angle, less, &c. Hence the same object appears greater or less as it is nearer the eye or farther off. And this is called the apparent magnitude.

But to judge of the real magnitude of an object, we must consider the distance; for since a near and a remote object may appear under equal angles, though the magnitudes be different, the distance must necessarily be estimated, because the magnitude is great or small according as the distance is great or small. So that the real magnitude is in the compound ratio of the distance and the apparent magnitude; at least when the subtended angle, or apparent magnitude, is very small; otherwise, the real magnitude will be in a ratio compounded of the distance and the sine of the apparent magnitude, nearly, or nearer still its tangent.

Hence, objects seen under the same angle, have their magnitudes in the same ratio as their distances. The chord of an arc of a circle appears of equal magnitude from every point in the circumference, though one point be vastly nearer than another. Or if the eye be fixed in any point in the circumference, and a right line be moved round so as its extremes be always in the periphery, it will appear of the same magnitude in every position. And the reason is, because the angle it subtends is always of the same magnitude. And hence also, the eye being placed in any angle of a regular polygon, the sides of it will all appear of equal magnitude; being all equal chords of a circle described about it.

If the magnitude of an object directly opposite to the eye be equal to its distance from the eye, the whole object will be distinctly seen, or taken in by the eye, but nothing more. And the nearer you approach an object, the less part you see of it.—The least angle under which an ordinary object becomes visible, is about one minute of a degree.

Of the Figure of Visible Objects.—This is estimated chiefly from our opinion of the situation of the several parts of the object. This opinion of the situation, &c, enables the mind to apprehend an external object under this or that figure, more justly than any similitude of the images in the retina with the object can; the images being often elliptical, oblong, &c, when the objects they exhibit to the mind, are circles, or squares, &c.

The laws of vision with regard to the figures of visible objects are, 1. That if the centre of the eye be exactly in the direction of a right line, the line will appear only as a point. 2. If the eye be placed in the direction of a surface, it will appear only as a line. 3. If a body be opposed directly towards the eye, so as only one plane of the surface can radiate on it, the body will appear as a surface. 4. A remote arch, viewed by an eye in the same plane with it, will appear as a right line. 5. A sphere, viewed at a distance, appears a circle. 6. Angular figures, at a distance, appear round. 7. If the eye look obliquely on the centre of a regular figure, or a circle, the true figure will not be seen; but the circle will appear oval, &c.

VISIBLE *Horizon, Place, &c.* See the substantives.

VISION, is the act of seeing, or of perceiving external objects by the organ of sight. When an object is so disposed, that the rays of light, coming from all parts of it, enter the pupil of the eye, and present its image on the retina, that object is then seen. This is proved by experiment; for if the eye of any animal be taken out, and the skin and fat be carefully stript off from the back part of it, till only the thin membrane, which is called the retina, remains to terminate it behind, and any object be placed before the front of the eye, the picture of that object will be seen figured as with a pencil on that membrane. There are thousands of experiments which prove that this is the mechanical effect of vision, or seeing, but none of them appear so conveniently as this, which is made with the very eye itself of an animal; an eye of an ox newly killed shows this happily, and with very little trouble. It will indeed appear singular in this, that the object is inverted, in the picture thus drawn of it, in the eye; and the case is the same in the eye of a living person.

Various other opinions however have been held concerning the means of vision among philosophers.

The Platonists and Stoics held vision to be effected by the emission of rays out of the eyes; conceiving that there was a kind of light thus darted out; which, with the light of the external air, taking hold as it were of the objects, rendered them visible; and thus returning back again to the eye, altered and new modified by the contact of the object, made an impression on the pupil, which gave the sensation of the object.

Our illustrious countryman, Roger Bacon, also assents to the opinion that visual rays proceed from the eye; giving this reason for it, that every thing in nature is qualified to discharge its proper functions by its own powers, in the same manner as the sun, and other celestial bodies. Opus Majus, pa. 289.

The Epicureans held, that vision is performed by the emanation of corporeal species or images from objects; or a kind of atomical effluvia continually flying off from the intimate parts of objects, to the eye.

The Peripatetics hold, with Epicurus, that vision is produced by the reception of species; but they differ from him in the circumstances; for they will have the species (which they call intentionales) to be incorporeal. It is true, Aristotle's doctrine of vision, delivered in his chapter De Aspectu, amounts to no more than this, that objects must have some intermediate body, that by this they may move the organ of sight. To which he adds, in another place, that when we perceive bodies, it is their species, not their matter, that we receive; as a seal makes an impression on wax, without the wax receiving any thing of the seal.

But this vague and obscure account the Peripatetics have thought proper to improve. Accordingly, what their master calls species, the disciples, understanding of real proper species, assert, that every visible object expresses a perfect image of itself in the air contiguous to it; and this image another, somewhat less, in the next air; and the third another; and so on till the last image arrives at the crystalline, which they are of opinion is the chief organ of sight, or that which immediately moves the soul. These images they call intentional species.

The modern philosophers however, as the Cartesians and Newtonians, give a better account of vision. They all agree, that it is performed by rays of light reflected from the several points of objects received in at the pupil, refracted and collected in their passage, through the coats and humours to the retina; and this striking, or making an impression, on so many points of it; which impression is conveyed, by the correspondent capillaments of the optic nerve, to the brain, &c.

Baptista Porta's experiments with the camera obscura, about the middle of the 16th century, convinced him that vision is performed by the intermission of something into the eye, and not by visual rays proceeding from the eye, as had been the general opinion before his time; and he was the first who fully satisfied himself and others on this subject; though several philosophers still adhered to the old opinion.

As for the Peripatetic series or chain of images, it is a mere chimera; and Aristotle's meaning is better understood without than with them. In fact, setting these aside, the Aristotelian, Cartesian, and Newtonian doctrines of vision, are very consistent with one another; for

Newton imagines that vision is performed chiefly by the vibrations of a fine medium (which penetrates all bodies) excited in the bottom of the eye by the rays of light, and propagated through the capillaments of the optic nerve, to the sensorium. And Descartes maintains, that the sun pressing the materia subtilis, with which the whole universe is every where filled, the vibrations and pulses of that matter reflected from objects, are communicated to the eye, and thence to the sensory: so that the action or vibration of a medium is equally supposed in all.

It is generally concluded then, that the images of objects are represented on the retina; which is only an expansion of the fine capillaments of the optic nerve, and from whence the optic nerve is continued into the brain. Now any motion or vibration, impressed on one extremity of the nerve, will be propagated to the other: hence the impulse of the several rays, sent from the several points of the object, will be propagated as they are on the retina (that is, in their proper colours, &c. or in particular vibrations, or modes of pressure, corresponding to them) to the place where those capillaments are interwoven into the substance of the brain. And thus is vision brought to the common case of sensation.

Experience teaches us that the eye is capable of viewing objects at a certain distance, without any mental exertion. Beyond this distance, no mental exertion can be of any avail: but, within it, the eye possesses a power of adapting itself to the various occasions that occur, the exercise of which depends on the volition of the mind. How this is effected, is a problem that has very much engaged the attention of optical writers; but it is doubted whether it has yet been satisfactorily explained. The first theory for the solution of this problem is that of Kepler. He supposes that the ciliary processes contract the diameter of the eye, and lengthen its axis by a muscular power. But Dr. Thomas Young (in some ingenious Observations on Vision in the Philos. Trans. 1793) observes, that these processes neither appear to contain any muscular fibres, nor have any attachment by which they can be capable of performing this action.

Descartes ascribed this contraction and elongation to a muscularity of the crystalline, of which he supposed the ciliary processes to be the tendons: but he neither demonstrated this muscularity, nor sufficiently considered the connexion with the ciliary processes.

De Lahire allows of no change in the eye, except the contraction and dilatation of the pupil: this opinion he founds on an experiment which Dr. Smith has shown to be fallacious. Hallet adopted his hypothesis, notwithstanding its inconsistency with the principles of optics and constant experience.

Dr. Pemberton supposes that the crystalline contains muscular fibres, by which one of its surfaces is flattened, while the other is made convex: but he has not demonstrated the existence of these fibres; and Dr. Jurin has proved that such a change as this is inadequate to the effect.

Dr. Porterfield conceives that the ciliary processes draw the crystalline forward, and make the cornea more convex. But the ciliary processes are incapable of this action; and it appears from Dr. Jurin's calculations, that a sufficient motion of this kind requires a very visible increase in the length of the axis of the eye; an increase which has never yet been observed.

Dr. Jurin maintains that the uvea, at its attachment to the cornea, is muscular; and that the contraction of this ring makes the cornea more convex. But this hypothesis is not sufficiently confirmed by observation.

Musschenbroek conjectures that the relaxation of this ciliary zone, which is nothing but the capsule of the vitreous humour where it receives the impression of the ciliary processes, permits the coats of the eye to push forward the crystalline and cornea. Such a voluntary relaxation however, Dr. Young observes, is wholly without example in the animal economy: besides, if it actually occurred, the coats of the eye could not act as he conceives; nor could they act in this manner without being observed. He adds, that the contraction of the ciliary zone is equally inadequate and unnecessary.

Dr. Young, having examined these theories, and some others of less moment, proceeds to investigate a more probable solution of this optical difficulty.—Adverting to the observation of Dr. Porterfield, that those who have been couched have not the power of accommodating the eye to different distances; and to the reflections of other writers on this subject; he was led to conclude that the rays of light, emitted by objects at a small distance, could only be brought to foci on the retina by a nearer approach of the crystalline to a spherical form; and he imagined that no other power was capable of producing this change, besides a muscularity of part or of the whole of its capsule:—but, on closely examining, first with the naked eye and then with a magnifier, the crystalline of an ox's eye turned out of its capsule, he discovered a structure which seemed to remove the difficulties that have long embarrassed this branch of optics.

"The crystalline of the ox," says he, "is composed of various similar coats, each of which consists of six muscles, intermixed with a gelatinous substance, and attached to six membranous tendons. Three of the tendons are anterior, and three posterior; their length is about two-thirds of the semidiameter of the coat; their arrangement is that of three equal and equidistant rays, meeting in the axis of the crystalline; one of the anterior is directed towards the outer angle of the eye, and one of the posterior towards the inner angle, so that the posterior are placed opposite to the middle of the interstices of the anterior, and planes passing through each of the six, and through the axis, would mark on either surface six regular equidistant rays. The muscular fibres arise from both sides of each tendon; they diverge till they reach the greatest circumference of the coat; and, having passed it, they again converge, till they are attached respectively to the sides of the nearest tendons of the opposite surface. The anterior or posterior portion of the six, viewed together, exhibits the appearance of three penniform-radiated muscles. The anterior tendons of all the coats are situated in the same planes, and the posterior ones in the continuations of these planes beyond the axis. Such an arrangement of fibres can be accounted for on no other supposition than that of muscularity. This mass is inclosed in a strong membranous capsule, to which it is loosely connected by minute vessels and nerves; and the connexion is more observable near its greatest circumference. Between the mass and its capsule is found a considerable quantity of an aqueous fluid, the liquid of the crystalline.

"When the will is exerted to view an object at a small distance, the influence of the mind is conveyed through the lenticular ganglion, formed from branches of the third and fifth pair of nerves by the filaments perforating the sclerotica, to the orbiculus ciliaris, which may be considered as an annular plexus of nerves and vessels; and thence by the ciliary processes to the muscle of the crystalline, which, by the contraction of its fibres, becomes more convex, and collects the diverging rays to a focus on the retina. The disposition of fibres in each coat is admirably adapted to produce this change; for, since the least surface that can contain a given bulk is that of a sphere (Simpson's Fluxions, pa. 486) the contraction of any surface must bring its contents nearer to a spherical form. The liquid of the crystalline seems to serve as a synovia in facilitating the motion, and to admit a sufficient change of the muscular part, with a smaller motion of the capsule."

Dr. Young proceeds to inquire whether these fibres can produce an alteration in the form of the lens sufficiently great to account for the known effects; and he finds, by calculation, that, supposing the crystalline to assume a spherical form, its diameter will be 642 thousandths of an inch, and its focal distance in the eye ·926. Then, disregarding the thickness of the cornea, we find (by Smith, art. 370) that such an eye will collect those rays on the retina, which diverge from a point at the distance of 12 inches and 8 tenths. This is a greater change than is necessary for an ox's eye; for if it be supposed capable of distinct vision at a distance somewhat less than 12 inches, yet it is probably far short of being able to collect parallel rays. The human crystalline is susceptible of a much greater change of form. The ciliary zone may admit of as much extension as this diminution of the diameter of the crystalline will require; and its elasticity will assist the cellular texture of the vitreous humour, and perhaps the gelatinous part of the crystalline, in restoring the indolent form.—Dr. Young apprehends that the sole office of the optic nerve is to convey sensation to the brain; and that the retina does not contribute to supply the lens with nerves.—As the human crystalline resembles that of the ox, it may reasonably be presumed that the action of both organs depends on the same general principles.

This theory of Dr. Young's however is strongly opposed by Dr. Hosack, (Philos. Trans. 1794, part 2, pa. 196). He contests the existence of the muscles, which Dr. Young has described, for several reasons. First, from the transparency they must possess; otherwise there would be some irregularity in the refraction of those rays which pass through the several parts, differing both in shape and density. Another circumstance is the number of these muscles. Dr. Young describes 6 in each lamina; and as Leuwenhoek makes 2000 laminæ in all, therefore the number of muscles must amount to 12 thousand, the action of which, Dr. Hosack apprehends, must exceed comprehension. But the existence of these muscles is still more doubtful, if the accuracy of Dr. Hosack's observations be admitted. With the assistance of the best glasses, and with the greatest attention, he could not discover the structure of the crystalline described by Dr. Young, but found it to be perfectly transparent. He first observed the lens in its viscid state, and then exposed different lenses to a moderate degree of heat, so that they became opaque and dry; and it was easy to separate the distinct layers described by Dr. Young. These were so numerous as not to admit of having, each of them six muscles. Another consideration, which seems to prove that these layers possess no distinct muscles, is that, in this opaque state, they are not visible, but consist of an almost infinite number of concentric fibres, not divided into particular bundles, but similar to as many of the finest hairs of equal thickness, arranged in similar order. This regular structure of layers, composed of concentric fibres, Dr. Hosack thinks is much better adapted to the transmission of the rays of light than the irregular structure of muscles. Besides, it ought to be considered that the crystalline lens is not the most essential organ in viewing objects at different distances; and if this be the case, the power of the eye cannot be owing to any changes in this lens. It is a fact, says Dr. Hosack, that we can, in a great degree, do without it; as is the case after couching or extraction, by which operation all its parts must be destroyed. Dr. Porterfield, however, and Dr. Young, on his authority, maintain that patients, after the operation of couching, have not the power of accommodating the eye to different distances of objects. On the whole, Dr. Hosack concludes that no such muscles, as Dr. Young has described, exist, and that he must have been deceived by some other appearances that resembled muscles: neither will he allow the effects ascribed to the ciliary processes in changing the shape or situation of the lens.

Dr. Hosack then proceeds to illustrate the structure and use of the external muscles of the eye; which are 6 in number, 4 called recti or straight, and 2 oblique, and by means of which he thinks the business is effected. The common purposes to which these muscles are subservient are well known: but besides these, Dr. Hosack suggests that it is not inconsistent with the general laws of nature, nor even with the animal economy to imagine that, from their combination, they should have a different action and an additional use. In describing the precise action of these muscles, he supposes an object to be seen distinctly, first at the distance of 6 feet; in which case the picture of it falls exactly on the retina. He then directs his attention to another object at the distance of 6 inches, as nearly as possible in the same line. While he is viewing this, he loses sight of the first object, though the rays proceeding from it still fall on the eye; and hence he infers that the eye must have undergone some change; so that the rays meet either before or behind the retina. But, as rays from a more distant object concur sooner than those from a nearer one, the picture of the more remote object must fall before the retina, while the others form a distinct image upon it. But yet the eye continued in the same place; and therefore the retina must, by some means, have been removed to a greater distance from the forepart of the eye, so as to receive the picture of the nearer object. This object, he contends, could not be seen distinctly, unless the retina were removed to a greater distance, or the refracting power of the media through which the rays passed were augmented :—but as the lens is the chief refracting medium, if we admit that this has no power of changing itself, we are under the necessity of adopting the first of these two suppositions.

The next object of inquiry is, how the external muscles are capable of producing these changes. The recti are strong, broad, and flat, and arise from the back part of the orbit of the eye; and, passing over the ball as over a pulley, they are inserted by broad flat tendons at the anterior part of the eye. The oblique are inserted towards the posterior part by similar tendons. When these different muscles act jointly, the eye being in the horizontal position, and every muscle in action contracting itself, the four recti by their combination must compress the various parts of the eye and lengthen its axis, while the oblique muscles serve to keep the eye in its proper direction and situation. The convexity of the cornea, by means of its great elasticity, is also increased in proportion to the degree of pressure, and thus the rays of light passing through it are necessarily more converged. The elongation of the eye serves also to lengthen the media, in the aqueous, crystalline, and vitreous humours through which the rays pass, so that their powers of refraction are proportionably increased. This is the general effect of the contraction of the external muscles, according to Dr Hosack's statement of it: to which it may be added, that we possess the same power of relaxing them in proportion to the greater distance of the object, till we arrive at the utmost extent of indolent vision. Dr. Hosack also illustrates this hypothesis by some experiments.

The misrepresentations of vision often depend on the distance of the object. Thus, if an opaque globe be placed at a moderate distance from the eye, the picture of it on the retina will be a circle properly diversified with light and shade, so that it will excite in the mind the sensation of a sphere or globe; but, if the globe be placed at a great distance from the eye, the distance between those lights and shades, which form the picture of a globe, will be imperceptible, and the globe will appear no otherwise than as a circular plane. In a luminous globe, distance is not necessary in order to take off the representation of prominent and flat; an iron bullet, heated very red hot, and held but a few yards distance from the eye, appears a plane, not a prominent body; it has not the look of a globe, but of a circular plane. It is owing to this misrepresentation of vision that we see the sun and moon flat by the naked eye, and the planets also, through telescopes, flat. It is in this light also that astronomers, when they speak of the sun, moon, and planets, as they appear to our view, call them the disks of the sun, moon, and planets, which we see.

The nearer a globe is to the eye, the smaller segment of it is visible, the farther off the greater, and at a due distance the half; and, on the same principle, the nearer the globe is to the eye, the greater is its apparent diameter, that is, under the greater angle it will appear; the farther off the globe is placed, the less is its apparent diameter. This is a proposition of importance, for, on this principle, we know that the same globe, when it appears larger, is nearer to our eye, and, when smaller, is further off from it. Therefore, as the globes of the sun and moon continue always of the same size, yet appear sometimes larger and sometimes smaller to us, it is evident, that they are sometimes nearer and sometimes farther off from the place whence we view them. Two globes, of different magnitude, may be made to appear of exactly the same diameter, if they be placed at different distances, and those distances be exactly proportioned to their diameters. To this it is owing, that we see the sun and moon nearly of the same diameter; they are, indeed, vastly different in real bulk, but, as the moon is placed greatly nearer to our eyes, the apparent magnitude of that smaller globe is nearly the same with that of the greater.

In this instance of the sun and moon (for there cannot be a more striking one) we see the misrepresentation of vision in two or three several ways. The apparent diameters of these globes are so nearly equal, that, in their several changes of place, they do, at times, appear to us absolutely equal, or mutually greater than each other. This is often to be seen, but it is at no time so obvious, and so perfectly evinced, as in eclipses of the sun, which are total. In these we see the apparent magnitudes of the two globes vary so much according to their distances, that sometimes the moon is large enough exactly to cover the disk of the sun, sometimes it is larger, and a part of it every where extends beyond the disk of the sun; and, on the contrary, sometimes it is smaller, and, though the eclipse be absolutely central, yet it is annular, or a part of the sun's disk is seen in the middle of the eclipsed part, enlightened, and surrounding the opaque body of the moon in form of a lucid ring.

When an object, which is seen above, without other objects of comparison, is of a known magnitude, we judge of its distance by its apparent magnitude; and custom teaches us to do this with tolerable accuracy. This is a practical use of the misrepresentation of vision, and, in the same manner, knowing that we see things which are near us, distinctly, and those which are distant, confusedly, we judge of the distance of an object by the clearness, or confusion, in which we see it. We also judge yet more easily and truly of the distance of an object by comparing it to another seen at the same time, the distance of which is better known, and yet more by comparing it with several others, the distances of which are more or less known, or more or less easily judged of. These are the circumstances which assist us, even by the misrepresentation of vision, to judge of distance; but, without one or more of these, the eye does not, in reality, enable us to judge concerning the distance of objects.

This misrepresentation, though it serves us on some occasions, yet is very limited in its effects. Thus, though it helps us greatly in distinguishing the distance of objects that are about us, both with respect to ourselves and them, and with respect to themselves with one another, yet it can do nothing with the very remote. We see that immense concave circle, in which we suppose the fixed stars to be placed, at all this vast remove from us, and no change of place that we could make to get nearer to it, would be of any avail for determining the distance of the stars from one another. If we look at three or four churches from a distance of as many miles, we see them stand in a certain position with regard to one another. If we advance a great deal nearer to them, we see that position differ, but, if we move forward only 8 or 10 feet, the difference is not perceptible.

Thus, during the last two centuries, numerous doubts and disputations have been held among anatomists and philosophers, on the immediate mode and means of vision by the eye: some ascribing it to the instrumentality of the retina spread over the bottom or posterior part of the eye; and others to the opaque choroides, immediately behind the retina. By dissecting the eyes of animals, to discover the nature and uses of the several humours and coats of that organ, it appears that the eye is justly considered as a natural achromatic instrument, or camera obscura, in which pictures of the external objects are exhibited as painted on the retina, by rays introduced through the aperture of the pupil. This was beautifully demonstrated by the celebrated discovery of Scheiner. By taking the eye of an ox, recently killed, and stripping the sclerotic coat with the choroides from its posterior portion, carefully preserving the retina as it lies upon the vitreous humour; then placing the eye in a suitable aperture in the window-shutter of a darkened chamber, with the cornea outwards, a transparent miniature painting of the external landscape, in all its variety of figures and colours, is exhibited on the retina: this experiment established the general idea that it is these pictures that we see, the sensations of which are conveyed to the sensorium by the optic nerve, the expansion of its substance forming the retina.

This discovery however introduced a new difficulty: the objects exhibited on the retina were found to be completely inverted, the upper side being undermost, and the right side changed to the left, and vice versa. Though this inversion might be the natural optical effect of the structure of the eye, how comes it that we do not usually see objects by our eyes inverted, but always in their natural position. This circumstance led to numerous disquisitions, mechanical, optical, and metaphysical, to account for it.—In the controversy relating to which are found the names of Kepler, Descartes, Newton, Hooke, Lahire, Berkeley, Porterfield, Smith, Reid, Michell, Priestley, Mariotte, Picard, Pacquet, and many others; most of these agreeing to the general idea of the retina being the chief cause of vision, but mostly endeavouring to account for the circumstance of the inversion of the images, while nevertheless the objects are seen in their due positions. At length it was accidentally discovered by Mariotte, that there is a particular part in the bottom of the eye on which no image is painted, or on which the rays have no effect, viz, the part where the optic nerve is inserted. Now if the retina is only the extension or continuation of this part, Mariotte inferred that if this were the cause of vision, the insertion of the nerve ought to be at least as sensible to the rays of light, as the rest of the retina, which is only a diffusion of the former. M. considering farther that the choroides lined the whole of the bottom of the eye, excepting the place of the insertion of the optic nerve, that is, the whole of the space exhibiting the painted images, he concluded that the choroides was the real seat and cause of vision, and not the retina.

This discovery and conclusion gave a new turn to the question, and the disquisitions of philosophers. Most of the before-mentioned persons entered into the dispute, some adopting the one opinion and some the other, but without coming to any settled and general decision. Dr. Porterfield agrees with the most part of optical writers, that the retina is the true seat of vision; and that though it is expanded over the whole concave surface of the eye as far as the ligamentum ciliare, yet it is not all equally sensible. While Mr. Walker, with several others, is of a contrary opinion: towards the close of his disquisition on vision this author adds, consistently with the theory just delivered, " I should conclude, that we have a decided proof that the posterior part of the retina is utterly insensible, since at the entrance of the nerve, where it exists in the greatest quantity, it can be demonstrated to be so; and that vision is wanting at this spot precisely, because where the nerve enters there is no choroides to reflect the rays to the sensible anterior portion." Dr. Reid, also decidedly concludes, "We have reason to believe that the rays of light make some impression on the retina; but we are not conscious of this impression; nor have anatomists or philosophers been able to discover the nature and effects of it; whether it produce a vibration in the nerve, or motion of some subtile fluid in the nerve, or something different from either, to which we cannot give a name." See an account of the several arguments of the different writers, at the end of Dr. Priestley's History of Optics.

After the contrary ideas and disquisitions of all the opticians and physiologists about the preference due to the one or the other of the two coats, the retina and the choroides, we have just seen a small pamphlet, part of a promised greater work, the production of a clergyman of the name of Horn, on " The Seat of Vision," in 8vo. 1813. In this little piece, after a neat and concise account of the different hypotheses and arguments of his predecessors in this line, the author relates some ingenious experiments accompanied with reflections on the subject, and finally deduces a theory which appears more rational and satisfactory than any of the former. He discharges the retina and the choroides each from the sole and exclusive office that had been assigned to it by the former contending parties, and assigns to each its necessary, but subordinate office, in the faculty of vision, the principle and ultimate part being performed by the optic nerve, which conveys the sensation immediately to the sensorium in the brain.

After some pertinent reflections this author adds, " Persuaded, therefore, that I had actually discovered the true origin of the retina, and that it had in consequence lost all claim to superior sensibility, and to the principle function in vision, I was induced, from a general survey of the organ, to conclude that the sole use of this transparent membrane, in the mechanism of vision, is to produce reflection, in a manner similar to the polished surface of a metallic reflector, or perhaps it might, with more propriety, be compared to glass, the choroides behind answering the purpose of the metallic coating upon the convex surface of a mirror.

" In prosecuting this inquiry, several circumstances contributed to direct my attention to the optic nerve, as the grand organ of vision. In surveying the general structure of the eye, I was particularly struck with the magnitude of this nerve, and the singular manner of its termination in the concave surface of the globe. The *optici* do not, like every other pair of nerves, terminate in branches; they are the largest in the system, yet the entire nervous substance *perforates* the globe *perpendicularly*, presenting in its concavity a well defined *circular base*, fringed with the choroides, and covered with the retina. The base was not only rendered remarkably distinct, by the following experiment; but at the same time I observed a beautiful effect produced by light upon the nerve. Having procured the eye of an ox recently killed, after dividing it transversely, and abstracting the vitreous humour from its posterior portion, leaving about 4 lines of the nerve attached, I placed the segment of the globe in a suitable aperture made in a window-shutter, with the concave surface inwards. Thus situated, having darkened the chamber, the base of the nerve exhibited, in its little hemisphere, an appearance beautifully distinct and luminous, having a striking resemblance to the sun, as seen through one of those brownish fogs with which the atmosphere is sometimes charged in the winter season. The light which produced this phenomenon, must have pervaded the whole extent of the nerve; for, being completely inclosed by the muscles and fat, it was impossible that any lateral light could have contributed to the appearance. The same phenomenon may be seen, though with less effect, by holding a similar portion of the globe between the eye and a lighted candle.

"The reader must have anticipated, and therefore will now readily comprehend the manner in which I conceive vision to be accomplished. Rays, from all points of such objects as are opposed to the organ, pass through the pupil, and, after refraction in the different humours, delineate perfect, but inverted pictures, on the retina at the bottom of the eye; these pictures are instantly reflected, in their various colours and shades, on the anterior portion of the concavity; another reflection from hence raises images of the external objects near the middle of the vitreous humour, in their natural order and position; these images make due impressions on the opposite base of the nerve, which are transmitted by it to the brain: thus the sensation is produced, and vision performed.

"Ever since Scheiner exhibited those beautiful pictures on the retina, philosophers have supposed the mind, *somehow,* affected by the impressions made on this membrane; but, mistaking the proper organ, they always found the optical phenomena, and the sensations of vision, at variance, and laboured in vain to reconcile them. However, having demonstrated, that neither the retina, nor the choroides, is the immediate seat of vision; and having restored the optic nerve to that dignified function in the theory which it naturally possesses in the organ, all the inferior instruments will be found harmoniously cooperating with it, in producing the various phenomena of vision.

"It is no longer a question, why the optic nerve has so very large a trunk bestowed on it; why the whole nervous substance enters the globe *perpendicularly*, and its *circular* base appears within, *destitute* of the choroides. If the medullary substance had not perforated the globe, or if the choroide membrane had covered the base of the nerve; in either case, it is evident, there could have been no impression made by the images in the eye on the nervous substance; consequently, in such a disposition of things, there would have been no vision.

"However, notwithstanding this surprising coincidence of things, in favour of the base of the nerve, as the immediate instrument of vision, those conversant with the subject may have foreseen what they deem an insuperable objection, which, as soon as it appears, they expect to find me drop, and the whole superstructure I have been raising come to the ground. In short, it is nothing less than the well-known fact, demonstrated by the experiment of Mariotte, that the organ is totally insensible to the impression of light, at the very spot that I have fixed on as the proper seat of vision.

"This phenomenon, I confess, appeared for some time a formidable obstacle; still, I felt a certain confidence powerfully inciting me to perseverance. More disposed to suspect some error in the conclusions drawn by philosophers from the experiment, than to doubt those principles, in the structure of the organ, by which the visual image is not only rectified, and other difficult phenomena solved, but upon which I conceived a satisfactory theory of vision might be established; I proceeded, the more anxiously, to seek another solution of this optical difficulty, than that commonly received.

"This insensible spot in the organ of vision is indeed the hidden rock, on which the most specious theories have been lost. Philosophers have been guilty of a fatal oversight—they have totally mistaken the real cause of this wonderful defect in vision; and consequently have left the most beautiful, if not the most important department of physical science, enveloped in mystery, and surrounded with difficulties, which they confess to be inexplicable. The following optical facts will at once dispel the darkness which has so long hung over this region of philosophy.

"If we take a convex lens, and place it in the window shutter of a dark room, and the eye be successively directed towards it, three effects will be produced. When the eye is situated farther from the lens than the focus of parallel rays, a very distinct, but diminished landscape, with all the objects inverted, is seen in the lens. On the contrary, if the eye be posited within the focal distance, the objects appear in their natural position, enlarged, but very indistinct. Now, undoubtedly, the medium distance between these two situations, in which the appearances of the objects are so very different, is the true focus of the lens, and the place where the images would be painted on a sheet of paper interposed. But when the eye is brought to occupy this point, no image whatever, in the lens, impresses the organ; a circular spot only is perceived, uniformly tinged with the prevailing colour of the landscape: for instance, if the ground be covered with snow the lens appears white; if the surrounding scenery consists of verdant fields, woods, &c, the colour exhibited by the lens is green; or if the prospect be upward to the sky, the lens in this case assumes an azure hue.

"Thus, the cause of that mysterious defect in the field of vision is detected; the above fact affording a clear demonstration of the effect produced on the base of the optic nerve, by the famous experiment with the patch upon the wall. Let the wall in this experiment be blue, or green, or any colour whatever, the paper is constantly lost in the general hue of the ground upon which it is fixed. But if the loss of the object proceeded from a real insensibility of the nerve, or retina at this place, whatever the colour of the wall might be, a very perceptible *dark* spot would, invariably, be substituted in its stead. So far is this, however, from being the effect produced by the experiment, that, when the wall happens to be white, and even a *black* paper is fixed upon it, no obscurity can be discerned: the black patch is entirely lost, and an uniform whiteness takes possession of its place.

"After this induction of facts, confirmed by the laws of optics, the conclusion can no longer be doubtful, that the surprising defect in vision, discovered by Mariotte, is neither to be attributed to any insensibility in the retina, nor to the nerve itself, which is the true seat of vision: the phenomenon proceeds solely from the pupil. When the base of the nerve is brought, by distorting the organ, into a straight direction with the pupil and the object, the pencils of rays, proceeding from the pupil, have their foci on the base of the nerve; and therefore, agreeably to the phenomenon of the lens above described, that portion of the cornea and humours in the axis of the eye, equal to the diameter of the pupil, is tinged with the colour of the ground upon which the paper is fixed; therefore, while the object, situated in a line with the pupil and base of the nerve, makes no impression upon this, still, the surrounding objects have their forms distinctly painted upon and reflected from the retina. The images, thus formed in the vitreous humour, make the same impressions upon the base of the nerve, as in ordinary vision: and hence a faithful representation is made to the mind of the whole scene, except that portion in the centre corresponding to the dimensions of the pupil, &c." See Mr. Horn's tract above referred to.

Vision, in Optics. The laws of vision, brought under mathematical demonstrations, make the subject of optics, taken in the greatest latitude of that word: for, among mathematical writers, optics is generally taken, in a more restricted signification, for the doctrine of direct vision: catoptrics, for the doctrine of reflected vision; and dioptrics, for that of refracted vision.

Direct or *Simple* Vision, is that which is performed by means of direct rays; that is, of rays passing directly, or in right lines, from the radiant point to the eye. Such is that explained in the preceding article.

Reflected Vision, is that which is performed by rays reflected from speculums, or mirrors. The laws of which, see under Reflection, and Mirror.

Refracted Vision, is that which is performed by means of rays refracted, or turned out of their way, by passing through mediums of different density; chiefly through glasses and lenses. The laws of this, see under the article Refraction.

Arch of Vision. See Arch.

Distinct Vision, is that by which an object is seen distinctly. An object is said to be seen distinctly, when its outlines appear clear and well defined, and the several parts of it, if not too small, are plainly distinguishable, so that they can easily be compared one with another, in respect to their figure, size, and colour.

In order to such distinct vision, it had commonly been thought that all the rays of a pencil, flowing from a physical point of an object, must be exactly united in a physical, or at least in a sensible point of the retina. But Dr. Jurin has made it appear from experiments, that such an exact union of rays is not always necessary to distinct vision. He shows that objects may be seen with sufficient distinctness, though the pencils of rays issuing from the points of them do not unite precisely in the same point on the retina; but that since, in this case, pencils from either point either meet before they reach the retina, or tend to meet beyond it, the light that comes from them must cover a circular spot on it, and will therefore paint the image larger than perfect vision would represent it. Whence it follows, that every object, placed either too near or too remote for perfect vision, will appear larger than it is by a penumbra of light, caused by the circular spaces, which are illuminated by pencils of rays proceeding from the extremities of the object.

The smallest distance of perfect vision, or that in which the rays of a single pencil are collected into a physical point on the retina in the generality of eyes, Dr. Jurin, from a number of observations, states at 5, 6, or 7 inches. The greatest distance of distinct and perfect vision he found was more difficult to determine; but by considering the proportion of all the parts of the eye, and the refractive power of each, with the interval that may be discerned between two stars, the distance of which is known, he fixes it, in some cases, at 14 feet 5 inches; though Dr. Porterfield had restricted it to 27 inches only, with respect to his own eye.

For other observations on this subject, see Jurin's Essay on Distinct and Indistinct Vision, at the end of Smith's Optics; and Robins's Remarks on the same, in his Math. Tracts, vol. 2, pa. 278 &c. See also an ingenious paper on Vision in the Philos. Trans. 1793, pa. 169, by Dr. Thomas Young.

Field of Vision. See Field.

VISUAL, relating to sight, or seeing.

Visual *Angle*, is the angle under which an object is seen, or which it subtends. See Angle.

Visual *Line*. See Line.

Visual *Point*, in Perspective, is a point in the horizontal line, where all the ocular rays unite. Thus, a person standing in a long straight gallery, and looking forward; the sides, floor, and cieling seem to meet and touch one another in this point, or common centre.

Visual *Rays*, are lines of light, conceived to come from an object to the eye.

VITELLIO, or Vitello, a Polish mathematician, of the 13th century, as he flourished about 1254. We have of his a large Treatise on Optics, the best edition of which is that of 1572. Vitello was the first optical writer of any consequence among the modern Europeans. He collected all that was given by Euclid, Archimedes, Ptolemy, and Alhazen; though his work is of but little use in the present day.

VITREOUS *Humour*, or *Vitreus Humor*, denotes the third or glassy humour of the eye; thus called from its resemblance to melted glass. It lies under the crystalline; by the impression of which, its fore part is rendered concave. It greatly exceeds in quantity both the aqueous and crystalline humours taken together, and consequently occupies much the greatest part of the cavity of the globe of the eye. Scheiner says, that the refractive power of this humour is a medium between those of the aqueous, which does not differ much from water, and of the crystalline, which is nearly the same with glass. Hawksbee makes its refractive power the same with that of water; and, according to Robertson, its specific gravity agrees nearly with that of water.

VITRUVIUS (Marcus Vitruvius Pollio), a celebrated Roman architect, of whom however nothing particular is known, but what is to be collected from his ten books De Architectura, still extant. In the preface to the sixth book he states, that he was carefully educated by his parents, and instructed in the whole circle of arts and sciences; a circumstance which he speaks of with much gratitude, laying it down as certain, that no man can be a complete architect, without some knowledge and skill in every one of them. And in the preface to the first book he informs us, that he was known to Julius Cæsar; that he was afterwards recommended by Octavia to her brother Augustus Cæsar; and that he was so favoured and provided for by this emperor, as to be out of all fear of poverty as long as he might live.

It is supposed that Vitruvius was born either at Rome or Verona; but it is not known which. His books of architecture are addressed to Augustus Cæsar, and not only show consummate skill in that particular science, but also very uncommon genius and natural abilities. Cardan, in his 16th book De Subtilitate, ranks Vitruvius as one of the 12 persons, whom he supposes to have excelled all men in the force of genius and invention; and would not have scrupled to have given him the first place, if it could be imagined that he had delivered nothing but his own discoveries. Those 12 persons were, Euclid, Archimedes, Apollonius Pergæus, Aristotle, Archytas of Tarentum, Vitruvius, Achindus, Mahomet Ibn Moses the inventor or improver of Algebra, Duns Scotus, Richard Suisset surnamed the Calculator, Galen, and Heber of Spain.

The architecture of Vitruvius has been often printed; but the best edition is that of Amsterdam in 1649. Perrault also, the noted French architect, gave an excellent French translation of the same, with the addition of notes and figures; the first edition of which was published at Paris in 1673, and the second, much improved, in 1684.—Mr. William Newton too, an ingenious architect, and late surveyor to the works at Greenwich Hospital, published in 1780 &c, curious commentaries on Vitruvius, illustrated with figures; to which is added a description, with figures, of the Military Machines used by the Ancients.

VIVIANI (Vincentio), a celebrated Italian mathematician, was born at Florence in 1621 or 1622. He was the last disciple of the illustrious Galileo, and lived with him from the 17th to the 20th year of his age. After the death of his great master, he passed two or three years more in prosecuting geometrical studies without interruption; and in this time it was that he formed the design of his Restoration of Aristeus. This ancient geometrician, who was contemporary with Euclid, had composed five books of problems De Locis Solidis, the bare propositions of which were collected by Pappus, but the books are entirely lost; which Viviani undertook to restore by the force of his genius.

He discontinued this work, however, before it was finished, in order to apply himself to another of the same kind; and that was, to restore the 5th book of Apollonius's Conic Sections. While he was engaged in this, the famous Borelli found, in the library of the grand duke of Tuscany, an Arabic manuscript, with a Latin inscription, which imported, that it contained the 8 books of Apollonius's Conic Sections; of which the 8th however was not found to be there. He carried this manuscript to Rome, in order to translate it, with the assistance of a professor of the Oriental languages. Viviani, very unwilling to lose the fruits of his labours, procured a certificate that he did not understand the Arabic language, and knew nothing of that manuscript: he was so jealous on this head, that he would not even suffer Borelli to send him an account of any thing relating to it. At length he finished his book, and published it, 1659, in folio, with this title, De Maximis & Minimis Geometrica Divinatio in quintum Conicorum Apollonii Pergæi. It was found that he had more than divined; as he seemed superior to Apollonius himself.

After this, Viviani was obliged to interrupt his studies for the service of his prince, in an affair of great importance, which was, to prevent the inundations of the Tiber, in which Cassini and he were employed for some time, though nothing was entirely executed.

In 1664 he had the honour of a pension from Louis the 14th, a prince to whom he was not subject, nor could indeed be useful. In consequence he resolved to finish his Divination on Aristeus, with a view to dedicate it to that prince; but he was interrupted in this task again by public works, and some negociations which his royal master intrusted to him.—In 1666 he was honoured by the grand duke with the title of his first mathematician.—He resolved three problems, which had been proposed to all the mathematicians of Europe, and dedicated the work to the memory of Mr. Chapelain, under the title of Enodatio Problematum &c.—He proposed the problem of the quadrable spherical surface, of which Leibnitz and l'Hospital gave solutions by the Calculus Differentialis.—In 1669, he was chosen to fill, in the Royal Academy of Sciences, a place among the 8 foreign associates. This new favour reanimated his zeal; and he published three books of his Divination on Aristeus, at Florence in 1701, which he dedicated to the king of France. It is a thin folio, entitled, De Locis Solidis secunda Divinatio Geometrica, &c. This was a second edition enlarged; the first having been printed at Florence in 1673.—Viviani employed the fortune, which he had raised by the bounties of his prince, in building a magnificent house at Florence; in which he placed a bust of Galileo, with several inscriptions in honour of that great man; and died in 1703, at 81 years of age.

Viviani had, says Fontenelle, that innocence and simplicity of manners which persons commonly preserve, who have less commerce with men than with books; without that roughness, and a certain savage fierceness, which those often acquire who have only to deal with books, not with men. He was affable, modest, a steady and faithful friend, and, what includes many virtues in one, he was grateful in the highest degree for favours.

ULLAGE, *of a Cask*, in Gauging, is so much as it wants of being full.

ULLOA (Don Antonio de), a learned Spaniard, was born in 1716, and died in 1795. His progress in science was so rapid, that at the age of 18 he was associated with George Juan and la Condamine, at the instance of Louis the 15th of France, and under the patronage of the king of Spain, to proceed to South America, to make observations for ascertaining the figure of the earth. He continued in America till 1744, when returning, he was taken prisoner, and brought to England, where he was elected a F. R. S. He was afterwards made governor of Louisiana. An account of his voyage was published at Madrid in 1748, in 5 vols. 4to.

ULTERIOR, in Geography, is applied to some part of a country or province, which, with regard to the rest of that country, is situate on the farther side of a river, or mountain, or other boundary, which divides the country into two parts.

ULTIMATE Ratios. See Prime, &c.

ULTRAMUNDANE, beyond the world, is that part of the universe supposed to be without or beyond the limits of our world or system.

UMBILICUS, and Umbilical *Point*, in Geometry, the same with focus.

UMBRA, a Shadow. See Light, Shadow, Penumbra, &c.

UNCIA, a term generally used for the 12th part of a thing; in which sense it occurs in Latin writers, both for a weight, called by us an ounce, and a measure called an inch.

UNCIÆ, in Algebra, first used by Vieta, are the numbers prefixed to the letters in the terms of any power of a binomial; now more usually, and generally, called coefficients. Thus, in the 4th power of $a + b$, viz, $a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$, the unciæ are 1, 4, 6, 4, 1.

Briggs first showed how to find these unciæ, one from another, in any power, independent of the foregoing powers. They are now usually found by what is called Newton's binomial theorem, which is the same rule as Briggs's, but in another form. See Binomial.

UNDECAGON, is a polygon of eleven sides.

If the side of a regular undecagon be 1, its area will be

$9{\cdot}3656399 = \frac{11}{4} \times$ tang. of $73\frac{7}{11}$ degrees; and therefore if this number be multiplied by the square of the side of any other regular undecagon, the product will be the area of that undecagon. See my Mensuration, pa. 85, &c, 4th edit.

UNDETERMINED, is sometimes used for INDETERMINATE.

UNDULATORY *Motion*, is applied to a motion in the air, by which its parts are agitated like the waves of the sea; as is supposed to be the case when the string of a musical instrument is struck. This undulatory motion of the air is supposed the matter or cause of sound.—Instead of the undulatory, some authors choose to call this a vibratory motion.

UNEVEN *Number*, the same as odd number, or such as cannot be divided by 2 without leaving 1 remaining. The series of uneven numbers are 1, 3, 5, 7, 9, &c. See NUMBER, and ODD *Number*.

UNFORMED *Stars*, are such as were not contained in the constellations. But on the modern celestial globes, the constellations are made to include those unformed stars.

UNGULA, in Geometry, is a part cut off a cylinder, cone, &c, by a plane passing obliquely through the base, and part of the curve surface; so called from its resemblance to the (ungula) hoof of a horse &c. For the contents &c of such ungulas, see my Mensuration, pa. 161, &c, 4th edition.

UNICORN, in Astronomy. See MONOCEROS.

UNIFORM or *Equable Motion*, is that by which a body passes always with the same celerity, or over equal spaces in equal times. See MOTION.—In uniform motions, the spaces described or passed over, are in the compound ratio of the times and velocities; but the spaces are simply as the times, when the velocity is given; and as the velocities, when the time is given.

UNIFORM *Matter*, in Natural Philosophy, is that which is all of the same kind and texture.

UNISON, in Music, is when two sounds are exactly alike, or the same note, or tone. What constitutes a unison, is the equality of the number of vibrations, made in the same time, by the two sonorous bodies.—It is a noted phenomenon in music, that an intense sound being raised, either with the voice or a sonorous body, another sonorous body near it, whose tone is either unison or octave to that tone, will sound its proper note, unison or octave, to the given note. The experiment is easily tried with the strings of two instruments; or with a voice and harpsichord; or a bell, or even a drinking-glass.

This phenomenon is thus accounted for: One string being struck, and the air put into a vibratory motion by it; every other string, within the reach of that motion, will receive some impression from it: but each string can only move with a determinate velocity of recourses or vibrations; and all unisons proceed from equal vibrations; and other concords from other proportions of vibration. The unison string then, keeping equal pace with the sounding string, as having the same measure of vibrations, must have its motion continued, and still improved, till at length its motion become sensible, and it give a distinct sound. Other concording strings have their motions propagated in different degrees, according to the frequency of the coincidence of their vibrations with those of the sounded string: the octave therefore most sensibly; then the 5th; after which, the crossing of the motions prevents any sensible effect.

This is illustrated, as Galileo first suggested, by the pendulum, which being set a-moving, the motion may be continued and augmented, by making frequent, light, coincident impulses; as blowing on it when the vibration is just finished: but if it be touched by any cross or opposite motion, and that frequently, the motion will be interrupted, and cease altogether. So, of two unison strings, if the one be forcibly struck, it communicates motion, by means of the air, to the other; and both performing their vibrations together, the motion of that other will be improved and heightened by the frequent impulses received from the vibrations of the first, because given precisely when the other has finished its vibration, and is ready to return: but if the vibrations of the chords be unequal in duration, there will be a crossing of motions, more or less, according to the proportion of the inequality; by which the motion of the untouched string will be so checked, as never to be sensible. And this we find to be the case in all consonances, except unison, octave, and the fifth.

UNIT, UNITE, or UNITY, in Arithmetic, the number one, or one single individual part of discrete quantity. See NUMBER.—The place of units, is the first place on the right hand in integer numbers.—According to Euclid, unity is not a number, for he defines number to be a multitude of units.

UNITY, the abstract or quality which constitutes or denominates a thing *one*.

UNIVERSE, a collective name, signifying the assemblage of heaven and earth, with all things in them. The Ancients, and after them the Cartesians, imagine the universe to be infinite; and the reason they give is, that it implies a contradiction to suppose it finite or bounded; since it is impossible not to conceive space beyond any limits that can be assigned; which space, according to the Cartesians, is body, and consequently part of the universe.

UNLIKE *Quantities*, in Algebra, are such as are expressed by different letters, or by different powers of the same letter. Thus, a, and b, and a^2, and ab are all unlike quantities.

UNLIKE *Signs*, are the different signs + and −.

UNLIMITED or *Indeterminate Problem*, is such a one as admits of many, or even of infinite answers. As, to divide a given triangle into two equal parts; or to describe a circle through two given points. See DIOPHANTINE and INDETERMINATE.

VOID *Space*, in Physics. See VACUUM.

VOLTAISM, or GALVANISM, is a curious and important branch of electricity, depending on metallic combinations first accidentally discovered by professor Galvani, of Bologna, about the year 1790. As we are indebted to that gentleman for the earliest insulated facts which paved the way to this science; so it appears we are indebted to professor Volta for their explanation, and application to purposes of real utility; and to sir Humphry Davy for the grand and simple law of nature by which they operate in the production of effects.

For a history of the dicovery and practice of this curious branch of electricity, see our article GALVANISM.

VOLUTE, in Architecture, a kind of spiral scroll, and used in the Ionic and Composite capitals; of which it makes the principal characteristic and ornament.

VORTEX, or **Whirlwind**, in Meteorology, a sudden, rapid, violent motion of the air, in circular whirling directions.

Vortex is also used for an eddy or whirlpool, or a body of water, in certain seas and rivers, which runs rapidly round, forming a sort of cavity in the middle.

Vortex, in the Cartesian Philosophy, is a system or collection of particles of matter moving the same way, and about the same axis.—Such vortices are the grand machines by which these philosophers attempt to solve most of the motions and other phenomena of the heavenly bodies. And accordingly, the doctrine of these vortices makes a great part of the Cartesian philosophy.

The matter of the world they hold to have been divided at the beginning into innumerable small equal particles, each endowed with an equal degree of motion, both about its own centre, and separately, so as to constitute a fluid. Several systems, or collections of this matter, they further state, have been endowed with a common motion about certain points, as common centres, placed at equal distances, and that the matters, moving round these, composed so many vortices.

Then, the primitive particles of the matter they suppose, by these intestine motions, to become, as it were, ground into spherical figures, and so to compose globules of divers magnitudes; which they call the matter of the second element: and the particles rubbed, or ground off them, to bring them to that form, they call the matter of the first element. And since there would be more of the first element than would suffice to fill all the vacuities between the globules of the second, they suppose the remaining part to be driven towards the centre of the vortex, by the circular motion of the globules; and that being there amassed into a sphere, it would produce a body like the sun.

This sun being thus formed, and moving about its own axis with the common matter of the vortex, would necessarily throw out some parts of its matter, through the vacuities of the globules of the second element constituting the vortex; and this especially at such places as are farthest from its poles; receiving, at the same time, in, by these poles, as much as it loses in its equatorial parts. And, by this means, it would be able to carry round with it those globules that are nearest, with the greater velocity; and the remoter, with less. And by this means, those globules which are nearest the centre of the sun, must be smallest; because, were they greater, or equal, they would, by reason of their velocity, have a greater centrifugal force, and recede from the centre. If it should happen, that any of these sun-like bodies, in the centres of the several vortices, should be so incrustated, and weakened, as to be carried about in the vortex of the true sun; if it were of less solidity, or had less motion, than the globules towards the extremity of the solar vortex, it would descend towards the sun, till it met with globules of the same solidity, and susceptible of the same degree of motion with itself; and thus, being fixed there, it would be for ever after carried about by the motion of the vortex, without either approaching any nearer to the sun, or receding from it; and so would become a planet.

Supposing then all this; we are next to imagine, that our system was at first divided into several vortices, in the centre of each of which was a lucid spherical body; and that some of these, being gradually incrustated, were swallowed up by others which were larger and more powerful, till at length they were all destroyed, and received by the largest solar vortex; except some few which were thrown off in right lines from one vortex to another, and so became comets.

But this doctrine of vortices is, at best, merely hypothetical. It does not pretend to show by what laws and means the celestial motions are effected, so much as by what means they possibly might, in case it should have so pleased the Creator. But we have another principle which accounts for the same phenomena as well, and far better than that of vortices; and which we plainly find has an actual existence in the nature of things: and this is gravity, or the weight of bodies.

There is, in the Philosophical Transactions, a Physico-mathematical demonstration of the impossibility and insufficiency of vortices to account for the Celestial Phenomena; by Mons. de Sigorne. See No. 457, sect. vi. pa. 409 et seq.—This author endeavours to show, that the mechanical generation of a vortex is impossible; and that it has only an axifugal force, and not a centrifugal and centripetal one; that it is not sufficient for explaining gravity and its properties; that it destroys Kepler's astronomical laws; and therefore he concludes, with Newton, that the hypothesis of vortices is fitter to disturb than explain the celestial motions. We must refer to the dissertation itself for the proof of these assertions. See Cartesian Philosophy.

But these vortices having long since been excluded by all philosophers, as utterly inconsistent with the laws and phenomena of the universe, it is useless to dwell longer upon them.

VOSSIUS (**Gerard John**), one of the most learned and laborious writers of the 17th century, was of a considerable family in the Netherlands; and was born in 1577, in the Palatinate near Heidelberg, at a place where his father, John Vossius, was minister. He first learned Latin, Greek, and Philosophy at Dort, where his father had settled, and died. In 1595 he went to Leyden, where he farther pursued these studies, in conjunction with mathematics, in which science he made a considerable progress. He became master of arts and doctor in philosophy in 1598; and soon after, director of the college at Dort; then, in 1614, director of the theological college just founded at Leyden; and, in 1618, professor of eloquence and chronology in the academy there, the same year in which appeared his History of the Pelagian Controversy. This history procured him much odium and disgrace on the continent, but an ample reward in England, where archbishop Laud obtained leave of king Charles the 1st for Vossius to hold a prebendary in the church of Canterbury, while he resided at Leyden: this was in 1629, when he came over to be installed, took a doctor of laws degree at Oxford, and then returned.—In 1633 he was called to Amsterdam to fill the chair of a professor of history; where he died in 1649, at 72 years of age; after having written and published as many works as, when they came to be collected and printed at Amsterdam in 1695 &c, made 6 volumes folio, writings which will long continue to be read with pleasure and profit. The principal of these are,—1. Etymologicon Linguæ Latinæ.—2. De Origine & Progressu Idololatriæ.—3. De Historicis Græcis.—4. De Historicis Latinis.—5. De Arte Grammatica.—6. De Vitiis Sermonis & Glossematis Latino-Barbaris.—7. Institutiones Oratoriæ.—8. Institutiones Poeticæ.—9. Ars Historica.—10. De quatuor Artibus popularibus, Grammatice, Gymnastice, Musice, & Graphice.—11. De Philologia.—12. De Universa Matheseos Natura & Constitutione.—13. De Philosophia.—14. De Philosophorum Sectis.—15. De Veterum Poetarum Temporibus.

Vossius (Denis), son of the foregoing, died at 22 years of age, a prodigy of learning, whose incessant studies brought on him so immature a death. There are of his, among other smaller pieces, Notes on Cæsar's Commentaries, and on Maimonides on Idolatry.

Vossius (Francis), brother of Denis and son of Gerard John, died in 1645, after having published a Latin poem in 1640, on a naval victory gained by the celebrated Van Tromp.

Vossius (Gerard), brother of Denis and Francis, and son of Gerard John, wrote Notes upon Paterculus, which were printed in 1639. He was one of the most learned critics of the 17th century; but died in 1640, like his two brothers, at a very early age, and before their father.

Vossius (Isaac), was the youngest son of Gerard John, and the only one that survived him. He was born at Leyden in 1618, and was a man of great talents and learning. His father was his only preceptor, and his whole time was spent in study. His merit recommended him to a correspondence with queen Christina of Sweden, who employed him in some literary commissions. At her request, he made several journeys into Sweden, where he had the honour to teach her the Greek language; though she afterwards discarded him on hearing that he intended to write against Salmasius, for whom she had a particular regard. In 1663 he received a handsome present of money from Louis the 14th of France, accompanied with a complimentary letter from the minister Colbert.—In 1670 he came over to England, when he was created doctor of laws at Oxford, and king Charles the 2d made him canon of Windsor; though he knew his character well enough to say, there was nothing that Vossius refused to believe, excepting the Bible. He appears indeed, by his publications, which are neither so numerous nor so useful as his father's, to have been a most credulous man, while he afforded many circumstances to bring his religious faith in question. He died at his lodgings in Windsor Castle, in 1689; leaving behind him the best private library, as it was then supposed, in the world; which, to the shame and reproach of England, was suffered to be purchased and carried away by the university of Leyden. His publications chiefly were:—1. Periplus Scylacis Caryandensis, &c, 1639.—2. Justin, with Notes, 1640.—3. Ignatii Epistolæ, & Barnabæ Epistola, 1646.—4. Pomponius Mela de Situ Orbis, 1648.—5. Dissertatio de vera Ætate Mundi, &c, 1659.—6. De Septuaginta Interpretibus, &c, 1661.—7. De Luce, 1662.—8. De Motu Marium & Ventorum.—9. De Nili & aliorum Fluminum Origine.—10. De Poematum Cantu & Viribus Rythmi, 1673.—11. De Sibyllinis aliisque, quæ Christi natalem præcessere, 1679.—12. Catullus, & in eum Isaaci Vossii Observationes, 1684.—13. Variarum Observationum liber, 1685, in which are contained the following pieces: viz, De Antiquæ Romæ & aliarum quarundam Urbium Magnitudine; De Artibus & Scientiis Sinarum; De Origine & Progressu Pulveris Bellici apud Europæos; De Triremium & Liburnicarum Constructione; De Emendatione Longitudinum; De patefacienda per Septentrionem ad Japonenses & Indos Navigatione; De apparentibus in Luna circulis; Diurna Telluris conversione omnia gravia ad medium tendere.

VOUSSOIRS, vault-stones, are the stones which immediately form the arch of a bridge, &c, being cut somewhat in the manner of a truncated pyramid or wedge, their under sides constituting the intrados, to which their joints or ends should be every where in a perpendicular direction.

The length of the middle voussoir, or key-stone, and which is the least of all, should be about $\frac{1}{15}$ or $\frac{1}{16}$th of the span of the arch; from hence these stones should be made larger and larger, all the way down to the impost; that they may the better sustain the great weight which rests upon them, without being crushed or broken, and that they may also bind the firmer together.

To find the just length of the voussoirs, or the figure of the extrados, when that of the intrados is given; see the Principles of Bridges in my Tracts, or Emerson's Construction of Arches, in his volume of Miscellanies.

URANIBURGH, or celestial town, the name of a celebrated observatory, in a castle in the little island Weenen, in the Sound; built by the celebrated Danish astronomer, Tycho Brahé, who furnished it with instruments for observing the course and motions of the heavenly bodies.—This observatory, which was finished about the year 1580, had not subsisted above 17 years when Tycho, who little thought to have erected an edifice of so short a duration, and who had even published the figure and position of the heavens, which he had chosen for the moment to lay the first stone in, was obliged to abandon his country. Soon after this, the persons to whom the property of the island was given, demolished the building: part of the ruins was dispersed into divers places: the rest served to build Tycho a handsome seat upon his ancient estate, which to this day bears the name of Uraniburgh; and it was here that Tycho composed his catalogue of the stars. Its latitude is 55° 54′ north, and longitude 12° 47′ east of Greenwich.

M. Picart, making a voyage to Uraniburgh, found that Tycho's meridian line, there drawn, deviated from the meridian of the world; which seems to confirm the conjecture of some persons, that the position of the meridian line may vary.

URANOLITE; the same as Aerolite.

URANUS, a new primary planet, discovered by Dr. Herschel at Bath, in the night of March 13, 1781. It is sometimes also called the Georgian Planet, and the New Planet, from its having been newly or lately discovered, also Herschel's Planet, from the name of its discoverer, and the Planet Herschel, or simply Herschel. The planet is denoted by this character ♅.

This planet is the remotest of all those that are yet known, though not the largest, being in point of magnitude less than Saturn and Jupiter. Its light, says Dr. Herschel, is of a blueish-white colour, and its brilliancy between that of Venus and the moon. With a telescope that magnifies about 300 times, it appears to have a very well defined visible disk; but with instruments of a small power, it can hardly be distinguished from a fixed star of between the 6th and 7th magnitude. In a very fine clear night, when the moon is absent, a good eye will perceive it without a telescope.

From the observations and calculations of Dr. Herschel and other astronomers, the elements and dimensions &c of this planet, have been collected as below.

Place of the node - - - - -	$2^{s}\ 11^{\circ}\ 49'\ 30''$
Place of the aphelion in 1795	11 23 33 55
Inclination of the orbit - - - · ·	43 35
Time of the perihelion passage,	Sept. 7, 1799
Excentricity of the orbit - - -	·8203
Half the greater axis - - -	19·0818 of Earth's dist.
Revolution - - - - - - - -	$83\frac{1}{3}$ sidereal years
Diameter of the planet - - -	34217 miles
Propor. of diam. to the earth's -	4·3177 to 1
Its bulk to the earth's - - -	80·4926 to 1
Its density as - - - - - - -	·2204 to 1
Its quantity of matter - - -	17·7406 to 1

And heavy bodies fall on its surface 18 feet 8 inches in one second of time. See PLANET, &c.

Dr. H. has also discovered 6 satellites belonging to this planet; the periodical revolutions of which are completed in the respective times following:—1st, 5d 21h 25m; 2d, 8d 17h 1m 9s; 3d, 10d 23h 4m; 4th, 13d 11h 5m 2s; 5th, 38d 1h 49m; 6th, 107d 16h 40m. The orbits of these satellites make very large angles with the ecliptic; and it has been asserted that their real motion is retrograde; but this is probably an optical illusion.

URSA, in astronomy, the Bear, a name common to two constellations of the northern hemisphere, near the pole, distinguished by Major and Minor.

URSA *Major*, or the Great Bear, one of the 48 old constellations, and perhaps more ancient than many of the others; being familiarly known and alluded to by the oldest writers, and is mentioned by Homer as observed by navigators. It is supposed that this constellation is that mentioned in the book of Job, under the name of Chesil, which our translation has rendered Orion, where it is said, "Canst thou loose the bands of Chesil (Orion)?" It is farther said that the ancients represented each of these two constellations under the form of a waggon drawn by a team of horses, and the Greeks originally called them waggons and two bears; they are to this day popularly called the wains, or waggons, and the greater of them Charles's Wain. Hence is remarked the propriety of the expression, "loose the bands &c," the binding and loosing being terms very applicable to a harness, &c.

Perhaps the Egyptians, or whoever else were the people that invented the constellations, placed those stars, which are near the pole, in the figure of a bear, as being an animal inhabiting towards the north pole, and making neither long journeys, nor swift motions. But the Greeks, in their usual way, have adapted some of their fables to it. They say this bear was Callisto, daughter of Lycaon, king of Arcadia; that being debauched by Jupiter, he afterwards placed her in the heavens, as well as her son Arcturus.

The Greeks called this constellation Arctos and Helice, from its turning round the pole. The Latins from the name of the nymph, as variously written, Callisto, Megisto, and Flemisto, and from the Arabians, sometimes Feretrum Majus, the Great Bier. And the Ursa Minor, they called Feretrum Minus, the Little Bier. The Italians have followed the same custom, and call them Cataletto. They spoke also of the Phenicians being guided by the Lesser Bear, but the Greeks by the Greater.

There are two remarkable stars in this constellation, viz, those in the middle of his body, considered as the two hindermost of the wain, and called the pointers, because they always point nearly in a direction towards the north pole star, and so are useful in finding out this star.

The stars in Ursa Major, are, according to Ptolemy's catalogue, 35; in Tycho's 56; in Hevelius's 73; but in the Britannic catalogue 87.

URSA *Minor*, the Little Bear, called also Arctos Minor, Phœnice, and Cynosura, one of the 48 old constellations, and near the north pole, the large star in the tip of its tail being very near to it, and thence called the pole-star.

The Phenicians guided their navigations by this constellation, for which reason it was called Phenice, or the Phenician constellation. It was also called Cynosura by the Greeks, because, according to some, that was one of the dogs of the huntress Callisto, or the Great Bear; but according to others Cynosura was one of the Idæan nymphs that nursed the infant Jupiter; and some say that Callisto was another of them, and that, for their care, they were taken up together to the skies.—Ptolemy places in this constellation 8 stars, Tycho 7, Hevelius 12, and Flamsteed 24.

URSUS (NICHOLAS RAIMARUS), a very extraordinary character, and distinguished in the science of astronomy, was born at Henstedt in Dithmarsen, in the duchy of Holstein, about the year 1550. He was a swineherd in his youth, and did not begin to read till he was 18 years of age; but then he employed all the hours he could spare from his daily labour, in learning to read and write. He afterwards applied himself to study the languages; and, having a strong genius, made a rapid progress in Greek and Latin. He quickly learned also the French language, the mathematics, astronomy, and philosophy; and most of them without the assistance of a master.

Having left his native country, he gained a maintenance by teaching; which he did in Denmark in 1584, and on the frontiers of Pomerania and Poland in 1585. It was in this place that he invented a new system of astronomy, very little different from that of Tycho Brahé. This he communicated, in 1586, to the landgrave of Hesse, which gave rise to a terrible dispute between him and Tycho. This celebrated astronomer charged him with being a plagiary; who, as he related, happening to come with his master into his study, saw there, drawn on a piece of paper, the figure of his system; and afterwards insolently boasted that he himself was the inventor of it. Ursus, on this accusation, wrote furiously against Tycho, called the honour of his invention in question, ascribing the system to Apollonius Pergæus; and in short abused him in so brutal a manner, that he was like to be prosecuted for it. Ursus was afterwards invited by the emperor to teach the mathematics in Prague; from which city, to avoid the presence of Tycho, he withdrew silently in 1589, and died soon after.

He made some improvement in trigonometry, and wrote several books, which discover the marks of his hasty studies; his erudition being indigested, and his style incorrect, as is almost always to be observed of persons that are late-learned.

VULPECULA *et* ANSER, the *Fox and Goose*, in Astronomy, one of the new constellations of the northern hemisphere, made out of the unformed stars by Hevelius, in which he reckons 27 stars; but Flamsteed counts 35.

W.

WAD, or WADDING, in Gunnery, a stopple of paper, hay, straw, old rope-yarn, or tow, rolled firmly up like a ball, or a short cylinder, and forced into a gun up to the powder, to keep it close in the chamber; or secure the shot from rolling out, as well as, according to some, to prevent the inflamed powder from dilating around the sides of the ball, by its windage, as it passes along the chace, which it was thought would much diminish the effort of the powder. But, from the accurate experiments lately made at Woolwich, it has not been found to have any such effect.

WADHOOK, or WORM, a long pole with a screw at the end, to draw out the wad, or the charge, or paper &c from a gun.

WAGGONER, in Astronomy, is the constellation Ursa Major, or the Great Bear, called also vulgarly Charles's Wain.

WAGGONER is also used for a routier, or book of charts, describing the seas, their coasts, &c.

WALES (WILLIAM), F.R.S. by his natural talents and close application, rose from a low situation, little connected with learning, to some of the first ranks in literary pursuits. We observe his early labours in the correspondence of the Ladies' Diary, that very useful little work, which has formed most of our eminent mathematicians. Here, and in some other periodical publications, for many years is observed the gradual improvement of Mr. W. in the various mathematical sciences. Mr. W. was deemed a fit person to be sent to a distant country (Hudson's Bay), to observe the transit of Venus over the sun 1769; and the manner in which he discharged that trust did honour to his talents. On his return he communicated to the Royal Society an excellent paper of observations made at that station, which was inserted in their Transactions, vol. for 1769; and the year following came out his general observations made at Hudson's Bay, in a large 4to volume. Mr. W. next, in the character of astronomer, accompanied Capt. Cook, in his first voyage, 1772—1774; and again in his other voyage of 1776—1779. In 1777 came out his Observations on a voyage with Capt. Cook; and in 1778 Remarks on Dr. Forster's Account of the Voyage, in which he showed considerable talents as a controversial writer. Soon after his return from the last voyage, Mr. W. was elected a F.R.S. where he proved a very useful member; and, on the death of Mr. Daniel Harris, he was appointed mathematical master to Christ's Hospital, London; and, some years after, secretary to the board of longitude; both which offices he held till the time of his death, which happened in 1798, at about 64 years of age.

In 1781, Mr. W. published an Enquiry into the State of the Population in England and Wales: and in 1794 his treatise on the Longitude by Timekeepers. Mr. W. published an ingenious restoration of one of the lost pieces of Apollonius: and it has been said he was author of one of the dissertations on the achronical rising of the Pleiades, annexed to Dr. Vincent's Voyage of Nearchus, 1797. Besides all these, Mr. W. wrote some ingenious papers in the Philos. Trans. and in various periodical publications, particularly the Ladies' Diaries, sometimes signed with his own name, and sometimes under certain fictitious signatures, as G, Celti, Felix M'Carthy, &c.

WALLIS (Dr. JOHN), an eminent English mathematician, was the son of a clergyman, and born at Ashford in Kent, Nov. 23, 1616. After being instructed, at different schools, in grammar learning, in Latin, Greek, and Hebrew, with the rudiments of logic, music, and the French language, he was placed in Emanuel college, Cambridge. About 1640 he entered into orders, and was chosen fellow of Queen's college. He kept his fellowship till it was vacated by his marriage, but quitted his college to be chaplain to sir Richard Darley; after a year spent in this situation, he passed two more as chaplain to lady Vere. While he lived in this family, he cultivated the art of deciphering, which proved very useful to him on several occasions: he met with rewards and preferment from the government at home for deciphering letters for them; and it is said, that the elector of Brandenburg sent him a gold chain and medal, for explaining for him some letters written in ciphers.

In 1643 he published Truth Tryed, or Animadversions on Lord Brooke's treatise, called The Nature of Truth &c; styling himself "a minister in London," probably of St. Gabriel Fenchurch, the sequestration of which had been granted to him.—In 1644 he was chosen one of the scribes or secretaries to the assembly of divines at Westminster.

Academical studies being much interrupted by the civil wars in both the universities, many learned men from them resorted to London, and formed assemblies there. Wallis belonged to one of these, the members of which met once a week, to discourse on philosophical matters; and this society was the rise and beginning of that which was afterwards incorporated by the name of the Royal Society, of which Wallis was one of the most early members.

The Savilian professor of geometry at Oxford being ejected by the parliamentary visitors, in 1649, Wallis was appointed to succeed him, and he opened his lectures there the same year. In 1650 he published some Animadversions on a book of Mr. Baxter's, entitled, "Aphorisms of Justification and the Covenant." And in 1653, in Latin, a Grammar of the English tongue, for the use of foreigners; to which was added, a tract De Loquela seu Sonorum Formatione, &c, in which he considers philosophically the formation of all sounds used in articulate speech, and shows how the organs being put into certain positions, and the breath forced out from the lungs, the person will thus be made to speak, whether he hear himself or not. Pursuing these reflections, he was led to think it possible, that a deaf person might be taught to speak, by being directed so to apply the organs of speech, as the sound of each letter required, which children learn by imitation and frequent attempts, rather than by art. He made a trial or two with success; and particularly on one Popham, which involved him in a dispute with Dr. Holder, of which some account has already been given in the life of that gentleman.

In 1654 he took the degree of doctor in divinity; and the year after became engaged in a long controversy with

Mr. Hobbes. This philosopher having, in 1655, printed his treatise De Corpore Philosophico, Dr. Wallis the same year wrote a confutation of it in Latin, under the title of Elenchus Geometriæ Hobbianæ; which so provoked Hobbes, that in 1656 he published it in English, with the addition of what he called, "Six Lessons to the Professors of Mathematics in Oxford." Upon this Dr. Wallis wrote an answer in English, entitled, "Due Correction for Mr. Hobbes; or School-discipline for not saying his Lessons right," 1656: to which Mr. Hobbes replied in a pamphlet called "ΣΤΙΓΜΑΙ, &c, or Marks of the absurd Geometry, Rural Language, Scottish Church-politics, and Barbarisms, of John Wallis, 1657." This was immediately rejoined to by Dr. Wallis, in Hobbiani Puncti Dispunctio, 1657. And here this controversy seems to have ended, at this time: but in 1661 Mr. Hobbes printed Examinatio & Emendatio Mathematicorum Hodiernorum in sex Dialogis; which occasioned Dr. Wallis to publish the next year, Hobbius Heautontimorumenos, addressed to Mr. Boyle.

In 1657 our author collected and published his mathematical works, in two parts, entitled, Mathesis Universalis, in 4to; and in 1658, Commercium Epistolicum de Quœstionibus quibusdam Mathematicis nuper habitum, in 4to; which was a collection of letters written by himself and many learned men, as Lord Brounker, Sir Kenelm Digby, Fermat, Schooten, and others.

Wallis was this year chosen Custos Archivorum of the university. On this occasion Mr. Stubbe, who, on account of his friend Mr. Hobbes, had before waged war also against Wallis, published a pamphlet, entitled, "The Savilian Professor's Case Stated," 1658. Dr. Wallis replied to this: and Mr. Stubbe republished his case, with enlargements, and a vindication against the exceptions of Dr. Wallis.

On the Restoration it appears he met with great respect; the king thinking favourably of him on account of some services he had done both to himself and his father Charles the first. He was therefore confirmed in his places, also admitted one of the king's chaplains in ordinary, and appointed one of the divines empowered to revise the Book of Common Prayer. He complied with the terms of the act of uniformity, and continued a steady conformist till his death. He was a very useful member of the Royal Society; and kept up a literary correspondence with many learned men. In 1670 he published his Mechanica; sive de Motu, 4to. In 1676 he gave an edition of Archimedis Syracusani Arenarius & Dimensio Circuli; and in 1682 he published from the manuscripts, Claudii Ptolomæi Opus Harmonicum, in Greek, with a Latin version and notes; to which he afterwards added, Appendix de veterum Harmonica ad hodiernam comparata, &c. In 1685 he published some theological pieces; and, about 1690, was engaged in a dispute with the Unitarians; also, in 1692, in another dispute about the sabbath. Indeed his books on subjects of divinity are very numerous, but nothing near so important as his mathematical works.

In 1685 he published his History and Practice of Algebra, in folio; a work replete with learned and useful matter. Besides the works above-mentioned, he published many others, particularly his Arithmetic of Infinites, a book of genius and good invention, and perhaps almost his only work that is so, for he was much more distinguished for his industry and judgment, than for his genius. Also a multitude of papers in the Philos. Trans. in almost every volume, from the 1st to the 25th volume. In 1697, the curators of the university press at Oxford thought it for the honour of the university to collect the doctor's mathematical works, which had been printed separately, some in Latin, some in English, and published them all together in Latin, in 3 vols. folio, 1699.

Dr. Wallis died at Oxford the 28th of October, 1703, in the 88th year of his age, leaving behind him one son and two daughters. We are informed that he was of a vigorous constitution, and of a mind which was strong, calm, serene, and not easily ruffled or discomposed. He speaks of himself, in his letter to Mr. Smith, in a strain which shows him to have been a very cautious and prudent man, whatever his secret opinions and attachments might be: he concludes, "It hath been my endeavour all along to act by moderate principles, being willing, whatever side was uppermost, to promote any good design, for the true interest of religion, of learning, and of the public good."

WALMESLEY (Charles), D.D. F.R.S. was an English Benedictine monk, and a Roman Catholic bishop; also senior bishop and vicar apostolic of the western district, as well as doctor of theology in the Sorbonne. He died at Bath in 1797, in the 76th year of his age, and the 41st of his episcopacy. Dr. W. was the last survivor of those eminent mathematicians, who were concerned in regulating the chronological style in England, which produced a change of the style in this country, in the year 1752. Besides some ingenious astronomical essays in the Philos. Trans. he published several separate works, both on mathematics and theology; as, 1. Analyse des Mesures des Rapports et des Angles, 4to, 1749; being an extension and explanation of Cotes's Harmonia Mensurarum. 2. Theorie du Monument des Apsides, 8vo, 1749. 3. De Inæqualitatibus Motuum Lunarium, 4to, 1758. An explanation of the Apocalypse, Esekiel's Vision, &c: By the fire at Bath at the time of the riots, several valuable manuscripts, which he had been compiling during a well-spent life of labour, and travelling through many countries, were irretrievably lost.

WARD (Dr. Seth), an English prelate, chiefly famous for his knowledge in mathematics and astronomy, was the son of an attorney, and born at Buntingford, Hertfordshire, in 1617 or 1618. From hence he was removed and placed a student in Sidney-college, Cambridge, in 1632. Here he applied with great vigour to his studies, particularly to the mathematics, and was chosen fellow of his college. In 1640 he was pitched upon by the vice-chancellor to be prævaricator, which at Oxford is called terræ-filius; whose office it was to make a witty speech, and to laugh at any thing or any body: a privilege which he exercised so freely, that the vice-chancellor actually suspended him from his degree; though he reversed the censure the day following.

The civil war now breaking out, Ward was involved not a little in the consequences of it. He was ejected from his fellowship for refusing the Covenant; against which he soon after joined with several others, in drawing up that celebrated treatise, which was afterwards printed. Being now obliged to leave Cambridge, he resided for some time with certain friends about London, and at other times at Aldbury in Surrey, with the noted mathematician Oughtred, where he prosecuted his mathematical studies. He afterwards lived for the most part, till 1649, with Mr.

Ralph Freeman at Aspenden in Hertfordshire, whose sons he instructed as their preceptor; after which he resided some months with lord Wenman, of Thame Park, in Oxfordshire.

He had not been long in this family before the visitation of the university of Oxford began; the effect of which was, that many learned and eminent persons were turned out, and among them Mr. Greaves, the Savilian professor of Astronomy: this gentleman laboured to procure Ward for his successor, whose abilities as an astronomer were universally known and acknowledged; and effected it; Dr. Wallis succeeding to the Geometry professorship at the same time. Mr. Ward then entered himself of Wadham college, for the sake of Dr. Wilkins, who was the warden; and he lost no time in bringing the astronomy lectures, which had long been neglected and disused, into repute again; and for this purpose he read them very constantly, never missing one reading day, during the time he held the lecture.

In 1654, both the Savilian professors did their exercises, in order to proceed doctors in divinity; and when they were to be presented, Wallis claimed precedency. This occasioned a dispute; which being decided in favour of Ward, who was really the senior, Wallis went out grand compounder, and so obtained the priority. In 1659, Ward was chosen president of Trinity college; but was obliged at the Restoration to resign that place. He had recompense made him, however, by being presented in 1660 to the rectory of St. Laurence Jewry. The same year he was also installed precentor of the church of Exeter. In 1661 he became fellow of the Royal Society, and dean of Exeter; and the year following he was advanced to the bishopric of the same church. In 1667 he was translated to the see of Salisbury: and in 1671 was made chancellor of the order of the garter; an honour which he afterwards procured to be permanently annexed to the see of Salisbury, after it had been held by laymen for above 150 years.

Dr. Ward was one of those unhappy persons who have the misfortune to survive their senses, which happened in consequence of a fever badly cured: he lived till the Revolution, but without knowing any thing of the matter; and died in January 1689, about 71 years of age. He was the author of several Latin works in astronomy and different parts of the mathematics, which were thought excellent in their day; but their use has been superseded by later improvements and the Newtonian philosophy. Some of these were,

1. A Philosophical Essay towards an Eviction of the Being and Attributes of God, &c. 1652.—2. De Cometis, &c; 4to, 1653.—3. In Ismaelis Bullialdi Astronomia Inquisitio; 4to, 1653.—4. Idea Trigonometriæ demonstratæ; 4to, 1654.—5. Astronomia Geometrica; 8vo, 1656. In this work, a method is proposed, by which the astronomy of the planets is geometrically resolved, either on the Elliptical or Circular motion; it being in the third or last part of this work that he proposes and explains what is called Ward's Circular Hypothesis.—6. Exercitatio epistolica in Thomæ Hobbii Philosophiam, ad D. Joannem Wilkins; 1656, 8vo.

But that by which he has chiefly signalized himself, as to astronomical invention, is his celebrated approximation to the true place of a planet, from a given mean anomaly, founded on an hypothesis, that the motion of a planet, though it be really performed in an elliptic orbit, may yet be considered as equable as to angular velocity, or with an uniform circular motion round the upper focus of the ellipse, or that next the aphelion, as a centre. By this means he rendered the praxis of calculation much easier than any that could be used in resolving what has been commonly called Kepler's problem, in which the coequate anomaly was to be immediately investigated from that of the mean elliptic one. His hypothesis agrees pretty well with those orbits which are elliptical but in a very small degree, as that of the Earth and Venus: but in others, that are more elliptical or excentric, as those of Mercury, Mars, &c, this approximation stood in need of a correction, which was made by Bulliald. Both the method, and the correction, are very well explained and demonstrated, by Keill, in this Astronomy, lecture 24.

WARGENTIN (Peter), an ingenious Swedish mathematician and astronomer, was born Sept. 22, 1717, and died Dec. 13, 1783. He became secretary to the Academy at Stockholm in 1749, when he was only 32 years of age; and he became successively a member of most of the literary academies in Europe, as London, Paris, Petersburg, Gottingen, Upsal, Copenhagen, Drontheim, &c. In this country he is probably most known on account of his tables for computing the eclipses of Jupiter's satellites, which are annexed to the Nautical Almanac of 1779. I know not that he has published any separate work; but his communications were very numerous to several of those Academies of which he was a member; as the Academy of Stockholm, in which are 52 of his memoirs; in the Philosophical Transactions, the Upsal Acts, the Paris Memoirs, &c.

WARING (Edward), M. D. and F. R. S. was born about the year 1736, near Shrewsbury, were also he died, August 15, 1798, in the sixty-third year of his age. After his early education at that place, he was sent to Magdalen college, Cambridge, in 1753. Here his talents for abstruse calculations soon distinguished him; so that, on taking his first degree, in 1757, he was ranked as senior wrangler, or the first student of the year, Mr. John Jebb being the second on the list. The Lucasian professorship of mathematics in the university becoming vacant, by the death of Mr. John Colson, in 1759, Mr. W. was elected to that office in Jan. 1760. On this occasion some remarkable circumstances took place. Before his election, Mr. W. gave a small specimen of his abilities, as a proof of his fitness for that office, by the publication of the first chapter of his Miscellanea Analytica. This specimen was attacked, and his election opposed, by Dr. Powell, of St. John's college, with the view of serving his friend Mr. Maseres (the present cursitor baron of the exchequer), then a candidate also for the vacant professorship. This opposition produced several curious pamphlets between the two parties, by Dr. Powell and Mr. Maseres on the one side, and by Mr. Waring, assisted by his friend Mr. Wilson (afterwards one of the judges, sir John Wilson), on the other side; which however ended in the success and election of the latter.

In 1762 Mr. W. published complete his Miscellanea Analytica, one of the most abstruse books, written on the abstrusest parts of Algebra; which might at least have the effect of extending the author's title to ingenuity. Mathematics however did not engross the whole of his attention: he could allow some part of his time to the study of medicine; and in 1767 he was admitted to the degree of M. D. though he never after practised as a physician. Mathematics again engaged his chief attention, and he successively produced a number of pieces, of a like abstruse kind as the former; several of which were inserted in different volumes of the Philos. Trans., and some he published in separate works; as, the Meditationes Analyticæ, in 1770; the Proprietates Algebraicarum Curvarum, in 1772; and the Meditationes Analyticæ, in 1776. To these might be added a work, written in his retirement, on morals and metaphysics; of which a few copies only were printed, and presented to his friends. As also a pamphlet published at Cambridge, in which algebraic quantities are translated into probable relations, and some theorems on probabilities thence deduced. In the same pamphlet are farther added some new propositions on chances, on the values of lives, on survivorships, &c.

Most of these essays give proofs of the strong powers of the author's mind, both in abstract science, and its application to philosophy: though they labour, in common with his other works, under the disavantage of being conveyed in a very unattractive form.

In his disposition and character, Dr. W. is represented as of inflexible integrity, great modesty, plainness, and simplicity of manners; of a meekness and a diffidence of mind to such a degree, as to be always embarrassed before strangers. His extreme short-sightedness too, joined to the natural want of order and method in his mind, which appeared remarkably even in his hand-writing, rendered his mathematical compositions so confused and embarrassed, that in manuscript they were often utterly inexplicable: a circumstance which may account for the numerous typographical errors in his publications.

Besides the works before-mentioned, Dr. Waring gave a number of valuable papers to the Philosophical Transactions of the Royal Society.

WATCH, a small portable machine, or movement, for measuring time; having its motion commonly regulated by a spiral spring. Perhaps, strictly speaking, watches are all such movements as show the parts of time; as clocks are such as proclaim them, by striking on a bell, &c. But commonly, the term watch is appropriated to such as are carried in the pocket; and clock to the large movements, whether they strike the hour or not.

Spring or *Pendulum* WATCHES stand on much the same principle with pendulum clocks. For if a pendulum, describing small circular arcs, make vibrations of unequal lengths, in equal times, it is because it describes the greater arc with a greater velocity; so a spring put in motion, and making greater and less vibrations, as it is more or less strong, and as it has a greater or less degree of motion given it, performs them nearly in equal times. Hence, as the vibrations of the pendulum had been applied to large clocks, to rectify the inequality of their motions; so, to correct the unequal motions of the balance in watches, a spring is added, by the isochronism of whose vibrations the correction is to be effected. The spring is usually wound into a spiral; that, in the little compass allotted it, it may be as long as possible; and may have strength enough not to be mastered, and displaced by the inequalities of the balance it is to regulate. The vibrations of the two parts, viz, the spring and the balance, should be of the same length; but so adjusted, as that the spring, being more regular in the length of its vibrations than the balance, may occasionally communicate its precision to the latter.

The Invention of Spring or Pocket Watches, is due to the 16th century. It is true, as we are informed, in the history of Charles the 5th, that a watch was presented to that prince: but this was probably no more than a kind of clock to be set on a table: some resemblance of which we have still remaining in the ancient pieces made before the year 1670. Some accounts also state, the first watches were made at Nuremberg in 1500, by Peter Hell, and were called Nuremberg eggs, on account of their oval form. And farther, that the same year, George Purbach, a mathematician of Vienna, employed a watch that pointed to seconds, for astronomical observations, which was probably a kind of clock. In effect, it is between Hooke and Huygens that the glory of this excellent invention lies: but to which of them it properly belongs, has been greatly disputed; the English ascribing it to the former, and the French, Dutch, &c, to the latter. Derham, in his Artificial Clockmaker, says positively, that Dr. Hooke was the inventor; and adds, that he contrived various means of regulation: one was with a loadstone; another with a tender straight spring, one end of which played backward and forward with the balance; so that the balance was to the spring as the ball of a pendulum, and the spring as the rod of the same: a third method was with two balances, of which there were divers kinds; some having a spiral spring to the balance for a regulator, and others without. But the way that prevailed, and which still continues in mode, was with one balance, and one spring running round the upper part of the verge of it: though this has a disadvantage, which those with two springs, &c were free from; in that, a sudden jerk, or confused shake, will alter its vibrations, and disturb it very much.

The time of these inventions was about the year 1658; as appears, among other evidences, from an inscription on one of the double-balance watches presented to king Charles the second, viz, Rob. Hooke inven. 1658. T. Tompion fecit, 1675. The invention soon came into repute both at home and abroad; and two of the machines were sent for by the dauphin of France. Soon after this, M. Huygens's watch with a spiral spring got abroad, and excited uncommon interest in England, as if the longitude could be found by it. It is certain however, that this invention was later than the year 1673, when his book De Horol. Oscillat. was published; in which there is no mention of this, though he speaks of several other contrivances in the same way.

One of these the lord Brounker sent for from France, where M. Huygens obtained a patent for them. This watch agreed with Dr. Hooke's, in the application of the spring to the balance; only that of Huygens had a longer spiral spring, and its pulses and beats were much slower; also the balance, instead of turning quite round as Dr. Hooke's did, turned several times every vibration. Huygens also invented divers other kinds of watches, some of them without any string or chain at all, which he called pendulum watches.

Mr. Derham suggests that he suspects Huygens's fancy was first set to work by some intelligence he might have of Hooke's invention from Mr. Oldenburg, or some other of his correspondents in England: though Mr. Oldenburg vindicates himself against that charge, in the Philos. Trans. Nos. 118 and 129.

Watches, since their first invention, have gone on in a continued course of improvement, and they have lately been brought to great perfection, both in England and in

France, but more especially the former, particularly owing to the great encouragement that has been given to them by the board of longitude. Some of the chief writers and improvers of watches, are, Le Roy, Cummins, Harrison, Mudge, Emery, and Arnold. See Derham's Artificial Clockmaker; Cummins's Principles of Clock and Watch work; Mudge's Thoughts on the Means of improving Watches, &c.

Striking WATCHES, are such as, besides the proper watch part, for measuring time, have a clock part, for striking the hours, &c. These are real clocks; only moved by a spring instead of a weight; and are properly called pocket-clocks.

Repeating WATCHES, are such as, by pulling a string, &c, repeat the hour, quarter, or minute, at any time of the day or night.—This repetition was the invention of Mr. Barlow, being first put in practice by him in larger movements or clocks, about the year 1676. The contrivance immediately set the other artists to work, who soon devised many ways of effecting the same. But its application to pocket-watches was not known before king James the second's reign; when the ingenious inventor above mentioned was soliciting a patent for it. The talk of a patent engaged Mr. Quare to resume the thoughts of a like contrivance, which he had in view some years before: he now effected it; and being pressed to endeavour to prevent Mr. Barlow's patent, a watch of each kind was produced before the king and council; on trial of which, the preference was given to Mr. Quare's. The difference between them was, that Barlow's was made to repeat by pushing in two pieces on each side the watch-box; one of which repeated the hour, and the other the quarter: whereas Quare's was made to repeat by a pin that stuck out near the pendant, which being thrust in (as is now done by forcing in the pendant itself) repeated both the hour and quarter with the same thrust.

Of the Mechanism of a WATCH.

Watches, as well as clocks, are composed of wheels and pinions, with a regulator to direct the quickness or slowness of the wheels, and of a spring which communicates motion to the whole machine. But the regulator and spring of a watch are vastly inferior to the weight and pendulum of a clock, neither of which can be employed in watches. Instead of a pendulum, therefore, they are obliged to use a balance (pl. 40, fig. 4) to direct the motion of a watch; and of a spring (fig. 6), which serves, instead of a weight, to give motion to the wheels and balance.

The wheels of a watch, like those of a clock, are placed in a frame, formed of two plates and four pillars. Fig. 3. represents the inside of a watch, after the plate (fig. 5) is taken off. A is the barrel which contains the spring (fig. 6); the chain is rolled about the barrel, with one end of it fixed to the barrel A, and the other to the fusee B.

When a watch is wound up, the chain which was upon the barrel winds about the fusee, and by this means the spring is stretched; for the interior end of the spring is fixed by a spring to the immoveable axis, about which the barrel revolves; the exterior end of the spring is also fixed to the inside of the barrel, which turns upon an axis. It is there easy to perceive how the spring extends itself, and how its elasticity forces the barrel to turn round, and consequently causes the chain which is upon the fusee to unfold and turn the fusee; the motion of the fusee is communicated to the wheel CC; then by means of the teeth, to the pinion *c*, which carries the wheel D; then to the pinion *d*, which carries the wheel E; then to the pinion *e*, which carries the wheel F; then to the pinion *f*, upon which is the balance-wheel G, whose pivot runs in the piece A, called the potance, and B called a follower, which are fixed on the plate fig. 5. This plate, of which only a part is represented, is applied to that of fig. 3, in such a manner, that the pivots of the wheels enter into holes made in the plate fig. 3. Thus the impressed force of the spring is communicated to the wheels: and the pinion *f* being then connected to the wheel F, obliges it to turn (fig. 7). This wheel acts on the pallats of the verge 1, 2. (fig. 4) the axis of which carries the balance HH (fig. 4). The pivot I, in the end of the verge, enters into the hole *e* in the potance A (fig. 5). In this figure the pallats are represented; but the balance is on the other side of the plate, as may be seen in fig. 11. The pivot 3 of the balance enters into a hole of the cock BC (fig. 10), as represented in fig. 12. Thus the balance turns between the cock and the potance *e* (fig. 5), as in a kind of cage. The action of the balance-wheel upon the pallats 1, 2, (fig. 4) is the same with that of the same wheel in the clock; i. e. in a watch the balance-wheel causes the balance to vibrate backwards and forwards like a pendulum.

At each vibration of the balance a pallat allows a tooth of the balance-wheel to escape; so that the quickness of the motion of the wheels is entirely determined by the celerity of the vibrations of the balance, and these vibrations of the balance and motion of the wheels are produced by the action of the spring.

But the quickness or slowness of the vibrations of the balance depends not solely on the action of the great spring, but chiefly on the action of the spring *abc*, called the spiral spring (fig. 13) situated under the balance H, and represented in (fig. 11); the exterior end of the spiral is fixed to the pin *a* (fig. 13). This pin is applied near the plate in *a* (fig. 11); the interior end of the spiral is fixed by a peg to the centre of the balance. Hence if the balance be turned upon itself, the plates remaining immoveable, the spring will extend itself, and make the balance perform one revolution. Now, after the spiral is thus extended, if the balance be left to itself, the elasticity of the spiral will bring back the balance, and in this manner the alternate vibrations of the balance are produced.

In fig. 7 all the wheels above described are represented in such a manner, that we may easily perceive at first sight how the motion is communicated from the barrel to the balance.

In fig. 8 are represented the wheels under the dial-plate, by which the hands are moved. The pinion *a* is made to fit tight on the prolonged pivot of the wheel D (fig. 7), and is called a cannon pinion. This wheel revolves in an hour. The end of the axis of the pinion *a*, upon which the minute hand is fixed, is square; the pinion (fig. 8) is indented into the wheel *b*, which is carried by the pinion *a*. Fig. 9 is a wheel fixed on a barrel, into the cavity of which the pinion *a* enters, and on which it turns freely. This wheel revolves in 12 hours, and carries along with it the hour-hand.

WATER, in Physiology, a clear, insipid, and colourless fluid, coagulable into a transparent solid substance, called ice, when placed in a temperature of 32° of Fahrenheit's thermometer, or lower, but volatile and fluid in every degree of heat above that; and when pure, or freed from heterogeneous particles, is reckoned one of the four elements.

By some late experiments of Messrs. Lavoisier, Watt, Cavendish, Priestley, Kirwan, &c, it appears, that water consists of dephlogisticated air, and inflammable air or phlogiston intimately united; or, as Mr. Watt conceives, of those two principles deprived of part of their latent heat. And in some instances it appears that air and water are mutually convertible into each other. Thus, Mr. Cavendish (Philos. Trans. vol. 74, pa. 128) recites several experiments, in which he changed common air into pure water, by decomposing it in conjunction with inflammable air. Dr. Priestley likewise, having decomposed dephlogisticated and inflammable air, by firing them together by the electric explosion, found a manifest decomposition of water, which, as nearly as he could judge, was equal in weight to that of the decomposed air. He also made a number of other curious experiments, which seemed to favour the idea of a conversion of water into air, without absolutely proving it. The difficulty which M. Deluc and others have found in expelling all air from water, is best accounted for on the supposition of the generation of air from water; and, admitting that the conversion of water into air is effected by the intimate union of what is called the principle of heat with the water, it appears sufficiently analogous to other changes, or rather combinations, of substances. Is not, says Dr. Priestley, the acid of nitre, and also that of vitriol, a thing as unlike to air as water is, their properties being as remarkably different? And yet it is demonstrable that the acid of nitre is convertible into the purest respirable air, and probably by the union of the same principle of heat. Philos. Trans. vol. 73, pa. 414 &c.

Indeed there seems to be water in all bodies, and particles of almost all kinds of matter in water; so that it is hardly ever sufficiently pure to be considered as an element. Water, if it could be had alone and pure, Boerhaave argues, would have all the requisites of an element, and be as simple as fire; but there is no expedient hitherto discovered for procuring it so pure. Rain water, which seems the purest of all those we know of, is replete with infinite exhalations of all kinds, which it imbibes from the air: so that, if filtered and distilled a thousand times, there still remain fæces. Besides this, and the numberless impurities it acquires after it is raised, by mixing with all sorts of effluvia in the atmosphere, and by falling upon and running over the earth, houses, and other places. There is also fire contained in all water; as appears from its fluidity, which is owing to fire alone. Nor can any kinds of filtering through sand, stone, &c, free it entirely from salts &c. Nor have all the experiments that have been invented by the philosophers, ever been able to derive water perfectly pure. Hence Boerhaave says, that he is convinced no person ever saw a drop of pure water; that the utmost of its purity known, only amounts to its being free from this or that kind of matter; and that it can never, for instance, be quite deprived of salt; since air will always accompany water, and air always contains salt.

Water seems to be diffused everywhere, and to be present in all space wherever there is matter. There are hardly any bodies in nature but what will yield water: it is even asserted that fire itself is not without it. A single grain of the fiery salt, which in a moment's time will penetrate through a man's hand, readily imbibes half its weight of water, and melts even in the driest air possible. Among innumerable instances, hartshorn, kept 40 years, and turned as hard and dry as any metal, so that it will yield sparks of fire when struck against a flint, yet being put into a glass vessel, and distilled, will afford $\frac{1}{8}$th part of its quantity of water. Bones dead and dried 25 years, and thus become almost as hard as iron, yet by distillation have yielded half their weight of water. And the hardest stones, ground and distilled, always discover a portion of it. But hitherto no experiment shows, that water enters as a principle into the combination of metallic matters, or even into that of vitrescible stones.

From such considerations, philosophers have been led to hold the opinion, that all things were made of water. Basil Valentine, Paracelsus, Van Helmont, and others have maintained, that water is the elemental matter or stamen of all things, and suffices alone for the production of all the visible creation. Thus too Newton: "All birds, beasts, and fishes, insects, trees, and vegetables, with their several parts, do grow out of water, and watery tinctures, and salts; and by putrefaction they all return again to watery substances." And the same doctrine is held, and confirmed by experiments, by Van Helmont, Boyle, and others.

But Dr. Woodward endeavours to show that the whole is a mistake.—Water containing extraneous corpuscles, some of which, according to him, are the proper matter of nutrition; the water being still found to afford so much the less nourishment, the more it is purified by distillation. So that water, as such, does not seem to be the proper nutriment of vegetables; but only the vehicle which contains the nutritious particles, and carries them along with it, through all the parts of the plant.

Helmont however carries his system still farther, and imagines that all bodies may be reconverted into water. His alkahest, he affirms, adequately resolves plants, animals, and minerals, into one liquor, or more, according to their several internal differences of parts; and the alkahest, being abstracted again from these liquors, in the same weight, and with the same virtues, as when it dissolved them, the liquors may, by frequent cohobations from chalk, or some other proper matter, be totally deprived of their seminal endowments, and at last return to their first matter; which is insipid water.

Spirit of wine, of all other spirits, seems freest from water: yet Helmont affirms, it may be so united with water, as to become water itself. He adds, that it is material water, only under a sulphureous disguise. And the same thing he observes of all salts, and of oils, which may be almost wholly changed into water.

No standard for the Weight and Purity of WATER.—Water hardly ever continues two moments exactly of the same weight; by reason of the air and fire contained in it. The expansion of water in boiling shows what effect the different degrees of fire have on the gravity of water. This makes it difficult to fix the specific gravity of water, in order to settle its degree of purity. However, the purest water we can obtain, according to the experiments of Hawskbee, is 850 times heavier than atmospheric air: or according to the experiments of Mr. Cavendish, the thermometer being at 50° and the barometer at 29$\frac{1}{4}$, about 800 times as heavy as air: and according to the experiments of sir Geo. Shuckburgh, when the barometer is at 29·27 and the thermometer at 53°, water is 836 times heavier than air; whence also may be deduced this general proportion, which may be accounted a standard, viz, that, when the barometer is at 30° and the thermometer at 55°, then water is 820 times heavier than air; also that in such a state the cubic foot of water weighs 1000 ounces avoirdupois, and that of air 1·222, or $1\frac{2}{9}$ nearly, also that of mercury 13600 ounces; and for other states of the thermometer and barometer, the allowance is after this rate, viz, that the column of mercury in the barometer varies its length by the 10 thousandth part of itself for a change of each single degree of temperature, and water changes by $\frac{3}{10000}$ part of its height or magnitude by each degree of the same. However, we have not any very exact standard in air; for water being so much heavier than air, the more water there is contained in the air, the heavier of course must the air be; as indeed a considerable part of the weight of the atmosphere seems to arise from the water that is contained in it.

Properties and Effects of WATER.—Water is a very volatile body. It is entirely reduced into vapours and dissipated, when exposed to the action of fire and unconfined.

Water heated in an open vessel, acquires no more than a certain determinate degree of heat, whatever be the intensity of the fire to which it is exposed; which greatest degree of heat is when it boils violently.

It has been found that the degree of heat necessary to make water boil, is variable, according to the purity of the water and the weight of the atmosphere. The annexed table shows the degree of heat at which water boils, at various heights of the barometer, being a medium between those resulting from the experiments of sir Geo. Shuckburgh and M. Deluc.

Height of the Barometer.	Heat of Boiling Water.
Inches.	°
26	205
26½	206
27	206·9
27½	207·7
28	208·5
28½	209·4
29	210·3
29½	211·2
30	212·0
30½	212·8
31	213·6

Water is found the most penetrative of all bodies, after fire, and the most difficult to confine; passing through leather, bladders, &c, which will confine air; making its way gradually through woods; and is only retainable in glass and metals; nay it was determined by experiment at Florence, that when shut up in a spherical vessel of gold, which was pressed with a great force, it made its way through the pores even of the gold itself.

Water, by this penetrative quality alone, may be inferred to enter the composition of all bodies, both vegetable, animal, fossil, and even mineral; with this particular circumstance, that it is easily, and with a gentle heat, separable again from bodies it had united with.

And yet the same water, as little cohesive as it is, and as easily separated from most bodies, will cohere firmly with some others, and bind them together in the most solid masses; as in the tempering of earth, or ashes, clay, or powdered bones, &c, with water, and then dried and burnt, when the masses become hard as stones, though without the water they would be mere dust or powder. Indeed it appears wonderful that water, which is otherwise an almost universal dissolvent, should nevertheless be a great coagulator.

Some have imagined that water is incompressible, and therefore non-elastic; founding their opinion on the celebrated Florentine experiment above mentioned, with the globe of gold; when the water being, as they say, incapable of condensation, rather than yield, transuded through the pores of the metal, so that the ball was found wet all over the outside; till at length making a cleft in the gold, it squirted out with great vehemence. But the truth of the conclusions drawn from this Florentine experiment has been very justly questioned; Mr. Canton having proved by accurate experiments, that water is actually compressed even by the weight of the atmosphere. See COMPRESSION.

Besides, the diminution of size which water suffers when it passes to a less degree of heat, sufficiently shows that the particles of this fluid are, like those of all other known substances, capable of approaching nearer together.

Ditch WATER, is often used as an object for the microscope, and seldom fails to afford a great variety of animalcules; often appearing of a greenish, reddish, or yellowish colour, from the great multitudes of them. And to the same cause is to be ascribed the green skim on the surface of such water. Dunghill water is also full of an immense crowd of animalcules.

Fresh WATER, is said of that which is insipid, or without salt, and inodorous; being the natural and pure state of the element.

Hard WATER, or *Crude* WATER, is that in which soap does not dissolve completely or uniformly, but is curdled. The dissolving power of hard water is less than that of soft; and hence its unfitness for washing, bleaching, dyeing, boiling kitchen vegetables, &c. The hardness of water may arise either from salts, or from gas. That which arises from salts, may be discovered and remedied by adding some drops of a solution of fixed alkali; but the latter by boiling, or exposure to the open air.—Spring waters are often hard; but river water soft. Hard waters are remarkably indisposed to corrupt; they even preserve putrescible substances for a considerable length of time: hence they seem to be best fitted for keeping at sea, especially as they are so easily softened by a little alkaline salt.

Putrid WATER, is that which has acquired an offensive smell and taste by the putrescence of animal or vegetable substances contained in it. This kind of water is in the highest degree pernicious to the human constitution, and capable of bringing on mortal diseases even by its smell. Quicklime put into water is useful to preserve it longer sweet; or even exposure to the air in broad shallow vessels. And putrid water may be in a great measure sweetened, by passing a current of fresh air through it, from bottom to top.

Rain WATER may be considered as the purest distilled water, but impregnated during its passage through the air with a considerable quantity of putrescent matter; whence it is superior to any other in fertilizing the earth. Hence also it is inferior for domestic purposes to spring or river water, even if it could be readily procured: but such as is obtained from spouts placed below the roofs of houses, the common way of procuring it in this country, is evidently very impure, and becomes putrid in a short time.

River or *Running* WATER, is next in purity to snow or distilled water; and for domestic purposes superior to both, in having less putrescent matter, and more fixed air. That however is much the purest that runs over a clean rocky or stony bottom. River waters generally putrefy sooner than those of springs. During the putrefaction, they throw off a part of their heterogeneous matter, and at length become sweet again, and purer than at first; after which they commonly preserve a long time: this is remarkably the case with the Thames water, taken up about London; which is commonly used by seamen, in their voyages.

Salt **Water**, such as has much salt in it, so as to be sensible to the taste.

Sea **Water**, or Water of the sea, is an assemblage of bodies, in which water can scarcely be said to have the principal part: it is an universal colluvies of all the bodies in nature, sustained and kept floating in water as a vehicle: being a solution of common salt, sal catharticus amarus, a selenitic substance, and a compound of muriatic acid with magnesia, mixed together in various proportions. It may be freshened by simple distillation without any addition, and thus it has sometimes been useful in long voyages at sea. Sea water by itself has a purgative quality, owing to the salt it contains; and has been greatly recommended in scrophulous disorders. Sea water is about 3 parts in 100 heavier than common water; and its temperature at great depths is from 34 to 40 degrees; but near the surface it follows more nearly the temperature of the air.

Snow **Water**, is the purest of all the common waters, when the snow has been collected pure. Kept in a warm place, in clean glass vessels, not closely stopped, but covered from dust, &c, snow water becomes in time putrid; though in well-stopped bottles it remains unaltered for several years. But distilled water suffers no alteration in either circumstance.

Spring **Water** is commonly impregnated with a small portion of imperfect neutral salt, extracted from the different strata through which it percolates. Some contain a vast quantity of stony matter, which they deposit as they run along, and thus form masses of stone; sometimes incrustating various animal and vegetable matters, which they are therefore said to petrify. Spring water is much used for domestic purposes, and on account of its coolness is an agreeable drink; but on account of its being usually somewhat hard, is inferior to that which has run for a considerable way in a channel. Spring water arises from the rain, and from the mists and moisture in the atmosphere. These falling upon hills and other parts of the earth, soak into the ground, and pass along till they find a vent out again, in the form of a spring.

Water-*Bellows*, in Mechanics, are bellows, for blowing air into furnaces, that are worked by the force of water; or air blown in by the fall of water.

Water-*Clock*. See **Clepsydra**.

Water-*Engine*, an engine for extinguishing fires; or any engine to raise water; or any engine moved by the force of water. See **Engine**, and **Steam**-*Engine*.

Water-*Gage*, an instrument for measuring the depth or quantity of any water. See **Gage**.

Water-*Level*, is the true level which the surface of still water takes, and is the most correct of any.

Water-*Logged*, in Sea-Language, denotes the state of a ship when, by receiving a great quantity of water into her hold, by leaking, &c, she has become heavy and inactive upon the sea, so as to yield without resistance to the effort of every wave rushing over her deck.

Water-*Machine*. See **Machine**.

Water-*Measure*. Salt, sea-coal, &c, while on board vessels in the pool, or river, are measured with the corn-bushel heaped up; or else 5 striked pecks are allowed to the bushel. This is called water-measure; and it exceeds Winchester measure by about 3 gallons in the bushel.

Water-*Microscope*. See **Microscope**.

Water-*Mill*. See **Mill**.

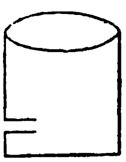
Motion of **Water**, in Hydraulics. The theory of the motion of running water is one of the principal objects of hydraulics, and to which many eminent mathematicians have paid their attention. But it were to be wished that their theories were more consistent with each other, and with experience. The inquisitive reader may consult Newton's Principia, lib. 2, pr. 36, with the comment. Dan. Bernoulli's Hydrodynamica. J. Bernoulli, Hydraulica, Oper. tom. 4, pa. 389. Dr. Jurin, in the Philos. Trans. No. 452. Gravesande, Physic. Elem. Mathem. lib. 3, par. 2. Maclaurin's Flux. art. 537. Poleni de Castellis, Ximenes, D'Alembert, Bossu, Buat, and many others.

But notwithstanding the labours of all these eminent authors, this intricate subject still remains in a great measure obscure and uncertain. Even the simple case of the motion of running water, when it issues from a hole in the bottom of a vessel, has never yet been determined, so as to give universal satisfaction to the learned. On this head, it is now pretty generally allowed, that the velocity of the issuing stream, is equal to that which a heavy body acquires by falling through the height of the fluid above the hole, as may be demonstrated by theory: but in practice, the quantity of the effluent water is much less than what is given by this theory, owing to the obstruction to the motion in the hole, partly from the sides of it, and partly from the different directions of the parts of the water in entering it, which thence obstruct each other's motion. And this obstruction, and the diminution in the quantity of water run out, is still the more in proportion as the hole is the smaller; in such sort, that when the hole is very small, the quantity is diminished in the ratio of $\sqrt{2}$ to 1 very nearly, which is the ratio of the greatest diminution; and for larger apertures, the diminution is always less and less. This fact is ascertained, or admitted by Newton, and all the other philosophers above mentioned, with some small variations.

That the velocity of the water in the hole, or at least some part of it, as that for example in the middle of the stream, is equal to that above-mentioned, is even evinced by experiment, by directing the stream either sideways, or upwards: for in the former case, it is found to range upon an horizontal plane, a distance that just answers to that velocity, by the nature of projectiles; and in the latter case, the jet rises nearly to the height of the water in the vessel; which it could not do, if its velocity were not equal to that acquired by the free descent of a body through that height. Hence it is evident then, that the particles of the water, which are in the hole at the same moment of time, do not all issue out with the same velocity; and, in fact, the velocity is found to decrease all the way from the middle of the hole, where it is greatest, towards the side or edge, where it is the least.

At a small distance from the hole, the diameter of the vein of water is much less than that of the aperture itself. Thus, if the diameter of the hole be 1, the diameter of the vein of water just without it, will be $\frac{21}{25}$, or 0·84, according to Newton's measure, who first observed this phenomenon; and according to Poleni's measure 0·78 nearly.

By the experiments of Buat (Principes d'Hydraulique), the quantity by theory is to that by experiment, for a small aperture made in the thin side of a reservoir, as 8 to 5. When a short pipe is added to the hole outwards, of the length of two or three times its diameter, that ratio is as 16 to 13. And when the short pipe is all within side the vessel, as in the margin, the same ratio becomes that of 3 to 2. Poleni also found that the quantity of water flowing through a pipe or tube, was much greater than that through a hole of the same diameter in the thin side or bottom of the vessel, the height of the head of water above each being the same. See also many other curious circumstances in Buat's Principes above-mentioned.



Some authors give this rule for finding the height due to the velocity in a flat orifice, or a medium among all the parts of it, such that this medium velocity being drawn into the area of the hole, shall give the quantity per second that runs through: viz, let A denote the area of the surface of the water in the vessel, *a* the area of the orifice by which the water issues, and H the height of the water above the orifice; then, as $2A - a : A :: H : h$, the height due to the medium velocity, or the height from which a body must freely descend, by the force of gravity, to acquire that mean velocity.

Authors are not yet agreed as to the force with which a vein of water, spouting from a round hole in the side of a vessel, presses on a plane directly opposed to the motion of the vein. Most authors agree, that the pressure of this vein, flowing uniformly, ought to be equal to the weight of a cylinder of water, whose base is equal to the orifice through which the water flows, and its height equal to the height of the water in the vessel above the hole. The experiments made by Mariotte, and others, seem to countenance this opinion. But Dan. Bernoulli rejects it, and estimates this pressure by the weight of a column of the fluid, whose diameter is equal to the contracted vein (according to Newton's observation above-mentioned), and the height of which is equal to double the altitude due to the real velocity of the spouting water; and this pressure is also equal to the force of repulsion, arising from the reaction of the spouting water on the vessel. The ingenious author remarks that he speaks only of single veins of water, the whole of which are received by the planes on which they press; for as to the pressures exerted by fluids surrounding the bodies they press upon, as the wind, or a river, the case is different, though confounded with the former by writers on this subject. Hydrodynamica, pa. 289.

Another rule however had been adopted by the Academicians of Paris, who made a number of experiments to confirm or establish it. Hist. Acad. Paris, ann. 1679, sect. 3, cap. 5.

D. Bernoulli, on the other hand, thinks his own theory is sufficiently established by the experiments he relates; for the particulars of which see the Acta Petropolitana, vol. 8, pa. 122. This ingenious author is also of opinion that his theory of the quantity of the force of repulsion, exerted by a vein of spouting water, might be usefully applied to move ships by pumping; and he thinks the motion produced by this repulsive force would fall little, if at all, short of that produced by rowing. He has given his reasons and computations at length in his Hydrodynamica, pa. 293 &c.

This science of the pressures exerted by water or other fluids in motion, is what Bernoulli calls *Hydraulico-statica*. This science differs from hydrostatics, which considers only the pressure of water and other fluids at rest; whereas hydraulico-statics considers the pressure of water in motion. Thus the pressure exerted by water moving through pipes, upon the sides of those pipes, is an hydraulico-statical consideration, and has been erroneously determined by many, who have given no other rules in these cases, but such as are applicable only to the pressure of fluids at rest. See Hydrodynam. pa. 256 &c.

WATER-*Poise*. See HYDROMETER, and AREOMETER.

Dr. Hooke contrived a water-poise, which may be of good service in examining the purity &c of water. It consists of a round glass ball, like a bolt head, about 3 inches diameter, with a narrow stem or neck, the 24th of an inch in diameter; which being poised with red lead, so as to make it but little heavier than pure sweet water, and thus fitted to one end of a fine balance, with a counterpoise at the other end; on the least addition of even the 2000th part of salt to a quantity of water, half an inch of the neck will emerge above the water. Philos. Trans. No. 197.

Raising of WATER, in Hydraulics. The great advantage of raising water by engines for the various purposes of life, is well known. Machines have in all ages been contrived with this view; a detail of the best of which, with the theory of their construction, would be very curious and instructive. M. Belidor has executed this in part in his Architecture Hydraulique. Dr. Desaguliers has also given a description of several engines to raise water, in his Course of Experimental Philosophy, vol. 2; and there are several other smaller works of the same kind.

Engines for raising water are either such as throw it up with a great velocity, as in jets; or such as raise it from one place to another by a gentle motion. For the general theory of these engines, see Bernoulli's Hydrodynamica. —Desaguliers has settled the maximum of engines for raising water, thus: A man with the best water engine cannot raise above one hogshead of water in a minute, 10 feet high, to hold it all day; but he can do almost twice as much for a minute or two.

WATER-*Spout*. See SPOUT.

WATER-*Wheel*, an engine for raising water in great quantity out of a deep well, &c. See PERSIAN-*Wheel*.

WATER-*Works*. See *Raising of* WATER.

WAVE, in Physics, a volume of water elevated by the action of the wind &c, upon its surface, into a state of fluctuation, and accompanied by a cavity. The extent from the bottom or lowest point of one cavity, and across the elevation, to the bottom of the next cavity, is the breadth of the wave.

Waves are considered as of two kinds, which may be distinguished from each other by the names of natural and accidental waves. The natural waves are those which are regularly proportioned in size to the strength of the wind which produces them. The accidental waves are those occasioned by the wind's reacting on itself by repercussion from hills or high shores, and by the dashing of the waves themselves, otherwise of the natural kind, against rocks and shoals; by which means these waves acquire an elevation much above what they can have in their natural state.

Boyle proved, by numerous experiments, that the most violent wind never penetrates deeper than 6 feet into the water: and it seems a natural consequence of this, that the water moved by it can only be elevated to the same height of 6 feet from the level of the surface in a calm; and these 6 feet of elevation being added to the 6 of excavation, in the part from whence that water so elevated was raised, should give 12 feet for the utmost elevation of a wave. This is a calculation that does great honour to its author; as many experiments and observations have proved that it is very nearly true in deep seas, where the waves are purely natural, and have no accidental causes to render them larger than their just proportion.

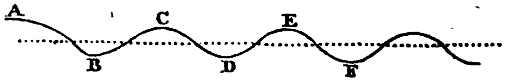
It is not to be understood however, that no wave of the sea can rise more than 6 feet above its natural level in open and deep water; for some vastly higher than these are formed in violent tempests in the great seas. These however are not to be accounted waves in their natural state, but as compound waves formed by the union of many others; for in these wide plains of water, when one wave is raised by the wind, and would elevate itself up to the exact height of 6 feet, and no more, the motion of the water is so great, and the succession of waves so quick, that while this is rising, it receives into it several other waves, each of which would have been at the same height with itself; these run into the first wave one after another, as it is rising; by which means its rise is continued much longer than it naturally would have been, and it becomes accumulated to an enormous size. A number of these complicated waves rising together, and being continued in a long succession by the continuation of the storm, make the waves so dangerous to ships, which the sailors in their phrase call mountains high.

Different waves do not disturb one another when they move in different directions. The reason is, that whatever figure the surface of the water has acquired by the motion of the waves, there may in that be an elevation and depression; as also such a motion as is required in the motion of a wave.

Waves are often produced by the motion of a tremulous body, which also expand themselves circularly, though the body goes and returns in a right line; for the water which is raised by the agitation, descending, forms a cavity, which is every where surrounded with a rising.

The Motion of the Waves, is a subject which makes an article in the Newtonian philosophy; that author having explained their motions, and calculated their velocity from mathematical principles, similar to the motion of a pendulum, and to the reciprocation of water in the two legs of a bent and inverted syphon or tube.

His proposition concerning such canal or tube is the 44th of the 2d book of his Principia, and is this: "If water ascend and descend alternately in the erected legs of a canal or pipe; and a pendulum be constructed, whose length between the point of suspension and the centre of oscillation, is equal to half the length of the water in the canal; then the water will ascend and descend in the same times in which the pendulum oscillates." The author hence infers, in prop. 45, that the velocity of waves is in the subduplicate ratio of their breadths; and in prop. 46, he proceeds "To find the velocity of waves," as follows: "Let a pendulum be constructed, whose length between the point of suspension and the centre of oscillation is equal to the breadth of the waves; and in the time that the pendulum will perform one single oscillation, the waves will advance forward nearly a space equal to their breadth. That which I call the breadth of the waves, is the transverse measure lying between the deepest part of the hollows, or between the tops of the ridges. Let ABCDEF represent the surface of stagnant water ascending and descending in successive waves; also let A, C, E, &c, be the tops of the waves; and B, D, F, &c, the intermediate hollows. Because the motion of the waves is carried on by the successive ascent and descent of the water, so that the parts of it, as A, C, E, &c, which are highest at one time, become lowest immediately after; and because the motive force, by which the highest parts descend and the lowest ascend, is the weight of the elevated water, that alternate ascent and descent will be analogous to the reciprocal motion of the water in the canal, and observe the same laws as to the times of its ascent and descent; and therefore (by prop. 44, above-mentioned) if the distances between the highest places of the waves A, C, E, and the lowest B, D, F, be equal to twice the length of any pendulum, the highest parts A, C, E, will become the lowest in the time of one oscillation, and in the time of another oscillation will ascend again. Therefore between the passage of each wave, the time of two oscillations will intervene; that is, the wave will describe its breadth in the time that the pendulum will oscillate twice; but a pendulum of 4 times that length, and which therefore is equal to the breadth of the waves, will just oscillate once in that time. *Q. E. I.*



"*Corol.* 1. Therefore waves, whose breadth is equal to 39⅛ inches, or 3 25/96 feet, will advance through a space equal to their breadth in one second of time; and therefore in one minute they will go over a space of 195⅝ feet; and in an hour a space of 11737 feet, nearly, or 2 miles and almost a quarter.

"*Corol.* 2. And the velocity of greater or less waves, will be augmented or diminished in the subduplicate ratio of their breadth.

"These things (Newton adds) are true on the supposition, that the parts of water ascend or descend in a right line; but in fact, that ascent and descent is rather performed in a circle; and therefore I propose the time defined by this proposition as only near the truth."

Stilling Waves *by means of Oil.* This curious property, though well known to the ancients, as appears from the writings of Pliny, was for many ages either quite unnoticed, or treated as fabulous by succeeding philosophers. Of late it has, by means of Dr. Franklin, again attracted the attention of the learned; though it appears, from some anecdotes, that seafaring people have always been acquainted with it. In Martin's description of the Western Islands of Scotland, we have the following passage: "The steward of Kilda, who lives in Pabbay, is accustomed, in time of a storm, to tie a bundle of puddings, made of the fat of sea-fowl, to the end of his cable, and let it fall into the sea behind his rudder. This, he says, hinders the waves from breaking, and calms the sea." Mr. Pennant, in his British Zoology, vol. iv, under the article Seal, takes notice, that when these animals are devouring a very oily fish, which they always do under water, the waves above are remarkably smooth; and by this mark the fishermen know where to find them. Sir Gilbert Lawson, who served long in the army at Gibraltar, assured Dr. Franklin, that the fishermen in that place are accustomed to pour a little oil on the sea, in order to still its motion, that they may be enabled to see the oysters lying at its bottom, which are there very large, and which they take up with a proper instrument. A similar practice obtains among fishermen in various other parts, and Dr. Franklin was informed by an old sea-captain, that the fishermen of Lisbon, when about to return into the river, if they saw too great a surf on the bar, would empty a bottle or two of oil into the sea, which would suppress the breakers, and allow them to pass freely.

The doctor having resolved in his mind all these pieces of information, became impatient to try the experiment himself. At last having an opportunity of observing a large pond very rough with the wind, he dropped a small quantity of oil into it. But having at first applied it on the lee-side, the oil was driven back again upon the shore. He then went to the windward side, and poured on about a tea-spoon-full of oil; this produced an instant calm over a space several yards square, which spread amazingly, and extended itself gradually till it came to the lee-side; making all that quarter of the pond, perhaps half an acre, as smooth as glass. This experiment was often repeated in different places, and always with success. Our author accounts for it in the following manner:

"There seems to be no natural repulsion between water and air, to keep them from coming into contact with each other. Hence we find a quantity of air in water; and if we extract it by means of the air pump, the same water again exposed to the air will soon imbibe an equal quantity.—Therefore air in motion, which is wind, in passing over the smooth surface of water, may rub as it were upon that surface, and raise it into wrinkles; which, if the wind continues, are the elements of future waves. The smallest wave once raised does not immediately subside and leave the neighbouring water quiet; but in subsiding raises nearly as much of the water next to it, the friction of the parts making little difference. Thus a stone dropped into a pool raises first a single wave round itself, and leaves it, by sinking to the bottom; but that first wave subsiding raises a second, the second a third, and so on in circles to a great extent.

"A small power continually operating, will produce a great action. A finger applied to a weighty suspended bell, can at first move it but little; if repeatedly applied, though with no greater strength, the motion increases till the bell swings to its utmost height, and with a force that cannot be resisted by the whole strength of the arm and body. Thus the small first raised waves being continually acted on by the wind, are, though the wind does not increase in strength, continually increased in magnitude, rising higher and extending their bases, so as to include a vast mass of water in each wave, which in its motion acts with great violence. But if there be a mutual repulsion between the particles of oil, and no attraction between oil and water, oil dropped on water will not be held together by adhesion to the spot whereon it falls; it will not be imbibed by the water; it will be at liberty to expand itself; and it will spread on a surface that, besides being smooth to the most perfect degree of polish, prevents, perhaps by repelling the oil, all immediate contact, keeping it at a minute distance from itself; and the expansion will continue, till the mutual repulsion between the particles of the oil is weakened and reduced to nothing by their distance.

"Now I imagine that the wind blowing over water thus covered with a film of oil cannot easily catch upon it, so as to raise the first wrinkles, but slides over it, and leaves it smooth as it finds it. It moves the oil a little indeed, which being between it and the water, serves to slide with, and prevents friction, as oil does between those parts of a machine that would otherwise rub hard together. Hence the oil dropped on the windward side of a pond proceeds gradually to leeward, as may be seen by the smoothness it carries with it quite to the opposite side. For the wind being thus prevented from raising the first wrinkles that I call the elements of waves, cannot produce waves, which are to be made by continually acting on and enlarging those elements; and thus the whole pond is calmed.

"Totally therefore we might suppress the waves in any required place, if we could come at the windward place where they take their rise. This in the ocean can seldom if ever be done. But perhaps something may be done on particular occasions to moderate the violence of the waves when we are in the midst of them, and prevent their breaking when that would be inconvenient. For when the wind blows fresh, there are continually rising on the back of every great wave a number of small ones, which roughen its surface, and give the wind hold, as it were, to push it with greater force. This hold is diminished by preventing the generation of those small ones. And possibly too, when a wave's surface is oiled, the wind, in passing over it, may rather in some degree press it down, and contribute to prevent its rising again, instead of promoting it.

"This, as mere conjecture, would have little weight, if the apparent effects of pouring oil into the midst of waves were not considerable, and as yet not otherwise accounted for.

"When the wind blows so fresh, as that the waves are not sufficiently quick in obeying its impulse, their tops, being thinner and lighter, are pushed forward, broken, and turned over in a white foam. Common waves lift a vessel without entering it; but these, when large, sometimes break above and pour over it, doing great damage.

"That this effect might in any degree be prevented, or the height and violence of waves in the sea moderated, we had no certain account; Pliny's authority for the practice of seamen in his time being slighted. But discoursing lately on this subject with his excellency Count Bentinck of Holland, his son the honourable captain Bentinck, and the learned professor Allemand (to all whom I showed the experiment of smoothing in a windy day the large piece of water at the head of the green park), a letter was mentioned which had been received by the count from Batavia, relative to the saving of a Dutch ship in a storm by pouring oil into the sea."

WAY *of a Ship*, is sometimes used for her wake or track. But more commonly the term is understood of the course or progress which she makes on the water under sail: thus, when she begins her motion, she is said to be under way; when that motion increases, she is said to have fresh way through the water; when she goes apace, they say she has a good way; and the account of her rate of sailing by the log, they call, keeping an account of her way. And because most ships are apt to fall a little to the leeward of their true course, it is customary, in casting up the log-board, to allow something for her leeward way, or leeway. Hence also a ship is said to have headway, and stern-way.

WAY-WISER, an instrument for measuring the road, or distance travelled; called also PERAMBULATOR, and PEDOMETER. See these two articles.

Mr. Lovell Edgworth communicated to the Society of

Arts, &c, an account of a way-wiser of his invention; for which he obtained a silver medal. This machine consists of a nave, formed of two round flat pieces of wood, 1 inch thick and 8 inches in diameter. In each of the pieces there are cut eleven grooves, $\frac{5}{8}$ of an inch wide, and $\frac{3}{8}$ deep; and when the two pieces are screwed together, they enclose eleven spokes, forming a wheel of spokes, without a rim; the circumference of the wheel is exactly one pole; and the instrument may be easily taken to pieces, and put up in a small compass. On each of the spokes there is driven a ferril, to prevent them from wearing out; and in the centre of the nave, there is a square hole to receive an axle. Into this hole is inserted an iron or brass rod, which has the thread of a very fine screw worked upon it from one end to the other; upon this screw hangs a nut which, as the rod turns round with the wheel, advances towards the nave of the wheel or recedes from it. The nut does this, because it is prevented from turning round with the axle, by having its centre of gravity placed at some distance below the rod, so as always to hang perpendicularly like a plummet. Two sides of this screw are filed away flat, and have figures engraved on them, to show by the progressive motion of the nut, how many circumvolutions of the wheel and its axle have been made: on one side the divisions of miles, furlongs, and poles are in a direct order, and on the other side the same divisions are placed in a retrograde order.

If the person who uses this machine places it at his right hand side, holding the axle loosely in his hands, and walks forward, the wheel will revolve, and the nut advance from the extremity of the rod towards the nave of the wheel. When two miles have been measured, it will have come close to the wheel. But to continue this measurement, nothing more is necessary than to place the wheel at the left hand of the operator; and the nut will, as he continues the course, recede from the axle-tree, till another space of two miles is measured.

It appears, from the construction of this machine, that it operates like circular compasses; and does not, like the common wheel way-wiser, measure the surface of every stone and molehill, &c, but passes over most of the obstacles it meets with, and measures the chords only, instead of the arcs of any curved surfaces upon which it rolls.

WEATHER, denotes the state or disposition of the atmosphere, with regard to heat and cold, drought and moisture, fair or foul, wind, rain, hail, frost, snow, fog, &c. See ATMOSPHERE, HAIL, HEAT, FROST, RAIN, &c.

There does not appear in all philosophy any circumstance of more immediate concern to us, than the state of the weather; as it is in, and by means of the atmosphere, that all plants are nourished, and all animals live and breathe; and as any alterations in the density, heat, purity, &c, of that, must necessarily be attended with proportionable ones in the state of these.

The great, but regular alterations, a little change of weather produces in many parts of inanimate matter, every person knows, from the common instance of barometers, thermometers, hygrometers, &c; and it is owing partly to our inattention, and partly to our unequal and intemperate course of life, that we also, like many other animals, do not feel as great and as regular ones in the tubes, chords, and fibres of our own bodies.

To establish a proper theory of the weather, it would be necessary to have registers carefully kept in divers parts of the globe, for a long series of years; from which we might be enabled to determine the directions, breadth, and bounds of the winds, and of the weather they bring with them; with the correspondence between the weather of divers places, and the difference between one kind and another at the same place. We might thus probably in time learn to foretell many great emergencies; as, extraordinary heats, rains, frosts, droughts, dearths, and even plagues, and other epidemical diseases, &c.

It is however but very few, and partial registers or accounts of the weather, that have been kept. The Royal Society, the French Academy, and a few particular philosophers, have at times kept such registers as their fancies have dictated, but never a regular and correspondent series in many different places, at the same time, followed with particular comparisons and deductions from the whole, &c. The most of what has been done in this way, is as follows: The volumes of the Philosophical Transactions from year to year; the same, for instructions and examples pertaining to the subject, vol. 65, part 2, art. 16; Eras. Bartholin has observations of the weather for every day in the year 1671; Mr. W. Merle made the like at Oxford, for 7 years; Dr. Plot did the same at the same place, for the year 1684; Mr. Hillier, at Cape Corse, for the years 1686 and 1687; Mr. Hunt and others at Gresham College, for the years 1695 and 1696; Dr. Derham at Upminster in Essex, for the years 1691, 1692, 1697, 1698, 1699, 1703, 1704, 1705; Mr. Townley, in Lancashire, in 1697, 1698; Mr. Cunningham, at Emin in China, for the years 1698, 1699, 1700, 1701; Mr. Locke, at Oats in Essex, 1692; Dr. Scheuchzer, at Zurich, 1708; and Dr. Tilly, at Pisa, the same year; professor Toaldo, at Padua, for many years; Mr. T. Barker, at Lyndon, in Rutland, for many years in the Philos. Trans.; Mr. Dalton for Kendal, and Mr. Crosthwaite for Keswick, in the years 1788, 1789, 1790, 1791, 1792, &c; and several others. The register now kept, for many years, in the Philos. Trans. contains an account, two times every day, of the thermometer, barometer, hygrometer, quantity of rain, direction and strength of the wind, and appearance of the atmosphere, as to fair, cloudy, foggy, rainy, &c. And if similar registers were kept in many other parts of the globe, and printed in such-like public transactions, they might readily be consulted, and a proper use made of them, for establishing this science on the true basis of experiment.

From many experiments, some general observations have been made, as follow: That barometers generally rise and fall together, even at very distant places, and a consequent conformity and similarity of weather; but this is the more uniformly so, as the places are nearer together, as might be expected. That the variations of the barometer are greater, as the places are nearer the pole; thus, for instance, the mercury at London has a greater range by 2 or 3 lines than at Paris; and at Paris, a greater than at Zurich; and at some places near the equator, there is scarcely any variation at all. That the rain in Switzerland and Italy is much greater in quantity, for the whole year, than in Essex; and yet the rains are more frequent, or there are more rainy days, in Essex, than at either of those places. That cold contributes greatly to rain; and this apparently by condensing the suspended vapours, and so causing them to descend: thus, very cold months, or seasons, are commonly followed immediately by very rainy ones; and cold summers are always wet ones. That high ridges of mountains, as the Alps, and the snows with which they are covered, not only affect the neighbouring places by the colds, rains, vapours, &c, which they produce; but even distant countries, as England, often partake of their effects. See a collection of ingenious and meteorological observations and conjectures, by Dr. Franklin, in his Experiments, &c, pa. 182, &c. Also a Meteorological Register kept at Mansfield Woodhouse, from 1784 to 1794, Nottingham 1795, 8vo; and Kirwin's ingenious papers on this subject in the Transactions of the Irish Academy, vol. 5. See also the articles EVAPORATION, RAIN, and WIND.

Other Prognostics and Observations, are as follow:

That a thick dark sky, lasting for some time, without either sun or rain, always becomes first fair, and then foul, i. e. it changes to a fair clear sky, before it turns to rain. And the reason is obvious: the atmosphere is replete with vapours which, though sufficient to reflect and intercept the sun's rays from us, yet want density to descend; and while the vapours continue in the same state, the weather will do so too: accordingly, such weather is commonly attended with moderate warmth, and with little or no wind to disturb the vapours, and a heavy atmosphere to sustain them; the barometer being commonly high; but when the cold approaches, and by condensing the vapours drives them into clouds or drops, then way is made for the sun beams; till the same vapours, by farther condensation, be formed into rain, and fall down in drops.

That a change in the warmth of the weather is followed by a change in the wind. Thus, the northerly and southerly winds, though commonly accounted the causes of cold and warm weather, are really the effects of the cold or warmth of the atmosphere; of which Dr. Derham assures us he had so many confirmations, that he makes no doubt of the fact. Thus, it is common to observe a warm southerly wind suddenly changed to the north, by the fall of snow or hail; or to have the wind, in a cold frosty morning, north, when the sun has well warmed the air, shift towards the south; and again turn northerly or easterly in the cold evening.

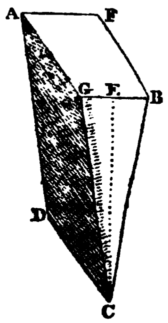
That most vegetables expand their flowers and down in sunshiny weather: and towards the evening, and against rain, close them again; especially at the beginning of their flowering, when their seeds are tender and sensible. This is visible enough in the down of dandelion, and other downs; and eminently so in the flowers of pimpernel; the opening and shutting of which make what is called the countryman's weather-wiser, by which he foretels the weather of the following day. The rule is, when the flowers are close shut up, it betokens rain, and foul weather; but when they are spread abroad, fair weather.

The stalk of trefoil, lord Bacon observes, swells against rain, and grows more upright; and the like may be observed, though less sensibly, in the stalks of most other plants. He adds, that in the stubble fields there is found a small red flower, called by the country people pimpernel, which, opening in a morning, is a sure indication of a fine day.

It is very conceivable that vegetables should be affected by the same causes as the weather, as they may be considered as so many hygrometers and thermometers, consisting of an infinite number of tracheæ, or air-vessels; by which they have an immediate communication with the air, and partake of its moisture, heat, &c.

Hence it is, that every kind of wood, even the hardest and most solid, swells in moist weather; the vapours easily insinuating themselves into the pores, especially of the lighter and drier kinds. And hence is derived a very extraordinary use of wood, viz, for breaking rocks or milstones. The method at the quarries is this: Having cut a rock into the form of a cylinder, the workmen divide it into several thinner cylinders, of horizontal courses, by making holes at proper distances round the great one; into these holes they drive pieces of sallow wood, dried in an oven; these in moist weather, imbibing the humidity from the air, swell, and, acting like wedges, they break or cleave the rock into several flat stones. And, in like manner, to separate large blocks of stone in the quarry, they wedge such pieces of wood into holes, forming the block into the intended shape, and then pour water upon the wedges, to produce the effect more immediately.

WEATHER-*Glasses*, are instruments contrived to show the state of the atmosphere, as to heat, cold, moisture, weight, &c; and so to measure the changes that take place in those respects; by which means we are enabled to predict the alteration of weather, as to rain, wind, frost, &c. —Under the class of weather-glasses, are comprehended barometers, thermometers, hygrometers, manometers, and anemometers.



WEDGE, in Geometry, is a solid, having a rectangular base, and two of its opposite sides ending in an acies or edge. Thus, AB is the rectangular base; and DC the edge; a perpendicular CE, from the edge to the base, is the height of the wedge. When the length of the edge DC is equal to the length of the base BF, which is the most common form of it, the wedge is equal to half a rectangular prism of the same base AB and height EC; or it is then a whole triangular prism, having the triangle BCG for its base, and AG or DC for its height. If the edge be more or less than AG, its solid content will also be more or less. But, in all cases of the wedge, the following is a general rule for finding the content of it, viz,

To twice the length of the base add the length of the edge, multiply the sum by the breadth of the base, and the product by the height of the wedge; then $\frac{1}{6}$ of the last product will be the solid content.

That is, $(2AG + DC) \times AF \times \frac{1}{6} EC =$ the content. See this rule demonstrated, and illustrated with examples, in my Mensuration, pa. 141, 4th edition.

WEDGE, in Mechanics, one of the five mechanical powers, or simple engines; being a geometrical wedge, or very acute triangular prism, applied to the splitting of wood, or rocks, or raising great weights.

This machine is made of iron, or some other hard matter, and applied to the raising of vast weights, or separating very firm blocks of wood or stone, by introducing the thin edge of the wedge, and driving it in by blows struck upon the back by hammers or mallets.

The wedge is the most powerful of all the simple machines, having an almost unlimited and double advantage over all the other simple mechanical powers; both as it may be made vastly thin, in proportion to its height; in which consists its own natural power; and as it is urged by the force of percussion, or of smart blows, which is a force incomparably greater than any mere dead weight or pressure, such as is employed on other machines. And accordingly we find it produces effects vastly superior to those of any other power whatever; such as the splitting and raising the largest and hardest rocks; or even the raising and lifting the largest ship, by driving a wedge under it; which a man can do by the blow of a mallet: and thus, the small blow of a hammer, on the back of a wedge, appears to be incomparably greater than any mere pressure, and will overcome it.

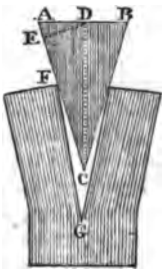
To the wedge may be referred all edge-tools, and instruments that have a sharp point, in order to cut, cleave, slit, split, chop, pierce, bore, or the like; as knives, hatchets, swords, bodkins, &c.

In the wedge, the friction against the sides is very great, at least equal to the force to be overcome; because the wedge retains any position to which it is driven; and therefore the resistance is at least doubled by the friction.

Authors have been of various opinions concerning the principle from which the wedge derives its power. Aristotle considers it as two levers of the first kind, inclined towards each other, and acting in opposite directions. Guido Ubaldi, Mersenne, &c, will have them to be levers of the second kind. But Delanis shows, that the wedge cannot be reduced to any lever at all. Others refer the wedge to the inclined plane. And others again, with Destair, will hardly allow the wedge to have any force at all in itself; ascribing much the greatest part to the mallet which drives it.

The doctrine of the force of the wedge, according to some writers, is contained in this proposition: "If a power directly applied to the head of a wedge, be to the resistance to be overcome, as the breadth of the back GB, is to the height EC; then the power will be equal to the resistance; and if increased, it will overcome it."

But Desaguliers has proved that, when the resistance acts perpendicularly against the sides of the wedge, the power is to the whole resistance, as the thickness of the back is to the length of both the sides taken together. And the same proportion is adopted by Wallis (Op. Math. vol. 1, pa. 1016), Keill (Intr. ad Ver. Phys.), Gravesande (Elem. Math. Lib. 1, cap. 14), and by almost all the modern mathematicians. Gravesande indeed distinguishes the mode in which the wedge acts, into two cases, one in which the parts of a block of wood, &c, are separated farther than the edge has penetrated to, and the other in which they have not separated farther: In his Scholium de Ligno findendo (ubi supra), he observes, that when the parts of the wood are separated before the wedge, the equilibrium will be when the force by which it is pushed in, is to the resistance of the wood, as the line DE drawn from the middle of the base to the side of the wedge but perpendicular to the separated side of the wood FG continued, is to the height of the wedge DC; but when the parts of the wood are separated no farther than the wedge is driven in, the equilibrium will be, when the power is to the resistance, as the half base AD, is to its side AC.



Mr. Ferguson, in estimating the proportion of equilibrium in the two cases last mentioned by Gravesande, agrees with this author, and other modern philosophers, in the latter case; but in the former he contends, that when the wood cleaves to any distance before the wedge, as it generally does, then the power impelling the wedge, will be to the resistance of the wood, as half its thickness, is to the length of either side of the cleft, estimated from the top or acting part of the wedge: for, supposing the wedge to be lengthened down to the bottom of the cleft, the power will be to the resistance, as half the thickness of the wedge is to the length of either of its sides. See Ferguson's Lect. pa. 40, &c, 4to. See also Desagu. Exp. Phil. vol. 1, pa. 107; and Ludlam's Essay on the Power of the Wedge, printed in 1770; &c.

The generally acknowledged property of the wedge, and the simplest way of demonstrating it, may be the following: When a wedge is kept in equilibrio, the power acting against the back, is to the force acting perpendicularly against either side, as the breadth of the back AB, is to the length of the side AC or BC.—*Demonstra.* For any three forces which sustain one another in equilibrio, are as the corresponding sides of a triangle that are drawn perpendicular to the directions in which the forces act. But AB is perpendicular to the force acting on the back, to drive the wedge forward; and the sides AC and BC are perpendicular to the forces acting on them; therefore the three forces are as the said lines AB, AC, BC.

Hence, the thinner a wedge is, the greater is its effect, in splitting any body, or in overcoming any resistance against the side of the wedge.

WEDNESDAY, the 4th day of the week, formerly consecrated by the inhabitants of the northern nations to Woden or Oden; who, being reputed the author of magic, and inventor of all the arts, was thought to answer to the Mercury of the Greeks and Romans, in honour of whom the same day was by them called dies Mercurii; and hence it is denoted by astronomers by the character of Mercury ☿.

WEEK, a division of time that comprises seven days.

The origin of this division of weeks, or of computing time by sevenths, is much controverted. It has often been thought to have taken its rise from the four quarters or intervals of the moon, between her changes of phases, which, being about 7 days distant, gave occasion to the division: but others more probably from the seven planets.

Be this however as it may, the division is certainly very ancient. The Syrians, Egyptians, and most of the oriental nations, appear to have used it from the earliest ages: though it did not get footing in the west till introduced by Christianity. The Romans reckoned their days not by sevenths, but by ninths; and the ancient Greeks by decads, or tenths; in imitation of which the new French calendar seems to have been framed.

The Jews divided their time by weeks, of 7 days each, as prescribed by the law of Moses; in which they were appointed to work 6 days, and to rest the 7th, in commemoration of the creation, which being effected in 6 days, God rested on the 7th.

Some authors will even have the use of weeks, among the other eastern nations, to have proceeded from the Jews; but with little appearance of probability. It is with better reason that others suppose the use of weeks, among the eastern nations, to be a remnant of the tradition of the creation, which they had still retained with divers others; or else from the number of the planets.

The Jews denominated the days of the week, the first, second, third, fourth, and fifth; and the sixth day they named the preparation of the sabbath, or 7th day, which answered to our Saturday. And the like method is still kept up by the christian Arabs, Persians, Ethiopians, &c.

The ancient heathens denominated the days of the week from the seven planets; which names are still mostly retained among the christians of the west: thus, the first day was called dies solis, sun-day; the 2d dies lunæ, moon-day; &c; a practice the more natural on Dion's principle, that the Egyptians took the division of the week itself from the seven planets.

In fact, the true reason for these denominations seems to be founded in astrology. For the astrologers distributing the government and direction of all the hours in the week among the seven planets, ♄ ♃ ♂ ☉ ♀ ☿ ☽, so as that the government of the first hour of the first day fell to Saturn, that of the second day to Jupiter, &c, they gave each day the name of the planet which, according to their doctrine, presided over the first hour of it, and that according to the order above stated. So that the order of the planets in the week bears little relation to that in which they follow in the heavens; the former being grounded on an imaginary power each planet has, in its turn, on the first hour of each day.

Dion Cassius gives another reason for the denomination, drawn from the celestial harmony. For it being observed, that the harmony of the diatessaron, which consists in the ratio of 4 to 3, is of great force and effect in music; it was thought proper to proceed directly from Saturn to the Sun; because, according to the old system, there are three planets between Saturn and the Sun, and 4 from the Sun to the Moon.

Our Saxon ancestors, before their conversion to Christianity, named the seven days of the week from the sun and moon and some of their deified heroes, to whom they were peculiarly consecrated, and representing the ancient gods or planets; which names we have received and still retain: Thus, Sunday was devoted to the Sun; Monday to the Moon; Tuesday to Tuisco; Wednesday to Woden; Thursday to Thor, the thunderer; Friday to Friga or Friya of Fræa, the wife of Thor; and Saturday to Seater. And nearly according to this order, the modern astronomers express the days of the week by the seven planets as annexed.

☉	Sunday
☾	Monday
♂	Tuesday
☿	Wednesday
♃	Thursday
♀	Friday
♄	Saturday

In the same order and number also do these obtain in the Hindu days of the week. See Kindersley's Specimens of Hindu Literature, just published, 8vo.

WEIDLER (John Frederick) was professor of mathematics at Wittemberg. Besides a number of communications to the Royal Society, contained in vols. 36, 38, 39, 40, 41, of the Philos. Trans. he was author of several separate works: as, 1. Institutiones Mathematicæ, in 8vo, 1725. This is a very thick volume, and contains a general, though concise course, of all the mathematical sciences. 2. Observationes Meteorologicæ et Astronomicæ, 1729. 3. Historia Astronomica, in 4to, 1741.

WEIGH, Way, or Wey, a weight of cheese, wool, &c, containing 256 pounds avoirdupois. Of corn, the weigh contains 40 bushels; of barley or malt, 6 quarters.

WEIGHT, or *Gravity*, in Physics, a quality in natural bodies, by which they tend towards the centre of the earth: and it is equal to the effort necessary to prevent a body from falling. See Gravity.

Weight, like gravity, may be distinguished into absolute specific, and relative. Newton demonstrates, 1. That the weights of all bodies, at equal distances from the centre of the earth, are directly proportional to the quantities of matter that each contains: Whence it follows, that the weights of bodies have no dependence on their shapes or textures; and that all spaces are not equally full of matter.

2. On different parts of the earth's surface, the weight of the same body is different; owing to the spheroidal figure of the earth, which causes the body on the surface to be nearer the centre in going from the equator towards the poles: and the increase in the weight is nearly in proportion to the versed sine of double the latitude; or, which is the same thing, to the square of the right sine of the latitude: the weight at the equator to that at the pole, being as 229 to 230; or the whole increase of weight from the equator to the pole, is the 229th part of the former.

3. That the weights of the same body, at different distances above the earth, are inversely as the squares of the distances from the centre. So that, a body at the distance of the moon, which is 60 semidiameters from the earth's centre, would weigh only the 3600th part of what it weighs at the earth's surface.

4. That at different distances within the earth, or below the surface, the weights of the same body are directly as the distances from the earth's centre: so that, at half way toward the centre, a body would weigh but half as much, and at the very centre it would have no weight at all.

5. A body immersed in a fluid, which is specifically lighter than itself, loses so much of its weight, as is equal to the weight of a quantity of the fluid of the same bulk with itself. Hence, a body loses more of its weight in a heavier fluid than in a lighter one: and therefore it weighs more in a lighter fluid than in a heavier one.

The weight of a cubic foot of pure water, is 1000 ounces, or 62½ pounds, avoirdupois. And the weights of the cubic foot of other bodies, are as set down under the article *Specific* Gravity.

In the Philos. Trans. (No. 458, pa. 457 &c) is contained some account of the analogy between English weights and measures, by Mr. Barlow. He states, that anciently the cubic foot of water was assumed as a general standard for liquids. This cubic foot, of 62½ lb, multiplied by 32, gives 2000, the weight of a ton: and hence 8 cubic feet of water made a hogshead, and 4 hogsheads a tun, or ton, in capacity and denomination, as well as weight.

Dry measures were raised on the same model. A bushel of wheat, assumed as a general standard for all sorts of grain, also weighed 62½lb. Eight of these bushels make a quarter, and 4 quarters, or 32 bushels, a ton weight. Coals were sold by the chaldron, supposed to weigh a ton, or 2000 pounds; though in reality it weighs perhaps upwards of 3000 pounds.

Hence a ton in weight is the common standard for liquids, wheat, and coals. Had this analogy been adhered to, the confusion now complained of would have been avoided.—It may reasonably be supposed that corn and other commodities, both dry and liquid, were first sold by weight; and that measures, for convenience, were afterwards introduced, as bearing some analogy to the weights before used.

Weight, *Pondus*, in Mechanics, denotes any thing to be raised, sustained, or moved by a machine; or any thing that in any manner resists the motion to be produced. In all machines, there is a natural and fixed ratio between the weight and the moving power; and if they be such as to balance each other in equilibrio, and then the machine be put in motion by any other force; the weight and power will always be reciprocally as the velocities of them, or of their centres of gravity; or their momentums will be equal, that is, the product of the weight multiplied by its velocity, will be equal to the product of the power multiplied by its velocity.

Weight, in Commerce, denotes a body of a known weight, appointed to be put into a balance against other bodies, whose weight is required to be known. These weights are usually of lead, iron, or brass; though in several parts of the East Indies common flints are used; and in some places a sort of little beans. The diversity of weights, in all nations, and at all times, makes one of the most perplexing circumstances in commerce, &c. And it would be a very great convenience if all nations could agree on a universal standard, and system, both of weights and measures.

Weights may be distinguished into ancient and modern, foreign and domestic.

Modern WEIGHTS, *used in the several parts of Europe, and the Levant.*

English **Weights**. By the 27th chapter of Magna Charta, the weights are to be the same all over England: but for different commodities there are two different kinds, viz, troy weight, and avoirdupois weight.

The origin from which both of these are raised, is the grain of wheat, gathered in the middle of the ear:

32 of these, well dried, made one pennyweight,
20 pennyweights - - - one ounce, and
12 ounces - - - - - one pound troy;

by Stat. 51 Hen. III; 31 Edw. I; 12 Hen. VII.

A learned writer has shown that, by the laws of assize, from William the Conqueror to the reign of Henry VII, the legal pound weight contained a pound of 12 ounces, raised from 32 grains of wheat; and the legal gallon measure contained 8 of those pounds of wheat, 8 gallons making the bushel, and 8 bushels the quarter.

Henry VII. altered the old English weight, and introduced the troy pound in its stead, being 3 quarters of an ounce only heavier than the old Saxon pound, or 1-16th heavier. The first statute that directs the use of the avoirdupois weight, is that of 24 Henry VIII; and the particular use to which this weight is thus directed, is simply for weighing butcher's meat in the market; though it is now used for weighing all kinds of coarse and large articles. This pound contains 7000 troy grains; while the troy pound itself contains only 5760 grains, and the old Saxon pound weight but 5400 grains. Philos. Trans. vol. 65, art. 3.

Hence there are now in common use in England, two different weights, viz, troy weight, and avoirdupois weight, the former being employed in weighing such fine articles as jewels, gold, silver, silk, liquors, &c: and the latter for coarse and heavy articles, as bread, corn, flesh, butter, cheese, tallow, pitch, tar, iron, copper, tin, &c, and all grocery wares. And Mr. Ward supposes that it was brought into use from this circumstance, viz, as it was customary to allow larger weight, of such coarse articles, than the law had expressly enjoined, and this he observes happened to be a 6th part more. Apothecaries buy their drugs by avoirdupois weight, but they compound them by troy weight, though under some little variation of name and divisions.

The troy or trone pound weight in Scotland, which by statute is to be the same as the French pound, is commonly supposed equal to 15¾ English troy ounces, or 7560 grains; but by a mean of the standards kept by the dean of gild of Edinburgh, it weighs 7599$\frac{1}{16}$ or 7600 grains nearly.

The following tables show the divisions of the troy and avoirdupois weights.

Table of Troy Weight, as used,

1. By the Goldsmiths, &c.

Grains Pennywt.
24 = 1 dwt.
480 = 20 = 1 oz.
5760 = 240 = 12 = 1 lb.

2. By the Apothecaries.

Grains Scruples
20 = 1 ℈
60 = 3 = 1 Dram ʒ
480 = 24 = 8 = 1 Ounce ℥
5760 = 288 = 96 = 12 = 1 lb.

Table of Avoirdupois Weight.

Drams Ounces
16 = 1
256 = 16 = 1 lb.
7168 = 448 = 28 = 1 quar.
28672 = 1792 = 112 = 4 = 1 cwt.
573440 = 35840 = 2240 = 80 = 20 = 1 ton.

Mr. Ferguson (Lect. on Mech. pa. 100, 4to) gives the following comparison between troy and avoirdupois weight.

175 troy pounds are equal to 144 avoirdup. pounds.
175 troy ounces are equal to 192 avoirdup. ounces.
1 troy pound contains 5760 grains.
1 avoirdupois pound contains 7000 grains.
1 avoirdupois ounce contains 437½ grains.
1 avoirdupois dram contains 27·34375 grains.
1 troy pound contains 13 oz. 2·651428576 drams avoirdupois
1 avoirdup. lb. contains 1 lb. 2 oz. 11 dwts 16 gr. troy

The moneyers, jewellers, &c, have a particular class of weights, for gold and precious stones, viz, carat and grain, the carat being 24 grains; and for silver, the pennyweight and grain. The first have also a peculiar subdivision of the troy grain: thus, dividing

the grain into 20 mites
the mite into 24 droits
the droit into 20 periots
the periot into 24 blanks.

The dealers in wool have likewise a particular set of weights; viz, the sack, weigh, tod, stone, and clove, the proportions of which are as below: viz,

the sack containing 2 weighs
the weigh - - 6½ tods
the tod - - - 2 stones
the stone - - 2 cloves
the clove - - 7 pounds.

Also 12 sacks make a last or 4368 pounds.

Farther,

56 lb of old hay, or 60 lb new hay, make a truss.

40 lb of straw make a truss.

36 trusses make a load, of hay or straw.

14 lb make a stone.

5 lb of glass a stone.

French WEIGHTS. The common or [illegible]ris old pound weight, is to the English troy pound, as 21 to 16, and to the avoirdupois pound as 27 to 25; it therefore contains 7560 troy grains; and it is divided into 16 ounces like the pound avoirdupois, but more particularly thus: the pound into 2 marcs; the marc into 8 ounces; the ounce into 8 gros, or drams; the gross or dram into 3 deniers, Paris scruples or pennyweights; and the pennyweight into 24 grains; the grain being an equivalent to a grain of wheat. So that the Paris ounce contains 472½ troy grains, and therefore it is to the English troy ounce as 63 to 64. But in several of the French provinces, the pound is of other different weights. A quintal is equal to 100 pounds.

Table of old French Weights with the Equivalent in Troy and Avoirdupois Weight.

French.	Troy.			Avoirdupois.		
	oz.	dwt.	grs.	oz.	drs.	hds.
1 Pound	15	14	20	17	4	34
1 Marc	7	17	10	8	10	17
1 Ounce		19	16¼	1	1	27
1 Gros		2	11		2	16
1 Scruple			19⅔			72
1 Grain			⅘			03

The new French weights are those which have for their measure a unit, called a gramme. This gramme is the absolute weight of the cube of the centimetre, or 100th part of the metre, or 3⅖ English feet, of distilled water, taken at its maximum of density; and answers to 19 Paris grains, or to 15⅘ English grains. The kilogramme, or the weight of a 1000 grammes, is equal to about 32⅛ troy ounces. See the article MEASURES.

The weights above enumerated under the two articles of English and French weights, are the same as are used throughout the greatest part of Europe; only under somewhat different names, divisions, and proportions. And besides, particular nations have also certain weights peculiar to themselves, of too little consequence here to be enumerated. But to show the proportion of these several weights to one another, there may be here added a reduction of the divers pounds in use throughout Europe, by which the other weights are estimated, to one standard pound, viz, the old pound of Amsterdam, Paris, and Bourdeaux; as they were accurately calculated by M. Ricard, and published in the new edition of his Traité de Commerce in 1722.

Proportion of the WEIGHTS *of the chief Cities in Europe, to that of Amsterdam, or the old French Pound.*

100 pounds of Amsterdam are equal to

108 lbs of	Alicant	95¼ lbs of	Bergen, Norw.
105	Antwerp	111	Bern
120	Archangel, or 3 poedes	100	Besançon
		100	Bilboa
105	Arschot	105	Bois le Duc
120	Avignon	151	Bologna
98	Basil	100	Bourdeaux
100	Bayonne	104	Bourg en Bresse
166	Bergamo	103	Bremen
97	Berg. op Zoom	125	Breslaw

WEIGHTS *continued.*

100 pounds of Amsterdam are equal to

105 lbs of	Bruges	114 lbs of	Madrid
105	Brussels	105	Malines
105	Cadiz	123½	Marseilles
105	Cologne	154	Messina
107½	Copenhagen	168	Milan
87	Constantinople	120	Montpelier
113½	Dantzic	125	Muscovy
100	Dort	100	Nantes
97	Dublin	100	Nancy
97	Edinburgh	169	Naples
143	Florence	98	Nuremberg
98	Franckfort, sur Maine	100	Paris
		112½	Revel
105	Gaunt	109	Riga
89	Geneva	100	Rochel
163	Genoa	146	Rome
102	Hamburg	100	Rotterdam
125	Kouingsberg	96	Rouen
105	Leipsic	100	S. Malo
106	Leyden	100	S. Sebastian
143	Leghorn	158⅐	Saragosa
105½	Liege	100	Seville
106	Lisbon	114	Smyrna
114	Lisle	110	Stetin
109	London, avoirdupois	81	Stockholm
		118	Tholouse
105	Louvain	151	Turin
105	Lubeck	158½	Valencia
141½	Lucca	182	Venice
116	Lyons		

Ancient WEIGHTS.

1. The weights of the ancient Jews, reduced to the English troy weights, will stand as below:

			lb	oz	dwt	gr
Shekel	-	-	0	0	9	2$\frac{4}{7}$
Manch	-	-	2	3	6	10$\frac{2}{7}$
Talent	-	-	113	10	1	10$\frac{2}{7}$

2. Grecian and Roman weights, reduced to English troy weight, are as in the following table:

			lb	oz	dwt	gr
Lentes	-	-	0	0	0	0$\frac{35}{112}$
Siliquæ	-	-	0	0	0	3$\frac{1}{28}$
Obolus	-	-	0	0	0	9$\frac{3}{28}$
Scriptulum		-	0	0	0	18$\frac{3}{14}$
Drachma	-	-	0	0	2	6$\frac{5}{14}$
Sextula	-	-	0	0	3	0$\frac{6}{7}$
Sicilicus	-	-	0	0	4	13$\frac{2}{7}$
Duella	-	-	0	0	6	1$\frac{5}{7}$
Uncia	-	-	0	0	18	5$\frac{1}{7}$
Libra	-	-	0	10	18	13$\frac{5}{7}$

The Roman ounce is the English avoirdupois ounce, which they divide into 7 denarii, as well as 8 drachms; and as they reckoned their denarius equal to the Attic drachm, this will make the Attic weights one-eighth heavier than the correspondent Roman weights. Arbuth.

Regulation of WEIGHTS *and Measures.*—This is a branch of the king's prerogative. For the public convenience, these ought to be universally the same throughout the nation, the better to reduce the prices of articles to equivalent values. But as weight and measure are things in their nature arbitrary and uncertain, it is necessary that they be reduced to some fixed rule or standard. It is however impossible to fix such a standard by any written law or oral proclamation; as no person can, by words only, give to another an adequate idea of a pound weight, or foot-rule. It is therefore expedient to have recourse to some visible, palpable, material standard; by forming a comparison with which all weights and measures may be reduced to one uniform size. Such a standard was anciently kept at Winchester: and we find in the laws of king Edgar, near a century before the conquest, an injunction that that measure should be observed throughout the realm.

Most nations have regulated the standard of measures of length from some parts of the human body; as the palm, the hand, the span, the foot, the cubit, the ell (*ulna* or arm), the pace, and the fathom. But as these are of different dimensions in men of different proportions, ancient historians inform us, that a new standard of length was fixed by our king Henry the first; who commanded that the ulna or ancient ell, which answers to the modern yard, should be made of the exact length of his own arm.

A standard of long measure being once obtained, all others are easily derived from it; those of greater length by multiplying that original standard, those of less by dividing it. Thus, by the statute called compositio ulnarum et perticarum, 5½ yards make a perch; and the yard is subdivided into 3 feet, and each foot into 12 inches; which inches will be each of the length of 3 barley-corns. But some, on the contrary, derive all measures, by composition, from the barley-corn.

Superficial measures are derived by squaring those of length; and measures of capacity by cubing them.

The standard of weights was originally taken from grains or corns of wheat, whence our lowest denomination of weights is still called a grain; 32 of which are directed, by the statute called compositio mensurarum, to compose a pennyweight, 20 of which make an ounce, and 12 ounces a pound, &c.

Under king Richard the first it was ordained, that there should be only one weight and one measure throughout the nation, and that the custody of the assize or standard of weights and measures, should be committed to certain persons in every city and borough; whence the ancient office of the king's ulnager seems to have been derived. These original standards were called pondus regis, and mensura domini regis, and are directed by a variety of subsequent statutes to be kept in the exchequer chamber, by an officer called the clerk of the market, except the wine gallon, which is committed to the city of London, and kept in Guildhall.

The Scottish standards are distributed among the oldest boroughs. The elwand is kept at Edinburgh, the pint at Stirling, the pound at Lanark, and the firlot at Linlithgow.

The two principal weights established in Great Britain, are troy weight and avoirdupois weight, as before mentioned. Under the head of the former it may farther be added, that

A carat is a weight of 4 grains; but when the term is applied to gold, it denotes the degree of fineness. Any quantity of gold is supposed divided into 24 parts. If the whole mass be pure gold, it is said to be 24 carats fine; if there be 23 parts of pure gold, and one part of alloy or base metal, it is said to be 23 carats fine, and so on.

Pure gold is too soft to be used for coin. The standard coin of this kingdom is 22 carats fine. A pound of standard gold is coined into 44½ guineas, and therefore every guinea should weigh 5 dwts $9\frac{39}{89}$ grains.

A pound of silver for coin contains 11 oz 2 dwts pure silver, and 18 dwts alloy: and standard silver-plate, 11 ounces pure silver, with 1 ounce alloy. A pound of standard silver is coined into 62 shillings; and therefore the weight of a shilling should be 3 dwts $20\frac{28}{31}$ grains.

Universal Standard for WEIGHTS *and Measures.*

Philosophers, from their habits of generalizing, have often made speculations for forming a general standard for weights and measures through the whole world. These have been devised chiefly of a philosophical nature, as best adapted to universality. After the invention of pendulum clocks, it first occurred that the length of a pendulum which should vibrate seconds, would be proper to be made a universal standard for lengths; whether it should be called a yard, or any thing else. But it was found, that it would be difficult in practice, to measure and determine the true length of such a pendulum, that is the distance between the point of suspension and the point of oscillation. Another cause of inaccuracy was afterwards discovered, when it was found that the seconds pendulum was of different lengths in all the different latitudes, owing to the spheroidal figure of the earth, which causes that all places in different latitudes are at different distances from the centre, and consequently the pendulums are acted on by different forces of gravity, and therefore require to be of different lengths. In the latitude of London this is found to be $39\frac{1}{8}$ inches.

The Society of Arts in London, among their many laudable and patriotic endeavours, offered a handsome premium for the discovery of a proper standard for weights and measures. This brought them many expedients, but none that merited any attention, except one, an improvement in the method of the pendulum, by Mr. Hatton, in the year 1779. This consisted in the application of a moveable point of suspension to one and the same pendulum, in order to produce the full and absolute effect of two pendulums, the difference of whose lengths was the intended measure. Here also the ratio of their lengths was easily determined, from observing the number of vibrations performed in a given time at each point of suspension. Whence there being two equations and two unknown quantities, the actual lengths of the pendulums themselves might be readily deduced by simple algebraic rules.

The late ingenious Mr. Whitehurst much improved on Mr. Hatton's original notion, in his essay published in 1787 under the title of "An attempt towards obtaining invariable Measures of Length, Capacity, and Weight, from the Mensuration of Time, &c." Mr. Whitehurst's proposal is to obtain a measure of the greatest length that conveniency will permit, from two pendulums whose vibrations are in the ratio of 2 to 1, and whose length coincide with the English standard in whole numbers. His numbers were chosen with considerable judgment and skill. On a supposition that the length of the seconds pendulum, in the latitude of London, is $39\frac{1}{5}$ inches, the length of one vibrating 42 times in a minute must be 80 inches; and that of another vibrating 84 times in a minute must be 20 inches; their difference 60 inches, or 5 feet is his standard measure. The difference of the lengths of two pendulums, however, resulting from his experiment, was 59·892 inches, instead of 60, the difference being occasioned by an error in the original assumption of 39·2 inches instead of 39·128 or $39\frac{1}{8}$ inches, as it is very nearly. Still, Mr. Whitehurst has accomplished a principal part of his grand design, by showing how an invariable standard may always be found for the same latitude. But this is by no means all that is wanted.

The French philosophers have gone much farther, and have very judiciously deduced the measures of capacity, and those of weight from the standard linear measure; confining themselves throughout to the decimal division.

But their system is liable to this heavy objection, that it depends on an accurate measure of a quarter meridian of the earth, at the same time that no such accurate measure has as yet been obtained; and at the same time probably that the meridians differ so widely among themselves as to leave no reasonable expectation that a correct medium length of a meridian will ever be found.

Some other method then must be resorted to, if we wish to obtain a universal measure, which at the same time that it shall be invariable, shall be easily recovered on the supposition that the actual standard is lost. Perhaps the least objectionable way would be to take for the length of the metre the length of a simple pendulum vibrating seconds at the equator, at a certain height above the surface of the sea, when the thermometer is at a fixed medium temperature: the length of the metre would then be about 39·027 English inches, instead of 39·37023, the metre of the new French system. The magnitude of the are, the stere, the gramme, &c, (or any other terms thought proper to introduce for similar purposes), might have the same relations to the metre as in the French system. Thus should we possess a standard taken from the gravitating force of the globe we inhabit, and which might be safely considered as invariable, so long as the constitution of the earth and its time of rotation remain the same.

The material standard itself also might be chosen of some shape that should possess the double advantage of being little affected by changes of temperature, and being a pendulum whose distance between the point of suspension and centre of oscillation, should be exactly equal to a fixed dimension of the pendulum that might readily be measured with exactness. Such a body we have in a right-angled cone, or one the diameter of whose base is equal to its altitude; for when this cone is suspended by its vertex as a centre of motion, the centre of oscillation is in the centre of the base; and when it is suspended by its base, the centre of oscillation coincides with the vertex of the solid; the length of the isochronous simple pendulum being in both cases equal to the altitude of the cone, or to the diameter of its base.

The universal standard for lengths being once established, those of weights, &c, evidently follow. For instance, a vessel of certain dimensions, being filled with distilled water, or some other homogeneous matter, the weight of that may be considered as a standard for weights. See also our article MEASURE.

WEIGHT of the Air, Water, &c. See those articles severally. See also SPECIFIC GRAVITY.

WERNER (JOHN), of Nuremberg, was born in 1468 and died 1528, and appears to have been the best mathematician of his time, being highly distinguished as an astronomer, a geometrician, &c, and well deserving of being better known than he is, having contributed by his writings on trigonometry, and other parts of the mathematics, to diffuse a taste for these sciences. It appears that he wrote 5 books on triangles. In his time the use of the cross-staff began to be introduced among seamen; this ancient instrument being described by him, in his Annotations on the first book of Ptolemy's Geography, printed in 1514; where he recommends it for observing the distance between the moon and some star, in order thence to determine the longitude. In 1522, he published Opera Mathematica at Nuremberg, in 4to, containing a specimen of the Conics, with some solid problems, and in which also he determined the precession of the equinox more exactly than it had before been done.

WERST, a Russian measure of length, equal to 3500 English feet, or $\frac{2}{3}$ of an English mile.

WEST, one of the cardinal points of the horizon, or of the compass, diametrically opposite to the east, or lying on the left hand when we face the north. Or west is strictly the intersection of the prime vertical with the horizon, on that side where the sun sets.

WEST *Wind*, is also called Zephyrus, and Favonius.

WEST *Dial*. See DIAL.

WESTERN *Amplitude*, *Horizon*, *Ocean*. See the several articles.

WESTING, in Navigation, is the quantity of departure made good to the westward from the meridian.

WEY. See WEIGH.

WHALE, in Astronomy, one of the constellations. See CETUS.

WHEEL, in Mechanics, a simple machine, consisting of a circular piece of wood, metal, or other matter, that revolves on an axis. This is otherwise called Wheel and Axle, or Axis in Peritrochio, as a mechanical power, being one of the most frequent and useful of any. In this capacity of it, the wheel is a kind of perpetual lever, and the axis another lesser one; or the radius of the wheel and that of its axis may be considered as the longer and shorter arms of a lever, the centre of the wheel being the fulcrum or point of suspension. Whence it is, that the power of this machine is estimated by this rule: As the radius of the axis is to the radius of the wheel or of the circumference; so is any given power, to the weight it will sustain.

Wheels, as well as their axes, are frequently dented, or cut into teeth, and are then of use on many occasions; as in jacks, clocks, mill-work, &c; by which means they are capable of moving and acting on one another, and of being combined together to any extent; the teeth either of the axis or circumference working in those of other wheels or axles; and thus, by multiplying the power to any extent, a very great effect is produced.

To compute the power of a combination of Wheels; the teeth of the axis of every wheel acting on those in the circumference of the next following. Multiply continually together the radii of all the axes, as also the radii of all the wheels; then it will be, as the former product is to the latter, so is a given power applied to the circumference, to the weight it can sustain. Thus, for example, in a combination of five wheels and axles, to find the weight a man can sustain, or raise, whose force is equal to 150 pounds, the radii of the wheels being 30 inches, and those of the axes 3 inches.

Here $3 \times 3 \times 3 \times 3 \times 3 = 243$,
and $30 \times 30 \times 30 \times 30 \times 30 = 24300000$,
therefore as 243 : 24300000 : : 150 : 15000000 lb, the weight he can sustain, which is more than 6696 tons weight. So prodigious is the increase of power in a combination of wheels!

But it is to be observed, that in this, as well as every other mechanical engine, whatever is gained in power, is lost in time; that is, the weight will move as much slower than the power, as the force is increased or multiplied, which in the example above is 100000 times slower.

Hence, having given any power, and the weight to be raised, with the proportion between the wheels and axles necessary to that effect; to find the number of the wheels and axles. Or, having the number of the wheels and axles given, to find the ratio of the radii of the wheels and axles. Here, putting

p = the power acting on the last wheel,
w = the weight to be raised,
r = the radius of the axles,
R = the radius of the wheels,
n = the number of the wheels and axles;

then, by the general proportion, as $r^n : R^n : : p : w$; therefore $pR^n = wr^n$ is a general theorem, from whence may be found any one of these five letters or quantities, when the other four are given. Thus, to find n the number of wheels: we have first $\frac{R^n}{r^n} = \frac{w}{p}$, then $n = \frac{\log. w - \log. p}{\log. R - \log. r}$. And to find the ratio of the wheel to the axle, it is $\frac{R}{r} = \sqrt[n]{\frac{w}{p}}$.

Wheels *of a Clock, &c*, are, the crown wheel, contrat wheel, great wheel, second wheel, third wheel, striking wheel, detent wheel, &c.

Wheels *of Coaches, Carts, Waggons, &c.* With respect to wheels of carriages, the following particulars are collected from the experiments and observations of Desaguliers, Beighton, Camus, Ferguson, Jacob, &c.

1. The use of wheels, in carriages, is twofold; viz, that of diminishing or more easily overcoming the resistance or friction from the carriage; and that of more easily overcoming obstacles in the road. In the first case, the friction on the ground is transferred in some degree from the outer surface of the wheel to its nave and axle; and in the latter, they serve easily to raise the carriage over obstacles and asperities met with on the roads. In both these cases, the height of the wheel is of material consideration, as the spokes act as levers, the top of an obstacle being the fulcrum, their length enables the carriage more easily to surmount them; and the greater proportion of the wheel to the axle serves more easily to diminish or to overcome the friction of the axle. See Jacob's Observations on Wheel Carriages, pa. 23 &c.

2. The wheels should be exactly round; and the fellies at right angles to the naves, according to the inclination of the spokes.

3. It is the most general opinion, that the spokes be somewhat inclined to the naves, so that the wheels may be dishing or concave. Indeed if the wheels were always to roll on smooth and level ground, it would be best to make the spokes perpendicular to the naves, or to the axles; because they would then bear the weight of the load perpendicularly. But because the ground is commonly uneven, one wheel often falls into a cavity or rut, when the other does not, and then it sustains much more of the weight than the other does; in which case it is best for the wheels to be dished, because the spokes become perpendicular in the rut, and therefore have the greatest strength when the obliquity of the road throws most of the weight on them; while those on the high ground have less weight to bear, and therefore need not be at their full strength.

4. The axles of the wheels should be quite straight, and perpendicular to the shafts, or to the pole. When the axles are straight, the rims of the wheels will be parallel to each other, in which case they will move the easiest, because they will be at liberty to proceed straight forwards. But in the usual way of practice, the ends of the axles are bent downwards; which always keeps the sides of the wheels that are next the ground nearer to each other than their upper sides are; and this not only makes the wheels drag sideways as they go along, and gives the load a much greater power of crushing them than when they are parallel to each other, but also endangers the overturning the carriage when a wheel falls into a hole or rut, or when the carriage goes on a road that has one side lower than the other, as along the side of a hill. Mr. Beighton however has offered several reasons to prove that the axles of wheels ought not to be straight; for which see Desaguliers's Exp. Phil. vol. 2, Appendix.

5. Large wheels are found more advantageous for rolling than small ones, both with regard to their power as a longer lever, and to the degree of friction, and to the advantage in getting over holes, rubs, and stones, &c. If we consider wheels with regard to the friction on their axles, it is evident that small wheels, by turning oftener round, and swifter about the axles, than large ones, must have much more friction. Again, if we consider wheels as they sink into holes or soft earth, the large wheels, by sinking less, must be much easier drawn out of them, as well as more easily over stones and obstacles, from their greater length of lever or spokes. Desaguliers has brought this matter to a mathematical calculation, in his Experim. Philos. vol. 1, pa. 171, &c. See also Jacob's Observ. pa. 63.

Hence it appears then, that wheels are the more advantageous as they are larger, provided they are not more than 5 or 6 feet diameter; for when they exceed these dimensions, they become too heavy; or if they are made light, their strength is proportionably diminished, and the length of the spokes renders them more liable to break; besides, horses applied to such wheels would not be capable of exerting their utmost strength, by having the axles higher than their breasts, so that they would draw downwards; which is even a greater disadvantage than small wheels have in occasioning the horses to draw upwards.

6. Carriages with 4 wheels, as waggons or coaches, are much more advantageous than carriages with 2 wheels, as carts and chaises; for, with 2 wheels it is plain the tiller horse carries part of the weight, in one way or other: in going down hill, the weight bears on the horse; and in going up hill, the weight falls the other way, and lifts the horse, which is still worse. Besides, as the wheels sink into the holes in the roads on different sides, the shafts strike against the tiller's sides, which occasions the death of many horses: moreover, when one of the wheels sinks into a hole or rut, half the weight falls that way, which endangers the overturning of the carriage.

7. It would be much more advantageous to make the 4 wheels of a coach or waggon large, and nearly of a

height, than to make the fore wheels of only half the diameter of the hind wheels, as is usual in many places. The fore wheels have commonly been made of a less size than the hind ones, both on account of turning short, and to avoid cutting the braces. Crane-necks have also been invented for turning yet shorter, and the fore wheels have been lowered, so as to go quite under the bend of the crane-neck.

It is accounted a great disadvantage in small wheels, that as their axle is below the bow of the horses' breasts, the horses not only have the loaded carriage to draw along, but also part of its weight to bear, which tires them soon, and makes them grow much stiffer in their hams, than they would if they drew on a level with the fore axle.

But Mr. Beighton disputes the propriety of fixing the line of traction on a level with the breast of a horse, and says it is contrary to reason and experience. Horses, he says, have little or no power to draw but what they derive from their weight; without which they could not take hold of the ground, and then they must slip, and draw nothing. Common experience also teaches, that a horse must have a certain weight on his back or shoulders, that he may draw the better. And when a horse draws hard, it is observed that he bends forward, and brings his breast near the ground; and then if the wheels are high, he is pulling the carriage against the ground. A horse tackled in a waggon will draw two or three ton, because the point or line of traction is below his breast, by the lowness of the wheels. It is also common to see, when one horse is drawing a heavy load, especially up hill, his fore feet will rise from the ground; in which case it is usual to add a weight on his back, to keep his fore part down, by a person mounting on his back or shoulders, which will enable him to draw that load, which he could not move before. The greatest stress, or main business of drawing, says this ingenious writer, is to overcome obstacles; for on level plains the drawing is but little, and then the horse's back need be pressed but with a small weight.

8. The utility of broad wheels, in amending and preserving the roads, has been so long and generally acknowledged, as to have occasioned the legislature to enforce their use. At the same time, the proprietors and drivers of carriages seem to be convinced by experience, that a narrow-wheeled carriage is more easily and speedily drawn by the same number of horses, than a broad-wheeled one of the same burthen: probably because they are much lighter, and have less friction on the axle.

On the subject of this article, see Jacob's Observ. &c. on Wheel-Carriages, 1773, pa. 81; Desagul. Exper. Phil. vol. 1, pa. 201; Martin's Phil. Brit. vol. 1, pa. 229; and Brewster's valuable edition of Ferguson's Lectures, both the work itself and the Appendix to the same, where several new observations &c. are given on this subject. See also the Report of the Committee of the House of Commons, on Acts regarding the use of Broad Wheels, and other matters relating to the Preservation of the Public Roads.—Abridged in the Repertory of Arts, No. 64, New Series.

Blowing Wheel, is a machine contrived by Desaguliers, for drawing the foul air out of any place, or for forcing in fresh, or doing both successively, without opening doors or windows. See Philos. Trans. No. 437. The intention of this machine is the same as that of Hales's ventilator, but not so effectual, nor so convenient. See Desag. Exper. Philos. vol. 2, pa. 563, 568.—This wheel is also called a centrifugal wheel, because it drives the air with a centrifugal force.

Water Wheel, of a Mill, that which receives the impulse of the stream by means of ladle-boards or float-boards. M. Parent, of the Academy of Sciences, has determined that the greatest effect of an undershot wheel, is when its velocity is equal to the 3d part of the velocity of the water that drives or keeps it in motion: but it ought to be the half of that velocity, as is fully shown in the article Mill, in this Dictionary. In fixing an undershot wheel, it ought to be considered whether the water can run clear off, so as to cause no back-water to stop its motion. Concerning this article, see Desagul. Exp. Philos. vol. 2, pa. 422. Also a variety of experiments and observations relating to undershot and overshot wheels, by Mr. Smeaton, in the Philos. Trans. vol. 51, pa. 100.

Aristotle's Wheel. See Rota *Aristotelica*.

Measuring Wheel. See Perambulator.

Orffyreus's Wheel. See Orffyreus.

Persian Wheel. See Persian.

Wheel-*Barometer*. See Barometer.

WHIRL-POOL, an eddy, vortex, or gulf, where the water is continually turning round.

Those in rivers are very common, from various accidents, and are usually very trivial, and of little consequence. In the sea they are more rare, but more dangerous. Sibbald has related the effects of a very remarkable marine whirlpool among the Orcades, which would prove very dangerous to strangers, though it is of no consequence to the people who are used to it. This is not fixed to any particular place, but appears in various parts of the limits of the sea among these islands. Wherever it appears it is very furious; and boats &c would inevitably be drawn in and perish with it; but the people who navigate them are prepared for the event, and always carry an empty vessel, a log of wood, or large bundle of straw, or some such thing, in the boat with them; as soon as they perceive the whirlpool, they toss this within its vortex, keeping themselves out; this substance, whatever it be, is immediately received into the centre, and carried under water; and as soon as this is done, the surface of the place where the whirlpool was becomes smooth, and they row over it with safety: and in about an hour they see the vortex begin again in some other place, usually at about a mile's distance from the first.

WHIRLING-TABLE, a machine contrived for representing several phenomena in philosophy and nature; as, the principal laws of gravitation, and of the planetary motions in curvilinear orbits.

The figure of this instrument is exhibited fig. 1, pl. 41: where AA is a strong frame of wood; B a winch fixed on the axis C of the wheel D, about which is the catgut string F, which also goes round the small wheels G and K, crossing between them and the great wheel D. On the upper end of the axis of the wheel G, above the frame, is fixed the round board *d*, to which may be occasionally fixed the bearer MSX. On the axis of the wheel H is fixed the bearer NTZ, and when the winch B is turned, the wheels and bearers are put into a whirling motion. Each bearer has two wires W, X, and Y, Z, fixed and screwed tight into them at the ends by nuts on the outside; and when the nuts are unscrewed, the wires may be drawn out in order to change the balls U, V, which slide upon the wires by means of brass loops fixed into the balls; and preventing their touching the wood below them. Through each ball there passes a silk line, which is fixed to it at any length from the centre of the bearer to its end, by a nut-screw at the top of the ball; the shank of the screw going into the centre of the ball, and pressing the line against the under side of the hole which it runs through. The line goes from the ball, and under a small pulley fixed in the middle of the bearer; then up through a socket in the round plate (S and T) in the middle of each bearer; then through a slit in the middle of the square top (O and P) of each tower, and going over a small pulley on the top comes down again the same way, and is at last fastened to the upper end of the socket fixed in the middle of the round plate above-mentioned. Each of these plates S and T has four round holes near their edges, by which they slide up and down on the wires, which make the corner of each lower. The balls and plates being thus connected, each by its particular line, it is plain that if the balls be drawn outward, or towards the end M and N of their respective bearers, the round plates S and T will be drawn up to the top of their respective towers O and P.

There are several brass weights, some of 2, some of 3, and others of 4 ounces, to be occasionally put within the towers O and P, on the round plates S and T: each weight having a round hole in the middle of it, for going on the sockets or axes of the plates, and being slit from the edge to the hole, that it may slip over the line which comes from each ball to its respective plate.

For a specimen of the experiments to be made with this machine, may be subjoined the following.

1. Removing the bearer MX, put the loop of the line *b* to which the ivory ball *a* is fastened over a pin in the centre of the board *d*, and turn the winch B; and the ball will not immediately begin to move with the board, but, on account of its inactivity, endeavour to remain in its state of rest. But when the ball has acquired the same velocity with the board, it will remain on the same part of the board, having no relative motion upon it. However, if the board be suddenly stopped, the ball will continue to revolve on it, until the friction stops its motion: so that matter resists every change of state, from that of rest to that of motion, and vice versa.

2. Put a longer cord to this ball; let it down through the hollow axis of the bearer MX and wheel G, and fix a weight to the end of the cord below the machine; and this weight, if left at liberty, will draw the ball from the edge of the whirling board to its centre. Draw off the ball a little from the centre, and turn the winch; then the ball will continue to revolve with the board, and gradually fly farther from the centre, raising up the weight below the machine. And thus it appears that all bodies, revolving in circles, have a tendency to fly off from those circles, and must be retained in them by some power proceeding from or tending to the centre of motion. Stop the machine, and the ball will continue to revolve for some time on the board; but as the friction gradually stops its motion, the weight acting upon it will bring it nearer and nearer to the centre in every revolution, till it brings it quite thither. Hence it appears, that if the planets met with any resistance in going round the sun, its attractive power would bring them nearer and nearer to it in every revolution, till they would fall into it.

3. Take hold of the cord below the machine with one hand, and with the other throw the ball upon the round board as it were at right angles to the cord, and it will revolve on the board. Then, observing the velocity of its motion, pull the cord below the machine, and thus bring the ball nearer the centre of the board, and the ball will be seen to revolve with an increasing velocity, as it approaches the centre: and thus the planets which are nearest the sun perform quicker revolutions than those which are more remote, and move with greater velocity in every part of their respective circles.

4. Remove the ball *a*, and apply the bearer MX, whose centre of motion is in its middle at *w*, directly over the centre of the whirling board *d*. Then put two balls (V and U) of equal weight on their bearing wires, and having fixed them at equal distances from their respective centres of motion *w* and *x* upon their silk cords, by the screw nuts, put equal weights in the towers O and P. Lastly, put the catgut strings E and F on the grooves G and H of the small wheels, which, being of equal diameters, will give equal velocities to the bearers above, when the winch B is turned; and the balls U and V will fly off toward M and N, and raise the weights in the towers at the same instant. This shows, that when bodies of equal quantities of matter revolve in equal circles with equal velocities, their centrifugal forces are equal.

5. Take away these equal balls, and put a ball of 6 ounces into the bearer MX, at a 6th part of the distance *wz* from the centre, and put a ball of one ounce into the opposite bearer, at the whole distance $xy = wz$; and fix the balls at these distances on their cords, by the screw nuts at the top: then the ball U, which is 6 times as heavy as the ball V, will be at only a 6th part of the distance from its centre of motion; and consequently will revolve in a circle of only a 6th part of the circumference of that in which V revolves. Let equal weights be put into the towers, and the winch be turned: as the catgut string is on equal wheels below, it will cause the balls to revolve in equal times; but V will move 6 times as fast as U, because it revolves in a circle of 6 times its radius, and both the weights in the towers will rise at once. Hence it appears, that the centrifugal forces of revolving bodies are in direct proportion to their quantities of matter multiplied into their respective velocities, or into their distance from the centres of their respective circular orbits.

If these two balls be fixed at equal distances from their respective centres of motion, they will move with equal velocities; and if the tower O has 6 times as much weight put into it as the tower P, the balls will raise their weights exactly at the same moment: i. e. the ball U, being 6 times as heavy as the ball V, has 6 times as much centrifugal force in describing an equal circle with an equal velocity.

6. Let two balls, U and V, of equal weights, be fixed on their cords at equal distances from their respective centres of motion *w* and *x*; and let the catgut string E be put round the wheel K (whose circumference is only half that of the wheel H or G) and over the pulley *s* to keep it tight, and let 4 times as much weight be put into the tower P as in the tower O. Then turn the winch B, and the ball V will revolve twice as fast as the ball U in a circle of the same diameter, because they are equidistant from the centres of the circles in which they revolve; and the weights in the towers will both rise at the same instant; which shows that a double velocity in the same circle will exactly balance a quadruple power of attraction in the centre of the circle: for the weights in the towers may be considered as the attractive forces in the

centres, acting on the revolving balls; which moving in equal circles, are as if they both moved in the same circle. Whence it appears that, if bodies of equal weights revolve in equal circles with unequal velocities, their centrifugal forces are as the squares of the velocities.

7. The catgut string remaining as before, let the distance of the ball V from the centre *x* be equal to 2 of the divisions on its bearer; and the distance of the ball U from the centre *w* be 3 and a 6th part; the balls themselves being equally heavy, and V making two revolutions by turning the winch, whilst U makes one; so that if we suppose the ball V to revolve in one second, the ball U will revolve in 2, the squares of which are 1 and 4: therefore, the square of the period of V is contained 4 times in the square of the period of U. But the distance of V is 2, the cube of which is 8, and the distance of U is $3\frac{1}{6}$, the cube of which is 32 very nearly, in which 8 is contained 4 times: and therefore, the squares of the periods V and U are to each other as the cubes of their distances from *x* and *w*, the centres of their respective circles. And if the weight in the tower O be 4 ounces, or equal to the square of 2, which is the distance of V from the centre *x*; and the weight in the tower P be 10 ounces, nearly equal to the square of $3\frac{1}{6}$, the distance of U from *w*; it will be found on turning the machine by the winch, that the balls U and V will raise their respective weights at very nearly the same instant of time. This experiment confirms the famous proposition of Kepler, viz, that the squares of the periodical times of the planets round the sun are in proportion as the cubes of their distances from him; and that the sun's attraction is inversely as the square of the distance from his centre.

8. Take off the string E from the wheels D and H, and let the string F remain on the wheels D and G; take away also the bearer MX from the whirling-board *d*, and instead of it put on the machine AB (fig. 2), fixing it to the centre of the board by the pins *c* and *d*, so that the end *ef* may rise above the board to an angle of 30 or 40 degrees. On the upper part of this machine, there are two glass tubes *a* and *b*, closely stopped at both ends, each tube being about three quarters full of water. In the tube *a* is a little quicksilver, which naturally falls down to the end *a* in the water; and in the tube *b* is a small cork, floating on the top of the water, and small enough to rise or fall in the tube. While the board *b* with this machine on it continues at rest, the quicksilver lies at the bottom of the tube *a*, and the cork floats on the water near the top of the tube *b*. But, on turning the winch and moving the machine, the contents of each tube fly off towards the uppermost ends, which are farthest from the centre of motion; the heaviest with the greatest force. Consequently, the quicksilver in the tube *a* will fly off quite to the end *f*, occupying its bulk of space, and excluding the water, which is lighter than itself: but the water in the tube *b*, flying off to its higher end *e*, will exclude the cork from that place, and cause it to descend toward the lowest end of the tube; for the heavier body, having the greater centrifugal force, will possess the upper part of the tube, and the lighter body will keep between the heavier and the lower part.

This experiment demonstrates the absurdity of the Cartesian doctrine of vortices; for, if a planet be more dense or heavy than its bulk of the vortex, it will fly off in it farther and farther from the sun; if less dense, it will descend to the lowest part of the vortex, at the sun: and the whole vortex itself, unless prevented by some obstacle, would fly quite off, together with the planets.

9. If a body be so placed on the whirling-board of the machine (fig. 1) that the centre of gravity of the body be directly over the centre of the board, and the board be moved ever so rapidly by the winch B, the body will turn round with the board, without removing from its middle; for, as all parts of the body are in equilibrio round its centre of gravity, and the centre of gravity is at rest in the centre of motion, the centrifugal force of all parts of the body will be equal at equal distances from its centre of motion, and therefore the body will remain in its place. But if the centre of gravity be placed ever so little out of the centre of motion, and the machine be turned swiftly round, the body will fly off towards that side of the board on which its centre of gravity lies. Then if the wire C (fig. 3) with its little ball B be taken away from the semi-globe A, and the flat side *f* of the semi-globe be laid on the whirling-board, so that their centres may coincide; if then the board be turned ever so quickly by the winch, the semi-globe will remain where it was placed: but if the wire C be screwed into the semi-globe at *d*, the whole becomes one body, whose centre of gravity is at or near *d*. Fix the pin *c* in the centre of the whirling-board, and let the deep groove *b* cut in the flat side of the semi-globe be put on the pin, so that the pin may be in the centre of A (see fig. 4), where the groove is to be represented at *b*, and let the board be turned by the winch, which will carry the little ball B (fig. 3) with its wire C, and the semi-globe A, round the centre pin *c i*; and then, the centrifugal force of the little ball B, weighing one ounce, will be so great as to draw off the semi-globe A, weighing two pounds, until the end of the groove at *c* strikes against the pin *c*, and so prevents A from going any farther: otherwise, the centrifugal force of B would have been sufficient to have carried A quite off the whirling-board. Hence we see that, if the sun were placed in the centre of the orbits of the planets, it could not possibly remain there; for the centrifugal forces of the planets would carry them quite off, and the sun with them; especially when several of them happened to be in one quarter of the heavens. For the sun and planets are as much connected by the mutual attraction subsisting between them, as the bodies A and B are by the wire C fixed into them both. And even if there were but one planet in the whole heavens to revolve about ever so large a sun in the centre of its orbit, its centrifugal force would soon carry off both itself and the sun: for the greatest body placed in any part of free space could be easily moved; because, if there were no other body to attract it, it would have no weight or gravity of itself, and consequently, though it could have no tendency of itself to remove from that part of space, yet it might be very easily moved by the force of any other substance.

10. As the centrifugal force of the light body B will not allow the heavy body A to remain in the centre of motion, even though it be 24 times as heavy as B; let the ball A (fig. 5) weighing 6 ounces be connected by the wire C with the ball B, weighing one ounce, and let the fork E be fixed into the centre of the whirling-board; then, hang the balls on the fork by the wire C in such a manner that they may exactly balance each other, which will be when the centre of gravity between them, in the wire at *d*, is supported by the fork. And this centre of gravity is as much nearer to the centre of the ball A than

to the centre B, as A is heavier than B; allowing for the weight of the wire on each side of the fork. Then, let the machine be moved, and the balls A and B will revolve about their common centre of gravity *d*, keeping their balance, because either will not allow the other to fly off with it. For, supposing the ball B to be only one ounce in weight, and the ball A to be six ounces; then, if the wire C were equally heavy on each side of the fork, the centre of gravity *d* would be 6 times as far from the centre of B as from the centre of A, and consequently B will revolve with 6 times the velocity of A, which will give B 6 times as much centrifugal force as any single ounce of A has; but then as B is only one ounce, and A six ounces, the whole centrifugal force of A will exactly balance that of B; and therefore, each body will detain the other, so as to retain it in its circle.

Hence it appears, that the sun and planets must all move round the common centre of gravity of the whole system, in order to preserve that just balance which takes place among them.

11. Remove the forks and balls from the whirling-board, and place the trough AB (fig. 6) thereon, fixing its centre to that of the board by the pin H. In this trough are two balls D and E of unequal weights, connected by a wire *f*, and made to slide easily on the wire stretched from end to end of the trough, and made fast by nut screws on the outside of the ends. Place these balls on the wire *c*, so that their common centre of gravity *g*, may be directly over the centre of the whirling-board. Then turn the machine by the winch ever so swiftly, and the trough and balls will move round their centre of gravity, so as neither of them will fly off; because, on account of the equilibrium, each ball detains the other with an equal force acting against it. But if the ball E be drawn a little more towards the end of the trough at A, it will remove the centre of gravity towards that end from the centre of motion; and then, upon turning the machine, the little ball E will fly off, and strike with a considerable force against the end A, and draw the great ball B into the middle of the trough. Or, if the great ball D be drawn towards the end B of the trough, so that the centre of gravity may be a little towards that end from the centre of motion; and the machine be turned by the winch, the great ball D will fly off, and strike violently against the end B of the trough, and will bring the little ball E into the middle of it. If the trough be not made very strong, the ball D will break through it.

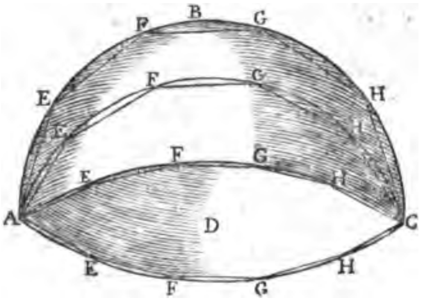
12. Mr. Ferguson has explained the reason why the tides rise at the same time on opposite sides of the earth, and consequently in opposite directions, by the following new experiment on the whirling-table. For this purpose, let *a b c d* (fig. 7) represent the earth, with its side *c* turned toward the moon, which will then attract the water so as to raise them from *c* to *g*: and in order to show that they will rise as high at the same time on the opposite side from *a* to *e*; let a plate AB (fig. 8) be fixed on one end of the flat bar DC, with such a circle drawn on it as *a b c d* (fig. 7) to represent the round figure of the earth and sea; and an ellipse as *e f g h* to represent the swelling of the tide at *e* and *g*, occasioned by the influence of the moon. Over this plate AB suspend the three ivory balls *e, f, g*, by the silk lines *h, i, k*, fastened to the tops of the wires H, I, K, so that the ball at *e* may hang freely over the side of the circle *e*, which is farthest from the moon M at the other end of the bar; the ball at *f* over the centre, and the ball at *g* over the side of the circle *g*, which is nearest the moon. The ball *f* may represent the centre of the earth, the ball *g* water on the side next the moon, and the ball *e* water on the opposite side. On the back of the moon M is fixed a short bar N parallel to the horizon, and there are three holes in it above the little weights *p, q, r*. A silken thread *o* is tied to the line *k* close above the ball *g*, and passing by one side of the moon M goes through a hole in the bar N, and has the weight *p* hung to it. Such another thread *m* is tied to the line *i*, close above the ball *f*, and, passing through the centre of the moon M and middle of the bar N, has the weight *q* hung to it, which is lighter than the weight *p*. A third thread *m* is tied to the line *h*, close above the ball *e*, and, passing by the other side, of the moon M through the bar N, has the weight *r* hung to it, which is lighter than the weight *q*. The use of these three unequal weights is to represent the moon's unequal attraction at different distances from her; so that if they are left at liberty, they will draw all the three balls towards the moon with different degrees of force, and cause them to appear as in fig. 9, in which case they are evidently farther from each other than if they hung freely by the perpendicular lines *h, i, k*. Hence it appears, that as the moon attracts the side of the earth which is nearest her with a greater degree of force than she does the centre of the earth, she will draw the water on that side more than the centre, and cause it to rise on that side: and as she draws the centre more than the opposite side, the centre will recede farther from the surface of the water on that opposite side, and leave it as high there as she raised it on the side next her. For, as the centre will be in the middle between the tops of the opposite elevations, they must of course be equally high on both sides at the same time.

However, upon this supposition, the earth and moon would soon come together; and this would be the case if they had not a motion round their common centre of gravity, to produce a degree of centrifugal force, sufficient to balance their mutual attraction. Such motion they have; for as the moon revolves in her orbit every month, at the distance of 240000 miles from the earth's centre, and of 234000 miles from the centre of gravity of the earth and moon, the earth also goes round the same centre of gravity every month at the distance of 6000 miles from it, i. e. from it to the centre of the earth. But the diameter of the earth being, in round numbers, 8000 miles, its side next the moon is only 2000 miles from the common centre of gravity of the earth and moon, its centre 6000 miles from it, and its farthest side from the moon 10000 miles. Consequently the centrifugal forces of these parts are as 2000, 6000, and 10000; i. e. the centrifugal force of any side of the earth, when it is turned from the moon, is 5 times as great as when it is turned towards the moon. And as the moon's attraction, expressed by the number 6000 at the earth's centre, keeps the earth from flying out of this monthly circle, it must be greater than the centrifugal force of the waters on the side next her; and consequently, her greater degree of attraction on that side is sufficient to raise them; but as her attraction on the opposite side is less than the centrifugal force of the water there, the excess of this force is sufficient to raise the water just as high on the opposite side.

To prove this experimentally, let the bar DC with its furniture be fixed on the whirling-board of the machine (fig. 1.) by pushing the pin P into the centre of the board; which pin is in the centre of gravity of the whole bar with its three balls, *e*, *f*, *g*, and moon M. Now if the whirling-board and bar be turned slowly round by the winch, till the ball *f* hangs over the centre of the circle, as in fig. 10, the ball *g* will be kept towards the moon by the heaviest weight *p* (fig. 8), and the ball *e*, on account of its greater centrifugal force, and the less weight *r*, will fly off as far to the other side, as in fig. 10. And thus, while the machine is kept turning, the balls *e* and *g* will hang over the ends of the ellipse *lfk*. So that the centrifugal force of the ball *e* will exceed the moon's attraction just as much as her attraction exceeds the centrifugal force of the ball *g*, while her attraction just balances the centrifugal force of the ball *f*, and makes it keep in its circle. Hence it is evident, that the tides must rise to equal heights at the same time on opposite sides of the earth. See Ferguson's Lectures on Mechanics, lect. 2, and Desag. Ex. Phil. vol. 1, lect. 5.

WHIRLWIND, a wind exceedingly rapid and impetuous, which moves in a spiral manner, as well as horizontally, which is but only of short duration. Dr. Franklin, in his Physical and Meteorological Observations, read to the Royal Society in 1756, supposes a whirlwind and a waterspout to proceed from the same cause, namely a stream of elastic matter rushing violently into the atmosphere out of the earth: their only difference being, that the latter passes over the water, and the former over the land. This opinion is corroborated by the observations of M. de la Pryme, and many others, who have remarked the appearances and effects of both to be the same. They have both a progressive as well as a circular motion; they usually rise after calms and great heats, and mostly happen in the warmer latitudes: the wind blows every way from a large surrounding space, both to the waterspout and whirlwind; and a waterspout has, by its progressive motion, passed from the sea to the land, and produced all the phenomena and effects of a whirlwind: so that there is no reason to doubt that they are meteors arising from the same general cause, and explicable on the same principles, furnished by electrical experiments and discoveries. See HURRICANE, and WATERSPOUT. For Dr. Franklin's ingenious method of accounting for both these phenomena, see his Letters and Papers, &c, vol. 1, pa. 191, 216, &c.

WHISPERING-*Places*, are places where a whisper, or other small noise, may be heard from one part to another, to a great distance. They depend on a principle, that the voice, &c, being applied to one end of an arch, easily passes by repeated reflections to the other. Thus, let ABC represent the segment of a sphere; and suppose a low voice uttered at A, the vibrations extending themselves every way, some of them will impinge on the points E, E, &c; and thence be reflected to the points F, F, &c; thence to G, G, &c; till at last they meet in C; where by their union they cause a much stronger sound than in any part of the segment whatever, even louder than at the point from which they set out. Accordingly, all the contrivance in a whispering-place is, that near the person who whispers, there be a smooth wall, arched either cylindrically, or elliptically, &c. A circular arch will do, but not so well.



Some of the most remarkable places for whispering, are the following: viz, The prison of Dionysius at Syracuse, which increased a soft whisper to a loud noise; or a clap of the hand to the report of a cannon, &c. The aqueducts of Claudius, which conducted a voice 16 miles: beside divers others mentioned by Kircher in his Phonurgia. In England, the most considerable whispering places are, the dome of St. Paul's church, London, where the ticking of a watch may be heard from side to side, and a very soft whisper may be sent quite round the dome: this Dr. Derham found to hold not only in the gallery below, but above on the scaffold, where a whisper would be carried over a person's head round the top of the arch, though there be a large opening in the middle of it into the upper part of the dome. And the celebrated whispering-place in Gloucester cathedral, which is only a gallery above the east end of the choir, leading from one side of it to the other. See Birch's Hist. of the Royal Soc. vol. 1. pa. 120.

WHISTON (WILLIAM), an English divine, philosopher, and mathematician, of uncommon abilities, learning, and extraordinary character, was born the 9th of December 1667, at Norton in the county of Leicester, where his father was rector. He was educated under his father till he was 17 years of age, when he was sent to Tamworth school, and two years after admitted of Clare-hall, Cambridge, where he pursued his studies, and particularly the mathematics, with great diligence. During this time he became afflicted with a great weakness of sight, owing to close study in a whitened room; which was in a good measure relieved by a little relaxation from study, and taking off the strong glare of light by hanging the place opposite his seat with green.

In 1693 he became master of arts and fellow of the college, and soon after commenced one of the tutors; but his ill state of health soon after obliged him to relinquish this profession. Having entered into orders, in 1694 he became chaplain to Dr. More, bishop of Norwich; and while in this station he published his first work, A New Theory of the Earth &c; in which he undertook to prove that the Mosaic doctrine of the earth was perfectly agreeable to reason and philosophy: which work, having much ingenuity, though it was written against by Mr. John Keill, brought considerable reputation to the author.

In the year 1698, bishop More gave him the living of Lowestoff in Suffolk, where he immediately went to reside, and devoted himself with great diligence to the discharge of that trust.—In the beginning of the last century he was made sir Isaac Newton's deputy, and afterwards his successor, in the Lucasian professorship of mathematics; when he resigned his living at Lowestoff, and went to reside at Cambridge. From this time his publications became very frequent, both in theology and mathematics. Thus, in 1702 he published, A Short View of the Chronology of the Old Testament, and of the Harmony of the four Evangelists.—In 1707, Prælectiones Astronomicæ; besides 8 Sermons on the Accomplishment of the Scripture Prophecies, preached at Boyle's lecture; and Newton's Arithmetica Universalis.—In 1708, Tacquet's Euclid, with select Theorems of Archimedes; the former of which had accidentally been his first introduction to the study of the mathematics.—In the same year he drew up an Essay on the Apostolical Constitutions, which the vice chancellor refused his licence for printing. The author informs us, he had read over the two first centuries of the church, and found that the Eusebian or Arian doctrine was chiefly the doctrine of those ages, which, though deemed heterodox, he thought it his duty to discover.—In 1709, he published a volume of Sermons and Essays on various subjects.—In 1710, Prælectiones Physico-Mathematicæ, which, with the Prælectiones Astronomicæ, were translated and published in English. And it may be said, with no small honour to the memory of Mr. Whiston, that he was one of the first who explained the Newtonian philosophy in a popular way, so as to be intelligible to the generality of readers.—Among other things also, he translated the Apostolical Constitutions into English, which favoured the doctrine of the supremacy of the Father and subordination of the Son, vulgarly called the Arian heresy: on which his friends began to be alarmed for him; and the consequence showed it was not groundless; for, Oct. 30, 1710, he was deprived of his professorship, and expelled the university of Cambridge, after he had been formally convened and interrogated for several successive days.—At the conclusion of this year, he wrote his Historical Preface, afterwards prefixed to his Primitive Christianity Revived, containing the reasons for his dissent from the commonly received notions of the Trinity, which work he published the next year, in 4 volumes 8vo, for which the Convocation fell upon him most vehemently.

In 1713, he and Mr. Ditton composed their scheme for finding the longitude, which they published the year following, a method which consisted in measuring distances by means of the velocity of sound; some more particulars of which are related in the life of Mr. Ditton.—In 1719, he published an ironical Letter of Thanks to doctor Robinson, bishop of London, for his late Letter to his clergy against the use of New Forms of Doxology. And, the same year, a Letter to the earl of Nottingham, concerning the Eternity of the Son of God, and his Holy Spirit.—In 1720, he was proposed by sir Hans Sloane and Dr. Halley to the Royal Society as a member; but was refused admittance by sir Isaac Newton the president.

On Mr. Whiston's expulsion from Cambridge, he went to London, where he conferred with Doctors Clarke, Hoadley, and other learned men, who endeavoured to moderate his zeal, which however he would not suffer to be tainted or corrupted, and many were not much satisfied with the authority of these constitutions, but approved his integrity. Mr. Whiston now settled in London with his family; where, without suffering his ardour to be intimidated, he continued to write, and to propagate his Primitive Christianity with as much fervency as if he had been in the most flourishing circumstances; which however were so bad, that, in 1721, a subscription was made for the support of his family, which amounted to 470l. For though he drew some profits from reading astronomical and philosophical lectures, and also from his publications, which were very numerous, yet these of themselves were very insufficient; nor, when joined with the benevolence and charity of those who loved and esteemed him for his learning, integrity, and piety, did they prevent his being frequently in great distress.—In 1722 he published an Essay towards restoring the true text of the Old Testament.—In 1724, The Literal Accomplishment of Scripture Prophecies.—Also, The Calculation of Solar Eclipses without Parallaxes.—In 1726, Of the Thundering Legion &c.—In 1727, A Collection of Authentic Records belonging to the Old and New Testament.—In 1730, Memoirs of the Life of Dr. Samuel Clarke.—In 1732, A Vindication of the Testimony of Phlegon, or an Account of the Great Darkness and Earthquake at our Saviour's Passion, described by Phlegon.—In 1736, Athanasian Forgeries, &c. And the Primitive Eucharist revived.—In 1737, The Astronomical Year, particularly of the Comet foretold by sir Isaac Newton.—Also the Genuine Works of Flavius Josephus.—In 1739, Mr. Whiston put in his claim to the mathematical professorship at Cambridge, then vacant by the death of Dr. Saunderson, in a letter to Dr. Ashton, the master of Jesus-college; but no regard was paid to it.—In 1745, he published his Primitive New Testament in English.—In 1748, his Sacred History of the Old and New Testament. Also, Memoirs of his own Life and Writings, which are very curious.

Whiston continued many years a member of the established church; but at length forsook it, on account of the reading of the Athanasian Creed, and went over to the Baptists; which happened while he was at the house of Samuel Barker, Esq. at Lindon in Rutlandshire, who had married his daughter; where he died, after a week's illness, the 22d of August 1752, at upwards of 84 years of age.—We have mentioned the principal of his writings in the foregoing memoir; to which may be added, Chronological Tables, published in 1750; and one paper only in the Philos. Trans. vol. 31, on two mock suns, and a halo seen in Oct. 1721.

The character of this conscientious and worthy man has been attempted by two very able personages, who were well acquainted with him, namely, bishop Hare and Mr. Collins, who unite in giving him the highest applauses, for his integrity, piety, &c.—Mr. Whiston left some children behind him; among them, Mr. John Whiston, who was for many years a very considerable bookseller in London.

WHITE, one of the colours of bodies. Though white cannot properly be said to be one colour, but rather a composition of all the colours together: for Newton has demonstrated that bodies only appear white by reflecting all the kinds of coloured rays alike; and that even the light of the sun is only white, because it consists of all colours mixed together.

This may be shown mechanically in the following manner: Take seven parcels of coloured fine powders, the same as the primary colours of the rainbow, taking such quantities of these as shall be proportional to the respective breadths of these colours in the rainbow, which are of red 45 parts, orange 27, yellow 48, green 60, blue 60, indigo 40, and of violet 80; then mix intimately together these seven parcels of powders, and the mixture will be a pretty white colour: this is only similar to the uniting the prismatic colours together again, to form a white ray or pencil of light of the whole of them. The same thing is performed conveniently thus: Let the flat upper surface of a top be divided into 360 equal parts, all around its edge; then divide the same surface into seven sectors in the proportion of the numbers above, by seven radii or lines drawn from the centre; next let the respective colours be painted in a lively manner on these spaces, but so as the edge of each colour may be made nearly like the colour next adjoining, that the separation may not be well distinguished by the eye; then if the top be made to spin, the colours will thus seem to be mixed all together, and the whole surface will appear of a uniform whiteness: if a large round black spot be painted in the middle, so as there may be only a broad flat ring of colours around it, the experiment will succeed the better. See Newton's Optics, prop. 6, book 1; and Ferguson's Tracts, pa. 296.

White bodies are found to take heat slower than black ones; because the latter absorb or imbibe rays of all kinds and colours, and the former reflect them. Hence it is that black paper is sooner put in flame, by a burning-glass, than white; and hence also black clothes, hung up in the sun by the dyers, dry sooner than white ones.

WHITEHURST (John), an ingenious English philosopher, was born at Congleton in the county of Cheshire, the 10th of April 1713, being the son of a clock and watch-maker there. Of the early part of his life but little is known. On his quitting school, where it seems the education he received was very defective, he was brought up by his father to his own profession, in which he soon gave hopes of his future eminence.

It was early in life that, from his vicinity to the many stupendous phenomena in Derbyshire, which were constantly presented to his observation, his attention was excited to inquire into the various causes of them.

At about the age of 21, his eagerness after new ideas carried him to Dublin, having heard of an ingenious piece of mechanism in that city, being a clock with certain curious appendages, which he was very desirous of seeing, and no less so of conversing with the maker. On his arrival however, he could neither procure a sight of the former, nor draw the least hint from the latter, concerning it. Thus disappointed, he fell upon an expedient for accomplishing his design; and accordingly took up his residence in the house of the mechanic, paying the more liberally for his board, as he thus had hopes of more readily obtaining the indulgence wished for. He was accommodated with a room directly over that in which the favourite piece was kept carefully locked up: and he had not long to wait for his gratification; for the artist, while one day employed in examining his machine, was suddenly called down stairs; which the young inquirer happening to overhear, softly slipped into the room, inspected the machine, and, presently satisfying himself as to the secret, escaped undiscovered to his own apartment. His end thus compassed, he shortly after bade the artist farewell, and returned to his father in England.

About two or three years after his return from Ireland, he left Congleton, and entered into business for himself at Derby, where he soon procured great employment, and distinguished himself very much by several ingenious pieces of mechanism, both in his own regular line of business, and in various other respects; as, in the construction of curious thermometers, barometers, and other philosophical instruments, as well as in ingenious contrivances for water-works, and the erection of various larger machines: being consulted in almost all the undertakings in Derbyshire, and in the neighbouring counties, where the aid of superior skill in mechanics, pneumatics, and hydraulics, was requisite.

In this manner his time was fully and usefully employed in the country, till, in 1775, when the act passed for the better regulation of the gold coin, he was appointed stamper of the money-weights; an office conferred on him, altogether unexpectedly, and without solicitation. On this occasion he removed to London, where he spent the remainder of his days, in the constant habits of cultivating some useful parts of philosophy and mechanism. And here his house became also the constant resort of the ingenious and scientific at large, of whatever nation or rank, and this to such a degree, as very often to impede him in the regular prosecution of his own speculations.

In 1778, Mr. Whitehurst published his Inquiry into the Original State and Formation of the Earth; of which a second edition appeared in 1786, considerably enlarged and improved; and a third in 1792. This was the labour of many years; and the numerous investigations necessary to its completion, were in themselves also of so untoward a nature, as at times, though he was naturally of a strong constitution, not a little to prejudice his health. When he first entered on this species of research, it was not altogether with a view to investigate the formation of the earth, but in part to obtain such a competent knowledge of subterraneous geography as might become subservient to the purposes of human life, by leading mankind to the discovery of many valuable substances which lie concealed in the lower regions of the earth.

May the 13th, 1779, he was elected and admitted a Fellow of the Royal Society. He was also a member of some other philosophical societies, which appointed him of their respective bodies, without his previous knowledge; but so remote was he from any thing that might savour of ostentation, that this circumstance was known only to a very few of his most confidential friends. Before he was admitted a member of the Royal Society, three several papers of his had been inserted in the Philosophical Transactions, viz, Thermometrical Observations at Derby, in vol. 57; An Account of a Machine for raising Water, at Oulton, in Cheshire, in vol. 65; and Experiments on Ignited Substances, vol. 66: which three papers were printed afterwards in the collection of his works in 1792.

In 1783 he made a second visit to Ireland, with a view to examine the Giants' Causeway, and other northern parts of that island, which he found to be chiefly composed of volcanic matter: an account and representations of which are inserted in the latter editions of his Inquiry. During this excursion, he erected an engine, for raising water from a well, to the summit of a hill, in a bleaching ground, at Tullidoi, in the county of Tyrone: it is worked by a current of water, and for its utility is perhaps unequalled in any country.

In 1787 he published, An Attempt toward obtaining Invariable Measures of Length, Capacity, and Weight, from the Mensuration of Time. His plan is, to obtain a measure of the greatest length that conveniency will permit, from two pendulums whose vibrations are in the ratio of 2 to 1, and whose lengths coincide nearly with the English standard in whole numbers. The numbers which he has chosen show much ingenuity. On a supposition that the length of a seconds pendulum, in the latitude of London, is 39⅕ inches, the length of one vibrating 42 times in a minute, must be 80 inches; and of another vibrating 84 times in a minute must be 20 inches; their difference, 60 inches, or 5 feet, is his standard measure. By the experiments however, the difference between the lengths of the two pendulum rods, was found to be only 59·892 inches, instead of 60, owing to the error in the assumed length of the seconds pendulum, $39\frac{1}{5}$ inches being greater than the truth, which ought to be $39\frac{1}{8}$ very nearly. By this experiment, Mr. Whitehurst obtained a fact, as accurately as may be in a thing of this nature, viz, the difference between the lengths of two pendulum rods whose vibrations are known: a datum from which may be obtained, by calculation, the true lengths of pendulums, the spaces through which heavy bodies fall in a given time, and many other particulars relating to the doctrine of gravitation, the figure of the earth, &c, &c.

Mr. Whitehurst had been at times subject to slight attacks of the gout, and he had for several years felt himself gradually declining. By an attack of that disease in his stomach, after a struggle of two or three months, it put an end to his laborious and useful life, on the 18th of February 1788, in the 75th year of his age, at his house in Bolt-court, Fleet-street, being the same house where another eminent self-taught philosopher, Mr. James Ferguson, had just before him lived and died.

For several years before his death, Mr. Whitehurst had been at times occupied in arranging and completing some papers, for a treatise on Chimneys, Ventilation, and Garden-stoves; which have since been collected and given to the public, by Dr. Willan, in 1794.

However respectable Mr. Whitehurst may have been in mechanics, and those parts of natural science which he more immediately cultivated, he was of still higher account with his acquaintance and friends on the score of his moral qualities. To say nothing of the uprightness and punctuality of his dealings in all transactions relative to business; few men have been known to possess more benevolent affections than he, or, being possessed of such, to direct them more judiciously to their proper ends. With regard to his person, he was above the middle stature, rather thin than otherwise, and of a countenance expressive at once of penetration and mildness. His fine gray locks, unpolluted by art, gave a venerable air to his appearance. In dress he was plain, in diet temperate, in his general intercourse with mankind easy and obliging. In company he was cheerful or grave alike, according to the dictate of the occasion; with now and then a peculiar species of humour about him, delivered with such gravity of manner and utterance, that those who knew him but slightly were apt to understand him as serious, when he was merely playful. But where any desire of information on subjects in which he was conversant was expressed, he omitted no opportunity of imparting it.

WHITSUNDAY, the 50th day or seventh Sunday from Easter.—The season properly called Pentecost, is popularly called Whitsuntide; because, it is said, in the primitive church, the newly baptized persons came to church between Easter and Pentecost in white garments.

WILKINS (Dr. John), a very ingenious and learned English bishop and mathematician, was the son of a goldsmith at Oxford, and born in 1614. After being educated in Greek and Latin, in which he made a very quick progress, he was entered a student of New Inn in that university, when he was but 13 years of age; but after a short stay there, he was removed to Magdalen Hall, where he took his degrees. Having entered into holy orders, he first became chaplain to William Lord Say, and afterwards to Charles Count Palatine of the Rhine, with whom he continued some time. Adhering to the parliament during the civil wars, they made him warden of Wadham college about the year 1648. In 1656 he married the sister of Oliver Cromwell, then lord protector of England, who granted him a dispensation to hold his wardenship, notwithstanding his marriage. In 1659, he was by Richard Cromwell made master of Trinity college in Cambridge; but ejected the year following, on the restoration. He was then chosen preacher to the society of Gray's Inn, and rector of St. Lawrence Jewry, London, on the promotion of Dr. Seth Ward to the bishoprick of Exeter. He was one of the first members of the Royal Society, was chosen of their council, being indeed their first chief secretary, and proved one of their most eminent members. He was afterwards made dean of Rippon, and in 1668 bishop of Chester; but died of the stone in 1672, at 58 years of age.

Bishop Wilkins was a man who thought it prudent to submit to the powers in being; he therefore subscribed to the solemn league and covenant, while it was enforced; and was equally ready to swear allegiance to king Charles when he was restored: this, with his moderate spirit towards dissenters, rendered him not very agreeable to the churchmen; and yet several of them could not but give him one of the best of characters. Burnet writes, that "he was a man of as great a mind, as true a judgment, as eminent virtues, and of as good a soul, as any he ever knew: that though he married Cromwell's sister, yet he made no other use of that alliance, but to do good offices, and to cover the university of Oxford from the sourness of Owen and Goodwin. At Cambridge, he joined with those who studied to propagate better thoughts, to divert men from parties, or from narrow notions, from superstitious conceits, and fierceness about opinions. He was also a great observer and promoter of experimental philosophy, which was then quite a new subject, and much sought after. He was naturally ambitious, but was the wisest clergyman I ever knew. He was a lover of mankind, and had a delight in doing good." The same historian mentions afterwards another quality which Wilkins possessed in a supreme degree, and which it was well for him he did, since he had great occasion for the use of it; and that was, says he, "a courage, which could stand against a current, and against all the reproaches with which ill-natured clergymen studied to load him."

Of his publications, which are all of them very ingenious and learned, and many of them particularly curious and entertaining, the first was in 1638, when he was only 24 years of age, viz, The Discovery of a New World; or, A Discourse to prove, that it is probable there may be another Habitable World in the Moon; with a Discourse concerning the Possibility of a Passage thither.—In 1640, A Discourse concerning a New Planet, tending to prove that it is probable our earth is one of the Planets.—In 1641, Mercury; or, the Secret and Swift Messenger; showing, how a man may with Privacy and Speed communicate his Thoughts to a Friend at any Distance, 8vo.—In 1648, Mathematical Magic; or, the Wonders that may be performed by Mathematical Geometry, 8vo. All these pieces were published entire in one volume 8vo, in 1708, under the title of, The Mathematical and Philosophical Works of the Right Rev. John Wilkins, &c; with a print of the author and general title page handsomely engraven, and an account of his life and writings. To this collection is also subjoined an abstract of a larger work, printed in 1668, folio, entitled, An Essay towards a Real Character and a Philosophical Language. These were all his mathematical and philosophical works; beside which, he wrote several tracts in theology, natural religion, and civil polity, which were much esteemed for their piety and moderation, and went through several editions. He was also the inventor of the Perambulator, or Measuring-wheel.

WILSON (ALEX.) M. D. was professor of astronomy in the university of Glasgow, and also very respectably learned in other arts and sciences, and was author of some ingenious papers in the Philos. Trans. He was also remarkably eminent as a founder of printing-types, an art which he carried to a high state of excellence. Dr. Wilson died Oct. 18, 1786, and was succeeded, in both his professions, by his ingenious and learned son.

WINCH, a popular term for a windlass. Also the bent handle for turning round wheels, grind-stones, &c.

WIND, a current or stream of air, especially when it is moved by some natural cause. Winds are denominated from the point of the compass or horizon they blow from; as the east wind, north wind, south wind, &c. Winds are also divided into several kinds; as general, particular, perennial, stated, variable, &c.

Constant or *Perennial* WINDS, are those that always blow the same way; such as the remarkable one between the two tropics, blowing constantly from east to west, called also the general trade-wind.

Stated or *Periodical* WINDS, are those that constantly return at certain times. Such are the sea and land breezes, blowing from land to sea in the morning, and from sea to land in the evening. Such also are the shifting or particular trade-winds, which blow one way during certain months of the year, and the contrary way the rest of the year.

Variable or *Erratic* WINDS, are such as blow without any regularity either as to time, place, or direction. Such are the winds in the interior parts of England, &c: though some of these claim their certain times of the day; as, the north wind is most frequent in the morning, the west wind about noon, and the south wind in the night.

General WIND, is such as blows at the same time the same way, over a very large tract of ground, most part of the year; as the general trade-wind.

Particular WINDS, include all others, except the general trade winds. Those peculiar to one little canton or province, are called topical or provincial winds. The winds are also divided, with respect to the points of the compass or of the horizon, into cardinal and collateral.

Cardinal WINDS, are those blowing from the four cardinal points, east, west, north, and south.

Collateral WINDS, are the intermediate winds between any two cardinal winds, and take their names from the point of the compass or horizon they blow from.

In navigation, when the wind blows gently, it is called a breeze; when it blows harder, it is called a gale, or a stiff gale; and when it blows very hard, a storm. For a particular account of the trade-winds, monsoons, &c, see Philos. Trans. No. 183, or Abridg. vol. 1, pa. 375. Also Robertson's Navigation, book 5, sect. 6.

A wind blowing from the sea, is always moist; as bringing with it the copious evaporation and exhalations from the waters: also, in summer, it is cool; and in winter warm. On the contrary, a wind from the continent, is always dry; warm in summer, and cold in winter. Our northerly and southerly winds however, which are usually accounted the causes of cold and warm weather, Dr. Derham observes, are really rather the effect of the cold or warmth of the atmosphere. Hence it is that we often find a warm southerly wind suddenly change to the north, by the fall of snow or hail; and in a cold frosty morning, we find the wind north, which afterward shifts about to the southerly quarter, when the sun has well warmed the air; and again in the cold evening, turns northerly, or easterly.

Physical Cause of WINDS. Some philosophers, as Descartes, Rohault, &c, account for the general wind, from the diurnal rotation of the earth; and from this general wind they derive all the particular ones. Thus, as the earth turns eastward, the particles of the air near the equator, being very light, are left behind; so that, in respect of the earth's surface, they move westwards, and become a constant easterly wind, as they are found between the tropics, in those parallels of latitude where the diurnal motion is swiftest. But yet, against this hypothesis, it is urged, that the air, being kept close to the earth by the principle of gravity, would in time acquire the same degree of velocity that the earth's surface moves with, as well in respect of the diurnal rotation, as of the annual revolution about the sun, which is about 30 times swifter.

Dr. Halley therefore substitutes another cause, capable of producing a like constant effect, not liable to the same objections, but more agreeable to the known properties of the elements of air and water, and the laws of the motion of fluid bodies. And that is the action of the sun's beams, as he passes every day over the air, earth, and water, combined with the situation of the adjoining continents. Thus, the air which is less rarefied or expanded by heat, must have a motion towards those parts which are more rarefied, and less ponderous, to bring the whole to an equilibrium; and as the sun keeps continually shifting to the westward, the tendency of the whole body of the lower air is that way. Thus a general easterly wind is formed, which being impressed on the air of a vast ocean, the parts impel one another, and so keep moving till the next return of the sun, by which so much of the motion as was lost, is again restored; and thus the easterly wind is made perpetual. But as the air towards the north and south is less rarefied than in the middle, it follows that from both sides it ought to tend towards the equator.

This motion, compounded with the former easterly wind, accounts for all the phenomena of the general trade-winds, which, if the whole surface of the globe were sea, would blow quite round the world, as they are found to do in the Atlantic and the Ethiopic oceans. But the large continents of land in this middle tract, being excessively heated, communicate their heat to the air above them, by which it is exceedingly rarefied, which makes it necessary that the cooler and denser air should rush in towards it, to restore the equilibrium. This is supposed to be the cause why, near the coast of Guinea, the wind always sets in on the land, blowing westerly instead of easterly.

From the same cause it happens, that there are such constant calms in that part of the ocean called the rains; for this tract being placed in the middle, between the westerly winds blowing on the coast of Guinea, and the easterly trade-winds blowing to the westward of it; the tendency of the air here is indifferent to either, and so stands in equilibrio between both; and the weight of the incumbent atmosphere being diminished by the continual contrary winds blowing hence, is the reason that the air here retains not the copious vapour it receives, but lets it fall in so frequent rains.

It is also to be considered, that to the northward of the Indian ocean there is every where land, within the usual limits of the latitude of 30°, viz, Arabia, Persia, India, &c, which are subject to excessive heats when the sun is to the north, passing nearly vertical; but which are temperate enough when the sun is removed towards the other tropic, because of a ridge of mountains at some distance within the land, said to be often in winter covered with snow, over which the air as it passes must needs be much chilled. Hence it happens that the air coming, according to the general rule, out of the north-east, to the Indian sea, is sometimes hotter, sometimes colder, than that which, by a circulation of one current over another, is returned out of the south-west; and consequently sometimes the under current, or wind, is from the north-east, sometimes from the south-west.

That this has no other cause, appears from the times when these winds set in, viz, in April: when the sun begins to warm these countries to the north, the south-west monsoons begin, and blow during the heats till October, when the sun having retired, and all things growing cooler northward, but the heat increasing to the south, the north-east winds enter, and blow all the winter, till April again. And it is doubtless from the same principle, that to the southward of the equator, in part of the Indian ocean, the north-west winds succeed the south-east, when the sun draws near the tropic of Capricorn. Philos. Trans. No. 183.

But some philosophers, not satisfied with Dr. Halley's theory above recited, or thinking it not sufficient for explaining the various phenomena of the wind, have had recourse to another cause, viz, the gravitation of the earth and its atmosphere towards the sun and moon, to which the tides are confessedly owing. They allege that, though we cannot discover aërial tides, of ebb or flow, by means of the barometer, because columns of air of unequal height, but different density, may have the same pressure or weight; yet the protuberance in the atmosphere, which is continually following the moon, must, say they, occasion a motion in all parts, and so produce a wind more or less to every place, which conspiring with, or being counteracted by, the winds arising from other causes, makes them greater or less. Several dissertations to this purpose were published, on occasion of the subject proposed by the Academy of Sciences at Berlin, for the year 1746. But Musschenbroek will not allow that the attraction of the moon is the cause of the general wind; because the east wind does not follow the motion of the moon about the earth; for in that case there would be more than 24 changes, to which it would be subject in the course of a year, instead of two. Introd. ad Phil. Nat. vol. 2, pa. 1102.

And Mr. Henry Eeles, conceiving that the rarefaction of the air by the sun cannot simply be the cause of all the regular and irregular motions which we find in the atmosphere, ascribes them to another cause, viz, the ascent and descent of vapour and exhalation, attended by the electrical fire or fluid; and on this principle he has endeavoured to explain at large the general phenomena of the weather and barometer. Philos. Trans. vol. 49, pa. 124.

Laws of the Production of WIND.

The chief laws concerning the production of wind, may be collected under the following heads.

1. If the spring of the air be weakened in any place more than in the adjoining places, a wind will blow through the place where the diminution is; because the less elastic or forcible will give way to that which is more so, and thence induce a current of air into that place, or a wind. Hence, because the spring of the air increases, as the compressing weight increases, and compressed air is denser than that which is less compressed; all winds blow into rarer air, out of a place filled with a denser.

2. Therefore, because a denser air is specifically heavier than a rarer; an extraordinary lightness of the air in any place must be attended with extraordinary winds, or storms. Now, an extraordinary fall of the mercury in the barometer showing an extraordinary lightness of the atmosphere, it is no wonder if that foretels storms of wind and rain.

3. If the air be suddenly condensed in any place, its spring will be suddenly diminished; and hence, if this diminution be great enough to affect the barometer, a wind will blow through the condensed air. But since the air cannot be suddenly condensed, unless it has before been much rarefied, a wind will blow through the air, as it cools, after having been violently heated.

4. In like manner, if air be suddenly rarefied, its spring is suddenly increased; and it will therefore flow through the air not acted on by the rarefying force. Hence a wind will blow out of a place, in which the air is suddenly rarefied; and on this principle probably it is, that the sun, by rarefying the air, must have a great influence on the production of winds.

5. Most caves are found to emit wind, either more or less. Musschenbroek has enumerated a variety of causes that produce winds, existing in the bowels of the earth, on its surface, in the atmosphere, and above it. See Introd. ad Phil. Nat. vol. 2, pa. 1116.

6. The rising and changing of the winds are determined by weathercocks, placed on the tops of high buildings, &c. But these only indicate what passes about their own height, or near the surface of the earth. And Wolfius assures us, from observations of several years, that the higher winds, which drive the clouds, are different from the lower ones, which move the weathercocks. Indeed it is no uncommon thing to see one tier of clouds driven one way by a wind, and another tier just over the former driven the contrary way, by another current of air, and that often with very different velocities. And the late experiments with air balloons have proved the frequent existence of counter winds, or currents of air, even when it was not otherwise visible, nor at all expected; by which they have been found to take very different and unexpected courses, as they have ascended to higher elevations in the atmosphere.

Laws of the Force and Velocity of the WIND.

Wind being only air in motion, and the motion of a fluid against a body at rest, creating the same resistance as when the body moves with the same velocity through the fluid at rest; it follows, that the force of the wind, and the laws of its action on bodies, may be referred to those of their resistance when moved through it; and as these circumstances have been treated pretty fully under the article RESISTANCE *of the Air*, there is no occasion here to make a repetition of them. We there laid down both the quantity and laws of such a force, on bodies of different shapes and sizes, moving with all degrees of velocity up to 2000 feet per second, and also for planes set at all degrees of obliquity, or inclination to the direction of motion; all these circumstances having, for the first time, been determined by real experiments.

As to the Velocity of the Wind: philosophers have made use of various methods for determining it. The method employed by Dr. Derham, was by letting light downy feathers fly in the air, and nicely observing the distance to which they were carried in any number of half seconds. He says that he thus measured the velocity of the wind in the great storm of August 1705, which he found moved at the rate of 33 feet in half a second, or 45 miles per hour: whence he concludes, that the most vehement wind does not fly at the rate of above 50 or 60 miles an hour; and that at a medium the velocity of wind is at the rate of 12 or 15 miles per hour. Philos. Trans. No. 313.

Mr. Brice observes however, that experiments with feathers are liable to much uncertainty; as they hardly ever go forward in a straight direction, but spirally, or else irregularly from side to side, or up and down.

He therefore considers the motion of a cloud, by means of its shadow over the surface of the earth, as a much more accurate measure of the velocity of the wind. In this way he found that the wind, in a considerable storm, moved at the rate of near 63 miles an hour; and when it blew a fresh gale, at the rate of 21 miles per hour; and in a small breeze it was near 10 miles an hour. Philos. Trans. vol. 56, pa. 226.

In the Philos. Trans. for 1759, pa. 165, Mr. Smeaton has given a table, communicated to him by a Mr. Rouse, for showing the force of the wind, with several different velocities, which is here inserted below, as I find the numbers nearly agree with my own experiments made on the resistance of the air, when the resisting surfaces are reduced to the same size, by a due proportion for the resistance, which is in a higher degree than that of the surfaces. The table of my results is printed under the article Anemometer.

A Table of the different Velocities and Forces of the Wind, according to their common appellations.

Velocity of the Wind.		Perpendicular force on 1 sq. foot, in avoirdupois pounds.	Common appellations of the Winds.
Miles in one hour.	= feet in one second.		
1	1·47	·005	Hardly perceptible.
2	2·93	·020	Just perceptible.
3	4·40	·044	
4	5·87	·079	Gentle pleasant wind.
5	7·33	·123	
10	14·67	·492	Pleasant brisk gale.
15	22·00	1·107	
20	29·34	1·968	Very brisk.
25	36·67	3·075	
30	44·01	4·429	High Winds.
35	51·34	6·027	
40	58·68	7·873	Very high.
45	66·01	9·963	
50	73·35	12·300	A storm or tempest.
60	88·02	17·715	A great storm.
80	117·36	31·490	A hurricane.
100	146·70	49·200	A hurricane that tears up trees, and carries buildings &c before it.

The velocity and force of the wind are also determined experimentally by various machines, called anemometers, wind-measurers, or wind-gages; the description of which see under these articles.

The force of the wind is nearly as the square of the velocity, or but little above it, in these velocities. But the force is much more than in the simple ratio of the surfaces, with the same velocity, and this increase of the ratio is the more, as the velocity is more. By accurate experiments with two planes, the one of 17¾ square inches, the other of 32, which are nearly in the ratio of 5 to 9, I found their resistances, with a velocity of 20 feet per second, to be, the one 1·196 ounces, and the other 2·542 ounces; which are in the ratio of 8 to 17, being an increase of between $\frac{1}{5}$ and $\frac{1}{6}$ part more than the ratio of the surfaces.

WINDAGE, *of a Gun,* is the difference between the diameter of the bore of the gun, and the diameter of the ball. Formerly the windage appointed in the English service, viz, $\frac{1}{20}$ of the diameter of the bore, of long usage, has been far too much, perhaps owing to the first want of roundness in the ball, or to rust, foulness, and irregularities in the bore of the gun. But lately a beginning has been made to diminish the windage, which cannot fail to be of great advantage; as the shot will go much truer, and have less room to bounce about from side to side, to the great damage of the gun: and besides, much less powder will serve for the same effect, as in some cases $\frac{1}{3}$ or $\frac{1}{2}$ the inflamed powder escapes by the windage. The French allowance of windage is $\frac{1}{25}$ of the diameter. For more on this subject, see the experiments described in my Tracts, vols. 2 and 3.

WINDLASS, or **Windlace,** a particular machine used for raising heavy weights, as guns, stones, anchors, &c.

This is a very simple machine, consisting only of an axis or roller, supported horizontally at the two ends by two posts and a pulley: the two posts meet at top, being placed diagonally, so as to prop each other; and the axis or roller goes through the two posts, and turns in them; the pulley being fastened at top, where the posts join. Lastly, there are two staves or handspikes, which go through the roller, to turn it by; and the rope, which goes over the pulley, is wound on and off the same.

Windlass, in a Ship, is an instrument in small ships placed upon deck, but just abaft the foremost. It consists of a stout piece of timber, in form of an axletree, placed horizontally on two pieces of wood at the ends, on which it is turned about by means of handspikes, put into holes made for that purpose. This instrument serves for weighing anchors, or hoisting any great weight in or out of the ship; it will purchase much more than any capstan, and that without any danger to those who heave; for if, in heaving the windlass about, any of the handspikes should happen to break, the windlass would stop of itself.—See fig. 15, pl. 40.

Wind-*Gage*, in Pneumatics, an instrument serving to determine the velocity and force of the wind. See Anemometer, Anemoscope, and the article just above, concerning the force and velocity of the wind.

Dr. Hales had various contrivances for this purpose. He found (Statical Essays, vol. 2, pa. 326) that the air rushed out of a smith's bellows, at the rate of 68¾ feet in a second of time, when compressed with a force of half a pound on every square inch lying on the whole upper surface of the bellows. The velocity of the air, as it passed out of the trunk of his ventilators, was found to be at the rate of 3000 feet in a minute, which is at the rate of 34 miles an hour. The same author observes, that the velocity with which impelled air passes out at any orifice, may be determined by hanging a light valve over the nose of a bellows, by pliant leathern hinges, which will be much agitated and lifted up from a perpendicular to a more than horizontal position by the force of the rushing air. There is also another more accurate way, he says, of estimating the velocity of air, viz, by holding the orifice of an inverted glass siphon full of water, opposite to the stream of air, by which the water will be depressed in one leg, and raised in the other, in proportion to the force with which the water is impelled by the air. Descrip. of Ventilators, 1743, pa. 12. And this perhaps gave Dr. Lind the idea of his wind-gage, mentioned below.

M. Bouguer contrived a simple instrument, by which may be immediately discovered the force which the wind exerts on a given surface. This is a hollow tube, AABB (fig. 14, pl. 40), in which a spiral spring CD is fixed, that may be more or less compressed by a rod FSD, passing through a hole within the tube at AA. Then having observed to what degree different forces or given weights are capable of compressing the spiral, mark divisions on the rod in such a manner, that the mark at S may indicate the weight requisite to force the spring into the situation CD: afterwards join at right angles to this rod at F, a plane surface EFE of any given area at pleasure; then let this instrument be opposed to the wind, so that it may strike the surface perpendicularly, or parallel to the rod; then will the mark at S show the weight to which the force of the wind is equivalent.

Dr. Lind has also contrived a simple and easy apparatus of this kind, nearly on the last idea of Dr. Hales mentioned above. This instrument is fully explained under the article ANEMOMETER, and a figure of it given, pl. 3, fig. 4.

Mr. Benjamin Martin, from a hint first suggested by Dr. Burton, contrived an anemoscope, or wind-gage, of a construction like a wind-mill, with four sails; but the axis which the sails turn, is not cylindrical, but conical, like the fusee of a watch; about this fusee winds a cord, having a weight at the end, which is wound always, by the force of the wind, on the sails, till the weight just balances that force, which will be at a thicker part of the fusee when the wind is strong, and at a smaller part of it when it is weaker. But though this instrument shows when a wind is stronger or weaker, it will neither show what is the actual velocity of the wind, nor yet its force upon a square foot of direct surface; because the sails are set at an uncertain oblique angle to the wind, and this acts at different distances from the axis or centre of motion. Martin's Phil. Brit. vol. 2, pa. 211. See the fig. 5. plate 3, vol. 1.

WIND-*Gun*, the same as AIR-*Gun*; which see.

WIND-*Mill*, a kind of mill which receives its motion from the impulse of the wind.—The internal structure of the windmill is much the same with that of watermills: the difference between them lying chiefly in an external apparatus, for the application of the power. This apparatus consists of an axis EF (fig. 11, pl. 41), through which pass perpendicular to it, and to each other, two arms or yards, AB and CD, usually about 32 feet long: on these yards are formed a kind of sails, vanes, or flights, in a trapezoid form, with parallel ends; the greater of which HI is about 6 feet, and the less FG are determined by radii drawn from the centre E, to I and H.

These sails are to be capable of being always turned to the wind, to receive its impulse: for which purpose there are two different contrivances, which constitute the two different kinds of windmills in common use.

In the one, the whole machine is supported upon a moveable arbor, or axis, fixed upright on a stand or foot; and turned round occasionally to suit the wind, by means of a lever.

In the other, only the cover or roof of the machine, with the axis and sails, in like manner turns round with a parallel or horizontal motion. For this purpose, the cover is built turret-wise, and encompassed with a wooden ring, having a groove, at the bottom of which are placed, at certain distances, a number of brass truckles; and within the groove is another ring, on which the whole turret stands. To the moveable ring are connected beams *ab* and *fe*; and to the beam *ab* is fastened a rope at *b*, having its other end fitted to a windlass, or axis-in-peritrochio: this rope being drawn through the iron hook G, and the windlass turned, the sails are moved round, and set fronting the wind, or with the axis pointing straight against the wind.

The internal mechanism of a windmill is exhibited in fig. 12; where AHO is the upper room, and HOZ the lower one; AB the axle-tree passing through the mill; STVW the sails covered with canvas, set obliquely to the wind, and turning round in the order of the letters; CD the cog-wheel, having about 48 cogs or teeth, *a*, *a*, *a*, &c, which carry round the lantern EF, having 8 or 9 trundles or rounds, *c*, *c*, *c*, &c, together with its upright axis GN; IK is the upper mill-stone, and LM the lower; QR is the bridge, supporting the axis or spindle GN; this bridge is supported by the beams *cd*, XY, wedged up at *c*, *d* and X; ZY is the lifting tree, which stands upright; *ab* and *ef* are levers, whose centres of motion are *z* and *e*; *fghi* is a cord, with a stone *i*, going about the pins *g* and *h*, and serving as a balance or counterpoise. The spindle *t*N is fixed to the upper millstone IK, by a piece of iron called the rynd, and fixed in the lower side of the stone, which is the only one that turns about, and its whole weight rests on a hard stone, fixed in the bridge QR at N. The trundle EF, and its axis G*t*, may be taken away; for it rests by its lower part at *t* by a square socket, and the top runs in the edge of the beam *w*. By bearing down the end *f* of the lever *fe*, *b* is raised, which also raises ZY, and this raises YX, which lifts up the bridge QR, with the axis NG, and the upper stone IK; and thus the stones are set at any distance. The lower or immoveable stone is fixed upon strong beams, and is broader than the upper one: the meal is conveyed through the tunnel *no* into a chest; P is the hopper, into which is put the corn, which runs through the spout *r* into the hole *t*, and so falls between the stones, where it is ground to meal. The axis G*t* is square, which shaking the spout *r*, as it goes round, makes the corn run out; *rs* is a string going about the pin *s*, and serving to move the spout nearer to the axis or farther from it, so as to make the corn run faster or slower, according to the velocity and force of the wind. And when the wind is strong, the sails are only covered in part, or on one side, or perhaps only one half of two opposite sails. Toward the end B of the axletree is placed another cogwheel, trundle, and millstones, with an apparatus like that just described; so that the same axis moves two stones at once;

and when only one pair is to grind, one of the trundles and its spindle are taken out: *xyl* is a girth of pliable wood, fixed at the end *x*; the other end *l* being tied to the lever *km*, moveable about *k*; and the end *m* being put down, draws the girth *xyl* close to the cogwheel, which gently and gradually stops the motion of the mill, when required: *pq* is a ladder for ascending to the higher part of the mill; and the corn is drawn up by means of a rope, rolled about the axis AB, when the mill is at work. See MILL.

Theory of the WINDMILL, *Position of the Sails, &c.*

Were the sails set square on their arms or yards, and perpendicular to the axletree, or to the wind, no motion would ensue, because the direct wind would keep them in an exact balance. But by setting them obliquely to the common axis, like the sails of a smoke-jack, or inclined like the rudder of a ship, the wind, by striking the surface of them obliquely, turns them about. Now this angle which the sails are to make with their common axis, or the degree of weathering, as the mill-wrights call it, so as that the wind may have the greatest effect, is a matter of nice inquiry, and has much occupied the thoughts of the mathematician and the artist.

In examining the compound motions of the rudder of a ship, we find that the more it approaches to the direction of the keel, or to the course of the water, the more weakly this strikes it; but, on the other hand, the greater is the power of the lever to turn the vessel about. The obliquity of the rudder therefore has, at the same time, both an advantage and a disadvantage. It has been a point of inquiry therefore to find the position of the rudder when the ratio of the advantage over the disadvantage is the greatest. And M. Renau, in his theory of the working of ships, has found, that the best situation of the rudder is when it makes an angle of about 55 degrees with the keel.

The obliquity of the sails, with regard to their axis, has precisely the same advantage, and disadvantage, with the obliquity of the rudder to the keel. And M. Parent, seeking by the new analysis the most advantageous situation of the sails on the axis, finds it the same angle of about 55 degrees. This obliquity has been determined by many other mathematicians, and found to be more accurately 54° 44′. See Maclaurin's Fluxions, pa. 733; Simpson's Fluxions, prob. 17, pa. 521; Martin's Philos. Britan. vol. 1, pa. 220, vol. 2, pa. 212; &c.

This inclination of the sails to their axis, however, is only that which gives the wind the greatest force to put the sail in motion, but not the angle which gives the force of the wind a maximum on the sail when in motion: for when the sail has a certain degree of velocity, it yields to the wind; and then that angle must be increased, to give the wind its full effect. Maclaurin, in his Fluxions, pa. 734, has shown also how to determine this angle.

It may be observed, that the increase of this angle should be different according to the different velocities from the axletree to the further extremity of the sail. At the beginning, or axis, it should be 54° 44′; and thence continually increasing, giving the vane a twist, and so causing all the ribs of the vane to lie in different planes.

It is farther observed, that the ribs of the vane or sail ought to decrease in length from the axis to the extremity, giving the vane a curvilinear form; so that no part of the force of any one rib be spent on the rest, but all move independent of each other. The twist above mentioned, and the diminution of the ribs, are exemplified in the wings of birds.

As the ends of the sail nearest the axis cannot move with the same velocity which the tips or farthest ends have, though the wind acts equally strong on them both, Mr. Ferguson (Lect. on Mech. pa. 52) suggests, that perhaps a better position than that of stretching them along the arms directly from the centre of motion, might be, to have them set perpendicularly across the farther ends of the arms, and there adjusted lengthwise to the proper angle: for in that case both ends of the sails would move with the same velocity; and being farther from the centre of motion they would have so much the more power, and in this case there would be no occasion for having them so large as they are generally made; which would render them lighter, and consequently there would be so much the less friction on the thick neck of the axle, when it turns in the wall.

Mr. Smeaton (Philos. Trans. 1759), from his experiments with windmill sails, deduces several practical maxims: as, 1. That when the wind falls on a concave surface, it is an advantage to the power of the whole, though every part, taken separately, should not be disposed to the best advantage. By several trials he has found that the curved form and position of the sails will be best regulated by the numbers in the following table:

6th Parts of the radius or sail.	Angle with the axis.	Angle with the plane of motion.
1	72°	18°
2	71	19
3	72	18 middle.
4	74	16
5	77½	12½
6	83	7 end.

2. That a broader sail requires a greater angle; and that when the sail is broader at the extremity, than near the centre, this shape is more advantageous than that of a parallelogram.

3. When the sails, made like sectors of circles, joining at the centre or axis, and filled up about 7-8ths of the whole circular space, the effect was the greatest.

4. The velocity of windmill sails, whether unloaded, or loaded so as to produce a maximum of effect, is nearly as the velocity of the wind; their shape and position being the same.

5. The load at the maximum is nearly, but somewhat less than, as the square of the velocity of the wind.

6. The effects of the same sails at a maximum, are nearly, but somewhat less than, as the cubes of the velocity of the wind.

7. In sails of a similar figure and position, the number of turns in a given time, are reciprocally as the radius or length of the sail.

8. The effects of sails of similar figure and position, are as the square of their length.

9. The velocity of the extremities of Dutch mills, as well as of the enlarged sails, in all their usual positions, is considerably greater than the velocity of the wind.

M. Parent, in considering what figure the sails of a windmill should have, to receive the greatest impulse from the wind, finds it to be a sector of an ellipsis, whose centre is that of the axletree of the mill; and the less semiaxis the height of 32 feet; as for the greater, it follows necessarily from the rule that directs the sail to be inclined to the axis in the angle of 55 degrees.

On this foundation he assumes four such sails, each being a quarter of an ellipse; which he shows will receive all the wind, and lose none, as the common ones do. These 4 surfaces, multiplied by the lever with which the wind acts on one of them, express the whole power the wind has to move the machine, or the whole power of the machine when in motion.

The same author also observes that a wind mill with 6 elliptical sails, would still have more power than one with only four. It would only have the same surface with the four; since the 4 contain the whole space of the ellipsis, as well as the 6. But the force of the 6 would be greater than that of the 4, in the ratio of 245 to 231. If it were desired to have only two sails, each being a semi-ellipsis, the surface would still be the same; but the power would be diminished by nearly 1-3d of that with 6 sails; because the greatness of the sectors would much shorten the lever on which the wind acts.

Mr. Parent has also considered which form, among the rectangular sails, will be most advantageous; i. e. that which shall have the product of the surface by the lever of the wind, the greatest. The result of this inquiry is, that the width of the rectangular sail should be nearly double its length; whereas usually the length is made almost 5 times the width.

The power of the mill, with four of these new rectangular sails, the same author shows, will be to the power of four elliptic sails, nearly as 13 to 23: which leaves a considerable advantage on the side of the elliptic ones; and yet the force of the new rectangular sails will still be considerably greater than that of the common ones.

M. Parent also considers what number of the new sails will be most advantageous; and finds that the fewer the sails, the more surface there will be, but the power the less. Farther, the power of a windmill with 6 sails is denoted by 14, that of another with 4 will be as 13, and another with 2 sails will be denoted by 9. That as to the common windmill, its power still diminishes as the breadth of the sails is smaller, in proportion to the length: and therefore the usual proportion of 5 to 1 is exceedingly disadvantageous.

WINDOW, q. d. *wind-door*, an aperture or opening in the wall of a house, to admit the air and light.

Before the use of glass became general, which was not till towards the end of the 12th century, the windows in England seem generally to have been composed of paper, oiled, both to defend it against the weather, and to make it more transparent; as now is sometimes used in workshops and unfinished buildings. Some of the better sort were furnished with lattices of wood or sheets of linen. These it seems were fixed in frames, called capsamenta, and hence our casements still so common in some of the counties.

The chief rules with regard to windows are, 1. That they be as few in number, and as moderate in dimensions, as may be consistent with other respects; inasmuch as all openings are weakenings of the structure.

2. That they be placed at a convenient distance from the angles or corners of the buildings: both for strength and beauty.

3. That they be made all equal one with another, in their rank and order; so that those on the right hand may answer to those on the left; and those above be right over those below, both on account of strength and beauty.

As to their dimensions, care is to be taken, to give them neither more nor less than is needful; regard being had to the size of the rooms, and of the building. The apertures of windows in middle-sized houses, may be from 4 to 5 feet; in the smaller ones less; and in large buildings more. And the height may be double their width at the least: but in lofty rooms, or large buildings, the height may be a 4th, or 3d, or half their breadth more than the double.

Such are the proportions for windows of the first story; and the breadth must be the same in the upper stories; but as to the height, the second story may be a 3d part lower than the first, and the third story a 4th part lower than the second.

WINDWARD, in Sea Language, denotes any thing towards that point whence the wind blows, in respect of a ship.

Sailing to Windward. See Sailing.

Windward *Tide*, a tide that runs against the wind.

WING (Vincent), a considerable mathematician and astrologer of the 17th century, who died about 1668. He was author of several popular astronomical and other works: as, 1. Astronomia Instaurata, fol. 1656.—2. Celestial Harmony of the Visible World, fol. 1657.—3. Astronomia Britannica, fol. 1669, a work of merit.—4. Ephemerides for 13 years, from 1659 to 1671.—5. Computatio Catholica, &c. Mr. Wing was much connected with the Stationers' Company, in the publication of their almanacs, one of which, in a broad sheet, is still continued in his name; and another, a book almanac, was only discontinued a very few years ago.

WINGATE (Edmund), one of the clearest writers on arithmetic &c in the English language, was the son of Roger Wingate, esq. of Bornend and Sharpenkoe, in Bedfordshire, but was born in Yorkshire in 1593. In 1610 he became a commoner of Queen's college, Oxford; but after taking a degree in arts, he removed to Gray's Inn, London, where he studied the law. But his chief inclination was to the mathematics, which he had studied with much success at college. In 1624 he was in France, where he published the Scale or Rule of Proportion, which had been invented by Edmund Gunter, of Gresham college. While in that country, he gave instructions in the English language to the princess Henrietta Maria, afterwards wife of Charles the first, and to her ladies. After his return to England, he became a bencher of Gray's Inn; and on the breaking out of the great rebellion, he joined the popular party, took the covenant, was made justice of the peace for the county of Bedford, where he resided at Woodend, in the parish of Harlington, and his name occurs in the register of Ampthill church, as a justice, in 1654, when, according to the republican custom of that period, marriages were celebrated by the civil magistrates. In 1650 he took the oath, commonly called the Engagement, became intimate with Cromwell, and was chosen into his parliament for Bedford. He was also appointed one of the commissioners, for that county, to eject from their situations, those loyal clergymen and schoolmasters who were accused as being scandalous and ignorant. He died in Gray's Inn, in 1656, and was buried in the parish church of St. Andrew, Holborn.

The works of Mr. Wingate, are,

1. The Use of the Proportional Rules in Arithmetic and Geometry. Also the Use of the Logarithms of Numbers, with those of Sines and Tangents. Printed in French, at Paris, in 1624, 8vo, and at London, in English, in 1626, 1645, and 1658.—In this book, Mr. W. speaks of having

been the first who carried the Logarithms to France; but an edition of Napier's Description and Construction of Logarithms was printed at Lyons in the year 1620, being 4 years earlier than Wingate's publication.

2. Of Natural and Artificial Arithmetic, or Arithmetic made easy; London, 1630, 8vo. It has also gone through numerous other editions, the best of which is that by Mr. Dodson.

3. Tables of Logarithms of the Sines and Tangents of all the Degrees and Minutes of the Quadrant. With the use and application of the same. London, 1633, 8vo.

4. The Construction and Use of Logarithms, with the Resolution of Triangles, &c.

5. Ludus Mathematicus; or an Explanation of the Description, Construction, and Use of the Numerical Table of Proportion. London, 1654, 8vo.

6. Tactometria, seu Tetagne-nometria, or the Geometry of Regulars, &c. 8vo.

7. The Exact Surveyor of Land, &c, 8vo.

8. An exact Abridgement of all Statutes in force and use from the Magna Charta, to 1641, 8vo.

9. The Body of the Common Law of England, &c; 8vo, 1655, &c.

10. Maxims of Reason, or the Reason of the Common Law of England; 1658, folio.

11. Statuta Pacis; or, the Table of all the Statutes which any way concern the office of a Justice of Peace, &c. 12mo.

12. An edition of Britton, a lawyer who wrote in the reign of Edward the 1st; a very useful law book, 1640, 12mo.

Mr. W. it is supposed, was also the editor of some other law books, which show equal judgment and industry; but he is now to be remembered only as a mathematician.

WINTER, one of the four seasons or quarters of the year.—Winter properly commences on the day when the sun's distance from the zenith of the place is the greatest, or when his declination is the greatest on the contrary side of the equator; and it ends on the day when that distance is a mean between the greatest and least, or when he next crosses the equinoctial.

At and near the equator, the winter, as well as the other seasons, return twice every year; but all other places have only one winter in the year; which in the northern hemisphere begins when the sun is in the tropic of Capricorn, and in the southern hemisphere when he is in the tropic of Cancer: so that all places in the same hemisphere have their winter at the same time.

Notwithstanding the coldness of this season, it is proved in astronomy, that the sun is really nearer to the earth in our winter than in summer: the reason of the defect of heat being owing to the lowness of the sun; or to the obliquity of his rays.

WITCHELL (GEORGE), F. R. S. a good astronomer and mathematician, was born in 1728. He was maternally descended from the celebrated clock and watch maker Daniel Quare, in which business he was himself brought up, and was educated in the principles of the Quakers, all his progenitors for many generations having been of that community, and whose simplicity of manners and integrity of character he practised through life. It appears that Mr. W. cultivated the study of astronomy at a very early age indeed, as he had a communication on that subject published in the Gentleman's Diary for 1741, which must have been written before he was 13 years of age. Soon after this he became a pretty constant writer in both the Diaries and the Gentleman's Magazine, a practice which he continued a long time, sometimes under his own name, but more frequently with the initials G. W. only. In 1764 Mr. W. published a map, exhibiting the passage of the moon's shadow over England in the great solar eclipse of April 1 that year; the exact correspondence of which to the observations gained him great reputation. In the following year he presented to the commissioners of longitude a plan for calculating the effects of refraction and parallax, on the moon's distance from the sun or a star, to facilitate the discovery of the longitude at sea. Having been elected F. R. S. and taught mathematics in London for many years with much reputation, he was, in 1767, appointed head master of the Royal Naval Academy at Portsmouth, on the recession of Mr. Robertson; where he died by a paralytic stroke in 1785, at 57 years of age, and was succeeded in that office by Mr. Bailey.

WOLFF, WOLFIUS, (CHRISTIAN,) baron of the Roman empire, privy counsellor to the king of Prussia, and chancellor to the university of Halle in Saxony, as well as member of many of the literary academies in Europe, was born at Breslau in 1679. After studying philosophy and mathematics at Breslau and Jena, he obtained permission to give lectures at Leipsic; which, in 1703, he opened with a dissertation, Philosophia Practica Universalis, Methodo Mathematica conscripta, which served greatly to enhance the reputation of his talents. He published two other dissertations the same year; the first De Rotis Dentatis, the other De Algorithmo Infinitesimali Differentiali; which obtained him the honourable appellation of Assistant to the Faculty of Philosophy at Leipsic.

He now accepted the professorship of mathematics at Halle, and was elected into the society at Leipsic, at that time engaged in publishing the Acta Eruditorum. After having inserted in this work many important pieces relating to mathematics and physics, he undertook, in 1709, to teach all the various branches of philosophy, beginning with a small logical treatise in Latin, being Thoughts on the Powers of the Human Understanding. He carried himself through these great pursuits with amazing assiduity and ardour: the king of Prussia rewarded him with the office of counsellor to the court in 1721, and augmented the profits of that post by very considerable appointments: he was also chosen a member of the Royal Society of London and of Prussia.

In the midst of all this prosperity however, Wolff raised an ecclesiastical storm against himself, by a Latin oration he delivered in praise of the Chinese philosophy: every pulpit immediately resounded against his tenets; and the faculty of theology, who entered into a strict examination of his productions, resolving that the doctrine he taught was dangerous to the last degree, an order was obtained in 1723 for displacing him, and commanding him to leave Halle in 24 hours.

Wolff now retired to Cassel, where he obtained the professorship of mathematics and philosophy in the university of Marbourg, with the title of Counsellor to the Landgrave of Hesse; to which a profitable pension was annexed. Here he renewed his labours with redoubled ardour; and it was in this retreat that he published the greatest part of his numerous works.

In 1725, he was declared an honorary professor of the academy of sciences at Petersburg, and in 1733 was admitted into that of Paris. The king of Sweden also declared him one of the council of regency; but the pleasing situation of his new abode, and the multitude of honours which he had received, were too alluring to permit him to accept of many advantageous offers; among which it has been said, was the office of president of the academy at Petersburg.

The king of Prussia too, who was now recovered from the prejudices he had been made to conceive against Wolff, wanted to re-establish him in the university of Halle in 1733, and made another attempt to effect it in 1739; which Wolff for a time thought proper to decline, but at last submitted: he returned therefore in 1741, invested with the characters of privy counsellor, vice-chancellor, and professor of the law of nature and of nations. The king afterwards, on a vacancy, raised him to the dignity of chancellor of the university; and the elector of Bavaria created him also a baron of the empire. He died at Halle in Saxony, of the gout in his stomach, in 1754, in the 76th year of his age, after a life filled up with a train of actions as wise and systematical as his writings, of which he composed in Latin and German more than 60 distinct pieces. The chief of his mathematical compositions, is his Elementa Matheseos Universæ, the best edition of which is that of 1732 at Geneva, in 5 vols 4to; which does not however comprise his Mathematical Dictionary in the German language, in 1 vol. 8vo, nor many other distinct works on different branches of the mathematics, nor his System of Philosophy, in 23 vols. in 4to.

WORKING *to Windward*, in Sea Language, is the operation by which a ship endeavours to make progress against the wind.

WREN (Sir Christopher), a great philosopher and mathematician, and one of the most learned and eminent architects of his age, was the son of the Rev. Christopher Wren, dean of Windsor, and was born at Knoyle in Wiltshire in 1632. He studied at Wadham college, Oxford; where he took the degree of master of arts in 1653, and was chosen fellow of Allsouls college there. Soon after, he became one of that ingenious and learned society, who then met at Oxford for the improvement of natural and experimental philosophy, and which at length produced the Royal Society.

When very young, he discovered a surprising genius for the mathematics, in which science he made great advances before he was 16 years of age.—In 1657 he was made professor of astronomy in Gresham college, London; and his lectures, which were much frequented, tended greatly to the promotion of real knowledge: in his inaugural oration, among other things, he proposed several methods by which to account for the shadows returning backward 10 degrees on the dial of king Ahaz, by the laws of nature. One subject of his lectures was upon telescopes, to the improvement of which he had greatly contributed; another was on certain properties of the air, and the barometer. In the year 1658 he read a description of the body and different phases of the planet Saturn; which subject he proposed to investigate while his colleague, Mr. Rooke, then professor of geometry, was prosecuting his observations on the satellites of Jupiter. The same year he communicated some demonstrations concerning cycloids to Dr. Wallis, which were afterwards published by the doctor at the end of his treatise on that subject. About that time also, he resolved the problem proposed by Pascal, under the feigned name of John de Montford, to all the English mathematicians; and returned another to the mathematicians in France, formerly proposed by Kepler, and then resolved likewise by himself, to which they never gave any solution.—In 1660, he invented a method for the construction of solar eclipses; and in the latter part of the same year, he with ten other gentlemen formed themselves into a society, to meet weekly, for the improvement of natural and experimental philosophy; being the foundation of the Royal Society.—In the beginning of 1661, he was chosen Savilian professor of astronomy at Oxford, in the room of Dr. Seth Ward; and where he was the same year created Doctor of Laws.

Among his other accomplishments, Dr. Wren had gained so considerable a skill in architecture, that he was sent for the same year, from Oxford, by order of king Charles the 2d, to assist sir John Denham, surveyor general of the works.—In 1663, he was chosen fellow of the Royal Society; being one of those who were first appointed by the Council after the grant of their charter. Not long after, it being expected that the king would make the society a visit, the lord Brounker, then president, by a letter requested the advice of Dr. Wren, concerning the experiments which might be most proper on that occasion; to whom the doctor recommended principally the Torricellian experiment, and the weather needle, as being not mere amusements, but useful, and also neat in their operation. Indeed on many occasions Dr. Wren did great honour to that illustrious body, by many curious and useful discoveries, in astronomy, natural philosophy, and other sciences, related in the History of the Royal Society, where Dr. Sprat has inserted them from the registers and other books of the society to 1665, also in Birch's history. Among others of his productions there enumerated, is a lunar globe; representing the spots and various degrees of whiteness on the moon's surface, with the hills, eminences and cavities: the whole contrived so, that by turning it round to the light, it shows all the lunar phases, with the various appearances that happen from the shadows of the mountains and valleys, &c: this lunar model was placed in the king's cabinet. Another of these productions, is a tract on the Doctrine of Motion that arises from the impact between two bodies, illustrated by experiments: also in the Philos. Trans. vol. 2, pa. 867. And a third is, The History of the Seasons, as to the temperature, weather, productions, diseases, &c, &c. For which purpose he devised also many curious machines, several of which kept their own registers, tracing out the lines of variations, so that a person might know what changes the weather had undergone in his absence: as windgages, thermometers, barometers, hygrometers, raingages, &c.—He made also great additions to the new discoveries on pendulums; and among other things showed, that there may be produced a natural standard for measure from the pendulum for common use.—He invented many ways to make astronomical observations more easy and accurate: he fitted and hung quadrants, sextants, and radii more commodiously than formerly; and also constructed two telescopes to open with a joint like a sector, by which observers may infallibly take a distance to half minutes, &c. He also made various kinds of retes, screws, and other devices, for improving telescopes to take small distances, and apparent diameters, to seconds: he made apertures to admit more or less light, as the observer pleases, by opening and shutting, the better to fit glasses for crepusculine observations.—He added much to the theory of dioptrics; and much also to the manufacture of grinding good glasses: he attempted, and not without success, the making of glasses of other forms than spherical. He exactly measured and delineated the spheres of the humours of the eye, the proportions of which to one another were only guessed at before: a discussion showing the reasons why we see objects erect, and that reflection conduces as much to vision as refraction. He displayed a natural and easy theory of refractions, which exactly answered every experiment. He fully demonstrated the whole doctrine of dioptrics in a few propositions, showing not only, as in Kepler's Dioptrics, the common properties of glasses, but the proportions by which the individual rays cut the axis, and each other, on which the charges of the telescopes, or the proportion of the eye-glasses and apertures, are clearly discovered.—He made constant observations on Saturn, and a true theory of that planet, before the printed discourse by Huygens on that subject appeared.—He also made maps of the Pleiades and other telescopic stars: and proposed methods to determine the great question as to the earth's motion or rest, by the small stars about the pole to be seen in large telescopes.—In navigation also our author made many improvements. He framed a magnetical terella, which he placed in the midst of a plane board with a hole, into which the terella is half immersed, till it be like a globe with the poles in the horizon: the plane is then dusted over with steel filings from a sieve: the dust, by the magnetical virtue, becomes immediately figured into furrows that bend like a sort of helix, proceeding as it were out at one pole, and returning in by the other; the whole plane becoming figured like the circles of a planisphere.—It being a question in his time among the problems of navigation, to what mechanical powers sailing against the wind was reducible; he showed it to be a wedge: he also demonstrated, how a transient force on an oblique plane would cause the motion of the plane against the first mover: and he made an instrument mechanically producing the same effect, and showed the reason of sailing on all winds. The geometrical mechanism of rowing, he showed to be a lever on a moving or cedent fulcrum: for this end, he made instruments and experiments, to find the resistance to motion in a liquid medium; with other circumstances that are the necessary elements for laying down the geometry of sailing, swimming, rowing, flying, and constructing of ships.—He invented a very speedy and curious way of etching. He started many things towards the emendation of water-works. He likewise made some instruments for respiration, and for straining the breath from fuliginous vapours, to try whether the same breath, so purified, will serve again.—He was the first inventor of drawing pictures by microscopical glasses. He invented perpetual, or at least long-lived lamps, for keeping a perpetual regular heat, in order to various uses, as hatching of eggs and insects, production of plants, chemical preparations, imitating nature in producing fossils and minerals, keeping the motion of watches equal, for the longitude and astronomical uses. He was also the first author of the anatomical experiment of injecting liquor into the veins of animals. By this operation, divers creatures were immediately purged, vomited, intoxicated, killed, or revived, according to the quality of the fluid injected. Hence arose many other new experiments, particularly that of transfusing blood, which has been prosecuted in many curious instances. This is a short account of the principal discoveries which Dr. Wren presented, or suggested, to the Royal Society, or were improved by him.

With respect to his architectural works: it has before been observed that he had been sent for to assist sir John Denham. In 1665 he travelled into France, to examine the most beautiful edifices and curious mechanical works there, when he made many useful observations. On his return home, he was appointed architect, and one of the commissioners for repairing St. Paul's cathedral. Within a few days after the fire of London, 1666, he drew a noble plan for a new city, and presented it to the king; but it was not approved of by the parliament. In this model, the chief streets were to cross each other at right angles, with lesser streets between them; the churches, public buildings, &c, so disposed as not to interfere with the streets, and four piazzas placed at proper distances.—On the death of sir John Denham in 1668, our author succeeded him in the office of surveyor-general of the king's works; and from this time he had the direction of a great many public edifices, by which he acquired the most distinguished reputation. He built the magnificent theatre at Oxford, St. Paul's cathedral, the Monument, the modern part of Hampton Court, Chelsea-college, one of the wings of Greenwich hospital, the churches of St. Stephen Walbrook, and St. Mary-le-bow, with upwards of 60 other churches and public works, which that dreadful fire rendered necessary. In the management of which business, he was assisted in the measurements, and laying out of private property, by the ingenious Dr. Robert Hooke. The variety of business in which he was by this means engaged, requiring his constant attendance and concern, he resigned his Savilian professorship at Oxford in 1673; and the year following he received from the king the honour of knighthood.—He was one of the commissioners who, on the motion of sir Jonas Moore, surveyor-general of the ordnance, had been appointed to select a proper place for erecting an observatory; and he proposed Greenwich, which was accordingly approved of; the foundation stone of which was laid the 10th of August 1675, and the building was presently finished under the direction of sir Jonas, with the advice and assistance of sir Christopher.

In 1680 our author was chosen president of the Royal Society; afterwards appointed architect and commissioner of Chelsea-college; and in 1684, principal officer or comptroller of the works in Windsor-castle. Sir Christopher sat twice in Parliament, as a representative for two different boroughs. While he continued surveyor-general, his residence was in Scotland-yard; but after his removal from that office, in 1718, he lived in St. James's-street, Westminster. He died the 25th of February 1723, at 91 years of age; and was interred with great solemnity in St. Paul's cathedral, in the vault under the south wing of the choir, near the east end.

The person of Sir Christopher Wren was of a low stature, and thin frame of body; but by temperance and skilful management he enjoyed a good state of health, to a very unusual length of life. He was modest, devout, strictly virtuous, and very communicative of his knowledge. Besides his peculiar eminence as an architect, his learning and knowledge were very extensive in all the arts and sciences, and especially in the mathematics.

Sir Christopher never printed any thing himself, but several of his works have been published by others: some in the Philosophical Transactions, and some by Dr. Wallis and other friends.—His posthumous works and draughts were published by his son.

WRIGHT (**Edward**), a distinguished English mathematician, who flourished in the latter part of the 16th century, and beginning of the 17th; dying in the year 1615. He was contemporary with Mr. Briggs, and much concerned with him in the business of the logarithms, the short time they were published before his death. He also contributed greatly to the improvement of navigation and astronomy. The following memoirs of him are translated from a Latin paper in the annals of Gonville and Caius-college in Cambridge, viz, "This year (1615) died at London, Edward Wright of Garveston in Norfolk, formerly a fellow of this college; a man respected by all for the integrity and simplicity of his manners, and also famous for his skill in the mathematical sciences: so that he was not undeservedly styled a most excellent mathematician by Richard Hackluyt, the author of an original treatise of our English navigations. What knowledge he had acquired in the science of mechanics, and how usefully he employed that knowledge to the public as well as private advantage, abundantly appear both from the writings he published, and from the many mechanical operations still extant, which are standing monuments of his great industry and ingenuity. He was the first undertaker of that difficult but useful work, by which a little river is brought from the town of Ware in a new canal, to supply the city of London with water; but by the tricks of others he was hindered from completing the work he had begun. He was excellent both in contrivance and execution, nor was he inferior to the most ingenious mechanic in the making of instruments, either of brass or any other matter. To his invention is owing whatever advantage Hondius's geographical charts have above others; for it was Wright who taught Jodocus Hondius the method of constructing them, which was till then unknown: but the ungrateful Hondius concealed the name of the true author, and arrogated the glory of the invention to himself. Of this fraudulent practice the good man could not help complaining, and justly enough, in the preface to his treatise of the Correction of Errors in the Art of Navigation; which he composed with excellent judgment, and after long experience, to the great advancement of naval affairs. For the improvement of this art he was appointed mathematical lecturer by the East-India Company, and read lectures in the house of that worthy knight sir Thomas Smith, for which he had a yearly salary of 50 pounds. This office he discharged with great reputation, and much to the satisfaction of his hearers. He published in English a book on the doctrine of the sphere, and another concerning the construction of sun dials. He also prefixed an ingenious preface to the learned Gilbert's book on the loadstone. By these and other his writings, he has transmitted his fame to latest posterity. While he was yet a fellow of this college, he could not be concealed in his private study, but was called forth to the public business of the nation, by the queen, about the year 1593. [Other accounts say 1589.] He was ordered to attend the earl of Cumberland in some maritime expeditions. One of these he has given a faithful account of, in the manner of a journal or ephemeris, to which he has prefixed an elegant hydrographical chart of his own contrivance. A little before his death he employed himself about an English translation of the book of logarithms, then lately discovered by lord Napier, a Scotchman, who had a great affection for him. This posthumous work of his was published soon after, by his only son Samuel Wright, who was also a scholar of this college. He had formed many other useful designs, but was hindered by death from bringing them to perfection. Of him it may truly be said, that he studied more to serve the public than himself; and though he was rich in fame, and in the promises of the great, yet he died poor, to the scandal of an ungrateful age." So far the memoir; other particulars concerning him, are as follow.

Mr. Wright first discovered the true method of dividing the meridian line, according to which the Mercator's charts are constructed, and upon which Mercator's sailing is founded. An account of this he sent from Caius-college, Cambridge, where he was then a fellow, to his friend Mr. Blondeville, containing a short table for that purpose, with a specimen of a chart so divided, together with the manner of dividing it. All which Blondeville published, in 1594, among his Exercises. And, in 1597, the reverend Mr. William Barlowe, in his Navigator's Supply, gave a demonstration of this division as communicated by a friend.

At length, in 1599, our author himself printed his celebrated treatise, entitled, The Correction of certain Errors in Navigation, which had been written many years before; where he shows the reason of this division of the meridian, the manner of constructing his table, and its uses in navigation, with other improvements. In 1610 a second edition of Mr. Wright's book was published, and dedicated to his royal pupil, prince Henry; in which the author inserted farther improvements; particularly, he proposed an excellent way of determining the magnitude of the earth; at the same time, recommending very judiciously, the making our common measures in some certain proportion to that of a degree on its surface, that they might not depend on the uncertain length of a barley-corn. Some of his other improvements were; The Table of Latitudes for dividing the meridian, computed as far as to minutes: An instrument, he calls the Sea-rings, by which the variation of the compass, the altitude of the sun, and the time of the day, may be readily determined at once in any place, provided the latitude be known: The correcting of the errors arising from the excentricity of the eye in observing by the cross-staff: A total amendment in the tables of the declinations and places of the sun and stars, from his own observations, made with a six-foot quadrant, in the years 1594, 95, 96, 97: A sea-quadrant, to take altitudes by a forward or backward observation; having also a contrivance for the ready finding the latitude by the height of the pole-star, when it is not on the meridian. And that this book might be the better understood by beginners, to this edition is subjoined a translation of Zamorano's Compendium; and he added a large table of the variation of the compass as observed in very different parts of the world, to show that it is not occasioned by any magnetical pole. The work has gone through several other editions since. Description and Use of the Sphere, in 1618. And, besides the books above mentioned, he published another on navigation, entitled, The Haven-finding Art, translated from the Dutch. Other accounts of our author state also, that it was in the year 1589 that he first began to attend the earl of Cumberland in his voyages. It is also said that he made, for his pupil, prince Henry, a large sphere with curious movements, which, by the help of spring-work, not only represented the motions of the whole celestial sphere, but showed also the particular systems of the sun and moon, and their circular motions, together with their places and possibilities of eclipsing each other: there are in this machine works for a motion of 17100 years, if it should not be stopt, or the materials fail. This sphere, though thus made at a great expense of money and ingenious industry, was afterwards in the time of the civil wars thrown aside, among dust and rubbish, where it was found, in the year 1646, by sir Jonas Moore, who at his own expense restored it to its first state of perfection, and deposited it at his own house in the Tower, among his other mathematical instruments and curiosities.

See Robertson's Navigation, Introd. pa. 12. Also the Philos. Magazine, vol. 21, pa. 164.

X.

XENOCRATES, an eminent philosopher among the ancient Greeks, was born at Chalcedon, and died 314 years before Christ, at about 90 years of age. He became early a disciple of Plato, studying under this great master at the same time with Aristotle, though he was not possessed of equal talents; the former wanting a spur, and the latter a bridle. He was fond of the mathematics; and permitted none of his scholars to be ignorant of them. There was something slovenly in the behaviour of Xenocrates; for which reason Plato frequently exhorted him to sacrifice to the graces. Seriousness and severity were always seen in his deportment: yet notwithstanding this severe cast of mind, he was very compassionate. There was also something very extraordinary in the rectitude of his morals: he was absolute master of his passions; and was not fond of pleasure, riches, or applause. Indeed, so great was his reputation for sincerity and probity, that he was the only person whom the magistrates of Athens dispensed from confirming his testimony with an oath: and yet he was so ill treated by them, as to be sold because he could not pay the poll-tax laid upon foreigners. Demetrius Phalereus bought Xenocrates, paid the debt to the Athenians, and immediately gave him his liberty. At Alexander's request, he composed a treatise on the Art of Reigning; 6 books on Nature; 6 books on Philosophy; one on Riches, &c; but none of them have descended to the present times.—His theology it seems was but poor stuff: Cicero refutes him in the first book of the Nature of the Gods.

XENOPHANES, a Greek philosopher, born in Colophon, was, according to some authors, the disciple of Archelaus; in which case he must have been contemporary with Socrates. Others relate that he was quite an autodidact, being entirely self-taught, and that he lived at the same time with Anaximander: according to which account he must have flourished before Socrates, and about the 60th Olympiad, as Diogenes Laertius affirms. He founded the Eleatic sect; and wrote several poems on philosophical subjects; as also a great many on the foundation of Colophon, and on that of the colony of Elea. He wrote also against Homer and Hesiod. He was banished from his country, withdrew to Sicily, and lived in Zanche and Catana. His opinion with regard to the nature of God differs not much from that of Spinoza. When he saw the Egyptians pour forth lamentations during their festivals, he thus advised them: "If the objects of your worship are Gods, do not weep: if they are men, offer not sacrifices to them." The answer he made to a man with whom he refused to play at dice, is highly worthy of a philosopher: This man calling him a coward, "Yes," replied he, "I am excessively so with regard to all shameful actions."

XENOPHON, a celebrated Greek general, philosopher, and historian, was born at Athens, and became early a disciple of Socrates, who, says Strabo, saved his life in battle. About the 50th year of his age he engaged in the expedition of Cyrus, and accomplished his immortal retreat in the space of 15 months. The jealousy of the Athenians banished him from his native city, for engaging in the service of Sparta and Cyrus. On his return therefore he retired to Scillus, a town of Elis, where he built a temple to Diana, which he mentions in his epistles, and devoted his leisure to philosophy and rural sports. But commotions arising in that country, he removed to Corinth, where it seems he wrote his Grecian History, and died at the age of 90, in the year 360 before Christ.

By his wife Philesia he had two sons, Diodorus and Gryllus. The latter rendered himself immortal by killing Epaminondas in the famous battle of Mantinea, but perished in that exploit, which his father lived to record.

The best editions of his works are those of Franckfort in 1674, and of Oxford, in Greek and Latin, in 1703, 5 vols. 8vo. Separately have been published his Cyropædia, Oxon. 1727, 4to, and 1736, 8vo. Cyri Anabasis, Oxon. 1735, 4to, and 1747, 8vo. Memorabilia Socratis, Oxon. 1741, 8vo.—His Cyri Anabasis has been admirably translated into English by Spelman.

XIPHIAS, in Astronomy, is the Dorado or Sword-fish, a constellation of the southern hemisphere; being one of the new constellations added by modern astronomers; and consisting of 6 stars only. See DORADO.

Y.

YARD, a lineal measure, or measure of length, used in England and Spain, chiefly to measure cloth, stuffs, &c. The yard was settled by Henry the 1st, from the length of his own arm.

The English yard contains 3 feet; and it is equal
to 4-5ths of the English ell,
to 7-9ths of the Paris ell,
to 4-3ds of the Flemish ell,
to 56-51sts of the Spanish vara or yard.

YARD, or *Golden* **YARD**, is also a popular name given to the 3 stars which compose the belt of Orion.

YEAR, in the full extent of the word, is a system or cycle of several months, usually 12. Others define year, in the general, a period or space of time, measured by the revolution of some celestial body in its orbit. Thus, the time in which the fixed stars make a revolution, is called the great year; and the times in which Jupiter, Saturn, the Sun, Moon, &c, complete their courses, and return to the same point of the zodiac, are respectively called the years of Jupiter, and Saturn, and the Solar, and Lunar years, &c.

As year denoted originally a revolution, and was not limited to that of the sun; accordingly we find by the oldest accounts, that people have, at different times, expressed other revolutions by it, particularly that of the moon; and consequently that the years of some accounts, are to be reckoned only months, and sometimes periods of 2, or 3, or 4 months. This will assist us greatly in understanding the accounts that certain nations give of their own antiquity, and perhaps also of the age of men. We read expressly, in several of the old Greek writers, that the Egyptian year, at one period, was only a month; and we are farther told that at other periods it was 3 months, or 4 months: and it is probable that the children of Israel followed the Egyptian account of their years. The Egyptians boasted, almost 2000 years ago, of having accounts of events 48 thousand years distance. A great deal must be allowed to fallacy, on the above account; but besides this, the Egyptians had, in the time of the Greeks, the same ambition which the Chinese have at present, and wanted to pass themselves on that people, as these do upon us, for the oldest inhabitants of the earth. They had recourse also to the same means, and both the present and the early impostors have pretended to ancient observations of the heavenly bodies, and recounted eclipses in particular, to vouch for the truth of their accounts. Since the time in which the solar year, or period of the earth's revolution round the sun, has been received, we may account with certainty; but for those remote ages, in which we do not precisely know what is meant by the term year, it is impossible to form any satisfactory conjecture of the duration of time in the accounts. The Babylonians pretend to an antiquity of the same romantic kind; they boast of 47 thousand years in which they had kept observations; but we may judge of these as of the others, and of the observations as of the years. The Egyptians speak of the stars having four times altered their courses in that period which they claim for their history, and that the sun set twice in the east. They were not such perfect astronomers, but, after a round-about voyage, they might perhaps mistake the east for the west, when they came in again.

YEAR, or **SOLAR YEAR**, properly, and by way of eminence so called, is the space of time in which the sun moves through the 12 signs of the ecliptic. This, by the observations of the best modern astronomers, contains 365 days, 5 hours, 48 min. 45½ seconds: the quantity assumed by the authors of the Gregorian calendar is 365 days, 5 hours, 49 min. But in the civil or popular account, this year only contains 365 days; except every 4th year, which contains 366.

The vicissitude of seasons seem to have given occasion to the first institution of the year. Man, naturally curious to know the cause of that diversity, soon found it was the proximity and distance of the sun; and therefore gave the name year to the space of time in which that luminary performed his whole course, by returning to the same point of his orbit. According to the accuracy of their observations, the year of some nations was more perfect than that of others, but none of them quite exact, nor whose parts did not vary with regard to the parts of the sun's course.

Herodotus informs us that it was the Egyptians who first formed the year, making it to contain 360 days, which they subdivided into 12 months, of 30 days each. Mercury Trismegistus added 5 days more to the account. And on this footing it is said that Thales instituted the year among the Greeks; though that form of the year did not obtain through all Greece. Also, the Jewish, Syrian, Roman, Persian, Ethiopic, Arabic, &c years, were all different. In fact, considering the imperfect state of astronomy in those ages, it is no wonder that different people should disagree in the calculation of the sun's course. We are even assured by Diod. Siculus, lib. 1. Plutarch, in Numa, and Pliny, lib. 7, cap. 48, that the Egyptian year itself was at first very different from that now represented.

The solar year is either astronomical or civil.

The *Astronomical Solar* **YEAR**, is that which is determined precisely by astronomical observations; and is of two kinds, tropical, and sidereal or astral.

Tropical, or *Natural* **YEAR**, is the time the sun takes in passing through the zodiac; which, as before observed, is 365d. 5h. 48m. 45½ sec. This is the only proper or natural year, because it always keeps the same seasons to the same months.

Sidereal or *Astral* **YEAR**, is the space of time the sun takes in passing from any fixed star, till his return to it again. This consists of 365d. 6h. 9m. 17 sec.; being 20m. 21½ sec. longer than the true solar year.

Lunar **YEAR**, is the space of 12 lunar months. Hence, from the two kinds of synodical lunar months, there arise two kinds of lunar years; the one astronomical, the other civil.

Lunar Astronomical **YEAR**, consists of 12 lunar synodical months; and contains 354d. 8h. 48m. 38 sec. and is therefore 10d. 21h. 0m. 7s. shorter than the solar year. A difference which is the foundation of the epact.

Lunar Civil **YEAR**, is either common or embolismic.

The *Common Lunar* YEAR consists of 12 lunar civil months; and therefore contains 354 days. And

The *Embolismic* or *Intercalary Lunar* YEAR, consists of 13 lunar civil months, and therefore contains 384 days.

Thus far we have considered years and months, with regard to astronomical principles, upon which the division is founded. By this, the various forms of civil years that have formerly obtained, or that do still obtain, in divers nations, are to be examined.

Civil YEAR, is that form of year which every nation has contrived or adopted, for computing their time by. Or the civil is the tropical year, considered as only consisting of a certain number of whole days: the odd hours and minutes being set aside, to render the computation of time, in the common occasions of life, more easy. As the tropical year is 365d. 5h. 49m. or almost 365d. 6h. which is 365 days and a quarter; therefore if the civil year be made 365 days, every 4th year it must be 366 days, to keep nearly to the course of the sun. And hence the civil year is either common or bissextile. The

Common Civil YEAR, is that consisting of 365 days; having seven months of 31 days each, four of 30 days, and one of 28 days; as indicated by the following well known memorial verses:

Thirty days hath September,
April, June, and November;
February twenty-eight alone,
And all the rest have thirty-one.

Bissextile or *Leap* YEAR, consists of 366 days; having one day extraordinary; called the intercalary, or bissextile day; and takes place every 4th year. This additional day to every 4th year, was first introduced by Julius Cæsar; who, to make the civil years keep pace with the tropical ones, contrived that the 6 hours which the latter exceeded the former, should make one day in 4 years, and be added between the 22d and 23d of February, which was their 6th of the calends of March; and as they then counted this day twice over, or had bis sexto calendas, hence the year itself came to be called bis sextus, and bissextile.

However, among us, the intercalary day is not introduced by counting the 23d of February twice over, but by adding a day at the end of that month, which therefore in that year contains 29 days.—A farther reformation was made in this year by Pope Gregory. See *Gregorian* YEAR, CALENDAR, BISSEXTILE, and LEAP-*Year*.

The civil or legal year, in England, formerly commenced on the day of the Annunciation, or 25th of March; though the historical year began on the day of the Circumcision, or 1st of January; on which day the German and Italian year also commences. The part of the year between these two terms was usually expressed both ways: as 1745-6, or 174$\frac{5}{6}$. But by the act for altering the stile, the civil year now commences with the 1st of January.

Ancient Roman YEAR. This was the lunar year, which, as first settled by Romulus, contained only ten months, of unequal numbers of days in the following order: viz, March 31; April 30; May 31; June 30; Quintilis 31; Sextilis 30; September 30; October 31; November 30; December 30; in all 304 days; which came short of the true lunar year by 50 days; and of the solar by 61 days. Hence, the beginning of Romulus's year was vague, and unfixed with regard to any precise season; to remove which inconvenience, that prince ordered so many days to be added yearly as would make the state of the heavens correspond to the first month, without calling them by the name of any month.

Numa Pompilius corrected this irregular constitution of the year, composing two new months, January and February, of the days that were used to be added to the former year. Thus Numa's year consisted of 12 months, of different days, as follow; viz,

January - 29; February - 28; March - - 31;
April - - 29; May - - - - 31; June - - - 29;
Quintilis 31; Sextilis - - - 29; September 29;
October - 31; November - 29; December 29;

in all 355 days; therefore exceeding the quantity of a lunar civil year by one day; that of a lunar astronomical year by $15^h\ 11^m\ 22^s$; but falling short of the common solar year by 10 days; so that its beginning was still vague and unstable.

Numa, however, desiring to have it begin at the winter solstice, ordered 22 days to be intercalated in February every 2d year, 23 every 4th, 22 every 6th, and 23 every 8th year. But this rule failing to keep matters even, recourse was had to a new way of intercalating; and instead of 23 days every 8th year, only 15 were to be added. The care of the whole was committed to the pontifex maximus; who however, neglecting the trust, suffered things to run to great confusion. And thus the Roman year stood till Julius Cæsar reformed it. See CALENDAR. And for the manner of reckoning the days of the Roman months, see CALENDS, NONES, and IDES.

Julian YEAR. This is in effect a solar year, commonly containing 365 days; though every 4th year, called bissextile, it contains 366. The months of the Julian year, with the number of their days, stood thus:

January - - 31; February - 28; March - - 31;
April - - - 30; May - - - 31; June - - 30;
July - - - 31; August - - 31; September 30;
October - 31; November 30; December 31.

But every bissextile year had a day added in February, making it then to contain 29 days.

The mean quantity therefore of the Julian year is 365¼ days, or $365^d\ 6^h$; exceeding the true solar year by somewhat more than 11 minutes; an excess which amounts to a whole day in almost 131 years. Hence the times of the equinoxes go backward, and fall earlier by one day in about 130 or 131 years. And thus the Roman year stood, till it was farther corrected by pope Gregory.

For settling this year, Julius Cæsar brought over from Egypt, Sosigenes, a celebrated mathematician; who, to supply the defect of 67 days, which had been lost through the neglect of the priests, and to bring the beginning of the year to the winter solstice, made one year to consist of 15 months, or 445 days; on which account that year was used to be called annus confusionis, the year of confusion. See *Julian* CALENDAR.

Gregorian YEAR. This is the Julian year corrected by this rule, viz, that instead of every secular or 100th year being a bissextile, as it would be in the former mode, in the new way three of them are common years, and only the 4th is bissextile.

The error of 11 minutes in the Julian year, by continual repetition, had accumulated to an error of 13 days from the time when Cæsar made his correction; by which means the equinoxes were greatly disturbed. In the year 1582, the equinoxes were fallen back 10 days, and the full moons 4 days, more backward than they were in the time of the Nicene council, which was in the year 325; viz, the former from the 20th of March to the 10th, and the latter from the 5th to the 1st of April. To remedy this increasing irregularity, pope Gregory the 13th, in the year 1582, called together the chief astronomers of his time, and concerted this correction, omitting the 10 days above mentioned. He exchanged the lunar cycle for that of the epacts, and made the 5th of October of that year to be the 15th; by that means restoring the vernal equinox to the 21st of March. It was also provided, by the omission of 3 intercalary days, in 400 years, to make the civil year keep pace nearly with the solar year, for the time to come. See CALENDAR.

In the year 1700, the error of 10 days was increased to 11; upon which, the protestant states of Germany, to prevent farther confusion, adopted the Gregorian correction. And the same was accepted also in England in the year 1752, when 11 days were thrown out after the 2d of September that year, by accounting the 3d to be the 14th day of the month: calling this the new style, and the former the old style. And the Gregorian, or new style, is now in like manner used in most countries of Europe.

Yet this last correction is still not quite perfect; for as it has been shown that, in 4 centuries, the Julian year gains $3^d\ 2^h\ 40^m$; and as it is only the 3 days that are kept out in the Gregorian year; there is still an excess of $2^h\ 40^m$ in 4 centuries, which amounts to a whole day in 36 centuries, or in 3600 years. See CALENDAR, *New* or *Gregorian* STILE, &c.

Egyptian YEAR, called also the year of Nabonassar, on account of the epoch of Nabonassar, is the solar year of 365 days, divided into 12 months, of 30 days each, beside 5 intercalary days, added at the end. The order and names of these months are as follow:

1. Thoth; 2. Paophi; 3. Athyr;
4. Chojac; 5. Tybi; 6. Mecheir;
7. Phamenoth; 8. Pharmuthi; 9. Pachon;
10. Pauni; 11. Epiphi; 12. Mesori.

As the Egyptian year, by neglecting the 6 hours, in every 4 years loses a whole day of the Julian year, its beginning runs through every part of the Julian year in the space of 1460 years; after which, they meet again; for which reason it is called the erratic year. And because this return to the same day of the Julian year, is performed in the space of 1460 Julian years, this circle is called the Sothic period.

This year was applied by the Egyptians to civil uses, till Anthony and Cleopatra were defeated; but the mathematicians and astronomers used it till the time of Ptolemy, who made use of it in his Almagest; so that the knowledge of it is of great service in astronomy, for comparing the ancient observations with the modern.

The ancient Egyptians, we are informed by Diodorus Siculus, (Plutarch, lib. 1, in the life of Numa; and Pliny, lib. 7, cap. 48) measured their years by the course of the moon. At first they were only one month, then 3, then 4, like that of the Arcadians; and then 6, like that of the people of Acarnania. Those authors add, that it is on this account that they reckon such a vast number of years from the beginning of the world; and that in the history of their kings, we meet with some who lived 1000, or 1200 years. The same thing is maintained by Kircher; Oedip. Egypt. tom. 2, pa. 252. And a late author observes, that Varro has affirmed the same of all nations, that has been quoted of the Egyptians. By which means many account for the great ages of the more ancient patriarchs; expounding the gradual decrease in their ages, by the successive increase of the number of months in their years.

On the Egyptians being subdued by the Romans, they received the Julian year, though with some alteration; for they still retained their ancient months, with the five additional days, and every 4th year they intercalated another day, for the 6 hours, at the end of the year, or between the 28th and 29th of August. Also, the beginning of their year, or the first day of the month Thoth, answered to the 29th of August of the Julian year, or to the 30th if it happened to be leap year.

The Ancient Greek YEAR.—This was a lunar year, consisting of 12 months, which at first had each 30 days, then alternately 29 and 30 days, computed from the first appearance of the new moon; with the addition of an embolismic month of 30 days, every 3d, 5th, 8th, 11th, 14th, 16th, and 19th year of a cycle of 19 years; in order to keep the new and full moons to the same terms or seasons of the year.

Their year commenced with that new moon which was nearest to the summer solstice. And the order of the months, with the number of their days, were as follow: 1. Εκατομϐαιων, of 29 days; 2. Μηταγειτνιων 30; 3. Βοηδρομιων 29; 4. Μαιμακτηριων 30; 5. Πυανεψιων 29; 6. Ποσειδεων 30; 7. Γαμηλιων 29; 8. Ανθεστηριων 30; 9. Ελαφηϐολιων 29; 10. Μενυχιων 30; 11. Θαργηλιων 29; 12. Σκιροφοριων 30.—But many of the Greek nations had other names for their months.

The Ancient Jewish YEAR.—This is a lunar year, usually consisting of 11 months, containing alternately 30 and 29 days. And it was made to agree with the solar year, by adding 11, and sometimes 12 days, at the end of the year, or by an embolismic month. The order and quantities of the months were as follow: 1. Nisan or Abib 30 days; 2. Jiar or Zius 29; 3. Siban or Sievan 30; 4. Thamuz or Tamuz 29; 5. Ab 30; 6. Elul 29; 7. Tifri or Ethanim 30; 8. Marchesvan or Bul 29; 9. Cisleu 30; 10. Tebeth 29; 11. Sabat or Schebeth 30; 12. Adar 30 in the embolismic year, but 29 in the common year.—In the defective year, Cisleu was only 29 days; and in the redundant year, Marchesvan was 30.

The Modern Jewish YEAR is likewise lunar, consisting of 12 months in common years, but of 13 in embolismic years; which, in a cycle of 19 years, are the 3d, 6th, 8th, 11th, 14th, 17th, and 19th. Its beginning is fixed to the new moon next after the autumnal equinox. The names and order of the months, with the number of the days, are as follow: 1. Tisri 30 days; 2. Marchesvan 29; 3. Cisleu 30; 4. Tebeth 29; 5. Schebeth 30; 6. Adar 29; 7. Veadar, in the embolismic year, 30; 8. Nisan 30; 9. Ilar 29; 10. Sivan 30; 11. Thamuz 29; 12. Ab 30; 13. Elul 29.

The Syrian YEAR, is a solar one, having its commencement fixed to the beginning of October in the Julian year; from which it only differs in the names of the months, the quantities being the same; as follow: 1. Tishrin, answering to our October, and containing 31 days; 2. Latter Tishrin, containing, like November, 30 days; 3. Canun 31; 4. Latter Canun 31; 5. Shabat 28, or 29 in a leap-year; 6. Adar 31; 7. Nisan 30; 8. Aiyar 31; 9. Haziram 30; 10. Thamuz 31; 11. Ab 31; 12. Elul 30.

The Persian YEAR, is also a solar one, of 365 days, consisting of 12 months of 30 days each, with 5 intercalary days added at the end. The months are as follow: 1. Asrudia meh; 2. Ardibascht meh; 3. Cardi meh; 4. Thir meh; 5. Merded meh; 6. Schabarir meh; 7. Mehar meh; 8. Aben meh; 9. Adar meh; 10. Di Meh; 11. Behen meh; 12. Assirer meh. This year is the same as the Egyptian Nabonassarean, and is called the Yezdegerdic year, to distinguish it from the fixed solar year, called the Gelalean year, which the Persians began to use in the year 1079, and which was formed by an intercalation, made six or seven times in 4 years, and then once every 5th year.

The Arabic, Mahometan, and Turkish YEAR, called also the year of the Hegira, is a lunar year, equal to 354^d 8^h 48^m, and consists of 12 months, containing alternately 30 and 29 days. Though sometimes it contains 13 months; the names &c being as follow: 1. Muharram of 30 days; 2. Saphar 29; 3. Rabia 30; 4. Latter Rabia 29; 5. Jomada 30; 6. Latter Jomada 29; 7. Rajab 30; 8. Shaaban 29; 9. Ramadan 30; 10. Shawal 29; 11. Dulkaadah 30; 12. Dulheggia 29, but in the embolismic year 30. An intercalary day is added every 2d, 5th, 7th, 10th, 13th, 15th, 18th, 21st, 24th, 26th, 29th, in a cycle of 29 years. The months commence with the first appearance of the new moons after the conjunction.

Ethiopic YEAR, is a solar year perfectly agreeing with the Actiac, except in the names of the months, which are; 1. Masçaram; 2. Tykympt; 3. Hydar; 4. Tyshas; 5. Tyr; 6. Jacatil; 7. Magabit; 8. Mijazia; 9. Ginbat; 10. Syne; 11. Hamel; 12. Hahase. Intercalary days 5. It commences with the Egyptian year, on the 29th of August of the Julian year.

The Year of the Native Americans.—In Humboldt's Researches, concerning the institutions and monuments of the ancient inhabitants of America, among many other things concerning the Mexican nation, &c, we find several particulars relating to their year and computation of time. It seems that the Mexican calendar possesses a degree of accuracy and refinement, that rises considerably above all the other marks of their civilization. It appears that a stone of porphyry, of an enormous bulk, dug up in the year 1790, and covered with sculpture, evidently relative to the calendar, has thrown considerable light on this curious subject. The sculpture is in relievo, and well polished; the concentric circles, with their numerous divisions and subdivisions, are traced with mathematical exactness. In the centre of the stone is sculptured the hieroglyphic of the sun, surrounded by eight triangular radii. The god Tonatiuh is figured, opening his large mouth, armed with teeth, which reminds us of a figure of a divinity in Hindustan, the image of Kala, or Time.

It appears that the civil year of the Mexicans, was a solar year of 365 days, and divided into 18 months, of 20 days each, with 5 days added at the end. The beginning of the day was reckoned, like that of the Persians and Egyptians, from sun-rising. It was divided into 4 intervals, determined by the rising and setting of the sun, and its two passages over the meridian circle. The hieroglyphic of the day was a circle divided into 4 equal parts. Each month, of 20 days, was divided into 4 weeks, or periods of 5 days. The Muescas, a nation south of the Isthmus, had weeks of 3 days; but it does not appear that any nation of the new continent was acquainted with the week or cycle of 7 days; which, with a few exceptions, is found all over the old world.

Thirteen Mexican years formed a cycle, to which they gave a particular name; and 4 of these, constituting a period of 52 years, was denoted by another term; and two of these periods of 52 years formed what they called an old age. At the end of 52 years, 13 days were added, which makes the Mexican year agree with the Julian, of 365¼ days. But Gama, an astronomer very learned in the chronology and history of the Mexicans, is of opinion that they intercalated only 25 days in 104 years; and this would give the length of the year = 365·24 days, which is very near the truth; being more accurate than that of Hipparchus, and is nearly the same as that which was determined by the astronomers of the caliph Almamoun.

The Mexicans were in possession of annals which went back 8½ centuries before the arrival of Cortes in the country of Anahuac. The reckoning of time was according to periods of 52 or 104 years; and along with the series of years and days, expressed by hieroglyphics, the migrations of the nations, the battles and remarkable events of each reign, were represented in the paintings of which these annals were composed. In the reckoning of time, however, a particular artifice was employed; for though the numbering of the years and months, from a given era, would have sufficiently ascertained the date of any event, just as with us, this simple method was rejected, and in its stead was substituted a contrivance, by which the name of the year determined its relative situation. This device, M. Humboldt thinks, was the work of the priests, and was effected by dividing the cycle of 13 years into smaller cycles of 4 years each, and distinguishing these years by particular names.

The symbolical writing of the Mexican nations exhibited simple signs for the number 20, and for its 2d and 3d powers, 400 and 8000. A small standard, or flag, represented 20 units; 400, the square of 20, was figured by a feather, because grains of gold, inclosed in a quill, were used in some places as money, or a sign for the purposes of exchange. The figure of a sack indicated 8000, or the cube of 20, and had the name that was given to a kind of purse that contained 8000 grains of cocoa. A flag, divided by two cross lines, and half coloured, denoted 10; and when three quarters were coloured, it denoted 15. The Mexican vocabulary afforded names for numbers as far as 48 millions, and derived, according to the strictest rules of analogy, from the decimal mode of reckoning. The units, as far as 10, or 20 sometimes, were marked by dots or points; thus, 23 was expressed by a flag followed by 3 dots, &c.

M. Humboldt remarks, that several of the names by which the Mexicans denoted the 20 days of their month, are those of the signs of a zodiac, in use from the remotest antiquity among the nations of eastern Asia. He compares the names of the Mexican symbols for the days, with the Tartarean, Japanese, and Tibetan names of the 12 signs, and also with the names of the lunar houses of the Hindus. In 8 of the hieroglyphics the analogy is very striking. Thus, Ath, the name of the first day, as also of water, is indicated by an hieroglyphic, the parallel or undulating lines of which remind us of the sign Aquarius. In the Tibetan zodiac this sign is marked by a rat, which is also used as an emblem of water. The rat is likewise an asterism in the Chinese zodiac. Seven other of the names or characters stand related nearly in the same manner. M. Humboldt also justly considers it a remarkable circumstance, that the ape is a character used in the Mexican calendar, as it is in the Tibetan zodiac, and in the lunar houses of the Hindus, though this animal does not exist in the high country of the Andes.

It appears that the Mexicans made astronomical observations by means of the gnomon; and knew from them, that in the first year of the cycle, the equinoxes fell on certain days of the 4th and 13th month. The Peruvians and Cousko regulated their intercalation, not by the shadow of the gnomons, which they however very assiduously measured, but, by marks placed in the horizon, to denote where the sun rose and set on the days of the solstices and equinoxes.

For the Hindu year, see the Philos. Trans. Abridg. vol. 16, pa. 742, &c, and vol. 17, pa. 250, &c; also our article CHRONOLOGY.

YELLOW, one of the primary or original colours of light.

YESDEGERDIC YEAR. See *Persian* YEAR.

YOUNG (MATTHEW) D. D., the very learned bishop of Clonfert and Kilmacduach, was of a respectable family in the county of Roscommon, was born in 1750, and died Nov. 28, 1800, at Whitworth in Lancashire, of a lingering and painful malady, a cancer in the tongue. He was admitted into the university of Dublin in 1766, and elected fellow of the college in 1775. In the prosecution of that object, his attention was necessarily directed to the Newtonian philosophy, of which he early became an enthusiastic admirer; and displayed, at the examination for his fellowship, an unexampled knowledge and comprehension of it. It continued to be his favourite study, but not his only one. His active mind embraced in rapid succession the most dissimilar objects; and these he pursued with unceasing ardour, amidst his various duties as a fellow and tutor, and the freest intercourse with society, which he was formed at once to delight and instruct. His love of literary conversation, and the advantages he experienced from it in the pursuit of science, led him early to engage in forming a society whose chief object was the improvement of its members in theological learning. It consisted of a small number of his most intimate college friends, and continued to exist for a series of years, with equal reputation and advantage. Out of this association grew another, somewhat more extensive, whose labours were directed to philosophical researches, and in the formation of which Mr. Young was also actively engaged: and this itself became the germ of the Royal Irish Academy; which owes its existence to the zeal and exertions of the members of that society, among whom Mr. Young was particularly distinguished. In the intervals of his severer studies, he applied himself to modern languages; and the result of his labours may be seen in the Transactions of the R. I. A., to which he also contributed largely on mathematical and philosophical subjects.

In the first volume of their Transactions; A Synthetical Demonstration of the Rule for the Quadrature of Simple Curves per æquationes terminorum numero infinitas; On the Extraction of Cubic and other Roots; Ancient Gaelic poems respecting the race of the Frians collected in the Highlands. In vol. 2nd; An Enquiry into the different modes of Demonstration by which the Velocity of Spouting Fluids has been investigated a priori. In vol. 3rd; The Origin and Theory of the Gothic Arch. In vol. 4th; Demonstration of Newton's Theorems for the Correction of spherical Errors in the Object-glasses of Telescopes. In the 5th and 6th volumes, nothing. Besides these, Dr. Young published the following learned and ingenious works: The Phenomena of Sounds and Musical Strings, 8vo, 1784: The Force of Testimony, &c, 4to: The Number of Primitive Colours in Solar Light: On the Precession of the Equinoxes: Principles of Natural Philosophy, 8vo, 1800, being his last publication, and containing the substance of his lectures in the college.

In 1786, when the professorship of philosophy in Trinity-college became vacant, he had attained so high reputation in that branch of science, that he was elected to the office without opposition. His Essay on Sounds had been published some years; and it was known he was engaged in the arduous task of illustrating the Principia of Newton. He now devoted himself to the duties of his professorship; and the college having been enriched with the excellent apparatus of Mr. Atwood, Dr. Y. improved the fortunate occasion of carrying his lectures to a degree of perfection unknown in the university of Dublin, and never perhaps exceeded in any other. He proceeded in the mean time in his great work, "The Method of Prime and Ultimate Ratios, illustrated by a Commentary on the first two books of the Principia," and had nearly completed it in English, when he was advised by his friends to publish it in Latin. He readily acquiesced, and thus had an opportunity, while translating it, of revising the whole, and rendering it fuller and more perfect. It was finished a year or two before his appointment to the see of Clonfert, at which time he was engaged in preparing for its publication. His attention was unavoidably diverted from it by the occupations attending so important a change; and, before he could return to it, the dreadful malady had commenced, under which he languished for 15 months, before its fatal termination; though in the midst of his sufferings his ardour for science was not abated.

The circumstances of his promotion to the episcopal bench reflect equal honour on himself and the noble person (lord Cornwallis) who conferred it. It was a favour as unsolicited as unexpected, unless the report made to his Excellency by his principal secretary, on being consulted as to the properest person to fill the vacant see, may be called solicitation. His report was, that "he believed Dr. Young to be the most distinguished literary character in the kingdom;" and he was recommended accordingly.

Z.

ZAMORANO (Rerico), a good Spanish mathematician, in the 16th century, being the royal lecturer on that science, at Seville, where he published an excellent compendium of navigation, in 1585; being a treatise written clearly and with brevity, not being encumbered with such idle speculations as abound in Medina and Cortes. Zamorano, it seems, contributed much to the reforming the sea charts, as we are informed by his successor, Anares Garcia de Cespedes, who himself also published a treatise on navigation at Madrid, in 1606.

ZENITH, in Astronomy, the vertical point, or point in the heavens directly overhead. Or, the zenith is a point in the surface of the sphere, from which a right line drawn through the place of any spectator, passes through the centre of the earth. The zenith of any place, is also the pole of the horizon, being 90 degrees distant from every point of it. And through the zenith pass all the azimuths, or vertical circles.

The point diametrically opposite to the zenith, is called the nadir, being the point in the sphere directly under our feet: and it is the zenith to our antipodes, as our zenith is their nadir.

Zenith *Distance*, is the distance of the sun or star from our zenith; and is the complement of the altitude, or what it wants of 90 degrees.

ZENO, Eleates, or of *Elea*, one of the greatest philosophers among the ancients, flourished about 500 years before the Christian æra. He was the disciple of Parmenides, and even, according to some writers, his adopted son. Aristotle asserts that he was the inventor of logic: but his logic seems to have been calculated and employed to perplex rather than to illustrate and decide any thing; for Zeno employed it only to dispute against all comers, and to silence his opponents, whether they argued right or wrong. Among many other subtleties and embarrassing arguments, he proposed some with regard to motion, denying that there was any such thing in nature; and Aristotle, in the 6th book of his physics, has preserved some of them, which are extremely subtile, especially the famous argument named Achilles; which was to prove this proposition; that the swiftest animal could never overtake the slowest, as a greyhound a tortoise, if the latter set out a little before the former: for suppose the tortoise to be 100 yards before the dog, and that this runs 100 times as fast as the other; then while the dog runs the first 100 yards, the tortoise runs 1, and is therefore 1 yard before the dog; again, while the dog runs over this yard, the tortoise will run the 100th part of a yard, and will be so much before the dog; and again, while the dog runs over this 100th part of a yard, the tortoise will have got the 100th part of that 100th part before him; and so on continually, says he, the dog will always be some small part behind the tortoise. But the fallacy will soon be detected, by considering where the tortoise will be when the dog has run over 200 yards; for as the former can have run only two yards in the same time, and therefore must then be 98 yards behind the dog, he consequently must have overtaken and passed the tortoise. It has been said that, to prove to him, or some disciple of his, that there is such a thing as motion, Diogenes the Cynic rose up and walked over the floor.—Zeno showed great courage in suffering pain; for having joined with others to endeavour to restore liberty to his country, which groaned under the oppression of a tyrant, and the enterprise being discovered, he supported with extraordinary firmness the sharpest tortures. It is even said that he had the courage to bite off his tongue, and spit it in the tyrant's face, for fear of being forced, by the violence of his torments, to discover his accomplices. Some relate that he was pounded to death in a mortar.

Zeno, a celebrated Greek philosopher, was born at Citium, in the Isle of Cyprus, and was the founder of the Stoics; a sect which had its name from that of a portico at Athens, where this philosopher chose to hold his discourses. He was cast on that coast by shipwreck; and he ever after regarded this as a great happiness, praising the winds for having so happily driven him into the port of Piræum.—Zeno was the disciple of Crates, and had a great number of followers. He made the sovereign good to consist in dying in conformity to nature, guided by the dictates of right reason. He acknowledged but one God; and admitted an inevitable destiny over all events. His servant taking advantage of this last opinion, cried, while he was beating him for dishonesty, "I was destined to steal;" to which Zeno replied, "Yes, and to be beaten too." This philosopher used to say, "That if a wise man ought not to be in love, as some pretended, none would be more miserable than beautiful and virtuous women, since they would have none for their admirers but fools." He also said, "That a part of knowledge consists in being ignorant of such things as ought not to be known: that a friend is another self: that a little matter gives perfection to a work, though perfection is not a little matter." He compared those who spoke well and lived ill, to the money of Alexandria, which was beautiful, but composed of bad metal.—It is said that being hurt by a fall, he took that as a sign he was then to quit this life, and laid violent hands on himself, about 264 years before Christ.

Cleanthes, Crysippus, and the other successors of Zeno maintained, that with virtue we might be happy in the midst even of disgrace and the most dreadful torments. They admitted the existence of but one God, the soul of the world, which they considered as his body, and both together forming a perfect being. It is remarked that, of all the sects of the ancient philosophers, this was one of those which produced the greatest men. We ought not to confound the two Zenos above mentioned, with

Zeno, a celebrated Epicurean philosopher, born at Sidon, who had Cicero and Pomponius Atticus for his disciples, and who wrote a book against the mathematics, which, as well as that of Possidonius's refutation of it, is lost: nor yet with several other Zenos mentioned in history.

ZENSUS, or Zenzus, in Arithmetic and Algebra, a name used by some of the older authors, especially in Germany, for a square number, or the 2d power: being a corruption from the Italic *censi*, of Pacioli, Tartalea, &c, or the Latin *census*, which signified the same thing.

ZETETICE, or Zetetic *Method*, in Mathematics, was the method made use of to investigate, or find out the solution of a problem; and was much the same thing as analytics, or the analytic method. Vieta has an ingenious work of this kind in 5 books; Zeteticorum libri quinque.

ZOCCO, Zoccolo, Zocle, or Socle, in Architecture, a square body, less in height than in breadth, placed under the bases of pedestals, statues, vases, &c. See Socle and Plinth.

ZODIAC, in Astronomy, an imaginary ring or broad circle, in the heavens, in form of a belt or girdle, within which the planets all make their excursions. In the very middle of it runs the ecliptic, or path of the sun in his annual course; and its breadth, comprehending the deviations or latitudes of the planets, is by some authors accounted 16°, some 18, and others 20 degrees.

The zodiac, cutting the equator obliquely, makes with it the same angle as the ecliptic, which is its middle line, which angle, continually varying, is now nearly equal to 23° 27′ 46″; which is called the obliquity of the zodiac or ecliptic, and is also the sun's greatest declination.

The zodiac is divided into 12 equal parts, of 30 degrees each, called the signs of the zodiac, being so named from the constellations which anciently passed through them. But, the stars having an apparent motion from west to east, arising from the precession of the equinoxes, those constellations do not now correspond to their proper signs. And therefore, when a star is said to be in such a sign of the zodiac, it is not to be understood of that constellation, but only of that dodecatemory or 12th part of it.

The zodiac appears to be very ancient, and to have passed from the ancient Hindus, successively westward, through Persia, Arabia, Assyria, Egypt, &c, to Europe; as specimens of the same kind of zodiac have been found in all those countries, with only some small variation in the figures of some of the constellations; accompanied also with appropriate emblematical figures of the sun and moon, with those of the planets, in their order.

Cassini has also observed a tract in the heavens, within whose bounds most of the comets, though not all of them, are observed to keep, and which he therefore calls the Zodiac of the comets. This he makes as broad as the other zodiac, and marks it with signs or constellations, like that; as Antinous, Pegasus, Andromeda, Taurus, Orion, the Lesser Dog, Hydra, the Centaur, Scorpion, and Sagittary.

ZODIACAL *Light*, a brightness sometimes observed in the zodiac, resembling that of the galaxy or milky way. It appears at certain seasons, viz, towards the end of winter and in spring, after sunset, or before his rising, in autumn and beginning of winter, resembling the form of a pyramid, lying lengthwise with its axis along the zodiac, its base being placed obliquely with respect to the horizon. This phenomenon was first described and named by the elder Cassini, in 1683. It was afterwards observed by Fatio, in 1684, 1685, and 1686; also by Kirch and Eimmart, in 1688, 1689, 1691, 1693, and 1694. See Mairan, Suite des Mem. de l'Acad. Royale des Sciences 1731, pa. 3.

The zodiacal light, according to Mairan, is the solar atmosphere, a rare and subtile fluid, either luminous by itself, or made so by the rays of the sun surrounding its globe; but in a greater quantity, and more extensively, about his equator, than any other part. Mairan observes also that it may be proved from many observations, that the sun's atmosphere sometimes reaches as far as the earth's orbit, and there meeting with our atmosphere, produces the appearance of an aurora borealis. The length of the zodiacal light varies sometimes in reality; and sometimes in appearance only, from various causes.

Cassini often mentions the great resemblance between the zodiacal light and the tails of comets. The same observation has been made by Fatio: and Euler endeavoured to prove that they were owing to similar causes. See Découverte de la Lumière Celeste que paroît dans le Zodiaque, art. 41. Lettre à M. Cassini, printed at Amsterdam in 1686. Euler, in Mem. de l'Acad. de Berlin, tom. 2.

This light seems to have no other motion than that of the sun itself: and its extent from the sun to its point, is seldom less than 50 or 60 degrees in length, and more than 20 degrees in breadth: but it has been known to extend to 100 or 103°, and from 8 to 9° broad.

It is now generally acknowledged, that the electric fluid is the cause of the aurora borealis, ascribed by Mairan to the solar atmosphere, which produces the zodiacal light, and which is thrown off chiefly and to the greatest distance from the equatorial parts of the sun, by means of the rotation on his axis, and extending visibly as far as the orbit of the earth, where it falls into the upper regions of our atmosphere, and is collected chiefly towards the polar parts of the earth, in consequence of the diurnal revolution, where it forms the aurora borealis. And hence it has been suggested, as a probable conjecture, that the sun may be the fountain of the electrical fluid, and that the zodiacal light, and the tails of comets, as well as the aurora borealis, the lightning, and artificial electricity, are its various and not very dissimilar modifications.

ZONE, in Geography and Astronomy, a division of the earth's surface, by means of parallel circles, chiefly with respect to the degree of heat in the different parts of that surface.

The ancient astronomers used the term Zone, to explain the different appearances of the sun and other heavenly bodies, with the length of the days and nights; and the geographers, as they used the climates, to mark the situation of places; using the term climate when they were able to be more exact, and the term zone when less so.

The zones were commonly accounted five in number; one a broad belt round the middle of the earth, having the equator in the very middle of it, and bounded, towards the north and south, by parallel circles passing through the tropics of Cancer and Capricorn. This they called the torrid zone, which they supposed not habitable, on account of its extreme heat. Though sometimes they divided this into two equal torrid zones, by the equator, one to the north, and the other south; and then the whole number of zones was accounted 6.

Next, from the tropics of Cancer and Capricorn to the two polar circles, were two other spaces called temperate zones, as being moderately warm; and these they supposed to be the only habitable parts of the earth.

Lastly, the two spaces beyond the temperate zones, about either pole, bounded within the polar circles, and having the poles in the middle of them, are the two frigid or frozen zones, and which they supposed not habitable, on account of the extreme cold there.

Hence, the breadth of the torrid zone is equal to twice the greatest declination of the sun, or obliquity of the ecliptic, equal to 46° 56′, or twice 23° 28′. Each frigid zone is also of the same breadth, the distance from the pole to the polar circle being equal to the same obliquity 23° 28′. And the breadth of each temperate zone is equal to 43° 4′, the complement of twice the same obliquity. See these zones exhibited in plate 40, fig. 16.

The difference of zones is attended with a great diversity of phenomena. 1. In the torrid zone, the sun passes through the zenith of every place in it twice a year; making as it were two summers in the year; and the inhabitants of this zone are called amphiscians, because they have their noon-day shadows projected different ways in different times of the year, northward at one season, and southward at the other.

2. In the temperate and frigid zones, the sun rises and sets every natural day of 24 hours. Yet every where, but under the equator, the artificial days are of unequal lengths, and the inequality is the greater, as the place is farther from the equator. The inhabitants of the temperate zones are called heteroscians, because their noon-day shadow is cast the same way all the year round, viz, those in the north zone toward the north pole, and those in the south zone toward the south pole.

3. Within the frigid zones, the inhabitants have their artificial days and nights extended to a great length; the sun sometimes skirting round a little above the horizon for many days together: and at another season never rising above the horizon at all, but making continual night for a considerable time. The inhabitants of these zones are called periscians, because sometimes they have their shadows going quite round them in the space of 24 hours.

ERRATA IN VOL. II.

Page 154, col. 1, line 39, for 395·7, read 3957.
— 233, — 1, line 3, for 2, read ·2.
— 248, — 2, line 37, dele and.
— 400, — 1, line 21, read 2 cos. *a*.
— 503, — 2, line 15 from the bottom, read plate 27.
— 557, — 1, line 9 from the bottom, for + ÿ, read *ẏ*.
— 582, — 2, line 2, for Celti, read Cetii.
— 294, — 1, line 34, for lieut.-governor &c, read inspector-general of the R. Mil. College, and commandant of the senior department of that institution, now established at Farnham.

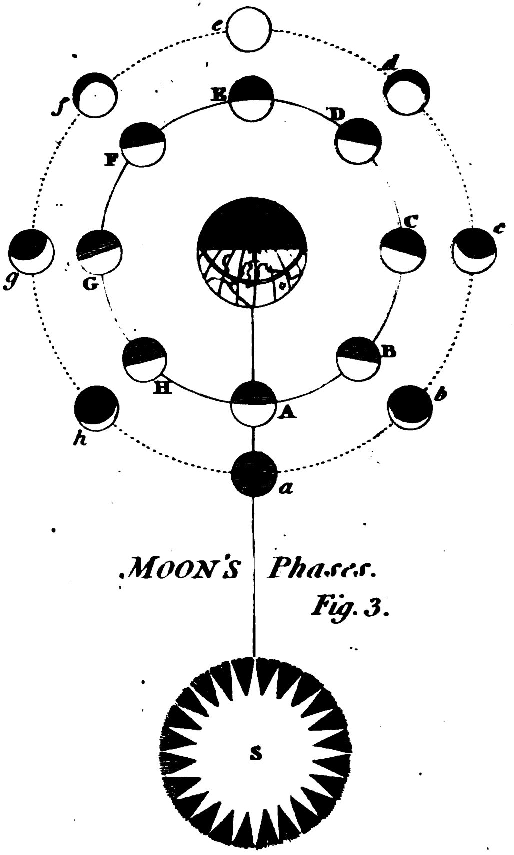
FINIS.

Printed by S. Hamilton, Weybridge, Surrey.

Fig. 1.

MAGIC Square of Squares.

226	255	18	15	194	223	50	47	162	191	82	79	130	159	114	111
32	1	240	241	64	33	208	209	96	65	176	177	128	97	144	145
239	242	31	2	207	210	63	34	175	178	95	66	143	146	127	98
17	16	225	256	49	48	193	224	81	80	161	192	113	112	129	160
228	253	20	13	196	221	52	45	164	189	84	77	132	157	116	109
30	3	238	243	62	35	206	211	94	67	174	179	126	99	142	147
237	244	29	4	205	212	61	36	173	180	93	68	141	148	125	100
19	14	227	254	51	46	195	222	83	78	163	190	115	110	131	158
230	251	22	11	198	219	54	43	166	187	86	75	134	155	118	107
28	5	236	245	60	37	204	213	92	69	172	181	124	101	140	149
235	246	27	6	203	214	59	38	171	182	91	70	139	150	123	102
21	12	229	252	58	44	197	220	85	76	165	188	117	108	133	156
232	249	24	9	200	217	56	41	168	185	88	73	136	153	120	105
26	7	234	247	58	39	202	215	90	71	170	183	122	103	138	151
233	248	25	8	201	216	57	40	169	184	89	72	137	152	121	104
23	10	231	250	55	42	199	218	87	74	167	186	119	106	135	154

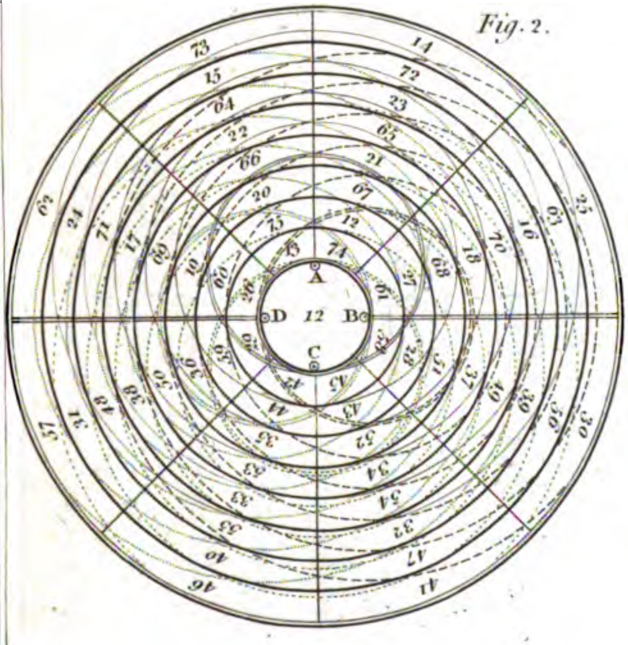


MOON'S Phases.

Fig. 3.

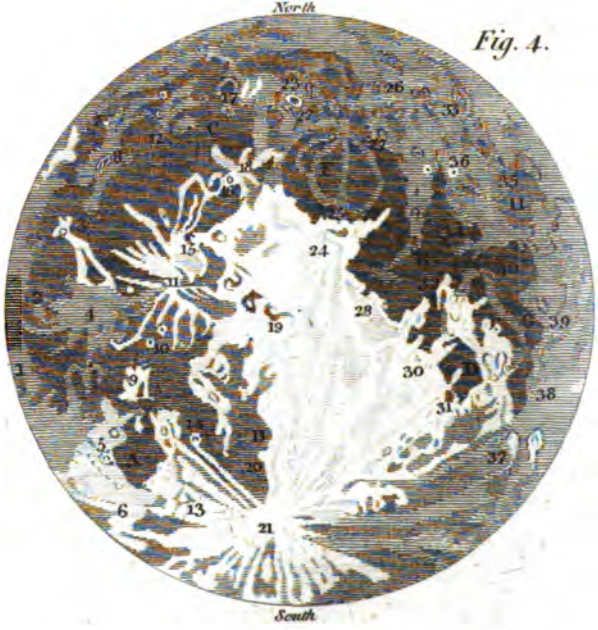
MAGIC Circle of Circles.

Fig. 2.

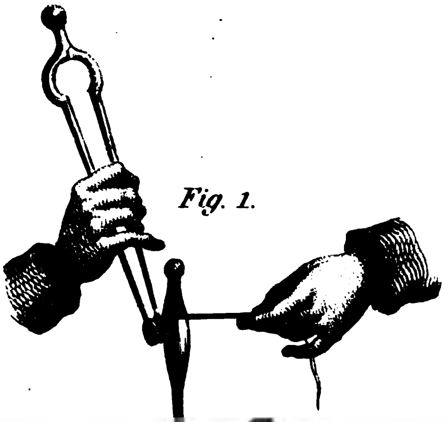


Face of the MOON.

Fig. 4.



ARTIFICIAL MAGNETS.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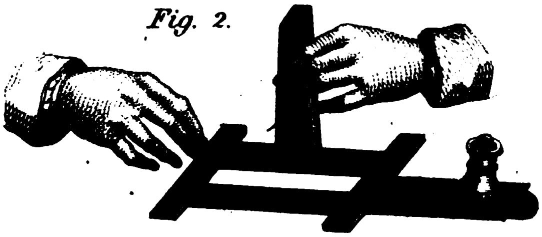
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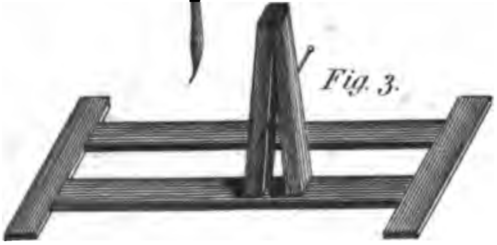
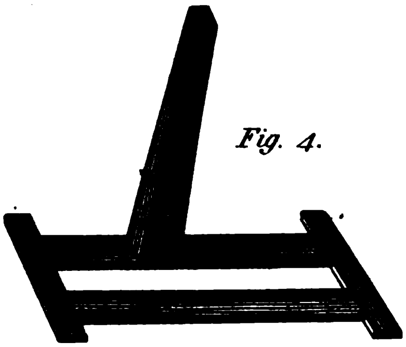


Fig. 3.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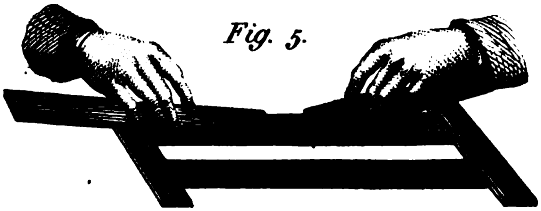


Fig. 5.

Fig. 6.

NEPER'S BONES.

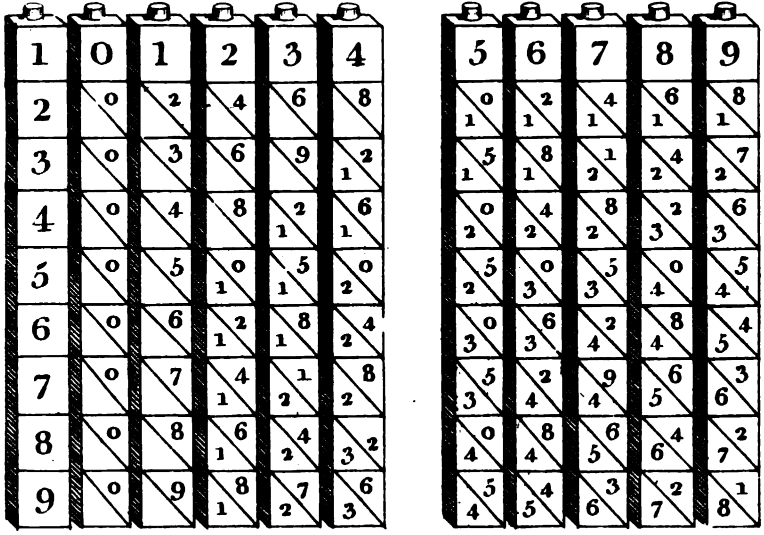


Fig. 7.

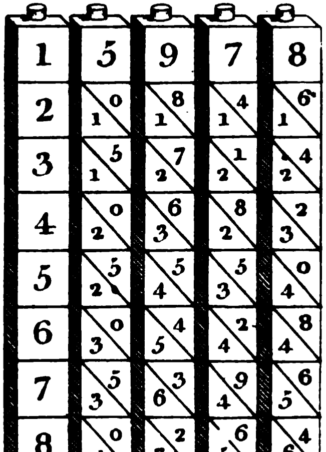


Fig. 8.

GEOGRAPHICAL MAPS.

Fig. 1.

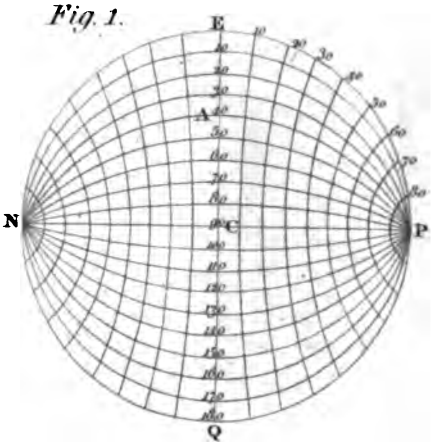
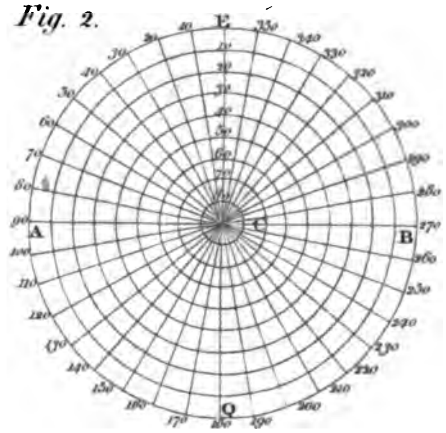


Fig. 2.



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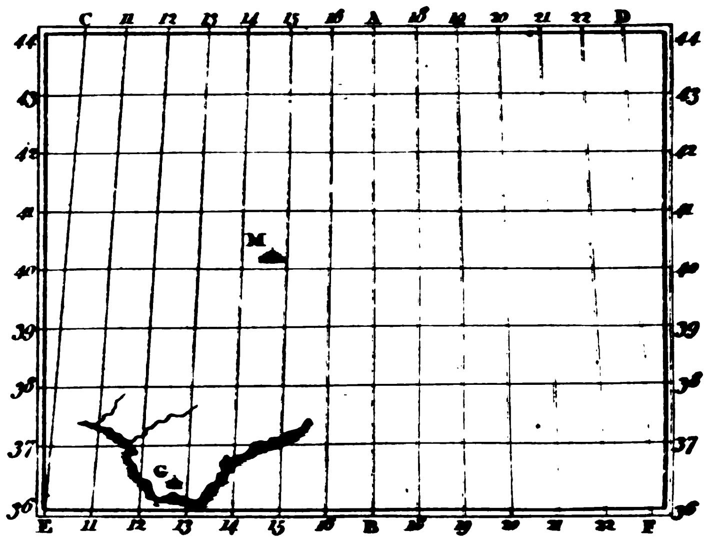

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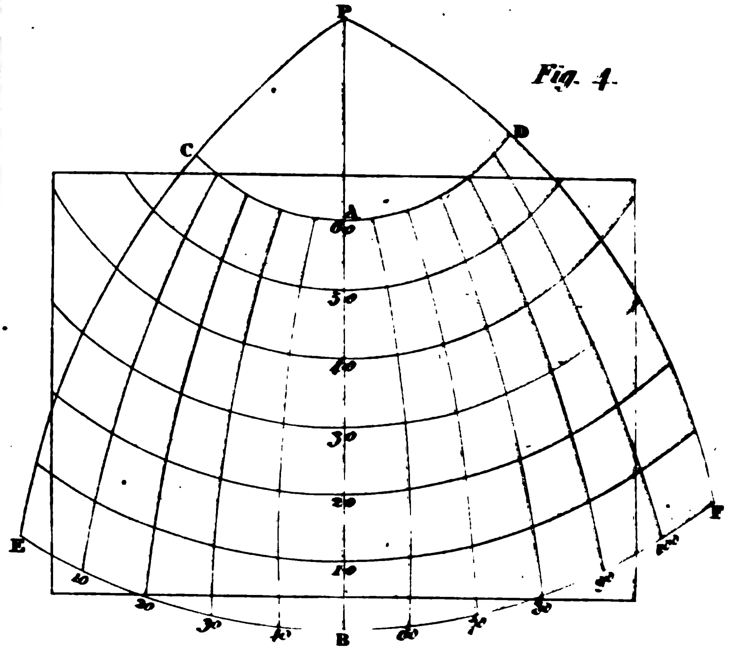
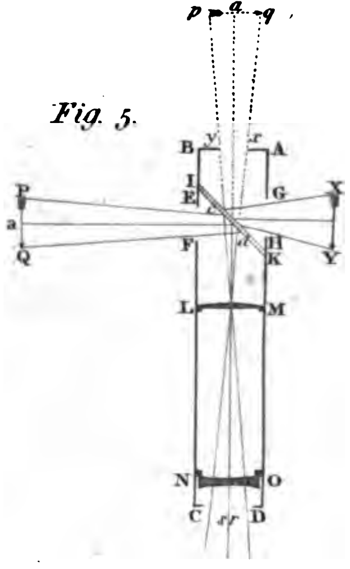


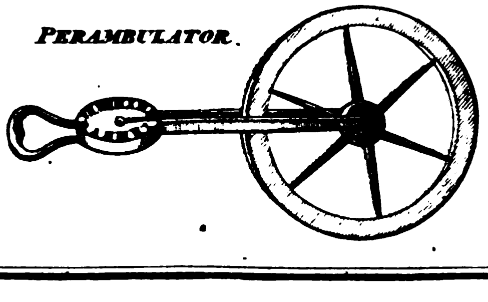
OPERA Glaſs

Fig. 5.

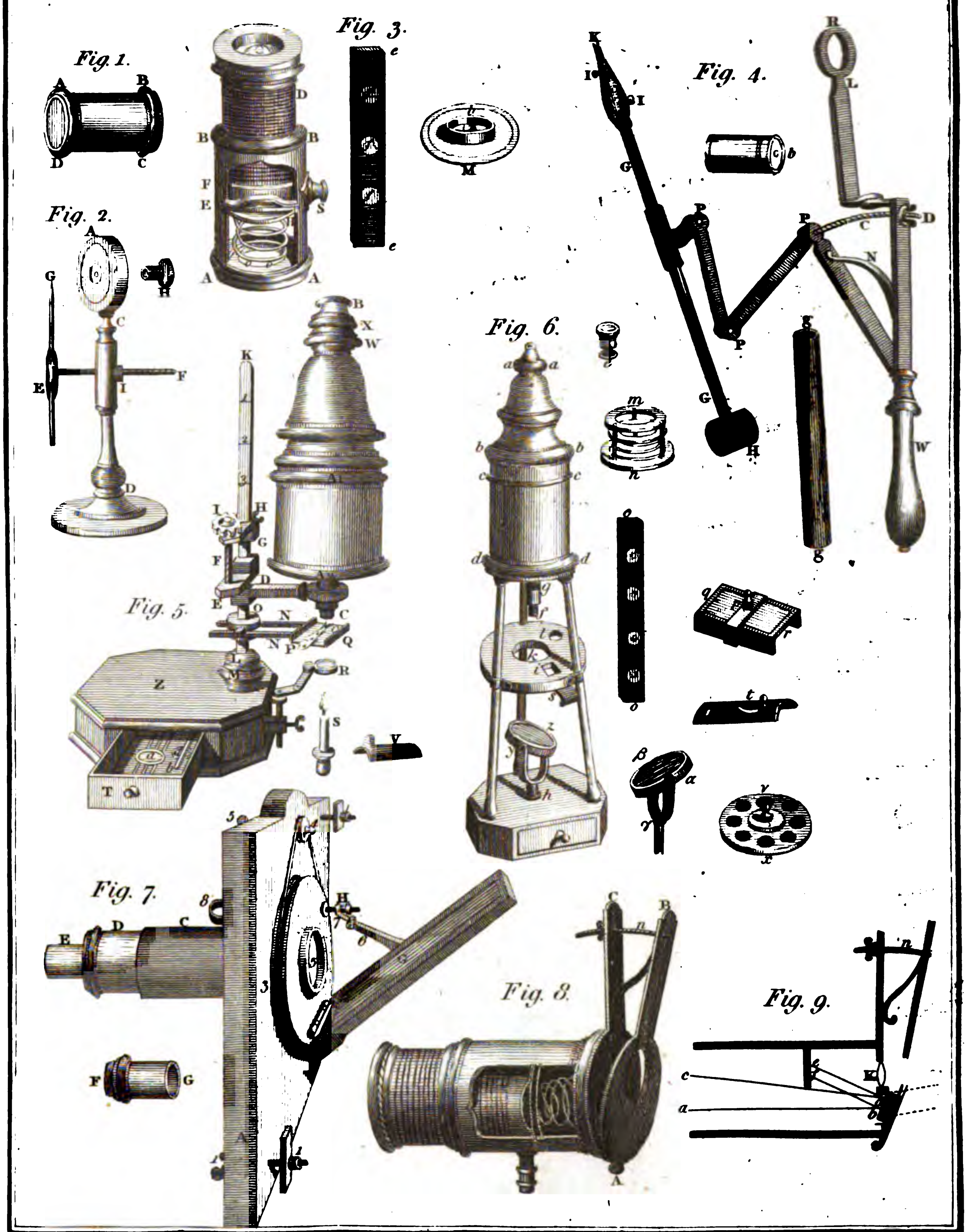


PERAMBULATOR.

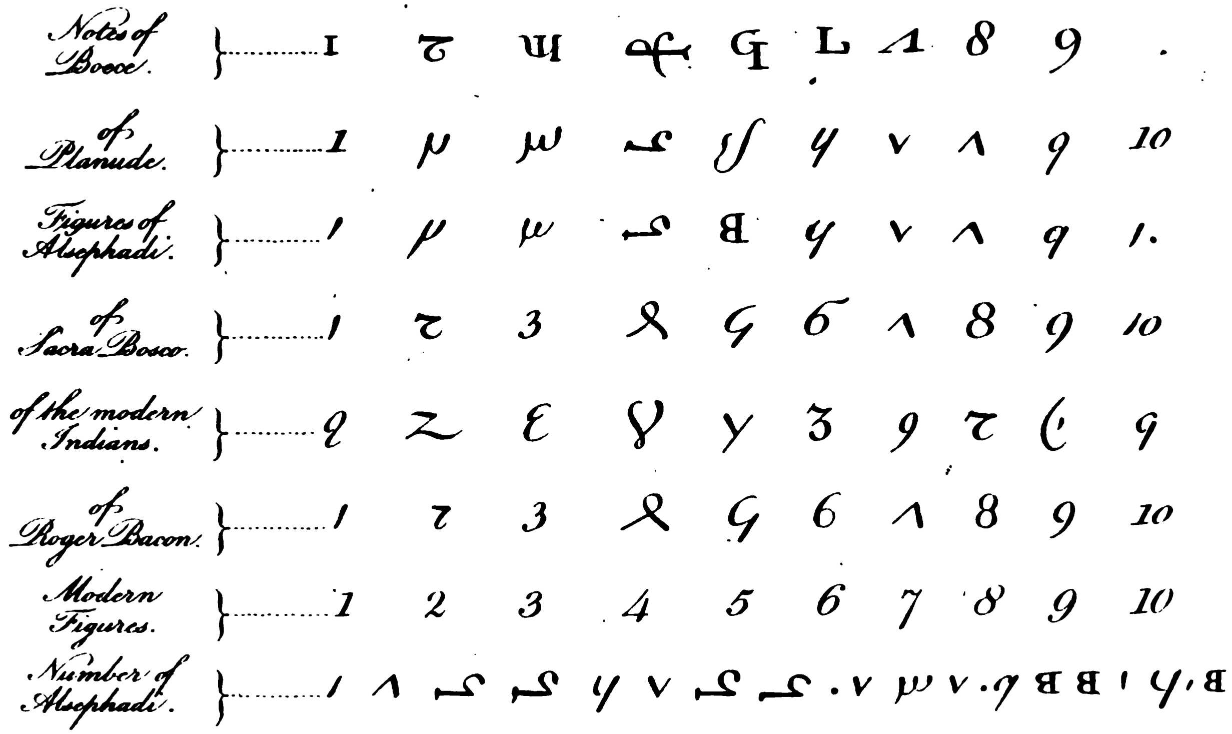
Fig. 6.



MICROSCOPES.

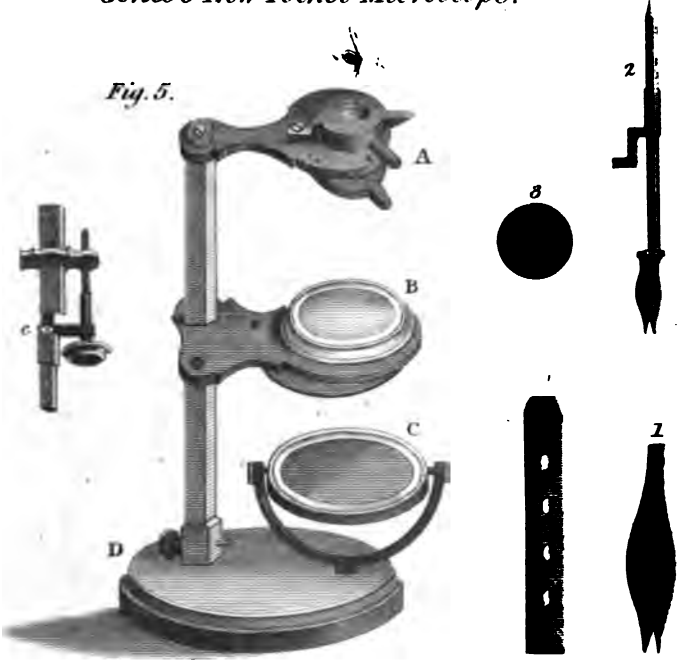


Ancient Arithmetical Characters.


Notes of Boece.
of Planude.
Figures of Alsephadi.
of Sacra Bosco.
of the modern Indians.
of Roger Bacon.
Modern Figures.
1 2 3 4 5 6 7 8 9 10
Number of Alsephadi.

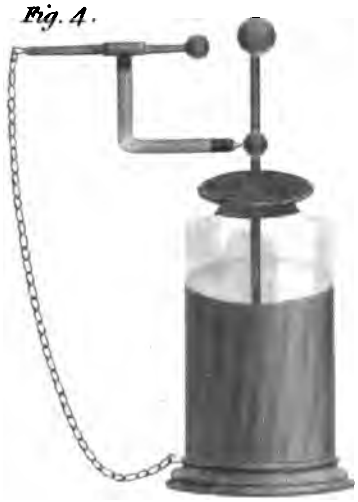
Jones's New Pocket Microscope.

Fig. 5.



Leyden Phial.

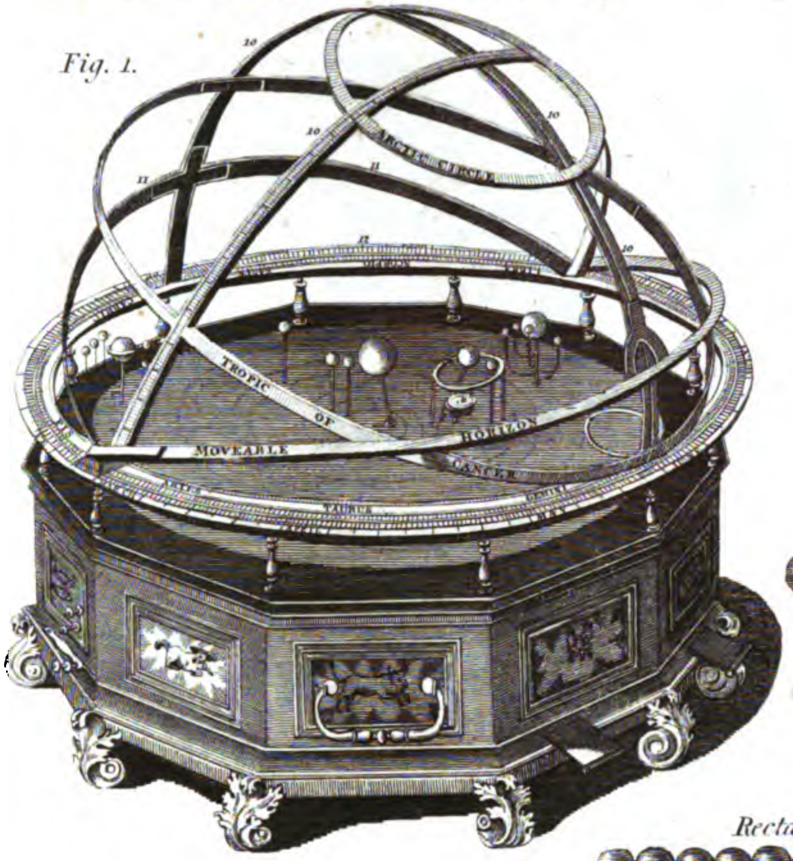
Fig. 4.



Mutlow sc. Russell Co.

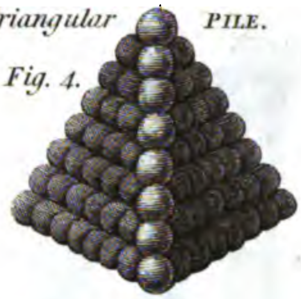
GRAND ORRERY, *by Graham & Rowley.*

Fig. 1.



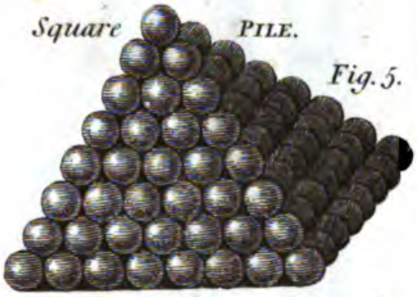
Triangular **PILE.**

Fig. 4.



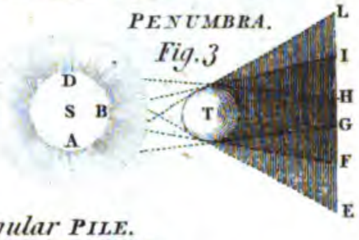
Square **PILE.**

Fig. 5.



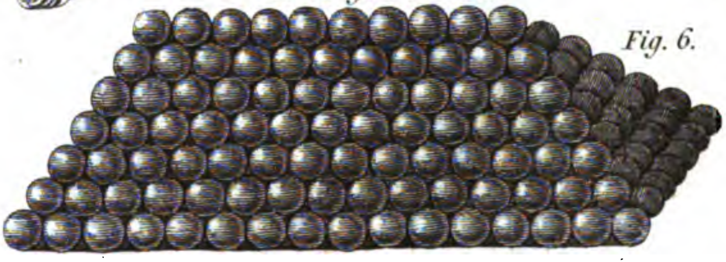
PENUMBRA.

Fig. 3



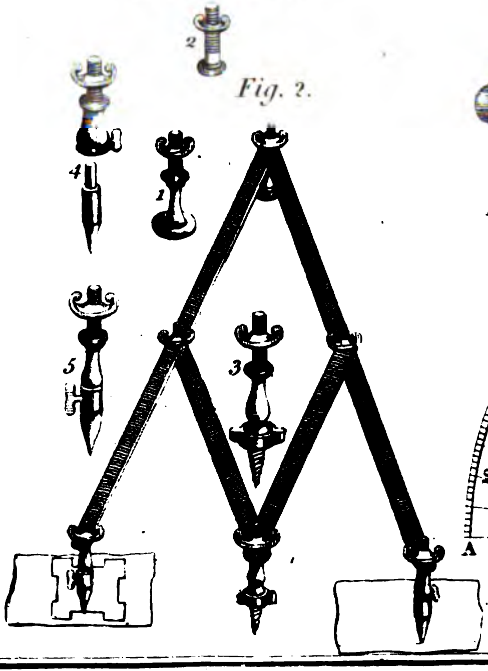
Rectangular **PILE.**

Fig. 6.



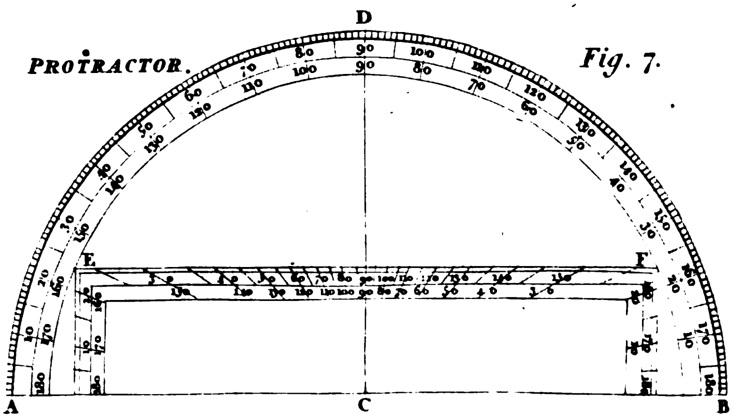
PENTAGRAPH.

Fig. 2.



PROTRACTOR.

Fig. 7.



PERSIAN *Wheel.*

Fig. 1.

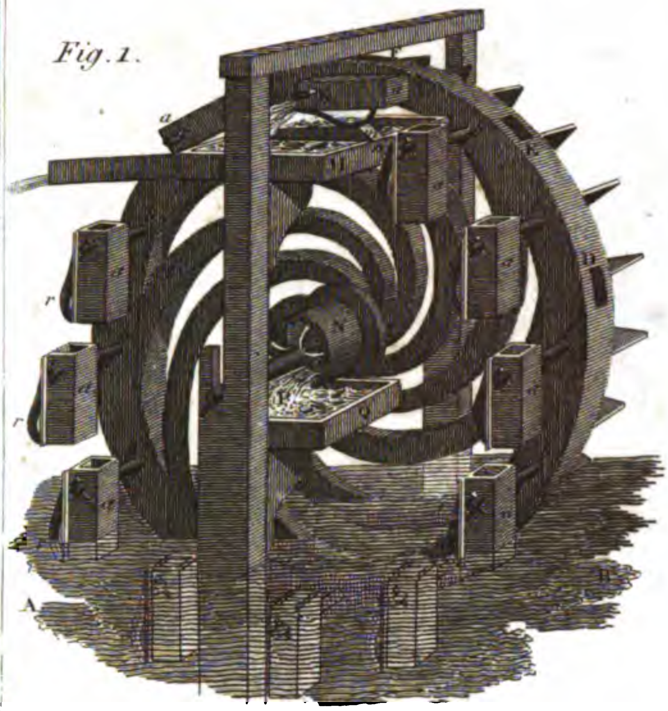
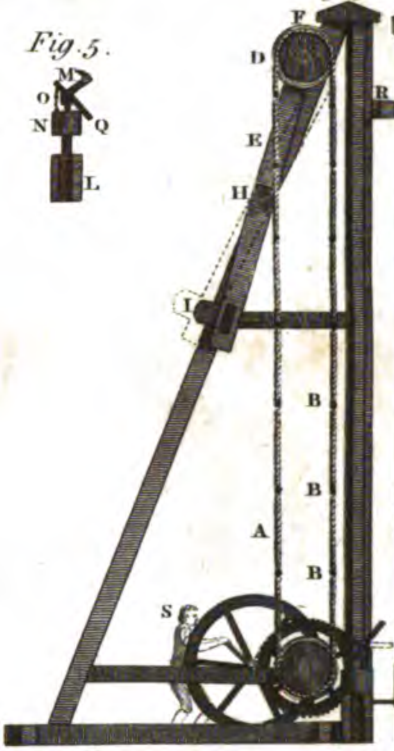


Fig. 5.

Bunce's **PILE** *Engine.*

Fig. 3.



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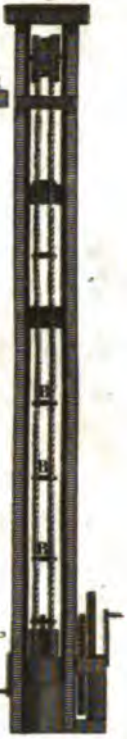


Vauloue's **PILE** *Engine.*

Fig. 2.

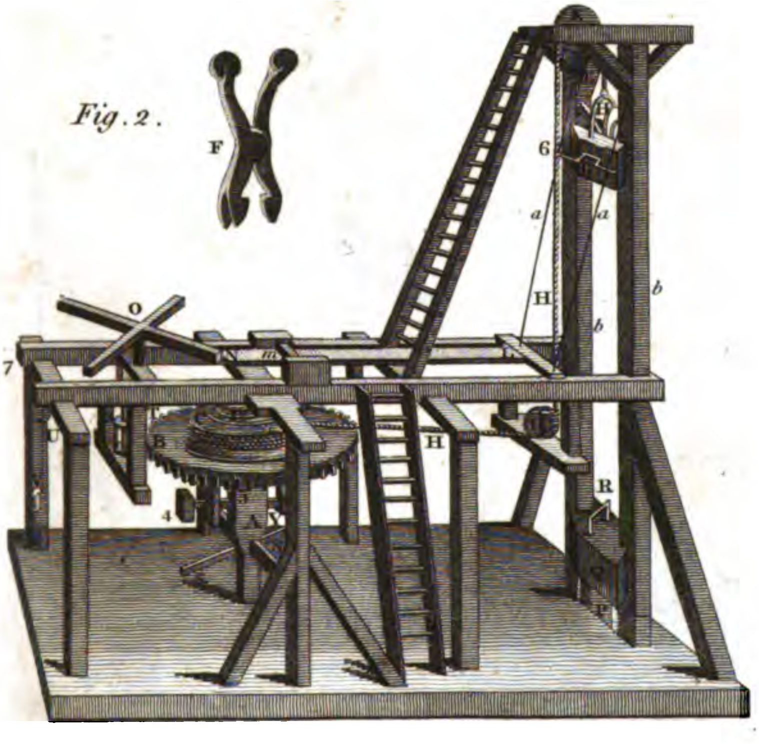
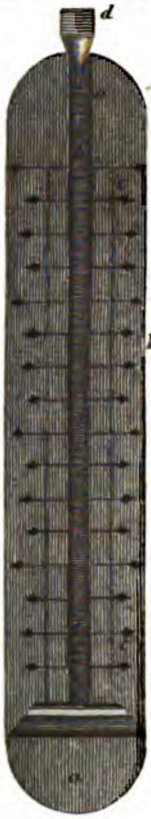


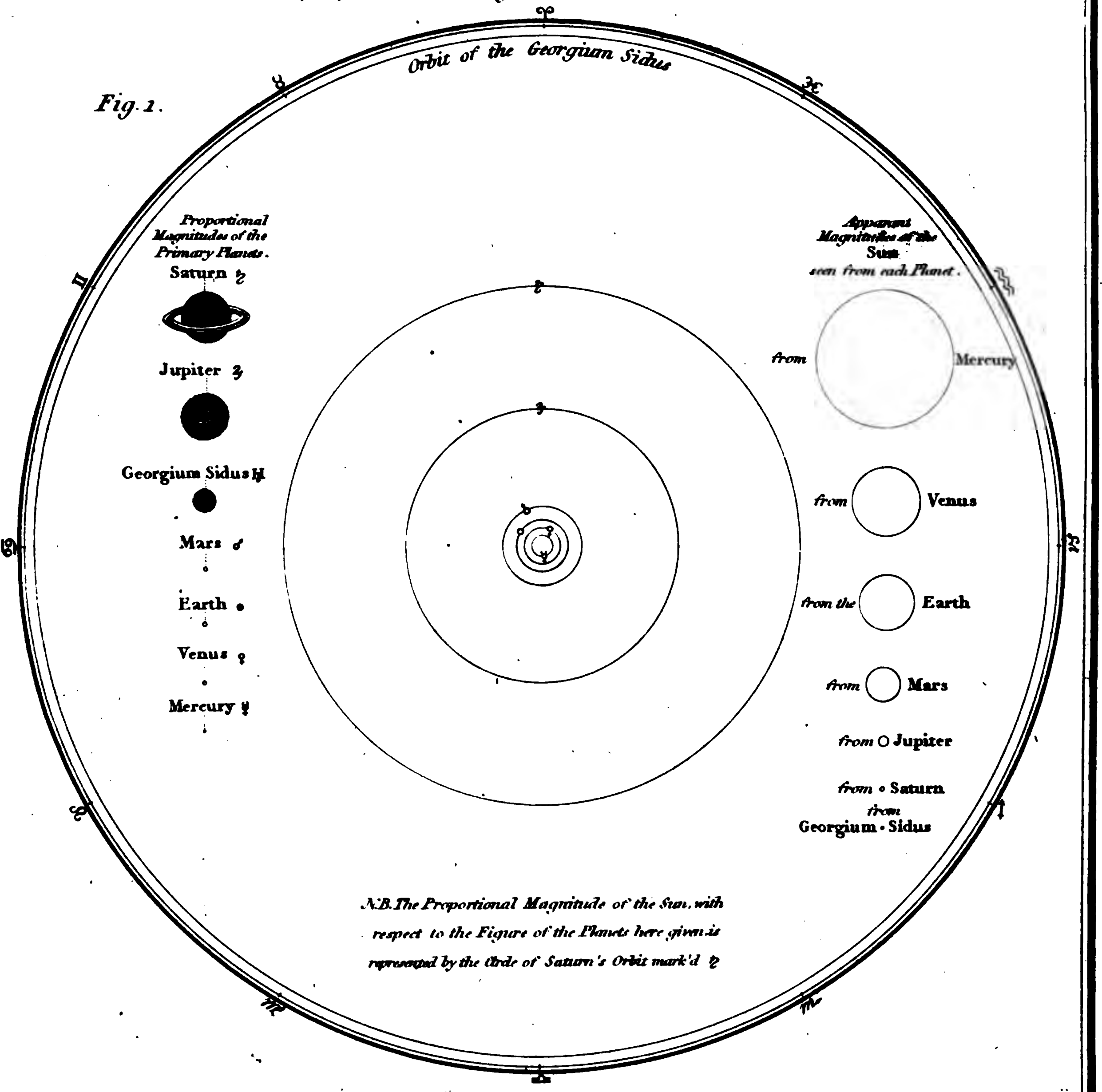
Fig. 6.



PLUVIAMETER

View of the proportional Magnitudes of the PLANETARY ORBITS

Fig. 1.



N.B. The Proportional Magnitude of the Sun, with respect to the Figure of the Planets here given is represented by the Circle of Saturn's Orbit mark'd ♄

PLANE SCALES. Fig. 2.

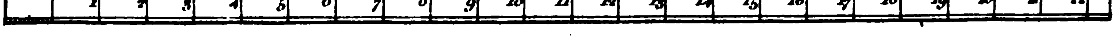
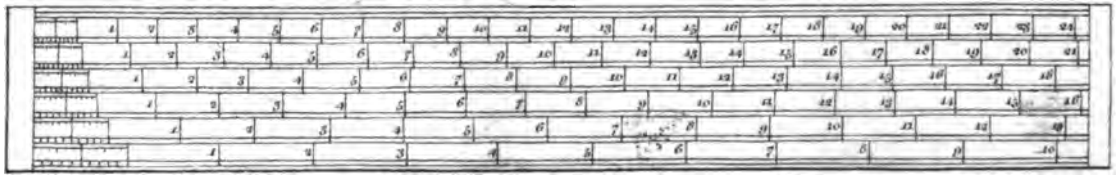


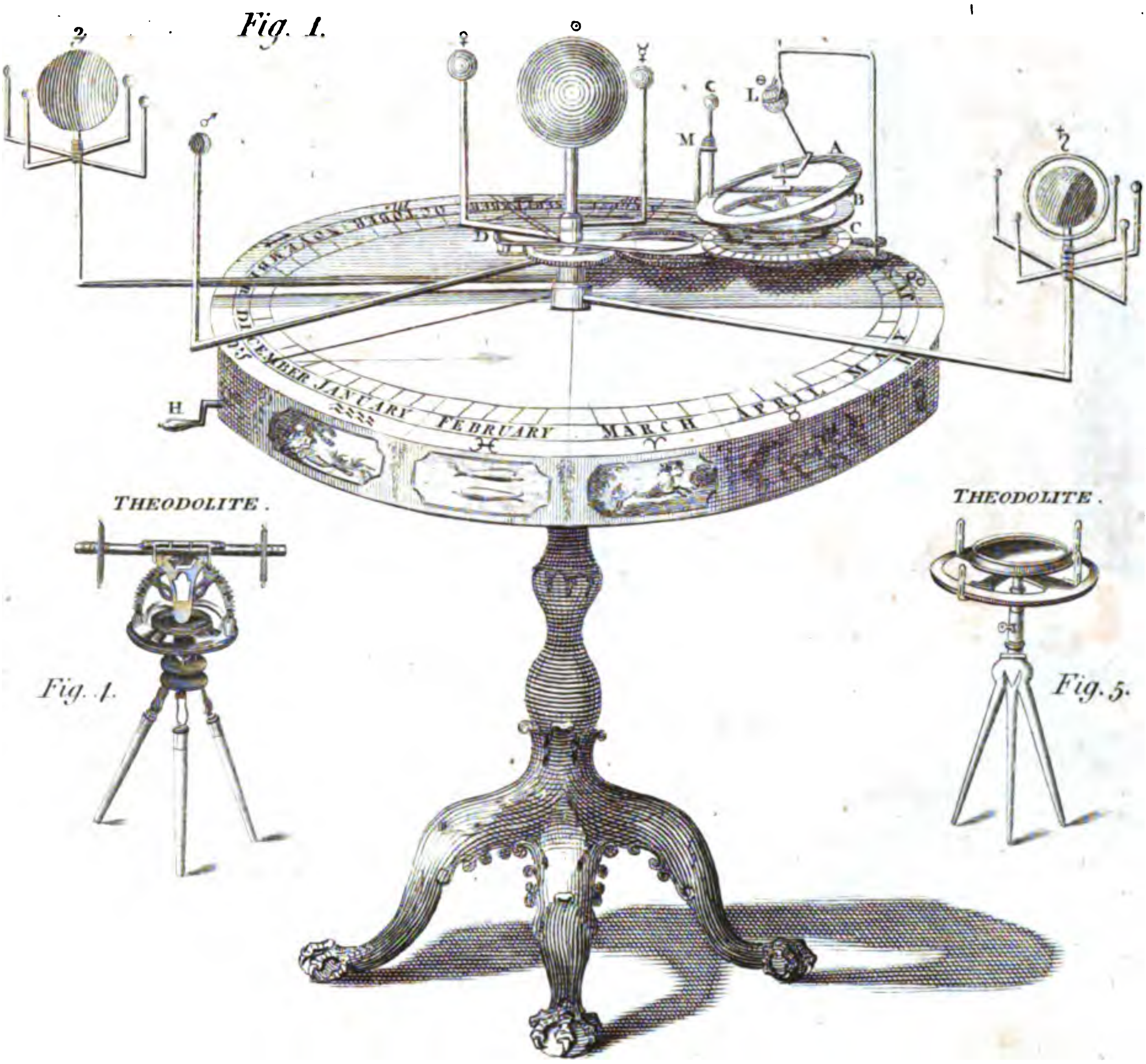
Fig. 3.



J. Eyre sculp.

PLANETARIUM by Jones.

Fig. 1.



THEODOLITE.

Fig. 4.

THEODOLITE.

Fig. 5.

PLANE DIAGONAL SCALES.

Fig. 2.

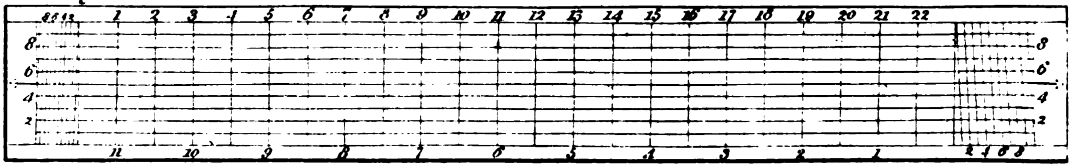
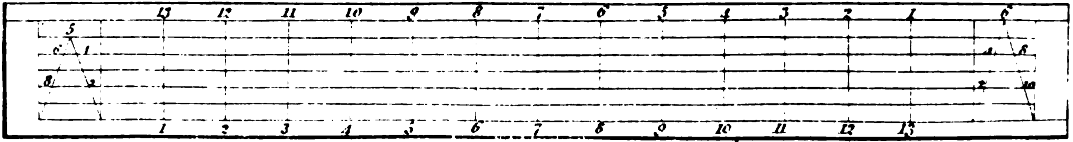
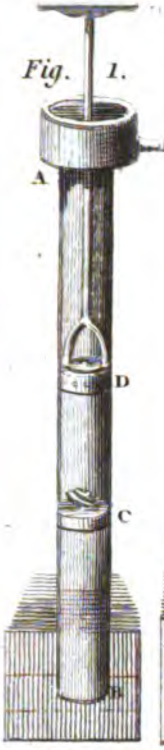


Fig. 3.



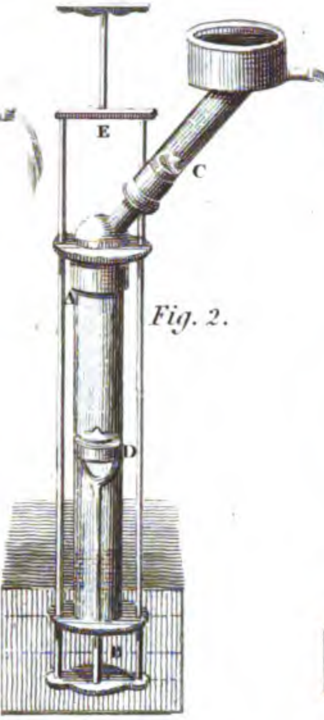
Sucking PUMP.

Fig. 1.



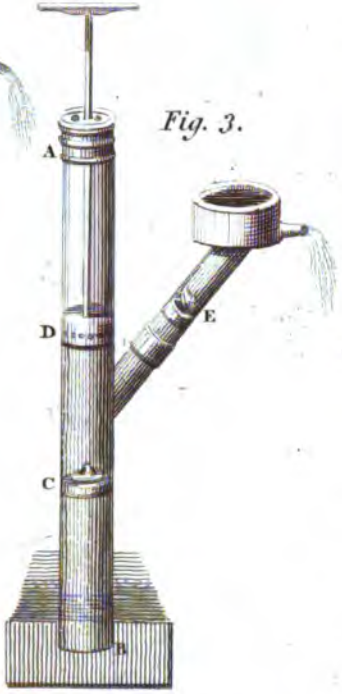
Lifting PUMP.

Fig. 2.



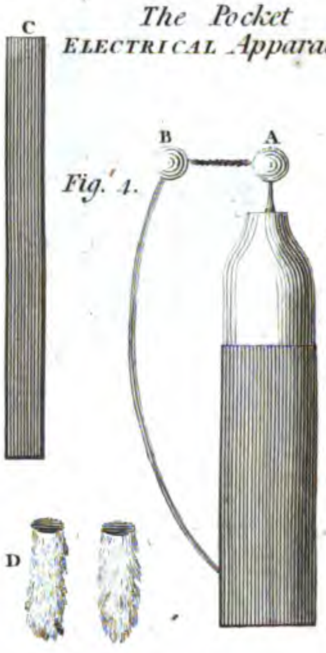
Forcing PUMP.

Fig. 3.



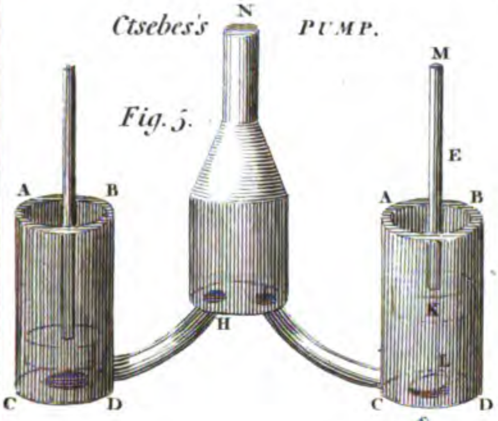
The Pocket ELECTRICAL *Apparatus.*

Fig. 4.



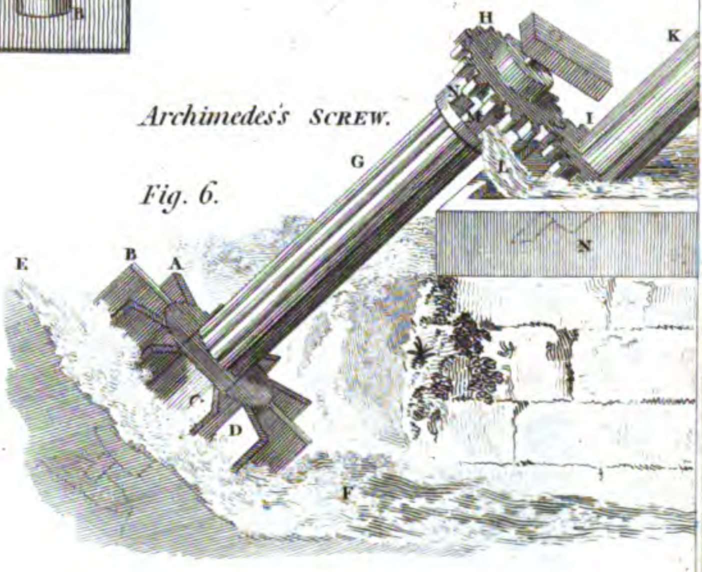
Ctsebes's PUMP.

Fig. 5.



Archimedes's SCREW.

Fig. 6.



Archimedes's SCREW.

Fig. 9.

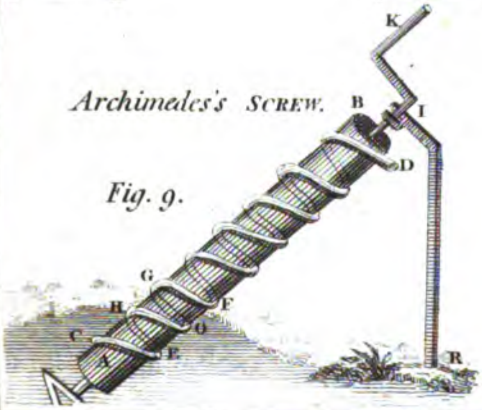


Fig. 7.

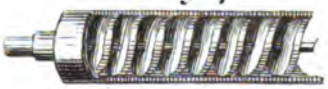
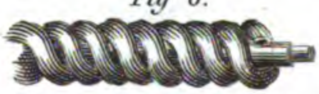
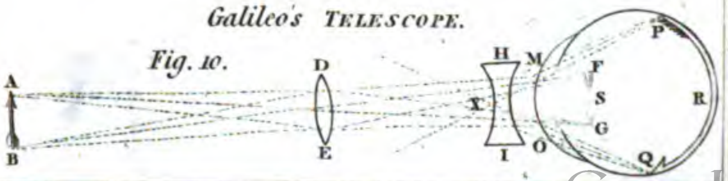


Fig 8.



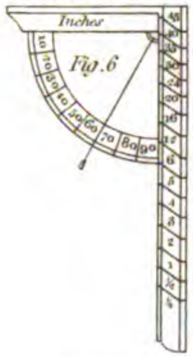
Galileo's TELESCOPE.

Fig. 10.

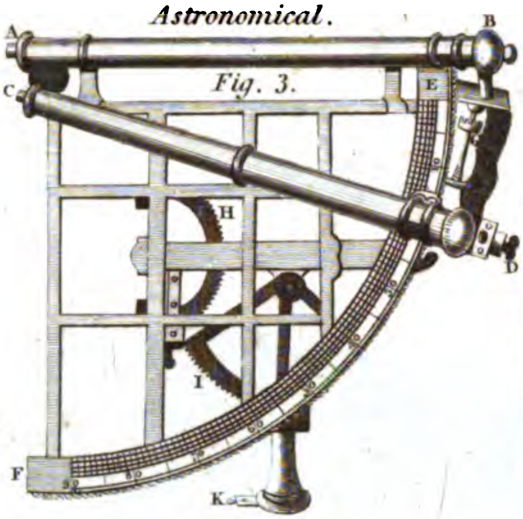


QUADRANTS.

Gunners



Astronomical.



Gunter's.

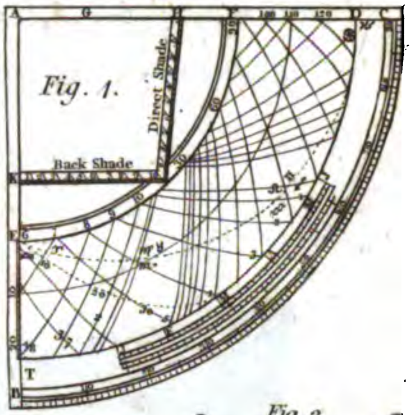
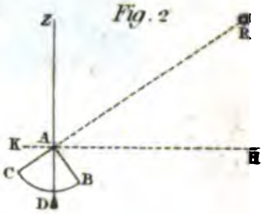
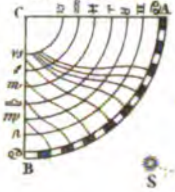


Fig. 2

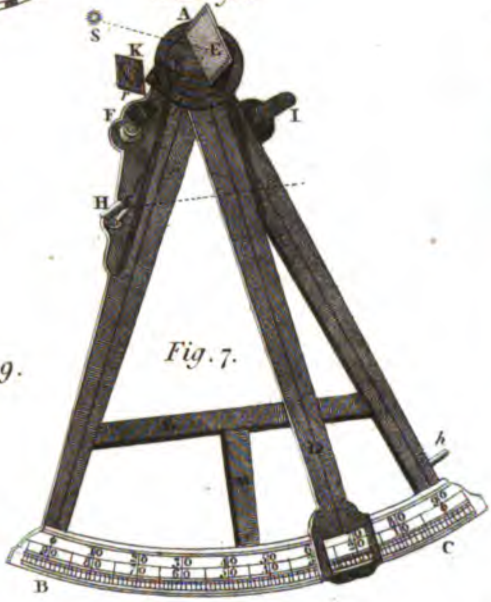


Horodotical

Fig. 5



Hadley's.



Coles

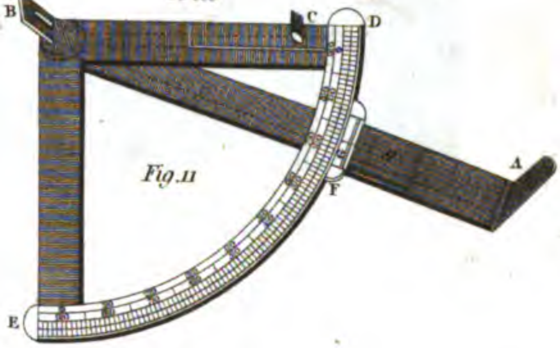
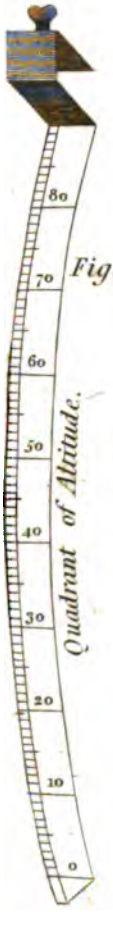
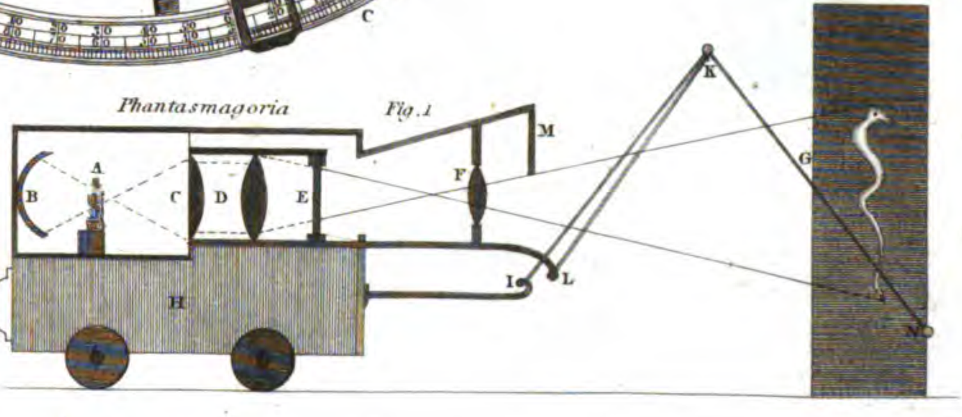


Fig. 9.



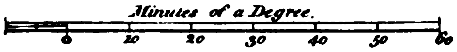
Quadrant of Altitude.

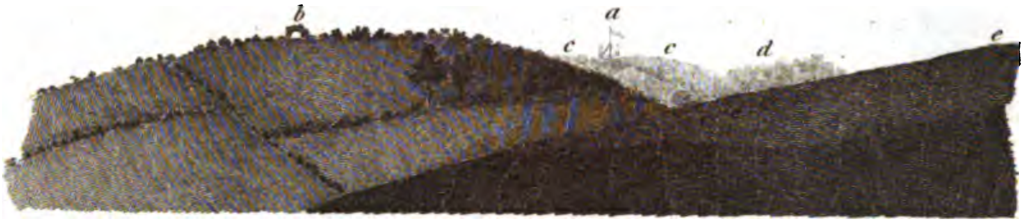
Phantasmagoria Fig. 1



Telescopic Appearances, of the Western Horizon, in three different States of the Atmosphere, taken from the Laurel Mount, at Traine, in Modbury Devonshire.

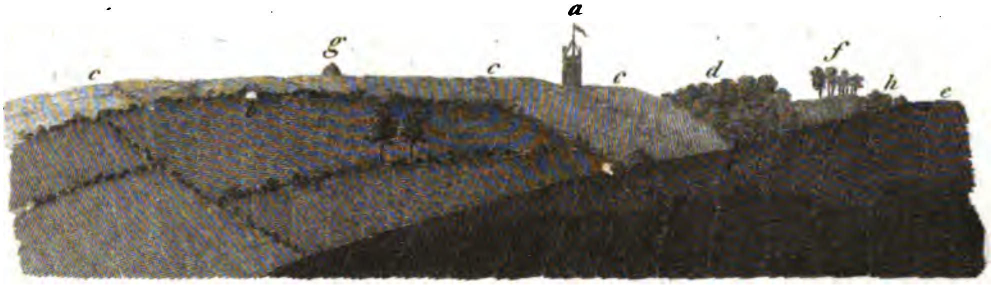
THE SCALE





Ordinary or common Appearance.

Appearance somewhat elevated by Refraction.

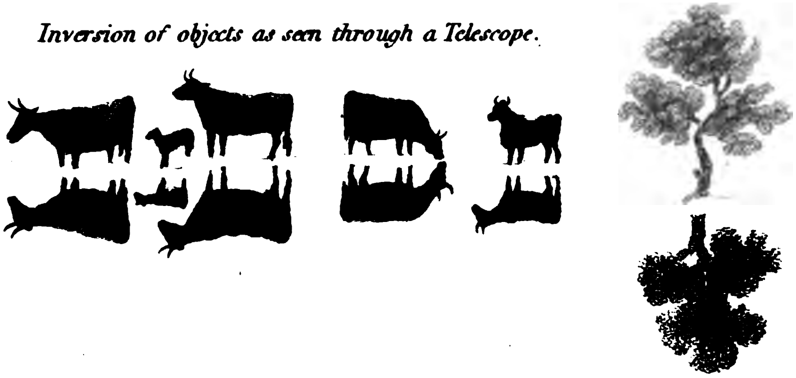


Appearance when more considerably elevated.

a. Maker tower, about 12¾ miles distant, in a straight line. b. Gate place (at present appearing like an Arch) on a Hill about 3¾ miles. cc. Ground about 9¾ miles. d. A wood in Mount Edgcumbe park, about 12¼ miles. ee. A hill about 3 miles. f. Trees in the Park. g. A mow, seen by the Refraction on the Ground. c c. h. Another set of Trees in the Park.

Inversion of objects as seen through a Telescope.

Fig. 1



Sir Howard Douglas's Reflecting Instrument.

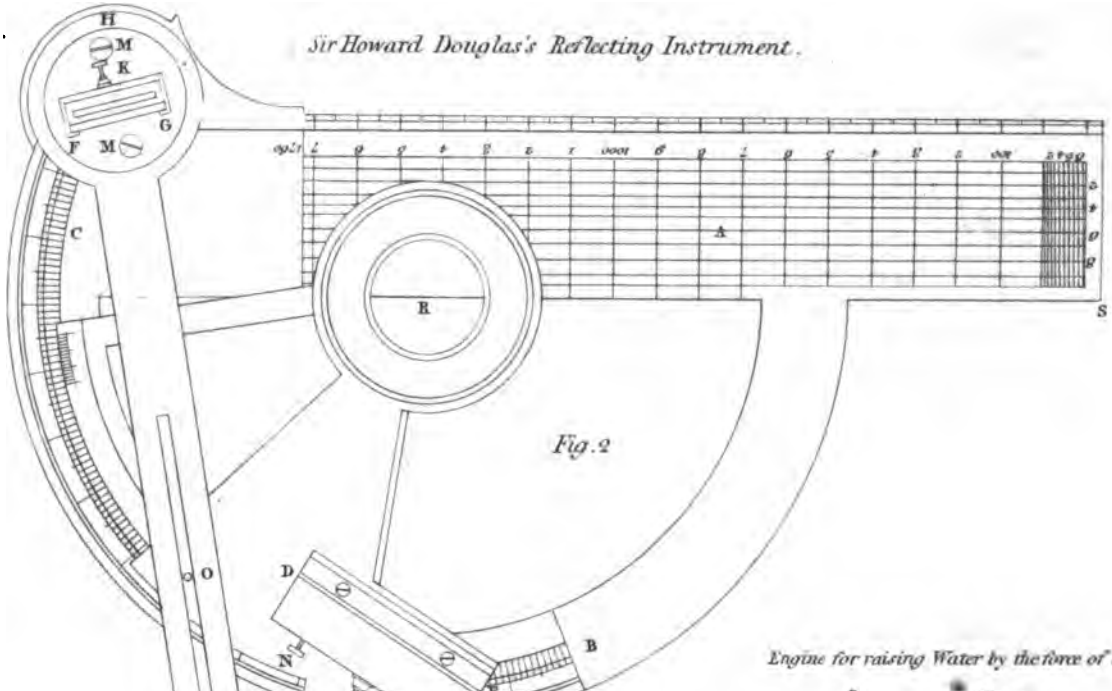
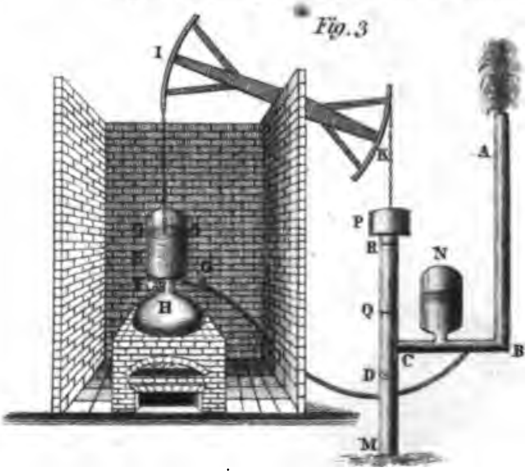


Fig. 2

Engine for raising Water by the force of Steam.

Fig. 3



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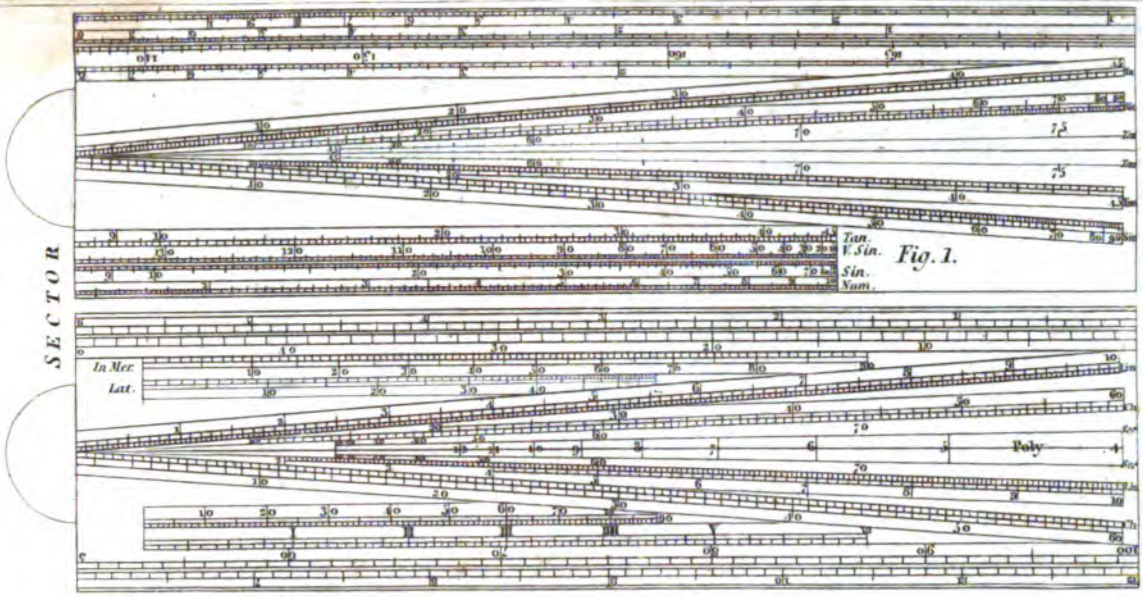


Fig. 1.

Astronomical SECTOR.

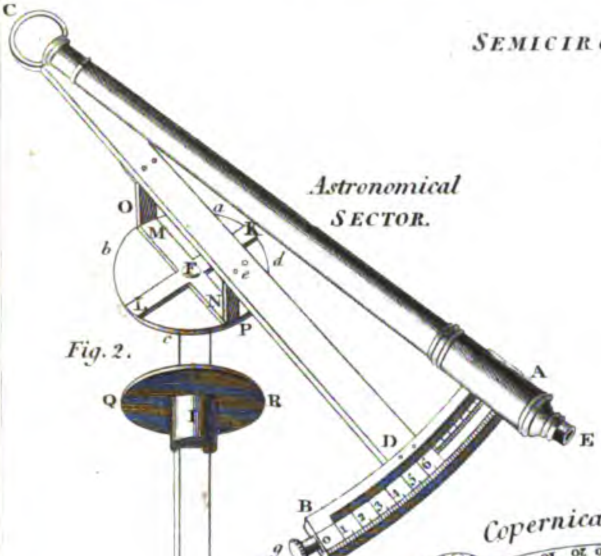


Fig. 2.

SEMICIRCLE.

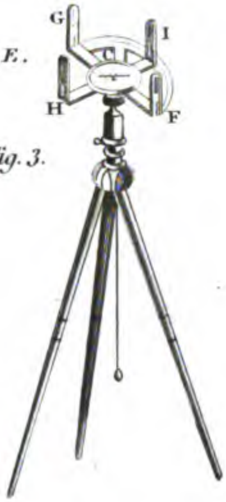
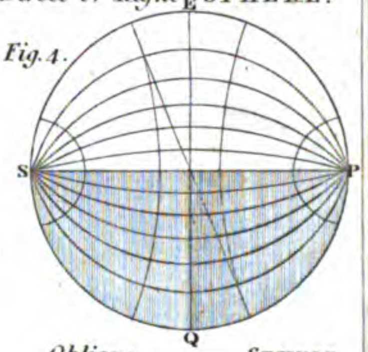


Fig. 3.

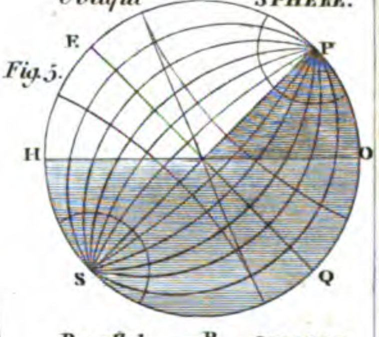
Direct or Right SPHERE.

Fig. 4.



Oblique SPHERE.

Fig. 5.



Parallel SPHERE.

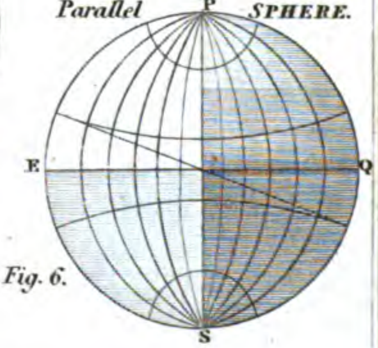


Fig. 6.

Copernican SPHERE.

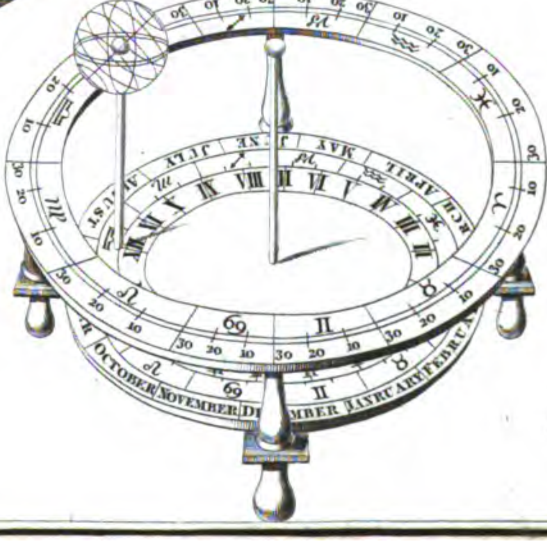


Fig. 7.

Water Spout

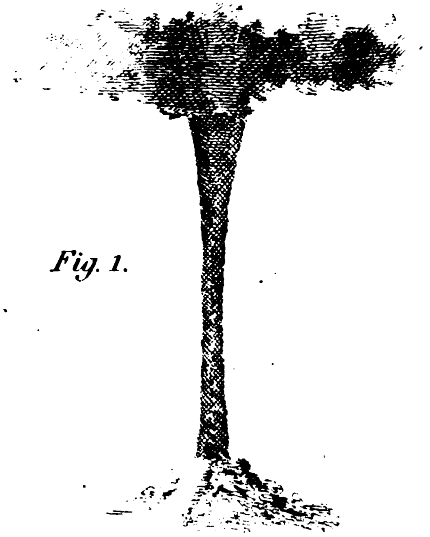
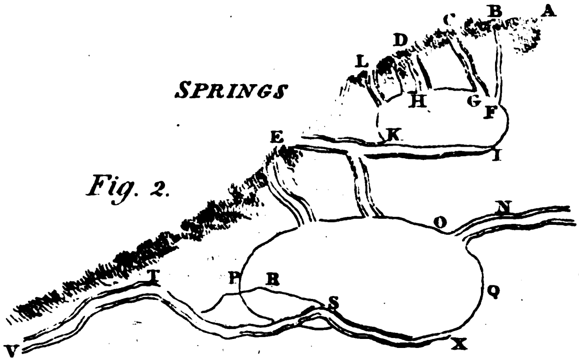


Fig. 1.

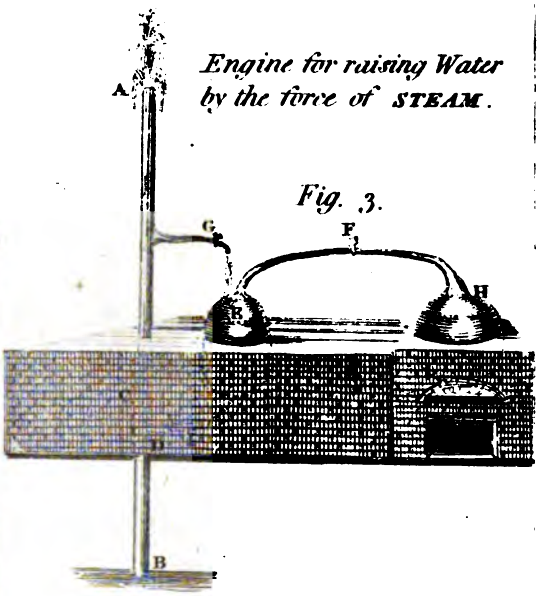
Springs

Fig. 2.



Engine for raising Water by the force of **Steam**.

Fig. 3.



Steam Engine.

Fig. 4.

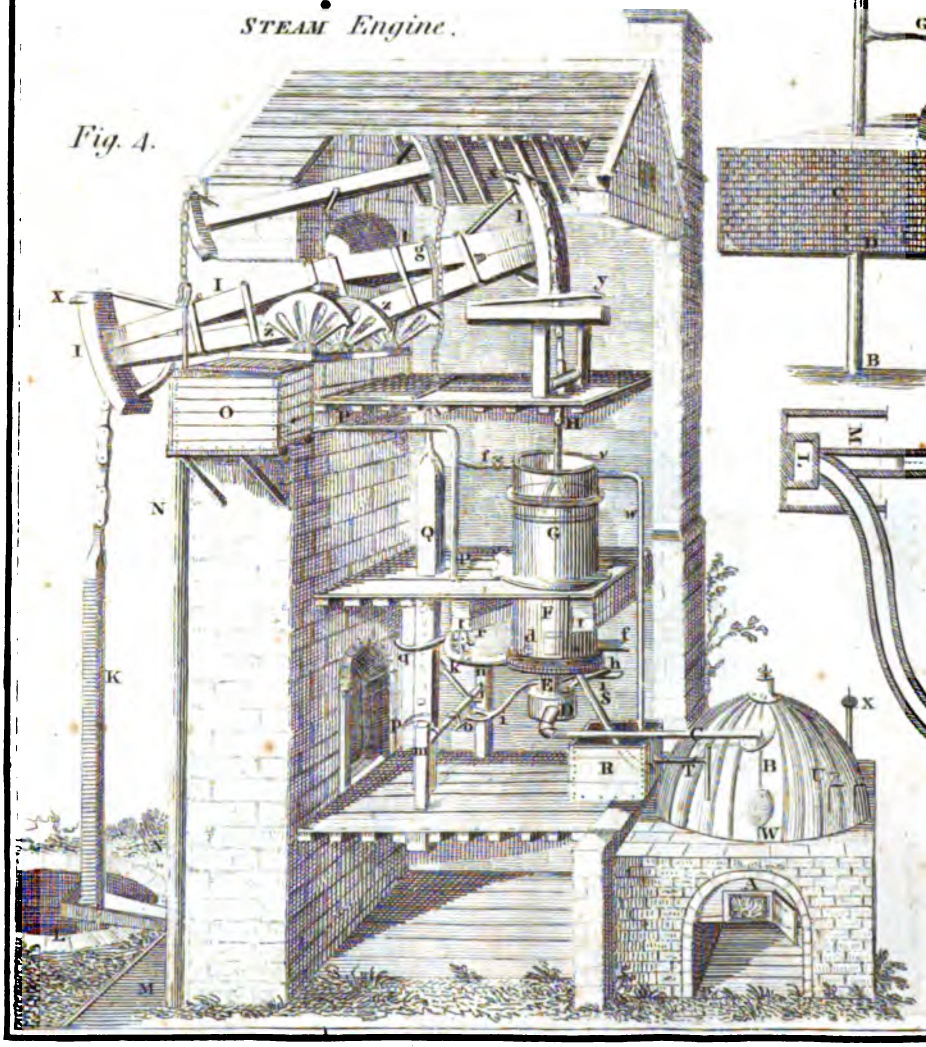
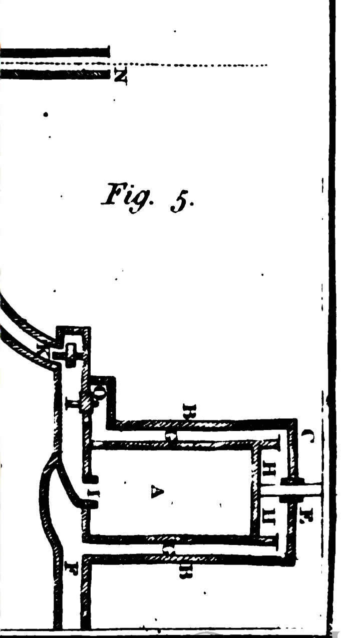


Fig. 5.



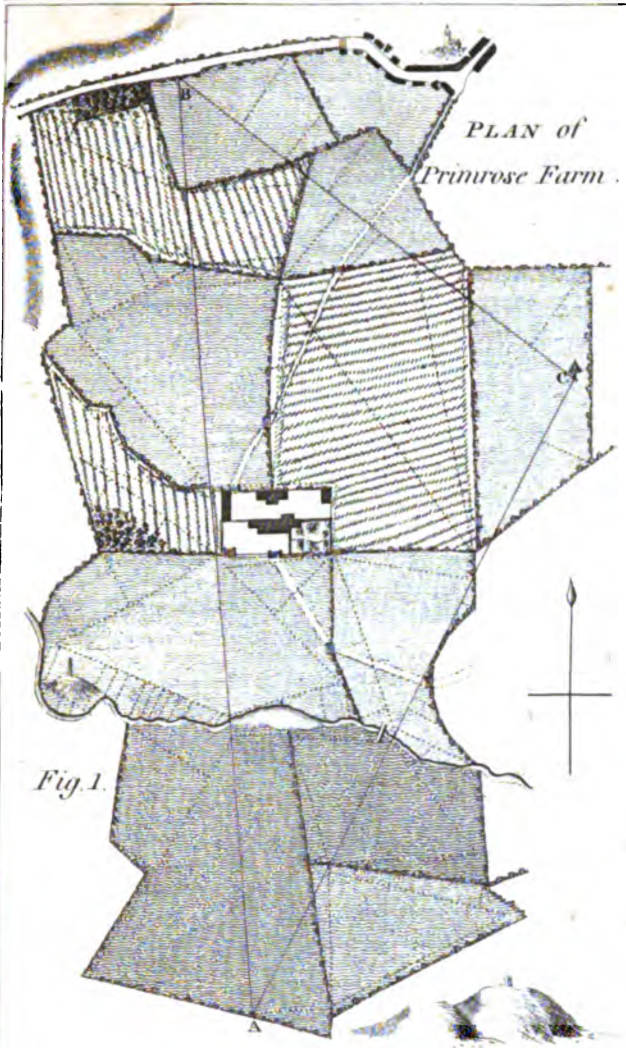
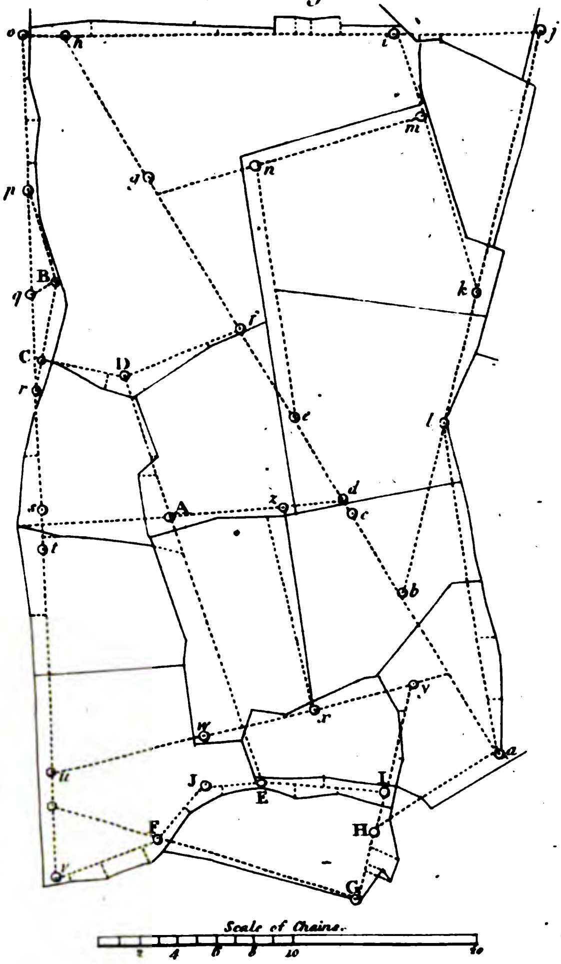
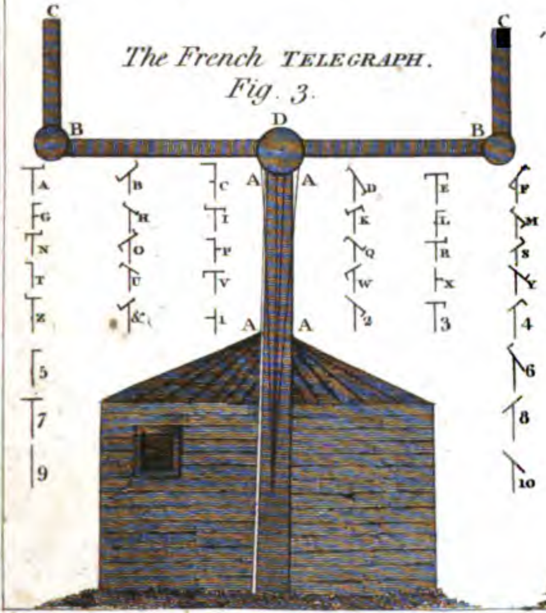


Fig. 1.

Fig. 2.

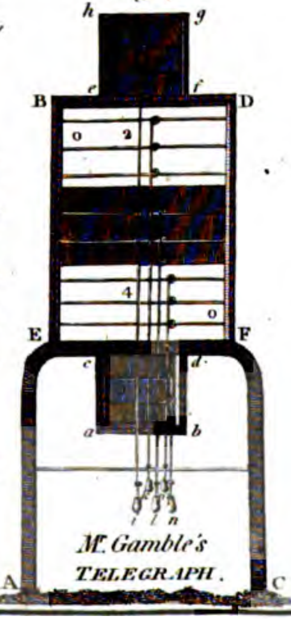




The French TELEGRAPH.

Fig. 3.

Fig. 4.



Mr. Gamble's TELEGRAPH.

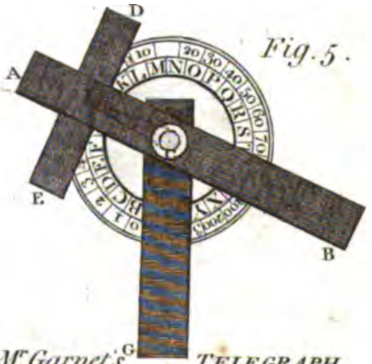


Fig. 5.

Mr. Garnet's TELEGRAPH.

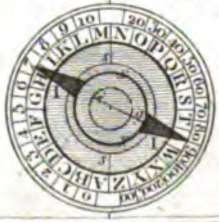
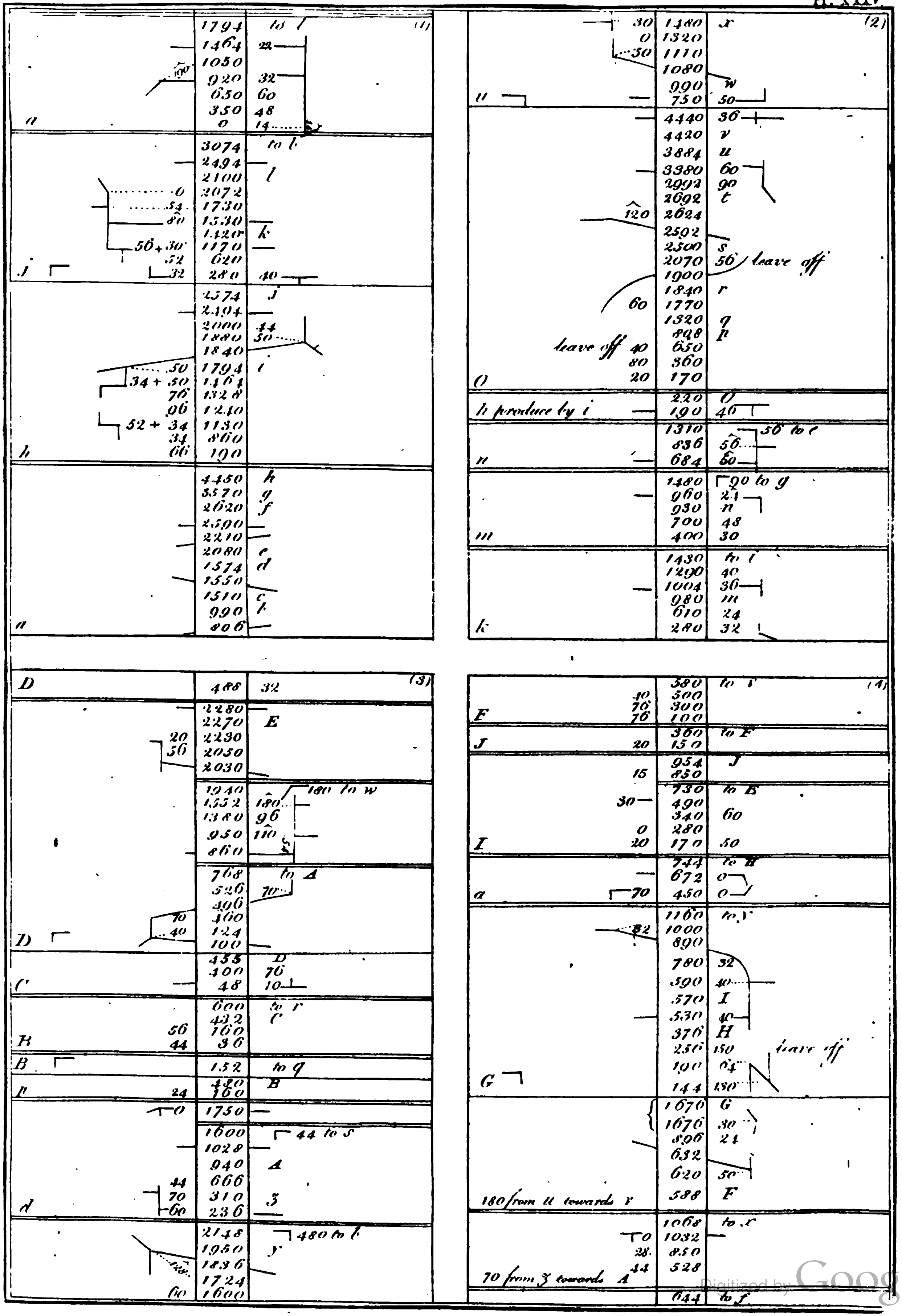


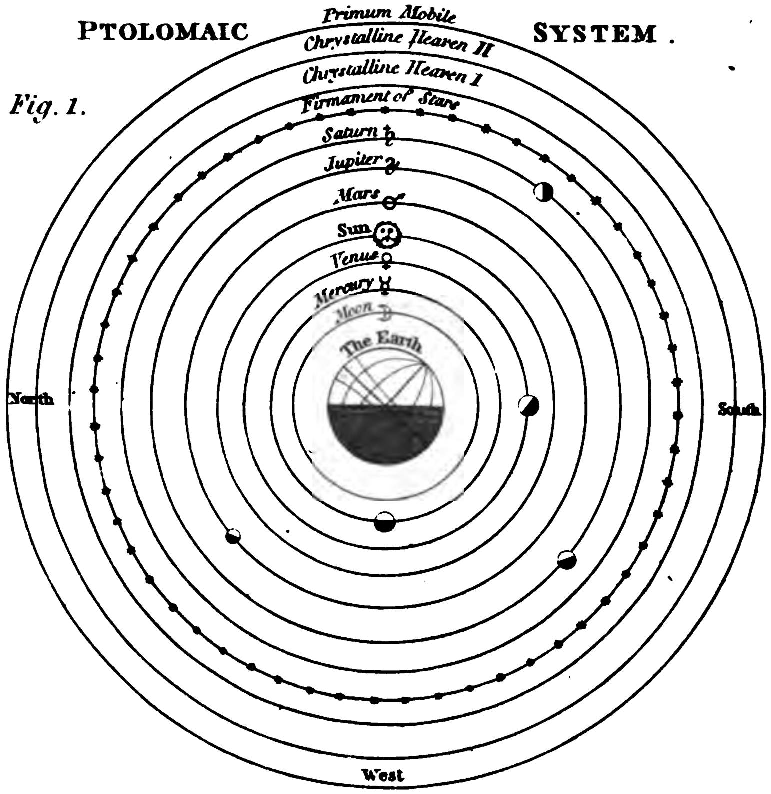
Fig. 6.

Vol. II. FIELD-BOOK TO THE NEW METHOD OF SURVEYING. Pl. XXXV.



PTOLOMAIC SYSTEM.

Fig. 1.



THEODOLITE.

Fig. 5.

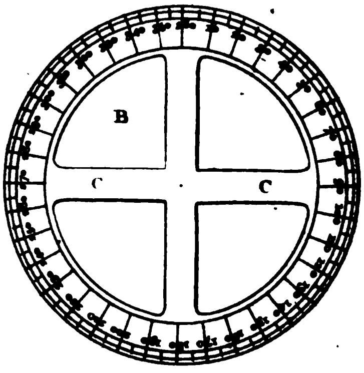
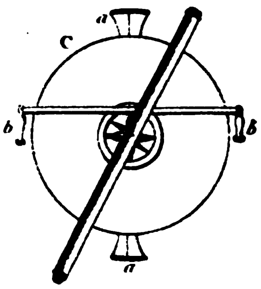
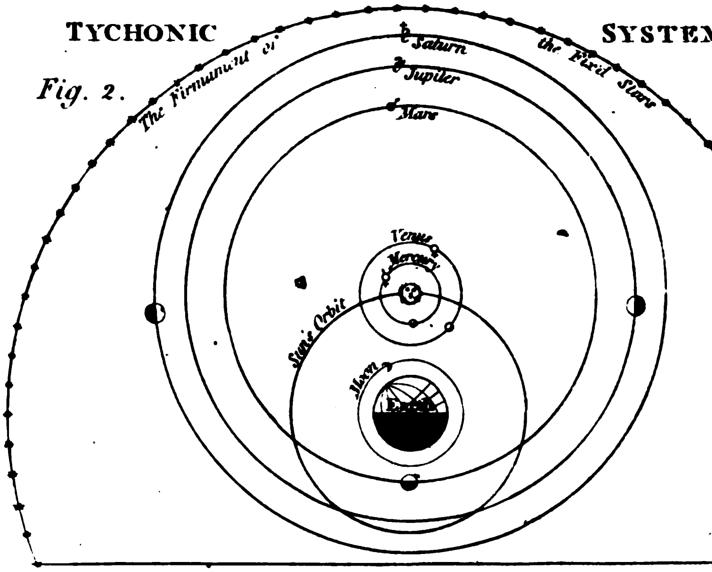


Fig. 6.



TYCHONIC SYSTEM.

Fig. 2.



Astronomical TELESCOPE.

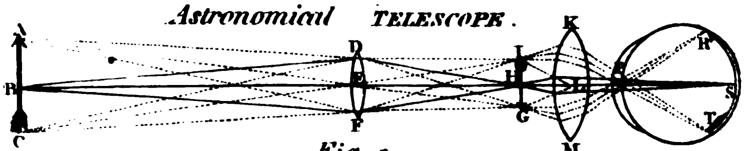


Fig. 3.

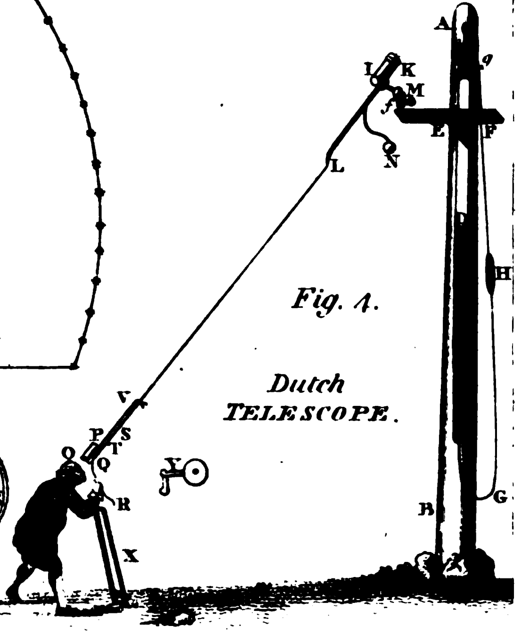
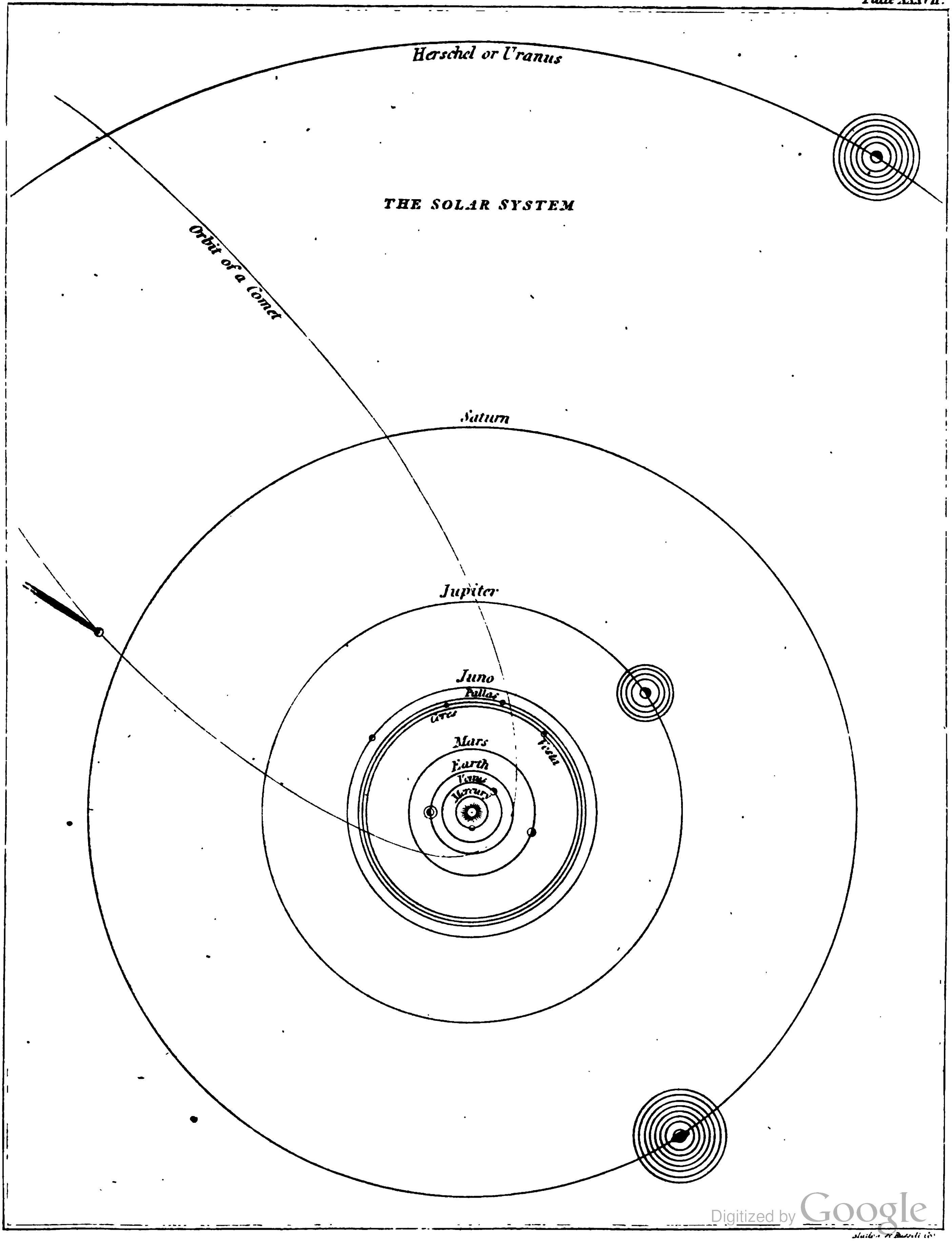


Fig. 4.

Dutch TELESCOPE.



THE SOLAR SYSTEM

[illegible]

Telescopes

Fig. 1.

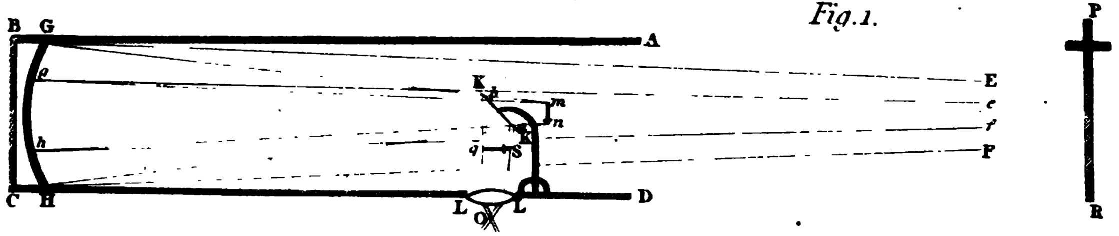


Fig. 2

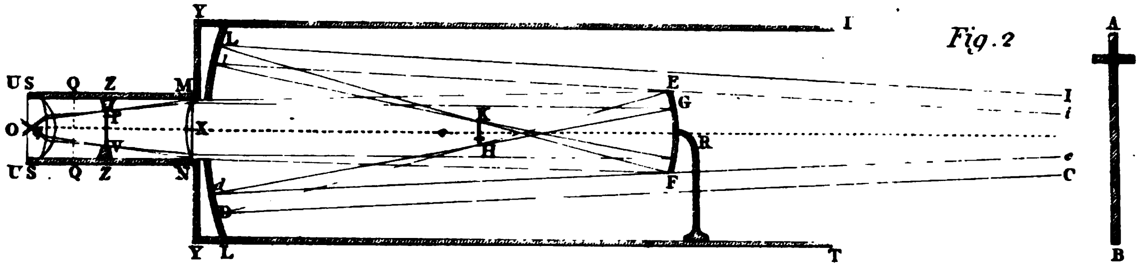


Fig. 3.

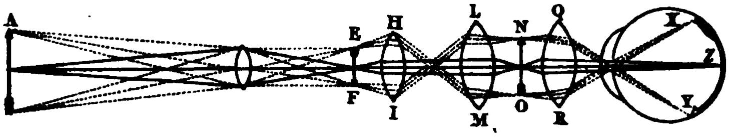
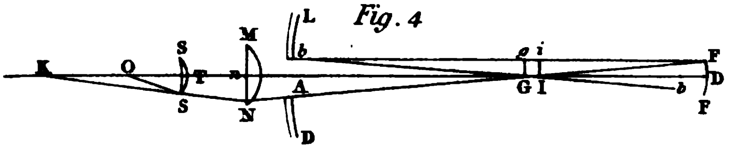


Fig. 4



Screw

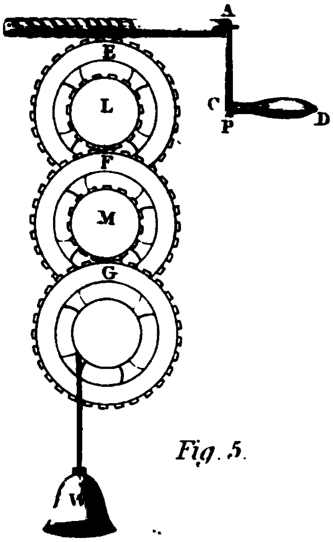


Fig. 5.

THERMOMETERS

Fig. 1

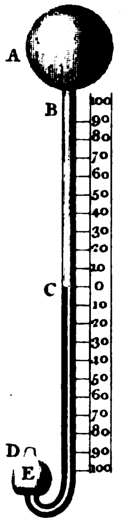


Fig. 7
Fahrenheit's.

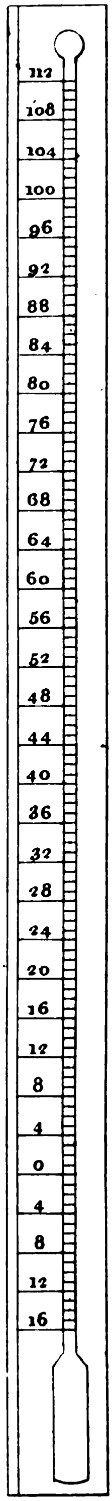


Fig. 8
Celsius's or the Centigrade.

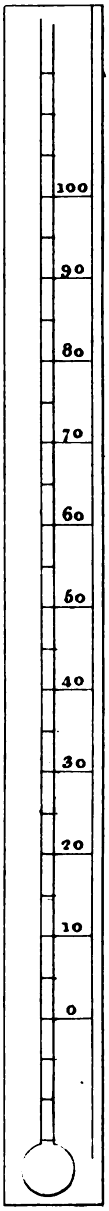


Fig. 4

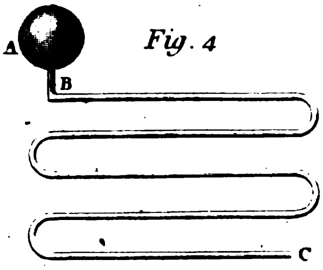


Fig. 6

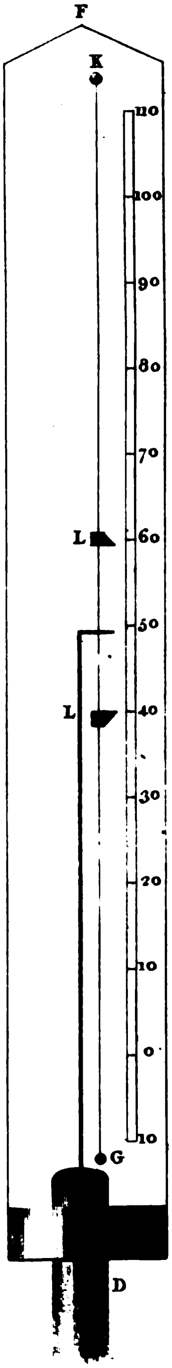


Fig. 3
Reaumer's

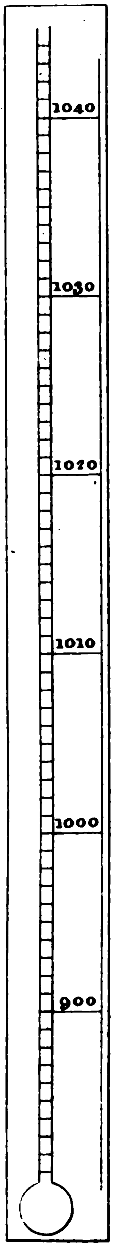


Fig. 5

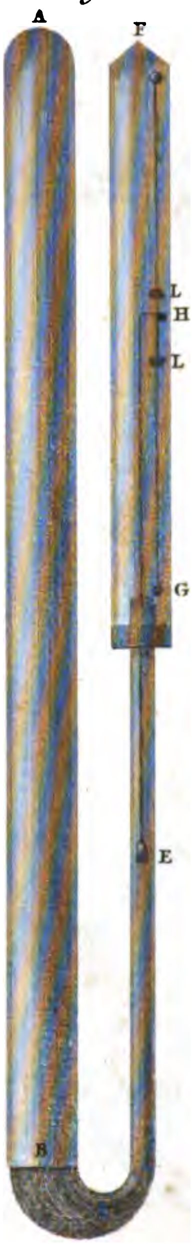
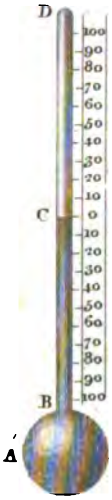


Fig. 2



Mutlow sc. Russell Co.

THUNDER House.

Fig. 1.

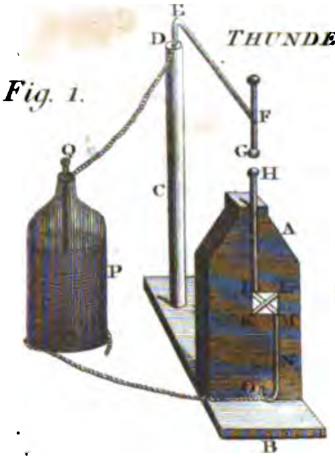
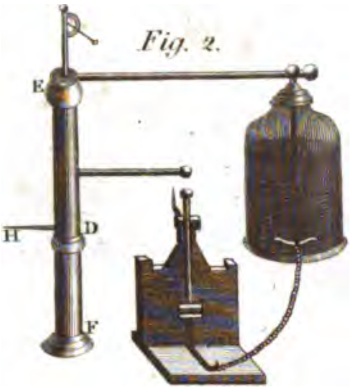
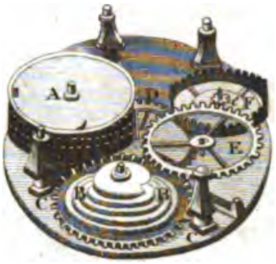


Fig. 2.



WATCH WORK.

Fig. 3.



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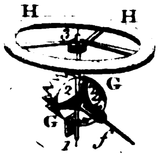

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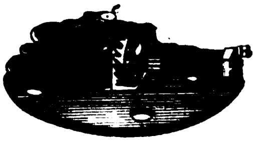

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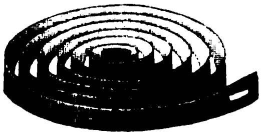

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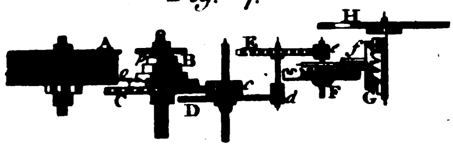

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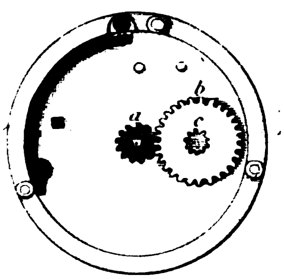

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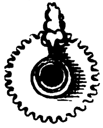

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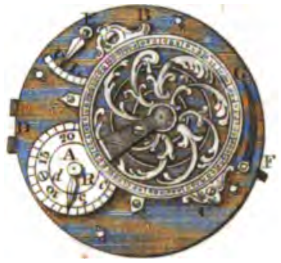

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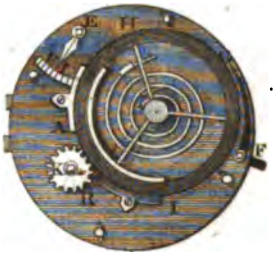

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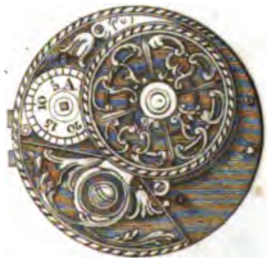
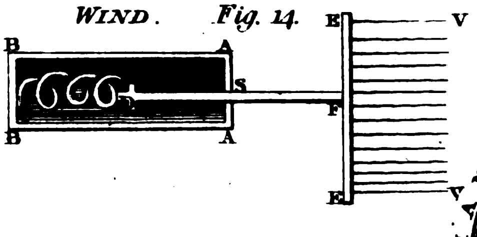


WIND. *Fig. 14.*



ZONES.

Fig. 16.

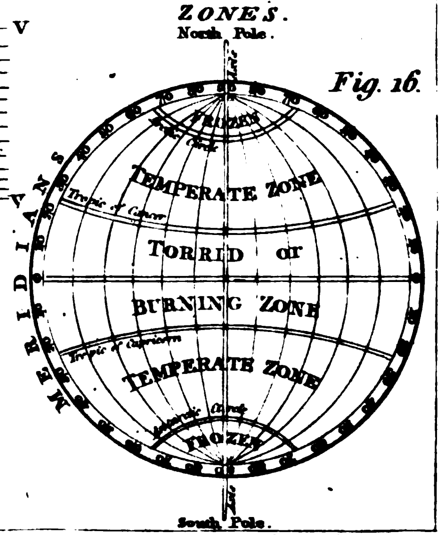
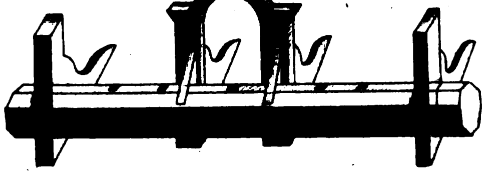


Fig. 13.

WINDLASS. *Fig. 15.*



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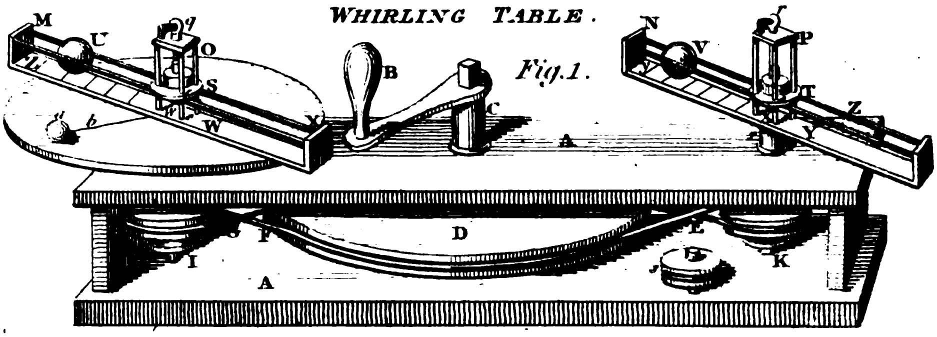

Fig. 1.

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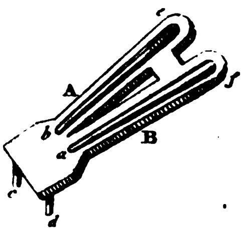

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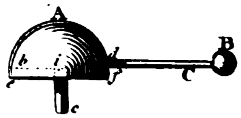


Fig. 4.

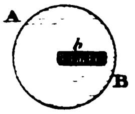
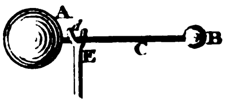


Fig. 5.



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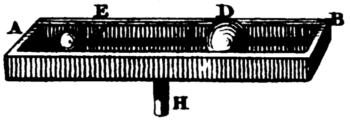


Fig. 7.

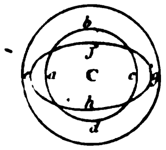
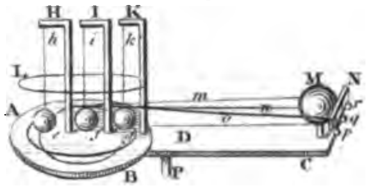


Fig. 8.



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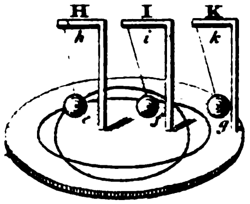

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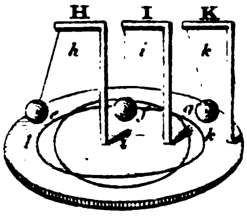

WIND - MILL

Fig. 11.

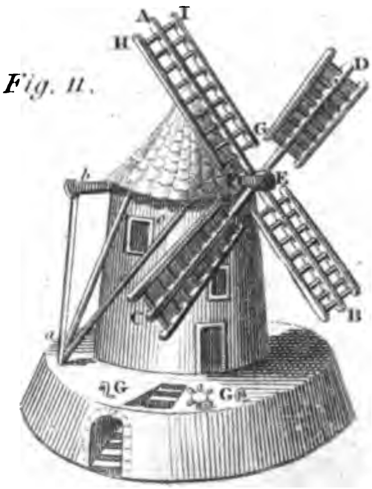

Fig. 12.